Nuclear deformation and stability

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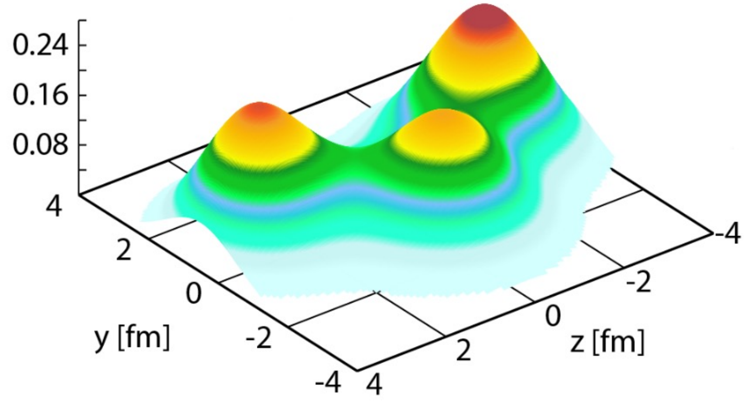
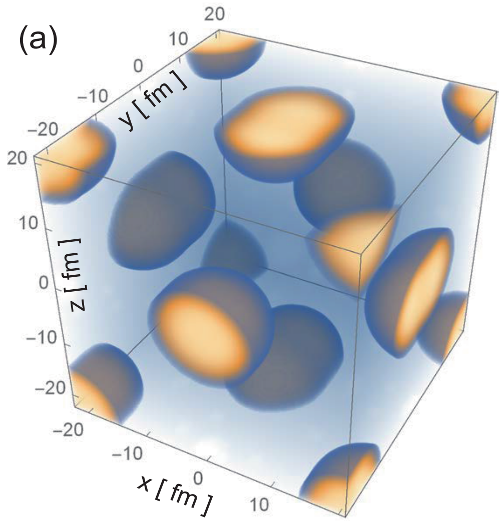
Nuclear Physics Group





- Faculty members
  - Nakatsukasa, Takashi
  - Shimizu, Noritaka
  - Hinohara, Nobuo
  - Yabana, Kazuhiro (Quantum condensed matter physics)

Overview

The nucleus can be described as a collection of Fermi particles (protons and neutrons) called nucleons. The mechanics to rule over this microscopic world, such as the quantum mechanics of many-particle systems and quantum field theory, is essential. Three of the four fundamental forces of nature—the strong force, electromagnetic force, and weak force—play important roles in the atomic nucleus, leading to a variety of aspects of reactions and structure that are related to the existence of the matter around us. For example, the sun and the stars in the night sky shine with atomic nuclei as fuel, and they are the lights of the factories that produce the elements. The burning (nuclear reaction) process, which depends on the nature of the forces involved and the nuclear structure, controls the brightness and lifetime of the star, as well as the type and quantity of elements produced.

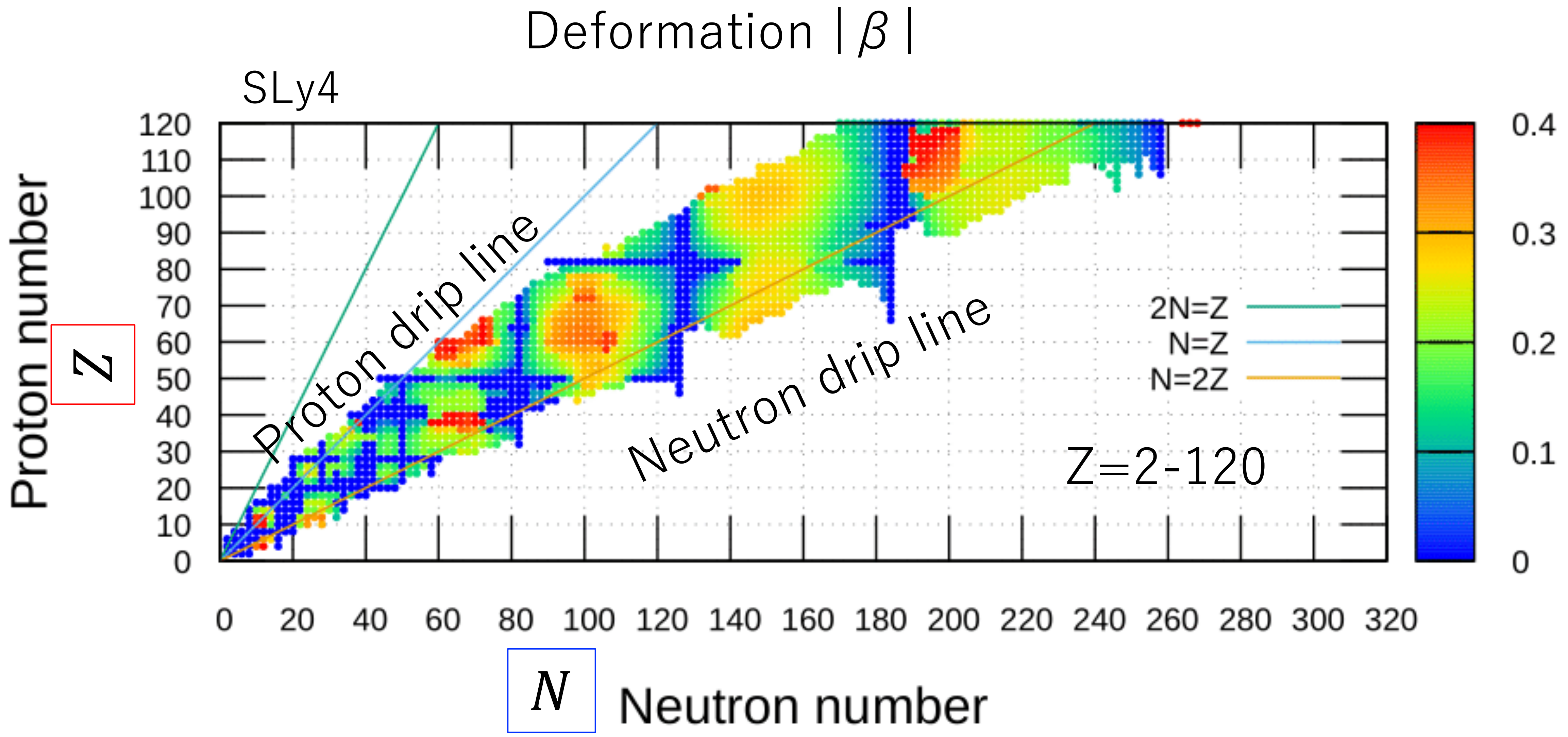
Nuclear physics has progressed through both experiments using accelerators and theoretical calculations using computers. Numerical calculations are indispensable for quantum many-body problems, such as nuclear problems. The Nuclear Physics division works on developing theories, models, and numerical methods based on quantum mechanics to clarify the nuclear structure, nuclear reactions, structure of stars, and quantum dynamics of matter.

Fundamental interactions in nuclei

- **Strong** interaction: The most important, the force to produce nucleons/hadrons, the origin of nuclear binding
- **Electromagnetic** interaction: Second most important, the reason of finite size, repulsive among protons
- **Weak** interaction: No effect on nuclear structure, but controls decay and weak process reactions
- **Gravity**: No effect on nuclei on Earth, but important to produce neutron stars

Quantum mechanical consequences of nuclear shape and stability

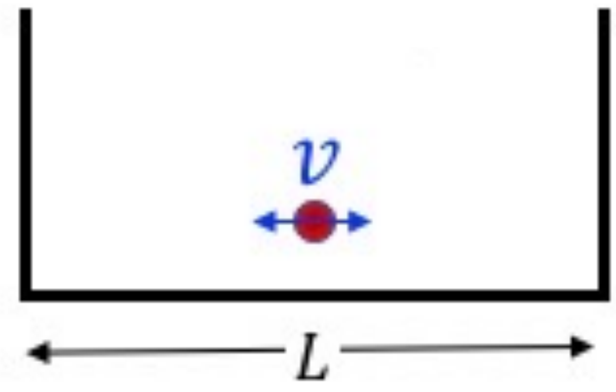
Systematic investigation of nuclear shape

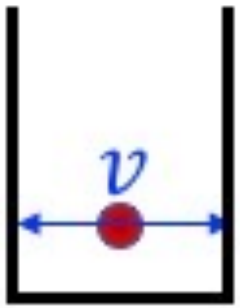


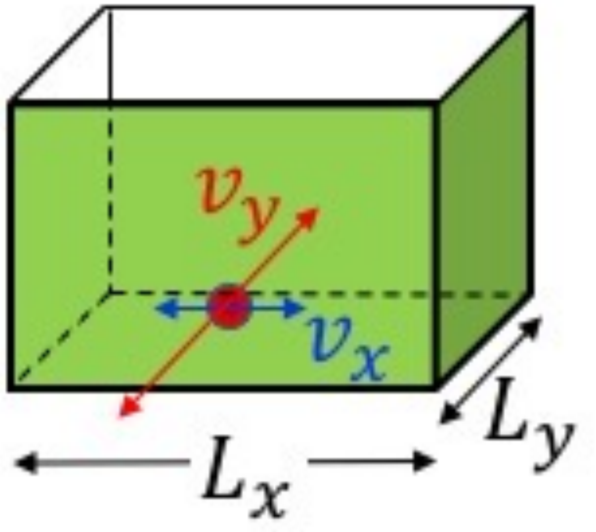
Uncertainty principle

$p \sim h/L$

h: Planck constant



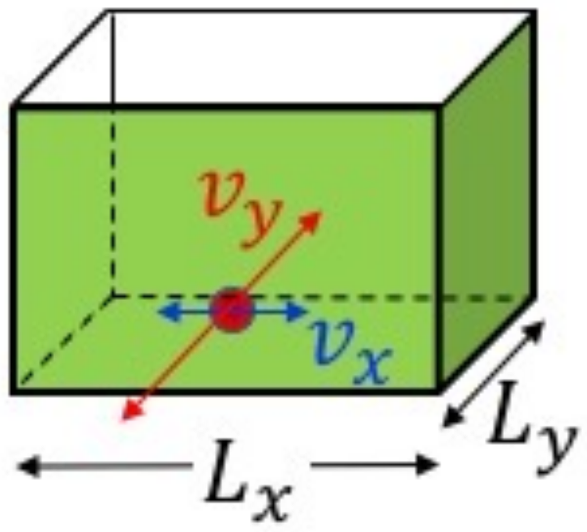




$\boxed{L_x > L_y \implies p_x < p_y}$

Non-zero kinetic energy $\frac{1}{2}m\boldsymbol{v}^2 = \frac{\boldsymbol{p}^2}{2m}$ [Zero-point oscillation]

Optimal shape of the box



Problem: Fixing the box size $L_x L_y = L^2$,
minimize $\frac{1}{2m}\boldsymbol{p}^2$

$$\boldsymbol{p}^2 = p_x^2 + p_y^2 \sim h^2\left(\frac{1}{L_x^2} + \frac{1}{L_y^2}\right) = \frac{h^2}{L^2}\left(\frac{L^2}{L_x^2} + \frac{L_x^2}{L^2}\right)$$

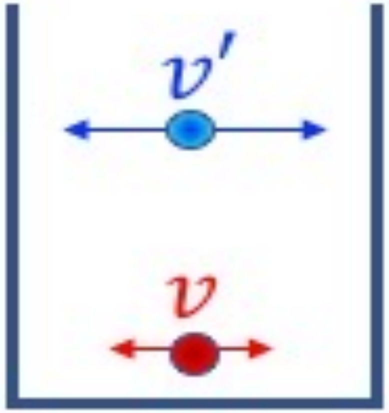
$$\delta\boldsymbol{p}^2 = 0 \ \rightarrow \ \boxed{L_x = L_y = L}$$

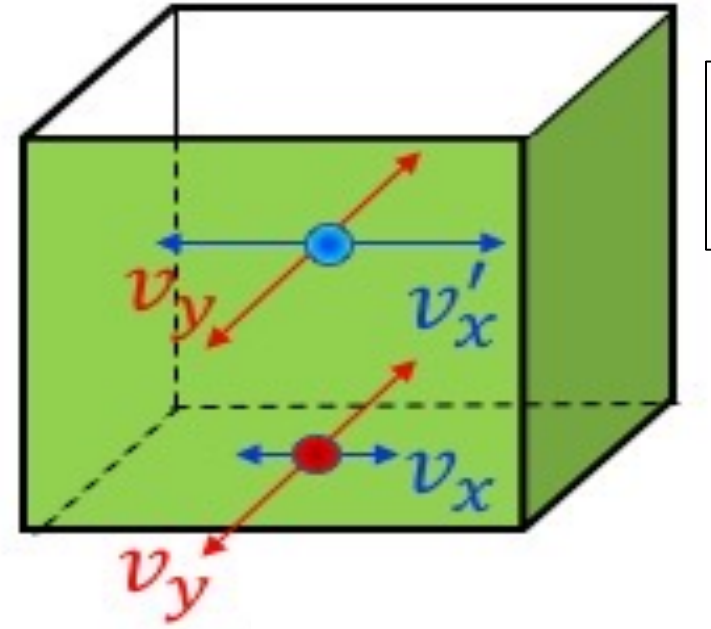
Isotropic shape ➤ Circular shape (2-dim)
➤ Spherical shape (3-dim)

Pauli's exclusion principle

The second particle cannot be in the same state as the first one.

$p' \sim 2p$





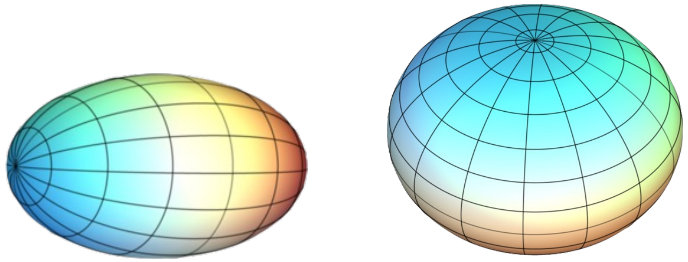
$p'_x \sim 2p_x$
$p'_y \sim p_y$

$$\boldsymbol{p}^2 + \boldsymbol{p}'^2 \sim h^2 \left(\frac{5}{L_x^2} + \frac{2}{L_y^2} \right)$$

➤ Ellipsoidal shape

$$\delta \boldsymbol{p}^2 = 0 \rightarrow \boxed{L_x^2 : L_y^2 = 5:2}$$

Shell effect!



Energy density functional method

Minimizing the total energy for a given nucleus (N, Z) with respect to normal and abnormal (pair) densities, $(\rho(\boldsymbol{r}), \kappa(\boldsymbol{r}))$

$$\delta\left(E[\rho,\kappa] - \lambda_n\left(\int \rho_n(\boldsymbol{r})d\boldsymbol{r} - N\right) - \lambda_p\left(\int \rho_p(\boldsymbol{r})d\boldsymbol{r} - Z\right)\right) = 0$$

HFB equation

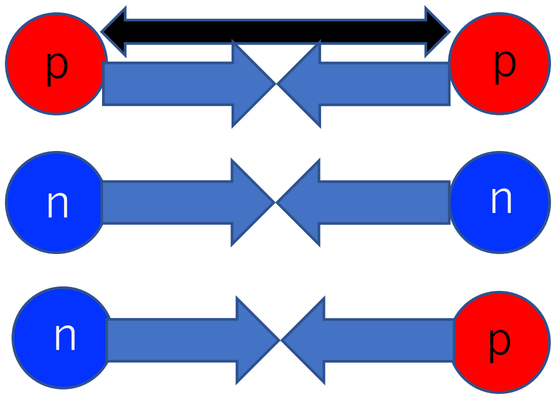
$$\boxed{\begin{pmatrix} h+\Gamma-\lambda & \Delta \\ -\Delta^* & -(h+\Gamma)^*+\lambda \end{pmatrix}\begin{pmatrix} U \\ V \end{pmatrix} = E\begin{pmatrix} U \\ V \end{pmatrix}}$$

Systematic calculation using HFBTHO code

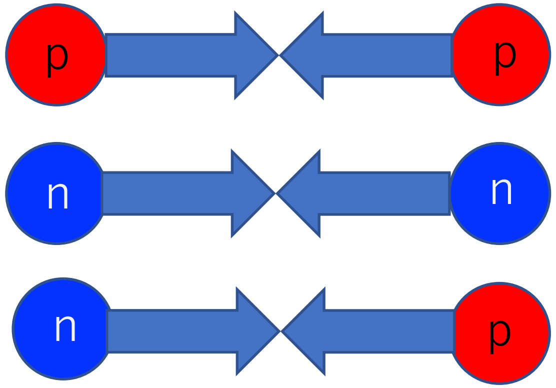
Effects of the electromagnetic interaction

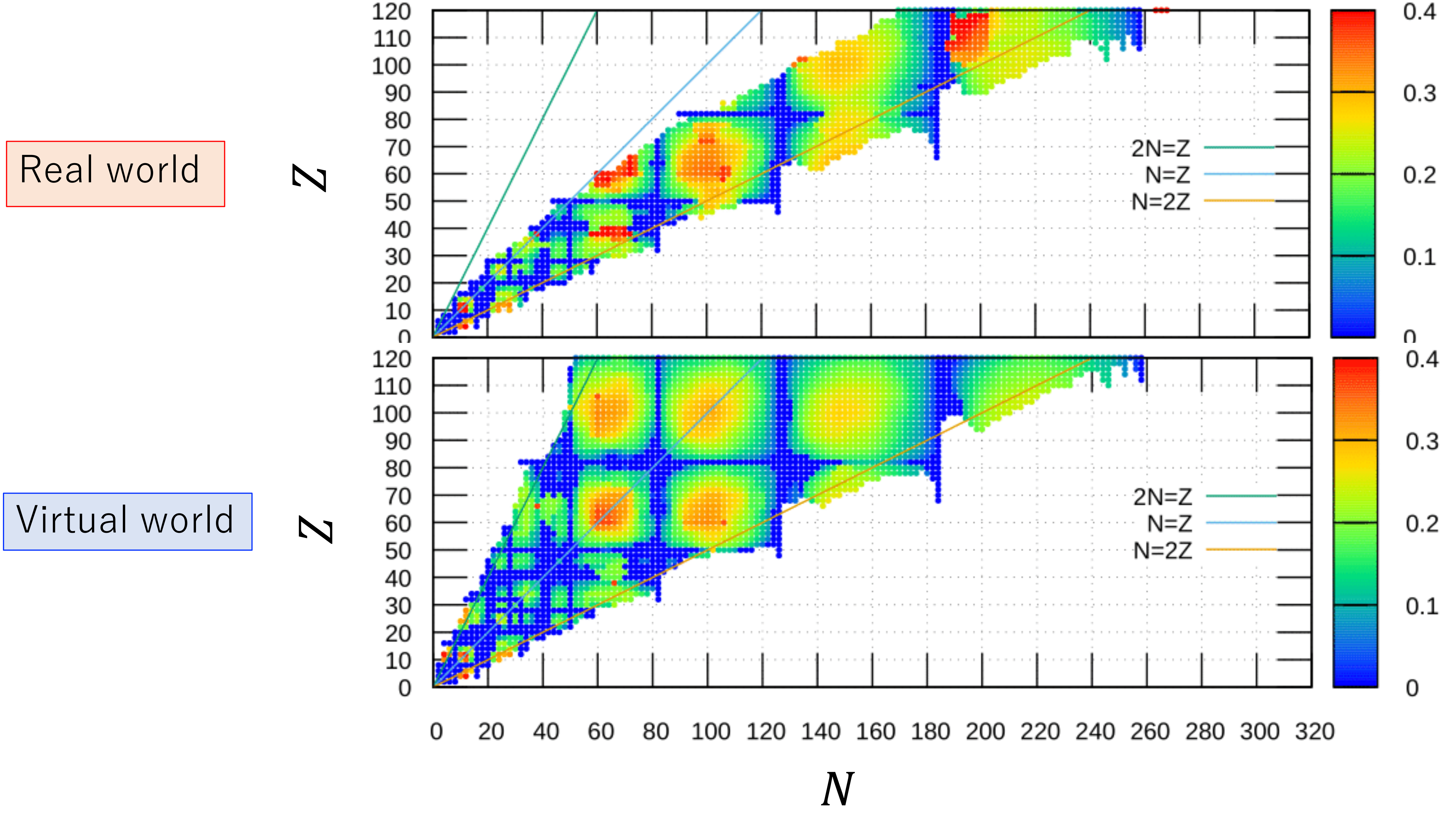
- Perform the systematic calculations with and without the Coulomb force between protons

  - Real world



Virtual world

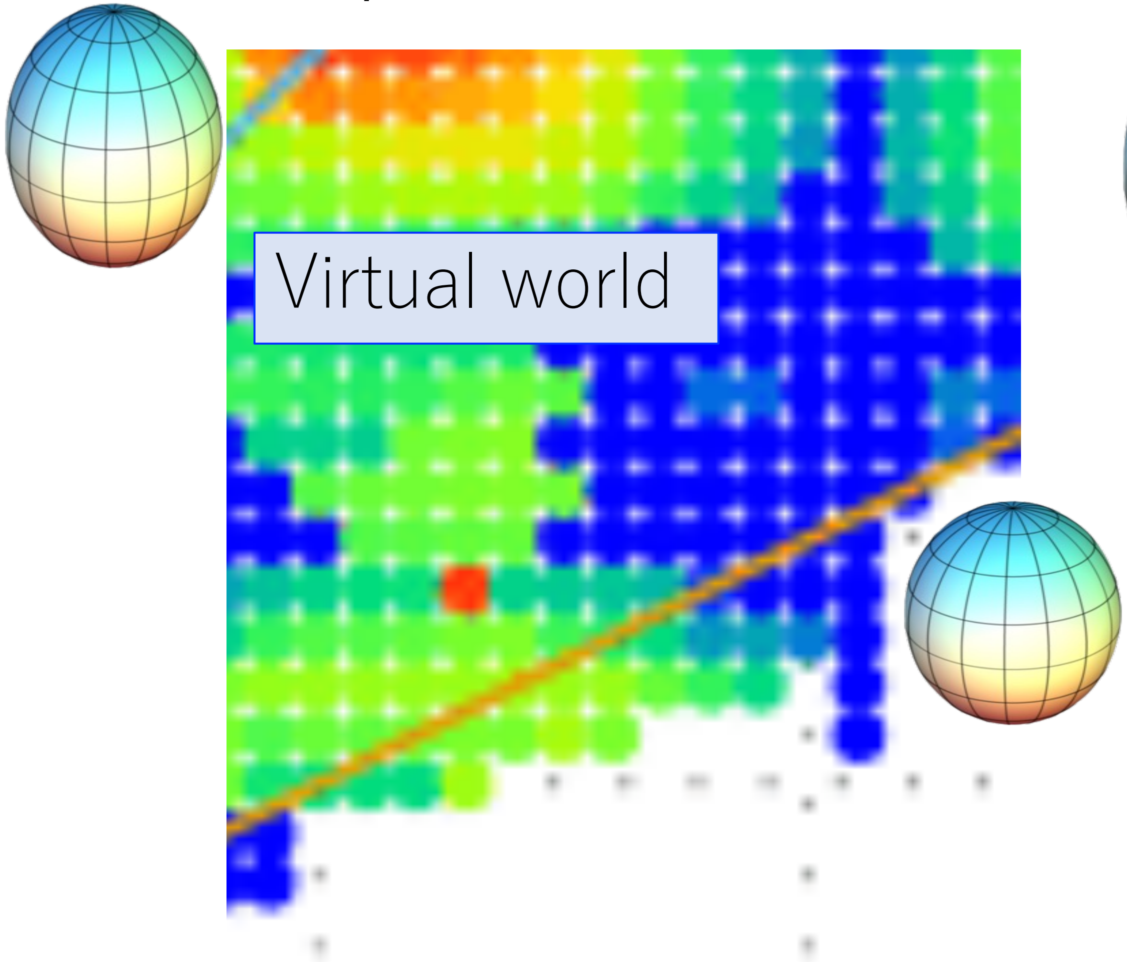


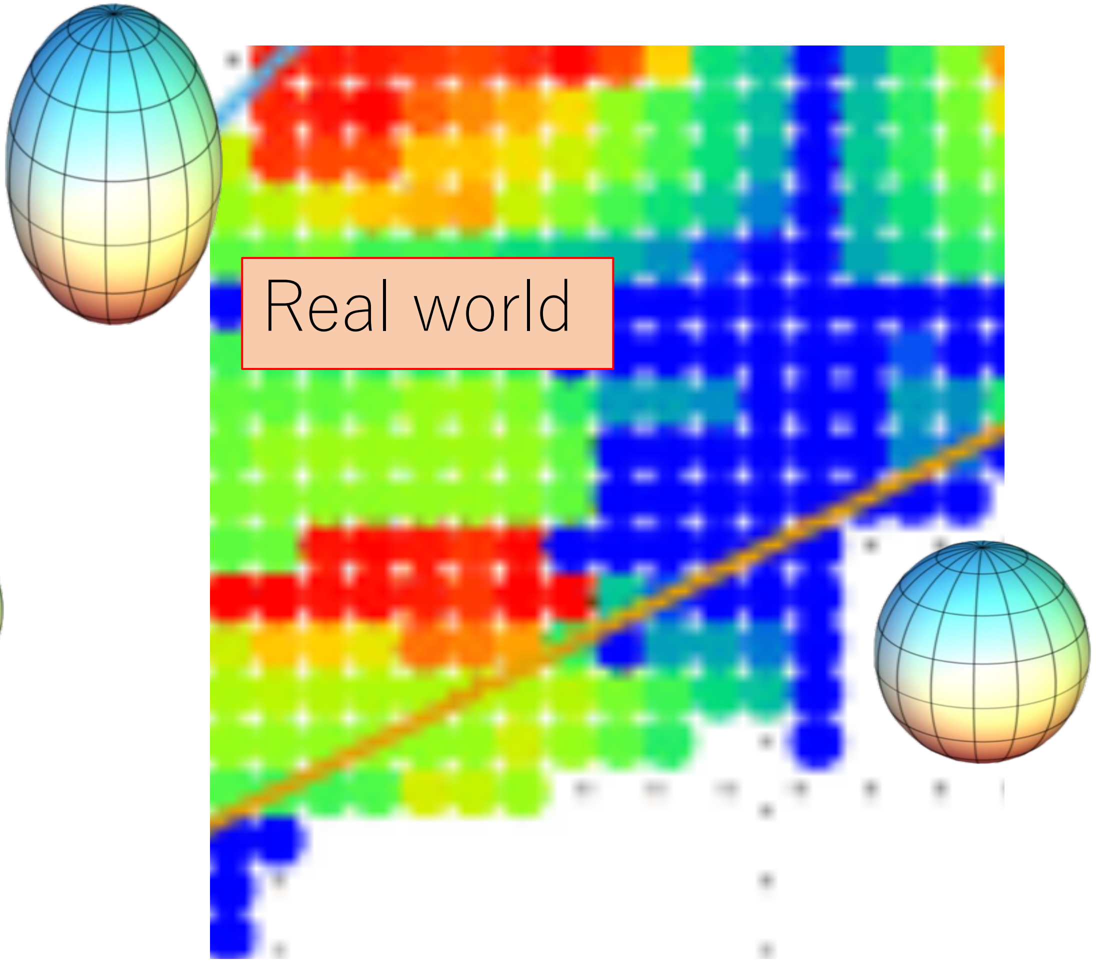


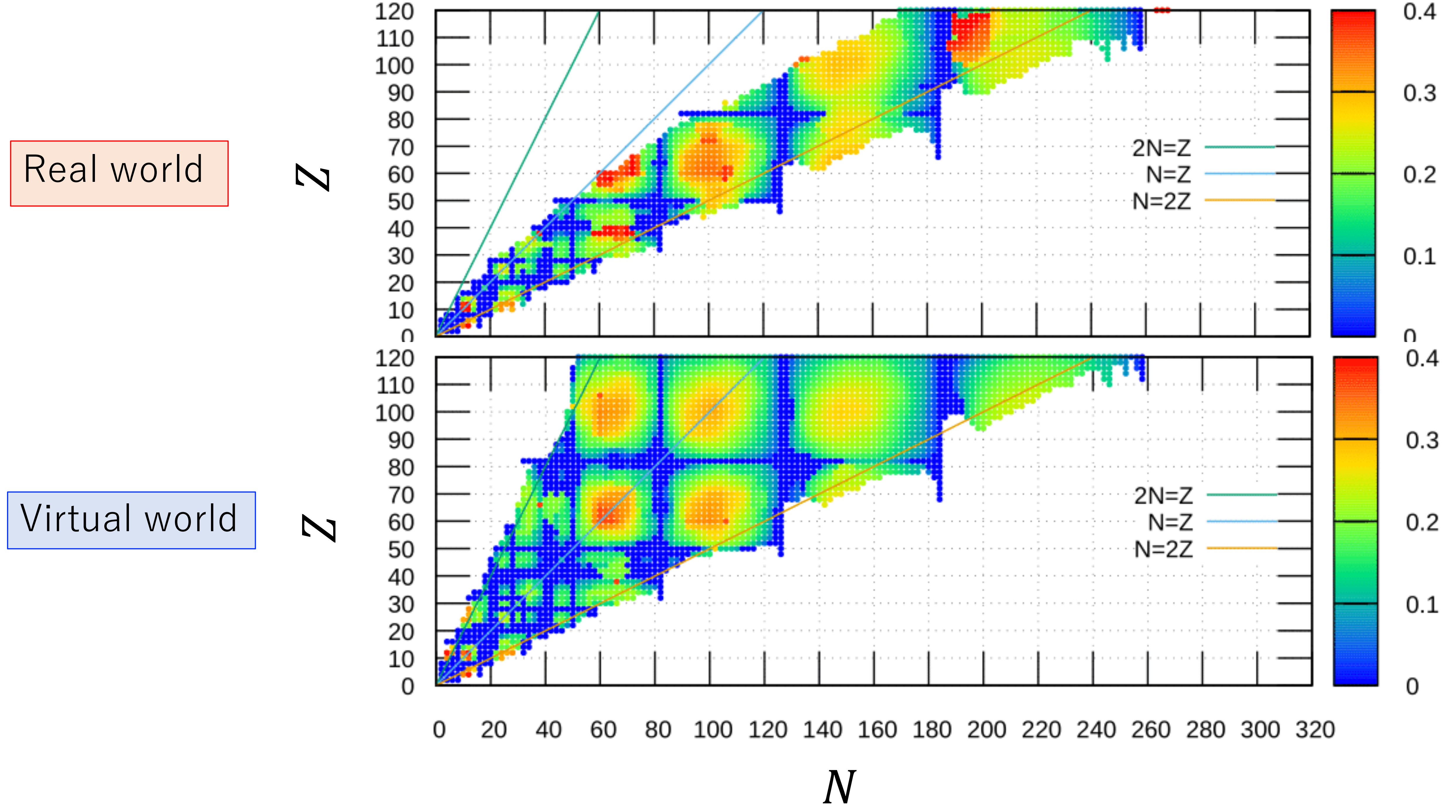
Real world
Virtual world
Z
Z
N
2N=Z
N=Z
N=2Z
2N=Z
N=Z
N=2Z
0
10
20
30
40
50
60
70
80
90
100
110
120
0
20
40
60
80
100
120
140
160
180
200
220
240
260
280
300
320
0
0.1
0.2
0.3
0.4

Coulomb increases deformation

- Deformation is principally produced by “shell effect”
- Repulsive Coulomb interaction favors larger deformation







Real world
Virtual world
Z
Z
N
2N=Z
N=Z
N=2Z
2N=Z
N=Z
N=2Z
0
10
20
30
40
50
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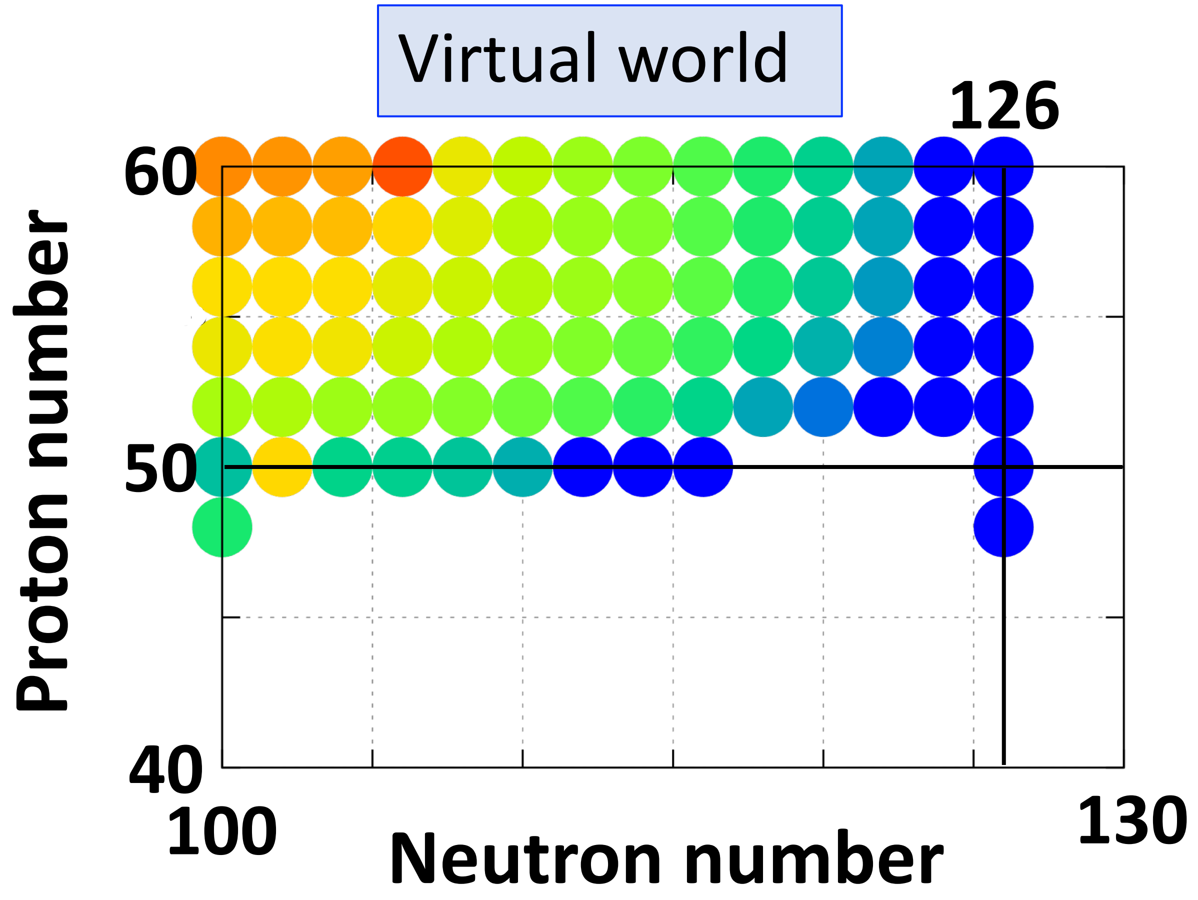
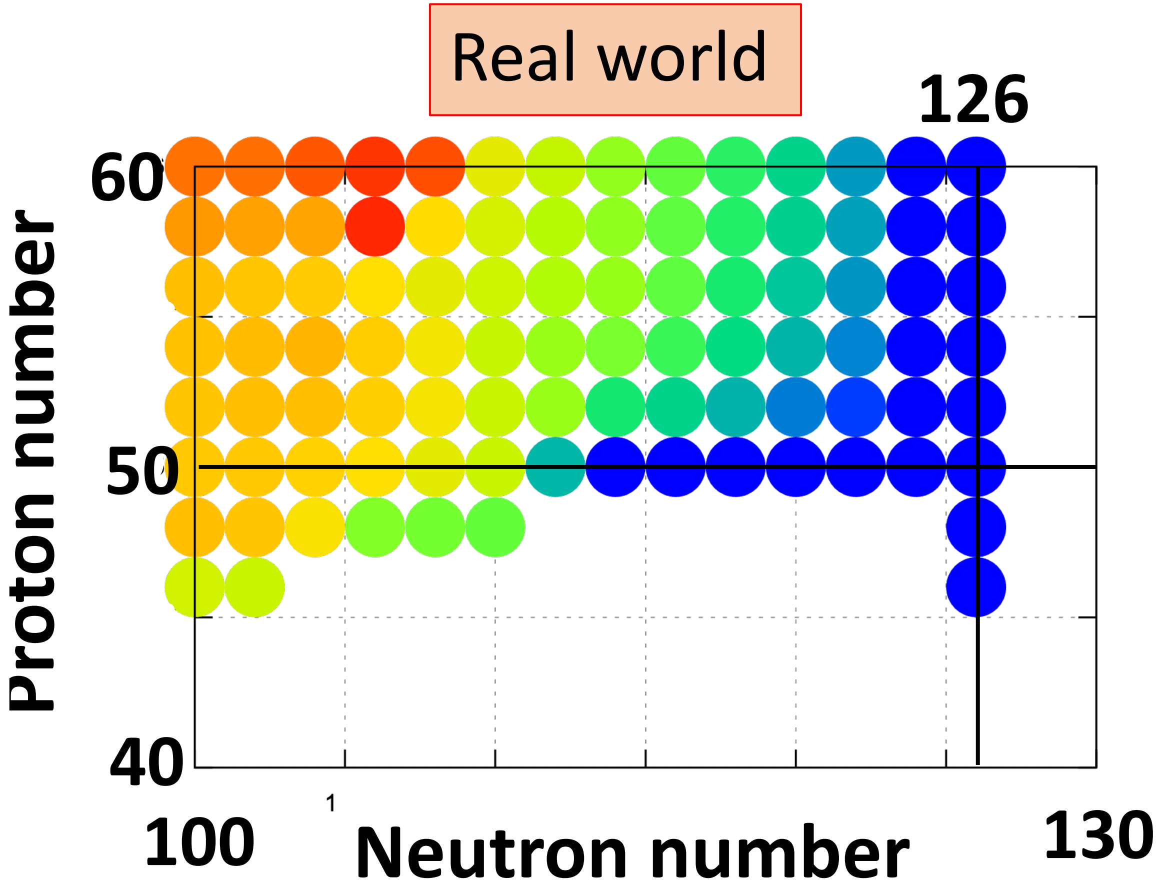
0.4

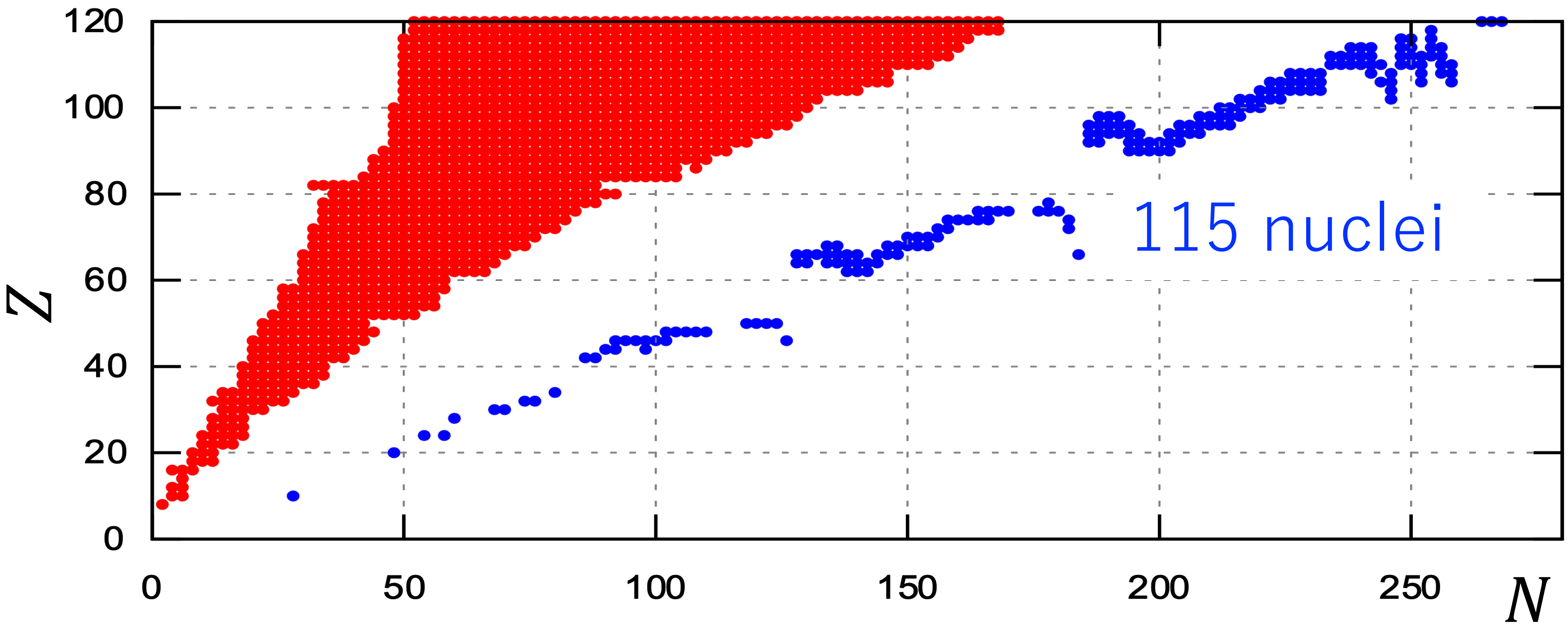
Near the neutron drip line





Coulomb stabilizing effect

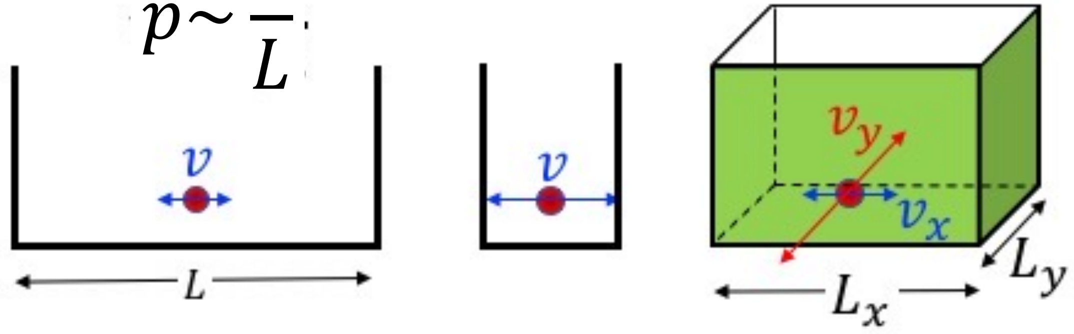
Shifts of drip lines ("Virtual" to "Real" worlds)



- **Do not exist in "real" world**
- **Do exist ONLY in "real" world!**

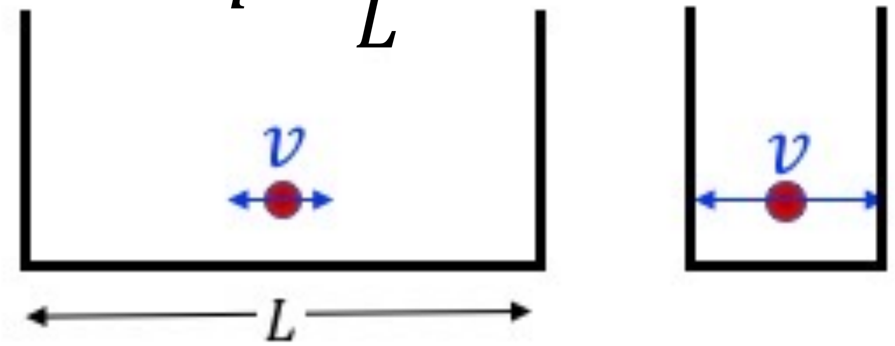
Why is the repulsive force able to provide additional binding?

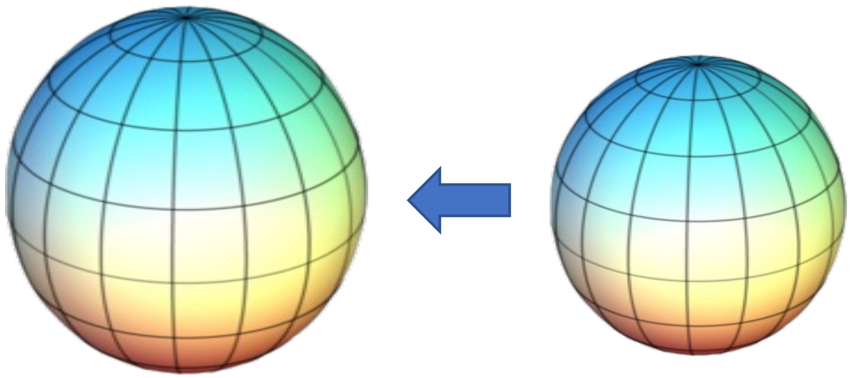
$$p \sim \frac{h}{L}$$



Coulomb stabilizing effect

$$p \sim \frac{h}{L}$$



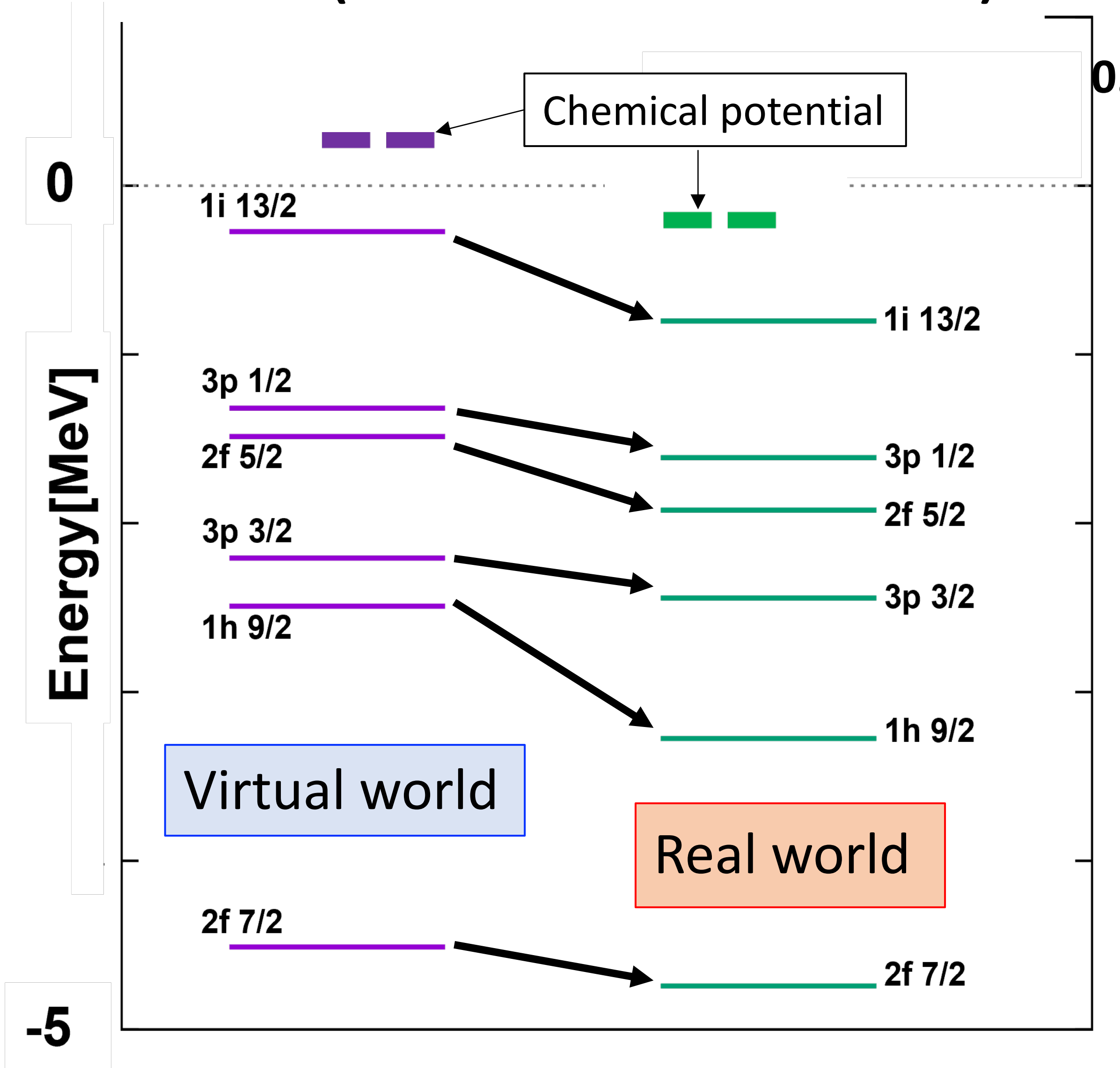


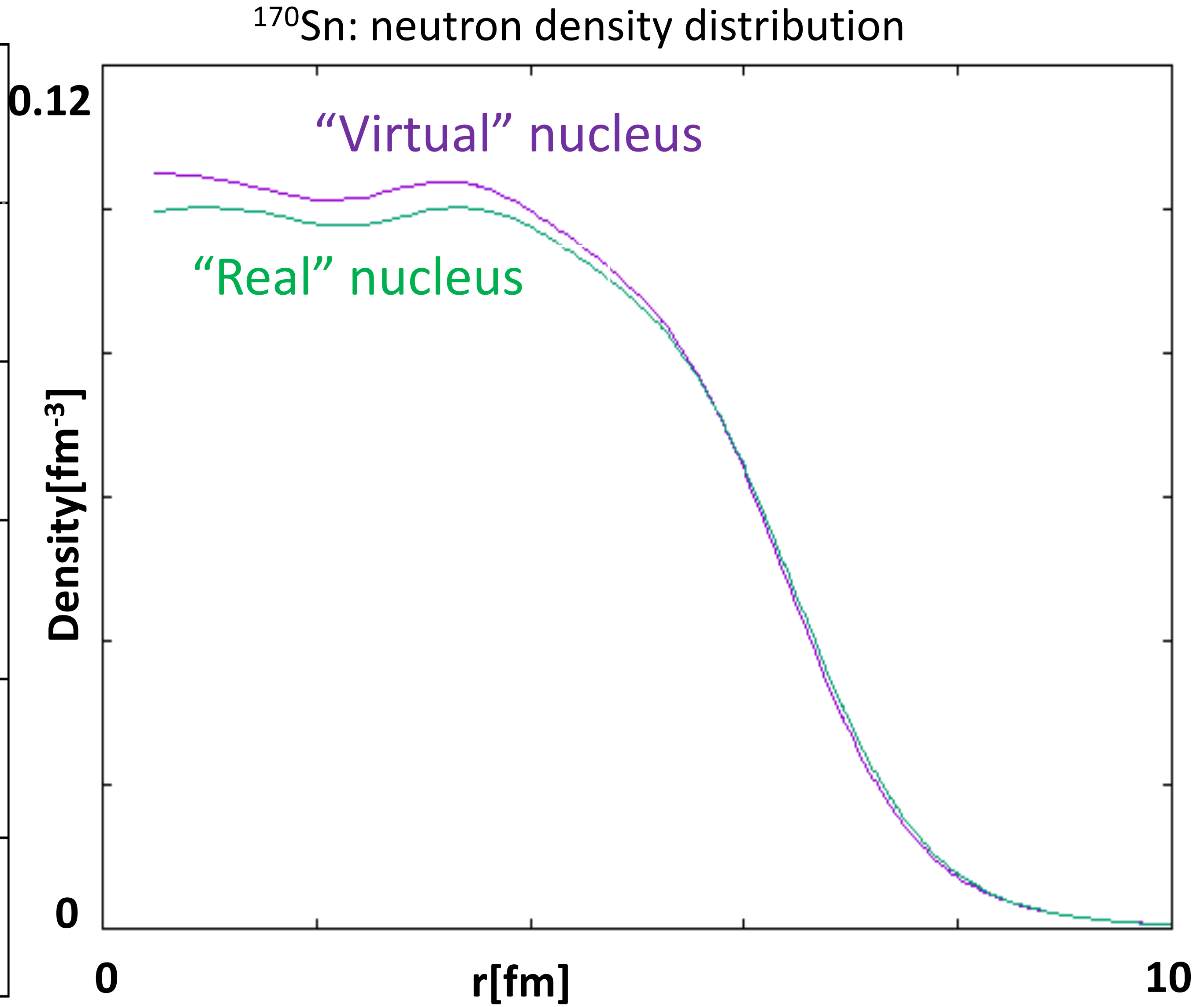
$E_{\mathrm{kin}} = \frac{\boldsymbol{p}^2}{2m}$: small large

1. Repulsive Coulomb among protons increases the nuclear size
2. Larger box leads to smaller zero-point kinetic energy
3. Stabilize nuclei at the neutron drip line

Neutron single-particle energy

$^{170}\mathrm{Sn}\ (Z = 50\ N = 120)$





Summary

- Effect of Electromagnetic interaction on nuclear structure
  - Enlarge deformation
  - Shift the proton drip line to the right
  - Shift the neutron drip line to the right
- Quantum mechanical stabilization effect
- A possible effect on the boundary between outer and inner crusts of neutron stars