Coupling Halo EFT to reaction models: a testcase study on $^{15}\mathrm{C}$ for transfer, breakup and radiative-capture

Laura Moschini, Jiecheng Yang and Pierre Capel







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Halo-EFT and transfer

Coulomb breakup of ¹⁵C on Pb 00000000 0000 Radiative capture

Conclusions

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Halo-EFT and transfer

Effective interaction Conditions for ANC determination Halo-EFT $^{14}C+n$ interactions at NLO

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Radiative capture

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Halo nuclei

Exotic nuclear structure far from stability



One-neutron halo ${}^{11}\text{Be} = {}^{10}\text{Be} + n$ ${}^{15}\text{C} = {}^{14}\text{C} + n$ **Two-neutron halo** ${}^{6}\text{He} = {}^{4}\text{He} + n + n$ ${}^{11}\text{Li} = {}^{9}\text{He} + n + n$

- Light, n-rich nuclei
- Low S_n or S_{2n}
- Cluster structure: core + halo
 ⇒ exibit large matter radius
- $\tau_{1/2}(^{15}C) = 2.4 \text{ s}$ \Rightarrow indirect techniques: reactions



Our goal

To provide good predictions for many reactions

- transfer
- o breakup at intermediate and high energies
- o radiative capture

using one Halo-EFT model of ¹⁵C

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Description of ¹⁵**C structure**



- ¹⁴C core
 - structureless
 - in its ground state
 - $(E_{e.s.} 6 \text{ MeV above } E_{g.s.})$
- loosely bound neutron

$$1/2^+$$
 $E_{g.s.} = -1.218$ MeV

 $H_0 = -\frac{\hbar^2}{2\mu_{14}}\Delta + V_{14}(\mathbf{r})$

 $\mu_{^{14}Cn} = m_{^{14}C}m_n/m_{^{15}C}$ reduced mass $V_{^{14}Cn}$ effective potential simulating their interaction

Halo-EFT and transfer ○●○ ○○○○○○○○○ Coulomb breakup of ¹⁵C on Pb

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Description of ¹⁵**C structure** $H_0 \varphi_{n'li}(E, \mathbf{r}) = E \varphi_{n'li}(E, \mathbf{r})$ eigenvalues and eigenfunctions

 $E_{n'lj} < 0 \rightarrow$ bound states, normed to unity, asymptotical reduced radial wave function behaviour

$$u_{n'lj}(r) \xrightarrow[r \to \infty]{} \frac{\mathcal{C}_{n'lj}}{r} e^{-k_{n'lj}r}$$

- $\hbar k_{n'lj} = \sqrt{2\mu_{14}} C_n |E_{n'lj}|$
- $C_{n'lj}$ single-particle asymptotic normalization constant ANC
- \circ strength of the exponential bound-state wave function tail \circ it depends on the geometry of the V_{1^4Cn} potential

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Description of ¹⁵**C structure**

 $H_0 \varphi_{n'lj}(E, \mathbf{r}) = E \varphi_{n'lj}(E, \mathbf{r})$ eigenvalues and eigenfunctions

$E > 0 \rightarrow {}^{14}\text{C-}n$ continuum reduced radial parts are normalized according to

$$u_{klj} \xrightarrow[r \to \infty]{} sin(kr - l\frac{\pi}{2} + \delta_{lj})$$

- δ_{IJ} phaseshift at energy E
- $\hbar k = \sqrt{2\mu_{14}} C_n E$

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Description of ¹⁵**C structure**

The interaction to calculate $\varphi_{14}{}_{Cn}$ is described by a phenomenological potential $V_{14}{}_{Cn}$ within Halo-EFT model

Halo nucleus \Rightarrow clear separation of scales:

- \circ small scale \leftrightarrow core radius \sim 8 fm
- \circ large scale \leftrightarrow halo range \sim 16 fm

⇒ provides an expansion parameter small scale/large scale upon which the Hamiltonian is expanded Hammer, Ji, Phillips JPG 44, 103002 (2017)

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Description of ¹⁵**C structure**

The interaction to calculate $\varphi_{{}^{14}Cn}$ is described by a phenomenological potential $V_{{}^{14}Cn}$ within Halo-EFT model

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We use narrow Gaussian potentials:

@LO
$$V_{14}_{Cn}(r) = V_0 e^{-\frac{r^2}{2r_0^2}}$$

 V_0 adjusted to fit binding energy (BE) in s wave ($V_{^{14}Cn}=0 ~\forall l>0)$

Description of ¹⁵C structure

The interaction to calculate φ_{14Cn} is described by a phenomenological potential V_{14C_n} within Halo-EFT model

Halo nucleus \Rightarrow clear separation of scales:

- small scale \leftrightarrow core radius \sim 8 fm
- \circ large scale \leftrightarrow halo range \sim 16 fm

 \Rightarrow provides an expansion parameter small scale/large scale upon which the Hamiltonian is expanded Hammer, Ji, Phillips JPG 44, 103002 (2017)

We use narrow Gaussian potentials: $@NLO V_{14}_{Cn}(r) = V_0 e^{-\frac{r^2}{2r_0^2}} + V_2 r^2 e^{-\frac{r^2}{2r_0^2}}$

 $V_2 \& V_0$ adjusted in s wave to fit BE and ANCs, and δ_p in p wave $(V_{14Cn} = 0 \ \forall l > 1)$

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Description of ¹⁵**C structure**

The interaction to calculate $\varphi_{14}{}_{Cn}$ is described by a phenomenological potential $V_{14}{}_{Cn}$ within Halo-EFT model

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We use narrow Gaussian potentials:

@NLO $V_{14}_{Cn}(r) = V_0 e^{-\frac{r^2}{2r_0^2}} + \frac{V_2 r^2 e^{-\frac{r^2}{2r_0^2}}}{r^2}$

 V_2 & V_0 adjusted in s wave to fit BE and ANCs, and δ_p in p wave ($V_{^{14}Cn}=0~\forall l>1)$

 r_0 to evaluate the sensitivity to short-range physics

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Conclusions

Extraction of ANC from transfer reactions

No direct measurement of ANC \Rightarrow we analyze a (d,p) reaction following work done for ¹¹Be by Yang and Capel PRC98,054602(2018)

They study the optimal experimental conditions that enable a safe ANC extraction

- low deuteron energy
- forward angles
- \Rightarrow peripheral process

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Extraction of ANC from transfer reactions

Two experiments satisfying the low energy condition

- *E_d* = 14 MeV University of Notre Dame Goss *et al.*, PRC12, 1730 (1975)
- $E_d = 17.06$ MeV Nuclear Physics Institute of the Czech Academy of Sciences Mukhamedzhanov *et al.*, PRC84, 024616 (2011)

The first one does not provide enough points at forward angles to constrain the ANC within a properly peripheral condition \Rightarrow we analyze the latter experiment

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${}^{14}C(d,p){}^{15}C$ transfer reaction

- Halo-EFT description of ¹⁵C
- FR-ADWA: finite-range adiabatic distorted wave approximation model

 \rightarrow one-step transition matrix between initial and final states, includes the breakup description Johnson and Tandy, NPA235, 56 (1974)

• transfer calculations performed using FRESCO



- Chappel-Hill global nucleons-nucleus potential
- Reid soft core potential for the deuteron bound state

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How we proceed:

- 1 determination of different potentials V_{14}_{Cn} @LO with different r_0
 - \Rightarrow adjusted on the $^{15}\mathrm{C}$ bound state energy
 - \Rightarrow related to as many ANCs
- 2 calculation of different wave functions $\varphi_{^{14}Cn}$ describing the $^{15}{
 m C}$ final state
- **3** with this input, computation of corresponding $\frac{d\sigma_{th}}{d\Omega}$
- 4 comparison with the experimental cross section
- 5 ¹⁵C ANC extraction

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Conclusions

1) Gaussian potentials @LO

with different widths r_0

$$V_{^{14}Cn}(r) = V_0 e^{-\frac{r^2}{2r_0^2}}$$

For each width r_0 the depth V_0 is adjusted to reproduce the **neutron binding energy**

 \Rightarrow each one generates a WF with a corresponding ANC

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2) The bound-state wave functions

obtained with the different potentials



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3) Theoretical cross section



(a) $d\sigma_{th}/d\Omega$ theoretical differential cross section for the transfer to the ¹⁵C g.s.

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3) Theoretical cross section



(b) transfer cross section scaled by ANC²

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 \Rightarrow the spread in the results is significantly reduced at forward angle

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3) Theoretical cross section



(c) To determine the angular range of purely peripheral process

$$\mathcal{R}_{r_0/1.4fm}(heta) = \ \left(rac{ANC^{(1.4fm)}}{ANC^{(r_0)}}
ight)^2 rac{d\sigma_{th}^{(r_0)}/d\Omega}{d\sigma_{th}^{(1.4fm)}/d\Omega} - 1$$

5% difference

 \Rightarrow peripherality condition $heta < 12^{\circ}$

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4) Comparison with data

We infer an ANC $C_{1/2^+}^{(r_0)}$ for each potential width r_0 from a χ^2 analysis



$$\chi^{2}_{r_{0}} = \sum_{i'} \frac{\left[\left(\frac{\mathcal{C}^{(r_{0})}_{n'l_{j}}}{b^{(r_{0})}_{n'l_{j}}} \right)^{2} \frac{d\sigma^{(r_{0})}_{th}}{d\Omega} |_{i'} - \frac{d\sigma_{exp}}{d\Omega} |_{i'} \right]^{2}}{(\delta_{exp}|_{i'})^{2}}$$

 $\circ \delta_{exp}|_{i'} \text{ exp. uncertainty at } \theta_{i'} \\ \circ \text{ sum limited to data } i' \ \theta < 12^{\circ}$

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5) ¹⁵C ANC extraction

We obtain $\mathcal{C}_{1/2^+} = 1.26 \pm 0.02 \ \mathrm{fm}^{-1/2}$ for g.s.

in analogous way for the e.s. $\mathcal{C}_{5/2^+}=0.056\pm0.001~\text{fm}^{-1/2}$



error bars \leftrightarrow uncertainty in χ^2 minimization

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Halo-EFT ¹⁴C+n interactions at NLO

Now we can adjust a NLO Halo-EFT Gaussian potential $V_{14}{}_{Cn}(r) = V_0 e^{-\frac{r^2}{2r_0^2}} + V_2 r^2 e^{-\frac{r^2}{2r_0^2}}$ to our ANC final result and binding energy

Note that we do not have info for p wave \rightarrow we assume it is small as for ¹¹Be: Calci *et al.*, PRL117, 242501 (2016)

We provide

• two ${}^{14}C+n$ potentials for the ground state with $r_0 = 1.2$ and 1.5 fm to check sensibility on short-range physics

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Halo-EFT ¹⁴C+*n* interactions at NLO ...and beyond

Now we can adjust a NLO Halo-EFT Gaussian potential $V_{^{14}Cn}(r) = V_0 e^{-\frac{r^2}{2r_0^2}} + V_2 r^2 e^{-\frac{r^2}{2r_0^2}}$

to our ANC final result and binding energy

Note that we do not have info for p wave \rightarrow we assume it is small as for ¹¹Be: Calci *et al.*, PRL117, 242501 (2016)

We provide

- two ${}^{14}C+n$ potentials for the ground state with $r_0 = 1.2$ and 1.5 fm to check sensibility on short-range physics
- one for the excited state (beyond NLO) with $r_0 = 1.5$ fm to understand if we should go to higher order than NLO

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The wavefunctions

adjusted on our Halo-EFT potential



As expected: both WFs have the same tail

Transfer cross section

using the Halo-EFT potential



As expected: both WFs reproduce cross section for $heta < 12^\circ$

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Coulomb breakup of ¹⁵C on Pb Experiments

605 AMeV GSI experiment: Datta Pramanik et al., PLB551 (2003) 63
68 AMeV RIKEN measurment: Nakamura et al., PRC79 (2009) 035805

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Coulomb breakup of ¹⁵C on Pb

The initial conditions in the laboratory frame of reference (LAB)

The three-body reaction model applied to ¹¹Be in Capel, Baye, Suzuki, PRC78 (2008) 054602 Moschini and Capel arXiv:1807.07537



- ²⁰⁸Pb target T at rest
- projectile P described within the Halo-EFT model
- bombarding energies $E_{LAB} = 605$ and 68 AMeV
 - $\rightarrow~\mathsf{P}$ has constant velocity
 - $\rightarrow~\mathsf{P}$ follows a straight line

 \Rightarrow ideal conditions to apply the Eikonal model!

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The Eikonal model

in the frame of reference of the P-T center of momentum (CM)

In the T-P CM system Klein-Gordon equation is

$$\left[(\hbar c)^2
abla^2 + (\hbar ck)^2 - 2EV_{PT}
ight]\Psi = 0$$

- $\hbar k$ is the relativistic momentum of P in CM
- $E = (M_P M_T c^2)/(M_P + M_T)$ reduced energy
- $M_P c^2$ and $M_T c^2$ are P and T total energies in the CM
- $M_P = \gamma_P m_P$ and $M_T = \gamma_T m_T$ are the relativistic masses

Satchler, Nucl. Phys. A 540 (1992) 533

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The Eikonal model

in the frame of reference of the P-T center of momentum (CM)

Klein-Gordon equation reduces to a Schroedinger equation

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 + V(\mathbf{R},\mathbf{r})\right]\Psi(\mathbf{R},\mathbf{r}) = E_{CM}\Psi(\mathbf{R},\mathbf{r})$$

for the scattering of two nuclei of masses M_P and M_T and a CM kinetic energy $E_{CM} = (\hbar k)^2/2\mu$ where $\mu = E/c^2 = M_P M_T/(M_P + M_T)$ plays the role of reduced mass

 \Rightarrow one can solve the usual nonrelativistic model provided one uses these kinematics prescriptions

P is initially bound in its ground state $\Phi_{l_0j_0m_0}$ of energy E_0 Satchler, Nucl. Phys. A 540 (1992) 533

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The Eikonal approximations

Adiabatic approximation: the collision occurs in a very brief time and the internal P coordinates are frozen during reaction

\circ no core excitation

Eikonal approximation: the wavefunction is factorized as

$$\Psi(\mathbf{b},z,\mathbf{r})=e^{ikz}\hat{\Psi}(\mathbf{b},z,\mathbf{r})$$

since $\hat{\Psi}$ is expected to vary weakly in **R** \rightarrow we assume ∇^2 negligible with respect to $\frac{\partial}{\partial z}$

$$i\hbar v \frac{\partial}{\partial_z} \hat{\Psi}(\mathbf{b}, z, \mathbf{r}) = V_{PT}(\mathbf{R}, \mathbf{r}) \hat{\Psi}(\mathbf{b}, z, \mathbf{r})$$

 $\mathbf{v} = \hbar k/\mu$ relative P-T velocity

The solution for a one-neutron halo nucleus

So we obtain the Eikonal expression

$$\hat{\Psi}(\mathbf{b}, z, \mathbf{r}) = e^{i\chi(\mathbf{b}, \mathbf{s})} \Phi_{I_0 j_0 m_0}(E_0, \mathbf{r})$$

where the Eikonal phase could be divided into its nuclear and Coulomb contributions:

$$\chi(\mathbf{b},\mathbf{s}) = \chi^{N}(\mathbf{b},\mathbf{s}) + \chi^{C}(\mathbf{b},\mathbf{s}) + \chi^{C}_{PT}(b)$$

The nuclear phase

The nuclear interaction is usually calculated using optical potentials

$$\chi^{N} = -rac{1}{\hbar v}\int_{-\infty}^{z}V_{CT}(\mathbf{b},z',\mathbf{r}) + V_{fT}(\mathbf{b},z',\mathbf{r})dz'$$

At higher energies

it is difficult to find data to fit optical potentials expecially for radioactive nuclei!

The nuclear phase

We apply the optical limit approximation of Glauber theory

$$\chi_{OLA}^{N}(\mathbf{b}) = i \int \int \rho_{T}(\mathbf{r}') \rho_{X}(\mathbf{r}'') \Gamma_{NN}(\mathbf{b} - \mathbf{s}' + \mathbf{s}'') d\mathbf{r}'' d\mathbf{r}'$$

- $\rho(\mathbf{r})$ is neutron or proton Fermi density
- $\Gamma_{NN}(\mathbf{b}) = \frac{1-i\alpha_{NN}}{4\pi\beta_{NN}} \sigma_{NN}^{tot} e^{-\frac{b^2}{2\beta_{NN}}}$ is a profile function that correspond to effective nucleon-nucleon interaction
 - σ_{NN}^{tot} total cross section for the NN collision
 - $\alpha_{\it NN}$ ratio of real to imag. part of the NN-scattering amplitude
 - β_{NN} slope of NN elastic differential cross section

Horiuchi, Suzuki, Capel and Baye, PRC 81 (2010)

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The Coulomb phases

Rutherford scattering between the P center-of-mass and the T

$$\chi_{PT}^{C} = 2\eta \ln(kb)$$

Coulomb tidal force

$$\chi^{C} = -\eta \int_{-\infty}^{\infty} \left(\frac{1}{|\mathbf{R} - \frac{m_{f}}{m_{P}}\mathbf{r}|} - \frac{1}{R} \right) dz$$

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The Coulomb phases

• Rutherford scattering between the P center-of-mass and the T

$$\chi_{PT}^{C} = 2\eta \ln(kb)$$

Coulomb tidal force

$$\chi^{C} = \eta \ln \left(1 - 2 \frac{m_f}{m_P} \frac{\mathbf{b} \cdot \mathbf{s}}{b} + \frac{m_f^2}{m_P^2} \frac{s^2}{b^2} \right)$$

→ Divergence due to the slow decrease of χ^{C} in *b* $e^{i\chi^{C}} = 1 + i\chi^{C} - \frac{1}{2}(\chi^{C})^{2} + ...$

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The Coulomb phases

• Rutherford scattering between the P center-of-mass and the T

$$\chi^{\rm C}_{\rm PT} \sim 2\eta \ln \left(\frac{b}{2z_{\rm max}}\right)$$

• Coulomb tidal force: \Rightarrow we make the replacement

$$e^{i\chi} = e^{i\chi^{N}} (e^{i\chi^{C}} - i\chi^{C} + i\chi^{FO}) e^{i\chi^{C}_{PT}}$$

first order term of the perturbation theory

$$\chi^{FO} = -\eta \int_{-\infty}^{\infty} e^{i\omega z/v} \left(\frac{1}{|\mathbf{R} - \frac{m_f}{m_P}\mathbf{r}|} - \frac{1}{R} \right) dz$$

where $\omega = (E - E_0)/\hbar$, and E C-f relative energy after dissociation

Margueron, Bonaccorso and Brink, Nucl. Phys. A 720 (2003)

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Coulomb breakup of ¹⁵C on Pb ○○○○○●○ ○○○○ Radiative capture

Conclusions

Changing frame of reference from the P-T CM frame to P rest frame

Dynamics is Lorentz invariant $\Leftrightarrow V_{PT(\mathbf{b},z,\mathbf{r})}$ is Lorentz invariant

 \Rightarrow it should transform as the time-like component of a Lorentz four-vector

$$V_{PT(\mathbf{b},z,\mathbf{r})} o \gamma V_{PT(\mathbf{b},\gamma z,\mathbf{r})}$$

where $\gamma = (1 - w^2/c^2)^{-1/2}$ and w the P velocity in P-T CM frame

This transformation is

- well established for electromagnetic field
- a conjecture for the nuclear interaction

Halo-EFT and transfer

Coulomb breakup of ¹⁵C on Pb ○○○○○○●○ ○○○○ Radiative capture

Conclusions

Changing frame of reference from the P-T CM frame to P rest frame

Let's apply the Lorentz boost:

- Nuclear phase χ^N and Coulomb phases χ^C_{PT} and χ^C are already Lorentz invariant in our model: no changes under the transformation $V(z) = \gamma V(\gamma z)$
- The phase χ^{FO} is not Lorentz invariant:

$$\chi^{FO} = -\eta \int_{-\infty}^{\infty} e^{i\omega z/\gamma v} \left(\frac{1}{|\mathbf{R} - \frac{m_f}{m_P}\mathbf{r}|} - \frac{1}{R} \right) dz$$

consistent with Winther and Alder's relativistic Coulomb excitation result

luction Halo-EFT and transfer Coulomb breakup of ¹⁵C on Pb Radiative 000 00000000 00000000 00000000 Conclusions

The breakup cross section

So the breakup amplitude is

$$S_{kljm}^{m_0}(b) \sim \langle \varphi_{ljm}(E) | e^{i\chi^N} (e^{i\chi^C} - i\chi^C + i\chi^{FO}) e^{i\chi^C_{PT}} | \varphi_{l_0j_0m_0}(E_0) \rangle$$

Breakup cross section as a function of C-f relative energy E after dissociation

$$\frac{d\sigma_{bu}}{dE} = \frac{4\mu_{cf}}{\hbar^2 K} \frac{1}{2j_0 + 1} \sum_{m_0} \sum_{ljm} \int_0^\infty bdb |S_{kljm}^{m_0}(b)|^2$$

where μ_{cf} and Kare C-f reduced mass and momentum in P rest frame no relativistic effects considered here ¹⁵C wavefunctions obtained with Halo-EFT @NLO

Coulomb breakup of ¹⁵C on Pb at 605 AMeV



Halo-EFT and transfer

Coulomb breakup of ¹⁵C on Pb

Radiative capture

Conclusions

Coulomb breakup of ¹⁵C on Pb at 605 AMeV

- relativistic corrections are important
- full calc. in good agreement with data



- band related to ANC uncertainty
- the reaction does not depend on WF inner part $r_0 = 1.2 \text{fm} \sim r_0 = 1.5 \text{fm}$
- no effect for the e.s. inclusion
 - \Rightarrow for this reaction model of Coulomb breakup the Halo-EFT expansion works fine at NLO

Coulomb breakup of ¹⁵C on Pb at 68 AMeV



Halo-EFT and transfer

Coulomb breakup of ¹⁵C on Pb

Radiative capture

Conclusions

Coulomb breakup of ¹⁵C on Pb at 68 AMeV

• excellent agreement with the data at all angles



- band related to ANC uncertainty
- the reaction does not depend on WF inner part $r_0 = 1.2 \text{fm} \sim r_0 = 1.5 \text{fm}$
- the result obtained taking into account the ¹⁵C e.s. is barely different from the others

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Radiative capture model ${}^{14}C(n,\gamma){}^{15}C$

¹⁴C and *n* merge to form ¹⁵C by emitting a photon \Rightarrow electromagnetic transition from ¹⁵C continuum (*E*) to one of its bound states (*E*_{nolo})

 $\sigma_{n_0 l_0}(E) \sim \sum_{\lambda \sigma} \mathcal{A}_{\lambda} \sum_{I} \frac{2l_0 + 1}{2l + 1} |\langle \varphi_{n_0 l_0}(E_{n_0 l_0})|| \mathcal{M}^{\sigma \lambda} ||\varphi_l(E)\rangle|^2$ $\hbar k_{\gamma} c = E - E_{n_0 l_0} \text{ photon energy}$ dominant term is $\sigma \lambda = E1 \Rightarrow \mathcal{M}_{\mu}^{E1} = e \frac{Z_{14_C}}{A_{14_C} + 1} r Y_{\mu}^{(1)}(\Omega)$ Capel and Nollet, PRC96, 015801 (2017)

Experiment: Reifarth et al., PRC77 (2008) 015804



Experimentally they cannot distinguish between the capture to g.s. and e.s. so we include our potential beyond NLO for e.s.



- fine agreement with data
- band related to ANC uncertainty
- cross section does not depend on wave function inner part

• Problem at low energy:

- experimental problem?
- new physics?
- \Rightarrow more research on this point is needed!

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Conclusions

We have developed **one Halo-EFT** description of ^{15}C adjusted on

- binding energy \rightarrow known from experiment
- ANC \rightarrow "measured" analysing transfer reaction

and we have **reproduced data** for **different reactions** transfer, radiative capture, and breakup (at intermediate and high energy)

In particular:

• Halo-EFT description works well @NLO

 \circ using the r_0 parameter we can test short-range physics which results to be important in transfer processes at high angles

Halo-EFT and transfer

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Conclusions

In the future...

Navrátil and collaborators are working on *ab initio* calculations on ^{15}C this could help us to \circ have some info to constrain the interaction in p wave \circ check our ANC result: $\mathcal{C}_{1/2^+} = 1.26 \pm 0.02 \ \text{fm}^{-1/2}$

We would also like to study other reactions involving ⁸B:

- ⁸B breakup
- ${}^{7}\text{Be}(p,\gamma){}^{8}\text{B}$ radiative capture

Thank you for your attention!

Appendix

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Satchler kinematical presctiptions

In the T-P CM system Klein-Gordon equation is

$$\left[(\hbar c)^2 \nabla^2 + (\hbar c k)^2 - 2 E V_{PT}\right] \Psi = 0$$

- $\hbar k$ is the relativistically correct CM momentum of P
- $E = (M_P M_T c^2)/(M_P + M_T) \rightarrow$ reduced energy function of P and T total energies in the CM frame: $M_P c^2$ and $M_T c^2$
- $M_P = \gamma_P m_P$ is the corrected projectile mass

-
$$\gamma_P = \frac{x + \gamma_L}{\sqrt{1 + x^2 + 2x\gamma_L}}$$
, $x = m_P/m_T$, $\gamma_L = 1 + (E_{LAB}/m_Pc^2)$
- E_{LAB} is the projectile bombarding energy in the LAB system

- E_{LAB} is the projectile bombarding energy in the LAB system
- same for M_T

•
$$\mu = E/c^2 = M_P M_T / (M_P + M_T) \rightarrow \text{reduced "mass"}$$

• $\Rightarrow k = \frac{m_P c}{\hbar} \sqrt{\gamma_P^2 - 1}$

Why ¹⁵C is an interesting nucleus?

¹⁵C has astrophysical interest

- the radiative capture ¹⁴C(n,γ)¹⁵C is part of the neutron-induced CNO cycle in the helium-burning shell of light AGB stars
- ¹⁵C plays a role in the primordial nucleosynthesis of intermediate-mass elements

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Wiescher, Görres, Schatz, *J. Phys. G* **25**, R133 (1999) Kajino, Mathews, Fuller, *Astrophys. J.* **364**, 7 (1990)

Comparison with ¹⁵C ANC in literature

We obtain ${\cal C}_{1/2^+}=1.26\pm0.02~{\rm fm}^{-1/2}$ in analogous way for the e.s. ${\cal C}_{5/2^+}=0.056\pm0.001~{\rm fm}^{-1/2}$

ANC^2 (fm ⁻¹)	reference
1.48 ± 0.18	Trache et al. Tex. A&M Cycl. Prog. Rep. 1 (2002) 16
1.89 ± 0.11	Timofeyuk <i>et al.</i> PRL96 (2006) 162501
2.14	Pang <i>et al.</i> PRC75 (2007) 024601
1.74 ± 0.11	Summers and Nunes PRC78 (2008) 069908
1.64 ± 0.26	Mukhamedzhanov <i>et al.</i> PRC84 (2011) 024616
1.88 ± 0.18	McCleskey <i>et al.</i> PRC89 (2014) 044605
1.59 ± 0.03	this work

¹¹Be model tested at RIKEN

¹¹Be+²⁰⁸Pb @ 69 AMeV and ¹¹Be+¹²C @ 67 AMeV



¹¹Be model tested at GSI energy

¹¹Be+²⁰⁸Pb and ¹¹Be+¹²C @ 520 AMeV





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What's the reason for the discrepancy?

Both cross sections can be reproduced using one structure model (and hence one E1 strength)

 \Rightarrow the discrepancy between RIKEN and GSI estimates

is most likely due to differences in the data analysis

• GSI analysis

proper treatment of relativity

 \circ Coulomb contribution evaluated by subtracting

a nuclear contribution estimate

extrapolated from the breakup on C

from the total breakup cross section on Pb

 \Rightarrow Coulomb and nuclear contributions interferences neglected! Typel and Shyam, PRC64 (2001) 024605

 RIKEN analysis is less sensitive to this issue because it focuses on a measurement at forward angles where the nuclear contribution is negligibly small