Multi-reference EDF theory alternative to GCM

Application to the pairing model

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Contents:

- Problems of GCM
- Alternative method based on TDHFB: ASCC + SPA
- Application to pairing model

GCM; success and failures

Generator Coordinate Method (GCM) is a powerful tool to investigate correlations beyond the mean field.

Generalized eigenvalue problem (Hill-Wheeler eq.)

$$|\Psi\rangle = \int d\alpha f(\alpha)|\phi(\alpha)\rangle \qquad \int d\alpha f^*(\alpha)\langle\phi(\alpha)|H - E|\phi(\alpha')\rangle = 0$$

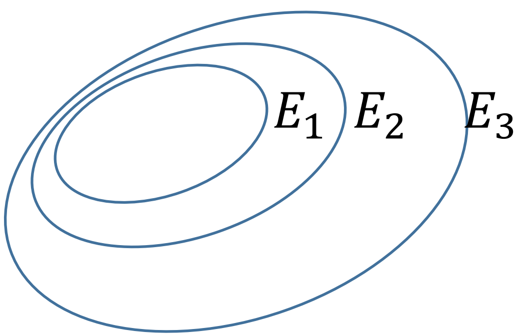
Quantum mechanical method applicable to LACM

- Problems
  - Continuum limit
    - Bonche, et al., NPA510, 466 (1990)
  - Singular behaviors for EDFs with fractional power of ρ
    - Anguiano, Egido, Robledo, NPA696, 467 (2001)
    - Dobaczewski et al., PRC76, 054315 (2007)
  - Choice of (complex) generator coordinates
    - Holzwarth, Yukawa, NPA219, 125 (1974) : Double projection
    - Peierls, Thouless, NP 38, 154 (1962) : Necessity of time-odd fields
  - 1D, 2D, complex GCM, ... where should we stop?

Quantization of TDHF(B)

- Advantages
  - Dynamics with time-odd mean fields
  - Small amplitude limit is guaranteed to be (Q)RPA
- Disadvantages
  - Missing quantum correlations/fluctuations
  - Energy eigenstates beyond (Q)RPA
  - Requantization requires periodic trajectories

$$\sum_{\alpha} \oint p_{\alpha} dq^{\alpha} = 2\pi\hbar n$$

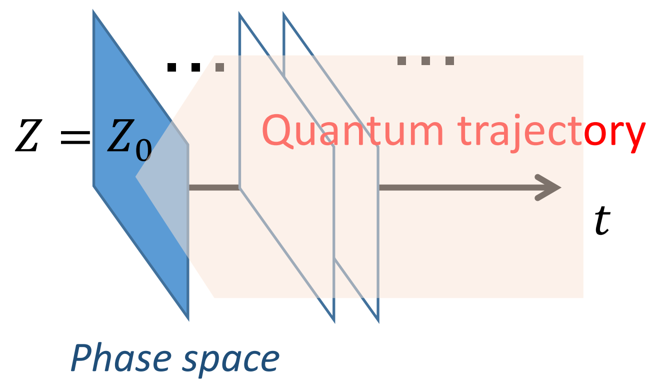


Stationary phase approximation（SPA)

Path integral expression

$$|\psi(t)\rangle = e^{-iHt/\hbar}|\psi(t=0)\rangle$$

$$= \int d\mu(Z)|Z\rangle \int D\mu(Z) e^{iS[Z]/\hbar}$$



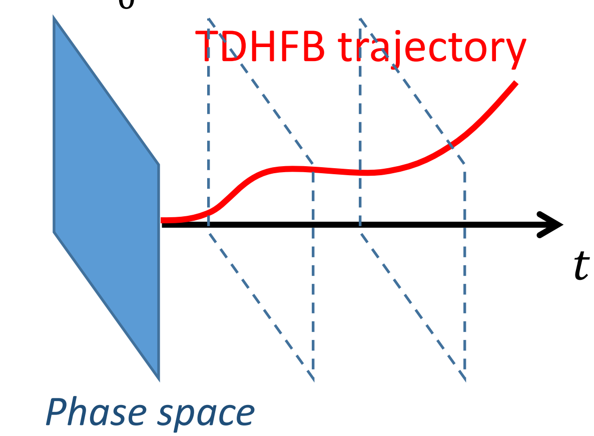
(Gen.) Slater det. $|Z(t)\rangle \propto \prod_{k<k'} e^{Z_{kk'}(t)\beta_k^+\beta_{k'}^+} |\phi_0\rangle$

$d\mu(Z)$: invariant measure with $\int d\mu(Z)|Z\rangle\langle Z| = 1$

SPA (classical trajectory) is nothing but TDHF(B)

$$\delta S_{cl} = 0$$

$$\boxed{|\tilde{\psi}(t)\rangle = \int d\mu(Z)|Z\rangle e^{iS_{cl}(Z)/\hbar}}$$



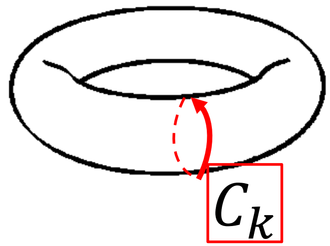
$S_{cl}(Z)$: Classical (TDHFB) action

Stationary states (energy eigenstates) correspond to periodic trajectories.

Integrable systems

Kuratsuji, Suzuki, PLB 92, 19 (1980)
Kuratsuji, PTP 65, 224 (1981)
Suzuki, Mizobuchi PTP 79, 480 (1988)

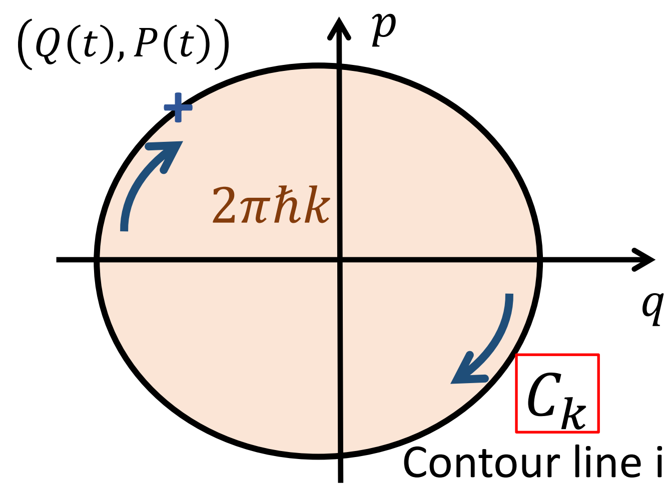
Separable with invariant tori



$$|\tilde{\psi}_k\rangle = \oint d\mu(Z_k)|Z_k\rangle e^{iT(Z_k)/\hbar}$$

Integration over a closed trajectory on invariant tori

2D Phase space



Contour line in k-th excited energy E_k

- EBK quantization condition

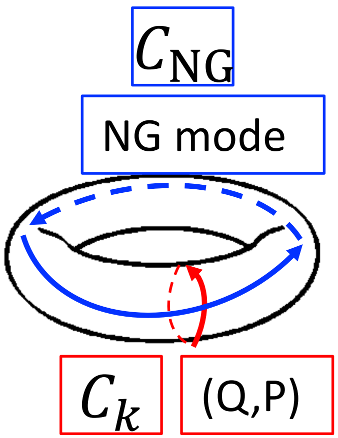
$$T_\circ = \oint_{C_k} \langle Z(t')|i\hbar\frac{\partial}{\partial t'}|Z(t')\rangle dt' = \oint_{C_k} \sum_\alpha p_\alpha dq^\alpha = 2\pi\hbar k \qquad k\text{: integer}$$

ASCC + SPA method

Adiabatic self-consistent collective coordinate (ASCC) Method

Determine a separable decoupled collective subspace

Solutions of ASCC provide a decoupled collective subspace associated with constants of motion (NG modes), and "non-trivial" collective variables, (Q, P).



SPA of the path integral quantization

EBK quantization and a wave function of a collective state

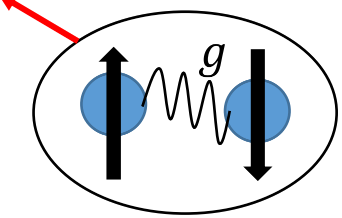
A closed trajectory on tori gives the energy E_k, action $T(Z_k)$, and wave functions, $|\tilde{\psi}_k\rangle$

$$|\tilde{\psi}_k\rangle = \oiint_{C_{NG}+C_k} d\mu(Z_k)|Z_k\rangle e^{iT(Z_k)/\hbar}$$

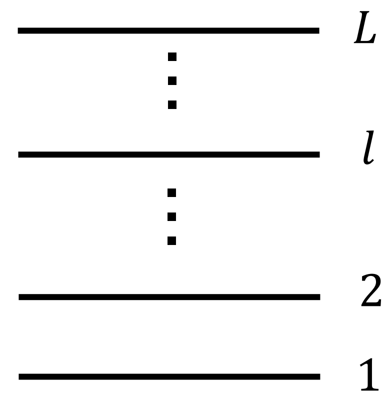
Pairing (Richardson) model

$$H=\sum_{l}\epsilon_l n_l - g\sum_{ll'} S_l^{+}S_{l'}^{-}$$

$$\begin{cases} n_l = \sum_m a_{lm}^{+}a_{lm} \\ S_l^{+} = \sum_{m>0} a_{lm}^{+}a_{l\bar{m}}^{+} \end{cases}$$



$J=0$



L s.p. level system

- TDHFB dynamics

$$\begin{cases} \dot{\chi}^{\alpha} = \dfrac{\partial\mathcal{H}}{\partial j_\alpha} \\ \dot{j}_\alpha = -\dfrac{\partial\mathcal{H}}{\partial\chi^{\alpha}} \end{cases}$$

$$|Z(t)\rangle = \prod_{\alpha}\frac{1}{(1+|Z_\alpha(t)|^2)^{S_\alpha}}\, e^{Z_\alpha(t)S_\alpha^{+}}\,|0\rangle$$

$$Z_\alpha \to (\chi^{\alpha}, j_\alpha) \qquad \langle Z|H|Z\rangle = \mathcal{H}\big(\chi(t), j(t)\big)$$

Tow-level pairing model is integrable.
Conserved quantities: *E* and *N*

Other quantization methods

Canonical quantization

Collective Hamiltonian $H_{\mathrm{coll}}(q,p) = \frac{(N-N_0)^2}{2\mathfrak{J}} + \frac{1}{2\bar{B}(q)} p^2 + \bar{V}(q)$

Pauli's prescription with $p = -i\partial_q$

Solve the collective Schroedinger eq.

Fourier decomposition

Energy given by the EBK quantization, the same as SPA

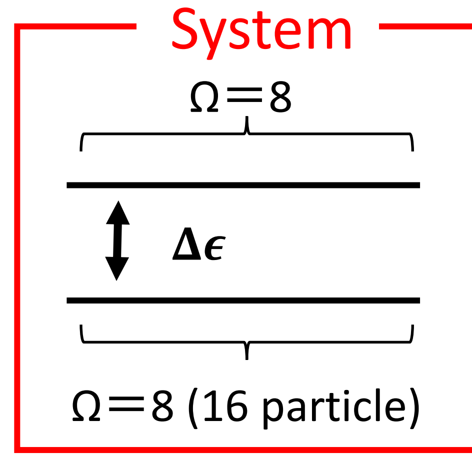
Time dependent expectation value; $F(t) = \langle \psi(t)|F|\psi(t)\rangle$

$$\langle k+\Delta k|F|k\rangle = F_{\Delta k}^{k}(\omega) = \frac{1}{T}\oint_0^T F(t) e^{i\omega t} dt$$

with $\omega = \frac{2\pi}{T}\Delta k$

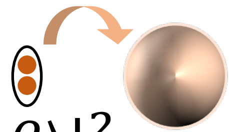
Two-level pairing model

F. Ni and TN, PRC **97**, 044310 (2018)

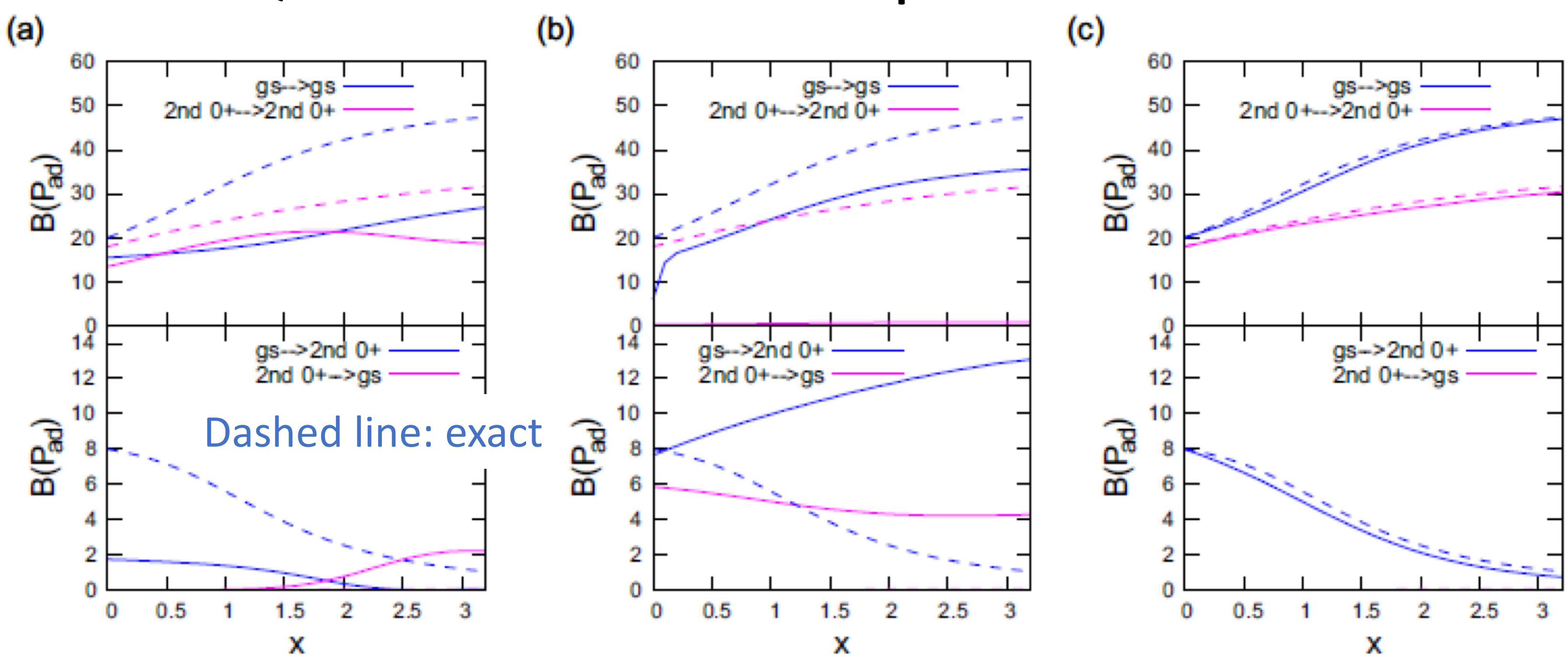


- Pair-additional transition

$$B(P_{ad}) = |\langle N=8,\alpha|S_+|N=6,\beta\rangle|^2$$



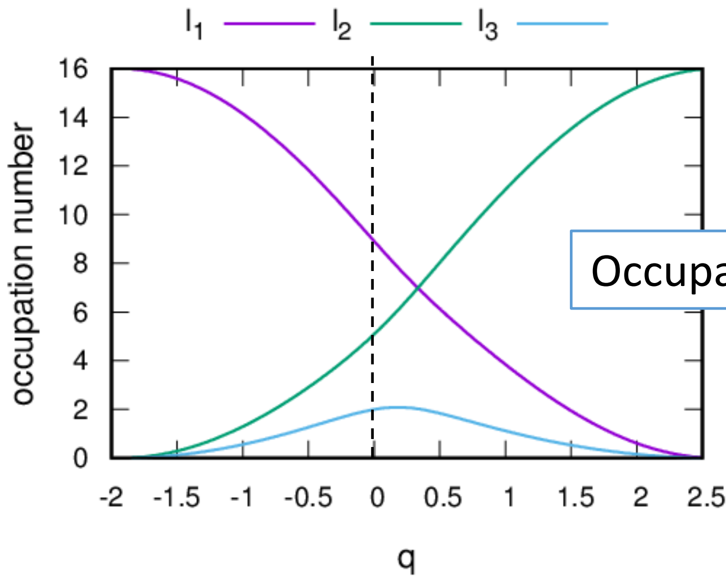
Canonical Quantization **Fourier Decomposition** **SPA**



Pairing collectivity is relatively weak.

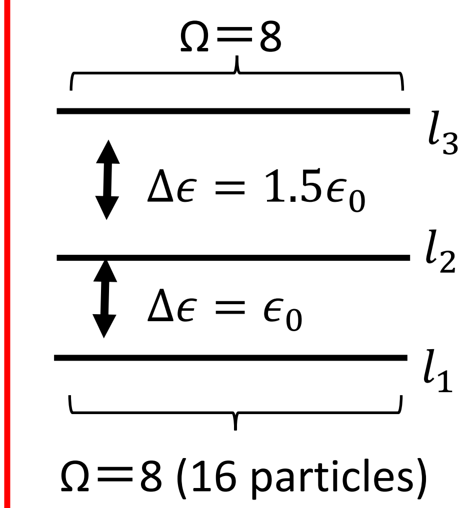
→ Superiority of SPA

Three-level pairing model



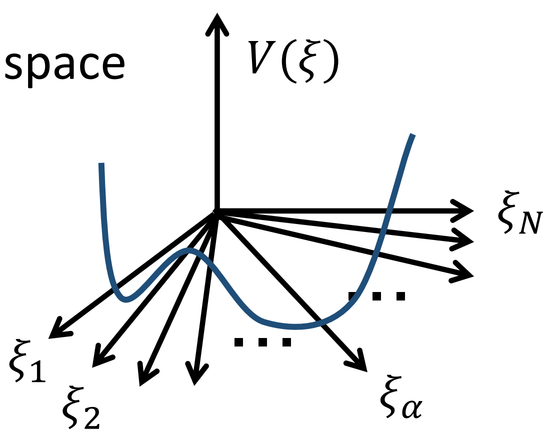
$N = 16$

Occupation number



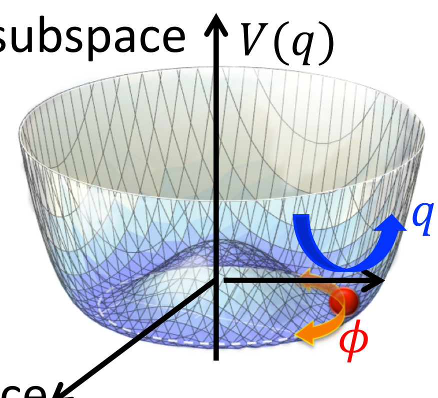
- Pairing strength

$$g = 0.2\epsilon_0$$



ASCC

1D pair rotation +
1D collective subspace



$$H(\xi, \pi) = \frac{1}{2} B^{\alpha\beta} \pi_\alpha \pi_\beta + V(\xi)$$

$$H_{\text{coll}}(q, p) = \frac{N^2}{2\mathfrak{I}} + \frac{1}{2\bar{B}(q)} p^2 + \bar{V}(q)$$

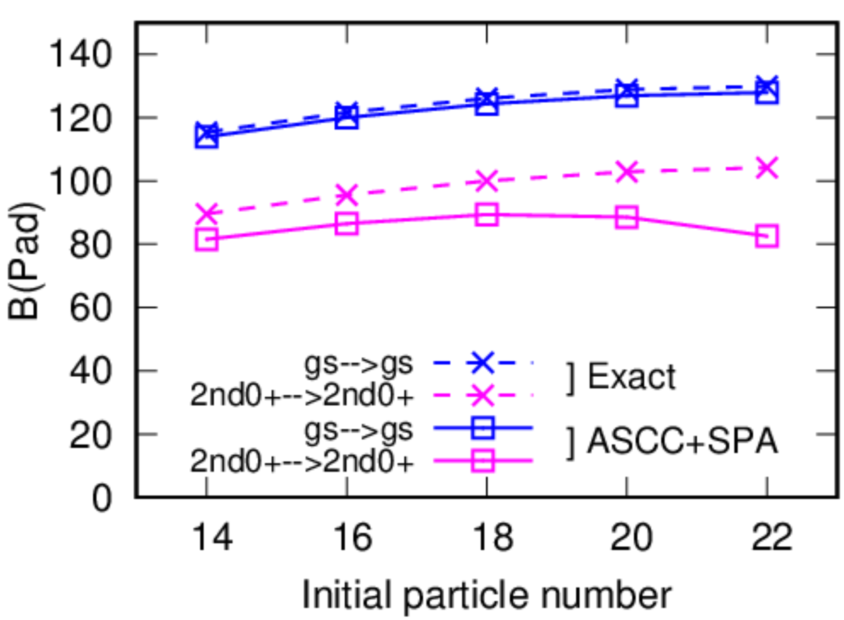
Three-level pairing model

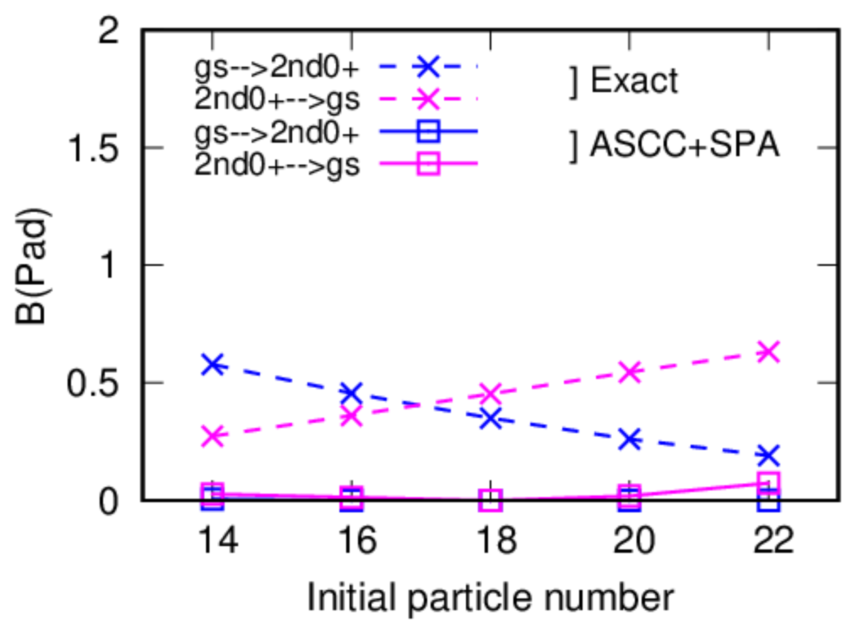
F. Ni, N.Hinohara, TN, arXiv:1811.02352

- Excitation energy (in units of ϵ_0)

		N=14	N=16	N=18	N=20	N=22	N=24
One-phonon (0_2^+)	**Exact**	4.09	4.13	4.20	4.30	4.44	4.60
	ASCC+SPA	3.87	3.90	3.97	4.09	4.23	4.33
Two-phonon (0_4^+)	**Exact**	7.65	7.71	7.88	8.15	8.49	8.74
	ASCC+SPA	7.42	7.42	7.60	7.92	8.26	8.47

- Two-particle addition transition $B(Pad) = |\langle N+2, \alpha | S_+ | N, \beta \rangle|^2$





Neutron pairing vibrations in Pb isotopes

F. Ni, N.Hinohara, TN, arXiv:1811.02352

Pairing vibration of neutron in Pb isotope

Input:

126

Neutron

s.p. level	Energy (MeV)
p1/2	-7.45
f5/2	-8.16
p3/2	-8.44
i13/2	-8.74
f7/2	-10.69
h9/2	-10.94

82

- $g = 0.138$ (MeV) is adopted so as to reproduce experimental pairing gap of $^{192}\mathrm{Pb}$ in three-point formula

- Results: Excitation energy of $|0_2^+\rangle$

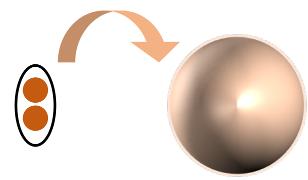
	Exact	ASCC+SPA
$^{188}\mathbf{Pb}$	2.44	2.31
$^{190}\mathbf{Pb}$	2.34	2.21
$^{192}\mathbf{Pb}$	2.25	2.12
$^{194}\mathbf{Pb}$	2.2	2.04

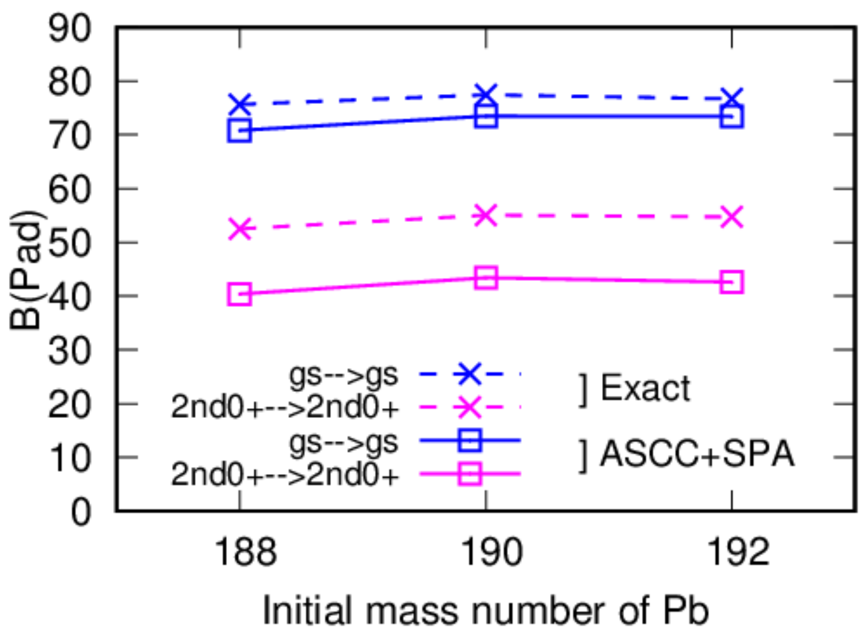
Pair transfer transition strengths

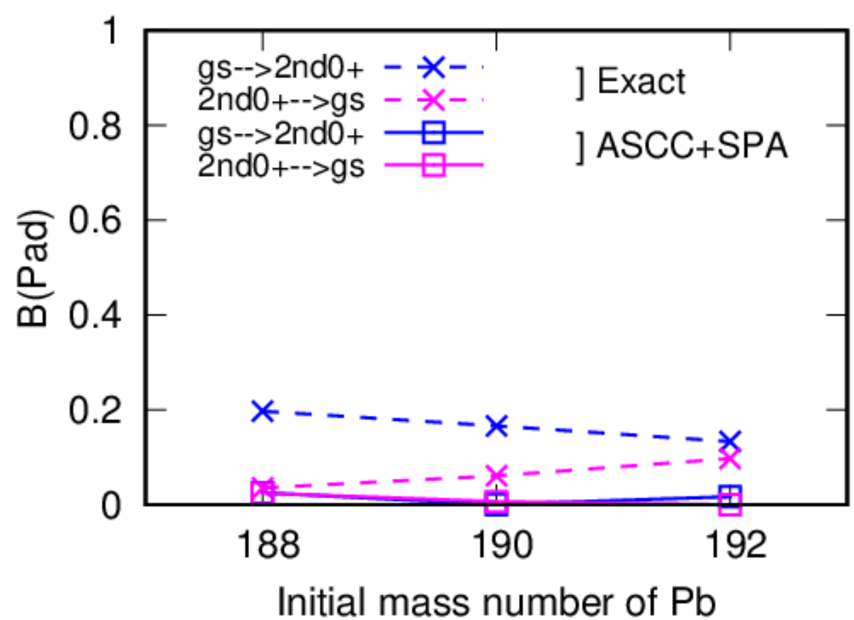
F. Ni, N.Hinohara, TN, arXiv:1811.02352

- Two-neutron additional transition

$$B(Pad) = \left|\langle {}^{A+2}\mathrm{Pb}, \alpha \middle| S_+ \middle| {}^{A}\mathrm{Pb}, \beta \rangle\right|^2$$







For $|0_2^+\rangle \rightarrow |0_2^+\rangle$, 20% smaller than exact solution.

Summary

- Requantization of TDHFB can be a possible alternative to GCM.
- Problems
  - The quantization is feasible only for integrable systems.
  - Realistic nuclei are non-integrable (of course).
- Possible solution
  - ASCC + SPA
  - Derive a collective subspace which is approximately "integrable".
- Application of the ASCC + SPA to multi-level pairing model
- Advantages of the method
  - Microscopic wave functions, like the GCM
  - No diagonalization needed, unlike the GCM
  - Applicable to states with weak collectivity
  - (Possiblely) Solutions to the problems of GCM