β-decay properties and Gamow-Teller strength distributions studied with shell-model calculations

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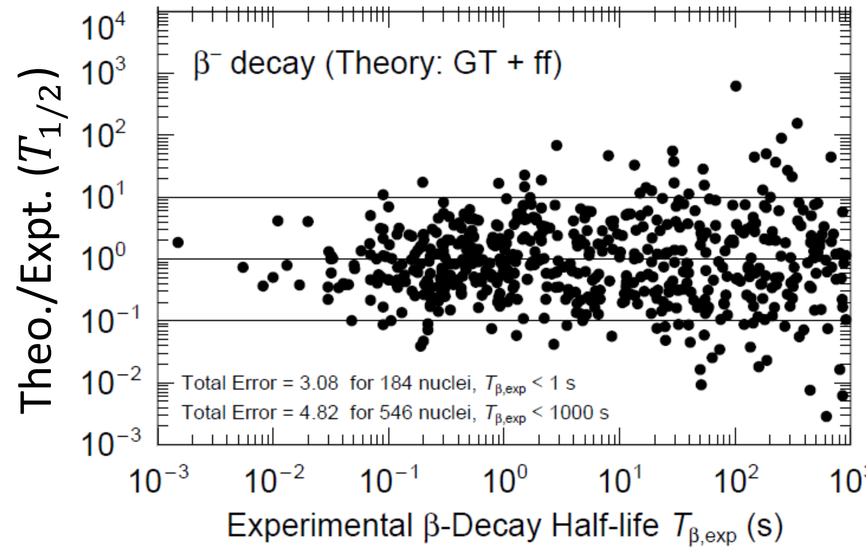
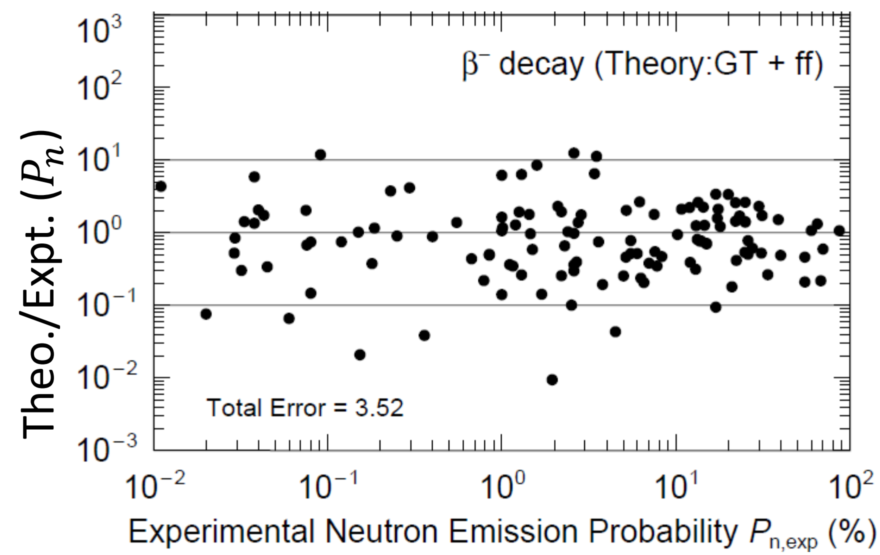
Aim of this study

- β-decay half-lives and delayed neutron emission probabilities
  - Strong need for applications
- Usual systematic calculations
  - QRPA: room for improvement
- Present study: S. Yoshida, Y. Utsuno et al., Phys. Rev. C 97, 054321 (2018).
  - To carry out systematic β-decay studies for $13 \le Z \le 18$, $22 \le N \le 34$
  - To demonstrate the descriptive power of the shell model
  - To discuss the systematics of Gamow-Teller distributions in n-rich nuclei

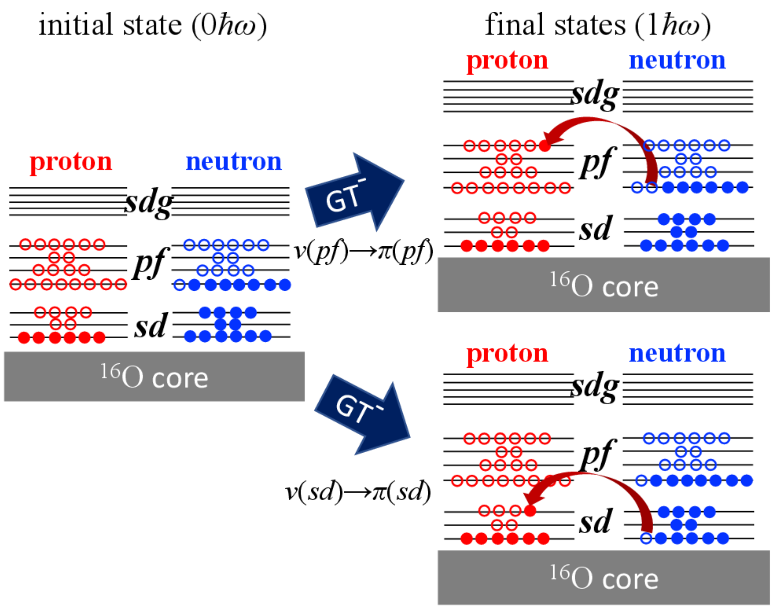


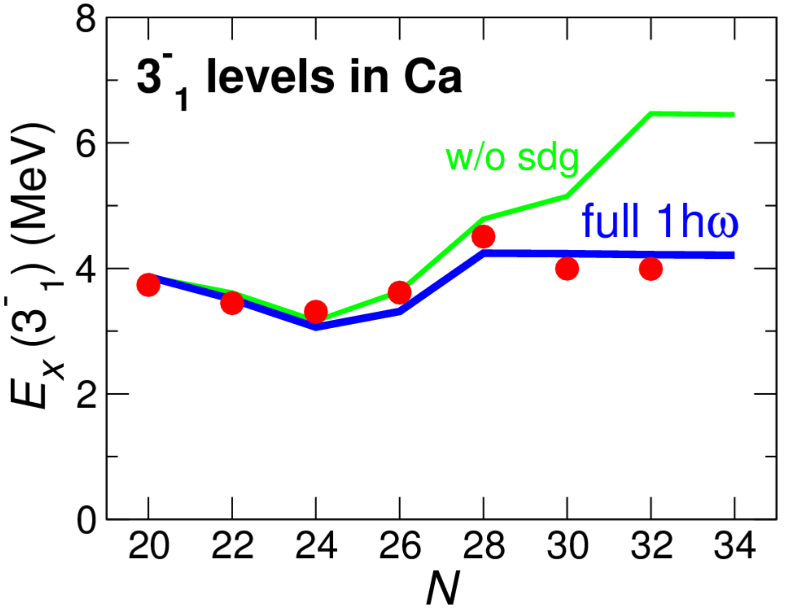


P. Möller et al., Phys. Rev. C 67, 055802 (2003)

Shell-model calculations: model space

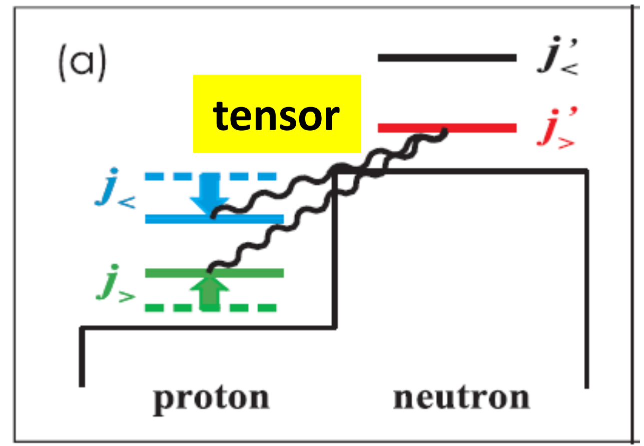
- Parent nuclei to be considered:
  $13 \le Z \le 18,\ 22 \le N \le 34$
  - Including odd-*A* and odd-odd nuclei
  - g.s.: dominated by $0\hbar\omega$ states
- Model space taken in this study
  - *sd* + *pf* + *sdg* valence shell
  - Natural-parity states: full $0\hbar\omega$ states
  - Unnatural-parity states: full $1\hbar\omega$ states
    - Daughter Gamow-Teller states
    - Satisfying the Ikeda sum rule
    - The *sdg* orbitals are needed for a better description of larger *N* nuclei.



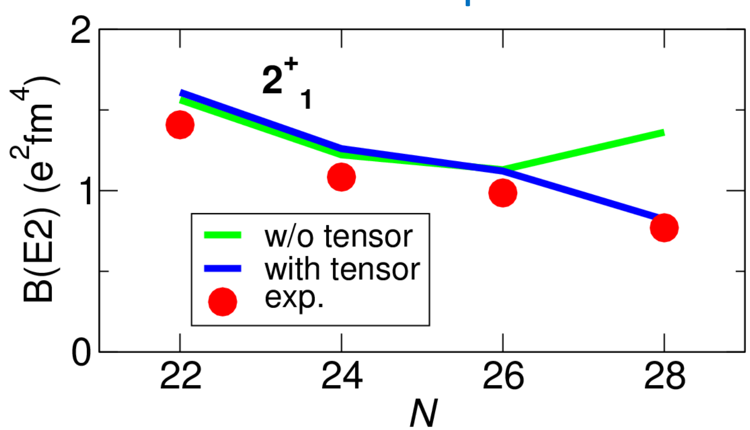


Shell-model calculations: effective interaction

- The SDPF-MU interaction is taken.
  - USD for *sd* shell, GXPF1B for *pf* shell
  - V_{MU} interaction for the other parts: aimed at a universal description of shell evolution in a simple functional form
    - Gaussian central force
    - Spin-orbit interaction from the M3Y interaction
    - Explicit inclusion of the tensor force as a π+ρ meson exchange potential
  - No adjustment specific to this study
- Some results
  - Breakdown of the N=28 magicity in ^{42}Si
  - K=4 isomeric state in ^{44}S



Si isotopes

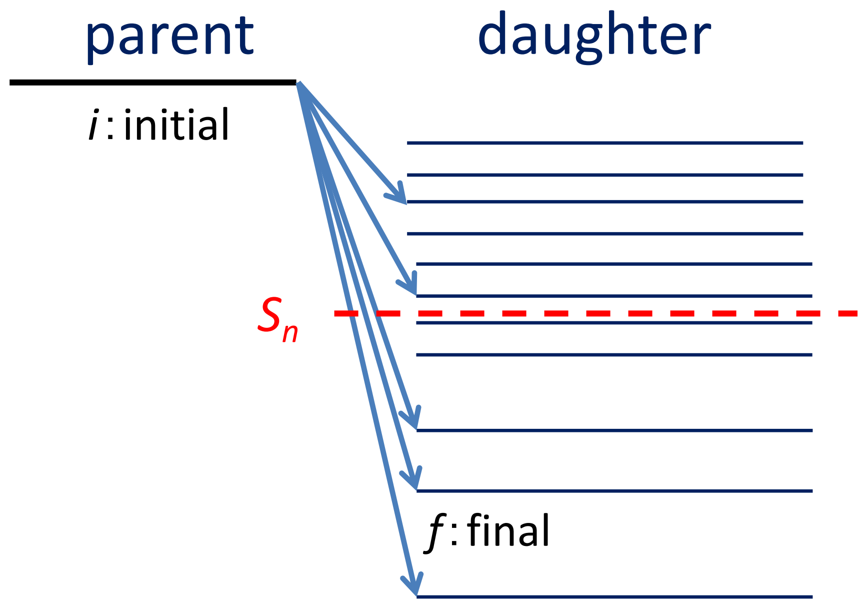


Y. Utsuno et al., PRC 86, 051301(R) (2012)

Y. Utsuno et al., PRL 114, 032501 (2015)

Calculating Gamow-Teller β decays

- Partial half-lives
  - $f_0 T_{1/2}(f) = \frac{6144 \text{ sec}}{(g_A/g_V)^2 \, B(\text{GT}; i \to f)}$
  - f_0: phase-space factor; roughly proportional to Q_β^5
  - Standard quenching factor 0.74 introduced for calculating B(GT)
- Total half-lives ($T_{1/2}$)
  - All the daughter states below Q_β
- β-delayed neutron emission probabilities (P_n)
  - Fraction of the feeding to the states above S_n
  - Good probe for investigating low-lying Gamow-Teller strengths



Lanczos method (for strength distribution)

- Calculating all of the eigenstates below Q_β is practically impossible.
- What one needs for the present purpose:

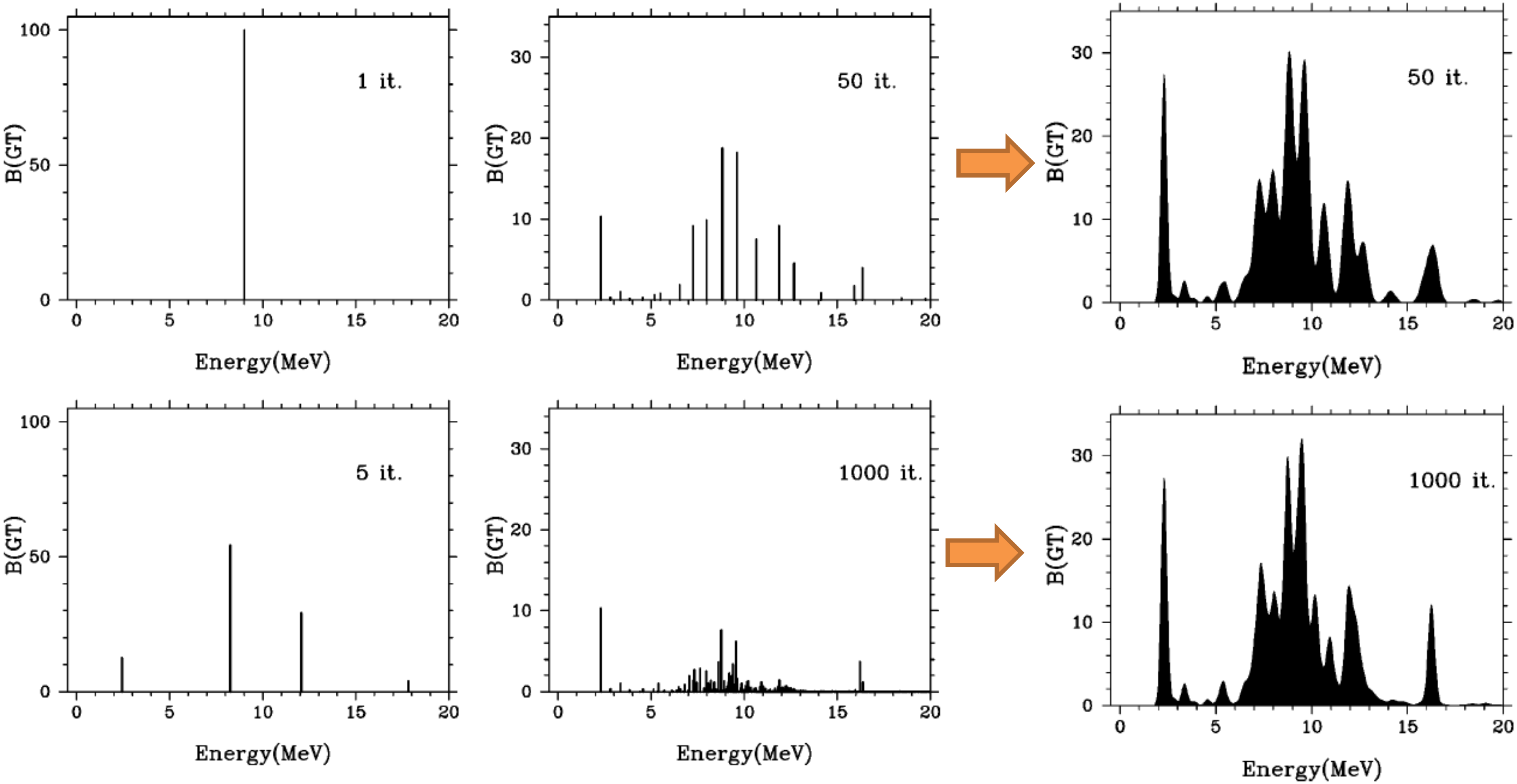
  To get a strength distribution of an operator $\hat{O}$ for an arbitrary state $|i\rangle$. In this situation, one does not care about each $\langle f|\hat{O}|i\rangle$.

  – $\hat{O}$ is not necessarily a one-body operator. See N. Shimizu et al., PRL 120, 142502 (2018) for a recent application to double Gamow-Teller resonances.

  – $|i\rangle$ can be an excited state.

- For this purpose, the moment $S_k = \sum_\nu (E_\nu - E_i)^k \left|\langle \nu|\hat{O}|i\rangle\right|^2$ up to a sufficiently large k should be calculated. The Lanczos algorithm guarantees the correct moments up to $k = 2n - 1$ with n Lanczos iterations starting with $\overrightarrow{u_1} = \hat{O}|i\rangle$.

Example



E. Caurier et al., Rev. Mod. Phys. 77, 427 (2005).

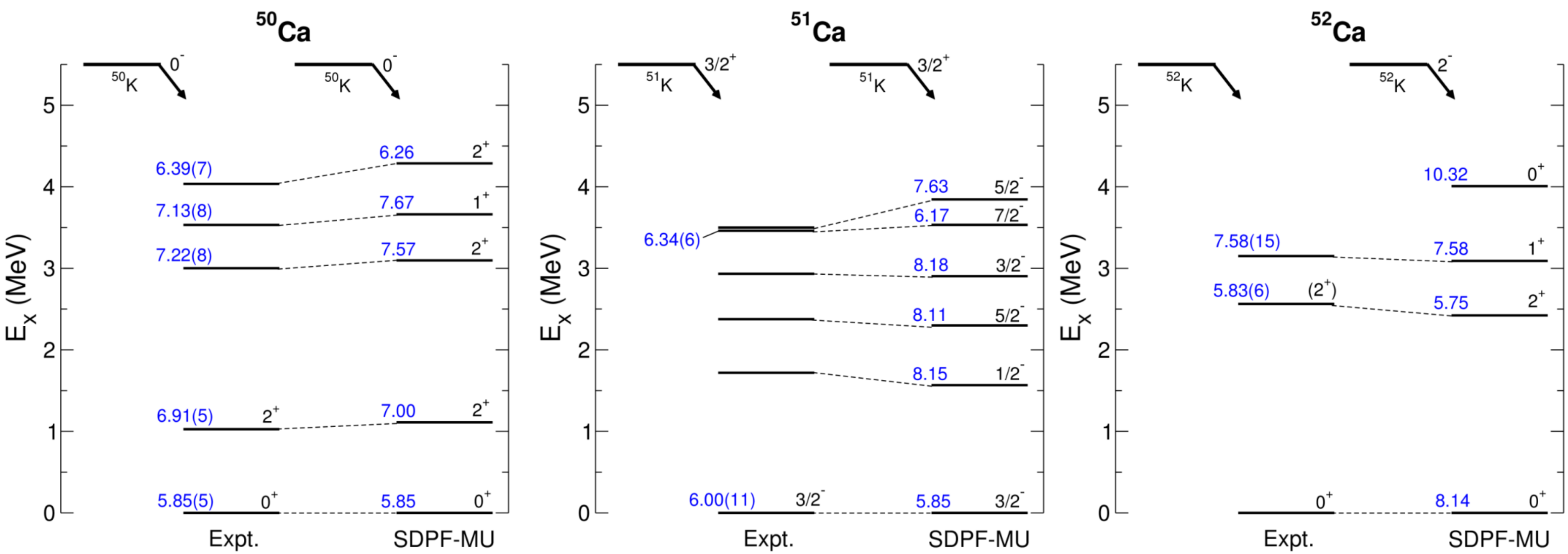
- Good distribution is obtained after 50 iterations, although individual strengths are quite different between 50 and 1000 iterations.

First-forbidden (FF) transitions

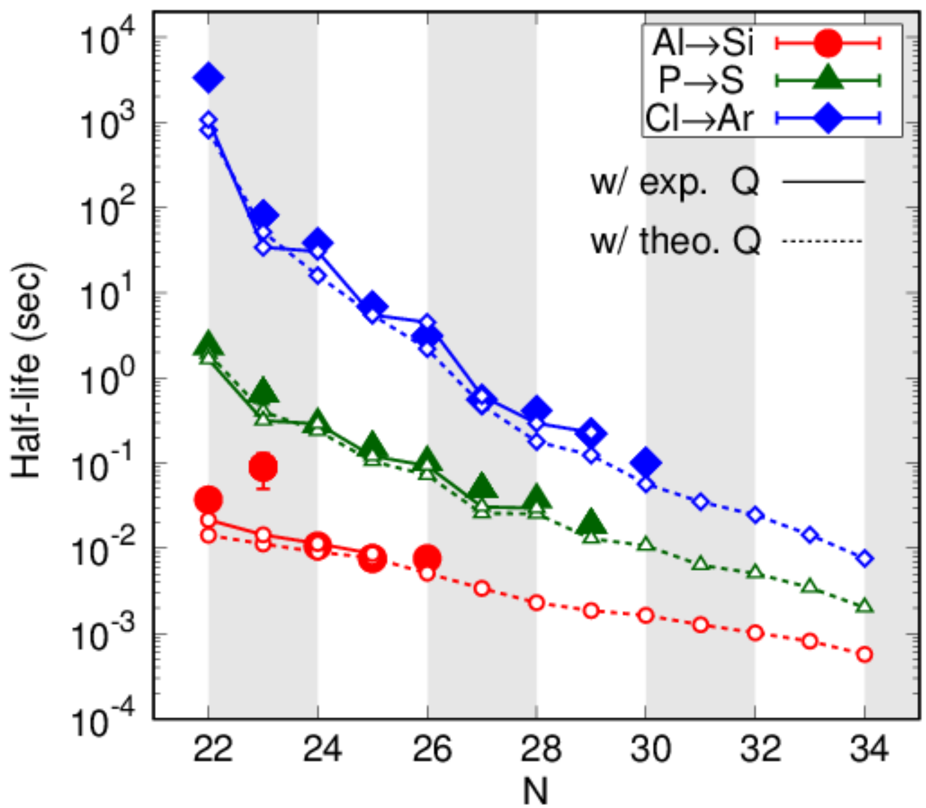
Matrix elements involved

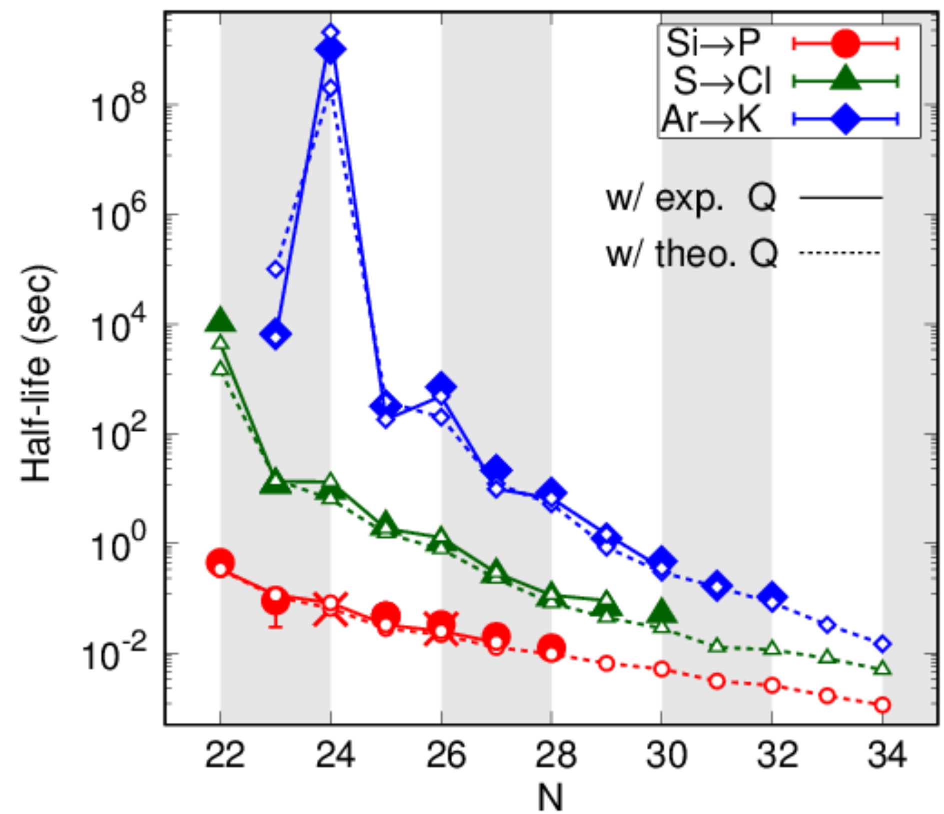
Rank		Matrix element (ME)	ME in nonrelativistic approx.
0	M_0^S	$\lambda\sqrt{3}\langle f\|\|ir[C_1\otimes\boldsymbol{\sigma}]^{(0)}t_-\|\|i\rangle C$	
	M_0^T	$\lambda\sqrt{3}\langle f\|\|\gamma_5 t_-\|\|i\rangle C$	$-\lambda\sqrt{3}\langle f\|\|(i/M_N)[\boldsymbol{\sigma}\otimes\boldsymbol{\nabla}]^{(0)}t_-\|\|i\rangle C$
1	x	$-\langle f\|\|irC_1 t_-\|\|i\rangle C$	
	$\xi' y$	$-\langle f\|\|\boldsymbol{\alpha}t_-\|\|i\rangle C$	$E_\gamma x$
	u	$\lambda\sqrt{2}\langle f\|\|ir[C_1\otimes\boldsymbol{\sigma}]^{(1)}t_-\|\|i\rangle C$	
2	z	$-2\lambda\langle f\|\|ir[C_1\otimes\boldsymbol{\sigma}]^{(2)}t_-\|\|i\rangle C$	

- Effective operators determined to reproduce FF decays of K→Ca



Results of half-lives





- Error analysis in terms of

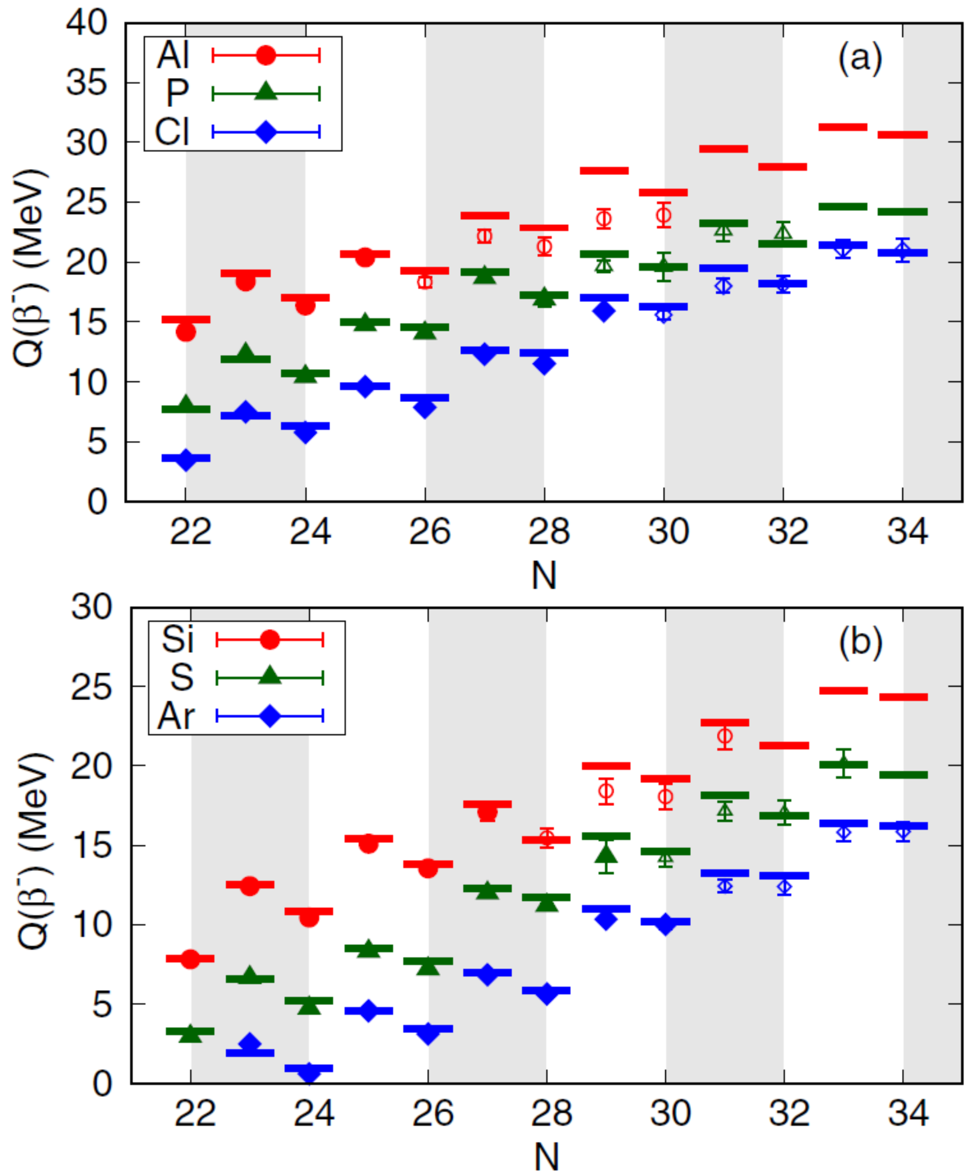
$$r = \log_{10}({T_{1/2}}^{\text{calc}}/{T_{1/2}}^{\text{exp}})$$

  - $\bar{r}$: mean value of r
  - σ: standard deviation of r

	This work		FRDM+QRPA
	Q (theory)	Q (exp.)	
$\bar{r}$	−0.18	−0.09	−0.07
σ	0.29	0.20	0.48
$10^{\bar{r}}$	0.66	0.81	0.85
10^{σ}	1.95	1.58	3.04
n	47	40	47

Comparison of Q_β values

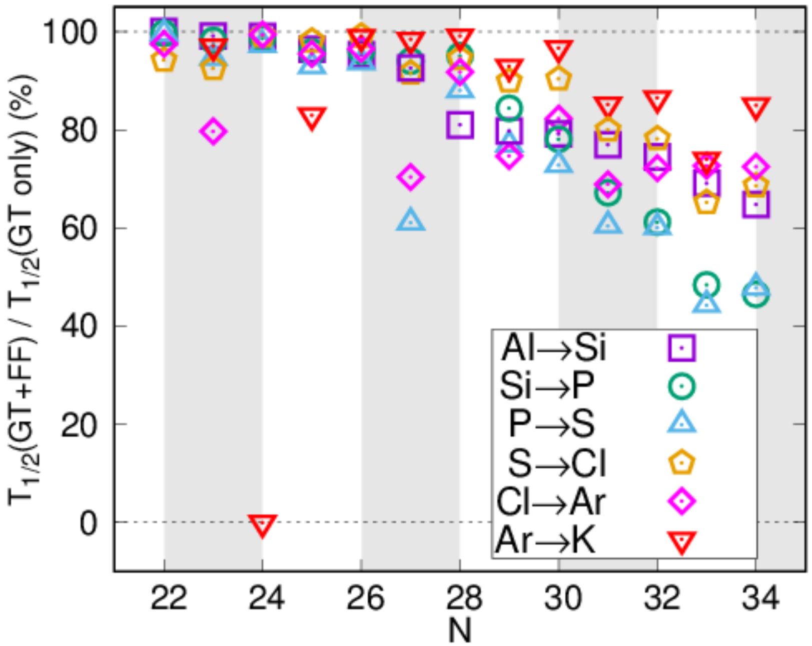
- Good agreement between experiment and theory
- Slightly overestimated by the calculations
  - Systematic shorter half-lives with the calculated Q_β values
- For some nuclei, large differences between the calculation and the evaluation
  - Impact on half-lives



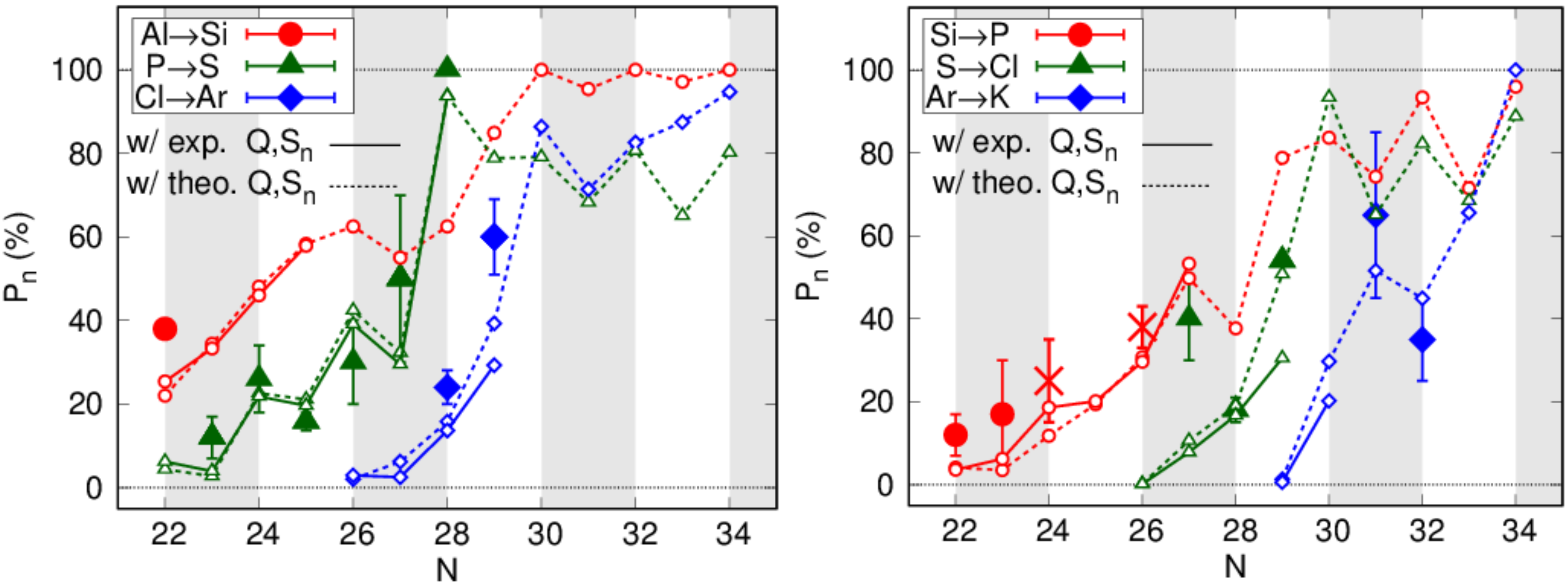
open symbols: estimated value in AME2016

Effect of FF transitions

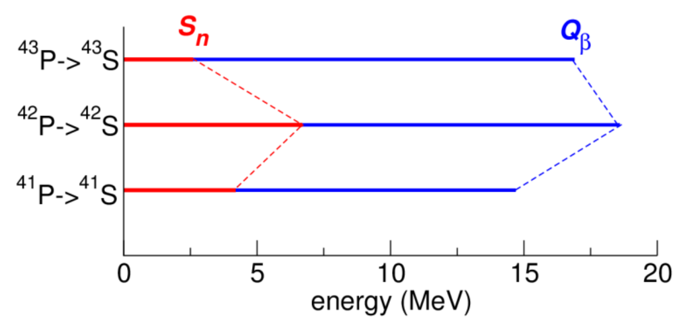
- Comparison of half-lives with and without the FF transitions included
  - More significant FF contributions for larger *N*, especially *N* > 28
- This is due to
  1. Reduction of low-lying GT strength
     - Absorbed by GT giant resonance
  2. Enhancement of low-lying FF matrix elements
     - $\Delta J = 0$ FF channels of $p_{3/2} \rightarrow d_{3/2}$ and $p_{1/2} \rightarrow s_{1/2}$ opens for nuclei with *N* > 28.



Results of β-delayed neutron emission prob.

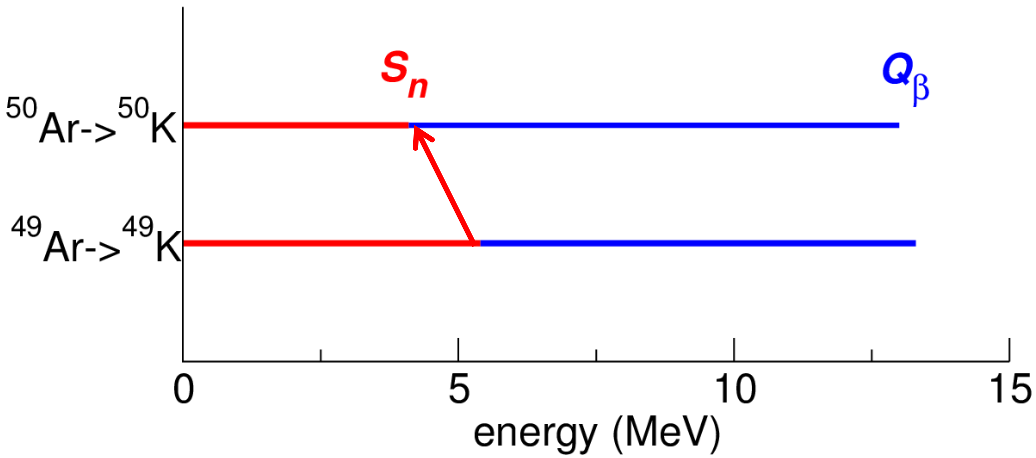


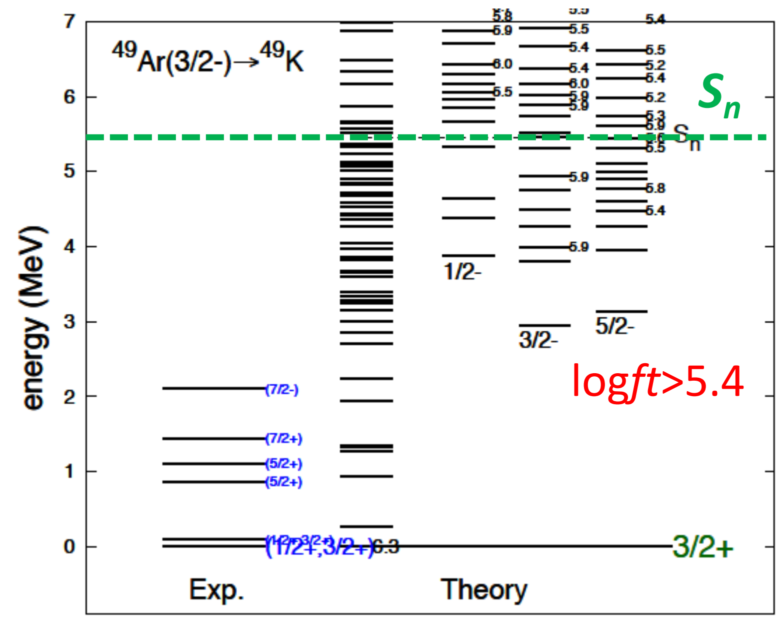
- Good agreement with experimental data
- Even-odd staggering
  - Clearly seen in P → S decays
  - Simply due to phase space

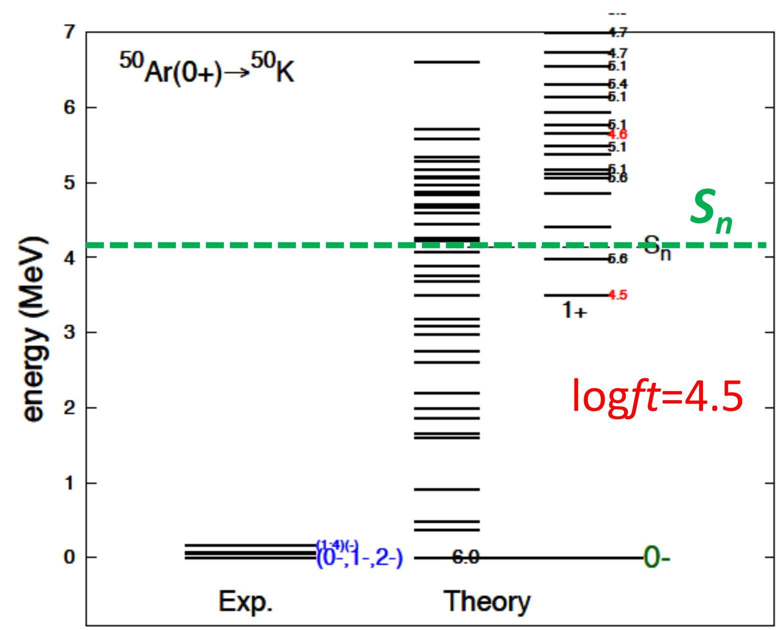


- Opposite even-odd staggering in ^{50}Ar (N=32) and ^{42}Si (N=28)?

Gamow-Teller decay of ^{49}Ar vs. ^{50}Ar

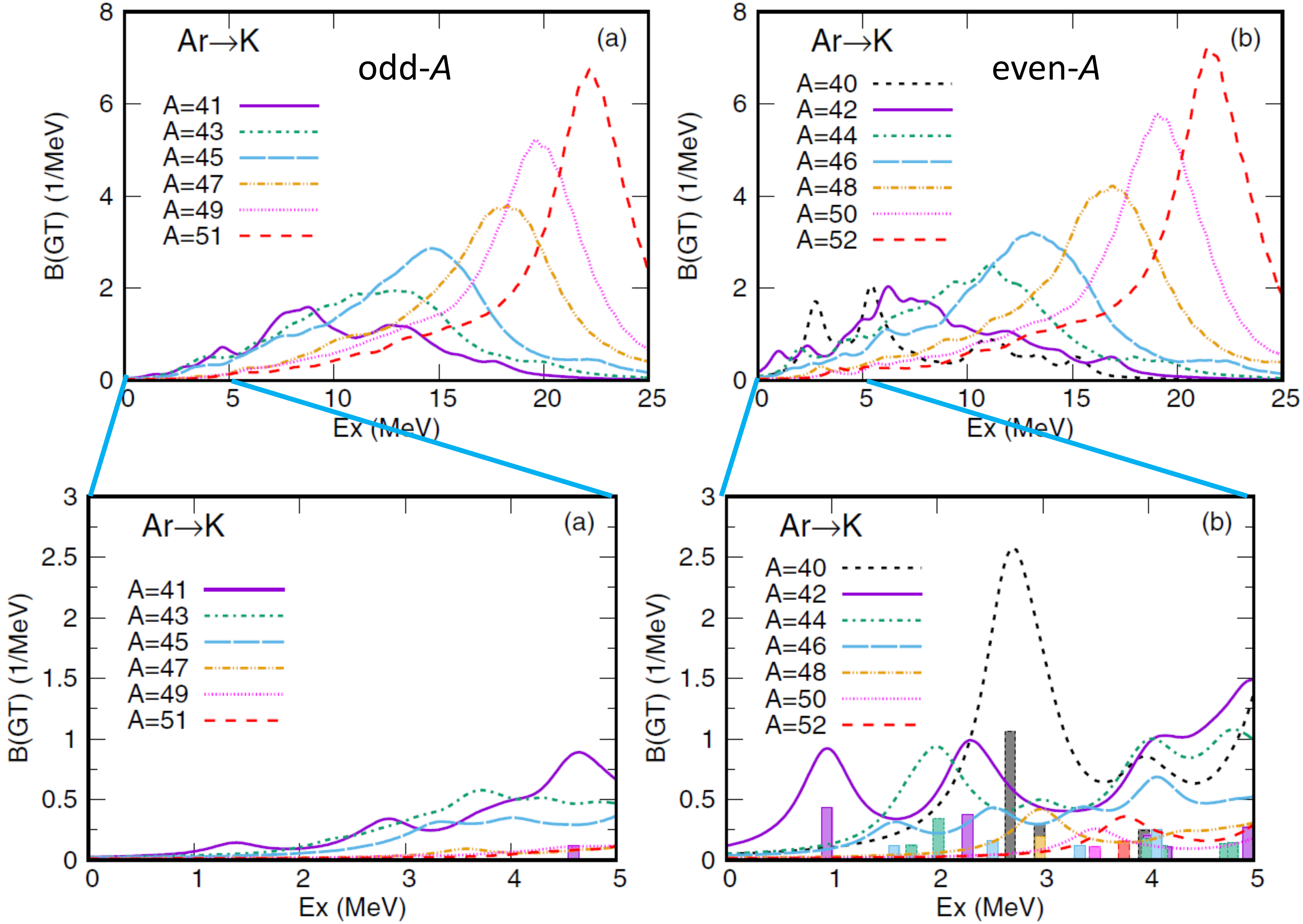






- Much more states below S_n for the decay of ^{49}Ar, but larger P_n obtained (or less transitions to bound levels).
- Existence of a strong transition to a low-lying state for the decay of ^{50}Ar
  - One order stronger or more
- How universal is this strong low-energy Gamow-Teller state?

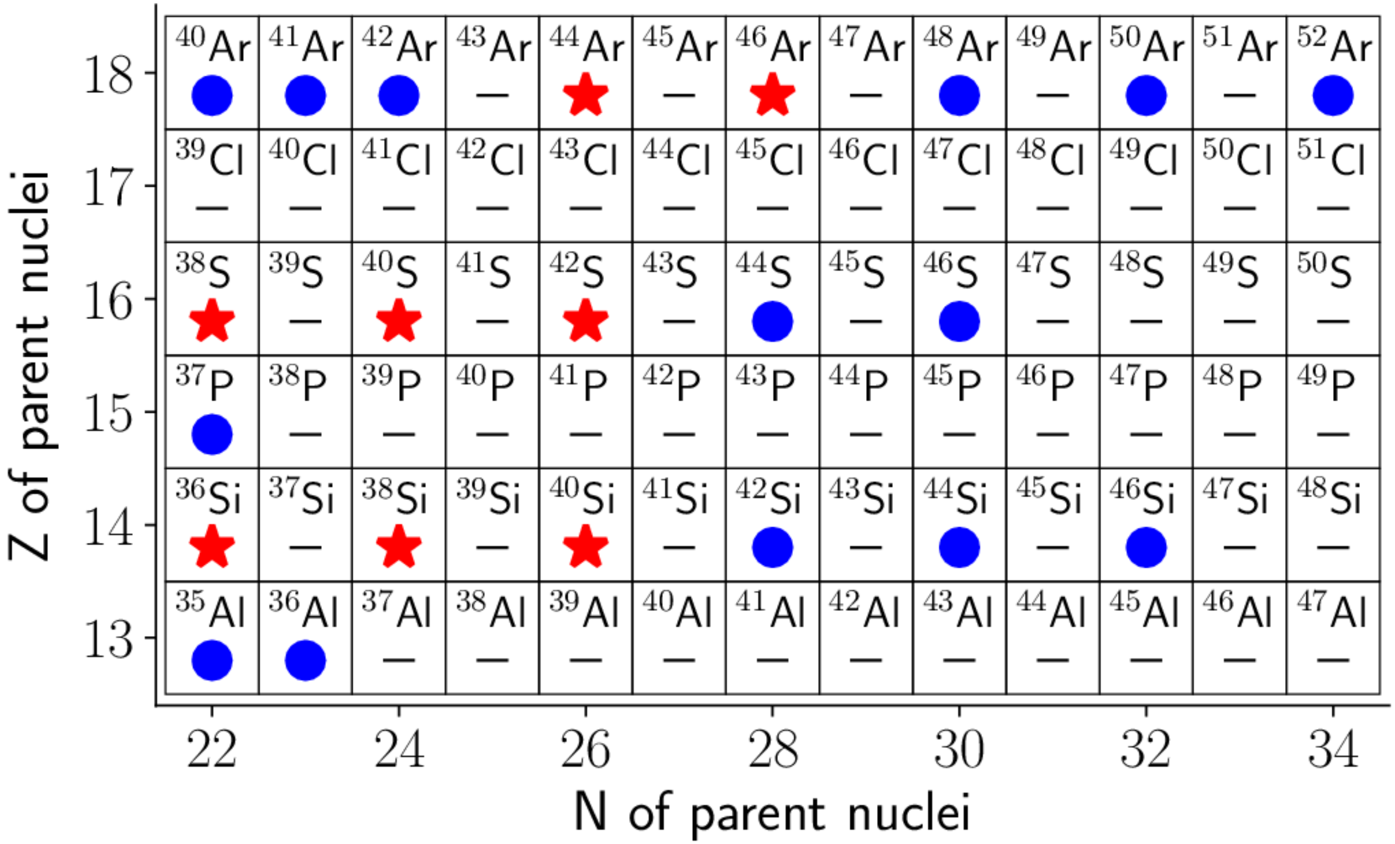
Systematics of GT strength distributions



Strong even-odd effect in low-energy GT strengths

Chart of low-energy strong GT transitions

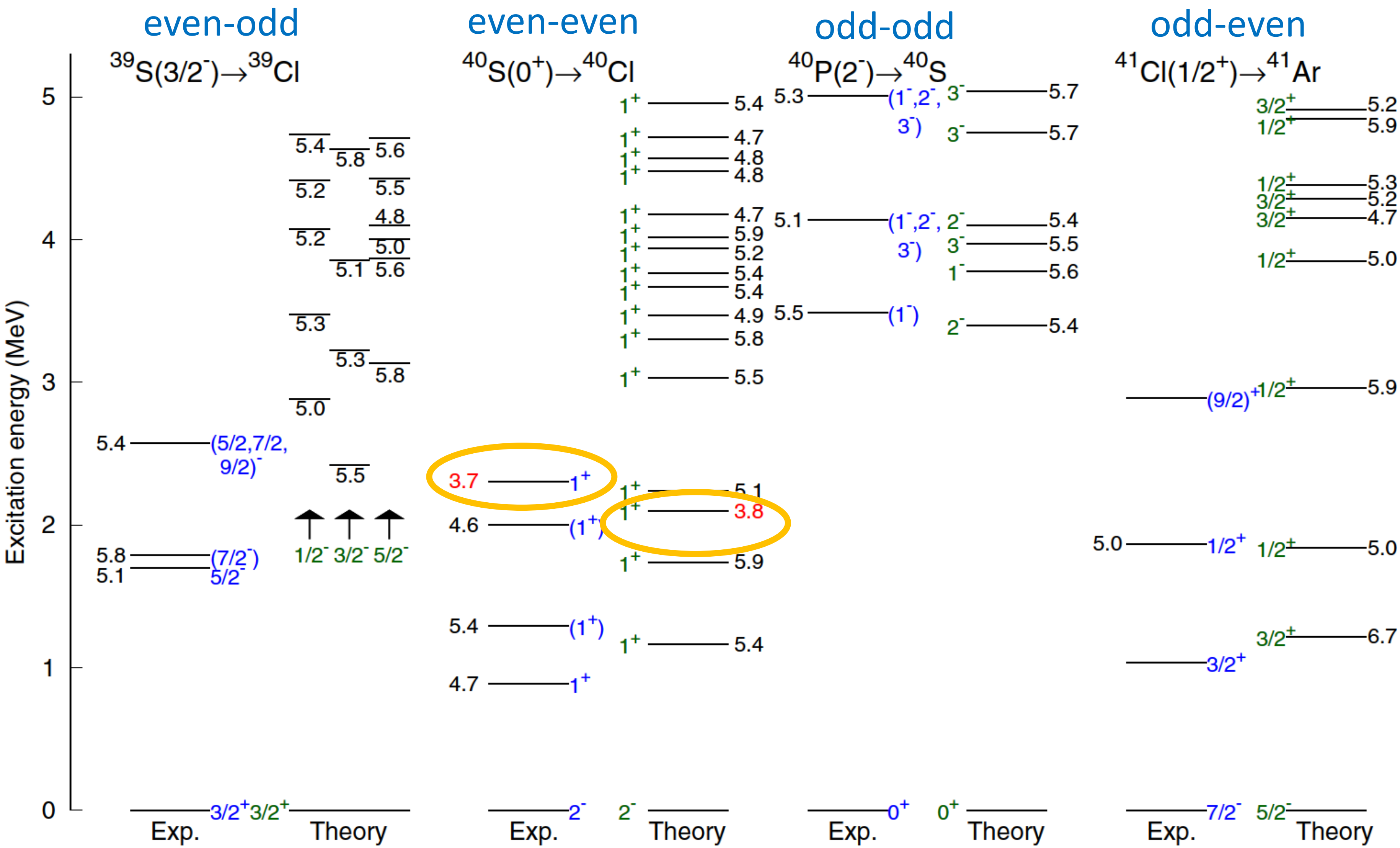
- Definition of "low-energy strong GT transition"
  - Energy level with B(GT) > 0.1 and $E_x < 5$ MeV



red: experimentally observed blue: predicted but no data

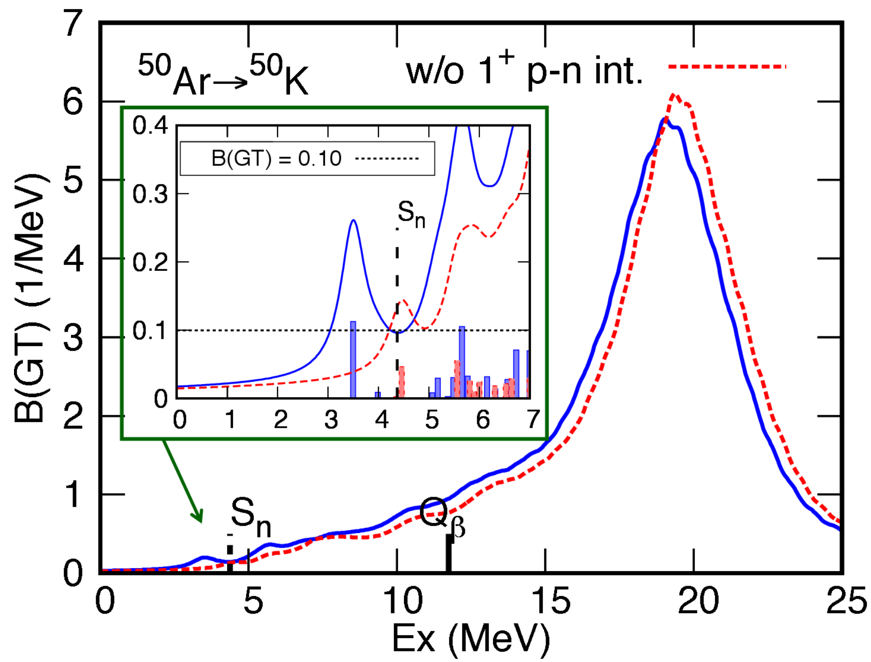
Widely seen in nuclei not very far from stability

Some low-lying data



Role of isoscalar pairing

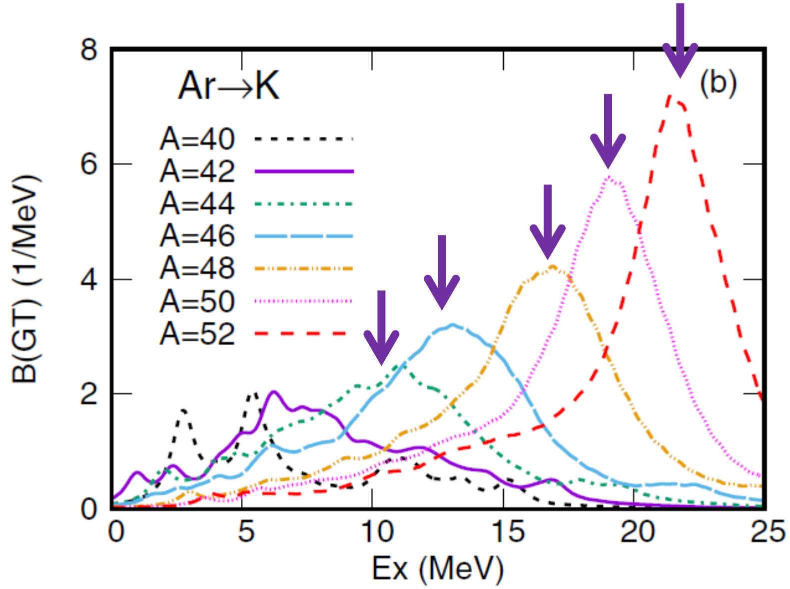
- The importance of isoscalar pairing on half-lives is known in terms of the QRPA calculations.
- We examine the role of isoscalar pairing in terms of the shell model:
  - All the $\langle a_\pi b_\nu J^\pi | V | c_\pi d_\nu J^\pi \rangle$ with $J^\pi = 1^+$ are set to zero.
  - To minimize the side effect, the diagonal matrix elements $\langle a_\pi b_\nu J^\pi | V | a_\pi b_\nu J^\pi \rangle$ are shifted to keep the monopole interaction of the original interaction.
    - The energies of the initial states are conserved.



Weaker strengths of low-energy GT transitions

Position of GT giant resonance (GTGR)

- Practically important to predict systematic half-lives based on the gross theory
- A good probe for the spin-isospin dependence of the effective interaction
  - $E_{\mathrm{GT}} - E_{\mathrm{IAS}} = \Delta E_{ls} + 4(\kappa_{\sigma\tau} - \kappa_{\tau})T_0$
    with $V = \sum_{i<j}(\kappa_\tau \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j + \kappa_\sigma \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + \kappa_{\tau\sigma} \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)$ and $T_0 = (N - Z)/2$



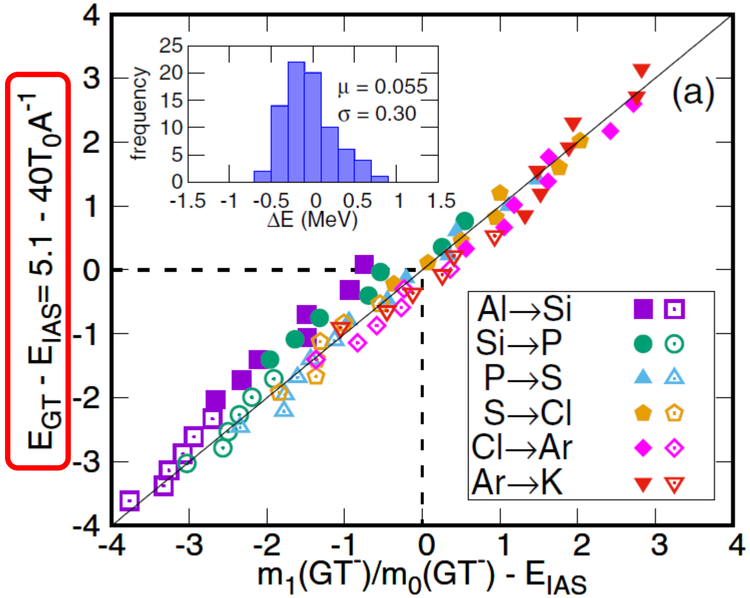
- Two different empirical formulas proposed on the basis of charge-exchange reactions of stable nuclei (in the small T_0 range)

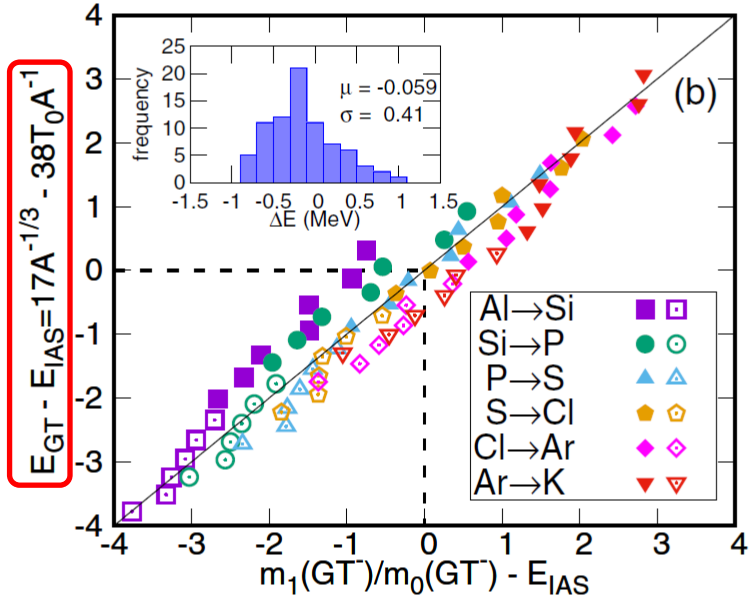
1. $E_{\mathrm{GT}} - E_{\mathrm{IAS}} = 6.7 - \mathbf{60}T_0 A^{-1}$ MeV
2. $E_{\mathrm{GT}} - E_{\mathrm{IAS}} = 26A^{-1/3} - \mathbf{37}T_0 A^{-1}$ MeV

quite different isospin dependence

Systematics obtained from the calculations

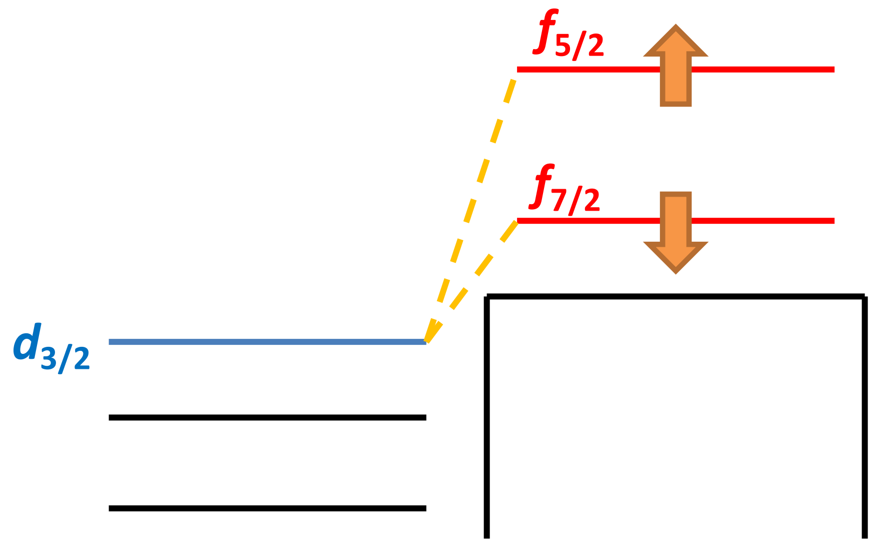
- Definition of E_{GT}: centroid energy
- Fitted to $E_{\mathrm{GT}} - E_{\mathrm{IAS}} = \Delta E_{ls} + CT_0A^{-1}$
  - ΔE_{ls}: constant or $A^{-1/3}$ dependence
- The resulting formulas for $E_{\mathrm{GT}} - E_{\mathrm{IAS}}$
  1. $5.1 - 40T_0A^{-1}$ MeV
  2. $17A^{-1/3} - 38T_0A^{-1}$ MeV
  - Good linearity for T_0
    - $E_{\mathrm{GT}} - E_{\mathrm{IAS}}$ can be negative for large T_0.
- Deviation from the formulas
  - Overestimate (underestimate) for smaller (larger) Z isotopes
  - Larger deviation for the formula with the ΔE_{ls} term of $17A^{-1/3}$ MeV





Effect of the shell evolution

- The calculations provide increasing ΔE_{ls} term with *A* (or *Z*) number as far as the $13 \leq Z \leq 18$ nuclei are concerned.
- This is well accounted for by shell evolution due to the tensor force.
  - For the $13 \leq Z \leq 18$ nuclei, the proton $d_{3/2}$ (plus $s_{1/2}$) orbital is predominantly occupied with increasing *Z*.
  - Since $d_{3/2}$ is a $j_<$ orbital, its occupation enlarges spin-orbit splitting.



Summary

- We carry out systematic calculations of β-decay half-lives and delayed neutron emission probabilities of 78 neutron-rich nuclei with $13 \leq Z \leq 18$, $22 \leq N \leq 34$.
  - Full $1\hbar\omega$ calculations in the *sd+pf+sdg* shell → Ikeda sum rule satisfied
  - Odd-*A* and odd-odd nuclei on the same footing
  - First forbidden transitions taken into account: non-negligible for N>28 nuclei
- Good agreement with the experimental data, especially by using the experimental Q_β values
- Systematics of Gamow-Teller strength distributions
  - Evolution of GT giant resonances (GTGR) with increasing N
  - Strong even-odd effect of low-energy GT strengths: isoscalar pairing
  - Position of GTGR: deducing isospin dependence and the influence of shell evolution due to the tensor force