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General relativistic radiation magnetohydrodynamics (GR-RMHD) simulations of black hole accretion flows based on solving the radiative transfer equation

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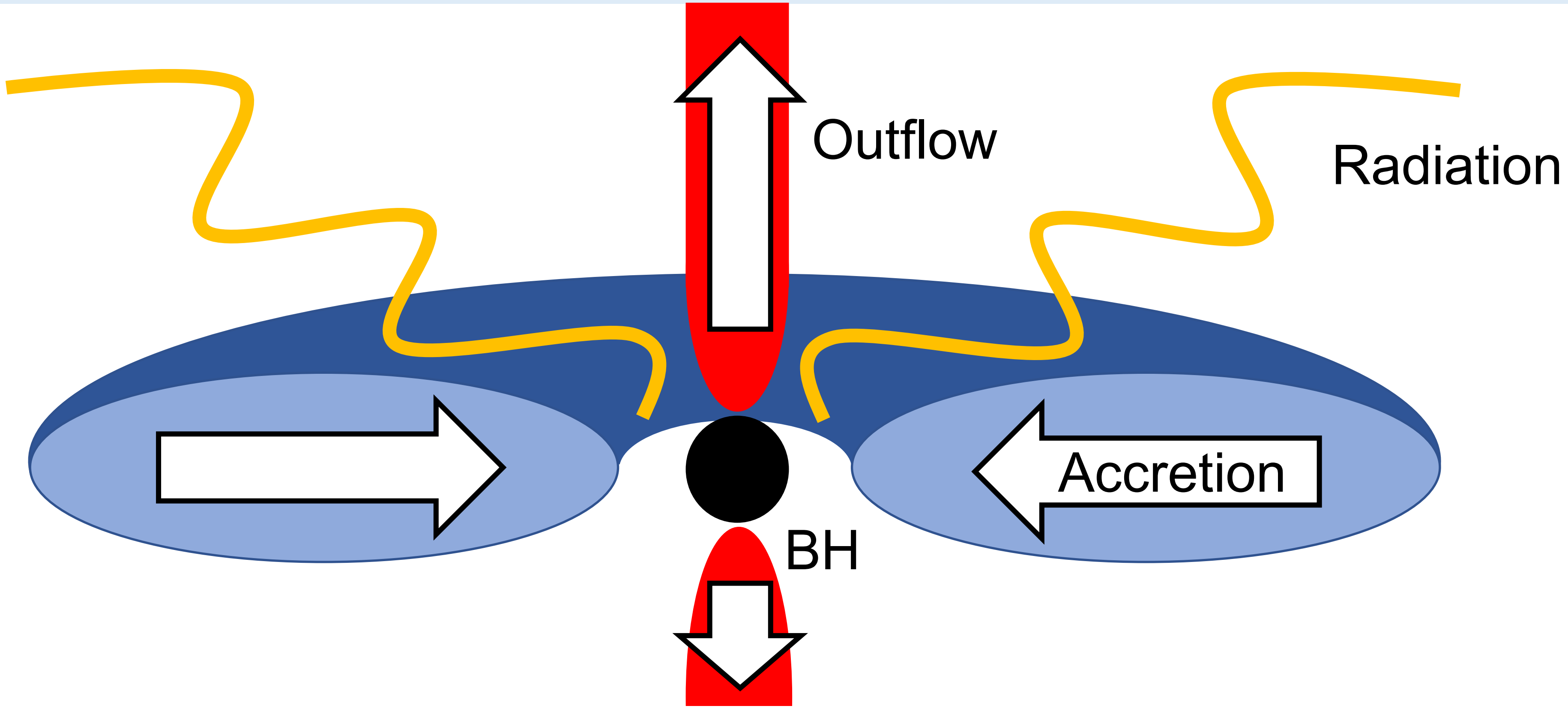
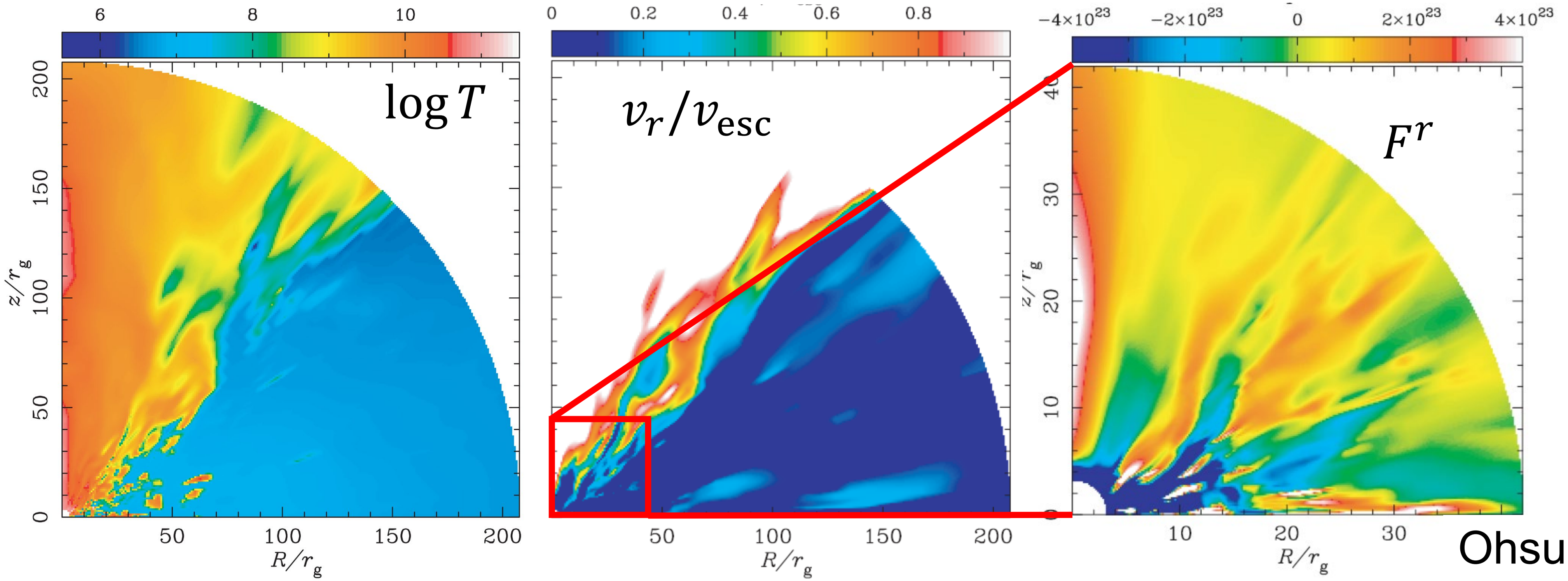
Black Hole (BH) Accretion Flows



- Rotating gas accretes into a compact object such as a BH forming an accretion flow
- A part of the gravitational energy is converted to the radiation
- The radiation force needs to be considered when the mass accretion rate is high

Radiation Hydrodynamics Simulations

Ohsuga et al. (2005) conducted 2D radiation HD simulations of the supercritical accretion flows around a black hole.

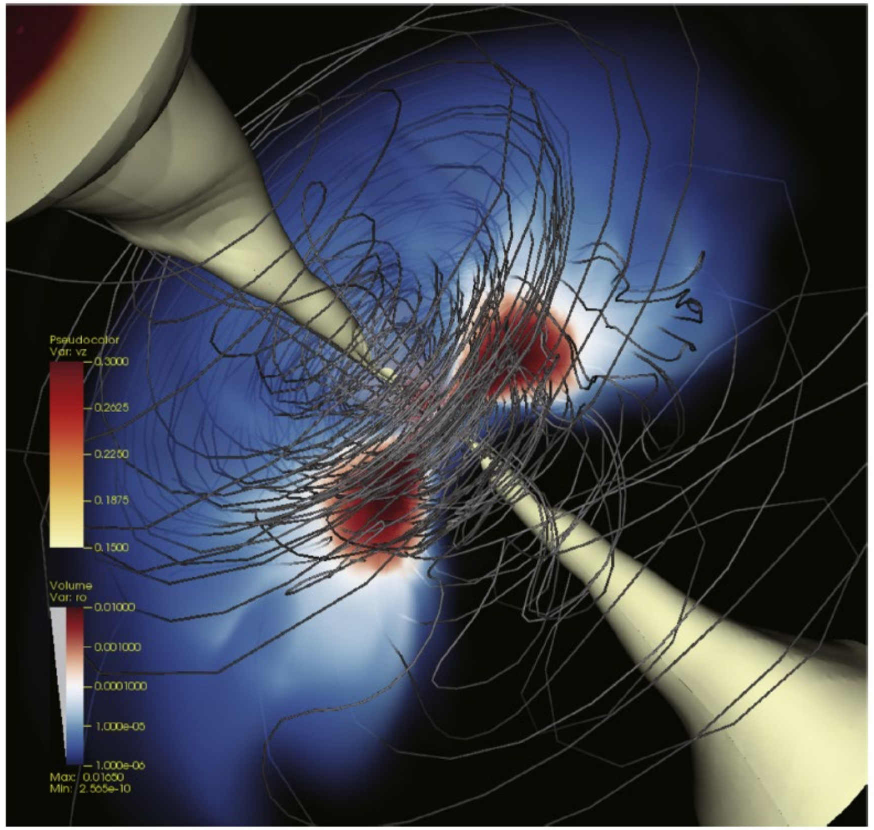


Ohsuga et al. (2005)

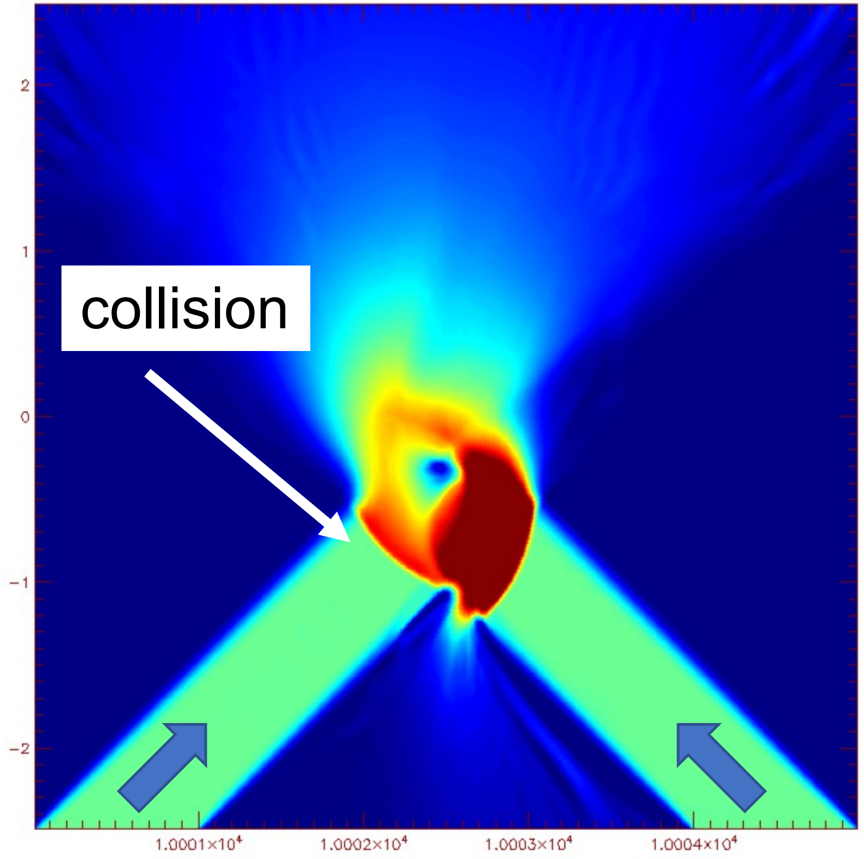
- Hot outflow with the velocity of 0.1c is formed around the rotation axis by strong radiation pressure
- The radiation is important for the outflow formation and structure of the accretion flow

General Relativistic Radiation Magnetohydrodynamics (GR-RMHD) Simulations

Takahashi et al. (2016) carried out 3D GR-RMHD simulations with the approximate method to solve the radiation transfer, which is called the M1 method. The strong radiation pressure accelerates the outflow.



Takahashi et al. (2016)



M1 method

Radiative Transfer Equation

Radiative transfer equation

$$\frac{\partial I}{\partial t} + \boldsymbol{n} \cdot \frac{\partial I}{\partial \boldsymbol{x}} = 0$$

Moment equations

$$\xrightarrow{\times \int \mathrm{d}\Omega} \quad \frac{\partial E_r}{\partial t} + \frac{\partial F^i}{\partial x^i} = 0$$

$$\xrightarrow[\times \boldsymbol{n} \int \mathrm{d}\Omega]{} \quad \frac{\partial F^i}{\partial t} + \frac{\partial P^{ij}}{\partial x^j} = 0$$

- In order to solve this equation, we need to obtain the radiation pressure $P^{ij} = \mathbb{D} E_r$
- M1 method assumes that the Eddington tensor is below in order to close these equations (Gonzalez et al. 2007)

$$\mathbb{D} = \frac{1-\chi}{2}\mathbb{I} + \frac{3\chi - 1}{2}\boldsymbol{n} \otimes \boldsymbol{n}, \quad \chi = \frac{3 + 4\|\boldsymbol{f}\|^2}{5 + 2\sqrt{4 - 3\|\boldsymbol{f}\|^2}}, \quad \boldsymbol{f} = \frac{\boldsymbol{F}_{\mathrm{r}}}{cE_{\mathrm{r}}}$$

Radiative Transfer Equation

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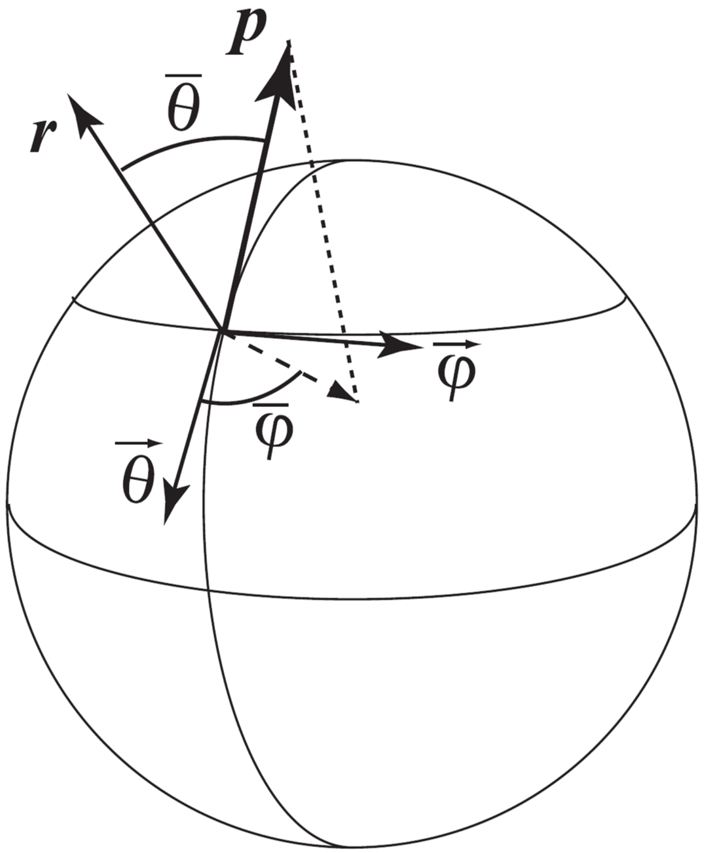
Radiative Transfer Equation solved in INAZUMA code

Conservation of the radiation energy and momentum

$$\partial_t(\sqrt{-g}R^t_\mu)+\partial_i(\sqrt{-g}R^i_\mu) = \sqrt{-g}R^\kappa_\lambda\Gamma^\lambda_{\mu\kappa}-\sqrt{-g}G_\mu$$

Frequency-integrated radiative transfer equation

$$\begin{aligned}&\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\alpha}\left[\left(e^\alpha_{(0)}+\sum_{i=1}^{3}l_{(i)}e^\alpha_{(i)}\right)\sqrt{-g}I\right]\\&\quad+\frac{1}{\sin\bar{\theta}}\frac{\partial}{\partial\bar{\theta}}\left(\sin\bar{\theta}\omega_{(\bar{\theta})}I\right)+\frac{1}{\sin^2\bar{\theta}}\frac{\partial}{\partial\bar{\phi}}\left(\omega_{(\bar{\phi})}I\right)+\omega_{(0)}I\\&\quad=-\gamma(1-\boldsymbol{v}\cdot\boldsymbol{l})\rho(\kappa_{\rm abs}+\kappa_{\rm sca})I+\gamma^{-3}(1-\boldsymbol{v}\cdot\boldsymbol{l})^{-3}\rho\left(\frac{j_0}{4\pi}+\kappa_{\rm sca}\frac{E_{\rm com}}{4\pi}\right)\end{aligned}$$



Shibata et al. (2014)

The computational cost becomes higher since we should consider the momentum space $(\bar{\theta}, \bar{\varphi})$.

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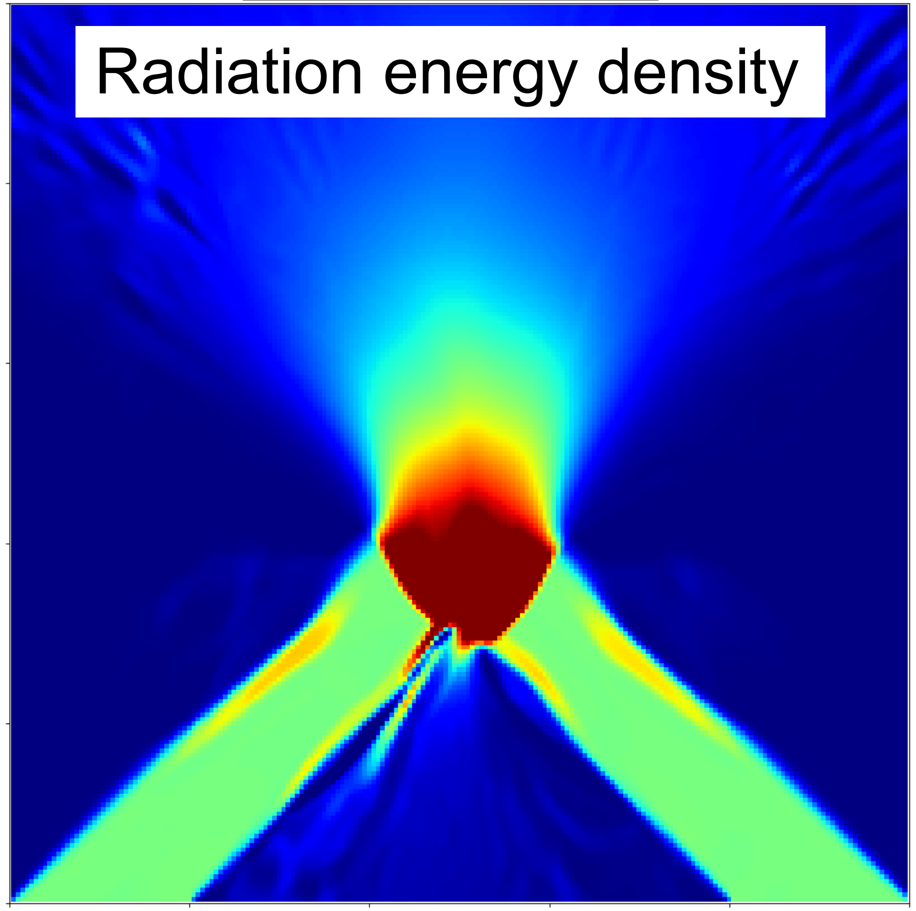
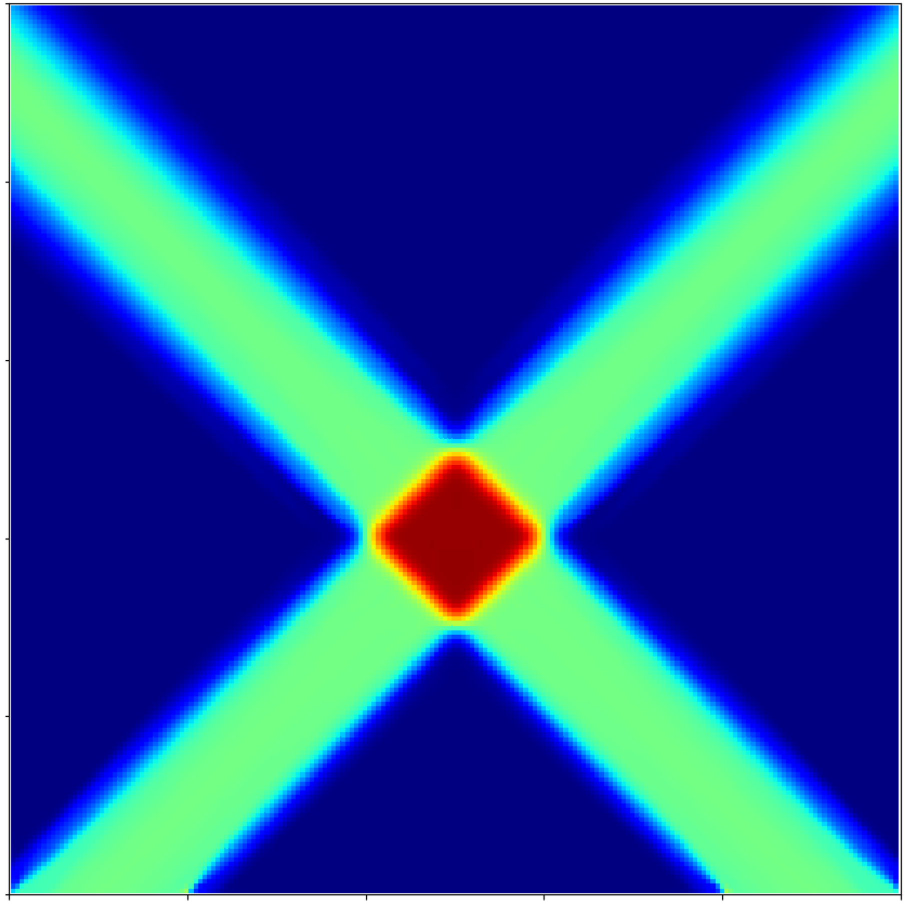
Beam Crossing Test

M1 method

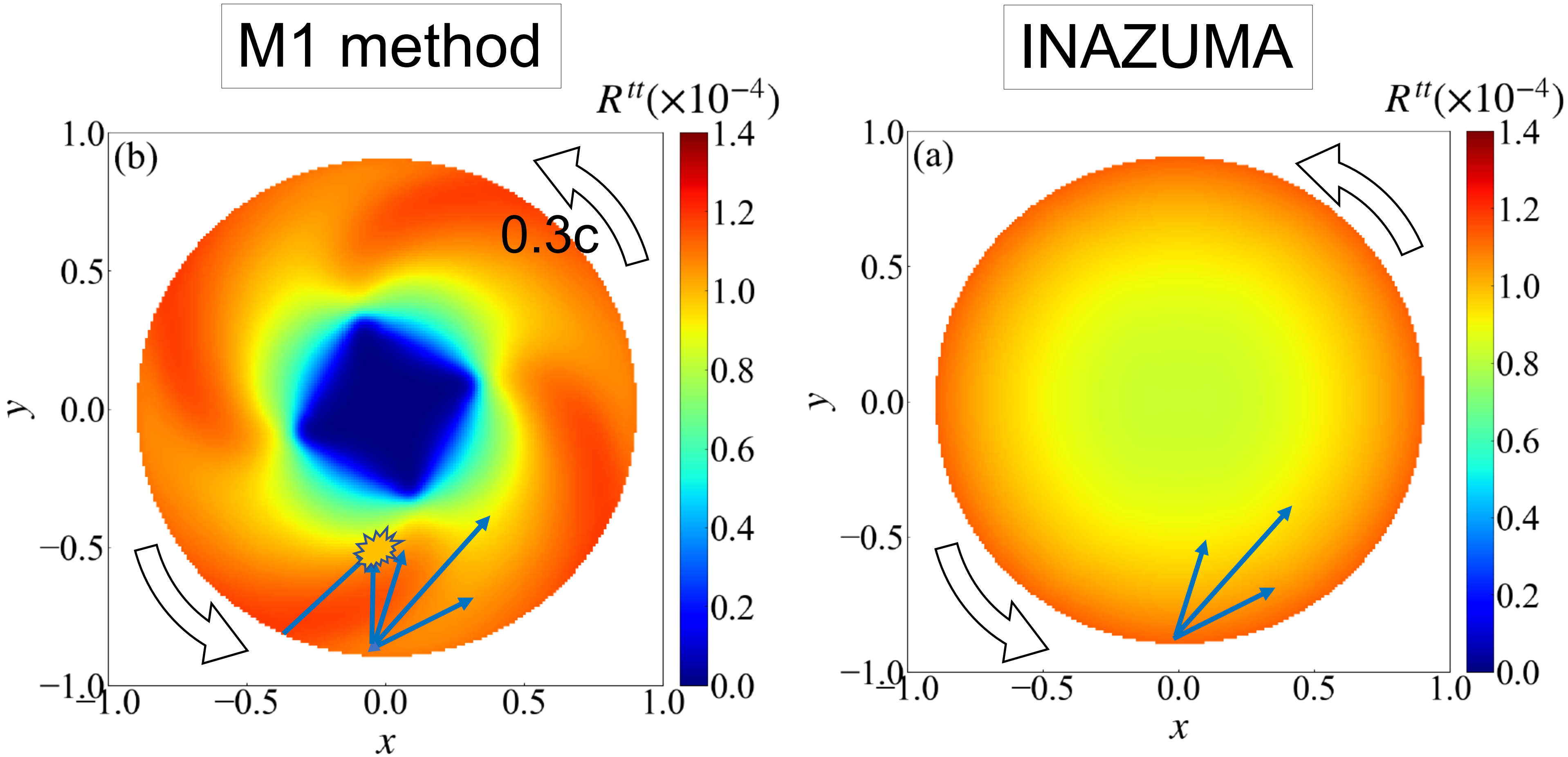


INAZUMA



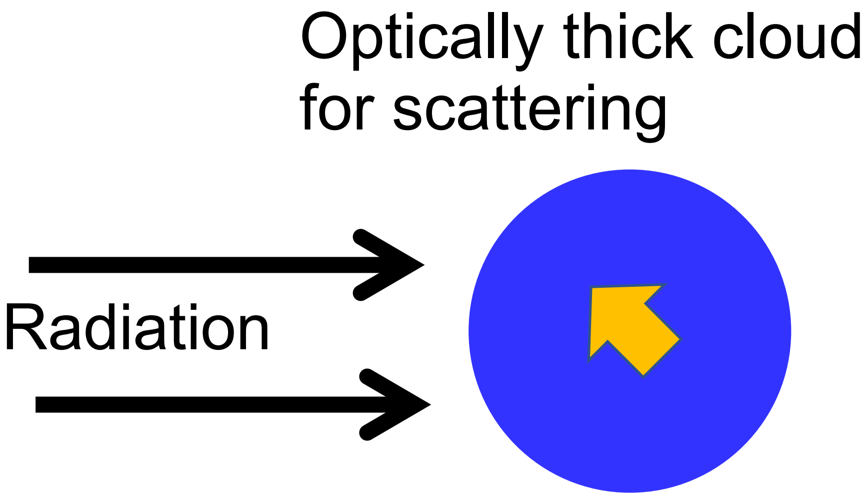
- The M1 method cannot solve the beam crossing since the radiation collides with each other
- The INAZUMA code can solve beam crossing successfully without collision

Radiation Propagation from Rotating Medium

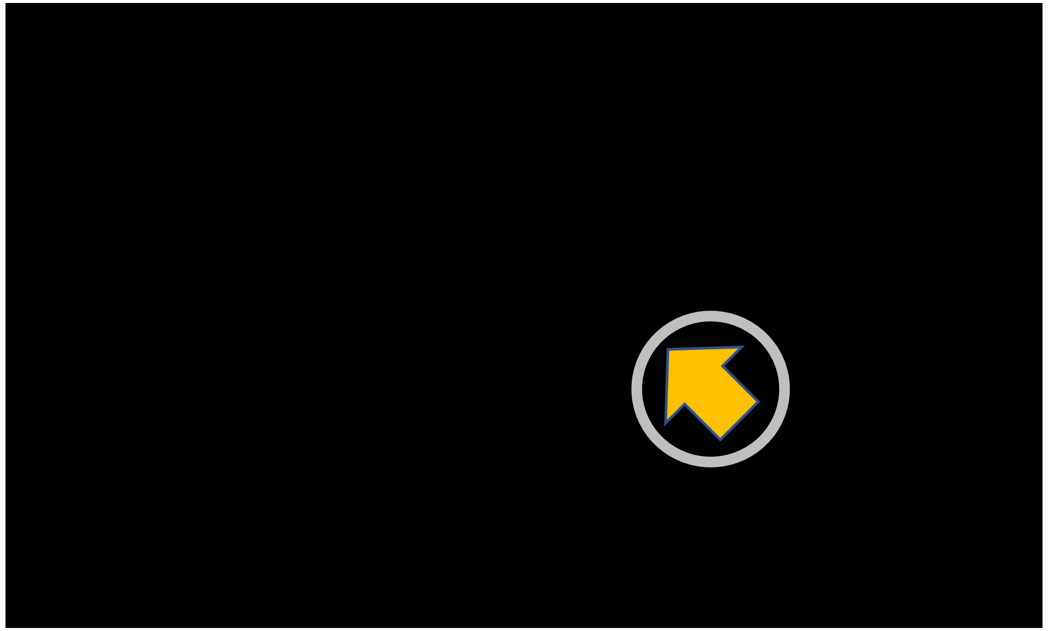


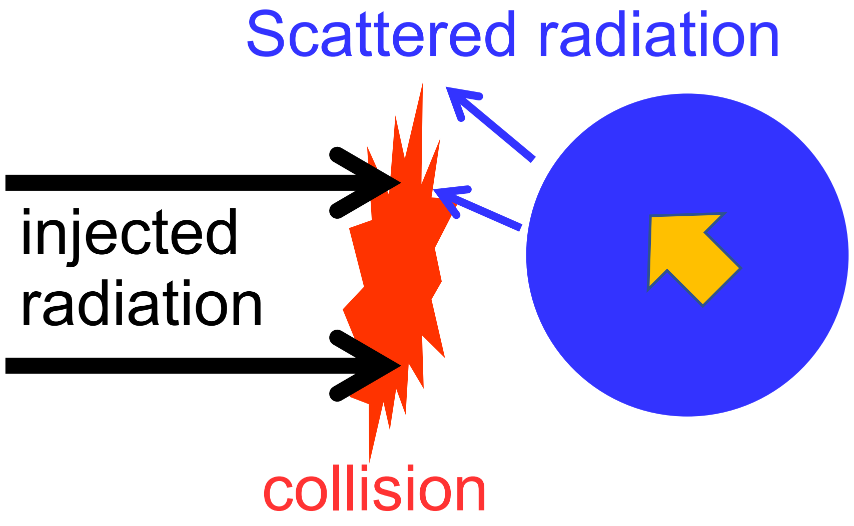
- The radiation emitted from the rotating medium cannot reach the center for the M1 method
- For the INAZUMA code, the radiation fulfill the center region

Interaction with Optically Thick Cloud (M1 method)

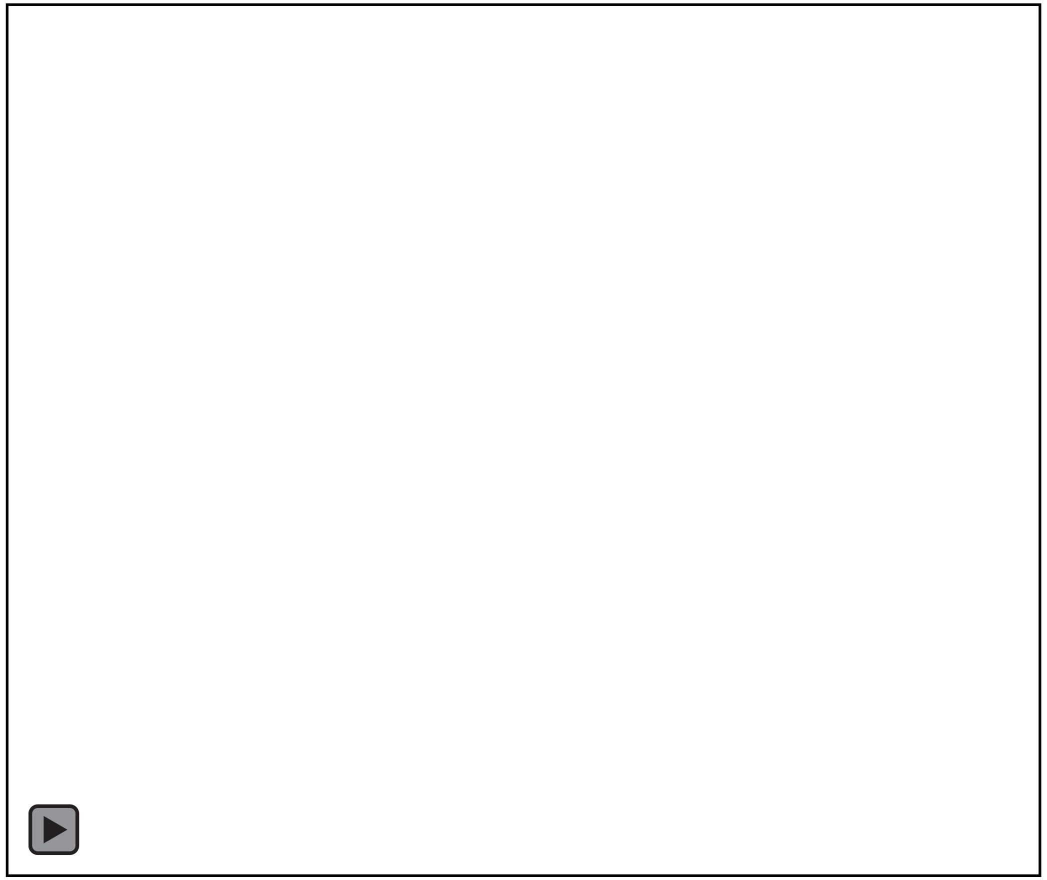


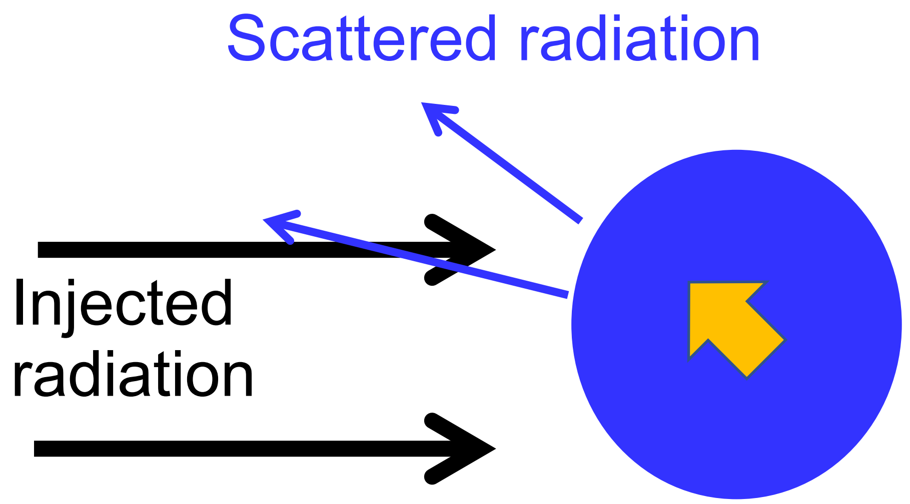
- Optically thick cloud for scattering with the velocity of 0.6c locates in the gray circle
- The scattered radiation collides with the injected radiation unphysically
- The M1 method cannot solve this test





Interaction with Optically Thick Cloud (INAZUMA)





Scattered radiation propagates to left hand side

- The scattered radiation can propagates in the upper left direction without unphysical radiation collision
- The INAZUMA code can solve the interaction of radiation with the optically thick cloud for scattering

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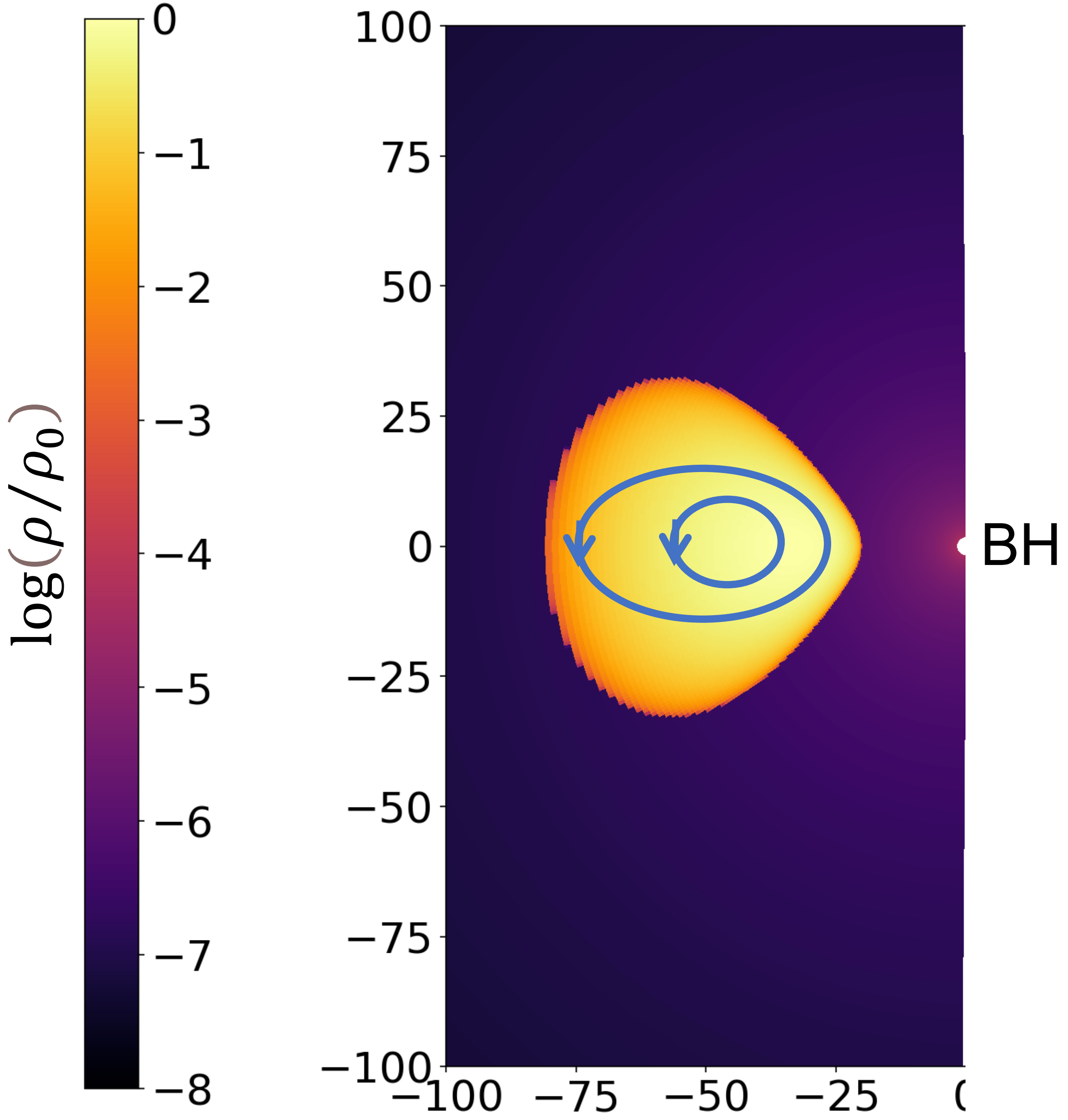
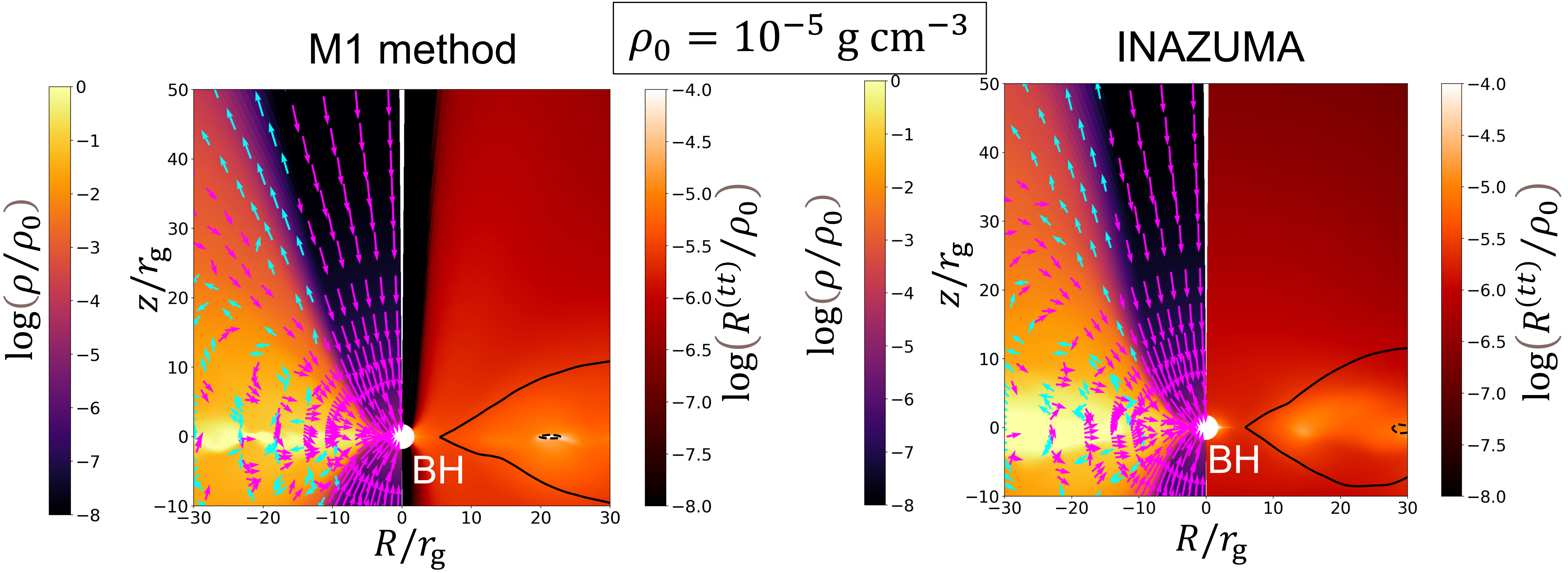
Application to BH Accretion Flows



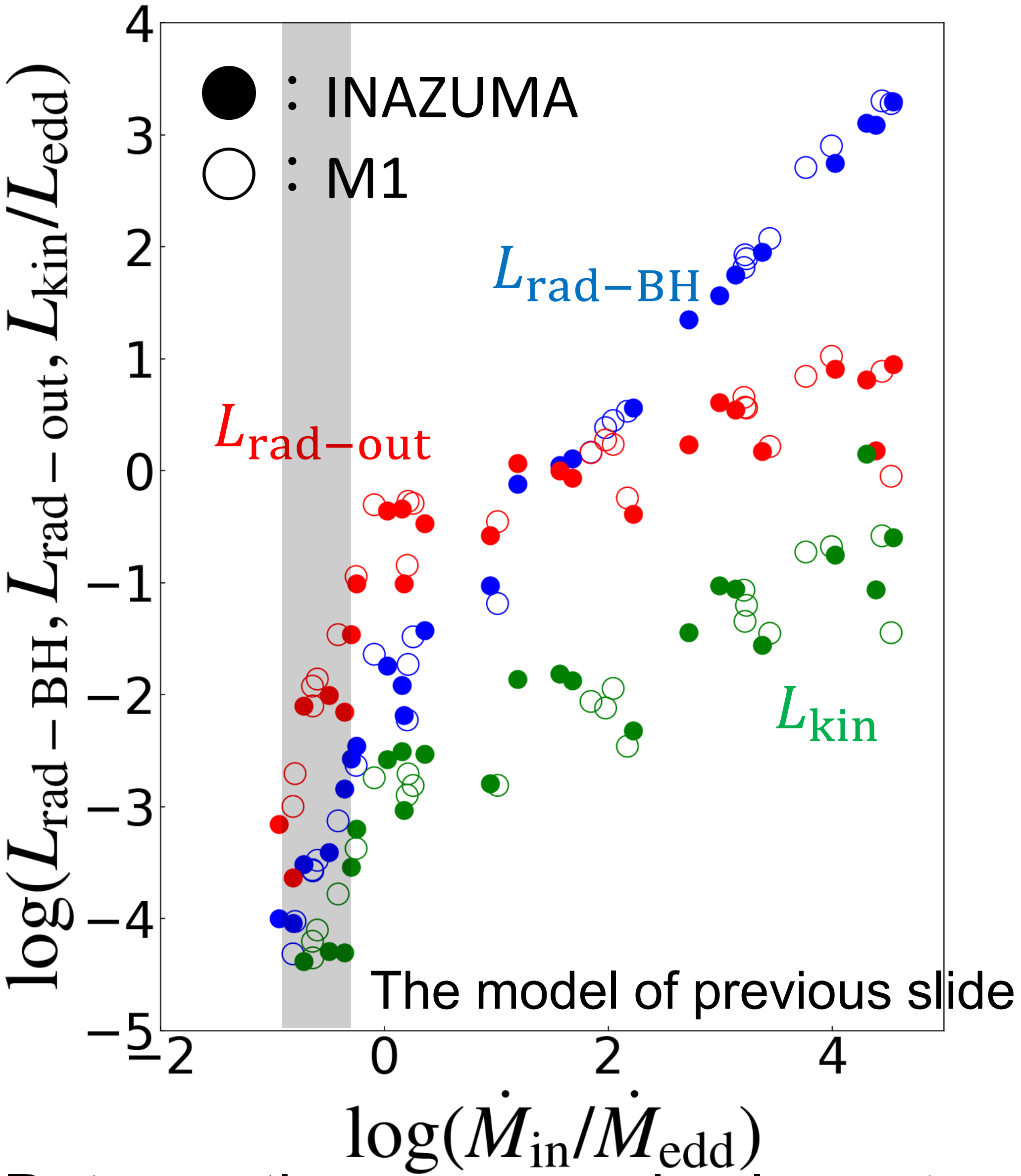
- We start simulations from an equilibrium torus given by Fishbone & Moncrief (1976)
- We simulate six models with the maximum density of the initial torus of $\rho_0 = 10^{-5,-4,-3,-2,-1,0}\ \mathrm{g\ cm^{-3}}$
- We assume the weak poloidal magnetic field in the torus (blue curves)
- Free-free emission/absorption and isotropic electron scattering are considered
- The radiation energy is assumed to be small and isotropic
- Grid points $(N_r, N_\theta, N_{\bar{\theta}}, N_{\bar{\varphi}})$ = (300, 300, 32, 16)

Profiles of the Density and Radiation Energy Density



- The density and radiation energy density profiles are similar globally between M1 and INAZUMA except around the axis
- The radiation energy density around the axis is small in the M1 method

Dependence of Radiation Luminosity and Kinetic Luminosity



Dots are time-averaged values at $100\, r_g$

$L_{\mathrm{rad-out}}$: Outward radiation luminosity

$L_{\mathrm{rad-BH}}$: Radiation energy flux falling into the BH

L_{kin} : Outward kinetic luminosity

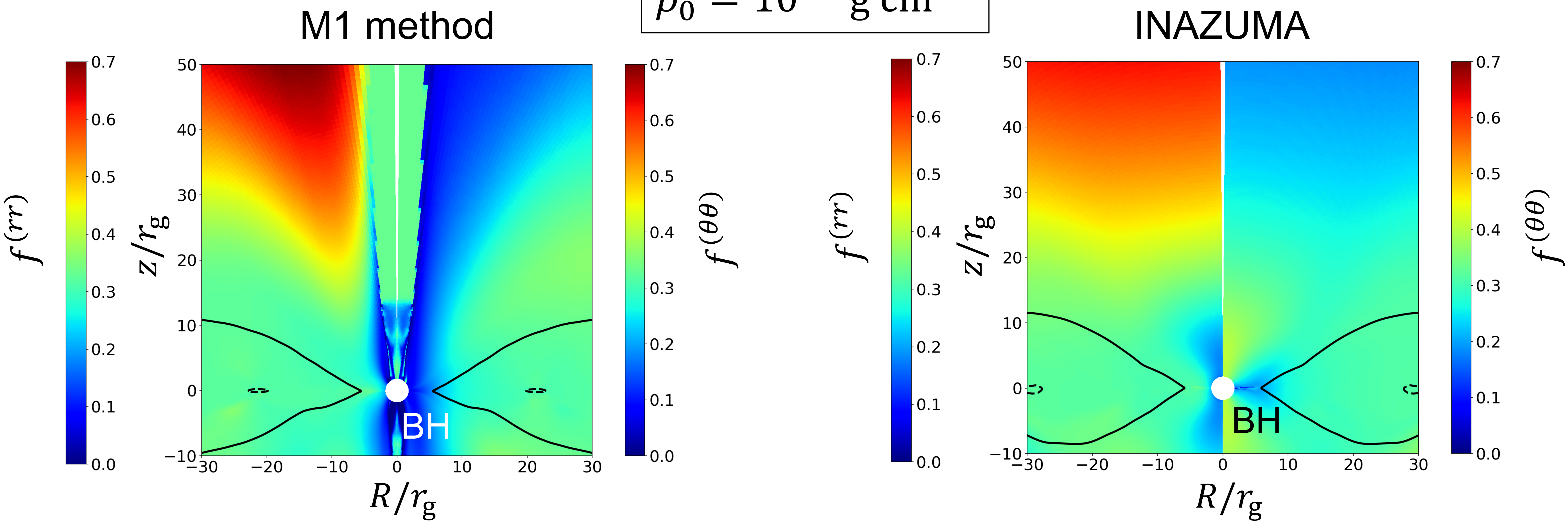
L_{edd} : The luminosity at which the radiation force is balanced by gravity in a spherically symmetric system

$\dot{M}_{\mathrm{edd}} = L_{\mathrm{edd}}/c^2$

- All luminosities tend to increase with an increase of $\dot{M}_{\mathrm{in}}$ since the released gravitational energy increases with the mass accretion rate
- The dependence is similar between the INAZUMA code and the M1 method

Difference of the Radiation Field (Low Accretion Rate Model)

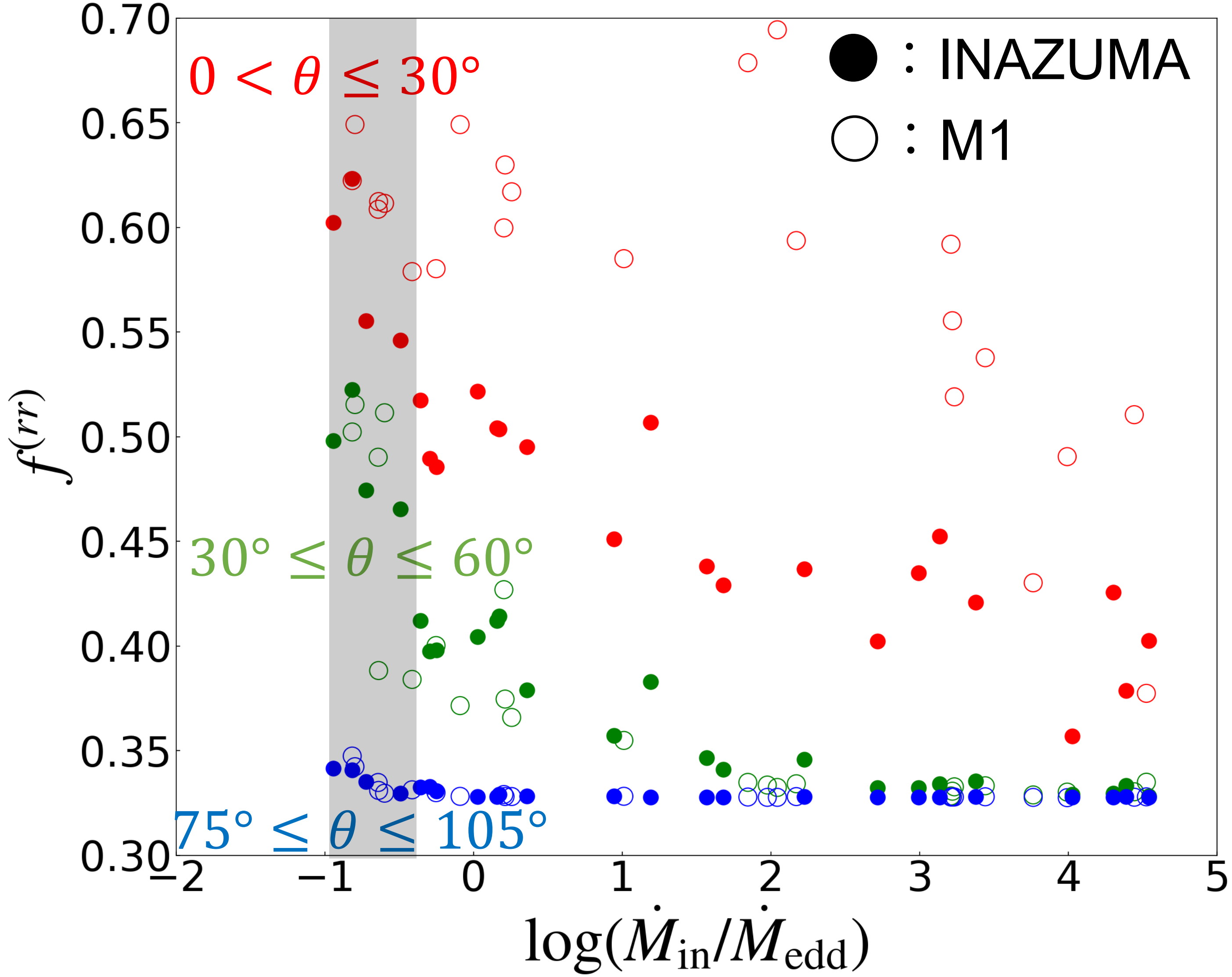
$\rho_0 = 10^{-5}\ \mathrm{g\ cm^{-3}}$



- The radiation emitted from the accretion flow can freely propagate radially outward
- $f^{(rr)}$ for the M1 method tends to be larger than that for INAZUMA

Dependence of the Radiation Field on the Mass Accretion Rate

The model of previous slide



Dots are time-averaged values at $50\ r_{\rm g}$

- As the mass accretion rate increases, the opacity also increases, so $f^{(rr)}$ approaches 1/3
- Except around the rotation axis, $f^{(rr)}$ is similar dependence between the M1 method and INAZUMA code since the opacity is large in these regions (blue and green symbols),
- Around the rotation axis, $f^{(rr)}$ for the M1 method is larger than that for INAZUMA code (red symbols)

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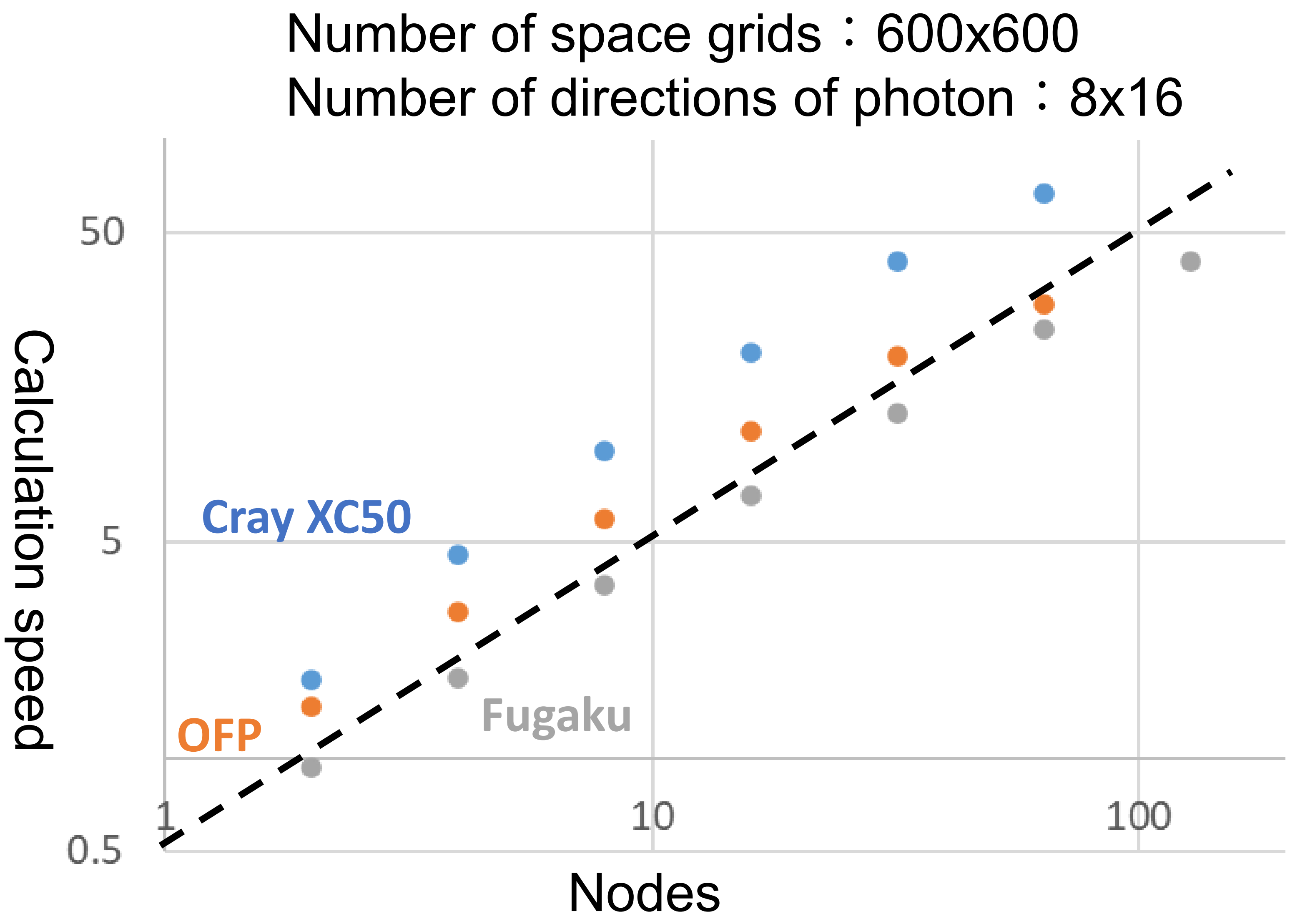
Strong Scaling



- INAZUMA code has been implemented in Oakforest-PACS at the CCS, Cray XC50 at National Astronomical Observatory of Japan, and Fugaku at the RIKEN Center for Computational Science
- Calculation speed is almost proportional to the number of nodes
- Parallelization efficiency is about 90%

Computational Cost Normalized by the Cost of the M1 Method

The number of grids in space is 300 x 300.

The number of photon directions ($N_{\bar{\theta}} \times N_{\bar{\varphi}}$)	Time	Δt ($\times 10^{-3} t_g$)	computational cost	Expected cost
M1 method	1	4.1	1	1
8x16	5.2	1.4	1.8	141/13 ~ 11
16x32	25	0.7	4.1	525/13 ~ 40
32x64	161	0.3	13	2,061/13 ~ 160

- For the M1 method, 13 equations are solved
- For INAZUMA code, $N_{\bar{\theta}} \times N_{\bar{\varphi}}$ equations are solved in addition to 13 equations (e.g., 128 equations for 8x16)
- The computational cost is much smaller than the expected cost
- The simulation time step (Δt) need to be small for the INAZUMA code since the advection speed in $\bar{\varphi}$-direction is larger than those in other directions

Why is the cost of solving the radiative transfer equation is low?

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\alpha}\left[\boxed{\left(e^\alpha_{(0)}+\sum_{i=1}^{3} l_{(i)} e^\alpha_{(i)}\right)\sqrt{-g}I}\right] + \frac{1}{\sin\bar{\theta}}\frac{\partial}{\partial\bar{\theta}}\left(\boxed{\sin\bar{\theta}\,\omega_{(\bar{\theta})}}I\right) + \frac{1}{\sin^2\bar{\theta}}\frac{\partial}{\partial\bar{\phi}}\left(\boxed{\omega_{(\bar{\phi})}}I\right) + \boxed{\omega_{(0)} I}$$
$$= \boxed{-\gamma(1-\boldsymbol{v}\cdot\boldsymbol{l})\rho(\kappa_{\rm abs}+\kappa_{\rm sca})I + \gamma^{-3}(1-\boldsymbol{v}\cdot\boldsymbol{l})^{-3}\rho\left(\frac{j_0}{4\pi}+\kappa_{\rm sca}\frac{E_{\rm com}}{4\pi}\right)}$$

Radiative transfer equation

- Since the advection velocity does not evolve with time, it is sufficient to calculate it **once** at the beginning of the simulation
- The source term can be solved **analytically**

Moment equations

- The advection velocity need to be calculated for **each time step**
- Since the source term is implicitly solved, the Newton-Raphson method is used to **iterate to converge the solution**

Summary

- We have developed the GR-RMHD code INAZUMA solving the radiative transfer equation
- We perform some test simulations and apply the INAZUMA code to the black hole accretion flow
- INAZUMA code can solve the radiative transfer exactly in the optically thin region far from the black hole and around the rotation axis
- The outward radiation luminosity, kinetic luminosity, and the luminosity falling into the BH are similar between the INAZUMA code and the M1 method
- The computational cost of the INAZUMA code is lower than the expected cost