



筑波大学  
*University of Tsukuba*



December 10-12, 2018

CCS workshop

## Impact of pear-shaped fission fragments on mass-asymmetric fission

Guillaume SCAMPS

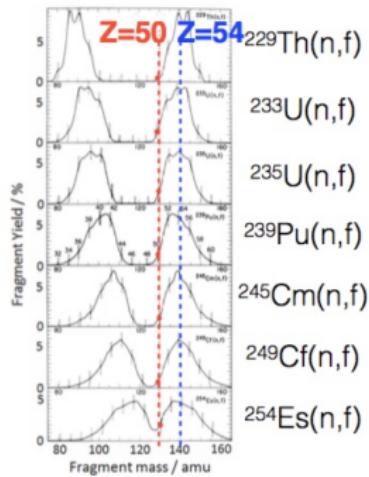


Center for Computational Sciences

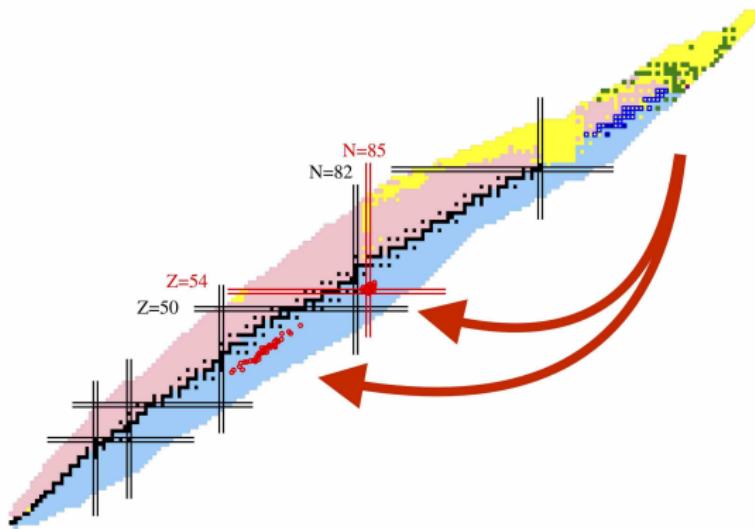
Collaboration : C. Simenel

# Motivation : understand the shell effects on fission

## Empirical behavior of actinide nuclei



J.P. Unik, J.E. Gindler, J.E. Glendenin et al. : Proc. Phys. and Chem. of Fission IAEA Vienna , Vol II, 20 (1974)

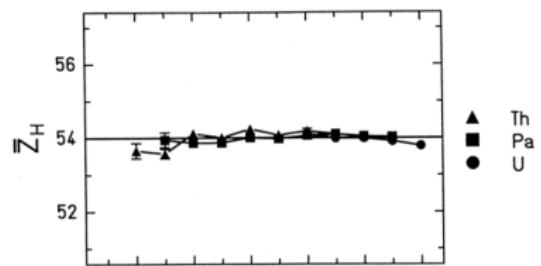


Data from D. A. Brown et al., Endf/b-viii.0, Nucl. Data Sheets 148, 1 (2018), (spontaneous and thermal neutron-capture).

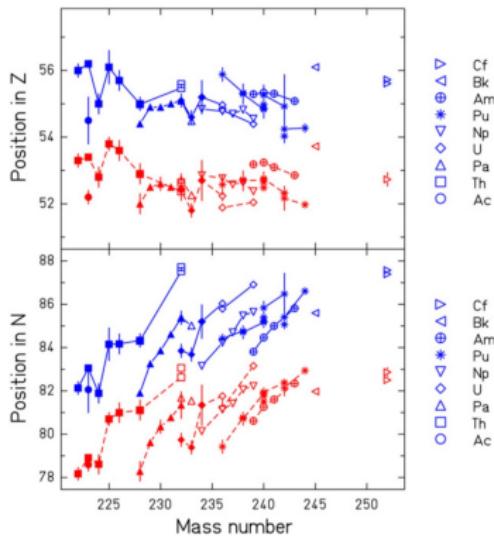
# Systematic comparison for actinide

## Empirical behavior of actinide nuclei

C. Böckstiegel et al. / Nuclear Physics A 802 (2008) 12–25



K.-H. Schmidt et al. Nuclear Physics A  
665 (2000)



## Motivation

How can we understand this behavior? Interplay between structure and reaction?

# Mean-field theory with pairing

## TDHF

- Independent particle
- Initialisation :  $\hat{h}_{MF} |\phi_i\rangle = \epsilon_i |\phi_i\rangle$
- Evolution :  
 $i\hbar \frac{d\rho}{dt} = [h_{MF}, \rho]$

## TDHFB

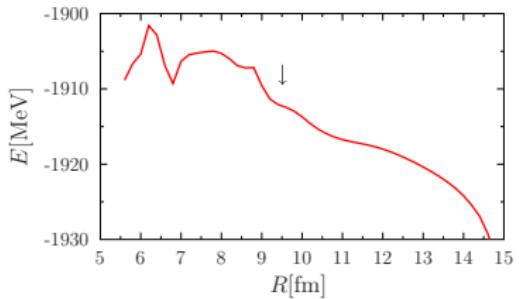
- Pairing correlation
- Quasi-particles :  $|\omega_\alpha\rangle = \begin{pmatrix} u_\alpha \\ v_\alpha \end{pmatrix}$
- Evolution :  
 $i\hbar \frac{d|\omega_\alpha\rangle}{dt} = \begin{pmatrix} h & \Delta \\ -\Delta^* & -h^* \end{pmatrix} |\omega_\alpha\rangle$

## TDHF+BCS

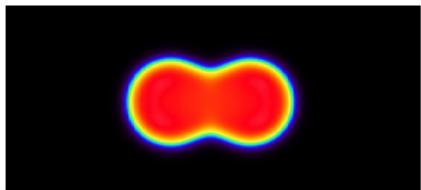
- Based on TDHFB with the approximation :  $\Delta_{ij} = \delta_{ij}\Delta_i$
- Evolution :  
 $i\hbar \frac{d\phi_i}{dt} = (\hat{h}_{MF} - \epsilon_i)\phi_i$   
 $i\hbar \frac{dn_i}{dt} = \Delta_i^* \kappa_i - \Delta_i \kappa_i^*$   
 $i\hbar \frac{d\kappa_i}{dt} = \kappa_i(\epsilon_i - \epsilon_i^-) + \Delta_i(2n_i - 1)$

# Why does we need pairing ?

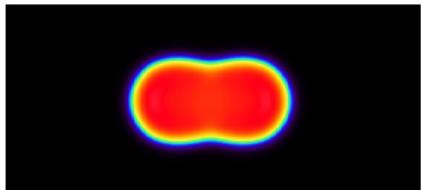
Fission barrier :  $^{258}\text{Fm}$



TDHF



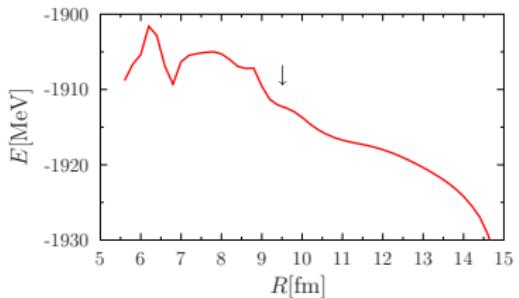
TDHF+BCS



G. Scamps, C. Simenel, D. Lacroix, PRC **92**, 011602(R) (2015).

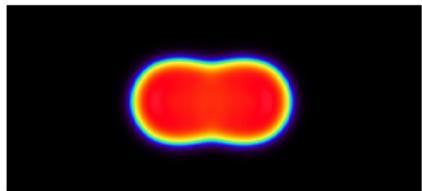
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Fission barrier :  $^{258}\text{Fm}$



TDHF

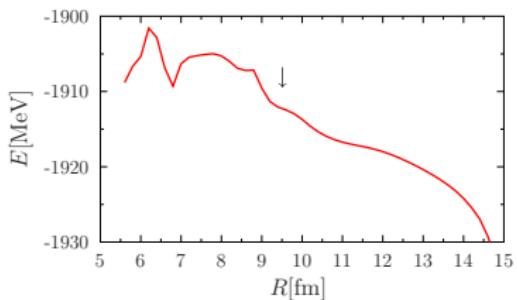
TDHF+BCS



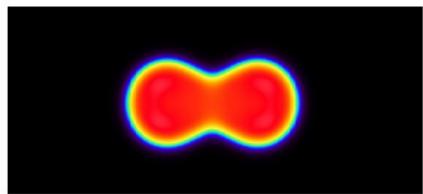
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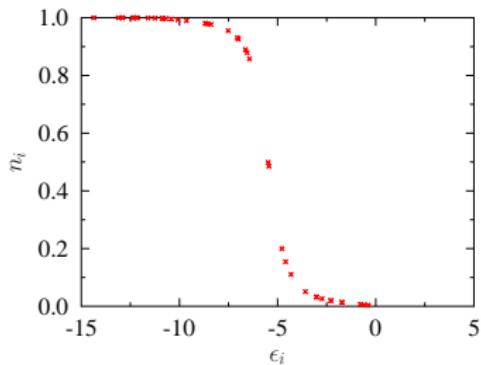
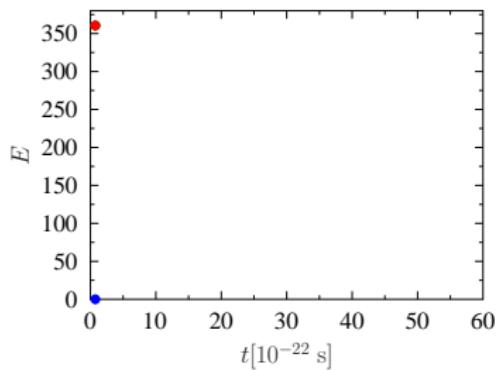
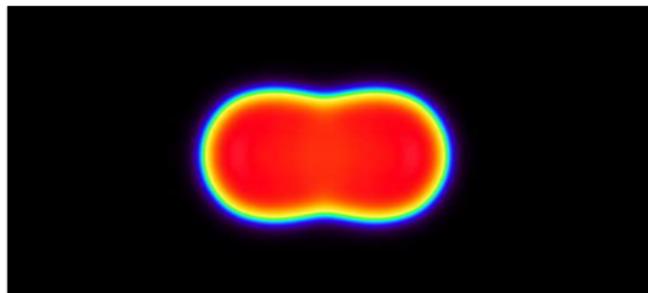
TDHF



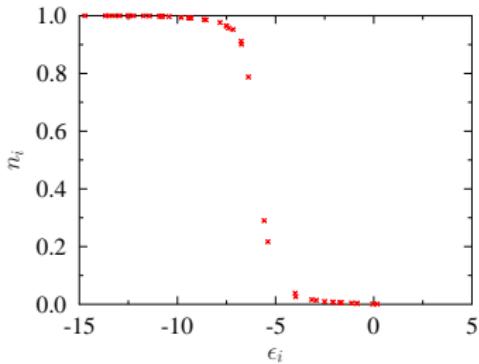
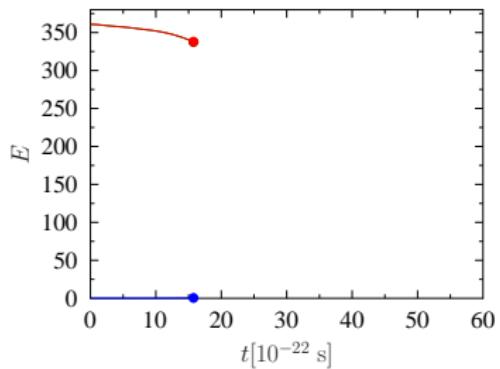
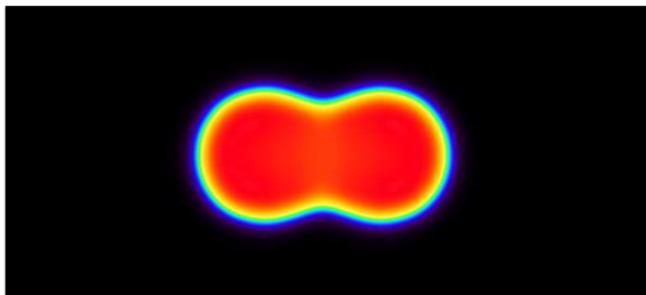
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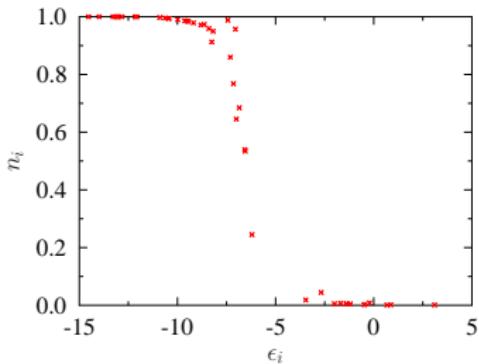
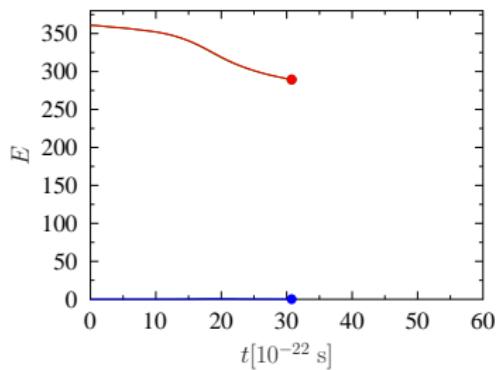
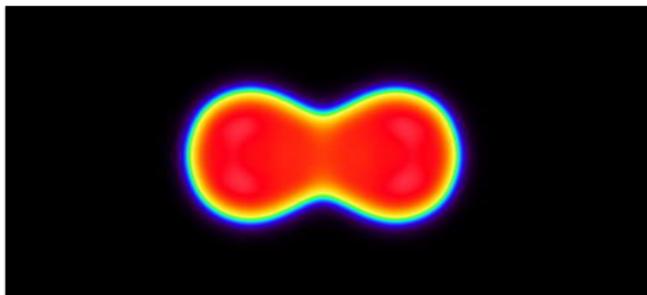
# Influence of pairing on fission process



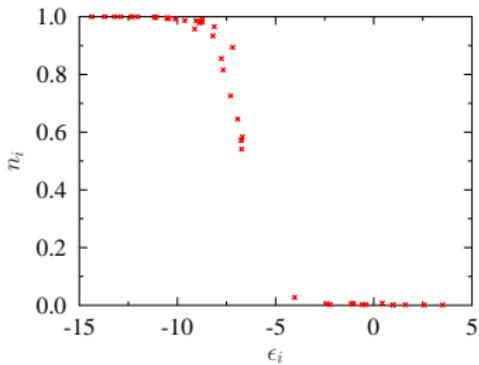
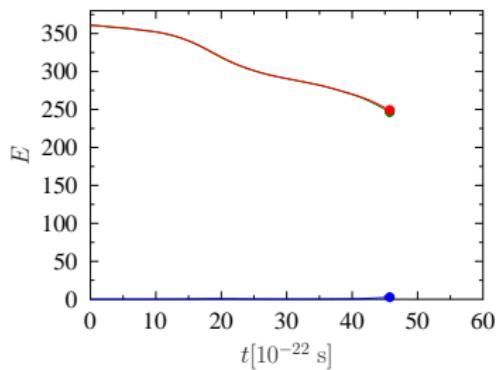
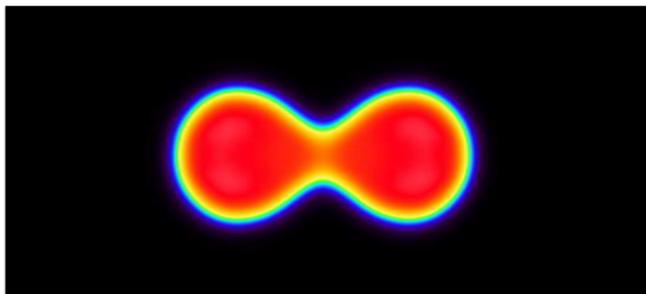
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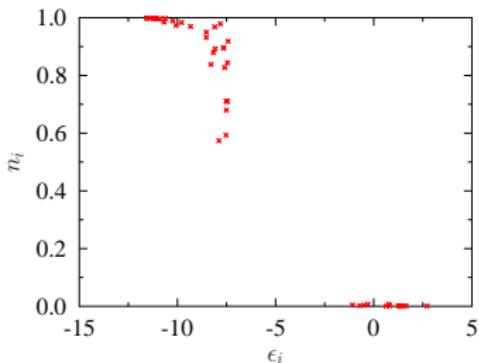
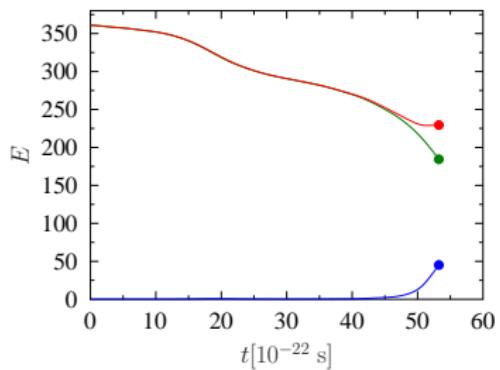
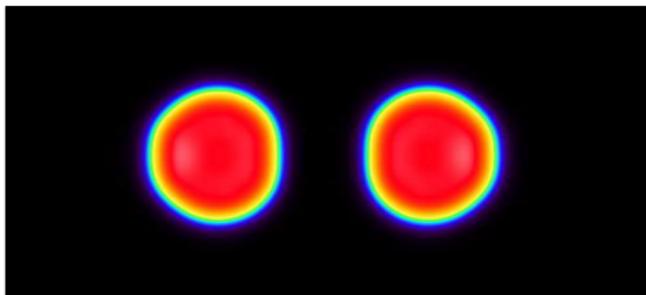
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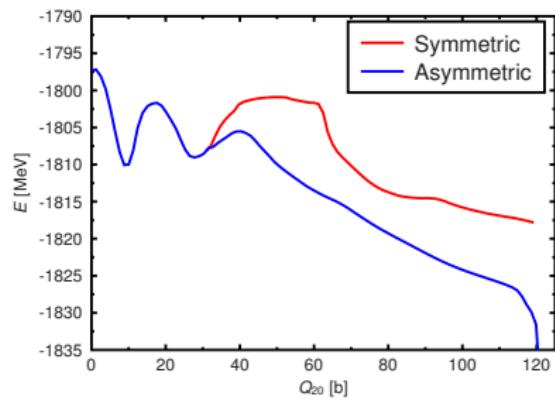
# Influence of pairing on fission process



# New systematic study

## First : CHF+BCS

Example :  $^{240}\text{Pu}$



## Second : TDHF+BCS



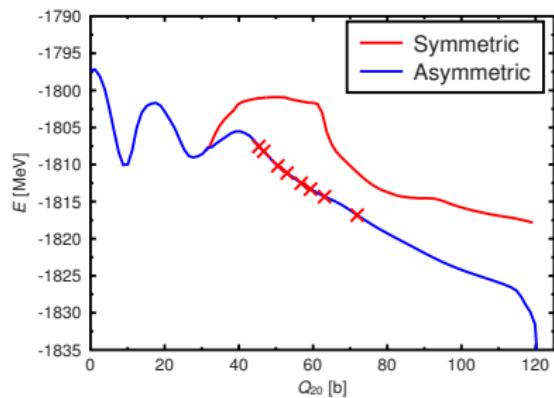
## Details of the calculation

- Skyrme functionnal Sly4d
- Surface pairing interaction
- $\Delta x = 0.8 \text{ fm}$
- Lattice :  $L_x \times L_y \times 2L_z = 40 \times 10.2 \times 10.2 \text{ fm}^3$

# New systematic study

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Example :  $^{240}\text{Pu}$



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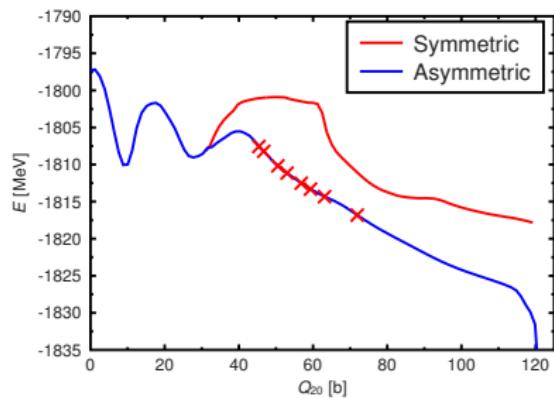
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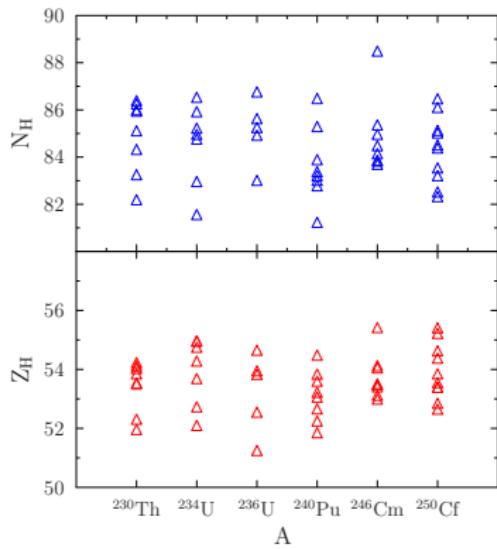
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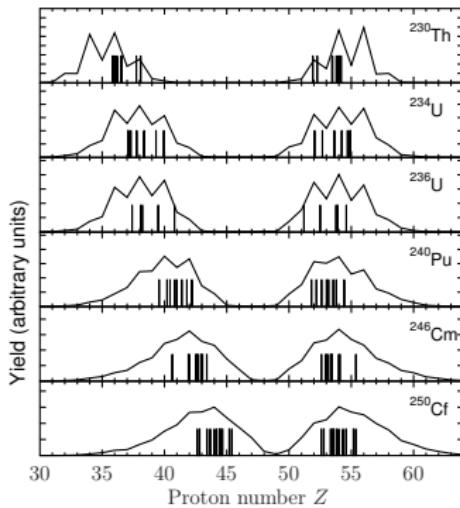
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# TDHF+BCS systematics results

## TDHF+BCS

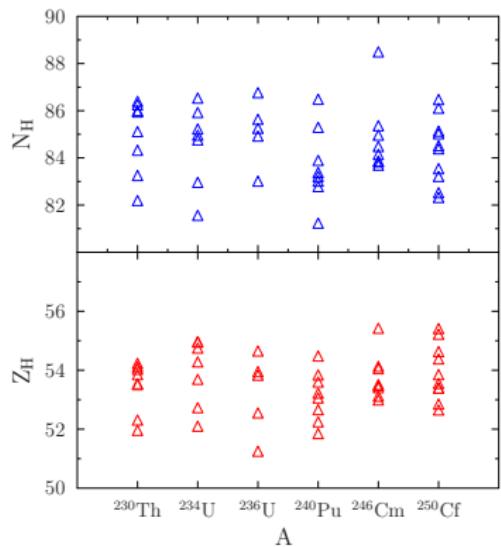


## Comparison with experimental data

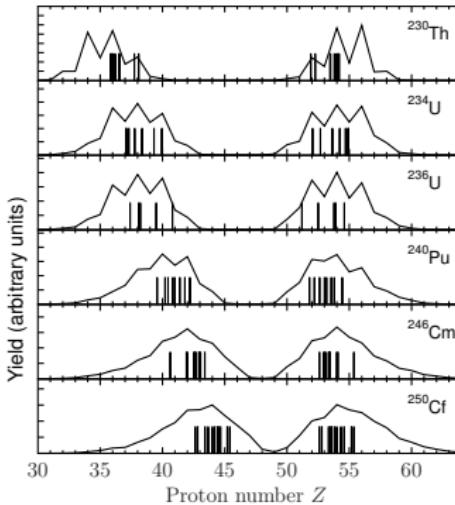


# TDHF+BCS systematics results

## TDHF+BCS



## Comparison with experimental data



## Conclusion :

The TDHF+BCS calculation reproduces well the  $Z=54$  behavior. But why ?

# Nucleon localization function

## Fermion localization function

$$\mathcal{C}_{q\sigma}(\mathbf{r}) = \left[ 1 + \left( \frac{\tau_{q\sigma}\rho_{q\sigma} - \frac{1}{4}|\nabla\rho_{q\sigma}|^2 - \mathbf{j}_{q\sigma}^2}{\rho_{q\sigma}\tau_{q\sigma}^{TF}} \right)^2 \right]^{-1}$$

A. D. Becke and K. E. Edgecombe, J. Chem. Phys.  
92, 5397 (1990).

Physical meaning :

$$\mathcal{C} \in [0 : 1]$$

$\mathcal{C}_{q\sigma}(\mathbf{r}) = 1$  Probability to find another particle with  
the same  $q$  and  $\sigma$  very low.

$\mathcal{C}_{q\sigma}(\mathbf{r}) = 0.5$  Limit of uniform-density Fermi gas.

Mask function :

$$\rightarrow \frac{\mathcal{C}_{q\sigma}(\mathbf{r})\rho_{q\sigma}}{\rho_{q\sigma}^{\max}}$$

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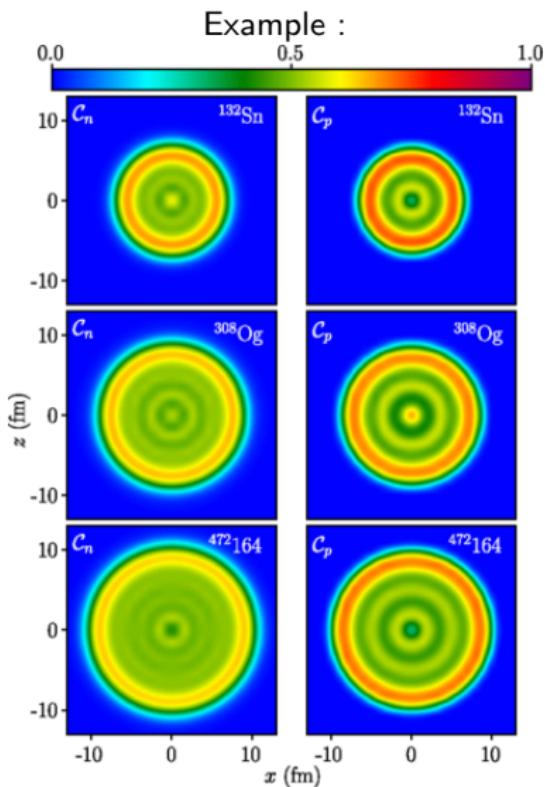
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P. Jerabek, B. Schuetrumpf, P. Schwerdtfeger, and W. Nazarewicz, Phys. Rev. Lett. **120**, 053001 (2018).

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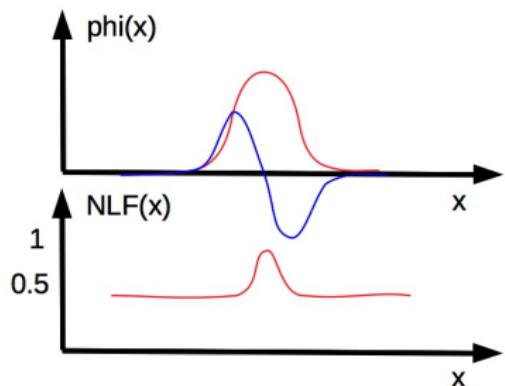
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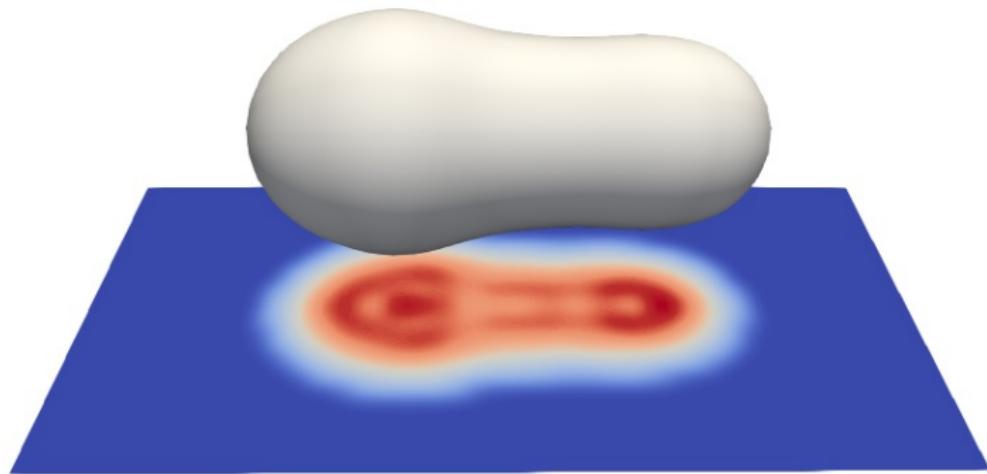
$$\rightarrow \frac{\mathcal{C}_{q\sigma}(\mathbf{r})\rho_{q\sigma}}{\rho_{q\sigma}^{\max}}$$

## Schematic system



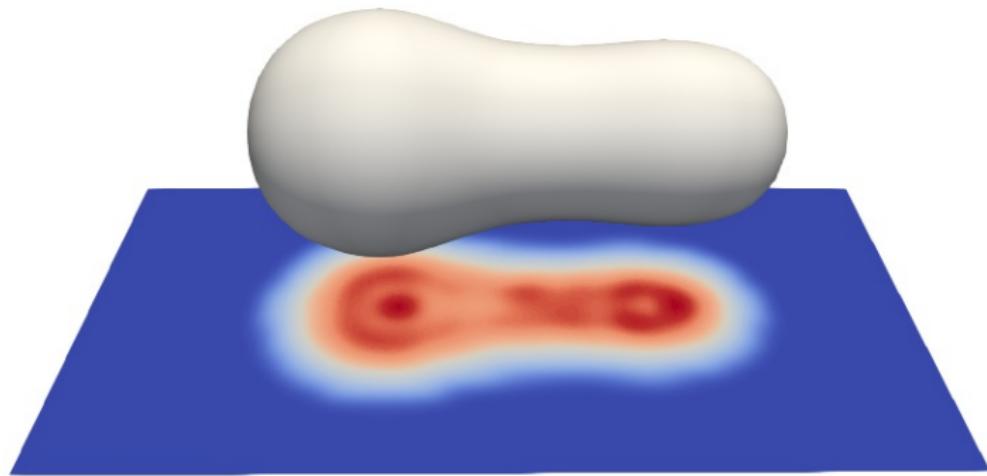
## Example of $^{240}\text{Pu}$

$^{240}\text{Pu}$



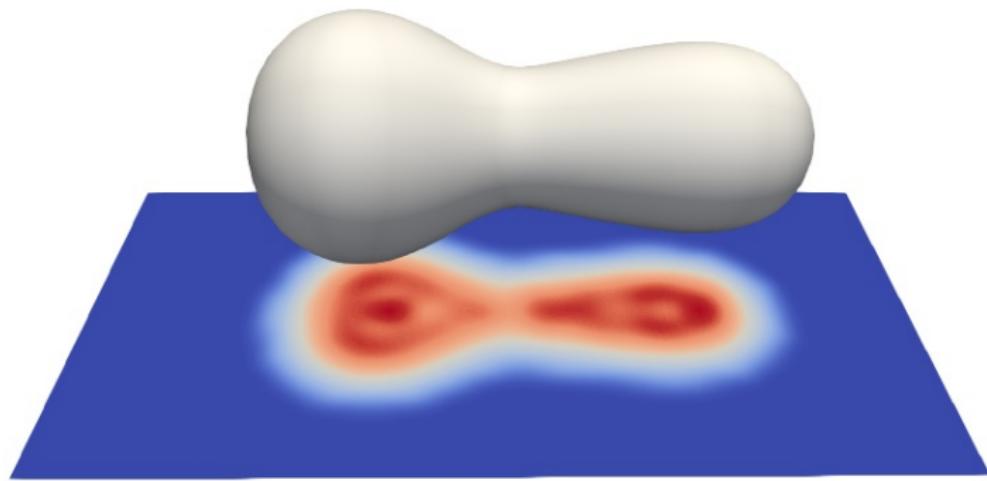
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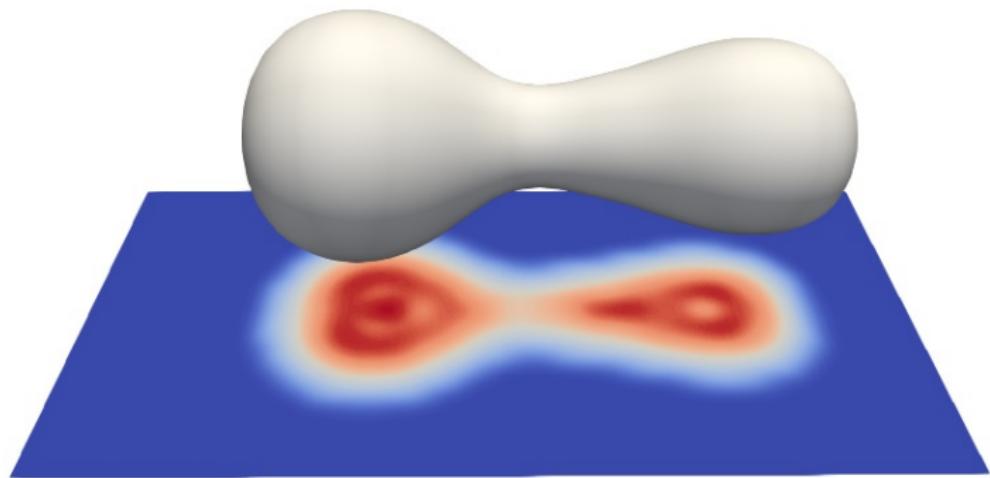
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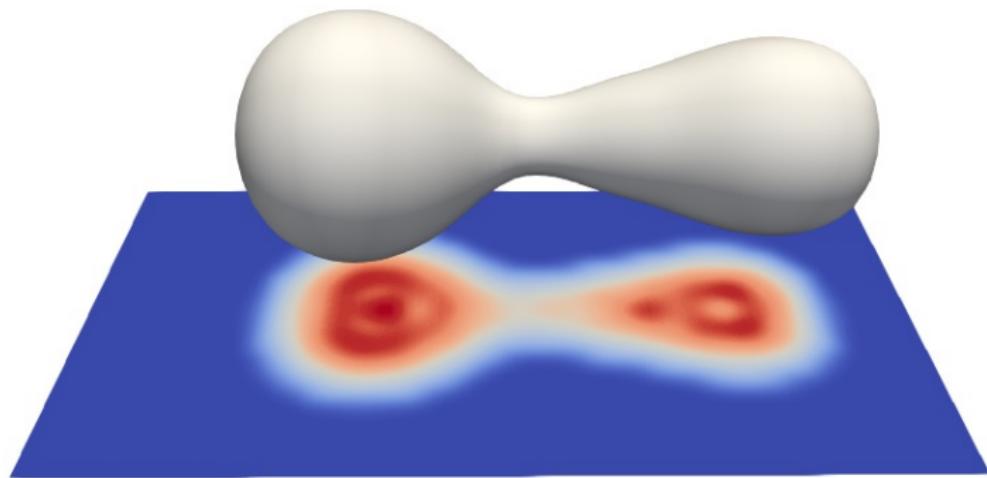
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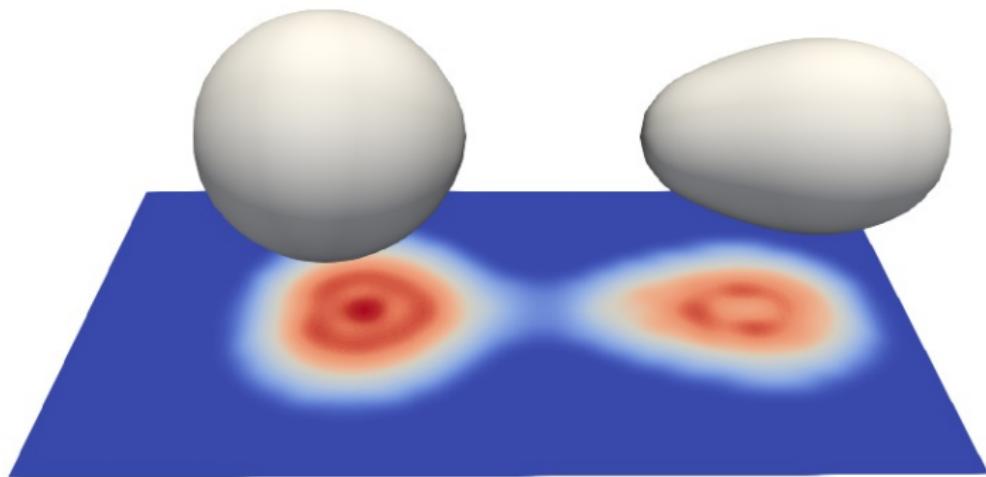
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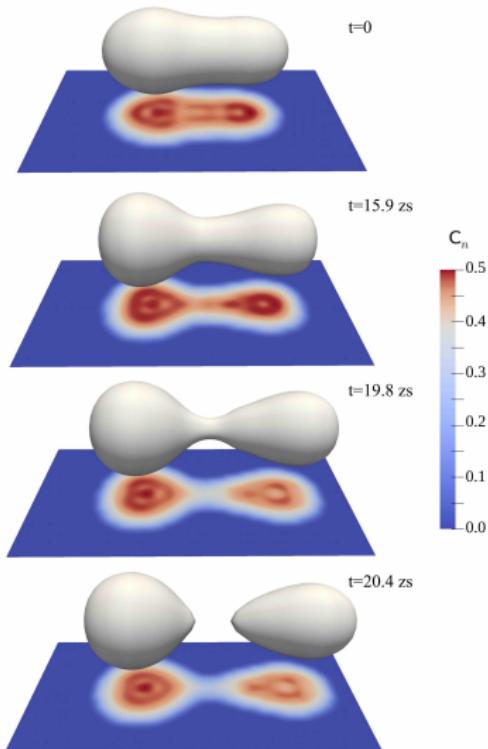


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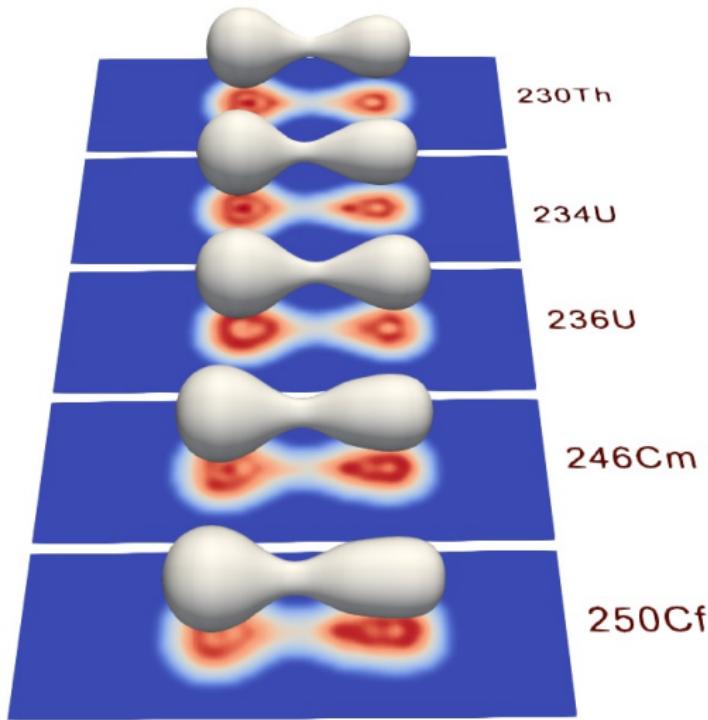
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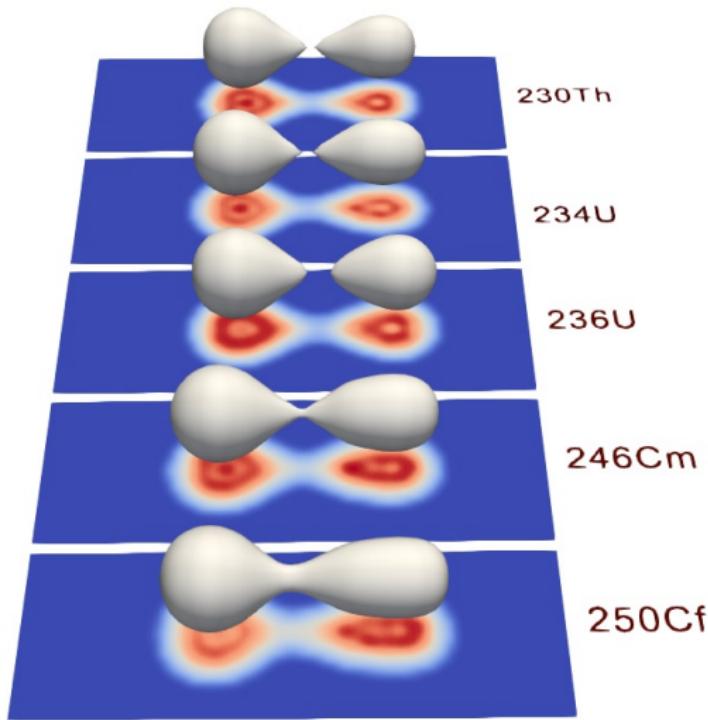
### Hypothesis

The octupole shell effects are important in the fission fragment

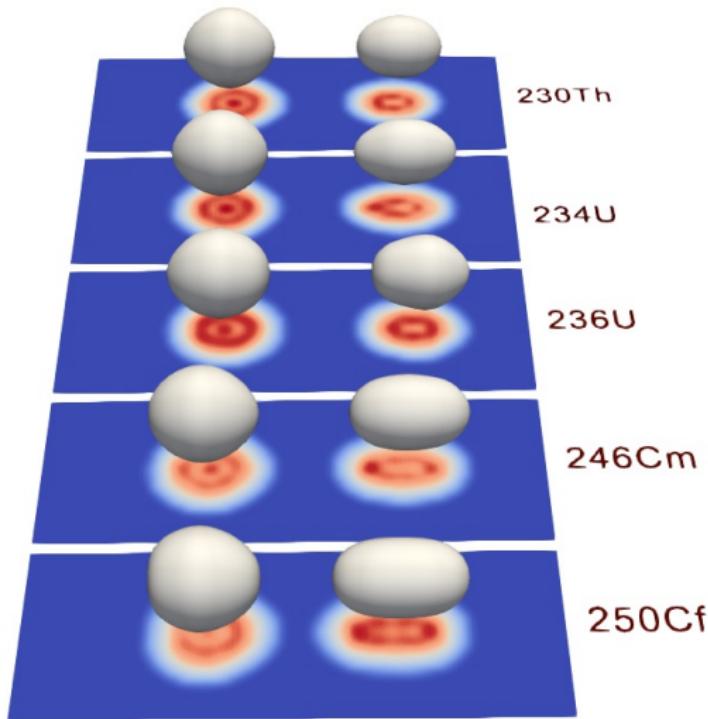
## Other systems



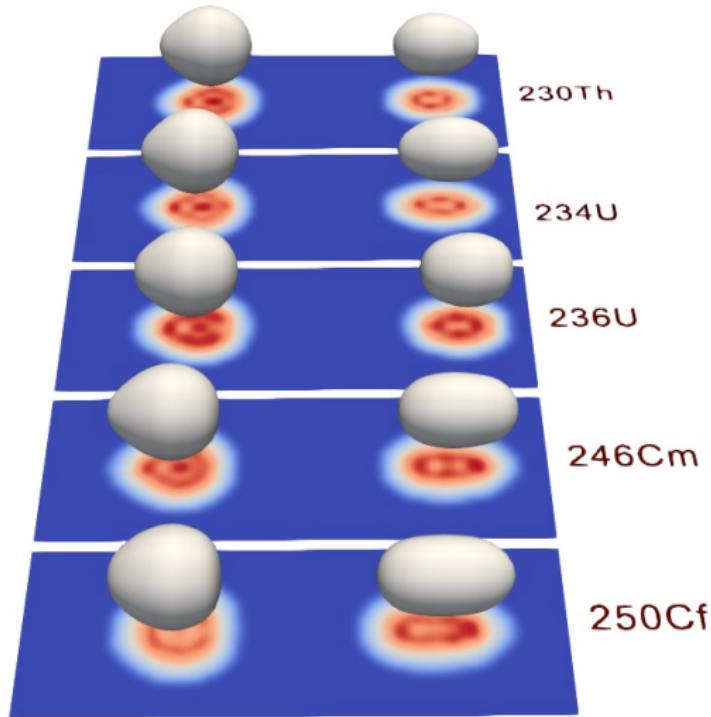
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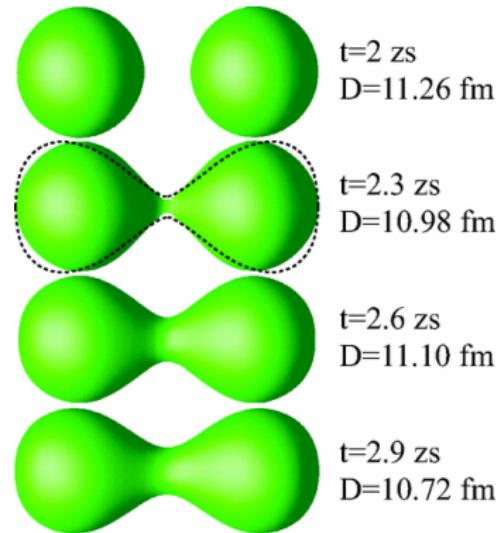
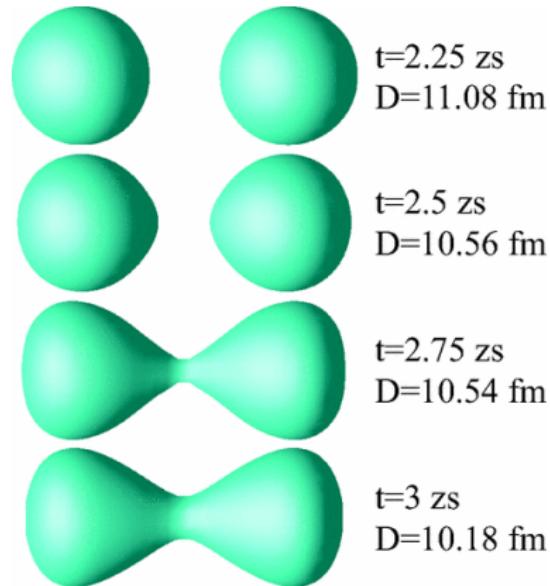


## Other systems



# Why the fragments have octupole deformation ?

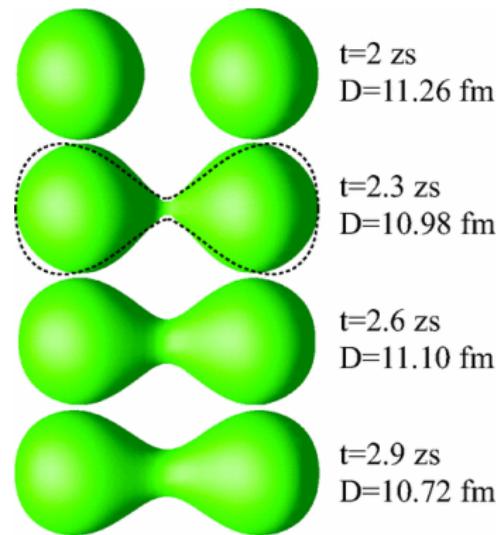
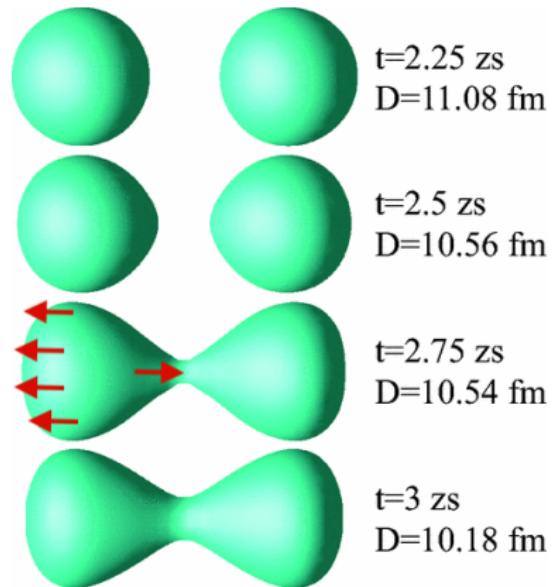
Similar effect on fusion reaction



C. Simenel, M. Dasgupta, D. J. Hinde, and E. Williams, Phys. Rev. C 88, 064604 (2013).

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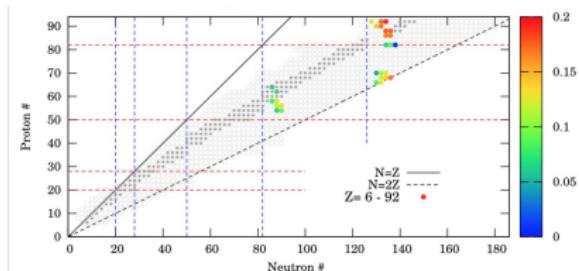
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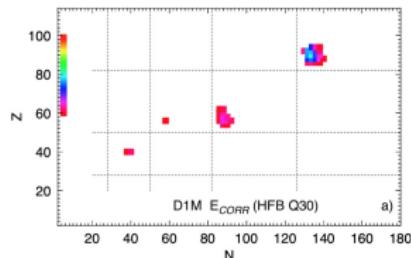
# Octupole deformation systematics

Skyrme Skm\*



S. Ebata, and T. Nakatsukasa, Phys. Scr. 92 (2017) 064005

Gogny D1S



LM Robledo - J. phys. G : Nucl. and Particle Physics, 2015

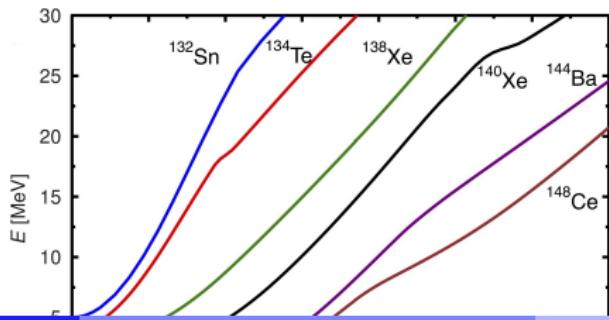
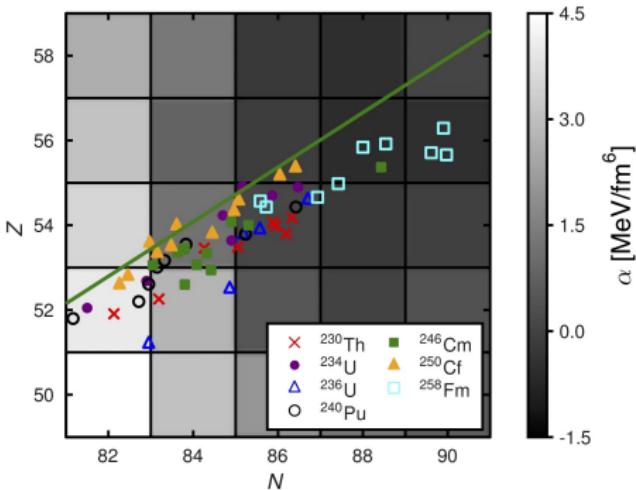
## Results from systematic calculation

In both calculations, the region  $Z \simeq 54$ ,  $N \simeq 88$  is favorable for octupole deformation .

## Experimental results

$^{144}\text{Ba}$  is found to be octupole in its ground state. Burcher et al. PRL 116 (2016).

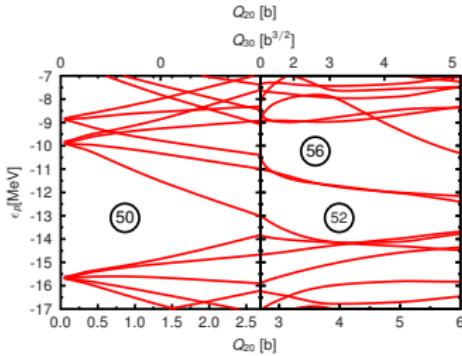
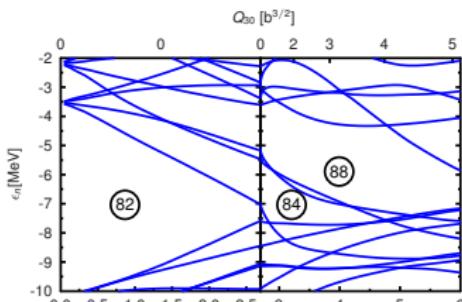
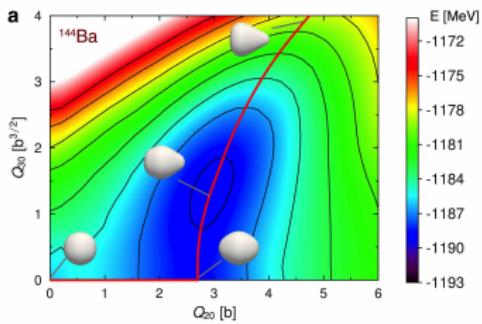
# Constraint HF+BCS octupole deformation with Sly4d



# Structure, $^{144}\text{Ba}$ , Z=56, N=88

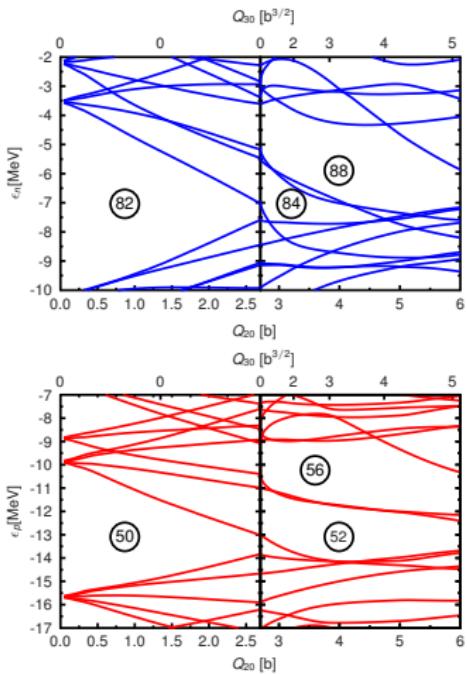
## Single particle energy

### $Q_2 - Q_3$ potential energy surface



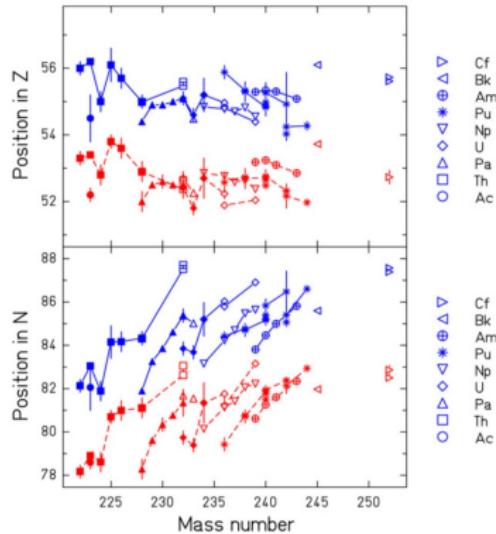
# Structure

## Single particle energies



## Experimental results

C. Böckstiegel et al. / Nuclear Physics A 802 (2008) 12–25



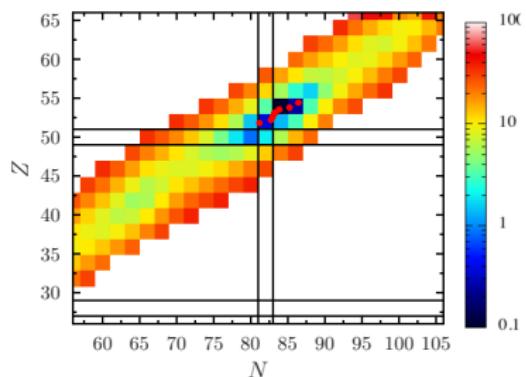
# Deformation energy at the scission. Simple scission point model

$$E(N, Z) = E_{\beta_3=0.35}(N, Z) + E_{\beta_2=0.8}(N_{\text{tot}} - N, Z_{\text{tot}} - Z) + e^2 \frac{Z(Z_{\text{tot}} - Z)}{D_{sc}} \quad (1)$$

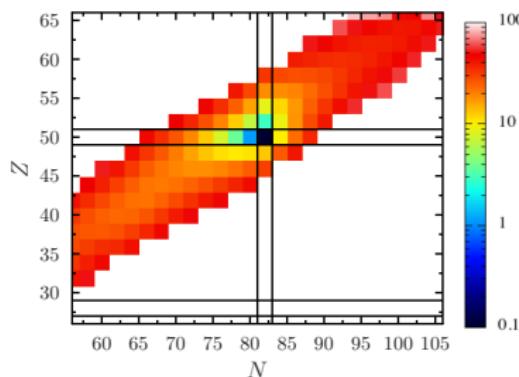
With  $D_{sc}=17$  fm. On the map,  $E(N, Z) - E_{\min}$  is shown.

For  $^{240}\text{Pu}$ ,  $N_{\text{tot}}=146$  and  $Z_{\text{tot}}=94$

Octupole deformation



Spherical heavy fragment



The energies have been calculated with the CHF+BCS theory Sly4d

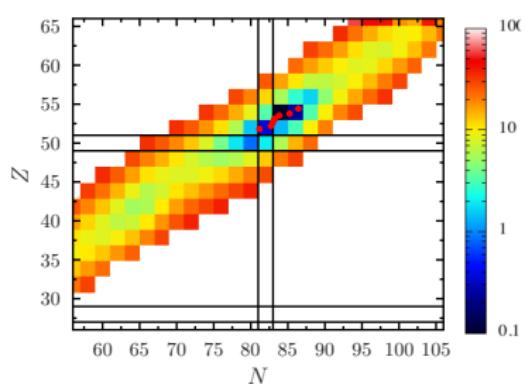
## Deformation energy at the scission. Simple scission point model

$$E(N, Z) = E_{\beta_3=0.35}(N, Z) + E_{\beta_2=0.8}(N_{\text{tot}} - N, Z_{\text{tot}} - Z) + e^2 \frac{Z(Z_{\text{tot}} - Z)}{D_{sc}} \quad (1)$$

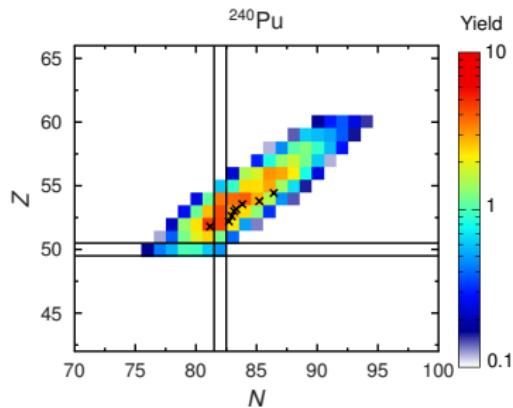
With  $D_{sc}=17$  fm. On the map,  $E(N, Z) - E_{\min}$  is shown.

For  $^{240}\text{Pu}$ ,  $N_{\text{tot}}=146$  and  $Z_{\text{tot}}=94$

Octupole deformation

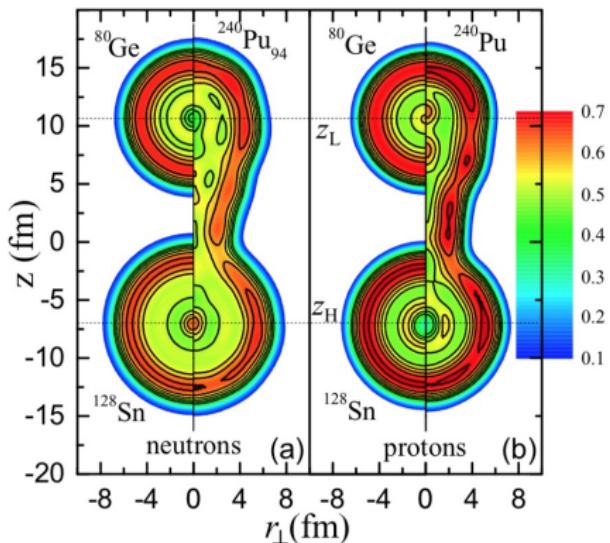


Experimental data



The energies have been calculated with the CHF+BCS theory Sly4d

# Identification method with the nucleon localisation function

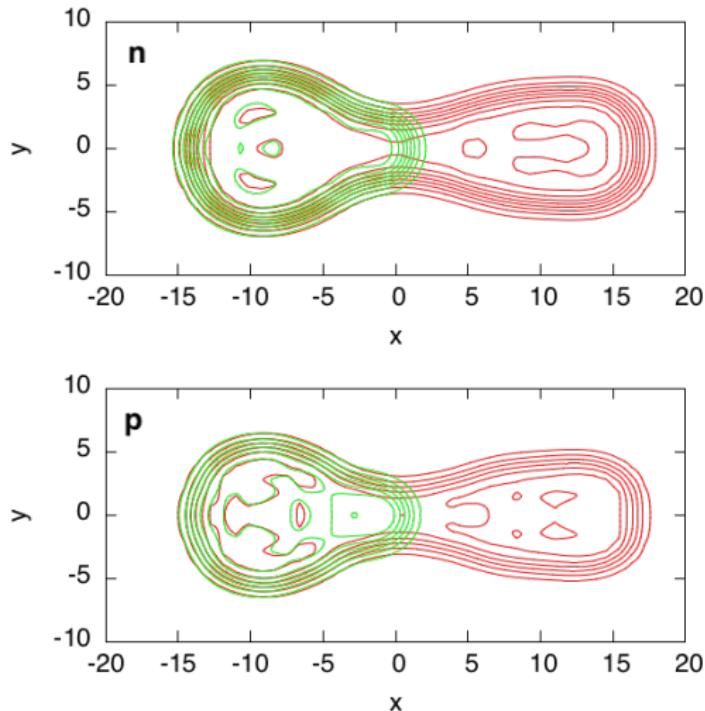


This method assumes that the pre-fragments have reflexion symmetry.

J. Sadhukhan, C. Zhang, W. Nazarewicz, and N. Schunck, PRC 96, 061301(R) (2017).

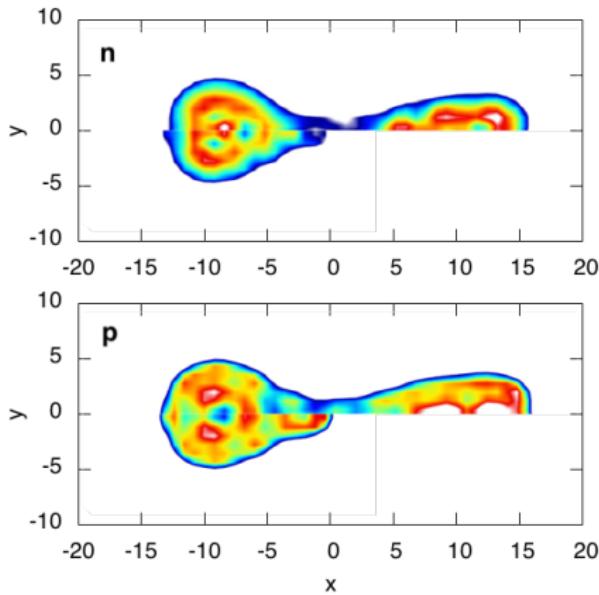
## Identification with density

Technique of : M. Warda, A. Staszczak, and W. Nazarewicz, PRC 86, 024601 (2012).



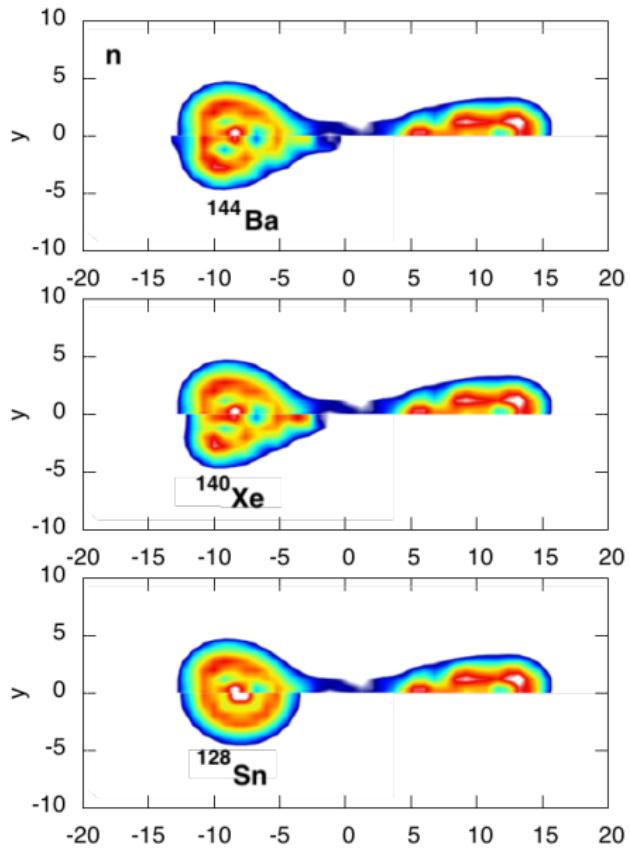
Green contour line : density of a  $^{144}\text{Ba}$  with a constraint  $\beta_3=0.42$   
Red contour line : density of a fissioning  $^{258}\text{Fm}$  (asymmetric mode)

## Identification with nucleon localisation function



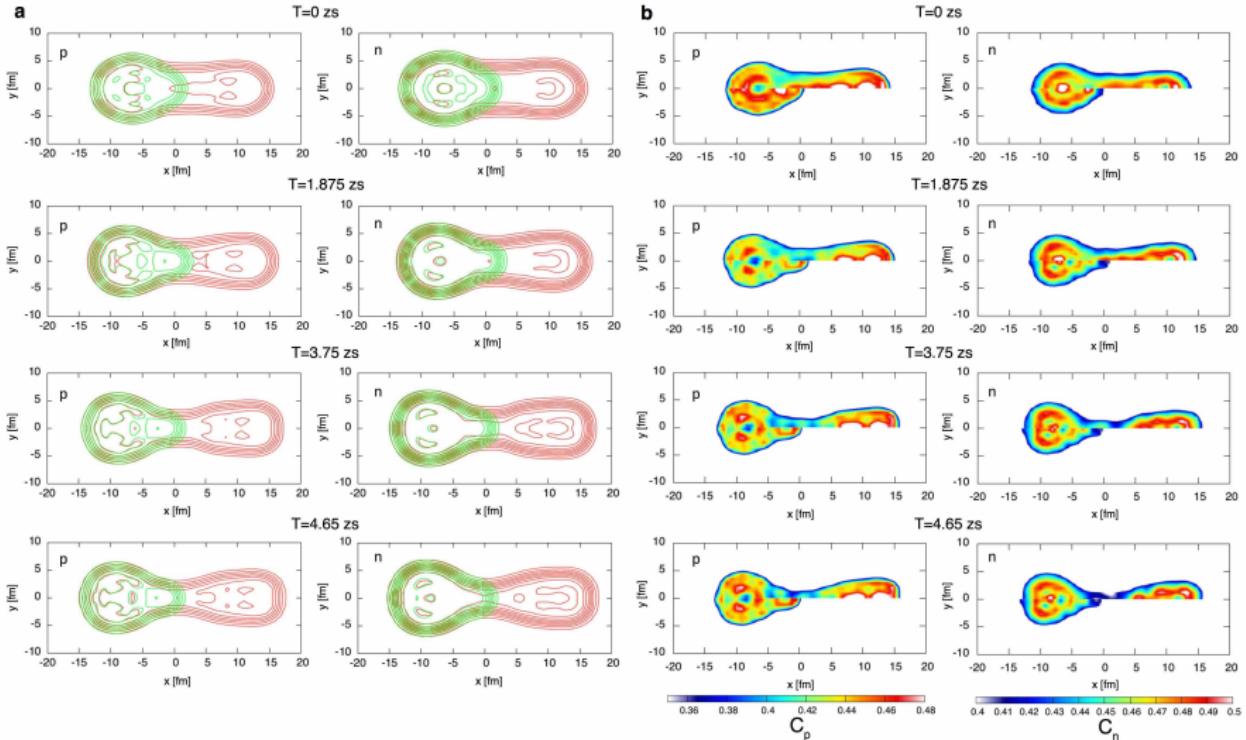
Top : NLF of a  $^{144}\text{Ba}$  with a constraint  $\beta_3=0.42$   
Bottom : NLF of a fissioning  $^{258}\text{Fm}$  (asymmetric mode)

# Identification with nucleon localisation function



# Identification method with octupole degree of freedom

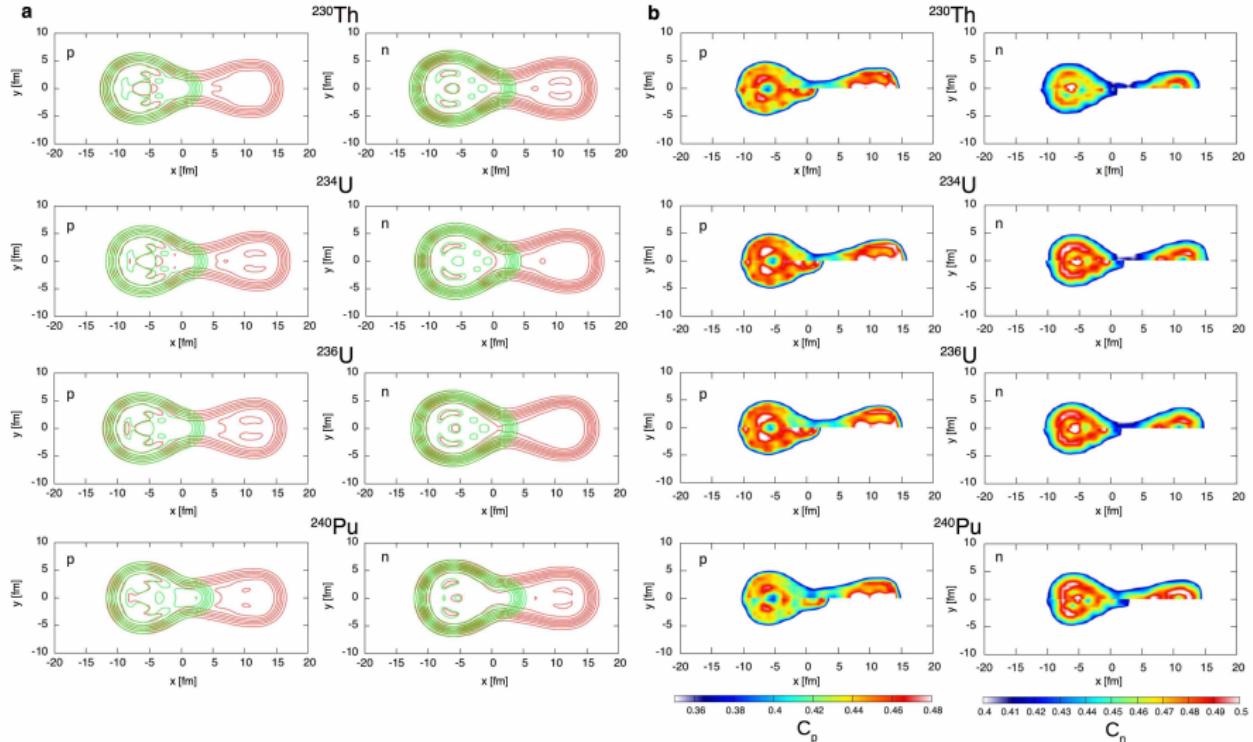
Identification of the fragments as a function of time for the fission of  $^{258}\text{Fm}$



All of the systems are identified as  $^{144}\text{Ba}$  with different  $\beta_3$  values (resp. 0.14, 0.39, 0.39 and 0.42)

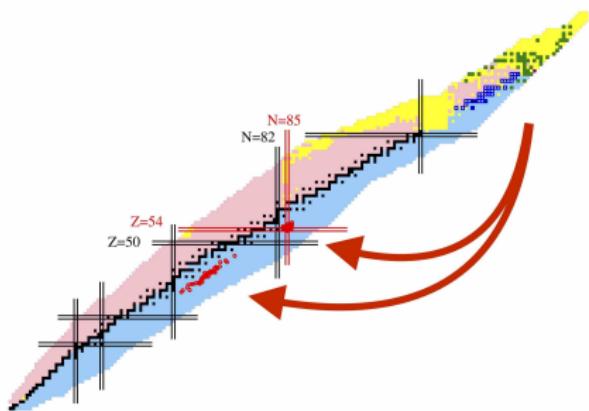
# Identification method with octupole degree of freedom

Identification of the fragments at the scission for the different elements.



All systems are identified as  $^{144}\text{Ba}$  with different  $\beta_3$  values (resp. 0.28, 0.28, 0.27 and 0.44)

# Conclusion



## Mechanism

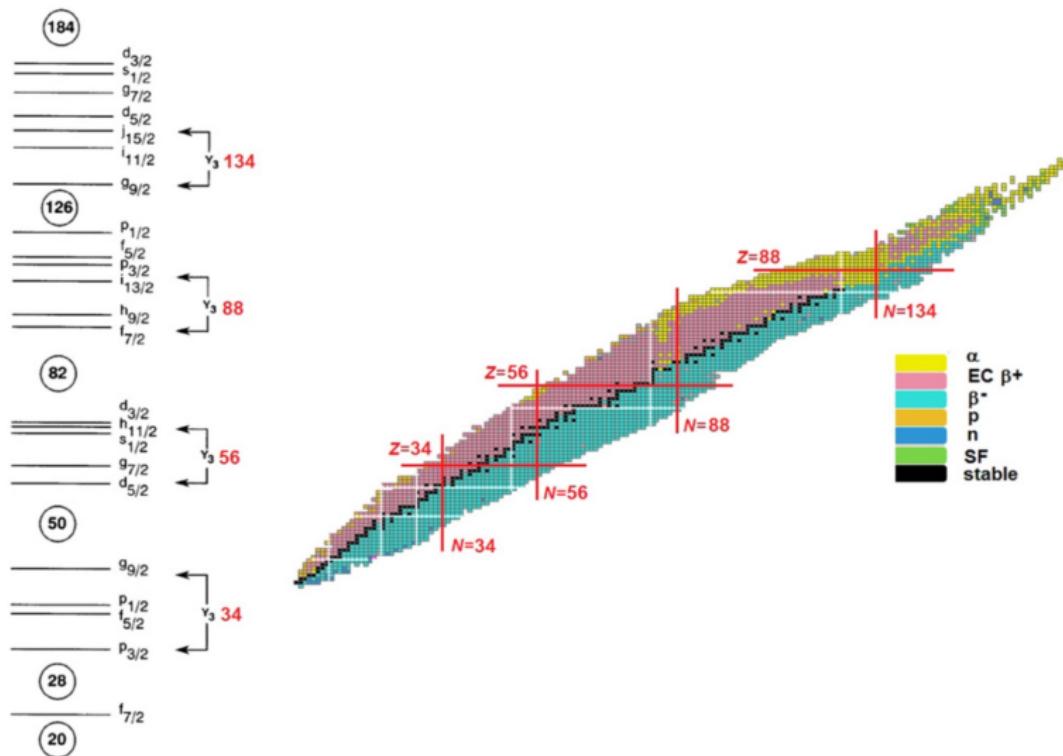
- The Nucleus-Nucleus interaction at the scission configuration favors the octupole shapes
- Shell structure favors octupole shape in the region  $Z \simeq 52-56$ ,  $N \simeq 84-88$
- Actinide fission fragments are driven in the region  $Z \simeq 54$ ,  $N \simeq 86$

G. Scamps, C. Simenel, arXiv :1804.03337 (2018).

# Similar effect for other systems ?

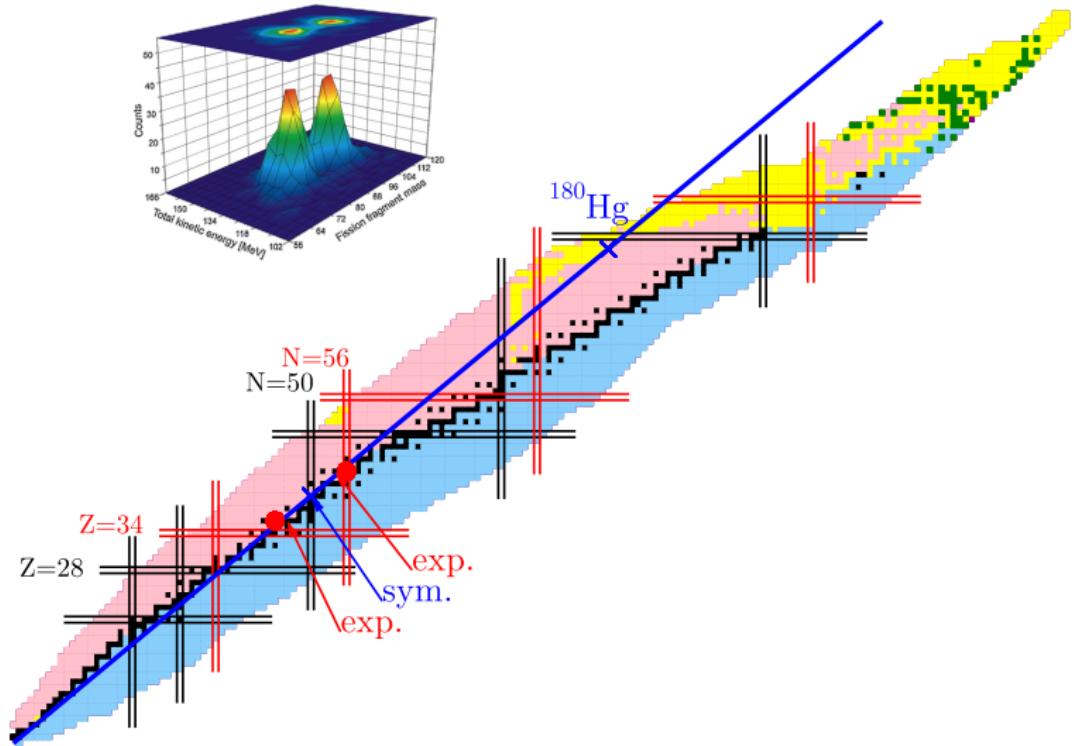
J. Phys. G: Nucl. Part. Phys. **43** (2016) 073002

Topical Review



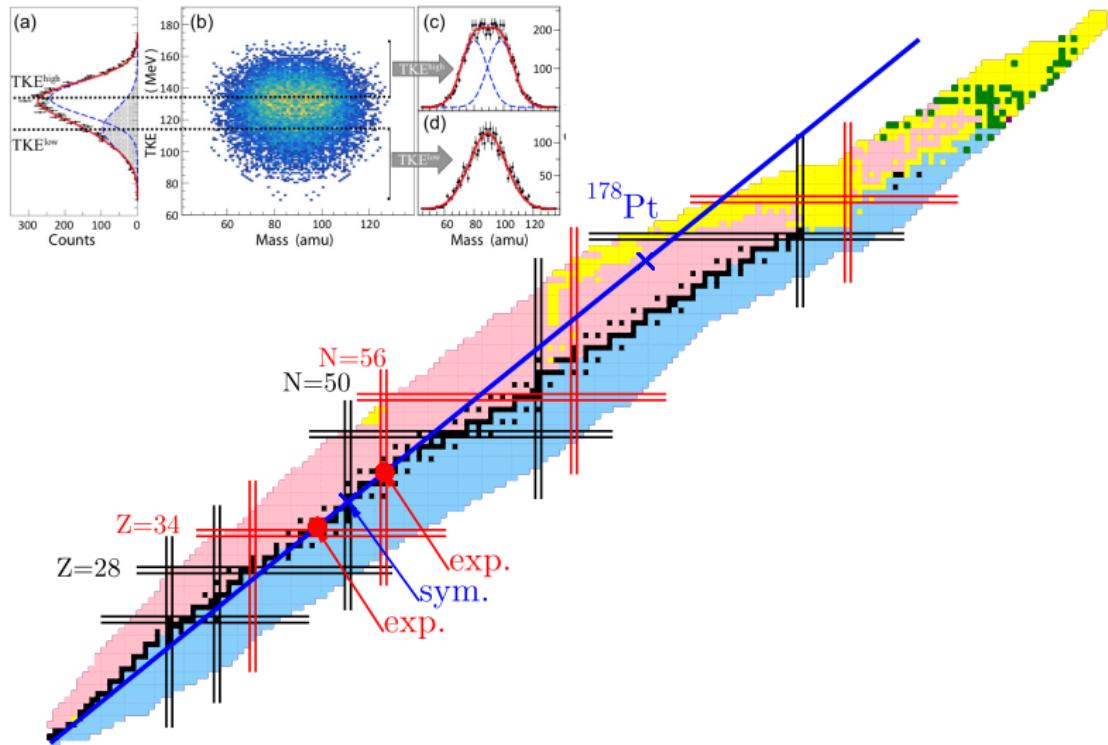
# Experimental data of $^{180}\text{Hg}$

A. N. Andreyev, et al., PRL 105, 252502 (2010)

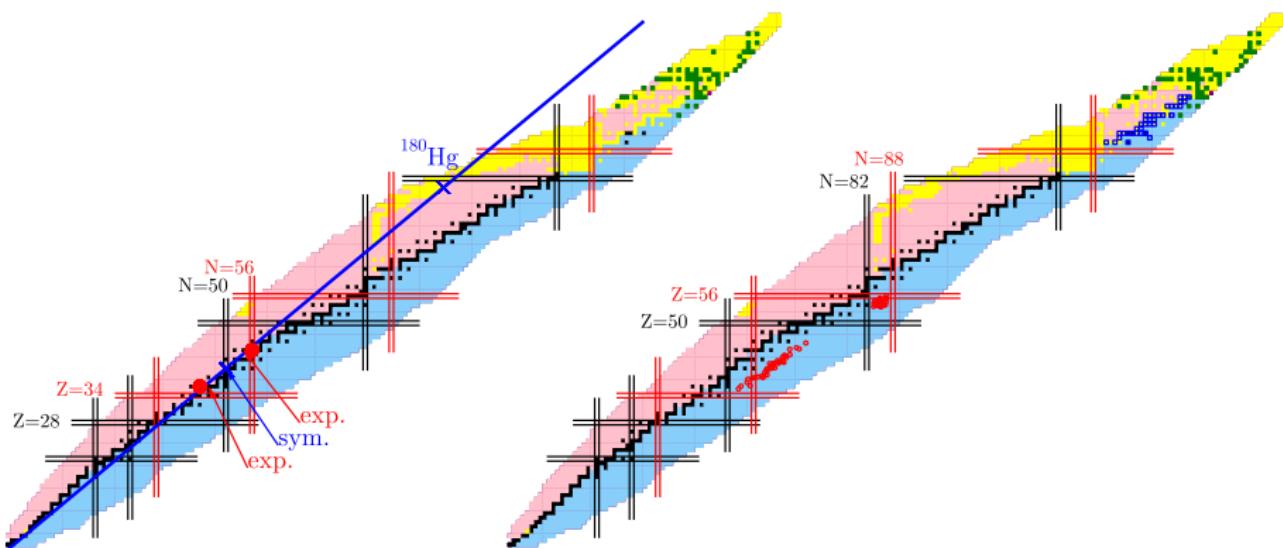


# Experimental data of $^{178}\text{Pt}$

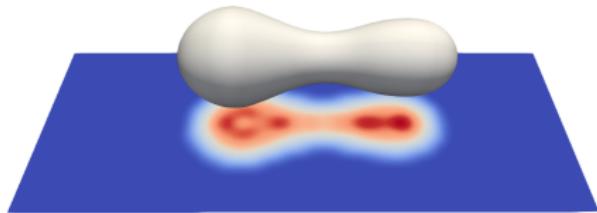
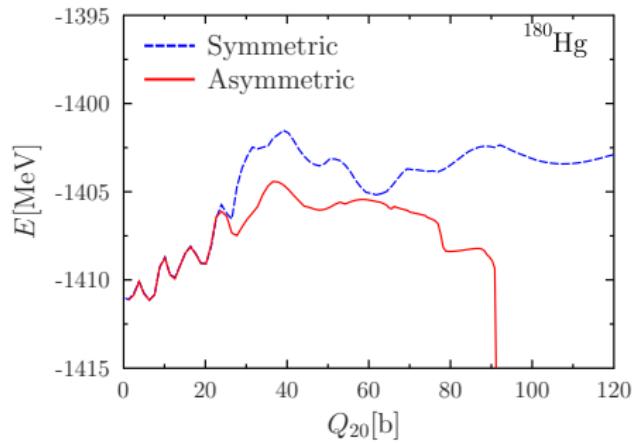
I. Tsekhanovich, ArXiv :1804.01832



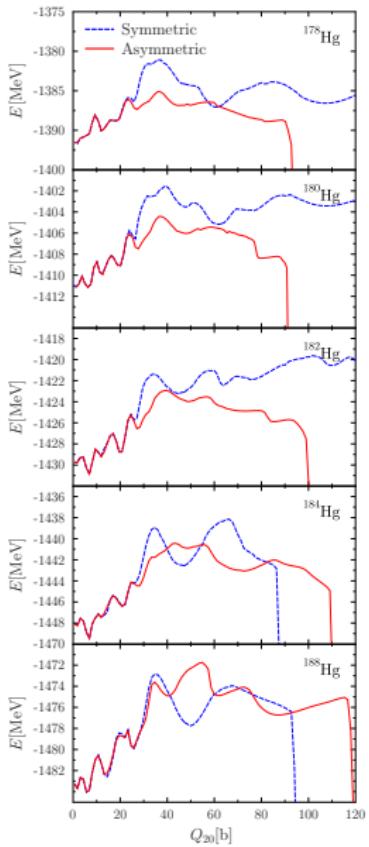
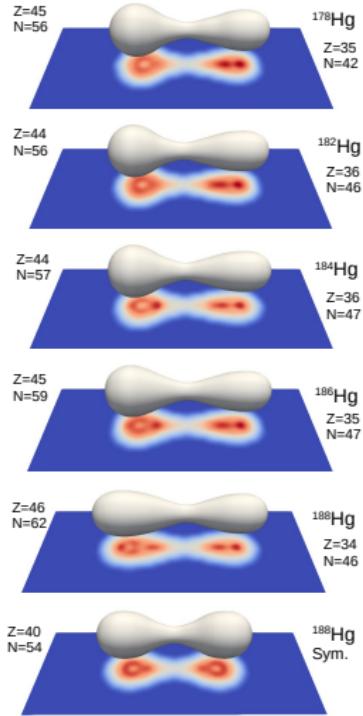
## Similar effect of the octupole deformation ?

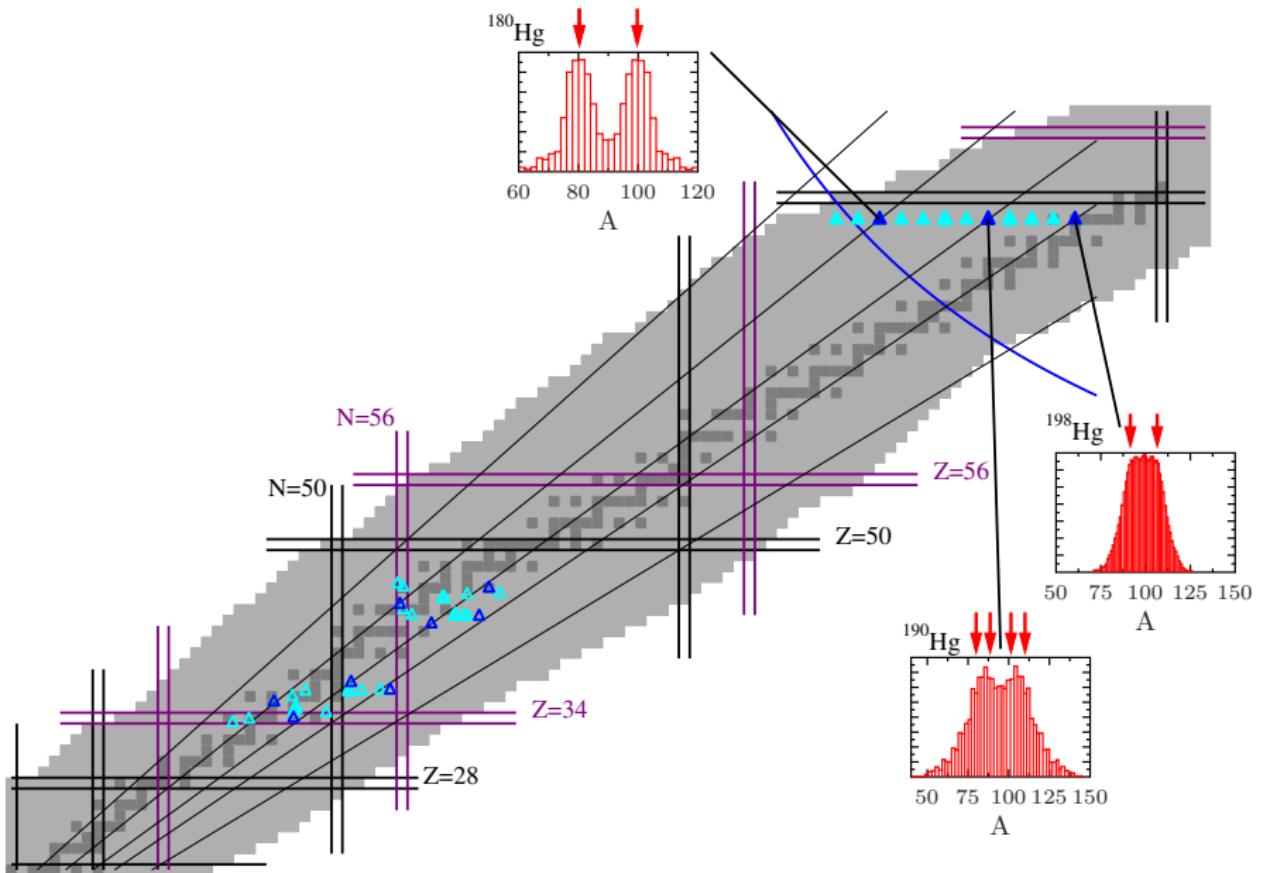


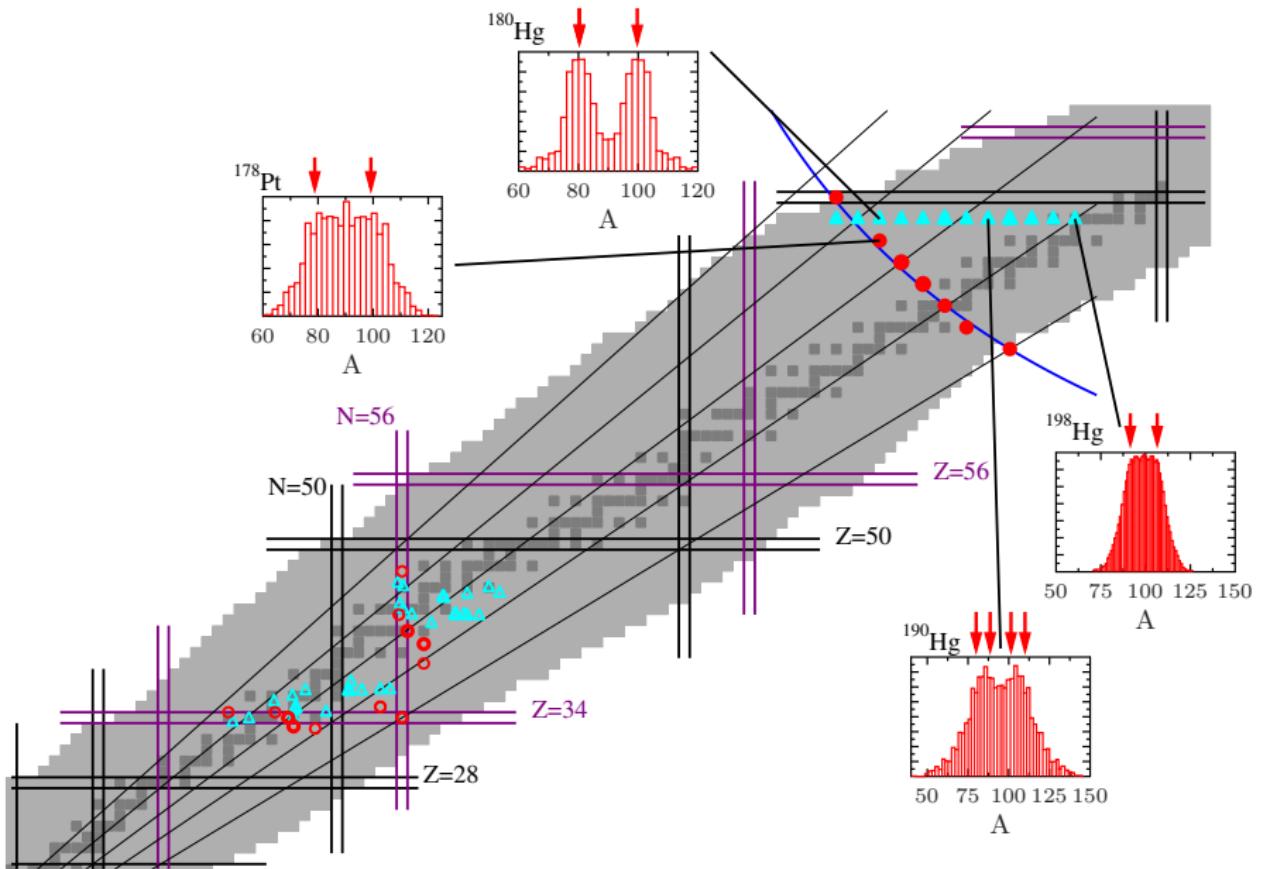
# CHF+BCS calculation



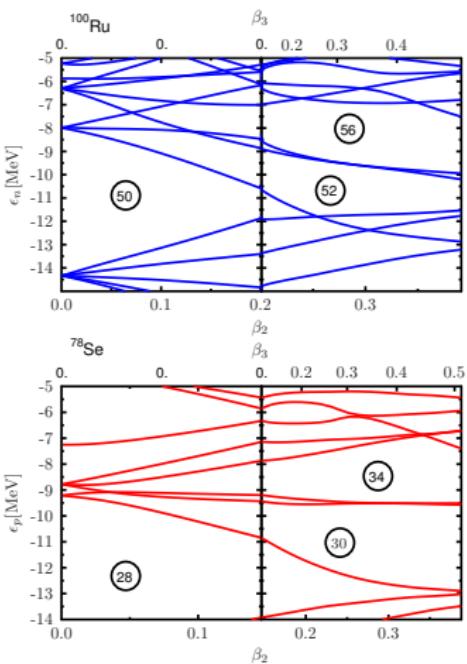
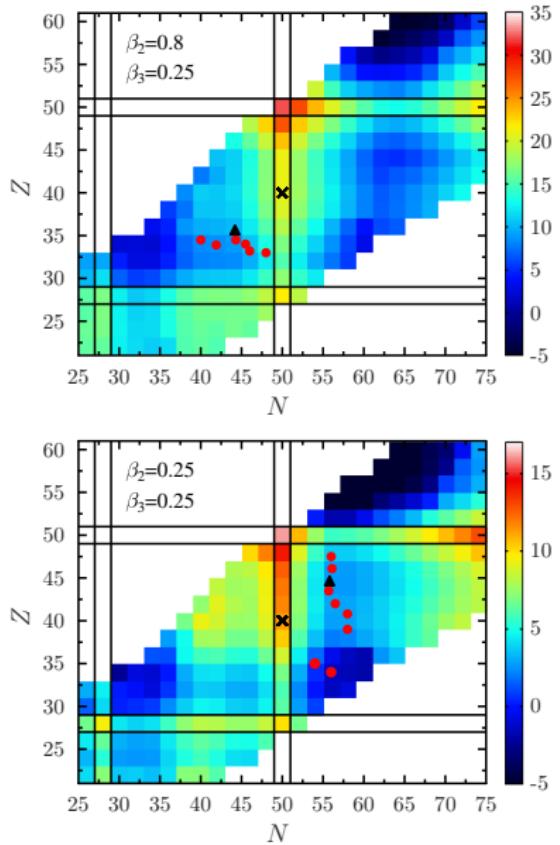
# CHF+BCS calculations : Hg isotopic chain





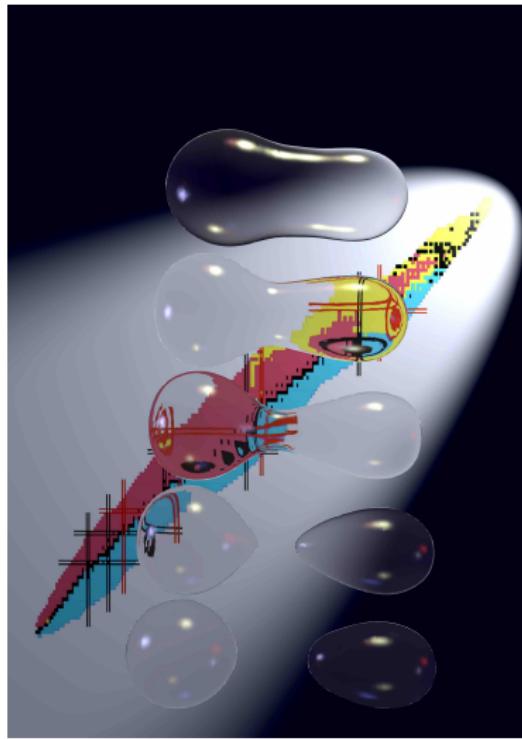


# Deformation energy of the fragments

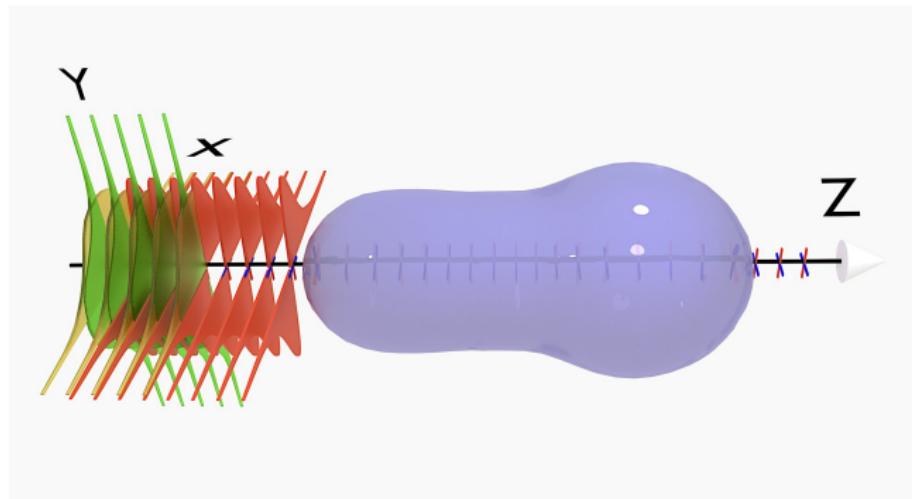


## Conclusion

The fission process magnify the octupole shell structure



# Preliminary Gogny-TDHFB calculation



- x and y direction :  
Harmonic oscillator  
basis  $n_x + n_y \leq N_{shell}$
- z direction : Cartesian  
mesh nz= 55
- $N_{base} \simeq 3000$
- full cartesian mesh  
about 100 000 degrees  
of freedom



Y. Hashimoto

# Preliminary test on $^{230}\text{Th}$

TDHFB  
(Gogny  
D1S)



TDHF+BCS  
(Sly4d)



# Preliminary test on $^{230}\text{Th}$

TDHFB  
(Gogny  
D1S)



TDHF+BCS  
(Sly4d)



## Preliminary test on $^{230}\text{Th}$

TDHFB  
(Gogny  
D1S)

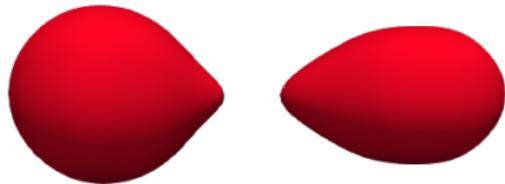


TDHF+BCS  
(Sly4d)



## Preliminary test on $^{230}\text{Th}$

TDHFB  
(Gogny  
D1S)

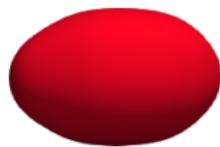


TDHF+BCS  
(Sly4d)

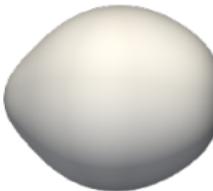


# Preliminary test on $^{230}\text{Th}$

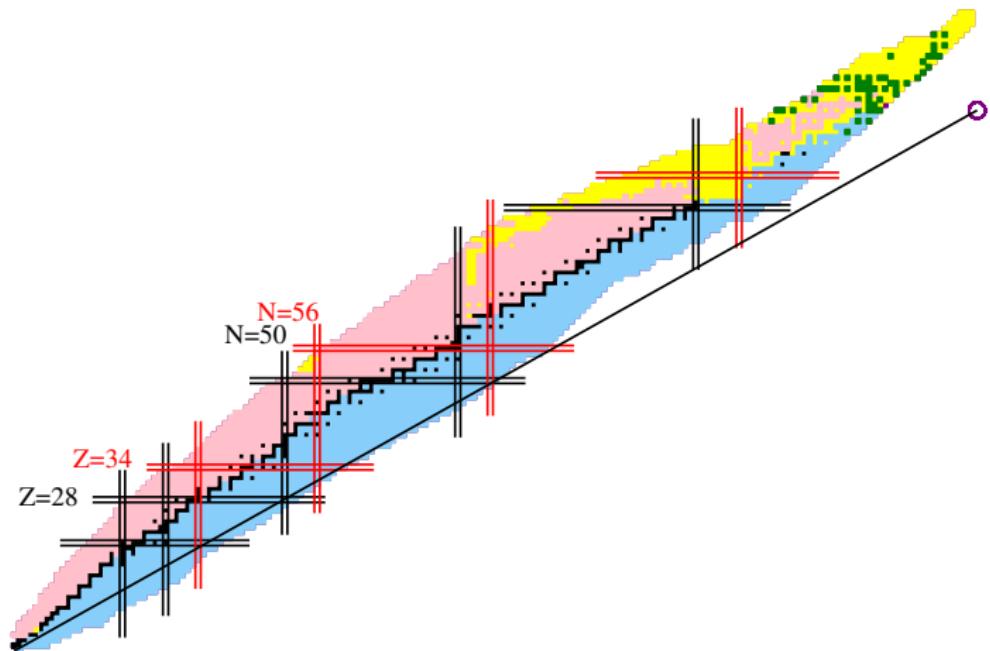
TDHFB  
(Gogny  
D1S)



TDHF+BCS  
(Sly4d)



## Outlook : Fission recycling



Region where BCS approximation is forbidden (due to continuum)

Thank you

# Comparison TDHFB - TDHF+BCS

## TDHF+BCS

- Based on TDHFB with the approximation :  $\Delta_{ij} = \delta_{ij}\Delta_i$
- Evolution :  $i\hbar \frac{d\varphi_i}{dt} = (\hat{h}_{MF} - \epsilon_i)\varphi_i$   
 $i\hbar \frac{dn_i}{dt} = \Delta_i^* \kappa_i - \Delta_i \kappa_i^*$   
 $i\hbar \frac{d\kappa_i}{dt} = \kappa_i(\epsilon_i - \epsilon_{\bar{i}}) + \Delta_i(2n_i - 1)$

## Theoretical difference

- Numerical cost : TDHFB requires 1000 times more numerical resources
- Treatment of continuum states : BCS gas problem
- Continuity equation
- Number of pairing degrees of freedom ( HFB  $\Delta(r)$ , BCS :  $\Delta_{i\bar{i}}$  )
- Spatial dependence of the pairing correlation

## Comparison

A. Bulgac, P. Magierski, K. J. Roche, and I. Stetcu, PRL 116, 122504 (2016).

S no.	$\eta$	$E^*$	$E_n$	$q_{zz}$	$q_{zzz}$	$t_{SS}$	$TKE^{syst}$	$TKE$	$A_L^{syst}$	$A_L$	$N_L^{syst}$	$N_L$	$Z_L^{syst}$	$Z_L$	
S1	0.75	8.05	1.52	1.78	-0.742	14 419	177.27	182	100.55	104.0	61.10	62.8	39.45	41.2	
S2	0.5	7.91	1.38	1.78	-0.737	4360	177.32	183	100.56	106.3	60.78	64.0	39.78	42.3	
S3	0	8.08	1.55	1.78	-0.737	14 010	177.26	180	100.55	105.5	60.69	63.6	39.81	41.9	
S4	0	6.17	-0.36	2.05	-0.956	12 751	177.92	181			103.9		62.6		41.3

TABLE – TDHF+BCS results for  $^{240}\text{Pu}$

#	$Q_0$ [b]	$E_0^*$ [MeV]	$T_{\text{fis}}$ [fm/c]	$Z_L$	$N_L$	$TKE$ [MeV]
1	45.4	1.46	6480	40.21	60.77	171.5
2	46.7	0.8	4830	40.83	62.68	181.8
3	50.5	-1.16	26970	42.2	64.83	181.8
4	53.0	-2.13	6750	41.39	63.05	177.9
5	56.8	-3.5	4800	40.99	62.85	177.2
6	59.3	-4.3	5400	40.45	62.17	178.4
7	63.1	-5.31	6630	39.55	59.58	162.7
8	71.9	-7.8	1020	41.8	63.28	179.9