New relativistic effective interaction for finite nuclei, infinite nuclear matter and neutron stars

Bharat Kumar

LIGO-India Group
IUCAA, Pune, India

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Questions

We have addressed the answers related to the following questions:

1. How to constraint the neutron-skin thickness of ^{208}Pb nuclei, and canonical radius of the NS?
   - PREX experiment has provided the neutron-skin thickness in ^{208}Pb: $\Delta r_{np} = 0.33^{+0.16}_{-0.18}$ fm, which gives real challenge to theory and experiment.
   - Recently, Fattoyev *et. al.* constrained the upper limit of $\Delta r_{np} \lesssim 0.25$ fm for the ^{208}Pb nucleus, and canonical radius of the NS of $R_{1.4M_\odot} < 13.76$ km with the help of GW170817 observation data [7].
2. How to constraint the emperical data of low-density of the pure neutron matter?
   - Low-density data is important for neutron-rich nuclei as well as for the inner crust of the NS.
3. How to reproduce the dimensionless tidal deformability and maximum mass of the recent observation GW170817 ?
   - GW170817 has been reported the $\tilde{\Lambda} \leq 800$ and maximum mass for nonrotating NS should be in the range $2.01 \pm 0.04 \lesssim M(M_\odot) \lesssim 2.16 \pm 0.03$ [8].

[7] F. J. Fattoyev *et al.*, Phys. Rev. Lett. **120**, 172702 (2018).
[8] L. Rezzolla *et al.*, ApJ Lett. **853**, L25 (2018).

Introduction

- In 1935 Yukawa proposed the meson theory of nuclear force which gives a path to finite range interaction of the nuclear force.
- Range of interaction, $r \propto \frac{1}{m_i}$ with m_i is the mass of the meson.
- $\pi-$ long range attraction ($m_\pi = 140$ MeV), $\rightarrow g_\pi \overline{\psi} i \gamma_5 \psi \pi_{ps}$
- $\sigma-$ (two-pion, s wave, $m_\sigma = 500 - 600$ MeV), intermediate range attraction , $\rightarrow g_s \overline{\psi} \psi \sigma$
- $\omega-$ ($3\pi-$ resonance state, p wave, $m_\omega = 783$ MeV) short range repulsion, $\rightarrow g_\omega \overline{\psi} \gamma^\mu \psi \omega_\mu$
- $\rho-$ ($2\pi-$ resonance state p wave, $m_\rho = 763$ MeV) short range repulsion$\rightarrow g_\rho \overline{\psi} \gamma^\mu \vec{\tau} \psi . \vec{R^\mu}$
- $\delta-$meson (4π-resonance, $m_\delta = 980$MeV), $\rightarrow g_\delta \overline{\psi} \psi \delta$
- NN-potential :

$$V(r) = -\frac{g_\delta^2}{4\pi} \frac{e^{-m_\delta r}}{r} + \frac{g_\omega^2}{4\pi} \frac{e^{-m_\omega r}}{r} + \frac{g_\rho^2}{4\pi} \frac{e^{-m_\rho r}}{r} - \frac{g_s^2}{4\pi} \frac{e^{-m_\sigma r}}{r}$$

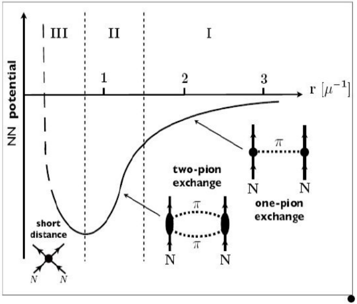


Fig. : Typical NN-potential due to the exchange of massive mesons

Walecka-model ($\sigma - \omega$)

The Lagrangian density of the model has the form [1]

$$\mathscr{L} = \overline{\psi}\{i\gamma^\mu \partial_\mu - M\}\psi + \frac{1}{2}\partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2}m_\sigma^2 \sigma^2 - g_s \overline{\psi}\psi\sigma + \frac{1}{2}m_\omega^2 \omega^\mu \omega_\mu - g_\omega \overline{\psi}\gamma^\mu \psi \omega_\mu - \frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu}$$

- It contains the baryon field ψ of mass M, isoscalar scalar-meson field σ and an isoscalar vector-meson field ω_μ, with the field tensor $\Omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$.
- Parameter sets: $C_s = M(g_s/m_s) = 16.34$, $C_\omega = M(g_\omega/m_\omega) = 13.99$
- In this model meson do not interact among themselves. Hence, the incompressibility (K_∞) of nuclear matter is found to be 550 MeV which is rather high comparison to experimental estimates range 200-260 MeV.
- Note: Walecka model has been renormalizable. Unfortunately, these renormalizable model have encountered difficulties due to large effects from loop integrals that incorporate the dynamics of the quantum vaccum. The effective theory is an alternative.
- In order to lower the value of K_∞ to acceptable range, the self-coupling terms in sigma meson are included by Boguta and Bodmer[2].

[1] S. A. Chin and J. D. Walecka, Phys. Lett. B **52** (1974) 24.
[2] J. Boguta and A. R. Bodmer, Nucl. Phys. A **292** (1977) 413.

Necessity of the non-linear coupling

Now, the Lagrangian density of the model has the form [2]

$$\mathscr{L} = \overline{\psi}\{i\gamma^{\mu}\partial_{\mu} - M\}\psi + \frac{1}{2}\partial^{\mu}\sigma\partial_{\mu}\sigma - \frac{1}{2}m_{\sigma}^{2}\sigma^{2} - \frac{1}{3}g_{2}\sigma^{3} - \frac{1}{4}g_{3}\sigma^{4} - g_{s}\overline{\psi}\psi\sigma + \frac{1}{2}m_{w}^{2}\omega^{\mu}\omega_{\mu} - g_{\omega}\overline{\psi}\gamma^{\mu}\psi\omega_{\mu} - \frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu}$$

- Addition of self-interaction of σ meson, reduction of the high incompressibility (K_{∞}) 550 MeV to ≈270 MeV of infinite nuclear matter.
- In addition to this, the self coupling of the σ meson (nonlinear terms)helps to generate the repulsive part of the NN potential at long range and reproduce finite nuclei properties remarkably [3].
- But it could not reproduce the equation of states upto satisfaction.

[3] F. Coester, S. Cohen, B. D. Day, and C. M. Vincent, Phys. Rev. C **1** (1970) 769.

Continue...

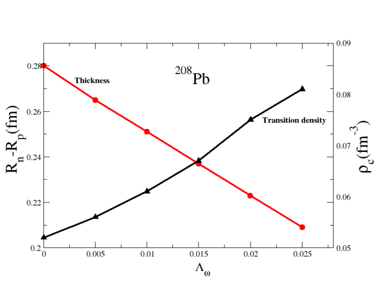
The Lagrangian density of the model ($\sigma - \omega - \rho$) has the form [4]

$$\mathscr{L} = \overline{\psi}\{i\gamma^{\mu}\partial_{\mu} - M\}\psi + \frac{1}{2}\partial^{\mu}\sigma\partial_{\mu}\sigma - \frac{1}{2}m_{\sigma}^{2}\sigma^{2} - \frac{1}{3}g_{2}\sigma^{3} - \frac{1}{4}g_{3}\sigma^{4} - g_{s}\overline{\psi}\psi\sigma$$
$$+\frac{1}{2}m_{w}^{2}\omega^{\mu}\omega_{\mu} - g_{\omega}\overline{\psi}\gamma^{\mu}\psi\omega_{\mu} - \frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \frac{\zeta_{0}}{4!}g_{\omega}^{4}(\omega_{\mu}\omega^{\mu})^{2} + \Lambda_{\omega}(g_{\rho}^{2}\vec{R}^{\mu}.\vec{R}_{\mu})(g_{\omega}^{2}\omega_{\mu}\omega^{\mu})$$
$$+\frac{1}{2}m_{\rho}^{2}\vec{R}^{\mu}.\vec{R}_{\mu} - g_{\rho}\overline{\psi}\gamma^{\mu}\vec{\tau}\psi.\vec{R^{\mu}} - \frac{1}{4}\vec{R}^{\mu\nu}.\vec{R}_{\mu\nu} - e\overline{\psi}\gamma^{\mu}\frac{(1-\tau_{3})}{2}\psi A_{\mu} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}.$$

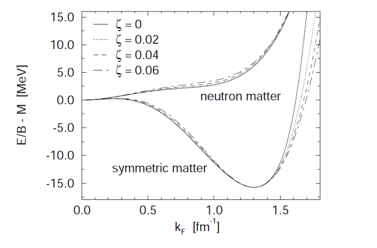
- The successful NL3[5] parameter set, it produces an equation of state stiffer and corresponding maximum neutron star mass larger but this parameter set timely reproduced ground state properties of finite nuclei.
- The value of ζ_0 may be used to efficiently tune the maximum neutron star mass.
- The cross-coupling of $\rho-$meson with ω allows one to vary the neutron-skin thickness in a heavy nucleus like ^{208}Pb over a wide range.
  Note: $(R_n - R_p)^{Exp.} = 0.33^{+0.16}_{-0.18}$ fm.
- The Λ_ω is highly sensitive to the density dependence of symmetry energy-and in particular to its slope at saturation density, which has important implications in structure and dynamics of neutron stars.

[4]Horst Mueller, Brian D. Serot, Nucl. Phys. A **606** (1996) 508.
[5] G. A. Lalazissis, J. Koning and P. Ring, Phys. Rev C **55** (1997) 540.

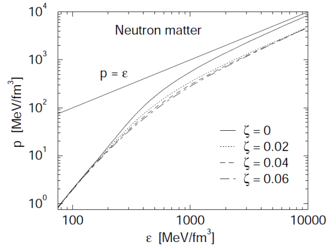
Examples...



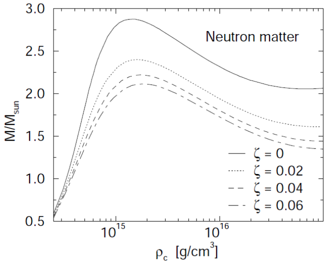
208Pb
Thickness
Transition density
R_n-R_p(fm)
ρ_c(fm^-3)
Λ_ω
0.2
0.22
0.24
0.26
0.28
0.05
0.06
0.07
0.08
0.09
0
0.005
0.01
0.015
0.02
0.025



ζ = 0
ζ = 0.02
ζ = 0.04
ζ = 0.06
neutron matter
symmetric matter
E/B - M [MeV]
k_F [fm^-1]
15.0
10.0
5.0
0.0
-5.0
-10.0
-15.0
0.0
0.5
1.0
1.5



Neutron matter
p = ε
ζ = 0
ζ = 0.02
ζ = 0.04
ζ = 0.06
p [MeV/fm^3]
ε [MeV/fm^3]
10^4
10^3
10^2
10^1
10^0
100
1000
10000



Neutron matter
ζ = 0
ζ = 0.02
ζ = 0.04
ζ = 0.06
M/M_sun
ρ_c [g/cm^3]
3.0
2.5
2.0
1.5
1.0
0.5
10^15
10^16

FSUGold Series

- After getting a idea from Muller et. al. paper, Florida group developed series of FSUGold parameter sets including terms like isoscalar and isovector parameters, i.e, ζ_0 and Λ_ω, respectively.
- First parameter FSUGold and published in PRL **95** (2005)122501.
  $K = 230$ MeV, $R_n - R_p = 0.21$ fm, $M = 1.72 M_\odot$ and a radius of $M_{1.4}$ is 12.66 km.
- Other parameter sets like FSUGold2 and FSU2R etc., but these parameter sets are fails to reproduce the sub-saturation density of the finite nuclei.
- Now, the question is that whether we include more parameter or not?
- Our answer is that yes, if additional parameters are taken into consideration it would be relevant from physics point of view. At subsatuartion densities, the $\delta-$meson softens the symmetry energy. At higher density, $\delta-$meson may affect the maximum mass of the neutron star. Similarly, we can expect that the addition of this coupling into the Lagrangian, may improve the quality of flow data and so on.

Our model

The Lagrangian density for nucleon-meson many body system can be written as [9]:

$$\mathscr{L} = \sum_{\alpha} \varphi_{\alpha}^{\dagger}(r) \Bigg\{ -i\alpha\cdot\nabla + \beta\left[M - \Phi(r) - \tau_3 D(r)\right] + W(r) + \frac{1}{2}\tau_3 R(r) + \frac{1+\tau_3}{2} A(r)$$

$$- \frac{i\beta\alpha}{2M}\cdot\left(f_{\omega}\nabla W(r) + \frac{1}{2} f_{\rho}\tau_3 \nabla R(r) + \lambda\nabla A\right) + \frac{1}{2M^2}\left(\beta_{\sigma} + \beta_{\omega}\tau_3\right)\Delta A \Bigg\} \varphi_{\alpha}(r)$$

$$+ \left(\frac{1}{2} + \frac{\kappa_3}{3!}\frac{\Phi(r)}{M} + \frac{\kappa_4}{4!}\frac{\Phi^2(r)}{M^2}\right)\frac{m_s^2}{g_s^2}\Phi^2(r) - \frac{\zeta_0}{4!}\frac{1}{g_{\omega}^2}W^4(r) + \frac{1}{2g_s^2}\left(1 + \alpha_1\frac{\Phi(r)}{M}\right)\left(\nabla\Phi(r)\right)^2$$

$$- \frac{1}{2g_{\omega}^2}\left(1 + \alpha_2\frac{\Phi(r)}{M}\right)\left(\nabla W(r)\right)^2 - \frac{1}{2}\left(1 + \eta_1\frac{\Phi(r)}{M} + \frac{\eta_2}{2}\frac{\Phi^2(r)}{M^2}\right)\frac{m_{\omega}^2}{g_{\omega}^2}W^2(r)$$

$$- \frac{1}{2e^2}\left(\nabla A(r)\right)^2 - \frac{1}{2g_{\rho}^2}\left(\nabla R(r)\right)^2 - \frac{1}{2}\left(1 + \eta_{\rho}\frac{\Phi(r)}{M}\right)\frac{m_{\rho}^2}{g_{\rho}^2}R^2(r)$$

$$- \Lambda_{\omega}\left(R^2(r)W^2(r)\right) + \frac{1}{2g_{\delta}^2}\left(\nabla D(r)\right)^2 + \frac{1}{2}\frac{{m_{\delta}}^2}{g_{\delta}^2}\left(D^2(r)\right)$$

$$- \frac{1}{2e^2}\left(\nabla A\right)^2 + \frac{1}{3g_{\gamma}g_{\omega}}A\Delta W + \frac{1}{g_{\gamma}g_{\rho}}A\Delta R.$$

$$\boxed{\frac{\partial\mathscr{L}}{\partial\psi} - \partial_{\mu}\left(\frac{\partial\mathscr{L}}{\partial(\partial_{\mu}\psi)}\right) = 0}$$

[9] Shailesh K. Singh *et. al.*, Phys. Rev. C **89** (2014) 044001.

Equation of motion.....

- The Dirac equation for the Lagrangian density eqn. (1) becomes

$$\Big\{-i\alpha\cdot\nabla+\beta[M-\Phi(r)-\tau_3 D(r)]+W(r)+\frac{1}{2}\tau_3 R(r)+\frac{1+\tau_3}{2}A(r)+\frac{1}{2M^2}(\beta_\sigma+\beta_\omega\tau_3)\Delta A$$

$$-\frac{i\beta\alpha}{2M}\cdot\left[f_\omega\nabla W(r)+\frac{1}{2}f_\rho\tau_3\nabla R(r)+\lambda\nabla A\right]\Big\}\varphi_\alpha(r)=\varepsilon_\alpha\,\varphi_\alpha(r).$$

- The mean field equations for Φ, W, R, D and A are given by

$$-\Delta\Phi(r)+m_s^2\Phi(r)=g_s^2\rho_s(r)-\frac{m_s^2}{M}\Phi^2(r)\left(\frac{\kappa_3}{2}+\frac{\kappa_4}{3!}\frac{\Phi(r)}{M}\right)+\frac{g_s^2}{2M}\left(\eta_1+\eta_2\frac{\Phi(r)}{M}\right)\frac{m_\omega^2}{g_\omega^2}W^2(r)$$

$$+\frac{\eta_\rho}{2M}\frac{g_s^2}{{g_\rho}^2}m_\rho^2R^2(r)+\frac{\alpha_1}{2M}[(\nabla\Phi(r))^2+2\Phi(r)\Delta\Phi(r)]+\frac{\alpha_2}{2M}\frac{g_s^2}{g_\omega^2}(\nabla W(r))^2,$$

$$-\Delta W(r)+m_\omega^2W(r)=g_\omega^2\left(\rho(r)+\frac{f_v}{2}\rho_{\rm T}(r)\right)-\left(\eta_1+\frac{\eta_2}{2}\frac{\Phi(r)}{M}\right)\frac{\Phi(r)}{M}m_\omega^2W(r)-\frac{1}{3!}\zeta_0W^3(r)$$

$$+\frac{\alpha_2}{M}[\nabla\Phi(r)\cdot\nabla W(r)+\Phi(r)\Delta W(r)]-2\,\Lambda_v{g_\omega}^2R^2(r)W(r),$$

$$-\Delta R(r)+m_\rho^2R(r)=\frac{1}{2}g_\rho^2\left(\rho_3(r)+\frac{1}{2}f_\rho\rho_{{\rm T},3}(r)\right)-\eta_\rho\frac{\Phi(r)}{M}m_\rho^2R(r)-2\,\Lambda_v{g_\rho}^2R(r)W^2(r),$$

$$-\Delta A(r)=e^2\rho_{\rm p}(r),-\Delta D(r)+{m_\delta}^2D(r)=g_\delta^2\rho_{s3},$$

Continue.....

- The scalar and vector potential is

$$S(r) = -g_s \Phi_0(r) - g_\delta D_0(r)$$

$$V(r) = g_\omega W_0(r) + \frac{1}{2} g_\rho \tau_3 R_0(r) + e\frac{(1-\tau_3)}{2} A_0(r)$$

- The set of coupled differential equations are solved self-consistently to describe the ground state properties of finite nuclei.
- The total binding energy is obtained by summing the individual contribution, which is given by:

$$\boxed{E_{total} = E_{part} + E_\sigma + E_\omega + E_\rho + E_{\rho\omega} + E_\delta + E_c + E_{pair} + E_{c.m.}}$$

Where E_{part} is the sum of the single particle energies of the nucleons.
E_σ, E_ω, E_ρ, E_δ, E_c are the contribution of the respective mesons and Coulomb fields.
The correction to the binding energy $E_{cm} = \frac{17.2}{A^{1/5}} \mathrm{M}eV$ and corresponding to the mean-square charge radius: $\langle r_c^2 \rangle = -\frac{3}{4}\frac{1}{(2MAE_{c.m.})} \mathrm{f}m^2$

Simulating Annealing Method(SAM)

What is simulated annealing?

Annealing is a process in which metal or liquid is heated to a specific temperature and then allowed to cool slowly to achieve minimum energy configuration of the system.

Unique Features:

1. Highly adaptive
2. Suitable for problems of large scale
3. Search desired global minimum hidden among local minima

- We have used the SAM[10] to determine the values of the new ERMF parameter set by searching for the global minimum in the hypersurface of the χ^2 function, given as

$$\chi^2 = \frac{1}{N_d - N_p} \sum_{i=1}^{N_d} \left(\frac{O_i^{exp} - O_i^{th}}{\sigma_i}\right)^2$$

- Here, N_d and N_p are the number of experimental data points and the number of fitted parameters, respectively. σ_i is the theoretical error, and O_i^{exp} and O_i^{th} are the experimental and theoretical values, respectively.

[10] B. K. Agrawal *et. al.*, Phys. Rev. C **72** (2005) 014310.

Strategy of fitting parameter set

For convenience, we define a vector v with the components as
$v \equiv (\mathscr{E}_0, K_\infty, \rho_0, M*/M, J, g_\delta, \eta_1, \eta_2, \eta_\rho, \Lambda_\omega, \alpha_1, \alpha_2, \beta_\sigma, \beta_\omega, \zeta_0, f_\rho, f_\omega, m_\sigma)$

We implemented the SAM algorithm by using the following basic steps,

1. We start with a guess value for the vector v and calculate χ^2 (say, χ^2_{old}) using Eq.(1) for a given set of the experimental data and corresponding ERMF results together with the theoretical errors.
2. We generate randomly a new set of ERMF parameters by using following steps.
   First, we use a uniform random number to select a component v_r of the vector v.
   Second, the randomly selected component v_r is then assigned a new value,

   $v_r \rightarrow v_r + \eta d_r$ where η is a uniform random number that lies within the range of -1 to +1.

   The second step is repeated until the new value of v_r is found within its allowed limit defined by v_0 and v_1.
3. The newly generated set of the ERMF parameters is accepted by use of the SAM algorithm as follows.
   We calculate the quantity $P(\chi^2) = e^{(\chi^2_{old} - \chi^2_{new})/T}$

   The new set of parameter ERMF parameters is accepted only if $P(\chi^2) > \beta$, $0 < \beta < 1$.

In present work we have employed the Cauchy annealing schedule given by $T(k) = T_i/(k+1)$ where, k=1,2,3....is the time index.

Parameter	v	v_0	v_1	d
$\mathscr{E}_0$	-16.02	-16.30	-15.70	0.025
K_∞	230.0	210.0	245.0	1.0
ρ_0	0.148	0.140	0.165	0.001
M^*/M	0.525	0.5	0.9	0.002
J	32.1	28.0	35.0	0.08
g_δ	2.0	0.0	15.0	0.2
η_1	0.410	0.4	0.8	0.002
η_2	0.10	0.09	0.12	0.002
$\eta\rho$	0.590	0.1	0.7	0.003
$\Lambda\omega$	0.03	0.02	0.09	0.002
α_1	1.73	1.0	2.0	0.005
α_2	-1.51	-1.65	-1.40	0.005
$\beta\sigma$	-0.083	-0.09	-0.08	0.00001
$\beta\omega$	-0.55	-0.6	-0.4	0.001
ζ_0	1.01	1.01	1.01	0.0
$f\rho/4$	3.0	0.0	6.0	0.03
$f\omega/4$	0.4	0.0	1.0	0.005
m_s	510.0	480.0	570.0	0.450

Table: The vector v_0 and v_1 contain the lower and upper limits of each of the components of the vector v. The vector d represents the maximum displacement allowed in a single step for the components of the vector v.

Results and discussions

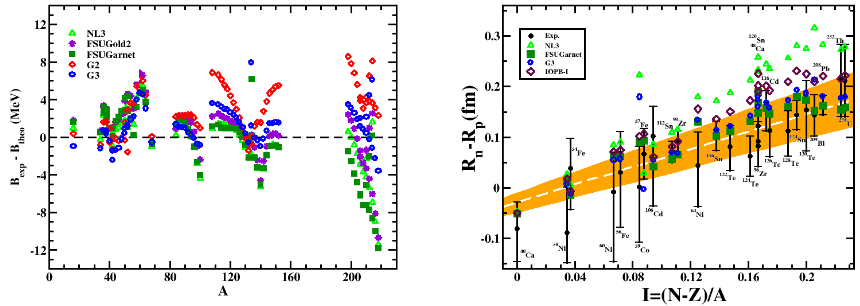
Table 1: The obtained new parameter set G3 along with NL3, FSUGarnet and IOPB-I sets are listed.

	NL3	FSUGarnet	G3	IOPB-I
m_s/M	0.541	0.529	0.559	0.533
m_ω/M	0.833	0.833	0.832	0.833
m_ρ/M	0.812	0.812	0.820	0.812
m_δ/M	0.0	0.0	1.043	0.0
$g_s/4\pi$	0.813	0.837	0.782	0.827
$g_\omega/4\pi$	1.024	1.091	0.923	1.062
$g_\rho/4\pi$	0.712	1.105	0.962	0.885
$g_\delta/4\pi$	0.0	0.0	0.160	0.0
k_3	1.465	1.368	2.606	1.496
k_4	-5.688	-1.397	1.694	-2.932
ζ_0	0.0	4.410	1.010	3.103
η_1	0.0	0.0	0.424	0.0
η_2	0.0	0.0	0.114	0.0
η_ρ	0.0	0.0	0.645	0.0
Λ_ω	0.0	0.043	0.038	0.024
α_1	0.0	0.0	2.000	0.0
α_2	0.0	0.0	-1.468	0.0
$f_\omega/4$	0.0	0.0	0.220	0.0
$f_\rho/4$	0.0	0.0	1.239	0.0
β_σ	0.0	0.0	-0.087	0.0
β_ω	0.0	0.0	-0.484	0.0
Nuclear matter properties				
ρ_0	0.148	0.153	0.148	0.149
$\mathcal{E}_0$	-16.29	-16.23	-16.02	-16.10
K_∞	271.5	229.5	243.9	222.6
M^*/M	0.595	0.578	0.699	0.593
J	37.40	30.95	31.8	33.30
L	118.6	51.04	49.31	63.58

Table 2: Bulk properties of finite nuclei.

Nucleus	Obs.	Expt.	NL3	FSUGarnet	G3	IOPB-I
^{16}O	B/A	7.976	7.917	7.876	8.037	7.977
	R_c	2.699	2.714	2.690	2.707	2.705
	R_n-R_p	-	-0.026	-0.028	-0.028	-0.027
^{40}Ca	B/A	8.551	8.540	8.528	8.561	8.577
	R_c	3.478	3.466	3.438	3.459	3.458
	R_n-R_p	-	-0.046	-0.051	-0.049	-0.049
^{48}Ca	B/A	8.666	8.636	8.609	8.671	8.638
	R_c	3.477	3.443	3.426	3.466	3.446
	R_n-R_p	-	0.229	0.169	0.174	0.202
^{68}Ni	B/A	8.682	8.698	8.692	8.690	8.707
	R_c	-	3.870	3.861	3.892	3.873
	R_n-R_p	-	0.262	0.184	0.190	0.223
^{90}Zr	B/A	8.709	8.695	8.693	8.699	8.691
	R_c	4.269	4.253	4.231	4.276	4.253
	R_n-R_p	-	0.115	0.065	0.068	0.091
^{100}Sn	B/A	8.258	8.301	8.298	8.266	8.284
	R_c	-	4.469	4.426	4.497	4.464
	R_n-R_p	-	-0.073	-0.078	-0.079	-0.077
^{132}Sn	B/A	8.355	8.371	8.372	8.359	8.352
	R_c	4.709	4.697	4.687	4.732	4.706
	R_n-R_p	-	0.349	0.224	0.243	0.287
^{208}Pb	B/A	7.867	7.885	7.902	7.863	7.870
	R_c	5.501	5.509	5.496	5.541	5.521
	R_n-R_p	-	0.283	0.162	0.180	0.221

Binding energy & Neutron-skin thickness



- The rms deviation are 2.977, 3.062, 3.696, 3.827 and 2.308 for NL3, FSUGold2, FSUGarnet, G2 and G3 respectively.
- The rms deviation on the binding energies for G3 parameter set is smaller in comparison to the other parameter set .
- The neutron-skin thickness is defined as [11]: $\Delta r_{np} = R_n - R_p$

[11] A. Trzcińska *et. al.*, Phys. Rev. Lett. **87** (2001) 082501.

Infinite nuclear matter

Energy and pressure density...

- By forming the energy-momentum tensor in the mean field approximation, one can calculated energy density and pressure of the system as a function of density.
- Energy density:

$$\mathscr{E} = \frac{2}{(2\pi)^3}\int d^3k E_i^*(k) + \rho W + \frac{m_s^2\Phi^2}{g_s^2}\left(\frac{1}{2}+\frac{\kappa_3}{3!}\frac{\Phi}{M}+\frac{\kappa_4}{4!}\frac{\Phi^2}{M^2}\right) - \frac{1}{2}m_\omega^2\frac{W^2}{g_\omega^2}\left(1+\eta_1\frac{\Phi}{M}+\frac{\eta_2}{2}\frac{\Phi^2}{M^2}\right)$$
$$-\frac{1}{4!}\frac{\zeta_0 W^4}{g_\omega^2}+\frac{1}{2}\rho_3 R-\frac{1}{2}\left(1+\frac{\eta_\rho\Phi}{M}\right)\frac{m_\rho^2}{g_\rho^2}R^2-\Lambda_\omega\left(R^2W^2\right)+\frac{1}{2}\frac{m_\delta^2}{g_\delta^2}\left(D^2\right),$$

- Pressure density:

$$P = \frac{2}{3(2\pi)^3}\int d^3k \frac{k^2}{E_i^*(k)} - \frac{m_s^2\Phi^2}{g_s^2}\left(\frac{1}{2}+\frac{\kappa_3}{3!}\frac{\Phi}{M}+\frac{\kappa_4}{4!}\frac{\Phi^2}{M^2}\right) + \frac{1}{2}m_\omega^2\frac{W^2}{g_\omega^2}\left(1+\eta_1\frac{\Phi}{M}+\frac{\eta_2}{2}\frac{\Phi^2}{M^2}\right)$$
$$+\frac{1}{4!}\frac{\zeta_0 W^4}{g_\omega^2}+\frac{1}{2}\left(1+\frac{\eta_\rho\Phi}{M}\right)\frac{m_\rho^2}{g_\rho^2}R^2+\Lambda_\omega\left(R^2W^2\right)-\frac{1}{2}\frac{m_\delta^2}{g_\delta^2}\left(D^2\right),$$

Note: The solution of the mean field equations is simplified significantly in the case of infinite nuclear matter, which we assume to be spatially uniform. For this uniform case, the meson fields are uniform (*i.e* constant throughout space) and the nucleon orbitals are plane-wave Dirac spinors with medium-modified effective mass and energies.

Symmetry Energy ...

- The binding energy per nucleon can be written in the form of asymmtery parameter $\alpha(=\frac{(\rho_n-\rho_p)}{(\rho_n+\rho_p)})$.

$$e(\rho,\alpha) = \frac{\mathscr{E}}{\rho_B} - M = e(\rho)\Big|_{\rho=\rho_0} + \alpha^2 S(\rho) + O(\alpha^4)$$

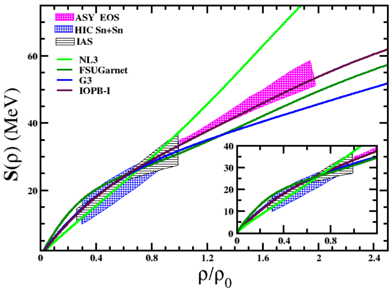
- The strong force which is binding in nuclei is symmetric under the exchange neutron to proton (or proton-to-neutron).
- The density dependent symmetry energy of the system:

$$S(\rho) = \frac{1}{2}\left[\frac{\partial^2 e(\rho,\alpha)}{\partial \alpha^2}\right]_{\alpha=0}$$

$$S(\rho) = \frac{k_F^2}{6E_F^*} + \frac{g_\rho^2 \rho}{8m_\rho^{*2}} - \frac{1}{2}\rho\frac{g_\delta^2}{m_\delta^2}\left(\frac{M^*}{E_F}\right)^2$$

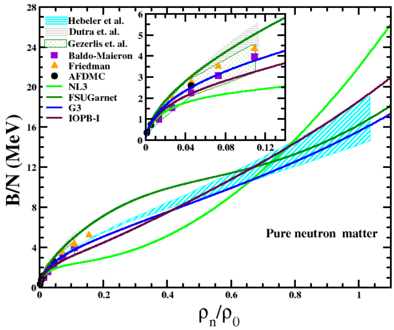
- To do so, we perform a Taylor series expansion around nuclear-matter saturation density ρ_0 .

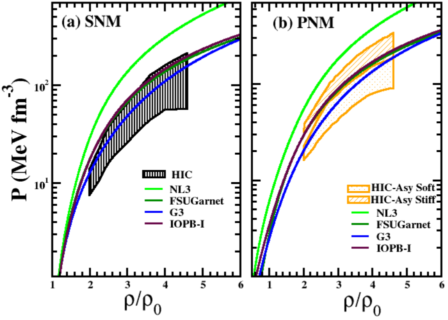
$$e(\rho) = \varepsilon_0 + \frac{1}{2}Kx^2 + \ldots,$$



- The symmetry energy determines how the energies of nuclei and nuclear matter depend on the difference between neutron and proton densities.
- The proton fraction in NS matter is controlled by the density dependence of the symmetry energy.

Low and high density matter for SNM & PNM

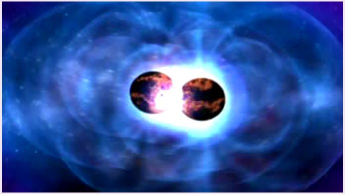




- NL3 model provides too much large pressure on SNM comparison to other EoSs.
- The non-linear terms in the Lagrangian can reduce the pressure in such models so as to be consistent with the present experimental constraints.
- The pressure in the actual NS environment is somewhat smaller than that for pure neutron matter, reflecting the small fraction of nucleons that are protons.

Neutron Star

Binary neutron star merger



Credit: LIGO team

Introduction

- Two neutron stars (NS) revolve about a common centre of mass. While rotating, they emit gravitational waves. In this process, the orbits lose energy and get close and closer, which is called inspiralling.
- When they approach, they come under the influence of each other and get distorted. The after effect is that tides are raised exactly the same way as tides are created on Earth due to Moon.
- The newly formed tides pick the energy out of the orbit resulting in the speedy motion of the inspiral. This can be detected and measured in the form of gravitational waves.
- Larger are the size of the neutron stars, bigger are the tides formed.
- From the equation of state (EoS) we can determine the size of NS alongwith its tidal deformation.
- In the experimental front, from the measurements of the NS masses and the extent of tidal deformation their size and EoS can be calculated.

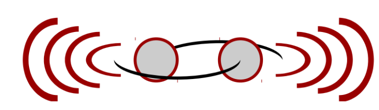


Figure: Compact stars; Credit: P. Landry

Tidal Love numbers

- In 1911, the mathematician A. E. H. Love introduced the dimensionless parameter in Newtonian theory. It is related to the tidal deformation of the Earth which is because of the gravitational attraction between the Moon and the Sun.
- In Newtonian gravity, the tidal Love number is a constant of proportionality between the tidal field applied to the body and the resulting multipole moment of its mass distribution.
- In case of quadrupole, the tidal field is characterized by tidal moment

$$\mathcal{E}_{ij} = -\partial u_{ext} = -\partial_{ij}\left(\frac{M_B}{r_{AB}}\right)$$

in which the external potential is generated by the rest of the universe.

- In the presence of a tidal field, the quadrupole moment is proportional to the tidal field

$$Q_{ij} = -\frac{2}{3}k_2 R^5 \mathcal{E}_{ij} = -\lambda \mathcal{E}_{ij}$$

where, k_2 and λ are the dimensionless tidal Love number, and the tidal deformability of the star which depend on the EoS. R is the radius of the star.

- In the absence of a tidal field the body would be spherical, and its quadrupole moment would vanish.
- In general relativity, two types of Love numbers: an electric type of Love number k_{el} that has direct analogy with the Newtonian Love number (the gravitational fields generated by masses), and a magnetic-type Love number k_{mag} (gravitational field generated by motion of masses) that has no analogue in Newtonian gravity [6].

[6] T. Binnington and E. Poisson, Phys. Rev. D **80**, 084018 (2009).

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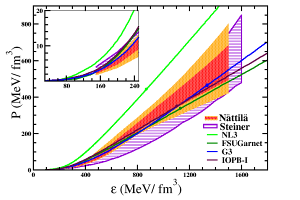
EoS



TOV equation

$$\frac{dP(r)}{dr} = -\frac{[\mathscr{E}(r)+P(r)][M(r)+4\pi r^3 P(r)]}{r^2(1-\frac{2M(r)}{r})},$$

$$\frac{dM(r)}{dr} = 4\pi r^2 \mathscr{E}(r).$$

- For a given EoS, the TOV equation must be integrated from the boundary conditions $P(0) = P_c$ and $M(0) = 0$, $P(R) = 0$ and $M(R) = M$.

TOV+$h_{\mu\nu}$

- To estimate the Love numbers k_2, alongwith the evolution of TOV equation, we have to compute $y = y_2(R)$ with initial boundary condition $y(0) = 2$ from the following differential equation iteratively:

$$r\frac{dy(r)}{dr} + y(r)^2 + y(r)F(r) + r^2Q(r) = 0,$$

$$F(r) = \frac{r - 4\pi r^3[\mathscr{E}(r) - P(r)]}{r - 2M(r)}, \quad Q(r) = \frac{4\pi r(5\mathscr{E}(r) + 9P(r) + \frac{\mathscr{E}(r)+P(r)}{\partial P(r)/\partial \mathscr{E}(r)} - \frac{6}{4\pi r^2})}{r - 2M(r)} - 4\left[\frac{M(r) + 4\pi r^3 P(r)}{r^2(1 - 2M(r)/r)}\right]^2.$$

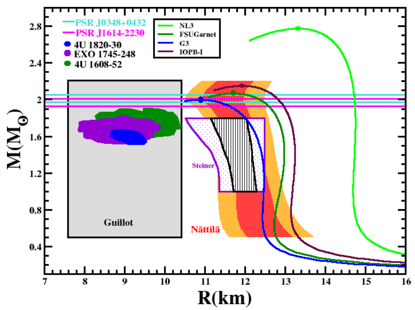
Quadrupole Love number

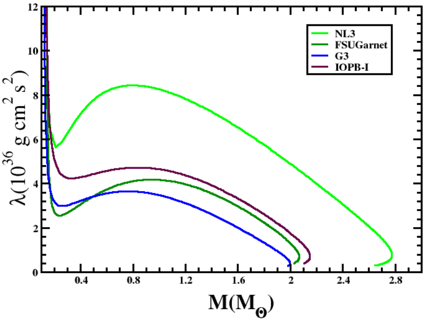
$$k_2 = \frac{8}{5}(1-2C)^2C^5[2C(y_2-1) - y_2 + 2]\Big\{2C(4(y_2+1)C^4 + (6y_2-4)C^3 + (26-22y_2)C^2 + 3(5y_2-8)C - 3y_2 + 6) - 3(1-2C)^2(2C(y_2-1) - y_2 + 2)log\left(\frac{1}{1-2C}\right)\Big\}^{-1},$$

Tidal deformability

$$\lambda_2 = \frac{2}{3}k_2R^5$$

Mass-radius and tidal deformability





- G3: $M_{max} = 2.0 M_\odot$, $R = 10.902$ km
- IOPB-I: $M_{max} = 2.15 M_\odot$, $R = 11.936$ km
- GW170817: $2.01 \pm 0.04 \lesssim M(M_\odot) \lesssim 2.16 \pm 0.03$ [12]

[12] L. Rezzolla *et al.*, ApJ Lett. **853**, L25 (2018).

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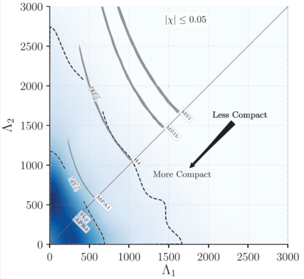
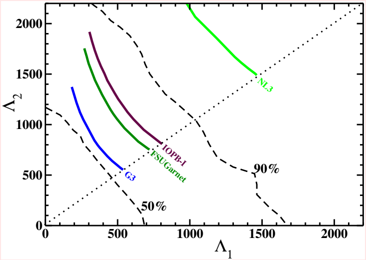
GW170817 results



Our results



- The LIGO/VIRGO limit on the tidal deformabilities favour soft EoSs, the $2M_{\odot}$ constraints requires the EoS to be stiff, thus setting a very restrictive bound for the quantity.
- The gravitational waveform depends on the weighted tidal deformability:

$$\tilde{\Lambda} = \frac{16}{13}\left(\frac{(M_1+12M_2)M_1^4}{(M_1+M_2)^5}\Lambda_1 + \frac{(M_2+12M_1)M_2^4}{(M_1+M_2)^5}\Lambda_2\right)$$

"Remember that all models are wrong; the practical question is how wrong do they have to be to not be useful."——Box

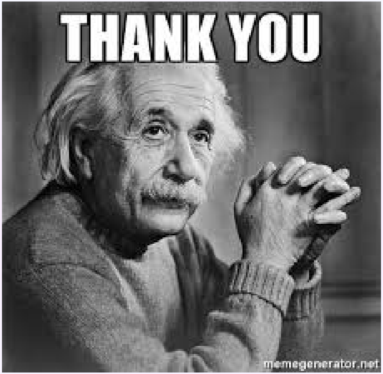
Conclusions

1. We have given two new parameter sets G3, and IOPB-I model.
2. The neutron-skin thickness for G3 and IOPB-I sets calculated for nuclei over a wide range of masses are in harmony with the available experimental data.
3. The neutron matter EoS at sub-saturation densities for G3/IOPB-I parameter set show reasonable improvement over the parameter considered.
4. The nuclear matter incompressibility coefficient and/or symmetry energy coefficient associated with earlier parametrizations of such ERMF model were little too large which has been taken care in our new parameter set G3/IOPB-I.
5. The maximum mass and tidal deformabilities for the neutron star of G3 and IOPB-I sets are compatible with the recent observation GW170817.
6. Our tidal deformability results will be useful for current as well as future observations.

Acknowledgment

Collaborators

1. Prof. S. K. Patra(*IOP, Bhubaneswar*)
2. Prof. B. K. Agrawal(*SINP, Kolkata*)
3. Dr. S. K. Singh(*IIT Roorki*)


THANK YOU
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