Collective mass in quadrupole collective Hamiltonian from local QRPA with Skyrme EDF

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Introduction: Quadrupole shape fluctuations in transitional nuclei

Goal: Bohr Hamiltonian with Skyrme EDF

Method: Finite amplitude method with Skyrme EDF

Result: Rotational moment of inertia by local FAM-QRPA

Result: Attempt to vibrational mass by local FAM-QRPA

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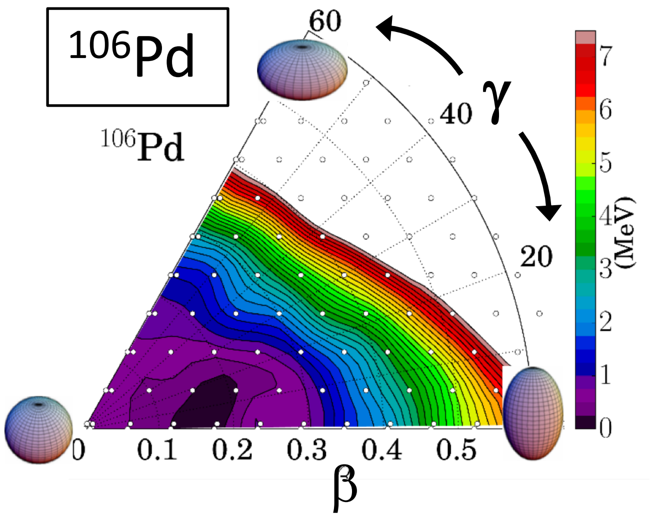
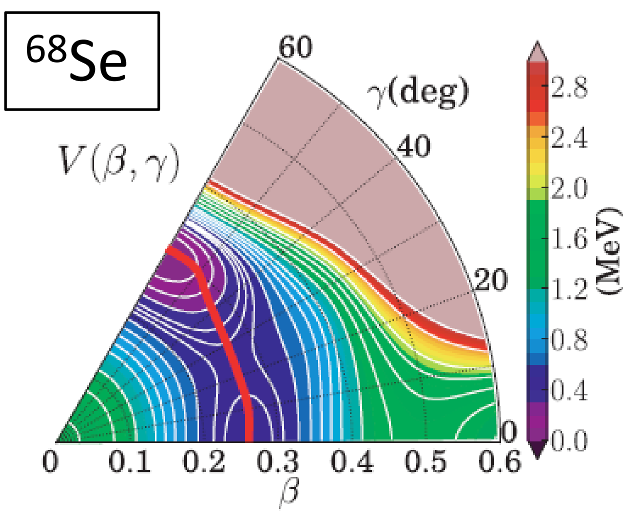
Shape fluctuations in transitional nuclei





Hinohara et al., PRC82 (2010) 064313

Potential is flat (V < 2 MeV) at large region

➡ **Shape fluctuation**

Two minima at oblate and prolate region

➡ **Shape coexistence**

Go **beyond mean field** to describe shape fluctuation

5D quadrupole collective (Bohr) Hamiltonian

Bohr Hamiltonian

$$\mathcal{H} = T_{\mathrm{vib}} + T_{\mathrm{rot}} + V(\beta, \gamma)$$

$$T_{\mathrm{vib}} = \frac{1}{2} D_{\beta\beta}(\beta, \gamma)\dot{\beta}^2 + D_{\beta\gamma}(\beta, \gamma)\dot{\beta}\dot{\gamma} + \frac{1}{2} D_{\gamma\gamma}(\beta, \gamma)\dot{\gamma}^2$$

$$T_{\mathrm{rot}} = \frac{1}{2} \sum_{k=1}^{3} \mathcal{J}_k(\beta, \gamma)\omega_k^2$$

Vibrational mass

Rotational moment of inertia

5D quadrupole collective (Bohr) Hamiltonian

HFB (V) + **Local QRPA** ($D_{\beta\beta},\ D_{\beta\gamma},\ D_{\gamma\gamma},\ \mathcal{J}_k$) with **P+Q force**

Hinohara et al., PRC82 (2010) 064313, 84 (2011) 061302, 85 (2012) 024323

HFB (V) + **Cranking approx. mass** ($D_{\beta\beta},\ D_{\beta\gamma},\ D_{\gamma\gamma},\ \mathcal{J}_k$)

with **Energy Density Functional** (Skyrme, Gogny, Relativistic)

Prochniak et al., NPA730 (2004) 59
Niksic et at., PRC79 (2009) 034303
Delaroche et al., PRC81 (2010) 014303, etc.

Goal: Combine the two approaches

HFB (V) + **Local QRPA** ($D_{\beta\beta},\ D_{\beta\gamma},\ D_{\gamma\gamma},\ \mathcal{J}_k$)

with **Skyrme EDF**

Aim of this talk

To construct 5D quadrupole collective Hamiltonian,

HFB (V) + Local QRPA ($D_{\beta\beta},\ D_{\beta\gamma},\ D_{\gamma\gamma},\ \mathcal{J}_k$), with Skyrme EDF

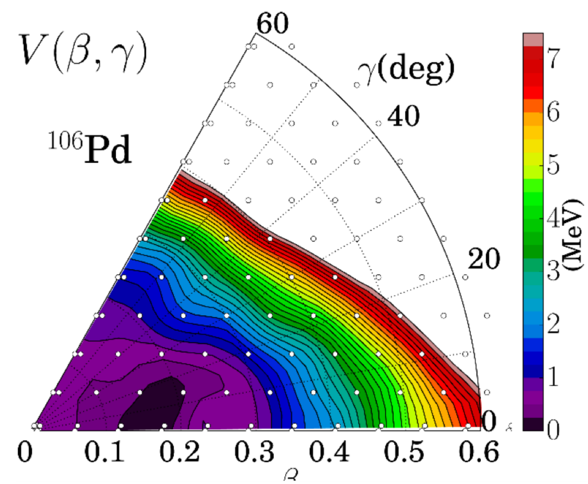
Step 1: Construct Skyrme QRPA for triaxial shapes

Finite amplitude method (FAM)

Step 2: Local FAM+QRPA at each β, γ

⇒ Collective mass

$$D_{\mu\nu}(\beta,\gamma),\ \ \mathcal{J}_k(\beta,\gamma)$$



Finite amplitude Method (FAM)

Nakatsukasa et al., PRC76 (2007) 024318
Avogadro & Nakatsukasa, PRC84(2011)014314
Stoitsov et al., PRC84 (2011) 041305
Liang et al., PRC87 (2013) 054310
Niksic et al., PRC88 (2013) 044327
Pei et al., PRC90 (2014) 051304
Kortelainen et al., PRC92 (2015) 015302

Linear response TDDFT

$$(E_\mu + E_\nu - \omega) X_{\mu\nu}(\omega) + \delta H^{20}_{\mu\nu}(\omega) = -F^{20}_{\mu\nu}$$
$$(E_\mu + E_\nu + \omega) Y_{\mu\nu}(\omega) + \delta H^{02}_{\mu\nu}(\omega) = -F^{02}_{\mu\nu}$$

$$\delta H_{\mu\nu} = \frac{\partial H_{\mu\nu}}{\partial \mathcal{R}_{\alpha\beta}} \partial \mathcal{R}_{\alpha\beta} \qquad \begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \omega \begin{pmatrix} X \\ Y \end{pmatrix}$$

- Residual part → finite difference
- No need of QRPA (*A,B)* matrices

QRPA*: N*^{2} *x N*^{2} matrix

FAM: *N x N* matrix

- FAM code from HF(B) code
- Equivalent to (Q)RPA

- iteratively solved at each ω

$$X, Y \to \delta\mathcal{R} \to \delta H^{20,02} \to \text{new } X, Y \ (\text{at fixed } \omega)$$

(modified Broyden method)

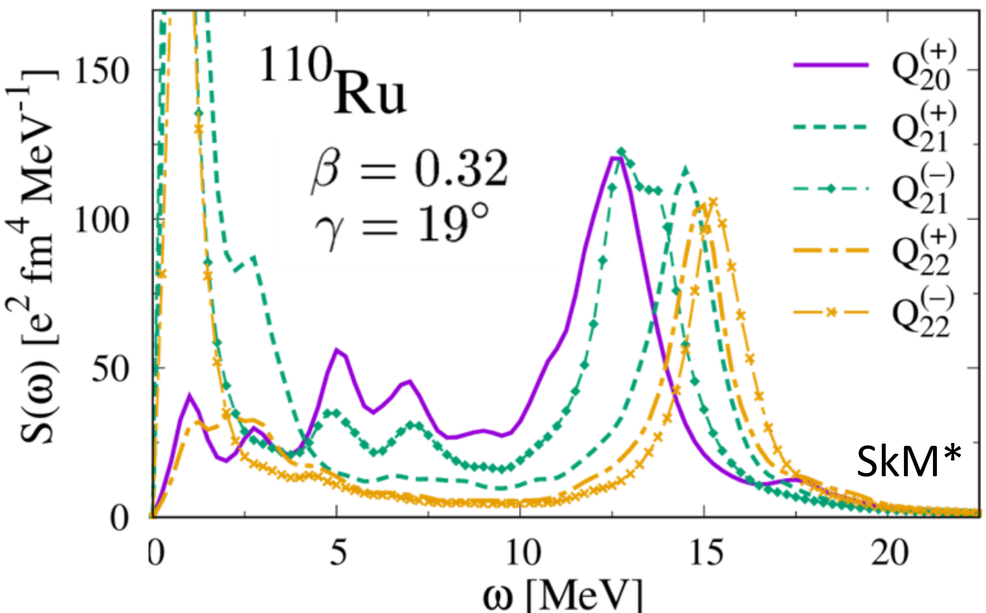
Result: Triaxial, superfluid nucleus

Based on 3D Cartesian coordinate HFB (cr8 code), SkM*, volume pairing

$$S(\omega) = -\frac{1}{\pi}\mathrm{Im}\left(\sum_{\mu<\nu} F^{20*}_{\mu\nu} X_{\mu\nu}(\omega) + F^{02*}_{\mu\nu} Y_{\mu\nu}(\omega)\right) \qquad \omega \to \omega + i\gamma$$

γ = 0.5 MeV

Isoscalar quadrupole response



KW, Nakatsukasa,
PRC96, 041304(R) (2017)

- Five different strength functions
- Three spurious rotations (x, y, z)

Computation:

15 min/ω (16threads)

3.5 GB memory

Numerical set up

^{110}Ru: 17^3 mesh, R_{max}=14.0fm, 1120 HF states

Aim of this talk

To construct 5D quadrupole collective Hamiltonian with

Skyrme EDF

Step 1: Construct Skyrme QRPA for triaxial shapes

Finite amplitude method (FAM)

Step 2: Local FAM+QRPA at each β, γ

⇒ Collective mass

$$D_{\mu\nu}(\beta,\gamma),\ \mathcal{J}_k(\beta,\gamma)$$

Moment of inertia from FAM

$$T_{\rm rot} = \frac{1}{2}\sum_{k=1}^{3}\mathcal{J}_k(\beta,\gamma)\omega_k^2$$

Spurious modes of QRPA ⟺ Nambu-Goldstone (NG) modes

⇧

Mean field breaks symmetries (translational, rotational etc.)

Relation between Thouless-Valatin inertia (M_{NG}) and FAM

$$S^{\rm FAM}(\hat{P}_{\rm NG},\omega=0) = \sum_{\mu<\nu} P^{20*}_{\mu\nu}X_{\mu\nu}(0) + P^{02*}_{\mu\nu}Y_{\mu\nu}(0)$$
$$= -M_{\rm NG}$$

Hinohara, PRC92(2015)034321

$\hat{P}_{\rm NG} = \hat{J}_k,\ M_{\rm NG} = \mathcal{J}_k^{\rm TV}$ for rotational moment of inertia

Strength at only ω = 0 ➡ Small computations

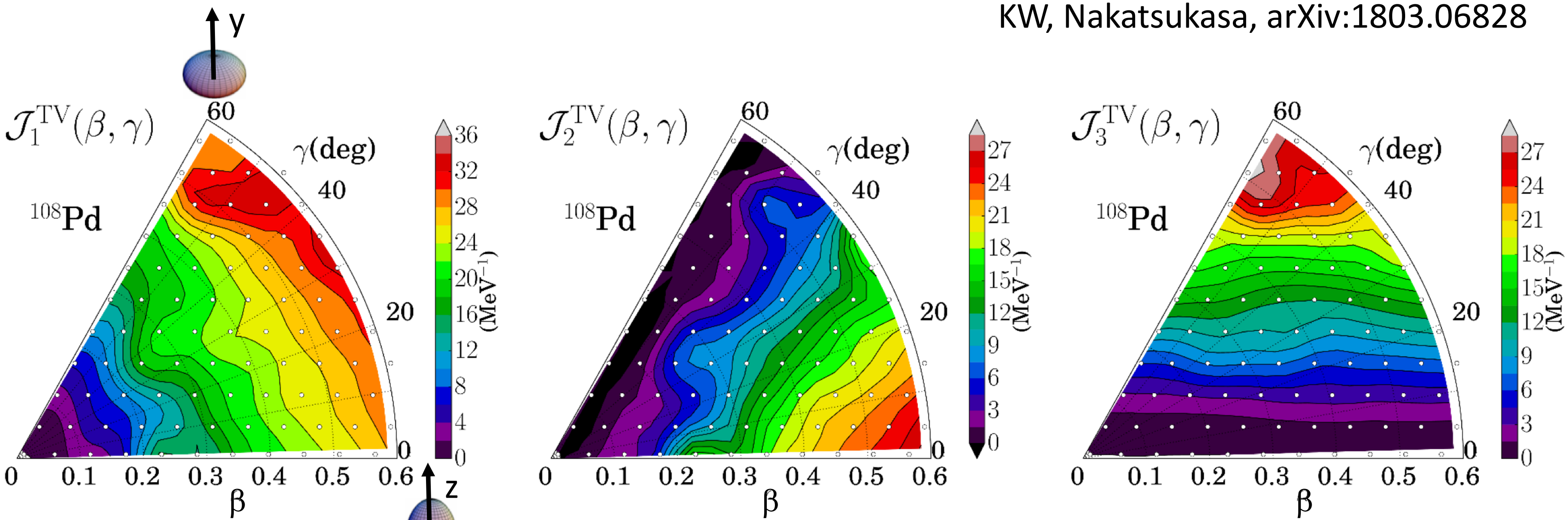
(a few minutes with 16 threads for one β–γ point)

Application to axially deformed nuclei

Petrik, Kortelainen, PRC97(2018)034321

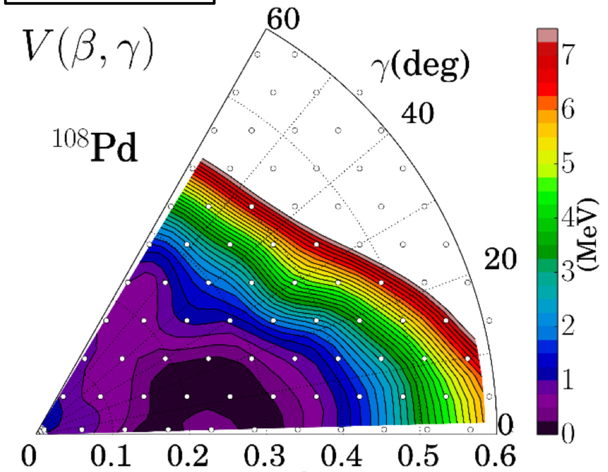
Result: Moment of inertia on β, γ plane

KW, Nakatsukasa, arXiv:1803.06828



SkM*

Potential



At β–γ 84 points (white dots), constrained HFB+ local FAM-QRPA with interpolation in between

Numerical set up

17^3 mesh, R_{max}=14.0fm, 1120 HF states, volume pairing

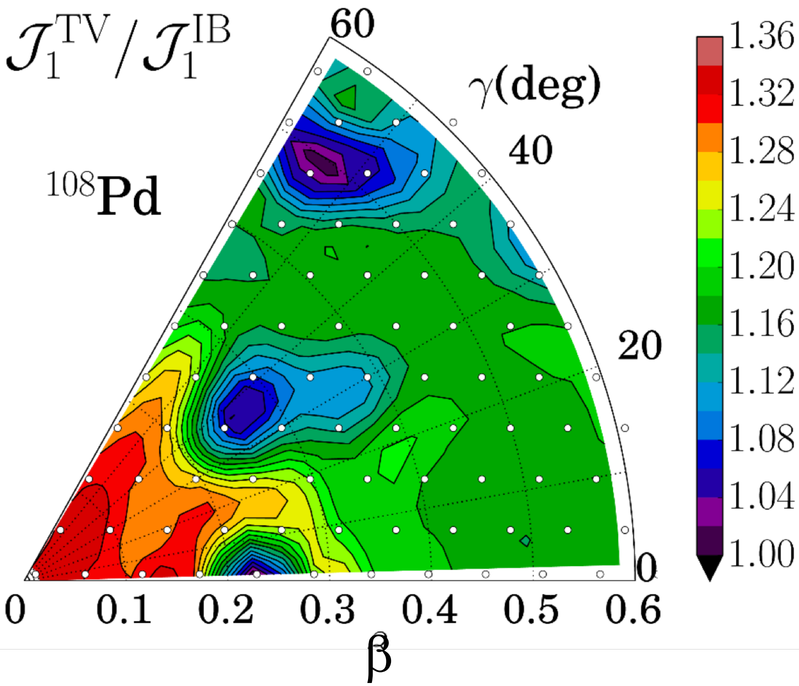
Importance of residual interaction in QRPA

Thouless-Valatin moment of inertia ⟵ With residual interaction

vs.

Inglis-Belyaev moment of inertia ⟵ Without residual interaction

(Cranking mass)



- Moment of inertia is increased by the residual interaction $\mathcal{J}^{TV}/\mathcal{J}^{IB} > 1$
- β–γ dependence is important
- Use of J^{IB} or constant factor on J^{IB} is not enough

KW, Nakatsukasa, arXiv:1803.06828

Vibrational mass $D_{\beta\beta}$, $D_{\beta\gamma}$, $D_{\gamma\gamma}$

QRPA for vibrational mass at each β, γ

Hinohara et al., PRC82 (2010) 064313

$$\delta\langle\phi(\beta,\gamma)|[\hat{H}_{\mathrm{CHFB}}(\beta,\gamma),\hat{Q}^i(\beta,\gamma)]-\frac{1}{i}\hat{P}^i(\beta,\gamma)|\phi(\beta,\gamma)\rangle=0$$

$$\delta\langle\phi(\beta,\gamma)|[\hat{H}_{\mathrm{CHFB}}(\beta,\gamma),\frac{1}{i}\hat{P}^i(\beta,\gamma)]-C_i(\beta,\gamma)\hat{Q}^i(\beta,\gamma)|\phi(\beta,\gamma)\rangle=0$$

➡ $Q^i_{\mu\nu},\ P^i_{\mu\nu},\ C_i=\Omega_i^2$ Low-lying discrete QRPA solutions

With FAM, how can we obtain discrete QRPA solutions?

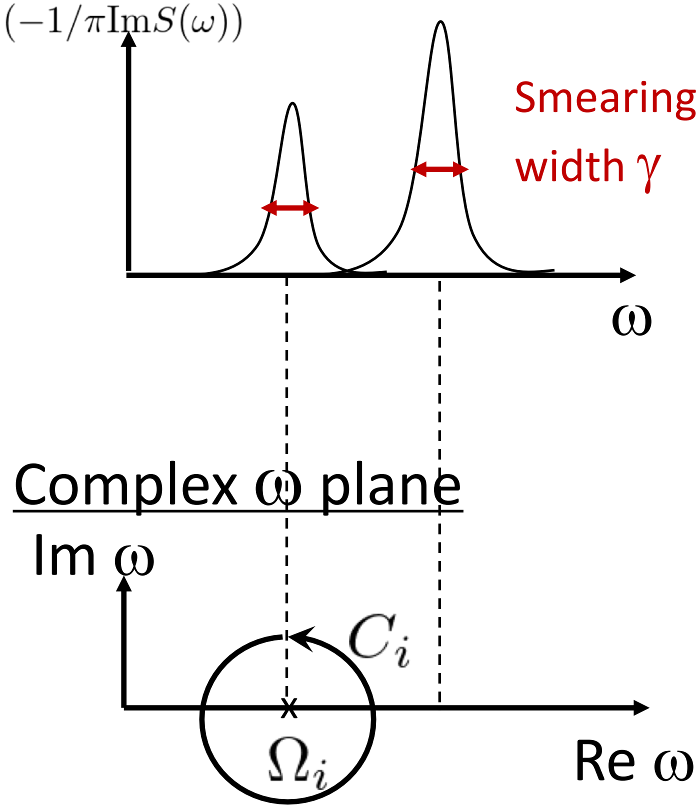
Standard FAM needs spreading width γ

Low-lying discrete states from FAM

Contour integral approach in complex ω plane

Hinohara et al., PRC87(2013)064309

FAM Strength



FAM strength

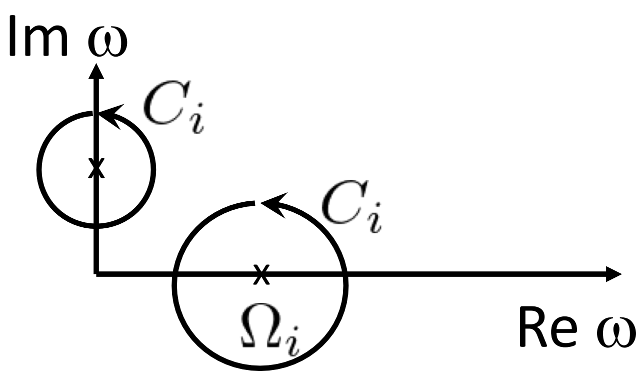
QRPA strength

$$S(\hat{F},\omega) = -\sum_i \left(\frac{|\langle i|\hat{F}|0\rangle|^2}{\Omega_i - \omega} + \frac{|\langle 0|\hat{F}|i\rangle|^2}{\Omega_i + \omega} \right)$$

$$\frac{1}{2\pi i}\oint_{C_i} d\omega S(\hat{F},\omega) = |\langle i|\hat{F}|0\rangle|^2$$

Contour integral in PQ representation

Local QRPA can have both real & imaginary solutions



PQ representation of FAM

Hinohara, PRC92(2015)034321

Hinohara, private communication

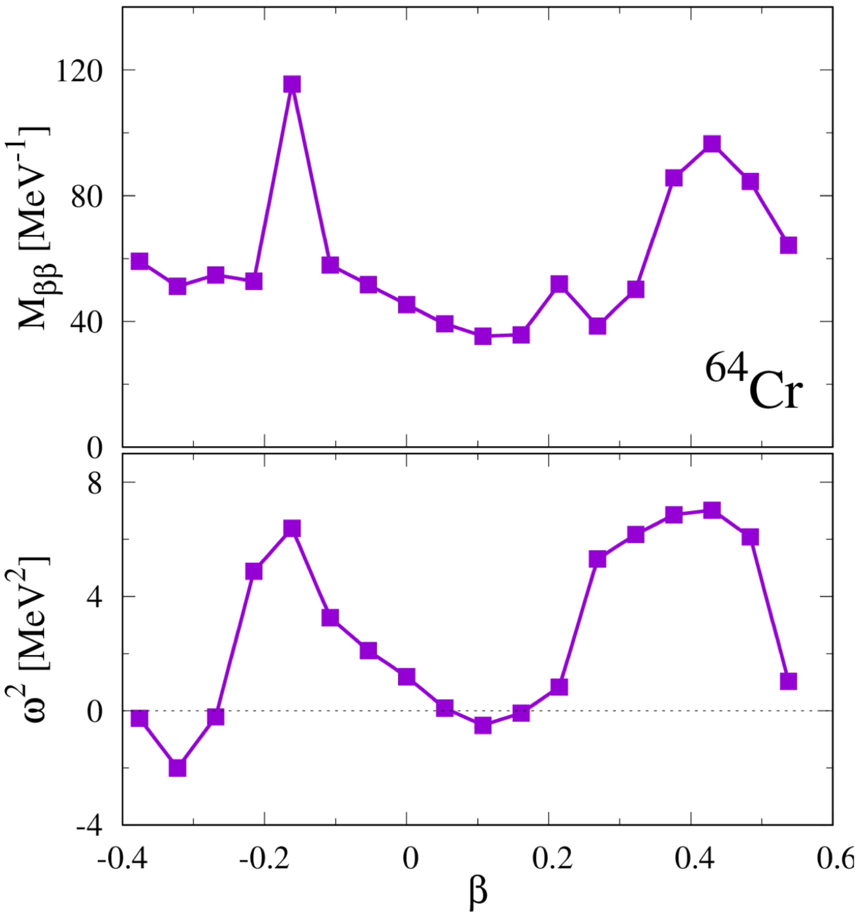
$$\frac{1}{2\pi i}\oint_{C_i} d\omega\, \omega S(\hat{F},\omega) = \frac{1}{2}|\langle P_i|\hat{F}|0\rangle|^2$$

$$\frac{\partial F}{\partial q^i} = \langle[\hat{F},\frac{1}{i}\hat{P}_i]\rangle = i\sum_{\mu<\nu} P^*_{i\mu\nu}F^{20}_{\mu\nu} - P_{i\mu\nu}F^{02}_{\mu\nu} = i\langle P_i|F|0\rangle$$

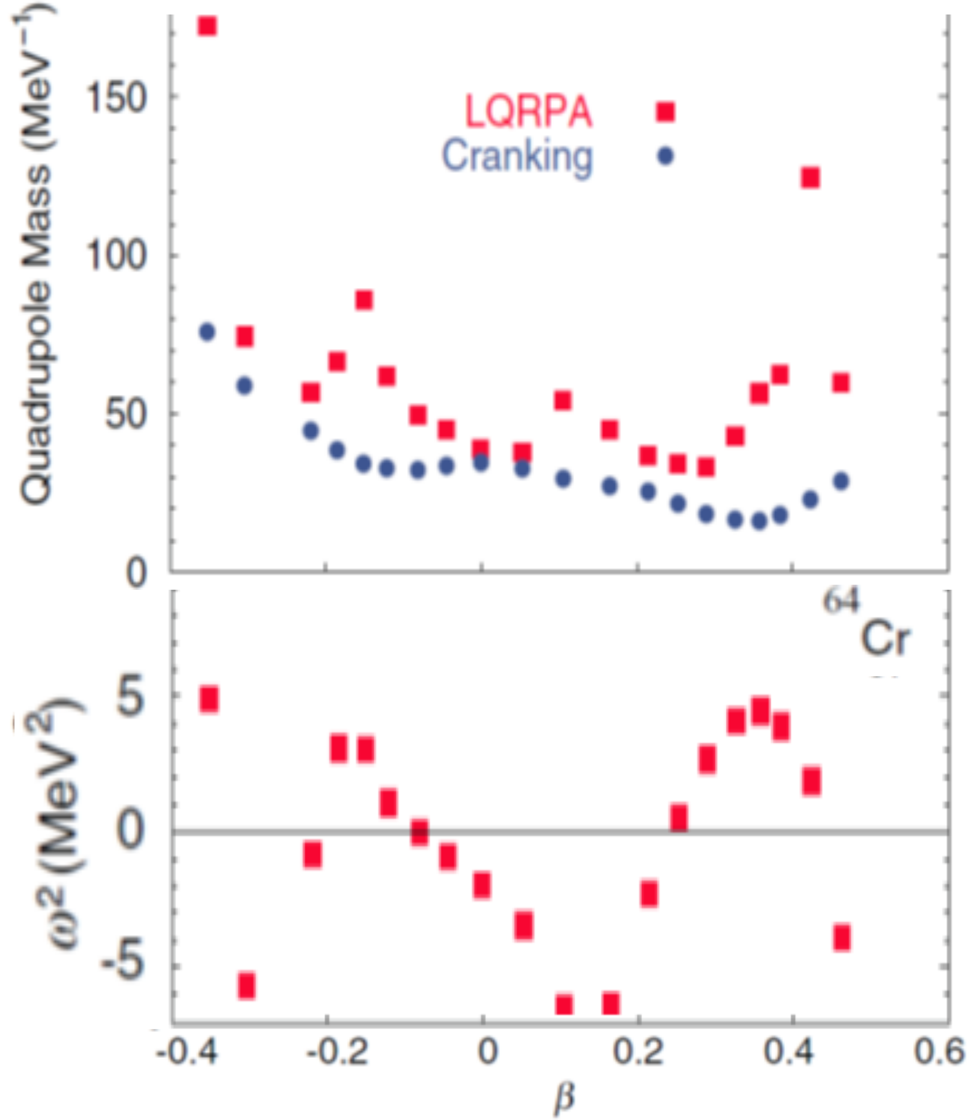
$$M_{mn}(\beta,\gamma) = \sum_{i=1,2}\frac{\partial q^i}{\partial Q_{2m}}\frac{\partial q^i}{\partial Q_{2n}} \qquad F = Q_{2m}$$

Vibrational mass from FAM

Test: Mass along β, axial case



Yoshida, Hinohara, PRC83, 061302(2011)



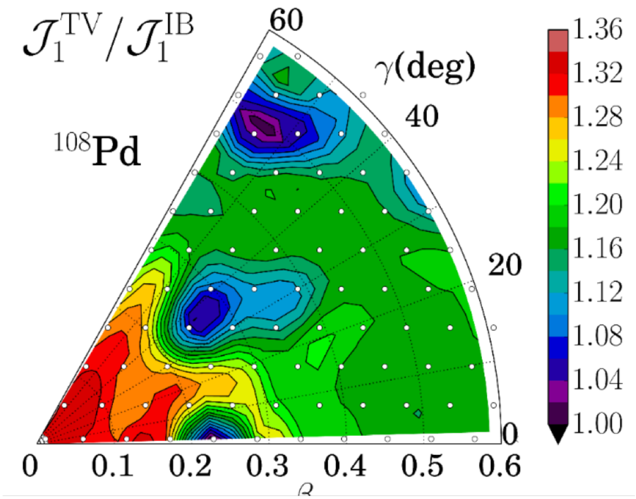
Summary

Quadrupole shape fluctuation

Goal: to construct Bohr Hamiltonian with full Skyrme EDF

3D FAM with Skyrme EDF for triaxial superfluid nuclei

Moment of inertia and vibrational mass
  by Local FAM+QRPA



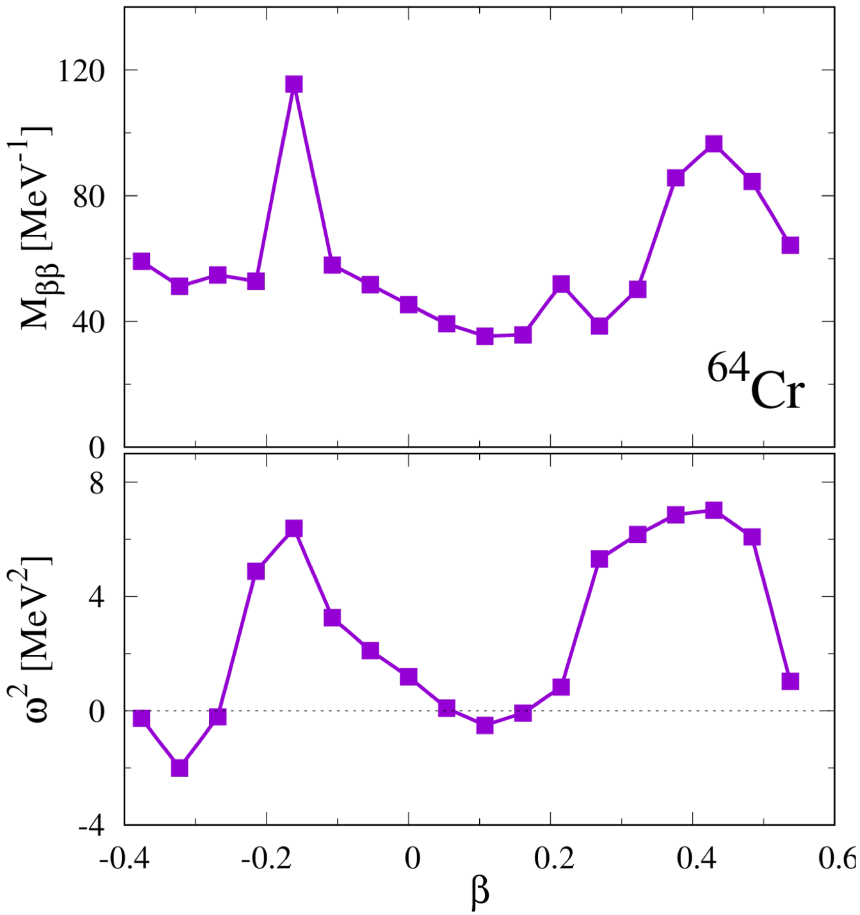
Future plan

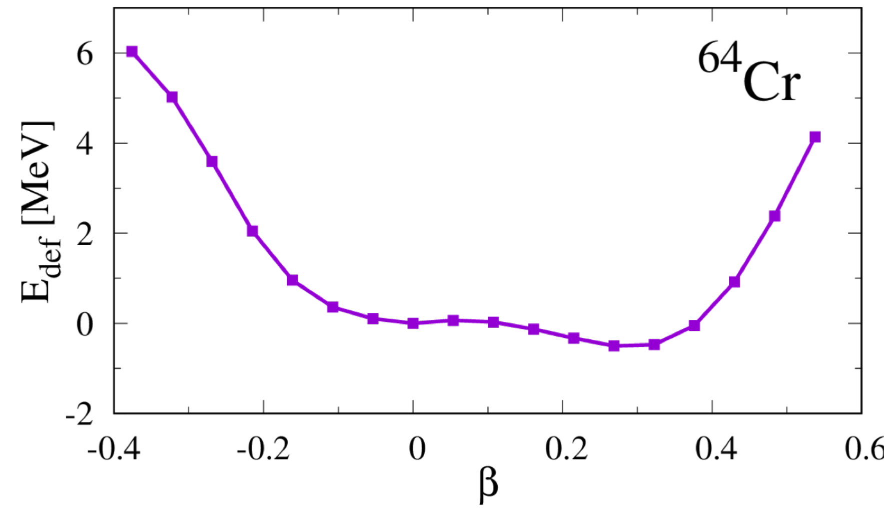
Local FAM+QRPA $D_{\beta\beta},\ D_{\beta\gamma},\ D_{\gamma\gamma}$

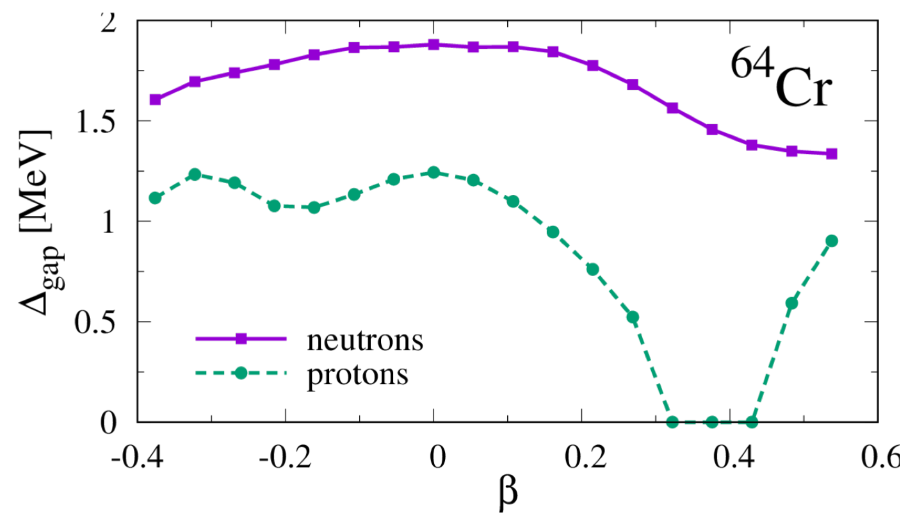
5D quadrupole collective (Bohr) Hamiltonian

Vibrational mass from FAM

Test: Mass along β, axial case







Test of Hinohara method 2 to 3D FAM

Lowest-lying (discrete) state in Yb isotopes

Isoscalar quadrupole K=0

	Ours $\lvert\langle i\lvert F\rvert 0\rangle\rvert^2$ (e^2fm^4)	Ours Ω_i (MeV)	Hinohara et al. $\lvert\langle i\lvert F\rvert 0\rangle\rvert^2$ (e^2fm^4)	Hinohara et al. Ω_i (MeV)
^{166}Yb	269.98	1.658	299.85	1.422
^{168}Yb	131.13	1.922	160.13	1.747
^{170}Yb	150.99	1.918		
^{172}Yb	90.38	1.529	93.27	1.306

Hinohara et al., PRC87.064309(2013)

ISQ K=2

	Ours B(E2) (e^2fm^4)	Ours Ω_i (MeV)	Terasaki & Engel B(E2) (e^2fm^4)	Terasaki & Engel Ω_i (MeV)
^{166}Yb	1169	1.326	1000	1.354
^{168}Yb	940	1.263	740	1.303
^{170}Yb	626	1.632	452	1.674
^{172}Yb	586	2.187	425	2.348

Terasaki & Engel, PRC84,014322(2011)