Lattice QCD studies on strong-interaction matter under extreme conditions of temperature and/or density

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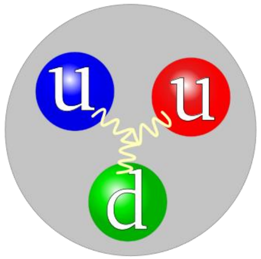
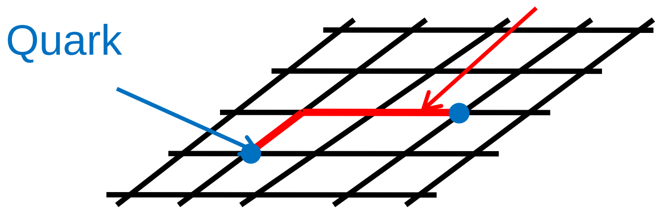
Lattice Quantum Chromodynamics (LQCD)

- QCD = theory of the strong interaction between quarks and gluons
  - Confinement, asymptotic freedom
  - Perturbation theory does not work in general.



- Lattice QCD
  - Discretized space-time
  - Non-perturbative effects can be taken into account
  - Large scale Monte Carlo simulations with supercomputers



Expectation value of observables

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}U \mathcal{O} \det M \, e^{-S_g} \rightarrow \frac{1}{N} \sum_{i=1}^{N} \mathcal{O}_i$$

4D space-time × internal d.o.f.
= O(10^9) dimensional integration

Generating U with $P \propto \frac{1}{Z} \det M \, e^{-S_g}$

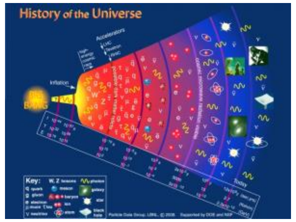
M: Fermion matrix←large sparse

Many matrix inversions needed
→ most expensive part of calculations

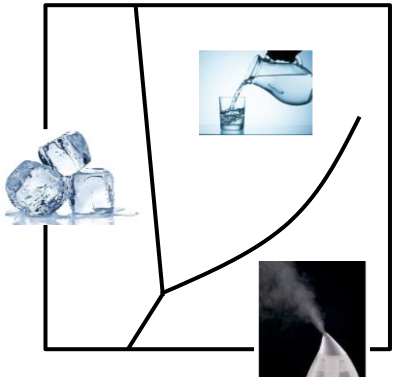
Quark Gluon Plasma (QGP)

- A new state of matter formed at extremely high temperature and/or density

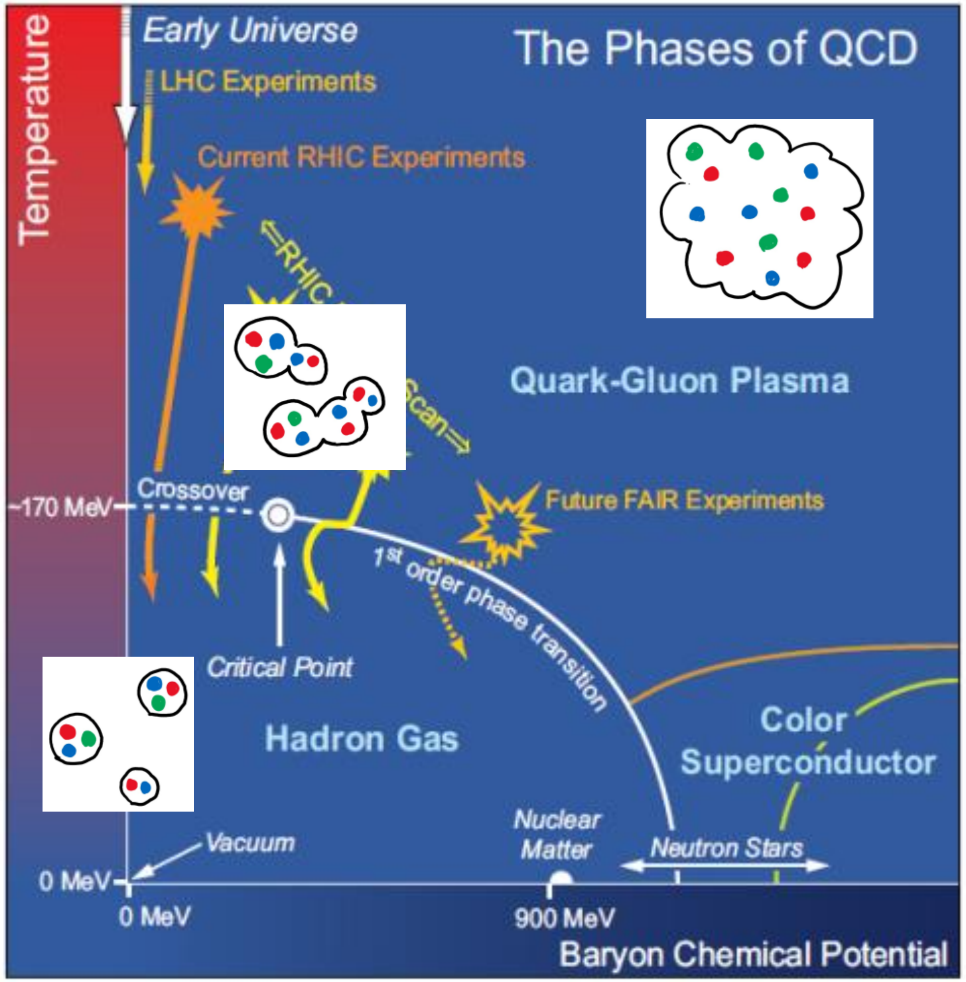
Early universe



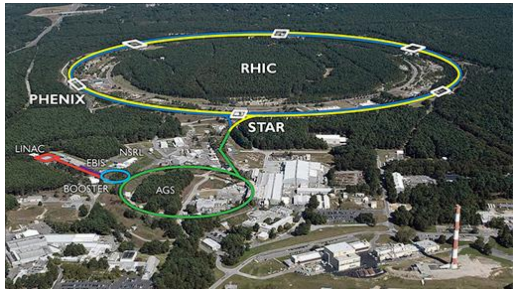
Pressure **Phases of water**

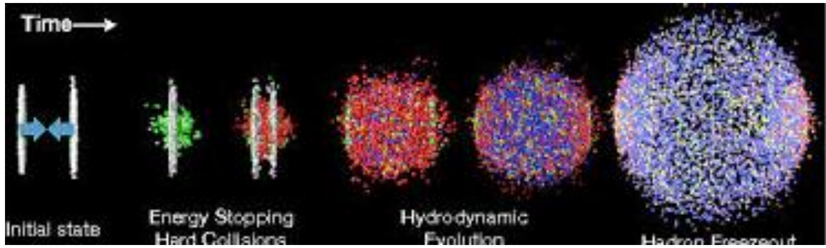


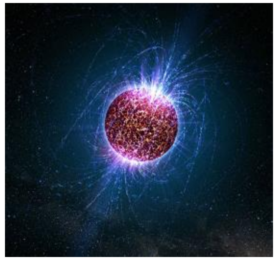
temperature



Heavy ion collision experiments



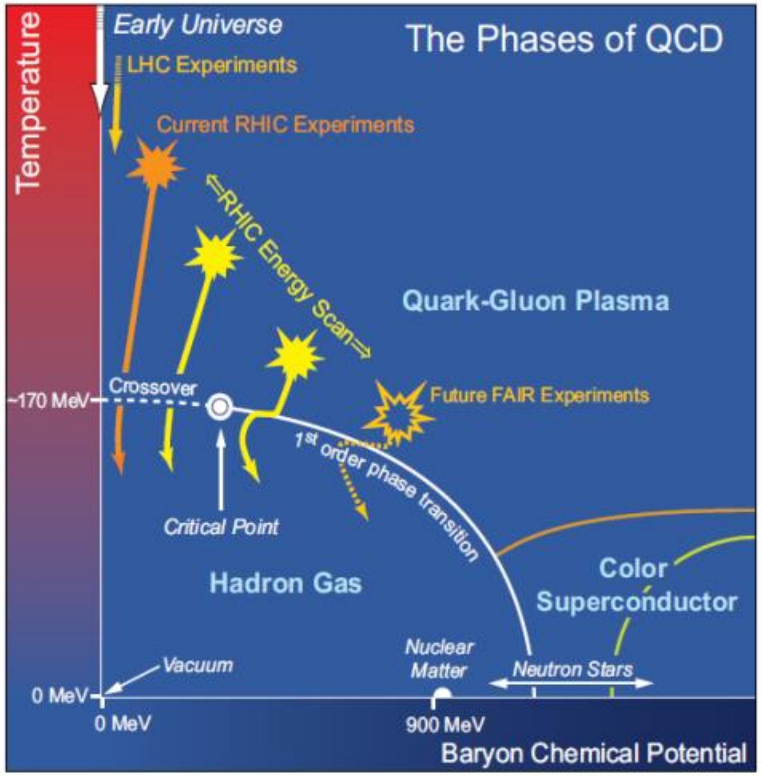




Inside of neutron stars

Topics shown in this talk

- Fluctuations and correlations of conserved charges
  - To search for the QCD critical point
- Quarkonium spectral function
  - Important to understand properties of QGP
    - Melting temperatures → thermometer
    - Transport properties



Fluctuations and correlations of conserved charges

Fluctuations and correlations of conserved charges

- Pressure

Bryon, electric charge, strangeness chemical potentials

Quark chemical potentials

$$\mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q$$

$$\mu_d = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q$$

$$\mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S$$

$$\frac{P(T,\vec{\mu})}{T^4} = \frac{1}{VT^3}\ln Z(T,\vec{\mu}), \qquad \vec{\mu} = (\mu_B, \mu_Q, \mu_S)$$

- Cumulants

$$\chi_n^X(T,\vec{\mu}) \equiv \frac{1}{VT^3}\frac{\partial^n}{\partial\hat{\mu}_X^n}\ln Z(T,\vec{\mu}), \qquad \hat{\mu}_X \equiv \frac{\mu_X}{T}, \qquad X = B, Q, S$$

$$R_{12}^X(T,\vec{\mu}) \equiv \frac{\chi_1^X(T,\vec{\mu})}{\chi_2^X(T,\vec{\mu})} = \frac{M_X}{\sigma_X^2} \quad R_{31}^X(T,\vec{\mu}) \equiv \frac{\chi_3^X(T,\vec{\mu})}{\chi_1^X(T,\vec{\mu})} = \frac{S_X\sigma_X^3}{M_X} \quad R_{42}^X(T,\vec{\mu}) \equiv \frac{\chi_4^X(T,\vec{\mu})}{\chi_2^X(T,\vec{\mu})} = \kappa_X\sigma_B^2$$

M_X : Mean $\quad \sigma_X$: Variance $\quad S_X$: Skewness $\quad \kappa_X$: Kurtosis

Higher order cumulants are more sensitive to correlation length
→ changing more rapidly around the critical point

Difficulty of lattice QCD simulations at finite density

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}U \mathcal{O} \boxed{\det M} e^{-S_g} \to \frac{1}{N} \sum_{i=1}^{N} \mathcal{O}_i$$

- Introducing the chemical potential μ_q → **sign problem**
  - Fermion determinant → complex
  - Impossible to perform Monte Carlo simulations

- Overcoming the sign problem
  - Taylor expansion w.r.t. μ_q, analytic continuation from imaginary μ_q, reweighting…
  - Complex Langevin algorithm, Lefschetz thimble method, Tensor network…

Comparison to experimental results: R_{31}, R_{42}

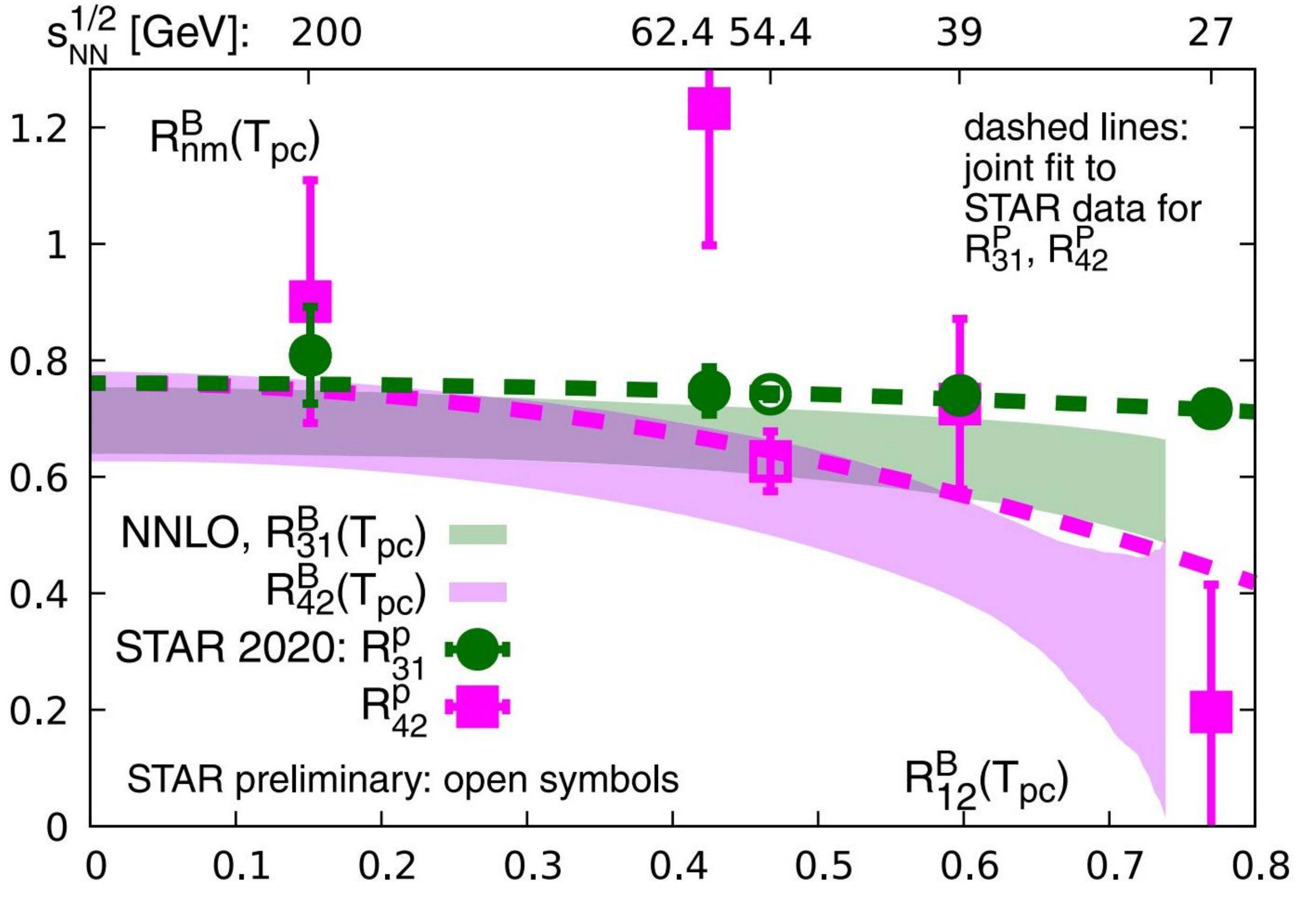
HotQCD Collaboration, Phys.Rev.D 101 (2020) 7, 074502

- Full QCD simulations (including effects of dynamical quarks) with physical quark masses.
- Taking the continuum limit.

- It was found that R^B_{12} is a monotonically increasing function of μ_B.

→ R^B_{12} can be used as an alternative of μ_B.

- Lattice QCD results are similar to the experimental ones.
- No sign of the criticality so far.



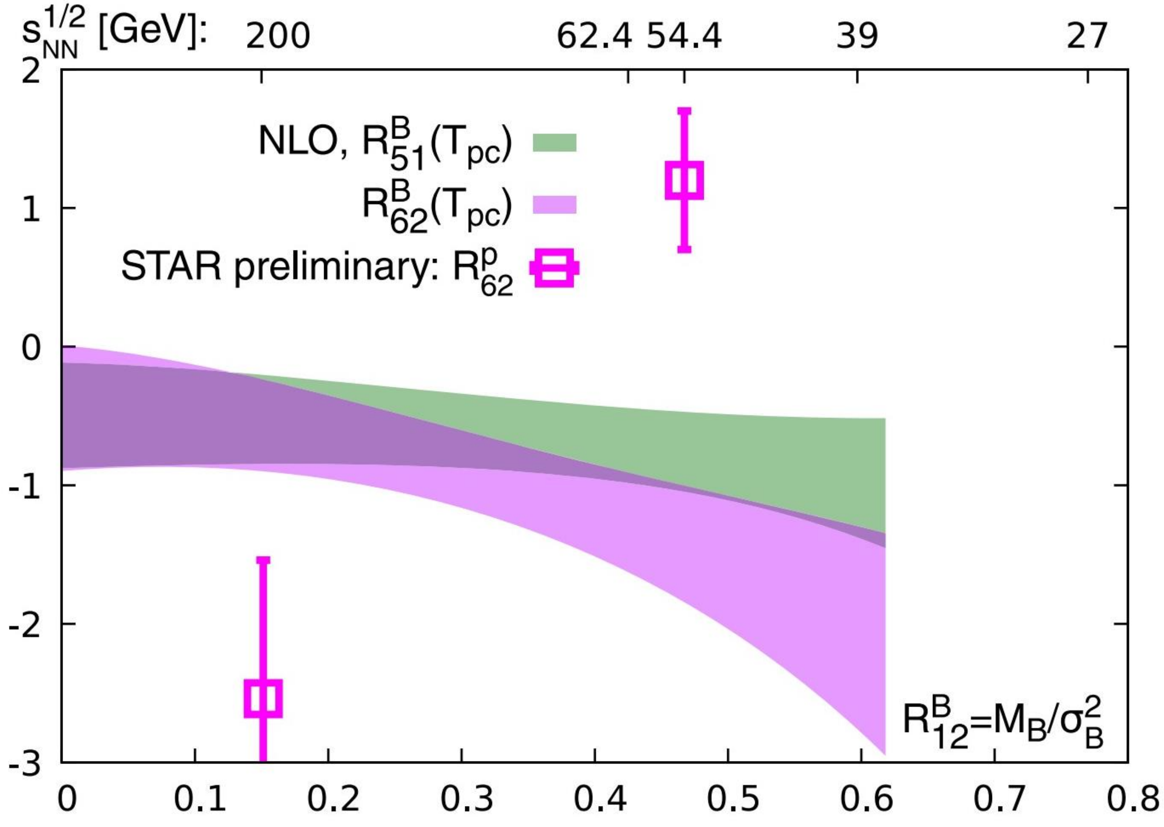
J. Adam et al. (STAR Collaboration), arXiv:2001.02852
T. Nonaka (STAR Collaboration), arXiv:2002.12505
A. Pandav (STAR Collaboration), arXiv:2003.12503.

Comparison to experimental results: R_{51}, R_{62}

HotQCD Collaboration, Phys.Rev.D 101 (2020) 7, 074502

- Lattice QCD results are quite different from the experimental ones.

- Higher expansion coefficients are needed to explain the experimental results.



J. Adam et al. (STAR Collaboration), arXiv:2001.02852
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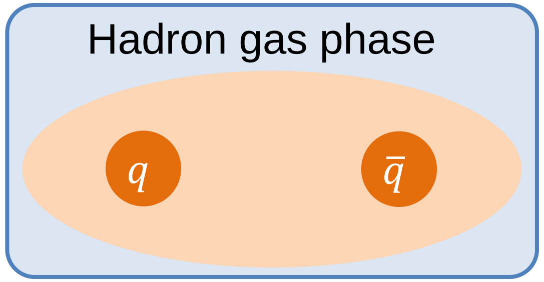
A. Pandav (STAR Collaboration), arXiv:2003.12503.

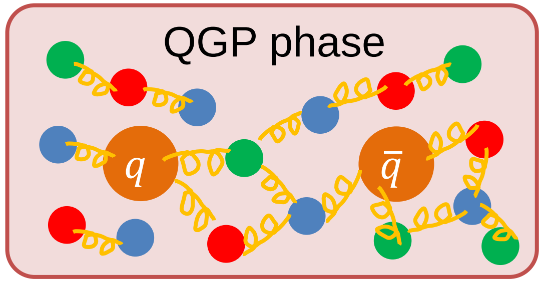
Quarkonium spectral function

Quarkonia

- Bound states of heavy (charm & bottom) quark and anti-quark
  - E.g., η_c, J/Ψ, χ_c, η_b, Υ, χ_b …
- Important probes to investigate QGP formed in heavy ion collision experiments.
  - Produced in the early stage of the collisions: experiencing entire evolution of QGP
  - A signal of QGP formation: color Debye screening in QGP

    → suppression of quarkonium production   T. Matsui and H. Satz, PLB 178 (1986) 416



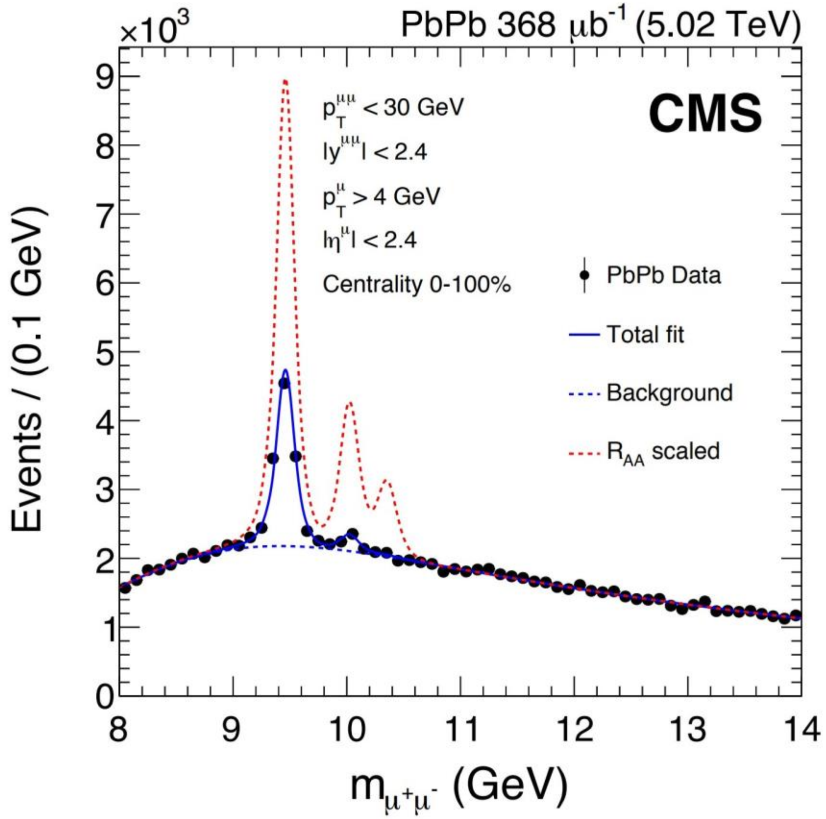


  - Sequential suppression: different binding energy for different bound state

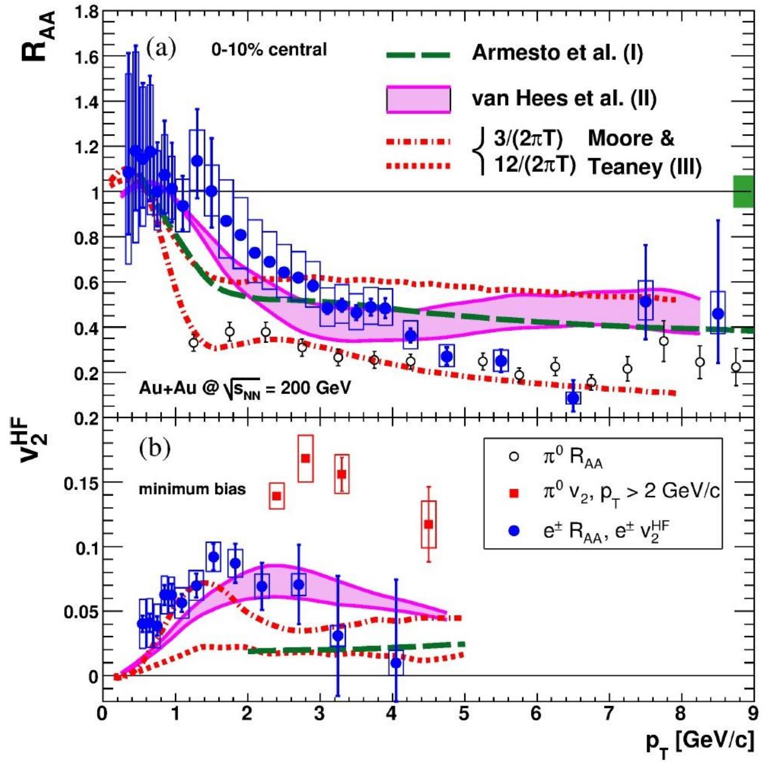
    → different melting temperature

    → QGP thermometer

What to understand



CMS Collaboration@QM2018



PHENIX Collaboration, PRL 98 (2007) 172301

Suppression patterns of quarkonia
→ **dissociation temperatures**

Inputs for transport models
→ **heavy quark diffusion coefficients**

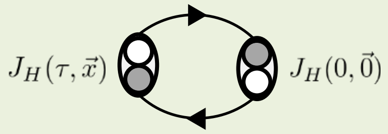
In-medium properties of heavy quarks ← All encoded in **spectral functions**

Spectral function

Euclidian mesonic correlation function (can be measured on the lattice)

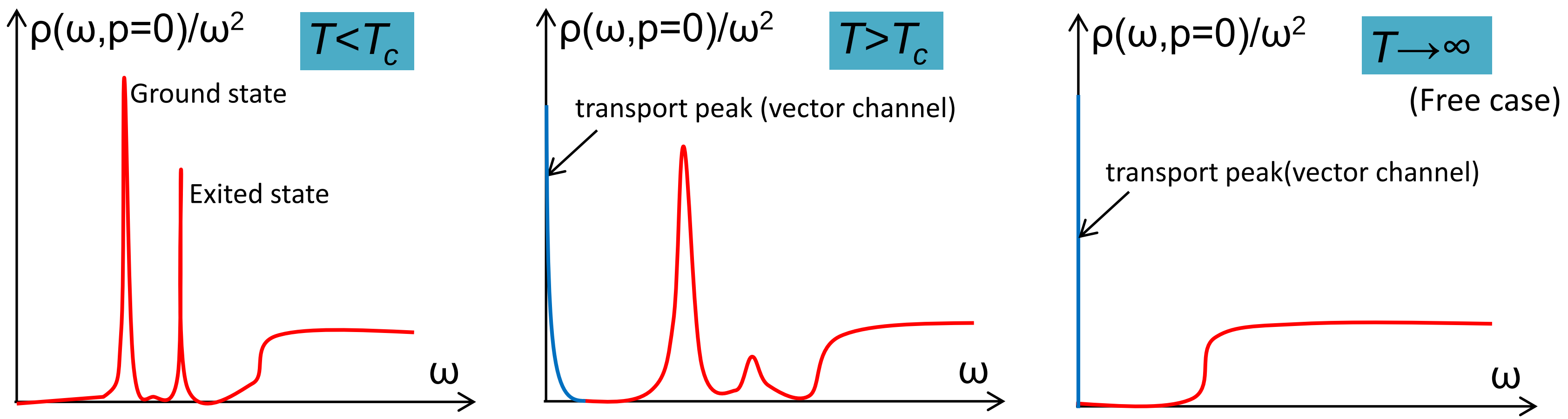
$$G_H(\tau,\vec{p}) \equiv \int d^3x e^{-i\vec{p}\cdot\vec{x}} \langle J_H(\tau,\vec{x}) J_H(0,\vec{0}) \rangle \qquad J_H(\tau,\vec{x}) \equiv \bar{\psi}(\tau,\vec{x})\Gamma_H \psi(\tau,\vec{x})$$

$$= \int_0^\infty \frac{d\omega}{2\pi} \boxed{\rho_H(\omega,\vec{p})} K(\omega,\tau)$$



Spectral function

$$K(\omega,\tau) \equiv \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}$$



Heavy quark diffusion coefficient

$$D = \frac{1}{6\chi_{00}} \lim_{\omega \to 0} \sum_{i=1}^{3} \frac{\rho^V_{ii}(\omega,\mathbf{0})}{\omega}$$

ρ^V_{ii} : Vector spectral function

χ_{00} : Quark number susceptibility

How to get spectral functions

- **Obtaining spectral functions is difficult: an ill-posted inverse problem!**
  - # of correlator data points << # of frequency bins of spectral functions
  - A naive χ^2-fiting gives infinite number of possible spectra within statistical uncertainties.

- **To overcome the difficulty**
  - Adding prior information
  - Bayesian inference: **Maximum Entropy Method (MEM), Bayesian Reconstruction (BR) Method, …**
  - Machine learning
  - **Phenomenologically motivated (perturbative) modeling of spectral functions**
  - …

Modeling quarkonium spectral functions

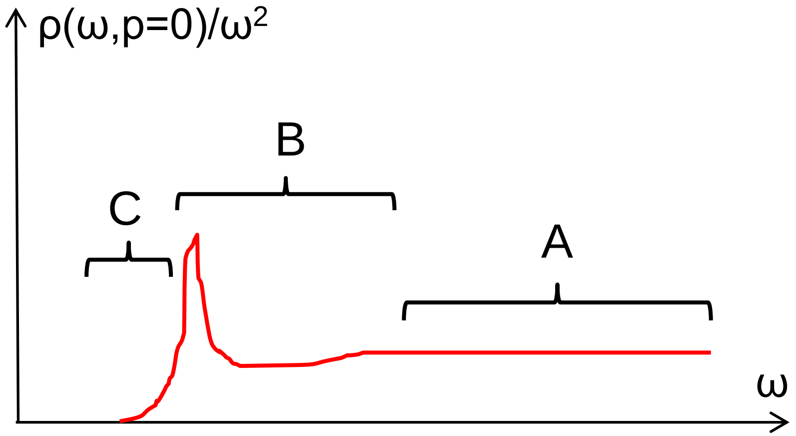
$$\rho^{pert}(\omega) = \underbrace{\rho^{vac}(\omega)}_{\text{Vacuum asymptotics}}\theta(\omega - \omega^{match}) + A^{match}\underbrace{\rho^{NRQCD}(\omega)}_{\text{pNRQCD}}\theta(\omega^{match} - \omega)\underbrace{\Phi(\omega)}_{\text{Suppression}}$$

Y. Burnier, H. -T. Ding, O. Kaczmarek, A. -L. Kruse, M. Laine, HO, H. Sandmeyer, JHEP11(2017)206

A) High energy ρ^{vac}: vacuum asymptotics Burnier, Laine, Eur.Phys.J.C 72 (2012) 1902

B) Threshold region ρ^{NRQCD}: pNRQCD Laine, JHEP 0705:028,2007

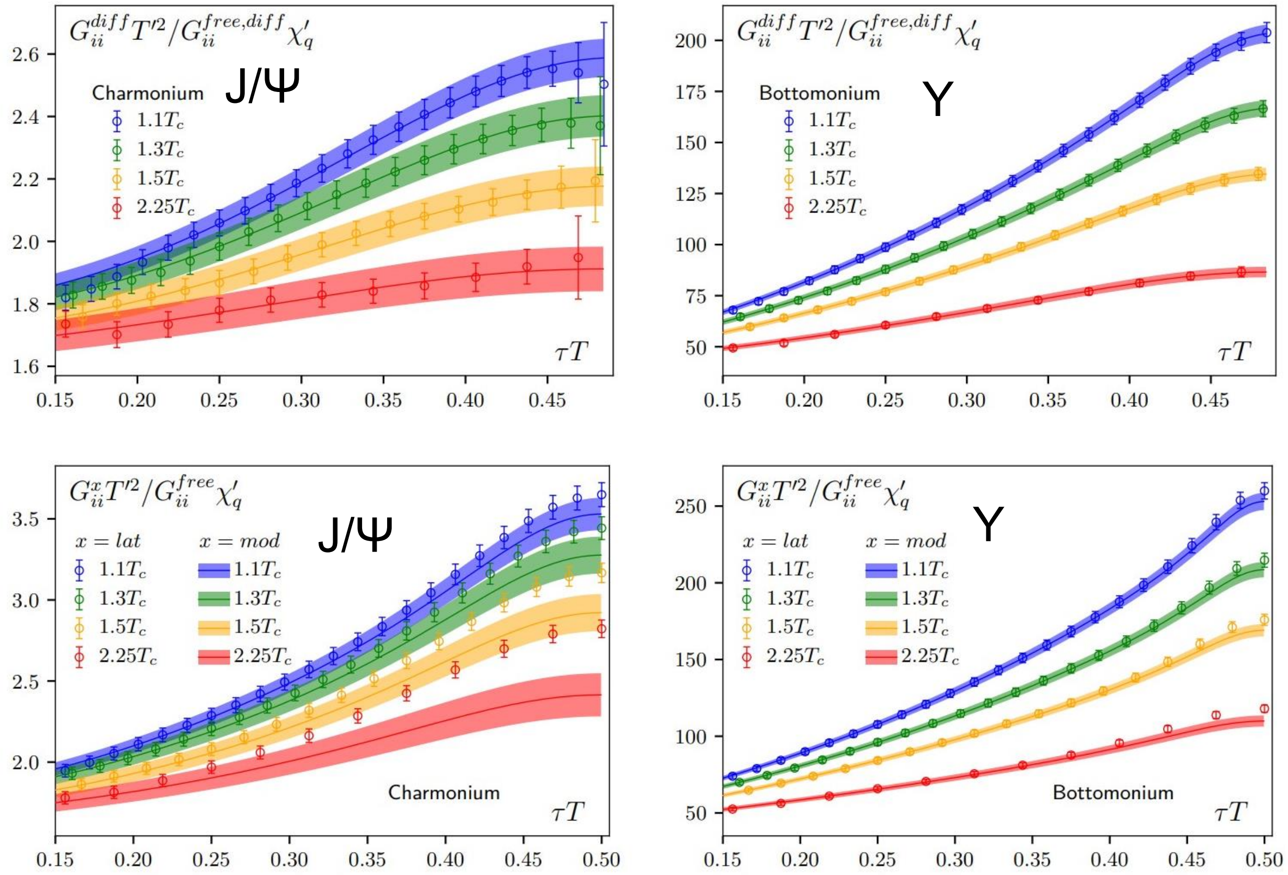
C) Low energy ϕ: suppression



Fitting lattice data to the model spectral function

- Quenched QCD (no dynamical quarks), continuum extrapolated, vector

H.-T. Ding, O. Kaczmarek, A.-L. Kurse, HO, H. Sandmeyer and H.-T. Shu, Phys. Rev. D **104** (2021) 11, 114508



T_c: transition temperature

$$\rho^{\mathbf{mod}}(\omega) = \mathbf{A}\rho^{\mathbf{pert}}(\omega - \mathbf{B})$$

Transport peak is not described by ρ^{pert}

Transport peak : ω ~ 0
→ τ independent contributions
→ can be removed by correlator difference

$$G_{ii}^{diff}(\tau/a) = G_{ii}(\tau/a+1) - G_{ii}(\tau/a)$$

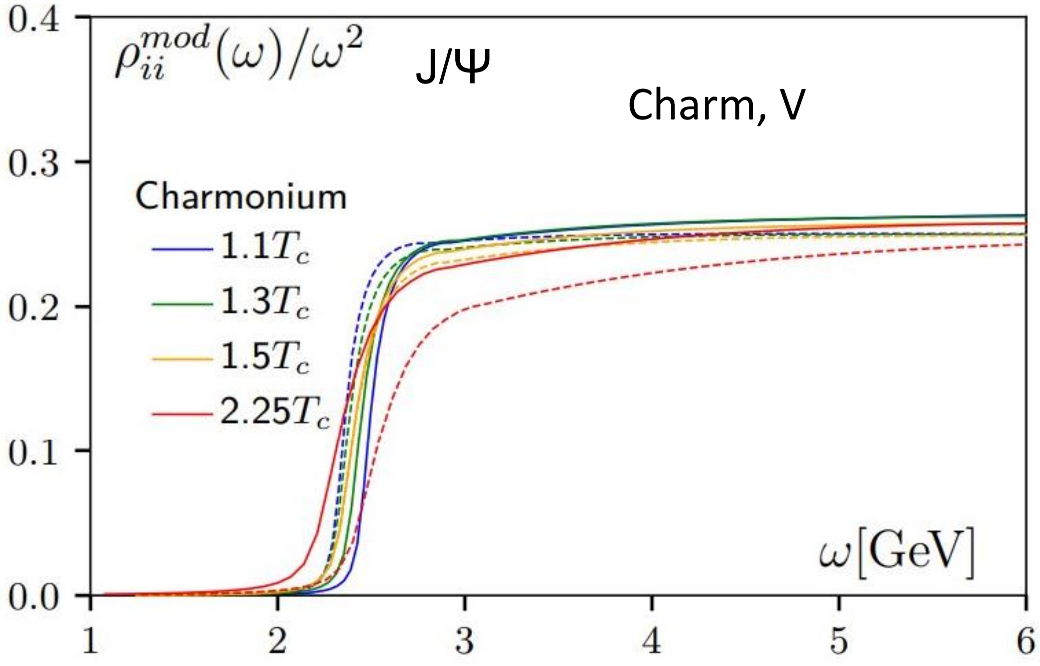
- **The model spectral function describes the correlator difference perfectly.**
- **Difference between the original lattice data and fit results**

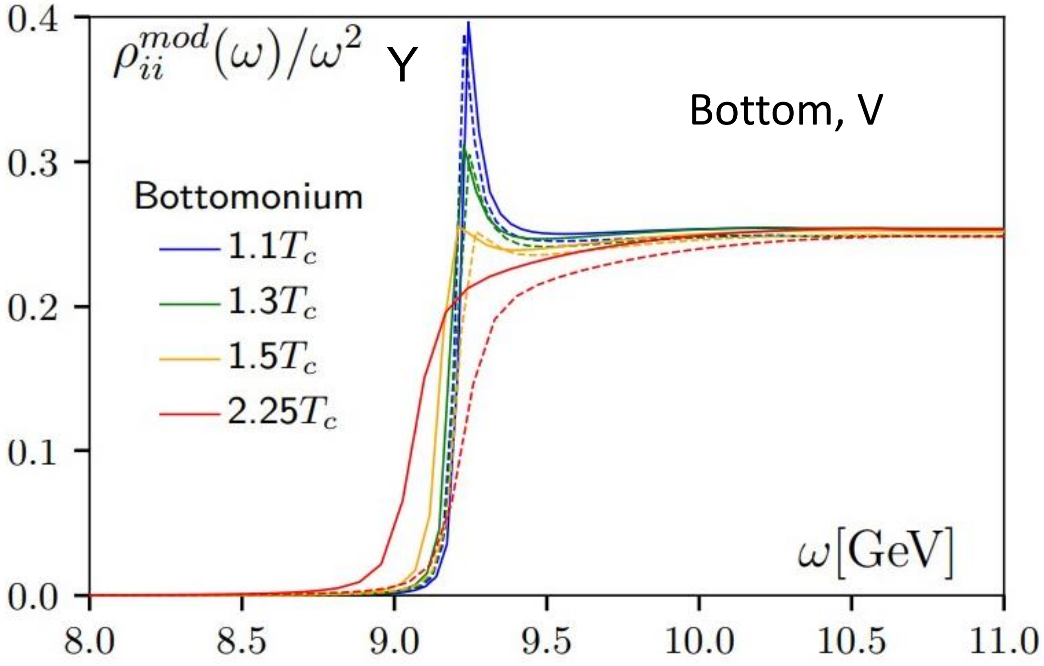
→ Indication of the transport contributions

Resulting spectral functions

- Quenched QCD (no dynamical quarks), continuum extrapolated, vector

H.-T. Ding, O. Kaczmarek, A.-L. Kurse, HO, H. Sandmeyer and H.-T. Shu, Phys. Rev. D **104** (2021) 11, 114508





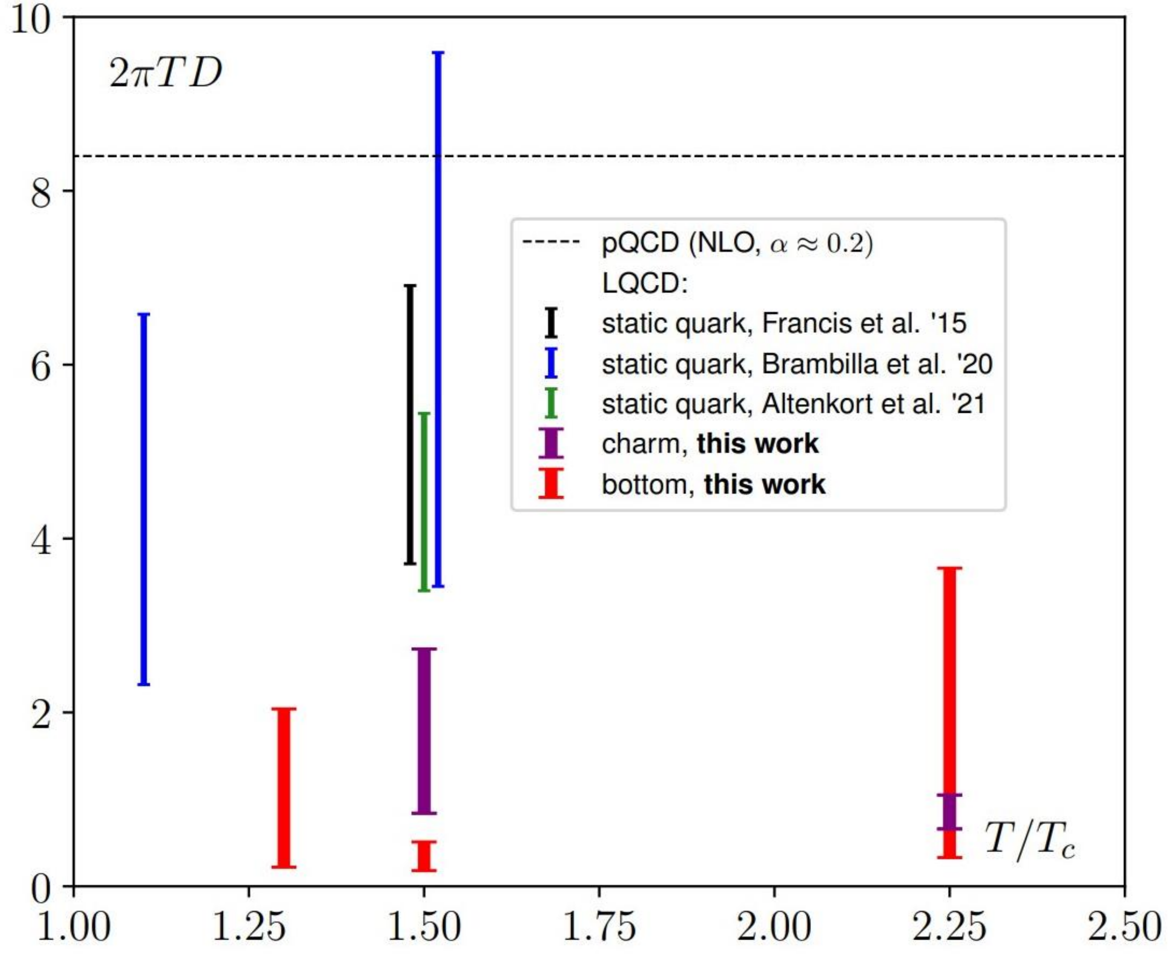
- **No resonance peak needed for charm.**
- **A resonance peak is needed for bottom at T ≲ 1.5 Tc.**

T_c: transition temperature

Estimation of the transport coefficient

- Quenched QCD, Clover Wilson, continuum extrapolated, vector

H.-T. Ding, O. Kaczmarek, A.-L. Kurse, HO, H. Sandmeyer and H.-T. Shu, Phys. Rev. D **104** (2021) 11, 114508



Summary and outlook

- Lattice QCD is a powerful tool to investigate strong-interaction matter at extremely high temperature and/or density non-perturbatively.

- Fluctuations and correlations of conserved charges
  - Studied with Taylor expansions w.r.t. quark chemical potentials to search for the QCD critical point.
  - Our lattice results are consistent with some experimental results.
  - No sign of the criticality so far.
  - Higher order expansion coefficients are needed for further investigations.

- Quarkonium spectral function
  - One of important quantities to investigate properties of quark-gluon plasma.
  - A phenomenologically motivated model can successfully reconstruct the spectral function from lattice data.
  - A future work: extension to full QCD.