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Effects of tensor force in the relativistic scheme

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December 12, 2018

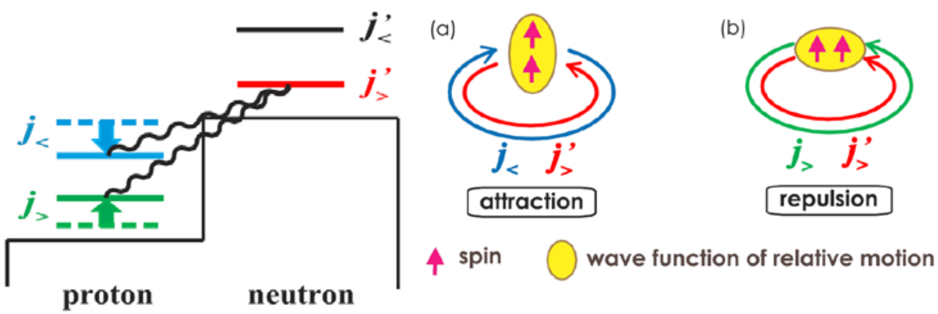
Contents

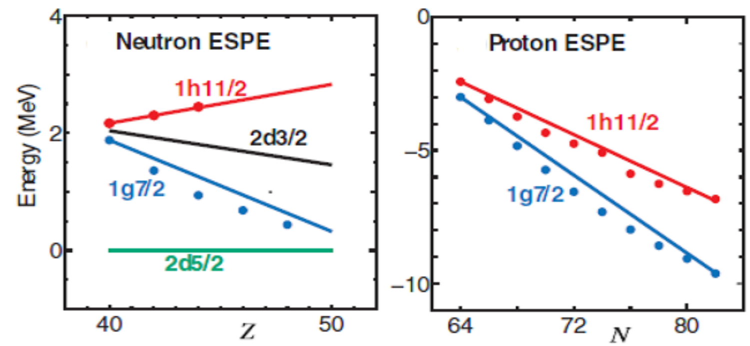
Zhiheng Wang
LZU & Tsukuba & RIKEN

Tensor force and its effects

Tensor force is an important component of **NN interaction**

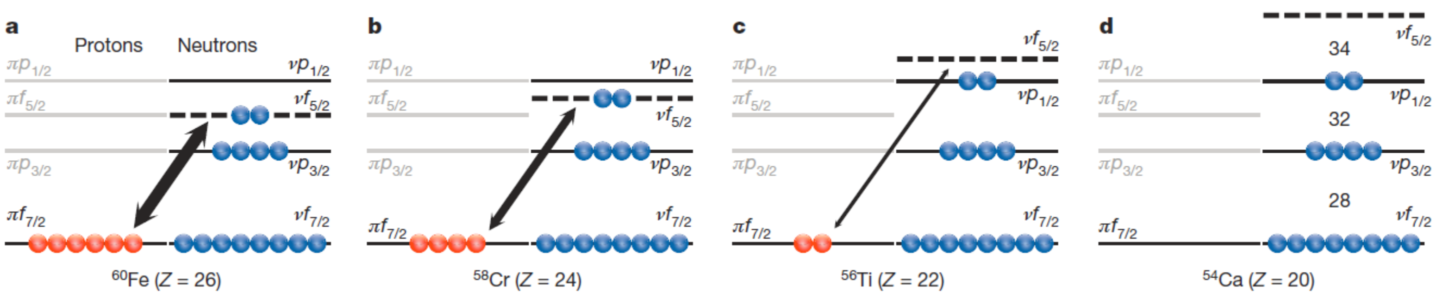
- Crucial for the **shell evolution** in exotic nuclei





Otsuka *et al.*, *PRL* **95**, 232502 (2005)

- Crucial for the **new magic numbers**

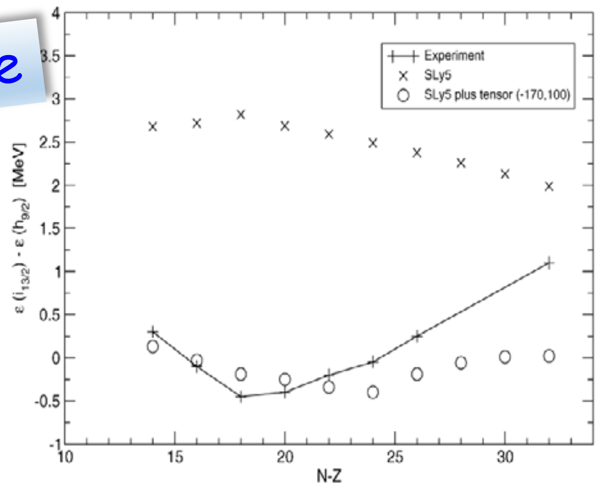


Steppenbeck *et al.*, *Nature* **502**, 207 (2013)

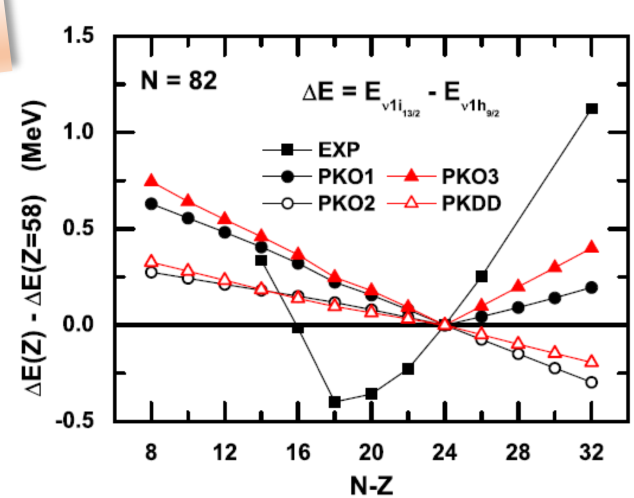
DFT and tensor effects

- **Shell evolution** (N = 82)

Skyrme



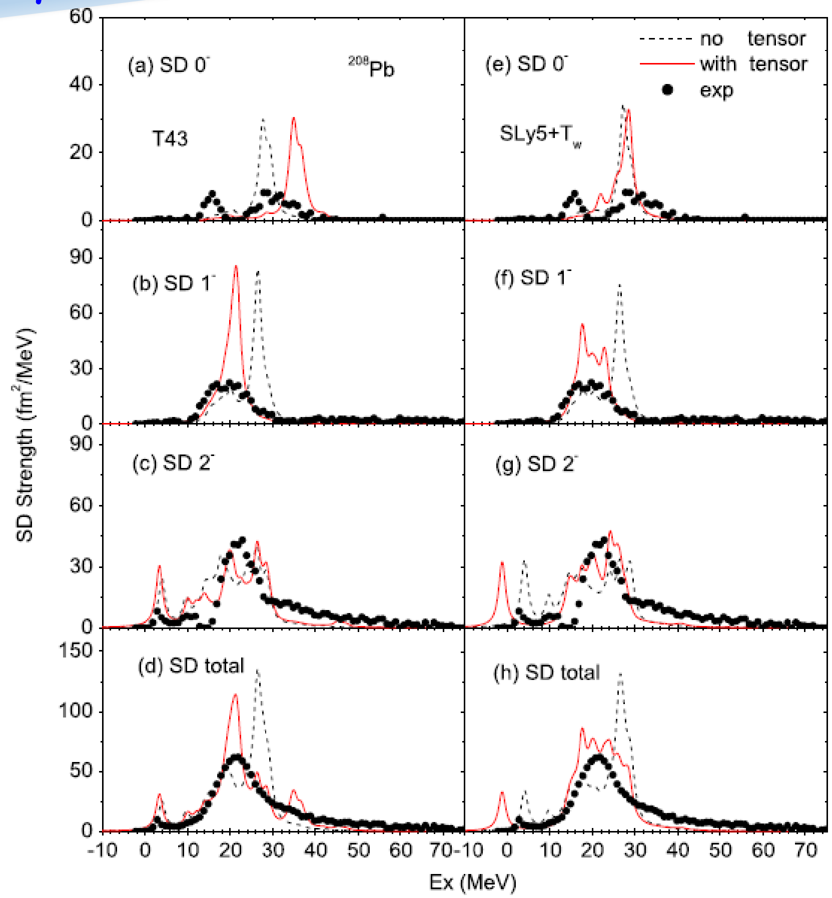
RHF



Colò, Sagawa, Fracasso, Bortignon, *PLB* **646**, 227 (2007)
Long, Sagawa, Meng, Giai, *EPL* **82**, 12001 (2008)
(exp) Schiffer et al., *PRL* **92**, 162501 (2004)

- **Giant resonances** (spin-dipole)

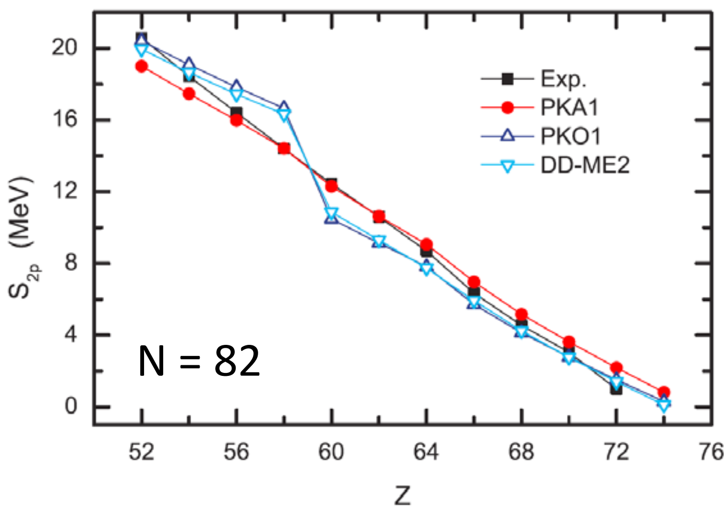
Skyrme



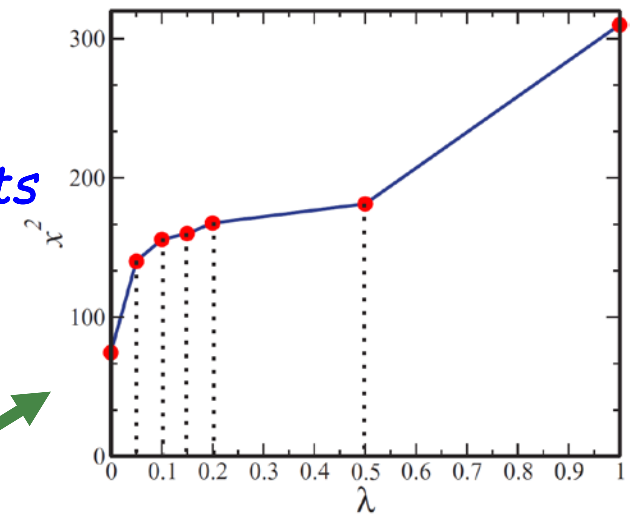
Bai, Zhang, Sagawa, Zhang, Colò, Xu, *PRL* **105**, 072501 (2010)
(exp) Wakasa et al., *PRC* **85**, 064606 (2012)

Tensor effects in relativistic DFT?

- Signals of tensor force in **relativistic DFT**? (nuclear mass)



some fingerprints



π meson (tensor) is not welcomed in CDFT

Long, Sagawa, Giai, Meng, *PRC* **76**, 034314 (2007)

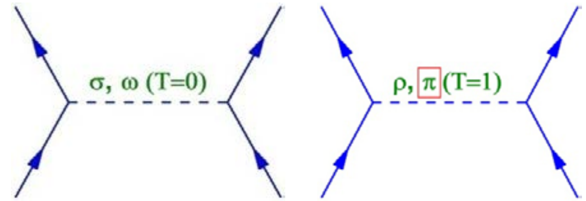
Lalazissis et al., *PRC* **80**, 041301(R) (2009)

- How to **identify** the tensor effects in relativistic DFT?
- Can we make **fair and quantitative comparison** with Skyrme, Gogny, etc.?
- What would be the proper ways to **determine the strengths** of tensor force?

Relativistic Hartree-Fock (RHF) theory

- **Lagrangian density**

Bouyssy *et al.*, *PRC* **36**. 380 (1987)
Long *et al.*, *PLB* **639**. 242 (2006)
HZL *et al.*, *PRL* **101**, 122502 (2008)



$$\mathscr{L} = \bar\psi\left(i\gamma_\mu\partial^\mu - M\right)\psi$$
$$+\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_\rho^2\vec\rho_\mu\cdot\vec\rho^\mu - \frac{1}{4}\vec R_{\mu\nu}\cdot\vec R^{\mu\nu} + \frac{1}{2}\partial_\mu\vec\pi\cdot\partial^\mu\vec\pi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$
$$-\bar\psi\left[g_\sigma\sigma + g_\omega\gamma^\mu\omega_\mu + g_\rho\gamma^\mu\vec\tau\cdot\vec\rho_\mu - \frac{f_\rho}{2M}\sigma^{\mu\nu}\vec\tau\cdot\partial_\nu\vec\rho_\mu + \frac{f_\pi}{m_\pi}\gamma_5\gamma^\mu\vec\tau\cdot\partial_\mu\vec\pi + e\gamma^\mu\frac{1-\tau_3}{2}A_\mu\right]\psi,$$

- **Hamiltonian**

$$H = \int d^3x\,\bar\psi(x)[-i\boldsymbol\gamma\cdot\boldsymbol\nabla + M]\psi(x) + \frac{1}{2}\sum_\phi\iint d^3x\,d^4y\,\bar\psi(x)\bar\psi(y)\Gamma_\phi(x,y)D_\phi(x,y)\psi(y)\psi(x)$$

interaction vertices **propagators**

According to interaction vertices, we have

σ-S, ω-V, ρ-V, ρ-T, ρ-VT, and π-PV couplings

Caution: **T** here indicates Dirac matrix $\boldsymbol{\sigma^{\mu\nu}}$, not the "tensor force"

- Their leading-order terms contain the **tensor force**. *Sagawa & Colò*, *PPNP* **76**, 76 (2014)
- **How about the next-to-the-leading-order terms? Are they important?**

Non-relativistic reduction

- Relativistic meson-exchange two-body interactions

$$\hat{V}_\phi(\boldsymbol{r}_1,\boldsymbol{r}_2) = \gamma_0(\boldsymbol{r}_1)\gamma_0(\boldsymbol{r}_2)\Gamma_\phi(\boldsymbol{r}_1,\boldsymbol{r}_2)D_\phi(\boldsymbol{r}_1,\boldsymbol{r}_2)$$

- Two-body interaction matrix elements

$$V_{\phi,\alpha\beta\gamma\delta} = \langle\varphi_\alpha\varphi_\beta|\hat{V}_\phi|\varphi_\gamma\varphi_\delta\rangle = \iint d\boldsymbol{r}_1\, d\boldsymbol{r}_2\, \varphi^\dagger_\alpha(\boldsymbol{r}_1)\varphi^\dagger_\beta(\boldsymbol{r}_2)\hat{V}_\phi(\boldsymbol{r}_1,\boldsymbol{r}_2)\varphi_\gamma(\boldsymbol{r}_1)\varphi_\delta(\boldsymbol{r}_2)$$

- **Non-relativistic reduced two-body interactions**

$$V_{\phi,\alpha\beta\gamma\delta} = \langle\varphi_\alpha\varphi_\beta|\hat{\mathcal{V}}_\phi\Pi_+|\varphi_\gamma\varphi_\delta\rangle = \langle\xi_\alpha\xi_\beta|\hat{\mathcal{V}}_\phi|\xi_\gamma\xi_\delta\rangle$$

ξ: upper component

$\hat{\mathcal{V}}$ can be expanded in the **powers of 1/*M***

(~ **Foldy-Wouthuysen method** for one-body problem)

Example: plane waves in finite-density systems

$$V_{\phi,abcd} = \bar{u}_{\boldsymbol{p}^*_a}(1)\bar{u}_{\boldsymbol{p}^*_b}(2)\frac{1}{m^2_\phi+\boldsymbol{q}^2}\Gamma_\phi(1,2)u_{\boldsymbol{p}^*_c}(1)u_{\boldsymbol{p}^*_d}(2)$$

with $$u_{\boldsymbol{p}^*_a} = \sqrt{\frac{M^*+\varepsilon^*_a}{2\varepsilon^*_a}}\begin{pmatrix} 1 \\ \frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}^*_a}{M^*+\varepsilon^*_a}\end{pmatrix}$$

$$\boldsymbol{p}^* \equiv \boldsymbol{p}+\hat{\boldsymbol{p}}\Sigma_V(p)$$
$$M^*(p) \equiv M+\Sigma_S(p)$$
$$\varepsilon^*(p) \equiv \varepsilon(p)-\Sigma_0(p)$$

Non-relativistic reduced interactions

- Up to the **$1/M^2$** order

$$S_{12} \equiv (\boldsymbol{\sigma}_1 \cdot \boldsymbol{q})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{q}) - \frac{1}{3}(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) q^2$$

$$\hat{\mathcal{V}}_{\sigma\text{-S}} = -g_\sigma(1) g_\sigma(2) \frac{1}{m_\sigma^2 + \boldsymbol{q}^2} \left[1 - \frac{1}{4} \frac{4\boldsymbol{k}^2 - \boldsymbol{q}^2 - 4i\boldsymbol{\sigma}_1 \cdot (\boldsymbol{k} \times \boldsymbol{q})}{4M^*(1)M^*(1)} - \frac{1}{4} \frac{4\boldsymbol{k}'^2 - \boldsymbol{q}^2 - 4i\boldsymbol{\sigma}_2 \cdot (\boldsymbol{q} \times \boldsymbol{k}')}{4M^*(2)M^*(2)} \right],$$

$$\hat{\mathcal{V}}_{\omega\text{-V}} = +g_\omega(1) g_\omega(2) \frac{1}{m_\omega^2 + \boldsymbol{q}^2} \left[1 + \frac{1}{4} \frac{4\boldsymbol{k}^2 - \boldsymbol{q}^2 - 4i\boldsymbol{\sigma}_1 \cdot (\boldsymbol{k} \times \boldsymbol{q})}{4M^*(1)M^*(1)} + \frac{1}{4} \frac{4\boldsymbol{k}'^2 - \boldsymbol{q}^2 - 4i\boldsymbol{\sigma}_2 \cdot (\boldsymbol{q} \times \boldsymbol{k}')}{4M^*(2)M^*(2)} \right]$$

$$- \frac{g_\omega(1) g_\omega(2)}{4M^*(1)M^*(2)} \frac{1}{m_\omega^2 + \boldsymbol{q}^2} \left[4\boldsymbol{k}\boldsymbol{k}' + 2i\boldsymbol{\sigma}_1 \cdot (\boldsymbol{q} \times \boldsymbol{k}') - 2i\boldsymbol{\sigma}_2 \cdot (\boldsymbol{q} \times \boldsymbol{k}) + \frac{2}{3}(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)\boldsymbol{q}^2 - S_{12} \right],$$

$$\hat{\mathcal{V}}_{\pi\text{-PV}} = -\vec{\tau}(1) \cdot \vec{\tau}(2) \frac{f_\pi(1) f_\pi(2)}{m_\pi^2} \frac{1}{m_\pi^2 + \boldsymbol{q}^2} \left[S_{12} + \frac{1}{3}(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)\boldsymbol{q}^2 \right],$$

$$\hat{\mathcal{V}}_{\rho\text{-T}} = +\vec{\tau}(1) \cdot \vec{\tau}(2) \frac{f_\rho(1) f_\rho(2)}{4M^2} \frac{1}{m_\rho^2 + \boldsymbol{q}^2} \left[S_{12} - \frac{2}{3}(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)\boldsymbol{q}^2 \right],$$

$$\hat{\mathcal{V}}_{\rho\text{-VT}} = +\vec{\tau}(1) \cdot \vec{\tau}(2) \frac{f_\rho(1) g_\rho(2)}{4MM^*(1)} \frac{1}{m_\rho^2 + \boldsymbol{q}^2} \left[-\boldsymbol{q}^2 + 2i\boldsymbol{\sigma}_1 \cdot (\boldsymbol{q} \times \boldsymbol{k}) \right]$$

$$+ \vec{\tau}(1) \cdot \vec{\tau}(2) \frac{f_\rho(1) g_\rho(2)}{4MM^*(2)} \frac{1}{m_\rho^2 + \boldsymbol{q}^2} \left[-2i\boldsymbol{\sigma}_1 \cdot (\boldsymbol{q} \times \boldsymbol{k}') - \frac{2}{3}(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)\boldsymbol{q}^2 + S_{12} \right]$$

$$+ \vec{\tau}(1) \cdot \vec{\tau}(2) \frac{g_\rho(1) f_\rho(2)}{4MM^*(2)} \frac{1}{m_\rho^2 + \boldsymbol{q}^2} \left[-\boldsymbol{q}^2 - 2i\boldsymbol{\sigma}_2 \cdot (\boldsymbol{q} \times \boldsymbol{k}') \right]$$

$$+ \vec{\tau}(1) \cdot \vec{\tau}(2) \frac{g_\rho(1) f_\rho(2)}{4MM^*(1)} \frac{1}{m_\rho^2 + \boldsymbol{q}^2} \left[+2i\boldsymbol{\sigma}_2 \cdot (\boldsymbol{q} \times \boldsymbol{k}) - \frac{2}{3}(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)\boldsymbol{q}^2 + S_{12} \right].$$

Finite density

Formalism is in agreement with **Bonn interaction**: Machleidt, *Adv. Nucl. Phys.* **19**, 189 (1989)

Operators for tensor force in RHF

" *t* " indicates the tensor force with operator S_{12}

- Tensor interactions in **Bonn** and **RHF**

Wang, HZL *et al.*, in preparation

$$\hat{\mathcal{V}}^t_{0,\phi} = \frac{1}{m_\phi^2 + \boldsymbol{q}^2}\mathcal{F}_{0,\phi}S_{12}$$

Coupling	$\mathcal{F}_{0,\phi}$	Ratio to π-PV	
		Bonn A	PKA1
ω-V	$\frac{g_\omega g_\omega}{4M^2}$	-0.02	-0.02
π-PV	$-\vec{\tau}\cdot\vec{\tau}\frac{f_\pi f_\pi}{m_\pi^2}$	1	1
ρ-V	$\vec{\tau}\cdot\vec{\tau}\frac{g_\rho g_\rho}{4M^2}$	-0.0009	-0.002
ρ-T	$\vec{\tau}\cdot\vec{\tau}\frac{f_\rho f_\rho}{4M^2}$	-0.03	-0.02
ρ-VT	$\vec{\tau}\cdot\vec{\tau}\frac{f_\rho g_\rho}{2M^2}$	-0.01	-0.01

Zero density

$$\hat{\mathcal{V}}^t_{\phi} = \frac{1}{m_\phi^2 + \boldsymbol{q}^2}\mathcal{F}_{\phi}S_{12}$$

Coupling	$\mathcal{F}_{\phi}$	Ratio to π-PV
ω-V	$\frac{g_\omega(1)g_\omega(2)}{4M^*(1)M^*(2)}$	-0.74
π-PV	$-\vec{\tau}\cdot\vec{\tau}\frac{f_\pi(1)f_\pi(2)}{m_\pi^2}$	1
ρ-V	$\vec{\tau}\cdot\vec{\tau}\frac{g_\rho(1)g_\rho(2)}{4M^*(1)M^*(2)}$	-0.03
ρ-T	$\vec{\tau}\cdot\vec{\tau}\frac{f_\rho(1)f_\rho(2)}{4M^2}$	-0.25
ρ-VT	$\vec{\tau}\cdot\vec{\tau}\frac{f_\rho(1)g_\rho(2)}{4MM^*(2)} + (1\leftrightarrow 2)$	-0.16

Finite density

- The **largest** tensor contribution comes from **pion**.
- **All other couplings** have **opposite and non-negligible** contributions.

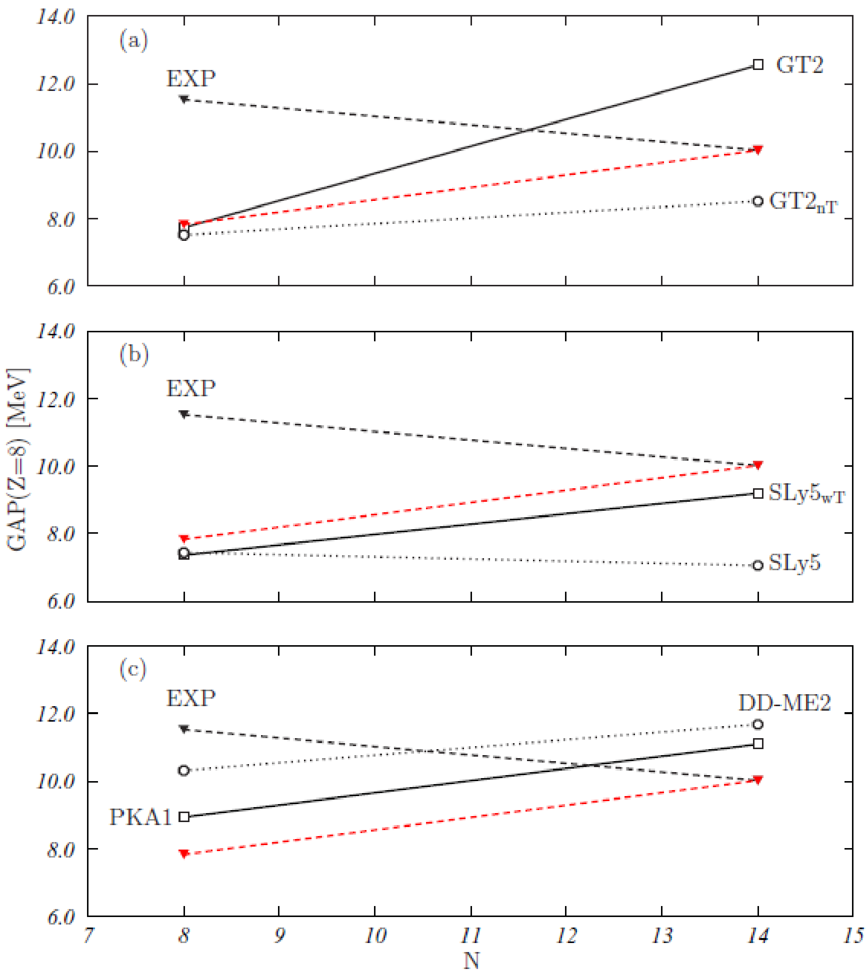
Tensor effects on shell evolution

PHYSICAL REVIEW C **81**, 064327 (2010)

Tensor effects in shell evolution at $Z, N = 8, 20$, and 28 using nonrelativistic and relativistic mean-field theory

M. Moreno-Torres,[1] M. Grasso,[2] H. Liang,[2,3] V. De Donno,[4] M. Anguiano,[1] and N. Van Giai[2]

Skyrme vs Gogny vs RHF



Gogny:

$GT2$ vs $GT2_{nT}$

Tensor effect

~ 4 MeV

Skyrme:

$SLy5_{wT}$ vs $SLy5$

Tensor effect

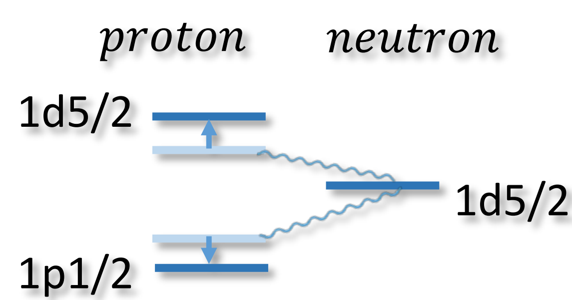
~ 2.5 MeV

RHF:

PKA1 vs DD-ME2

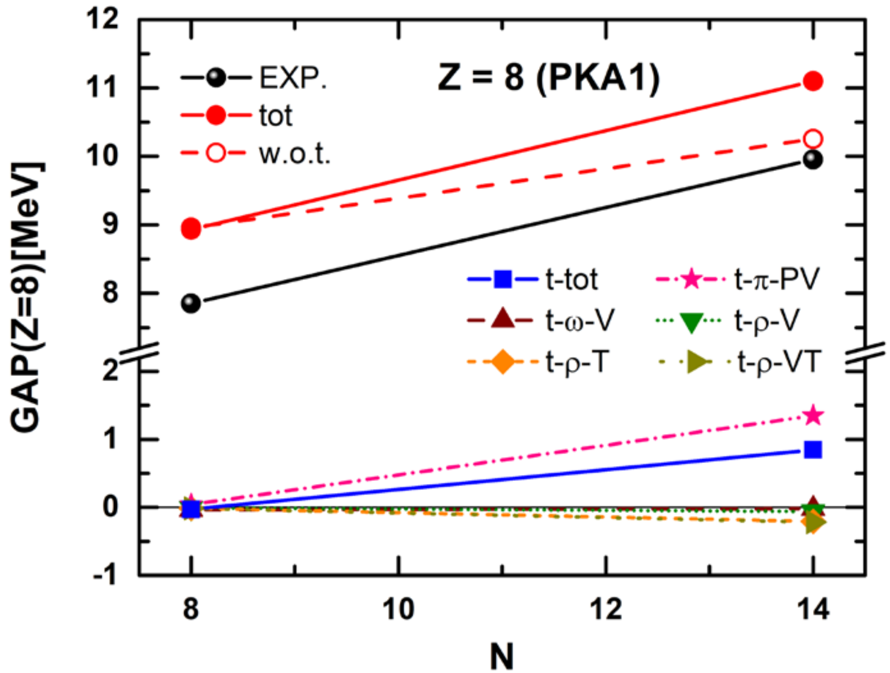
Tensor effect

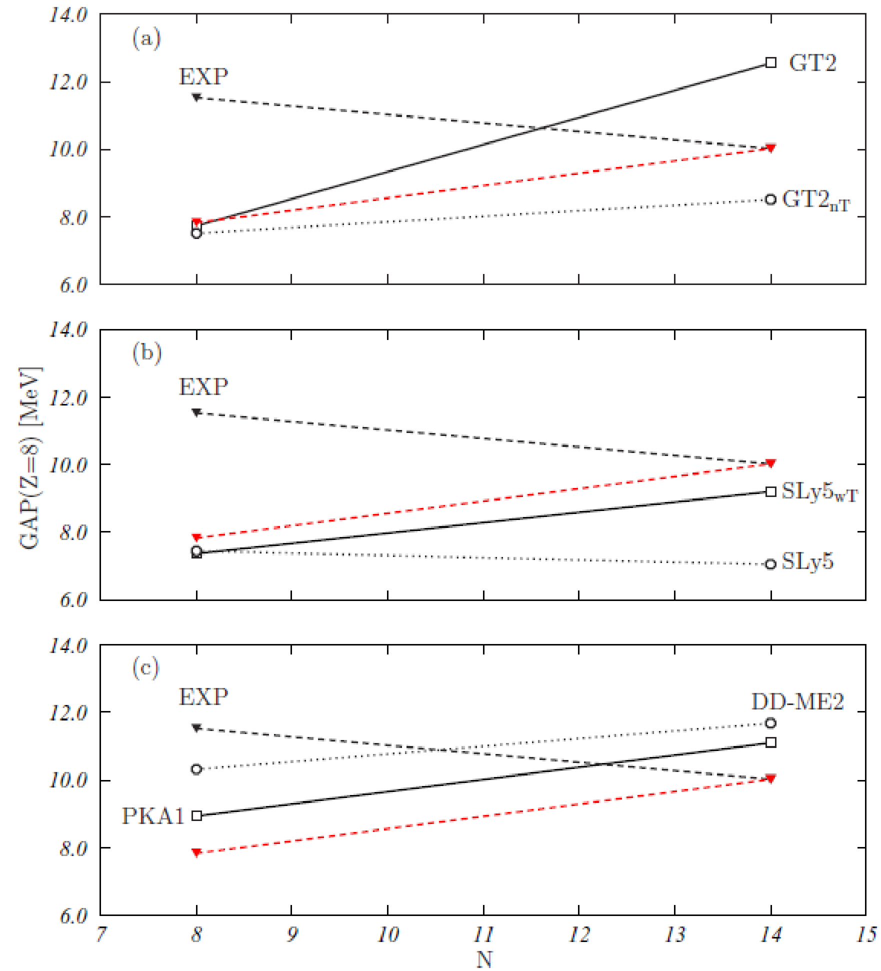
? MeV



From ^{16}O to ^{22}O

Tenser effects on gap Z=8





PKA1 vs $PKA1_{nT}$

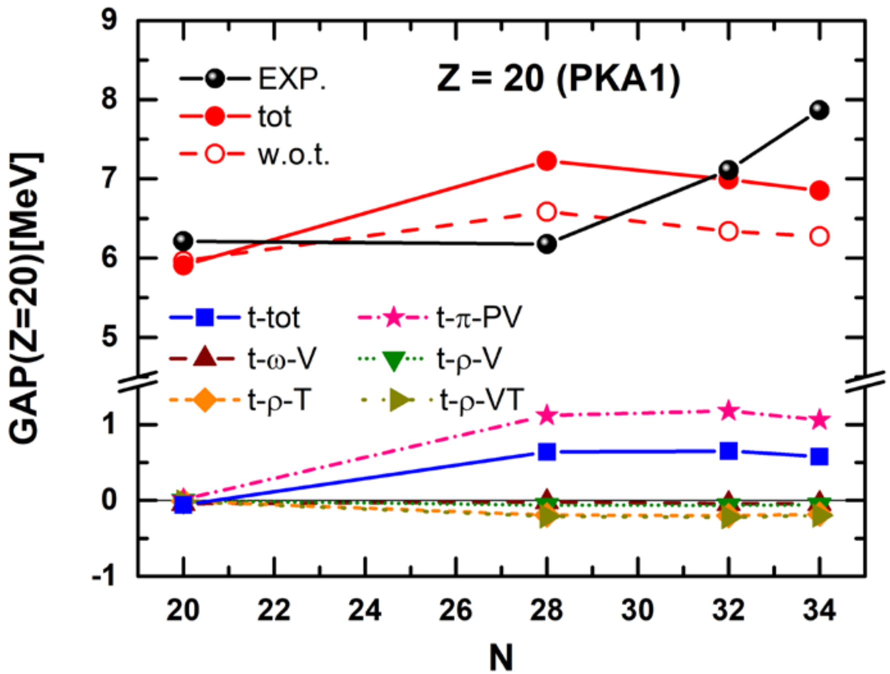
Tensor effect **~ 1 MeV**

first fair comparison

- The **same behavior** of tensor force but **weaker** than Skyrme and Gogny

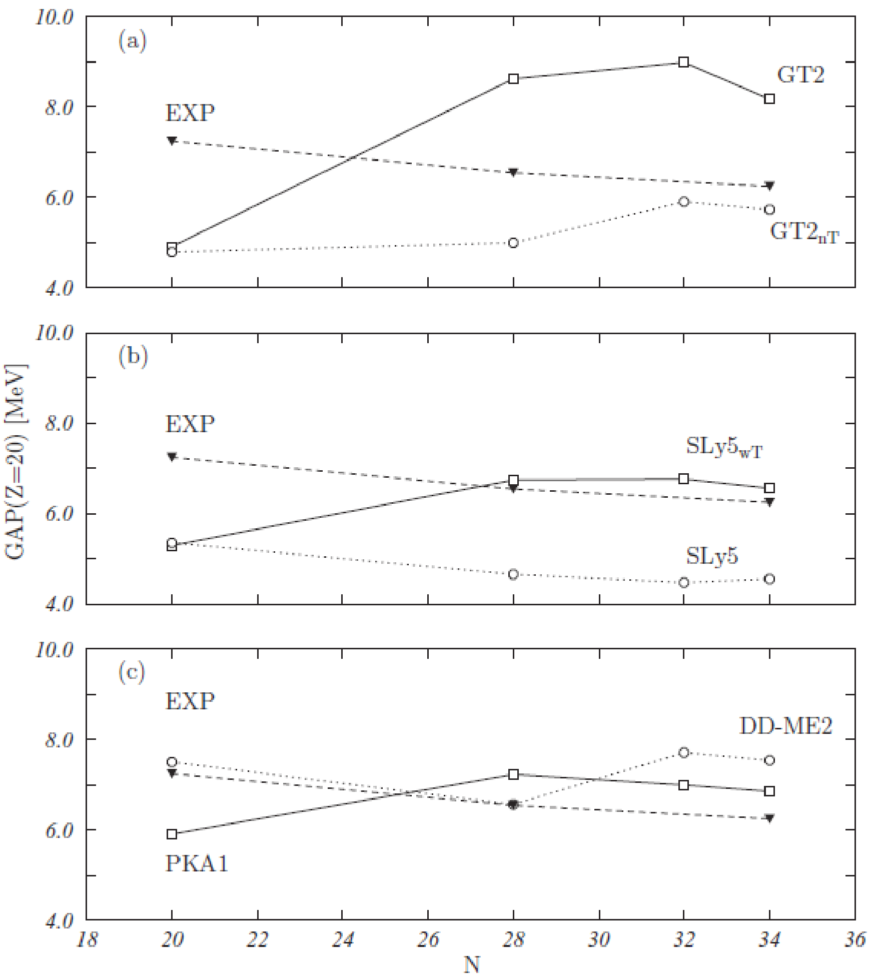
Wang, Zhao, HZL, Long, *PRC* **98**, 034313 (2018)

Tenser effects on gap Z=20



Wang, Zhao, HZL, Long, *PRC* **98**, 034313 (2018)

- **Quite similar behavior** as Skyrme
- But **too weak?**

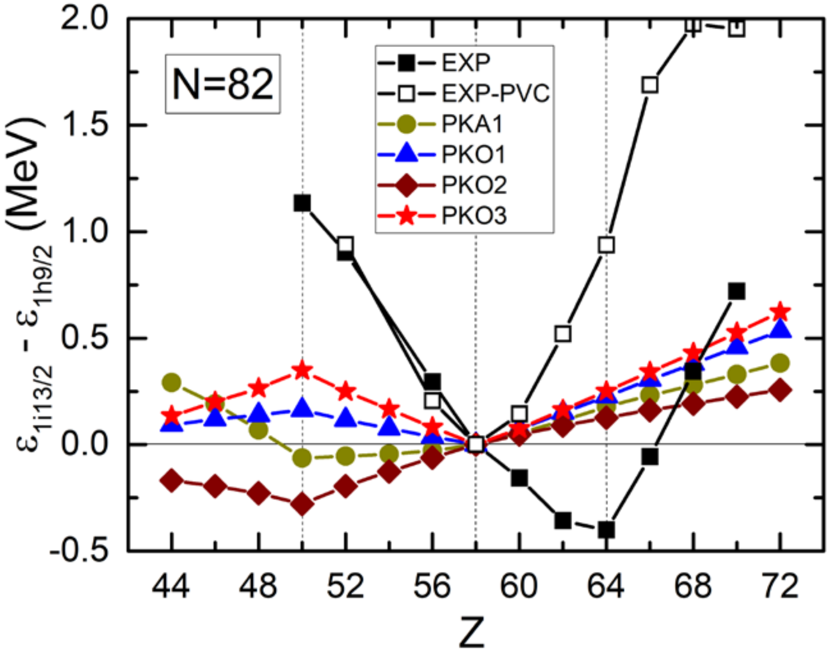


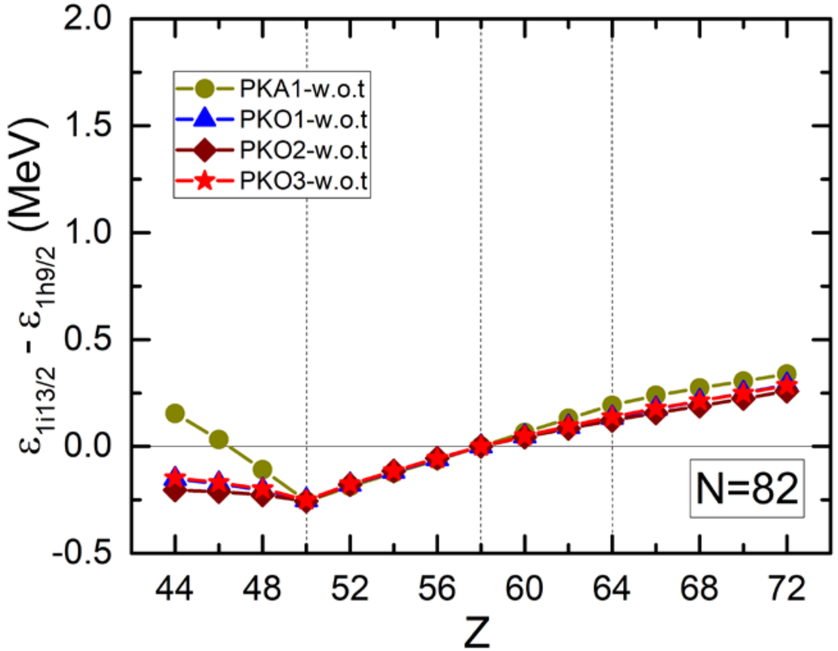
Moreno-Torres *et al.*, *PRC* **81**. 064327 (2010)

What are proper benchmarks?

Tenser effects on N=82 isotones

W/I & W/O tensor force in CDFT





EXP: Original data

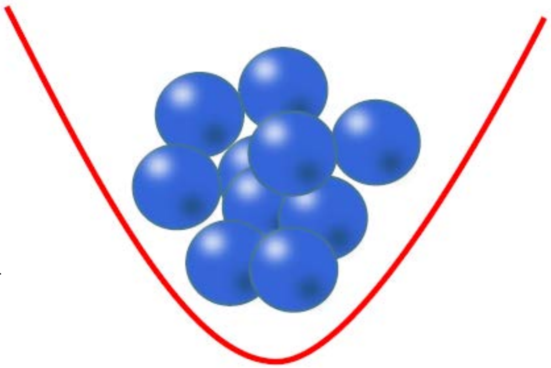
EXP-PVC: subtracting PVC by RMF (as ref.) Afanasjev & Litvinova, *PRC*, **92**, 044317 (2015)

Strengths of tensor force: PKO3>PKO1>PKA1 >PKO2

- Different **strengths of tensor force** → Different **results**

Neutron drops

Neutron drop is a multi-neutron system confined in an external field.



Why neutron drops?

- Simple, can be accessed by many *ab initio* methods
- An ideal environment for studying neutron rich system

Pudliner et al., *PRL* **76**, 2416 (1996)
Gandolfi, Carlson, Pieper, *PRL* **106**, 012501 (2011)
Maris et al., *PRC* **87**, 054318 (2013)
Potter et al., *PLB* **739**, 445 (2014)
Zhao & Gandolfi, *PRC* **94**, 041302(R) (2016)
Shen, Colò , Roca-Maza, arXiv:1810.09691
……

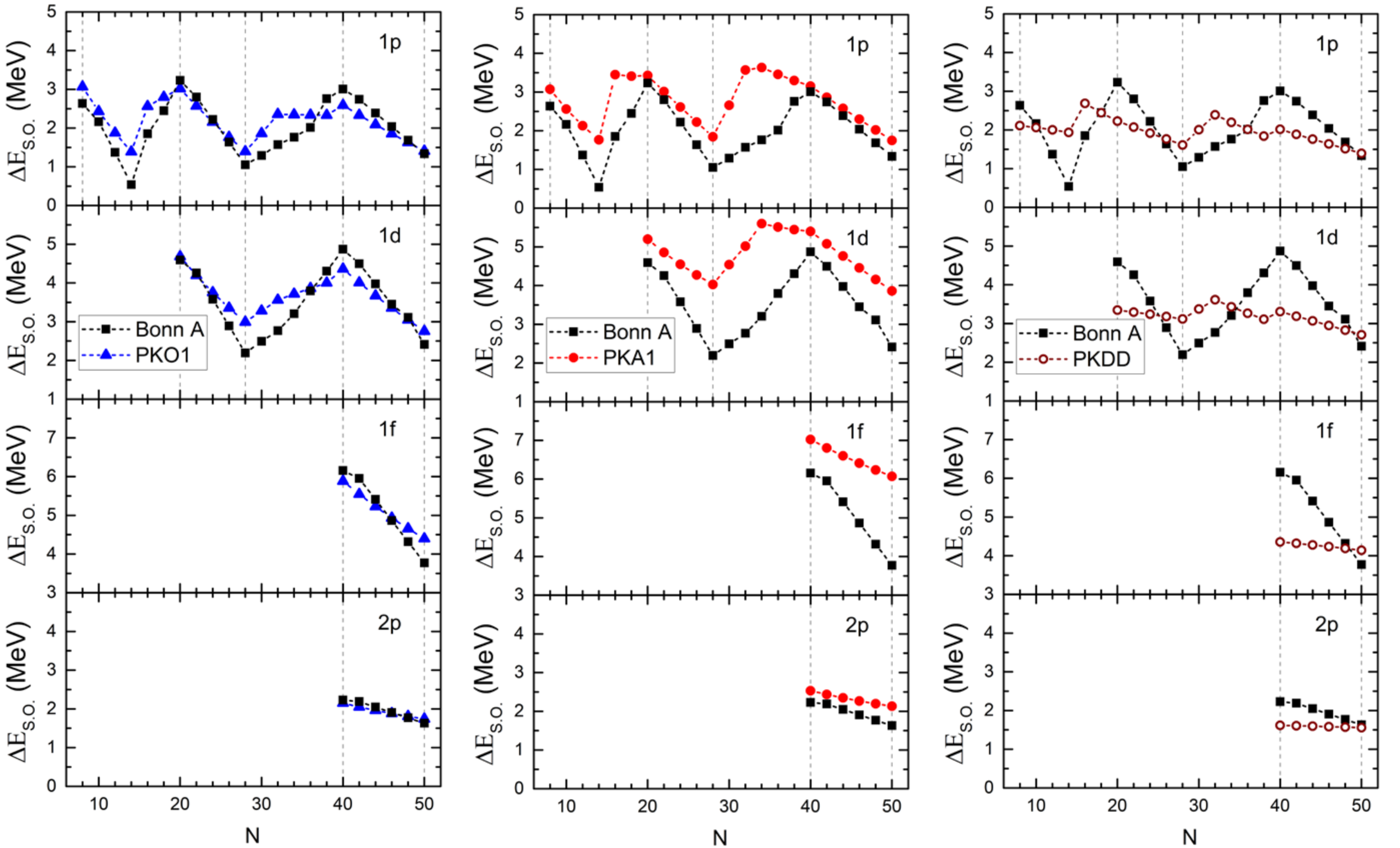
Relativistic Brueckner-Hartree-Fock (RBHF) theory for neutron drops

- Neither RBHF nor CDFT includes **beyond-mean-field effects**

  ➔ it is a fair comparison!

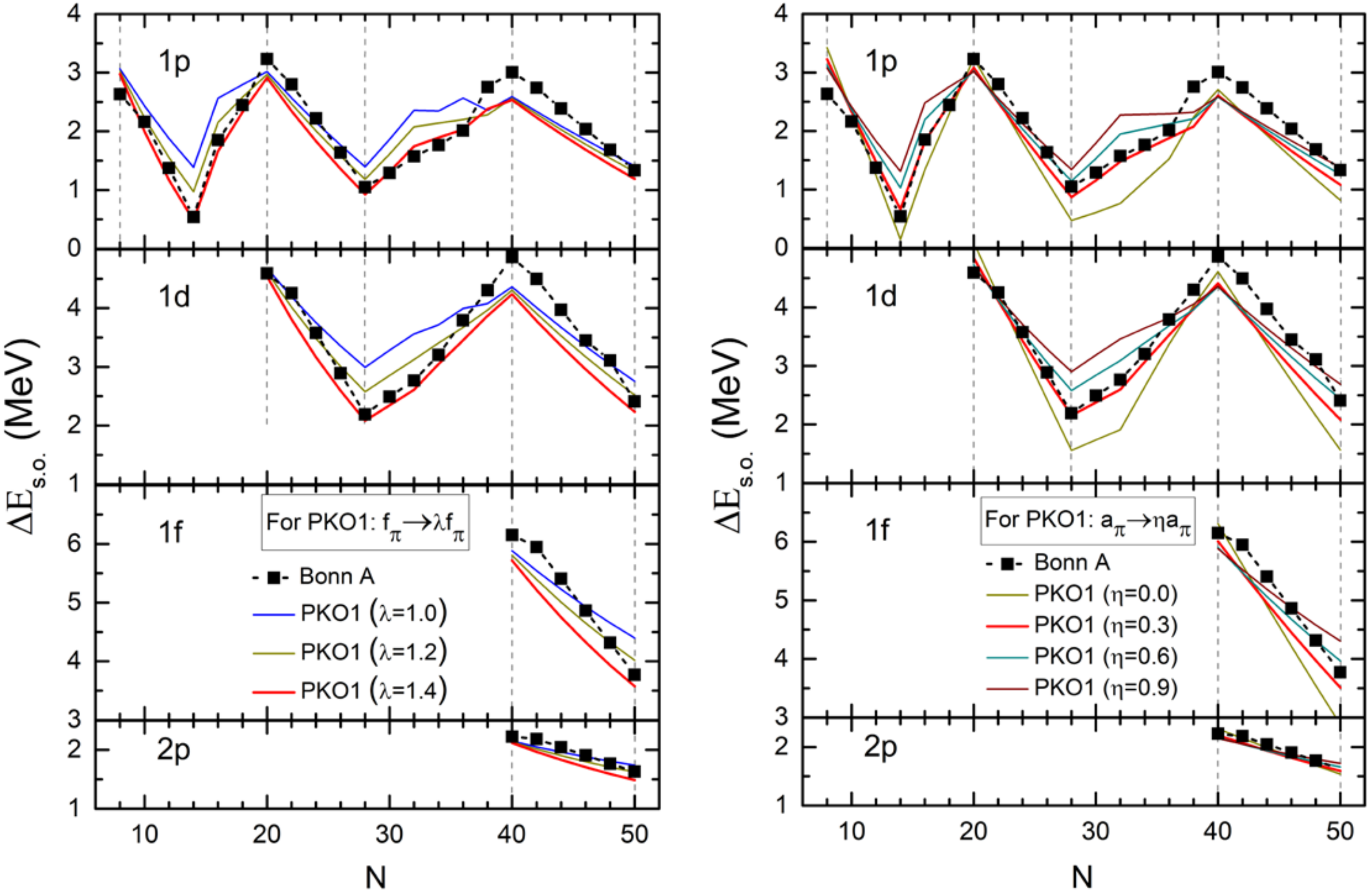
Shen, HZL, Hu, Meng, Ring, Zhang, *CPL* **33**, 102103 (2016)
Shen, HZL, Meng, Ring, Zhang, *PRC* **96**, 014316 (2017)
Shen, HZL, Meng, Ring, Zhang, *PLB* **778**, 344 (2018)
Shen, HZL, Meng, Ring, Zhang, *PRC* **97**, 054312 (2018)
Shen, HZL, Meng, Ring, Zhang, *PLB* **781**, 227 (2018)

Spin-orbit splitting in neutron drops



Shen, HZL, Meng, Ring, Zhang, *PLB* **778**, 344 (2018)

Tensor effects on spin-orbit splitting



$$f_\pi(\xi) = f_\pi(0)e^{-a_\pi \xi}$$

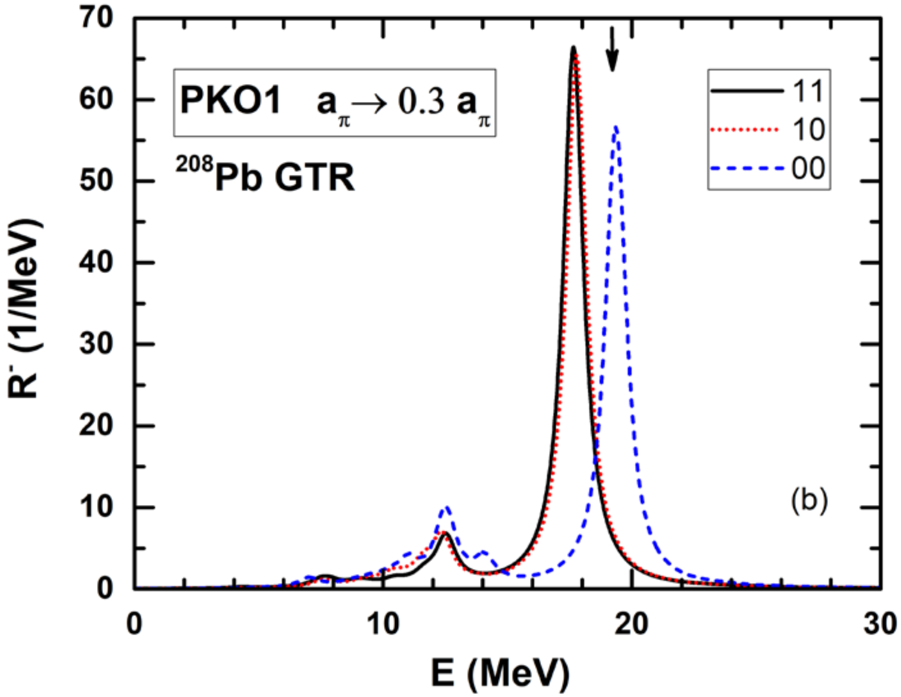
$$\xi \equiv \rho_b/\rho_{\mathrm{sat.}}$$

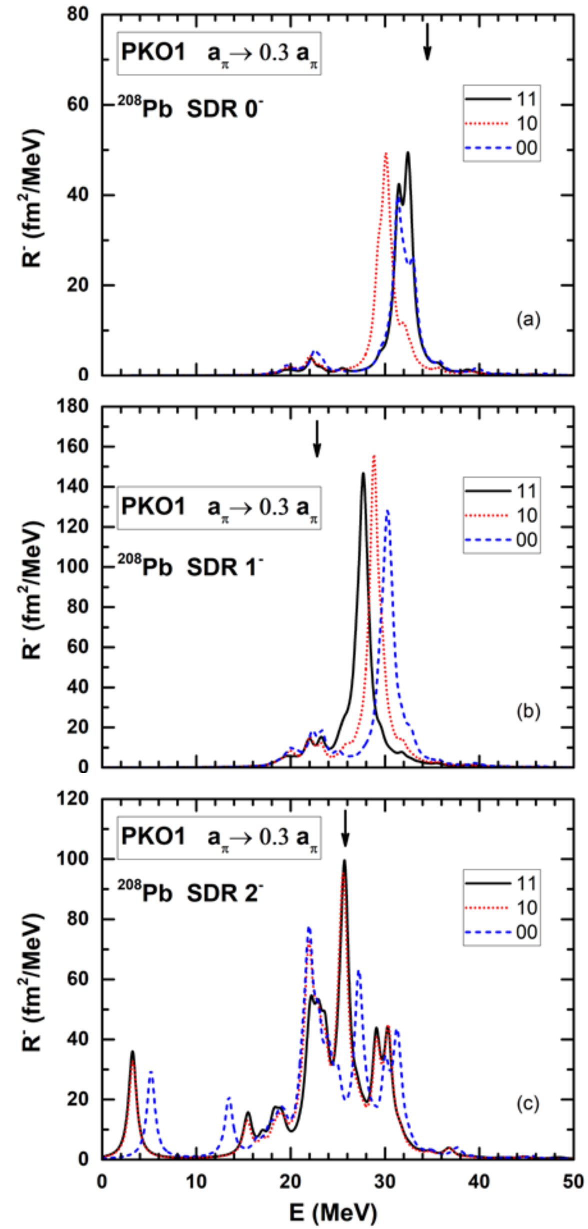
- Increasing the strengths of tensor force: $f_\pi \to 1.4 f_\pi$ or $a_\pi \to 0.3 a_\pi$
- Small a_π → "Tensor renormalization persistency"

cf. Tsunoda *et al.*, *PRC*, **84**, 044322 (2011)

Tensor effects on spin-isospin resonances

- Gamow-Teller and spin-dipole resonances





- GTR: E_{GTR} shifts down
- SDR: $E_{SDR}(0^-)$ & $E_{SDR}(2^-)$ shifts up,
  $E_{SDR}(1^-)$ shifts down

Summary

- **A first quantitative analysis of tensor effects** in the relativistic Hartree-Fock theory
  - ✓ It allows **fair and direct comparisons** with Skyrme, Gogny, etc.
  - ✓ We have investigated the tensor effects on shell evolutions, spin-orbit splitting, spin-isospin resonances, etc.
  - ✓ The strengths of tensor force in present EDFs seem too weak.
- **Remarkable tensor effects on spin-orbit splitting in neutron drops**
  - ✓ **Important guideline** for phenomenological EDF

Thank you!