Solving Kinematics of Cosmological Neutrinos in the 6D Phase Space on FUGAKU

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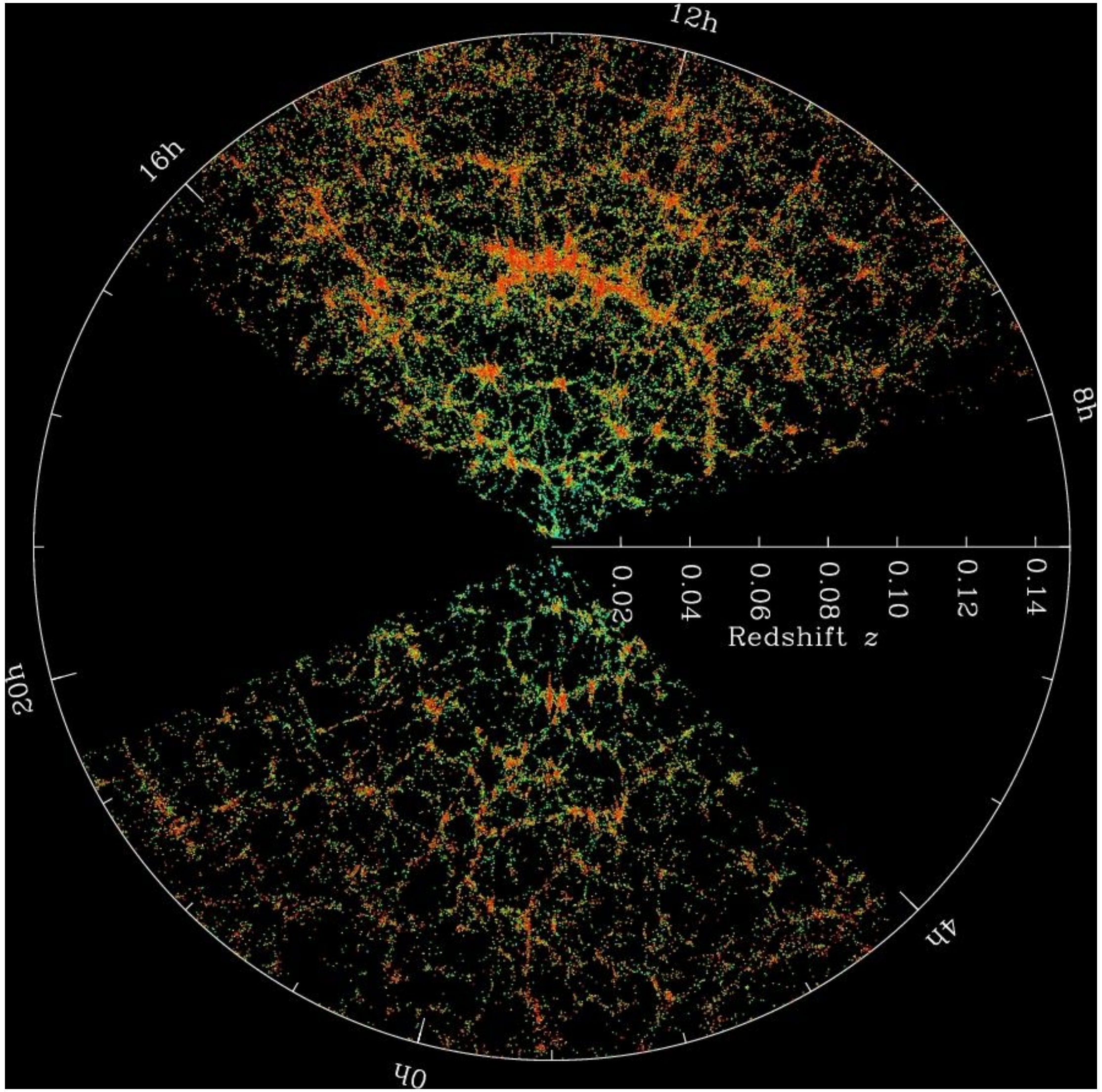
Large-Scale Structure (LSS) in the Universe



(SDSS survey : www.sdss.org)

- inhomogeneous distribution of galaxies in the Universe
- gravitational growth of small density fluctuation found in the early Universe.
- contains lots of valuable physical information about the origin and contents of our Universe.
  - amount of dark matter, baryons and dark energy
  - expansion rate of the Universe
  - very early stage of the Universe
  - mass and mass hierarchy of neutrinos

Massive Neutrinos

- ubiquitous in huge quantities in the Universe.

$$\bar{n}_\nu = \frac{3}{11}\bar{n}_\gamma = 113\,\mathrm{cm}^{-3}$$

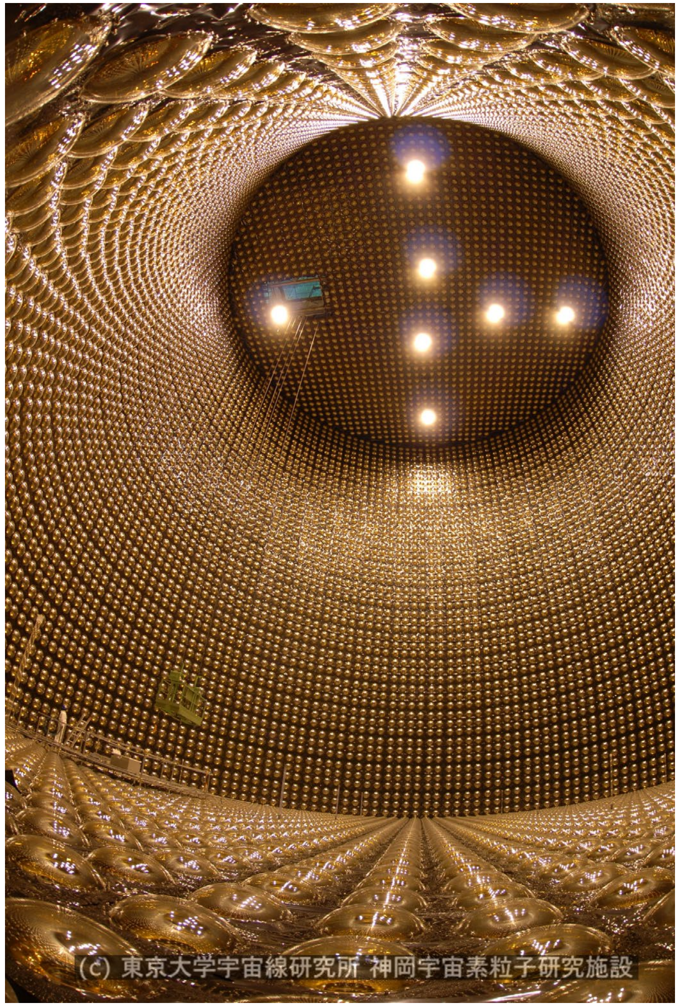
- they are thought to be massless in the standard model of elementary particle physics

- discovery of neutrino oscillation reveals that they are massive.

  Dynamical effect of massive neutrinos on the large-scale structure formation

- their absolute mass and mass hierarchy are still unknow, though they are both important for the new physics beyond standard model

Super-Kamiokande



Dynamical effect of massive neutrinos

- collisionless damping (analogue to Landau damping)
  - dumping of density fluctuation below the damping scale

$$\lambda_{\rm d} = \sqrt{\frac{\pi\sigma_{\rm v}^2}{G\bar{\rho}(t)a(t)^2}} \sim 2.02\times10^3\left(\frac{m_\nu}{0.1{\rm eV}}\right)^{-1/2} h^{-1}{\rm Mpc}$$

  - degree of damping depends on the neutrino mass.

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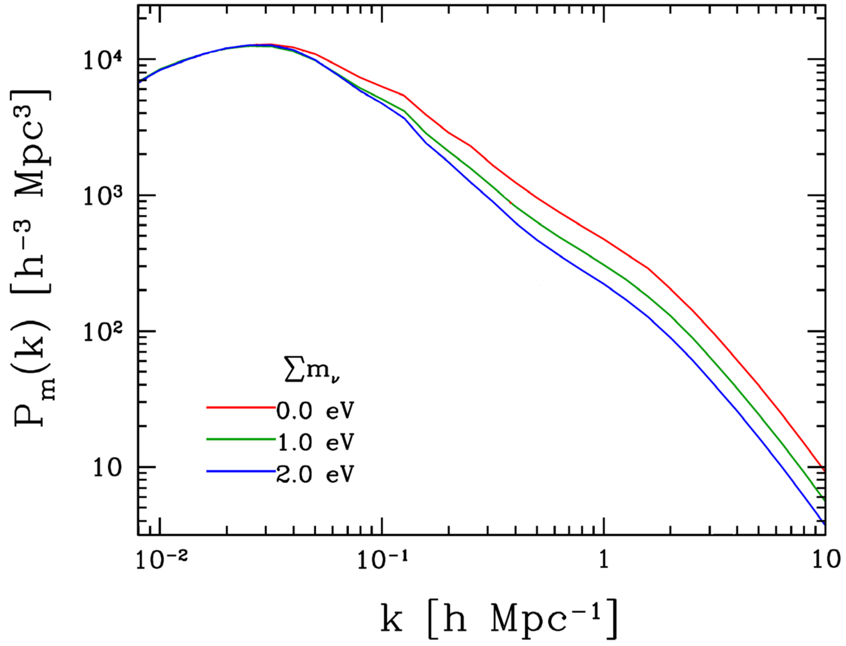
estimation of neutrino mass with
astronomical observation of the large-scale structure

power spectra of the density fluctuation



- non-linear behavior of massive neutrinos in the smaller scales is poorly known.

we need numerical simulations of massive neutrinos to be confronted with observations

N-body simulation ?

- So far, most of numerical simulations of dark matter in the large-scale structure have been performed with the N-body method.

- Can we apply the N-body simulation to massive neutrinos?

- possible drawbacks of N-body simulations
  - statistical sampling of matter distribution in 6D phase space (***x***, ***p***)
  - shot-noise contamination in numerical results, especially in smaller scales
  - not very good at handling collisionless damping, in which matter in the tails of velocity distribution function plays an important role.

- We adopt an alternative approach to the particle-based N-body method to avoid such drawbacks.

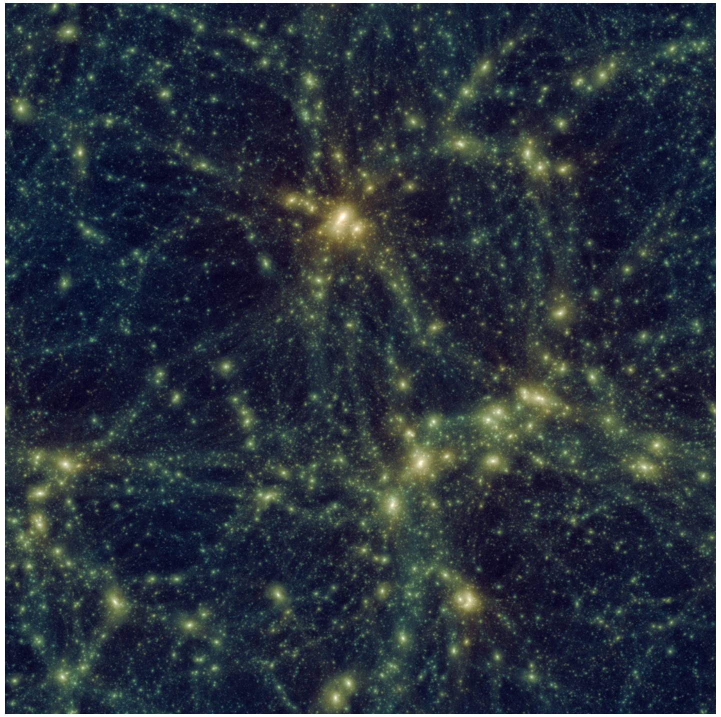
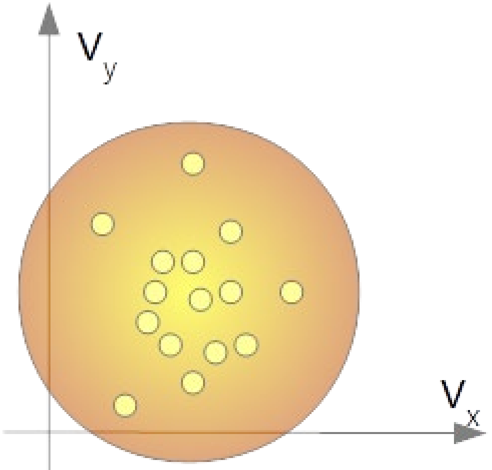


image courtesy of Tomoaki Ishiyama



Vlasov-Poisson simulation

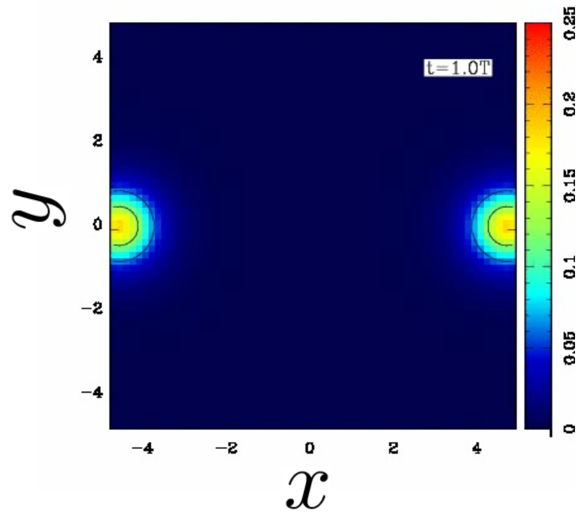
- we directly solve the collisionless Boltzmann equation (aka Vlasov equation) in 6-D phase space (***x***, ***p***) in a finite volume manner.

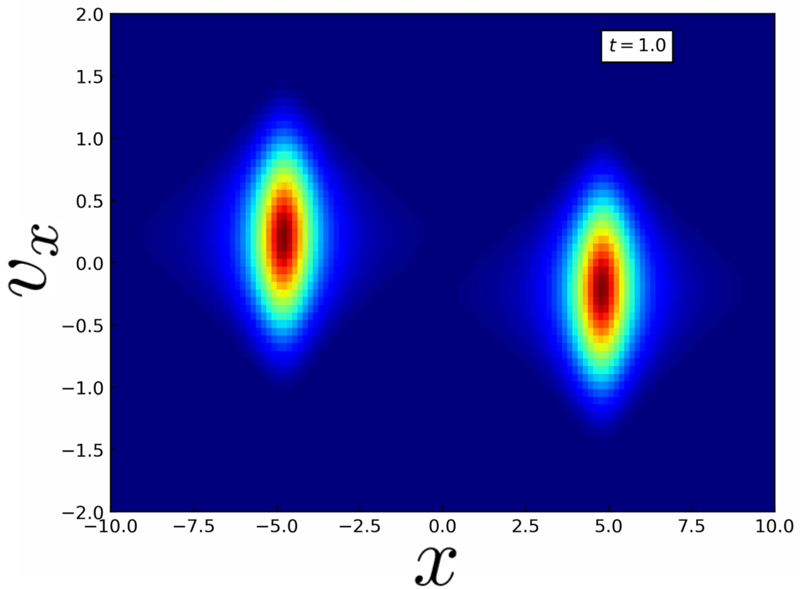
$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \frac{\partial f}{\partial \boldsymbol{x}} - \nabla \phi \cdot \frac{\partial f}{\partial \boldsymbol{v}} = 0 \qquad \nabla^2 \phi = 4\pi G \rho = 4\pi G \int f d^3 \boldsymbol{v}$$

$f(\boldsymbol{x}, \boldsymbol{v}, t)$: density of matter in the 6-Dphase space

- mass distribution as a continuum matter
- free from shot noise contamination
- good at simulating collisionless damping
- requires huge amount of memory and computational cost

- head-on merger of two self-gravitating spheres





Large-scale structure formation with dark matter and massive neutrinos

- hybrid of N-body / Vlasov simulation of cold dark matter and neutrinos

  - "cold" dark matter has very small extension in the velocity space

    ➡ conventional N-body approach is adopted for the cold dark matter.

$$\frac{\mathrm{d}^2 \boldsymbol{x}_i}{\mathrm{d}t^2} + 2H(t)\frac{\mathrm{d}\boldsymbol{x}_i}{\mathrm{d}t} = -\frac{\nabla\phi(\boldsymbol{x}_i)}{a(t)^2}$$

  - neutrinos are followed by Vlasov-Poisson simulation

$$\frac{\partial f}{\partial t} + \frac{\boldsymbol{p}}{a^2}\cdot\frac{\partial f}{\partial \boldsymbol{x}} - \frac{\partial \phi}{\partial \boldsymbol{x}}\cdot\frac{\partial f}{\partial \boldsymbol{p}} = 0$$

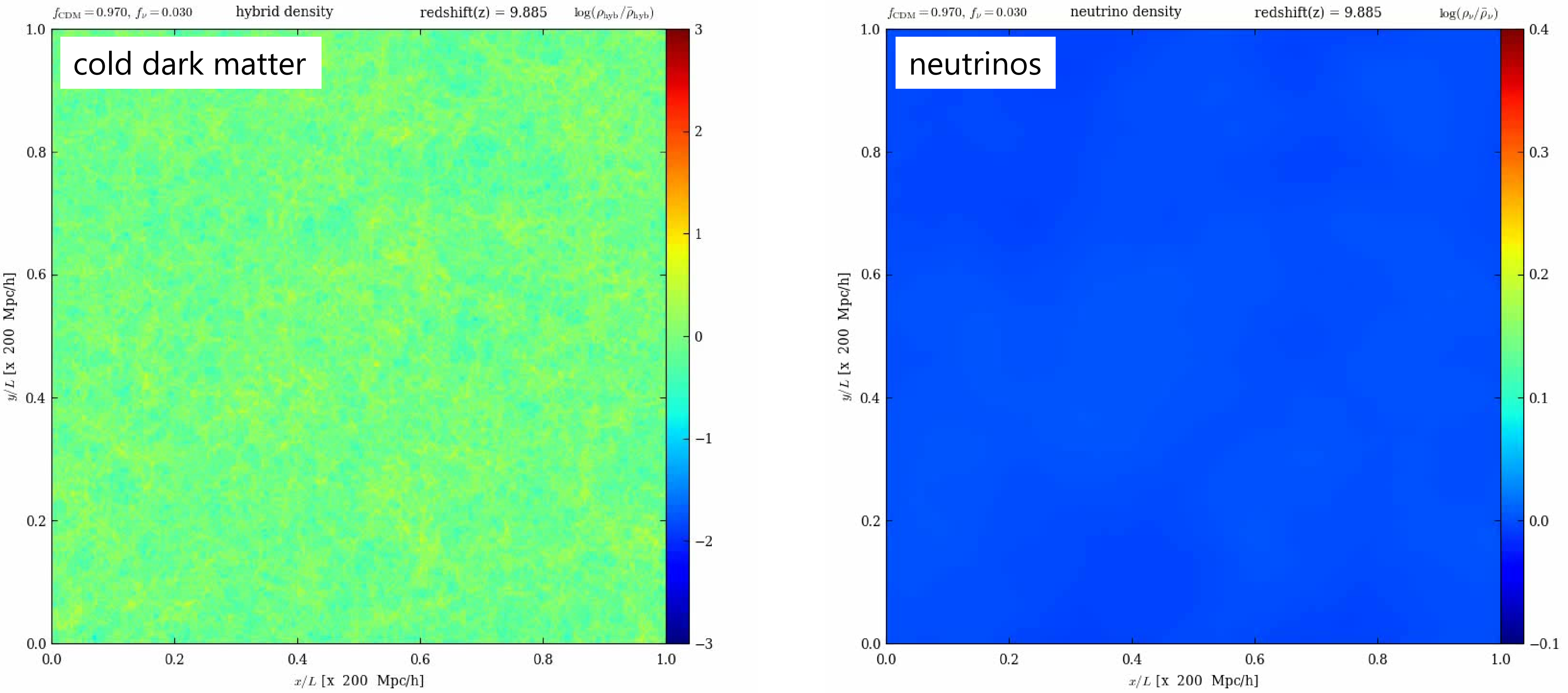
spatially 7th-order SL-MPP7 scheme

Tanaka, Yoshikawa, Minoshima, Yoshida (2017)

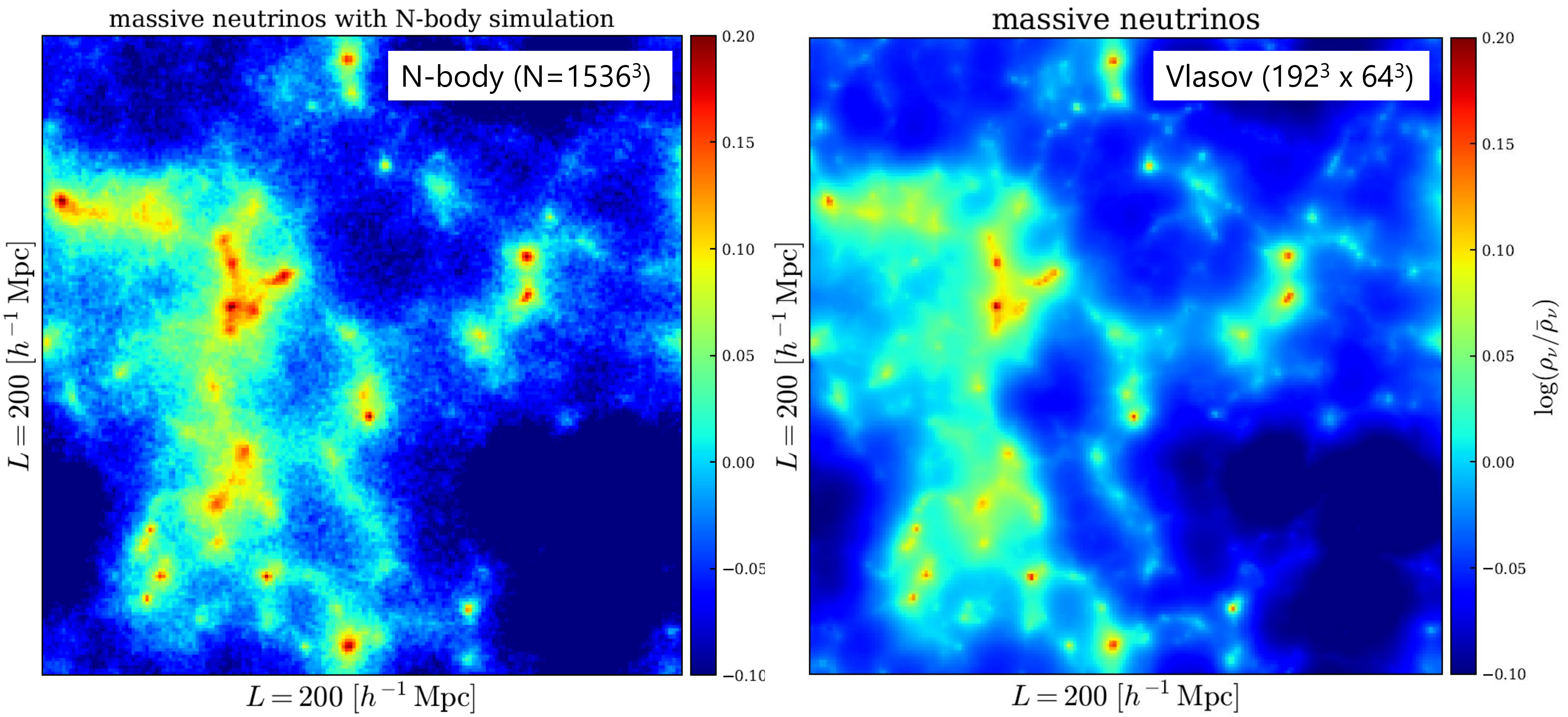
  - both of cold dark matter and neutrinos are subject to the same grav. potential

$$\nabla^2\phi = 4\pi G\bar{\rho}a^2(f_{\mathrm{cdm}}\boxed{\delta_{\mathrm{cdm}}} + f_\nu\boxed{\delta_\nu})$$

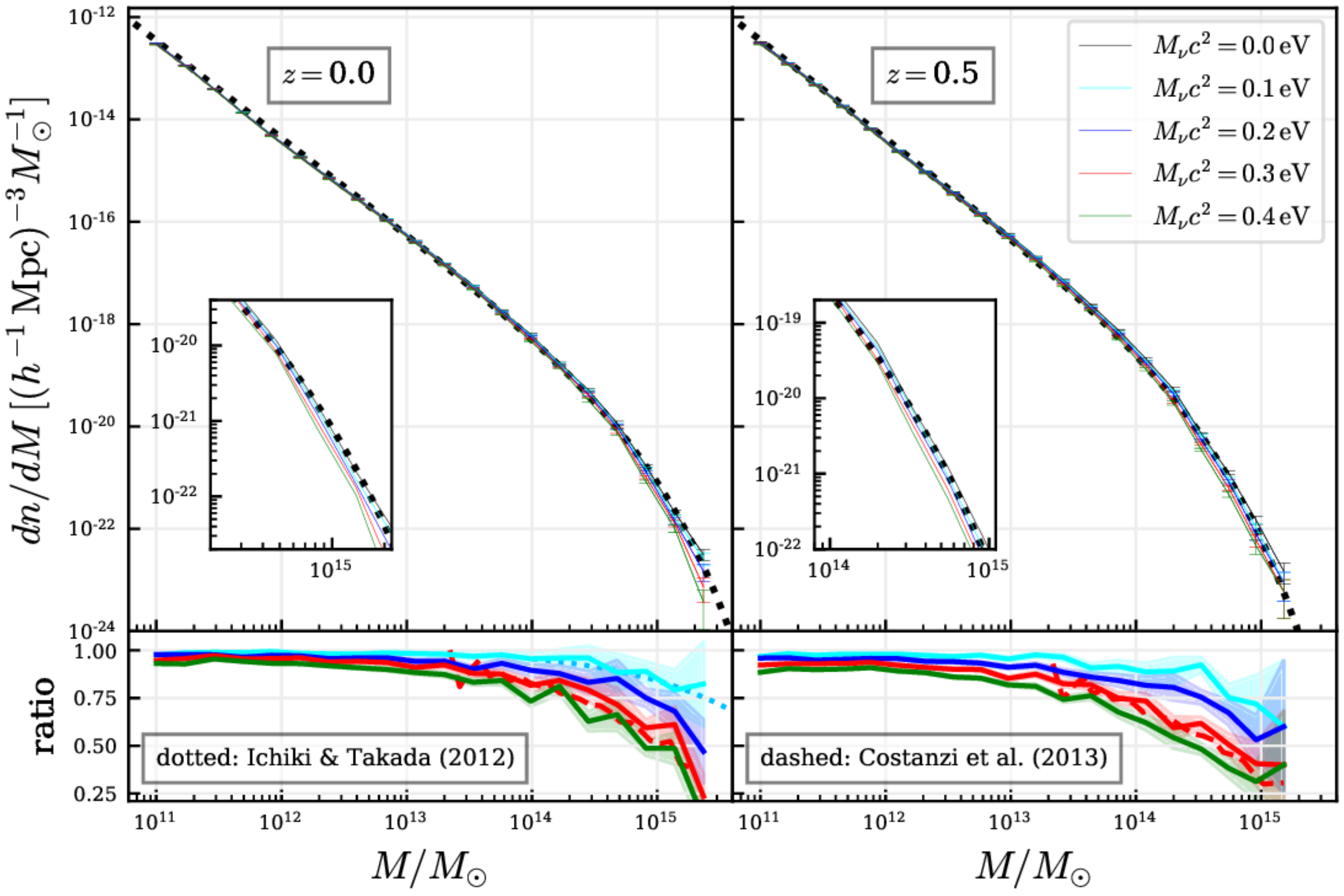
N-body + Vlasov Hybrid Simulation



N-body vs Vlasov

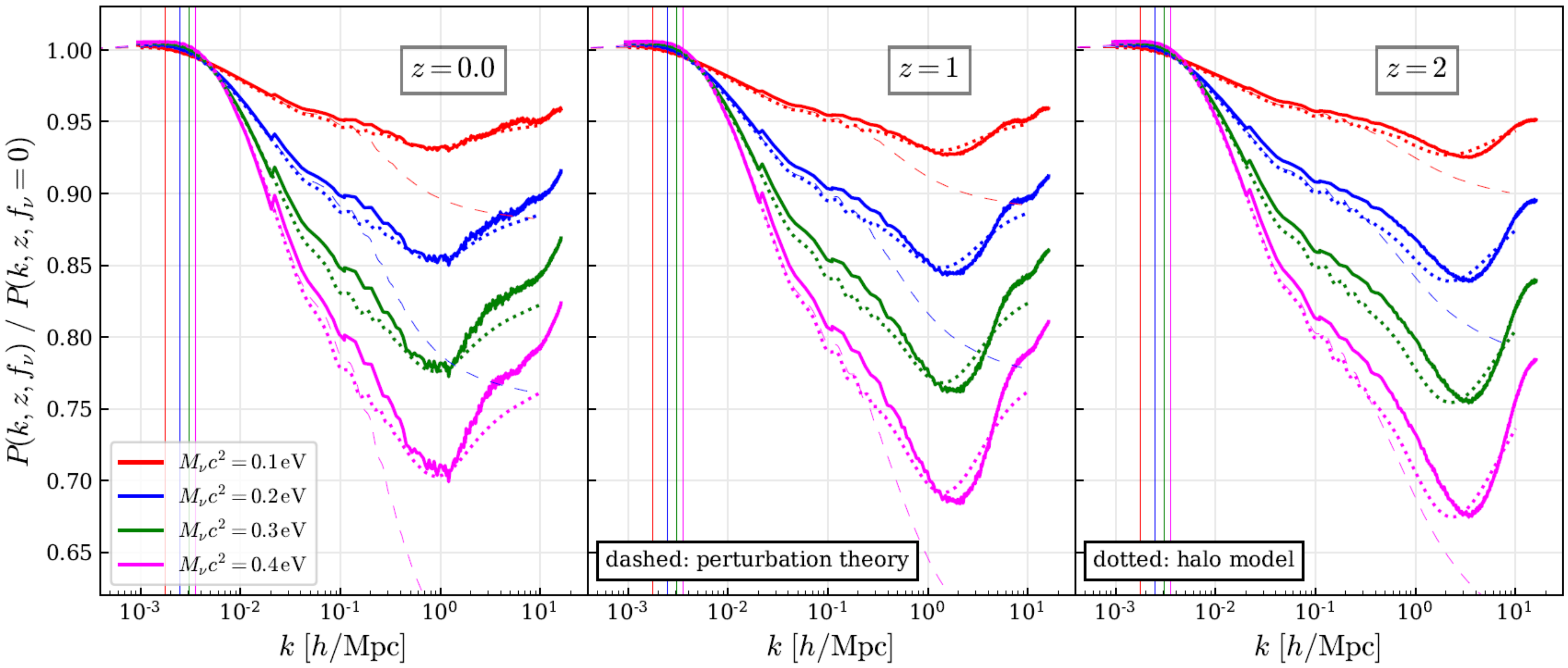


Mass Functions of Dark Matter Halos



- mass distribution of gravitationally bound objects (dark matter halos) in the large-scale structure.
- massive neutrinos suppress the formation of dark matter halos, especially at the scale of galaxy clusters.

Power Spectrum of Density Fluctuation



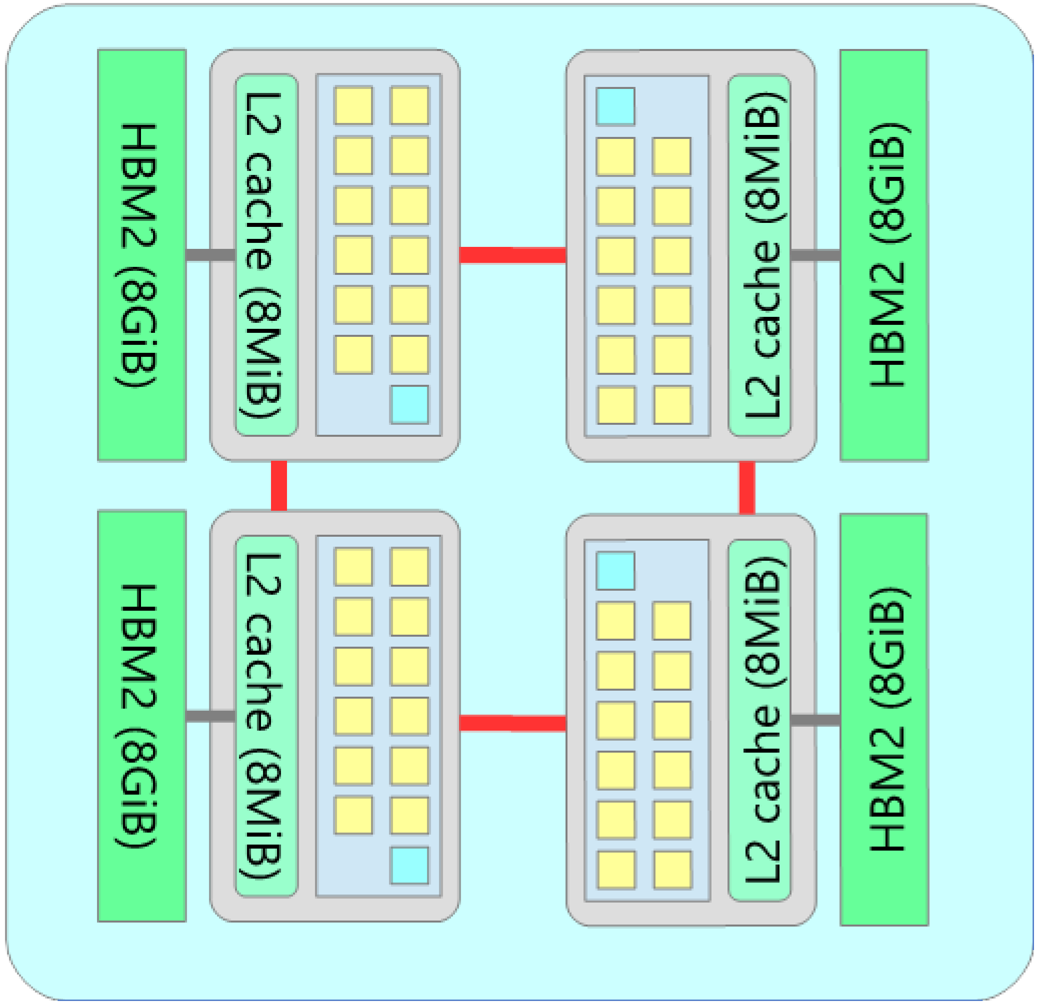
- the power of density fluctuation is also suppressed by the massive neutrinos as expected
- the non-linear features are found in the small scales.

Performance Tuning on FUGAKU

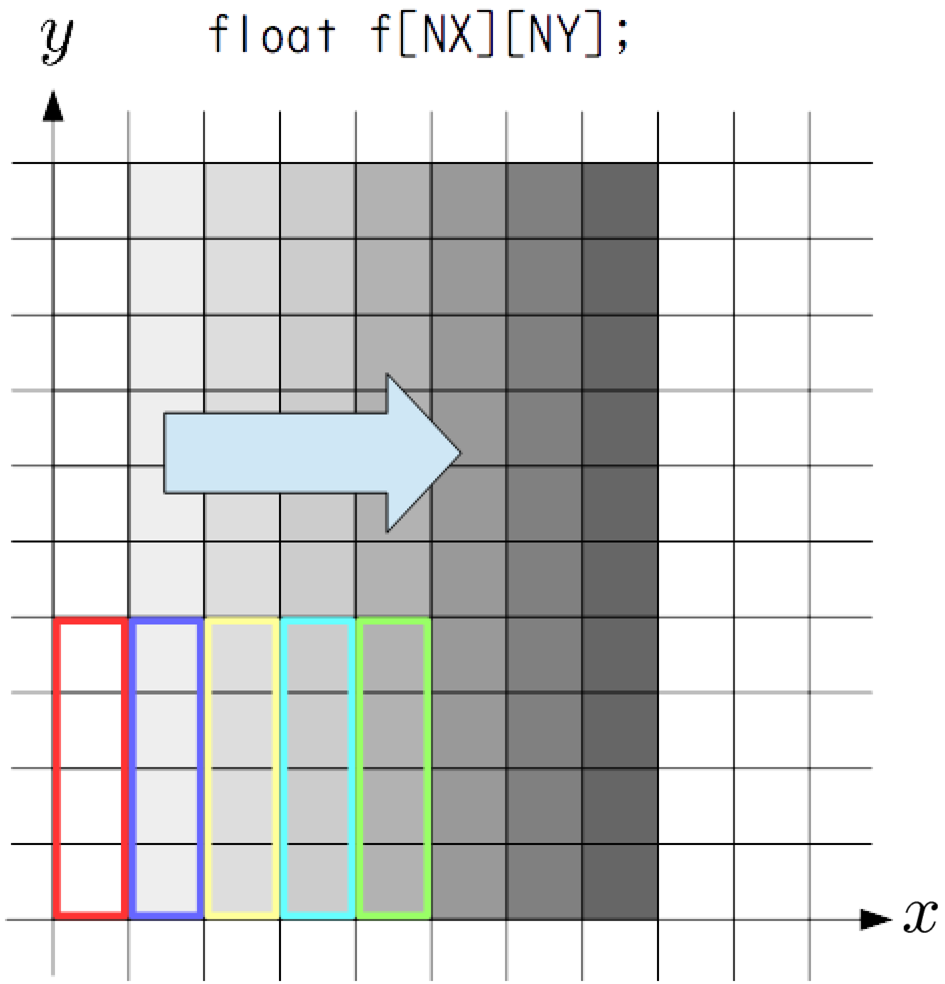
- a A64FX processor composed of four core-memory group (CMG)
  - 12 compute cores + L2C(8MiB) + HBM2(8GiB) / CMG
  - 4 CMGs / node

    ⇓

    48 compute cores + 32GiB HBM2 memory / node
- 512-bit wide SIMD instruction set (SVE)
  - 32 SIMD registers + 16 predicate registers / core
  - 2 FMA units / core
- ACLE (ARM C-Language Extension)
  - set of APIs to exploit SVE instructions in C/C++ languages.



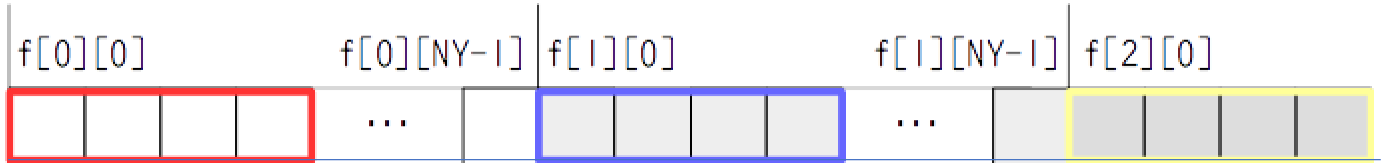
SIMD optimization on the A64FX processor



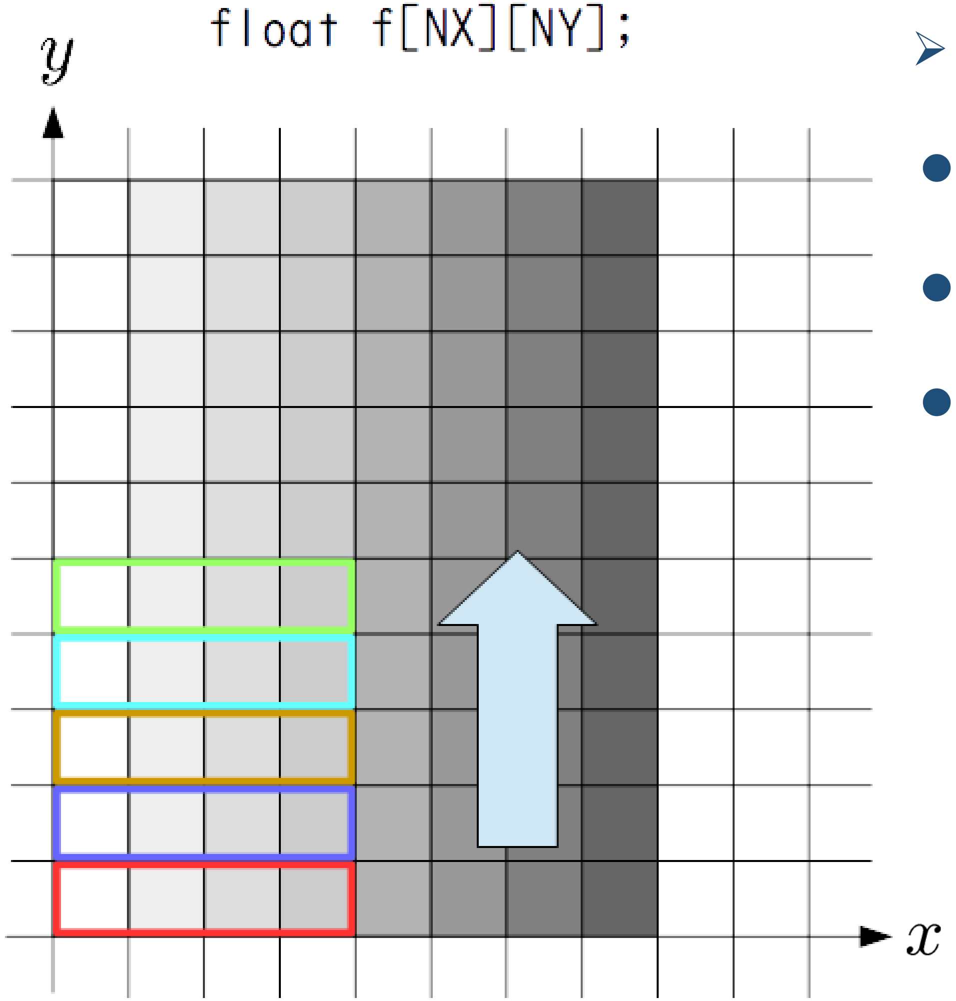
- solving multiple 1-D advection equations with SIMD instructions

$$\frac{\partial f}{\partial t} + v_i \frac{\partial f}{\partial x_i} = 0$$

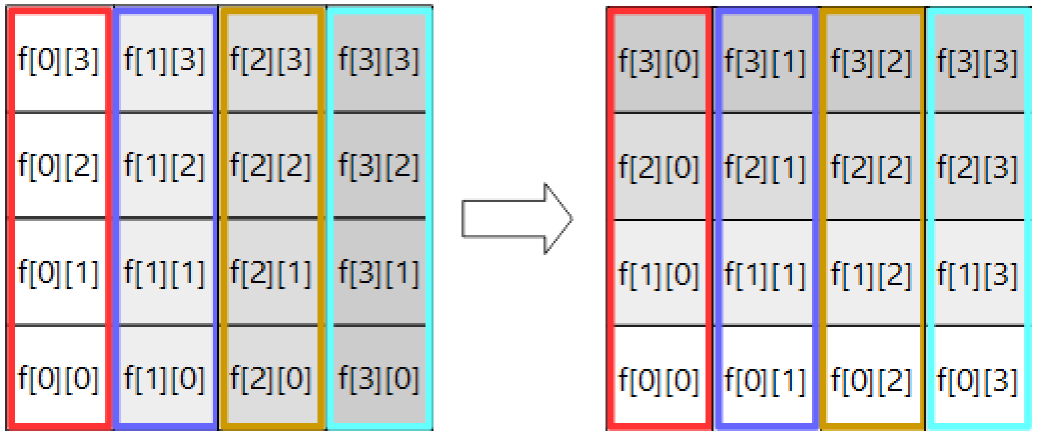
- advection along the x-axis
  - load a sequence of data f[i][j:j+3] with a single instruction
  - solve multiple advection equations along multiple y-coordinates using SIMD instruction set in a straightforward manner

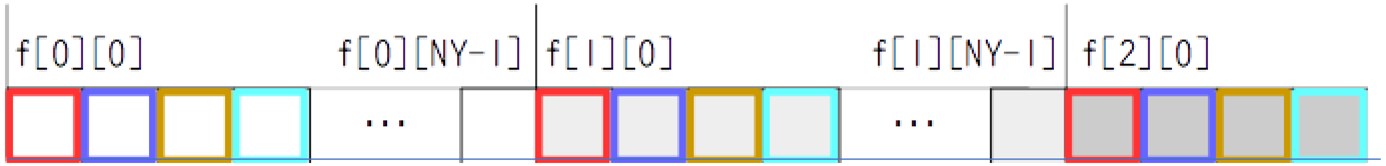


SIMD optimization on the A64FX processor



- ➢ advection along the y-axis
- a set of data along the x-axis is not contiguous in address
- re-arrangement of the data requires lots of memory operations
- transpose of the data on SIMD registers after loading along y-axis





Performance of the Numerical Advection

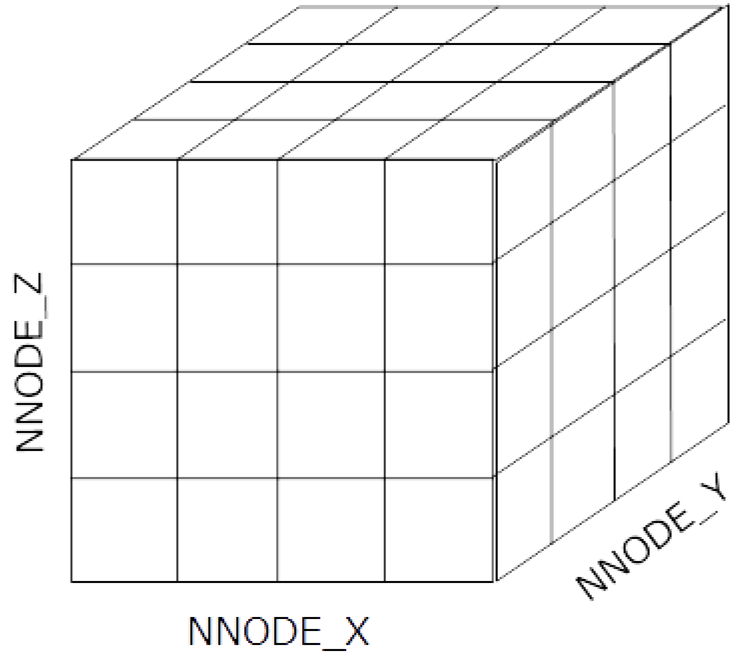
Performance / CMG

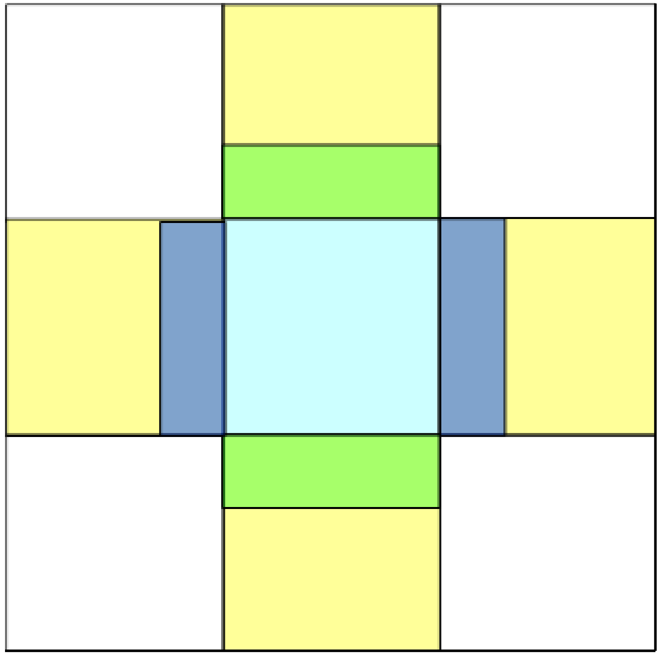
direction	w/o SIMD inst. [Gflops]	w/ SIMD inst. [Gflops]	w/ transpose [Gflops]
x	4.8	176.7	-
y	7.1	233.3	-
z	7.4	17.9	224.2

theoretical peak performance for SP : 1.5Tflops / CMG

- performance of the advection along z-axis is much improved by the transpose scheme.
- performance efficiency w.r.t. the peak performance is 15%

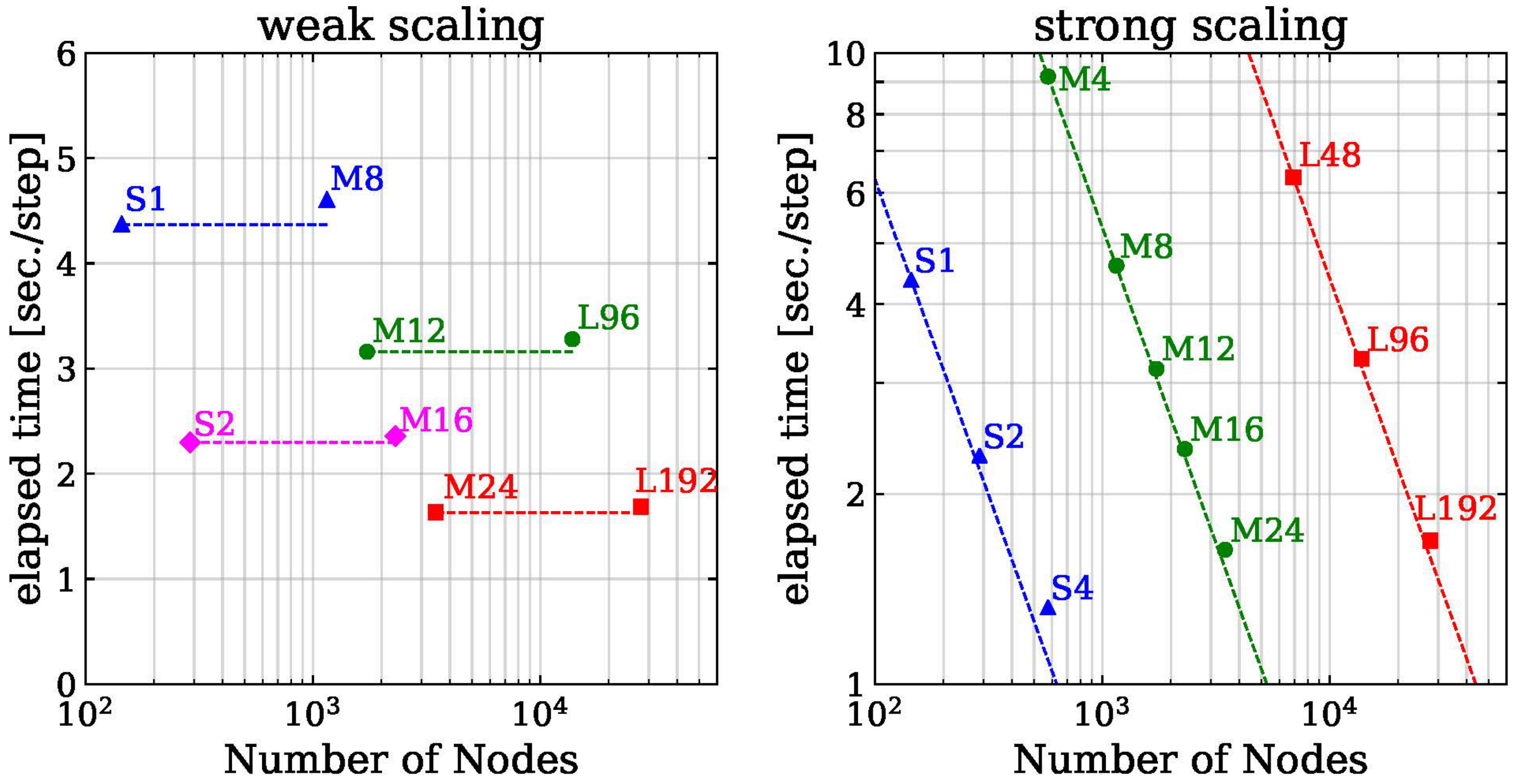
Parallelization





- Only physical (spatial) grids are decomposed among computational nodes.
- Each spatial grids contains the entire velocity (momentum) grids
- Data commnication at the boundary of decomposed domains

Weak and strong scaling efficiencies



Prospects with FUGAKU

- Brief history of 6D Vlasov simulations
  - We launched this project in 2011 and perform 6D Vlasov simulations of self-gravitating objects with 64^3 x 32^3 mesh grids for the first time in the world on T2K-Tsukuba.
  - With Oakforest-PACS system and K-computer, we performed 6D Vlasov simulations of cosmological neutrinos with 128^3 x 64^3 mesh grids in 2017-2019.
  - With 27648 nodes of Fugaku, we conduct 6D Vlasov simulations with 384^3 x 64^3 mesh grids.
- Now, we can perform 6D Vlasov simulations for scientifically meaningful problems
- Further applications of Vlasov simulations
  - dark matter beyond cold dark matter (warm / self-interacting)
  - astrophysical magnetized plasma (magnetic reconnection / collisionless shock / particle acceleration)