# Finite-amplitude method for double-beta decay

Nobuo Hinohara

Center for Computational Sciences, University of Tsukuba, Japan

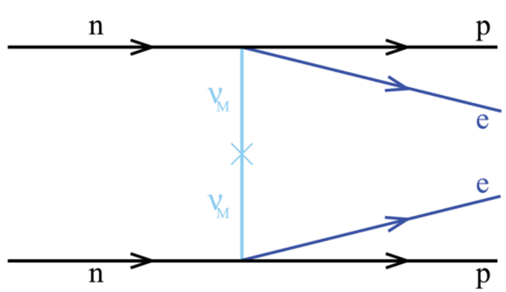
# Double-beta decay and nuclear matrix elements

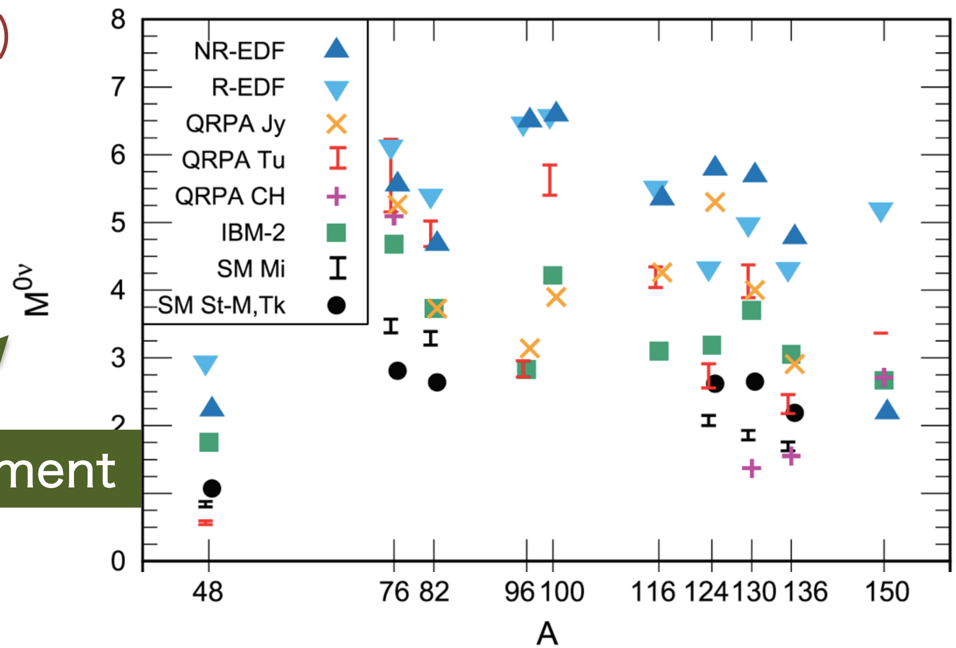
## neutrino mass

- neutrino mass is zero within the standard model
- neutrino oscillation measurements clarifies the finite mass of neutrino

**Neutrino oscillation cannot determine the neutrino mass**
**Double-beta decay may determine it if the neutrino is Majorana particle**

## neutrinoless double-beta decay(0νββ)




Engel and Menéndez, Rep. Prog. Phys. **80**, 046301 (2017)

nuclear matrix element

half life of 0νββ
(from experiments)

$$(T^{0\nu}_{1/2})^{-1} = G_{0\nu}(Q_{\beta\beta},Z)|M_{0\nu}|^2\langle m_{\beta\beta}\rangle^2$$

phase space factor

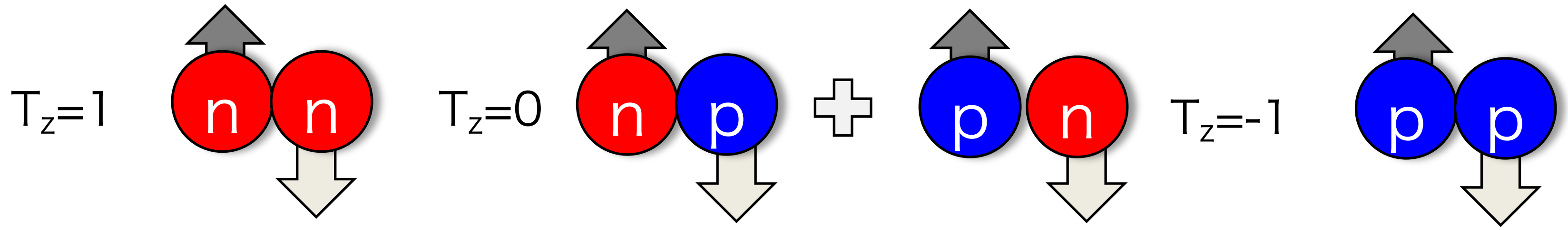
effective mass of electric neutrino

- Precise value of the NME is necessary to determine the neutrino mass
- Theoretical values of NMEs are within a factor of 2-3

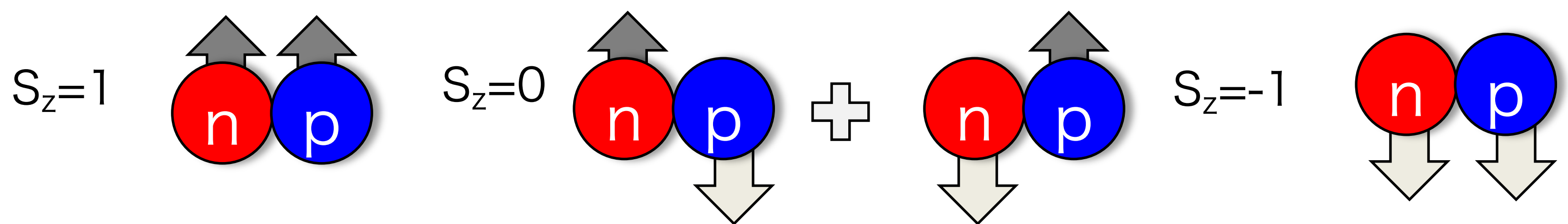
# Nuclear matrix element and $g_{pp}$

## neutron-proton pairing

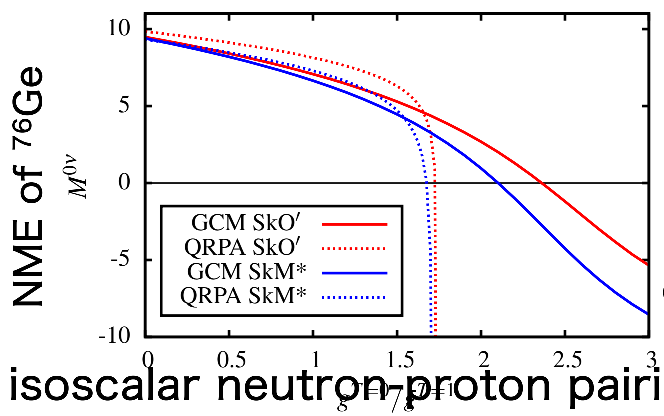
Isovector (T=1, S=0) pairings: isospin symmetric, spin antisymmetric




Isoscalar (T=0, S=1) pairings: isospin antisymmetric, spin symmetric




Suppression of the NME by the neutron-proton pairing

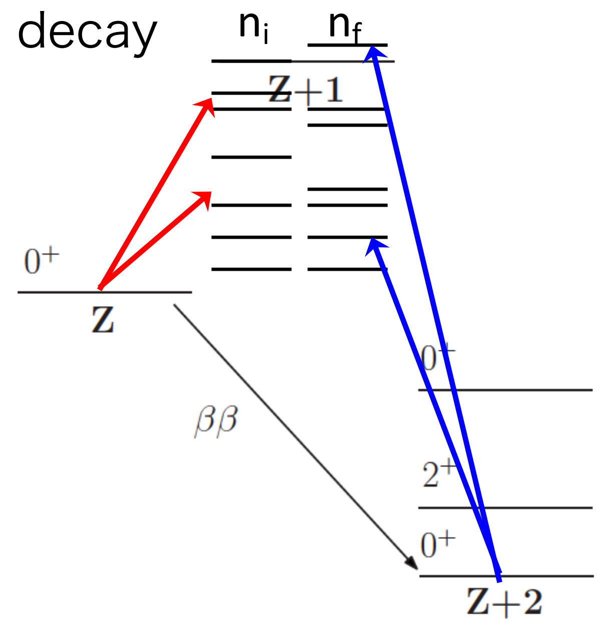



GCM:
NH and Engel, Phys. Rev. C **90**. 031301 (2014)

$g_{pp}$ cannot be determined from the nuclear ground state properties

# QRPA for double-beta decay

- $g_{pp}$ can be determined from β decay and 2νββ decay




2νββ

$$M^{2\nu} = M^{2\nu}_{\mathrm{GT}} - \frac{g_V^2}{g_A^2} M^{2\nu}_{\mathrm{F}},$$

$$M^{2\nu}_{\mathrm{F}} = \sum_n \frac{\langle 0_f^+ | \sum_a \tau_a^- | n \rangle \langle n | \sum_b \tau_b^- | 0_i^+ \rangle}{E_n - \frac{M_i + M_f}{2}},$$

$$M^{2\nu}_{\mathrm{GT}} = \sum_n \frac{\langle 0_f^+ | \sum_a \boldsymbol{\sigma}_a \tau_a^- | n \rangle \cdot \langle n | \sum_b \boldsymbol{\sigma}_b \tau_b^- | 0_i^+ \rangle}{E_n - \frac{M_i + M_f}{2}}$$

two pn QRPA calculations (initial and final)

$$\begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix} \begin{pmatrix} X^n \\ Y^n \end{pmatrix} = \Omega_n \begin{pmatrix} X^n \\ -Y^n \end{pmatrix}$$

dimension~$10^6$

$$|n_i\rangle = \hat{O}^\dagger_{n_i} |0_i^+\rangle \qquad |n_f\rangle = \hat{O}^\dagger_{n_f} |0_f^+\rangle \qquad \hat{O}^\dagger_n = \sum_{\mu\nu} X^n_{\mu\nu} a^{n\dagger}_\mu a^{p\dagger}_\nu - Y^n_{\mu\nu} a^p_\nu a^n_\mu$$

Two sets of intermediate states do not agree because the QRPA is an approximation
→ matching of the intermediate states

$$\langle n_f, K | n_i, K \rangle = \sum_{pn} \sum_{p'n'} [X^{f,K*}_{pn} X^{i,K}_{p'n'} - \alpha Y^{f,K*}_{pn} Y^{i,K}_{p'n'}] \mathcal{O}_{p'p}(\alpha) \mathcal{O}_{n'n}(\alpha)$$

O($\alpha$) is a quantity from the two HFB states

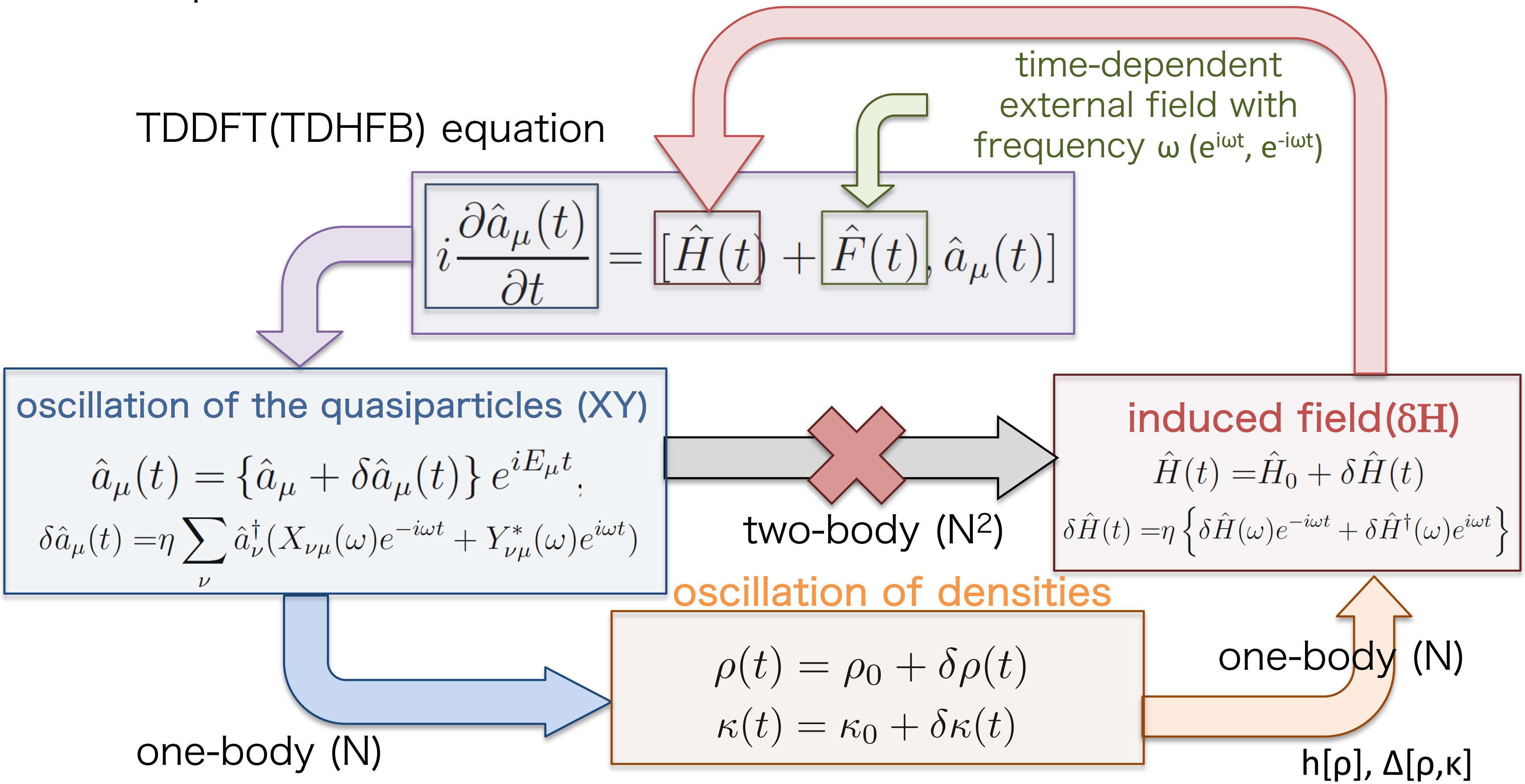
- AB matrix diagonalization: large-scale calculation
- Skyrme-EDF QRPA is available for selected nuclei ($^{76}$Ge, $^{130}$Te, $^{136}$Xe, $^{150}$Nd)

Mustonen and Engel, Phys. Rev. C **87**,064302 (2013)

# Finite-amplitude method (FAM)

linear response formalism of the QRPA

Nakatsukasa et al., Phys. Rev. C **76**, 024318 (2007)

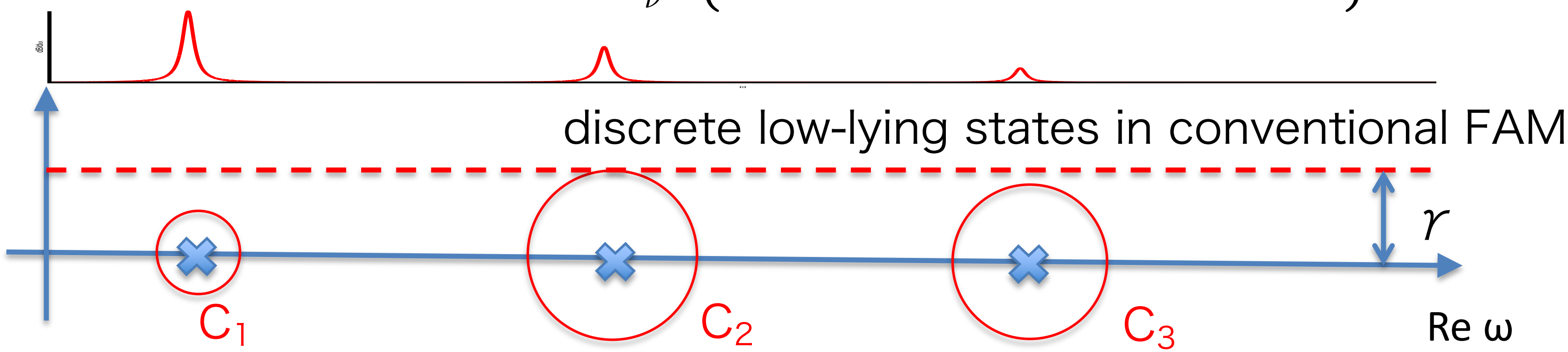



- efficient iterative solution to the QRPA problem
- one-body induced field not through two-body AB matrices
- easy implementation on top of existing mean-field codes
- initial applications are limited to giant resonances with finite width

# FAM for discrete low-lying states

strength function $S(\hat{F},\omega) = \sum_{\mu<\nu} F^{20}_{\mu\nu} X_{\mu\nu}(\omega) + F^{02}_{\mu\nu} Y_{\mu\nu}(\omega)$

$$\frac{dB}{d\omega}(\hat{F},\omega) = -\frac{1}{\pi}\mathrm{Im}S(\hat{F},\omega) = \frac{\gamma}{\pi}\sum_{\nu}\left\{\frac{|\langle\nu|\hat{F}|0\rangle|^2}{(\Omega_i-\omega)^2+\gamma^2} - \frac{|\langle 0|\hat{F}|\nu\rangle|^2}{(\Omega_i+\omega)^2+\gamma^2}\right\}$$




FAM amplitudes and strength functions have first-order poles at QRPA energies

$$X_{\mu\nu}(\omega) = -\sum_i \left\{\frac{X^i_{\mu\nu}\langle i|\hat{F}|0\rangle}{\Omega_i-\omega} + \frac{Y^{i*}_{\mu\nu}\langle 0|\hat{F}|i\rangle}{\Omega_i+\omega}\right\} \qquad Y_{\mu\nu}(\omega) = -\sum_i \left\{\frac{Y^i_{\mu\nu}\langle i|\hat{F}|0\rangle}{\Omega_i-\omega} + \frac{X^{i*}_{\mu\nu}\langle 0|\hat{F}|i\rangle}{\Omega_i+\omega}\right\}$$

Contour integration can extract each QRPA solution ($X^\lambda$, $Y^\lambda$, and $\Omega_\lambda$)

| $\Omega_i$ (MeV) | | strength(isoscalar monopole) | | |
|---|---|---|---|---|
| MQRPA | FAM | MQRPA | FAM-C | FAM-D |
| 1.3185 | 1.3183 | 5.729(−4) | 5.776(−4) | 5.781(−4) |
| 1.3731 | 1.3731 | 1.539(−2) | 1.510(−2) | 1.511(−2) |
| 2.4582 | 2.4581 | 0.1796 | 0.1784 | 0.1783 |
| 2.5998 | 2.5975 | 2.957(−3) | 3.060(−3) | 3.057(−3) |
| 3.6687 | 3.6657 | 0.5776 | 0.5788 | 0.5788 |
| 5.1185 | 5.1212 | 3.539(−4) | 4.360(−4) | 4.345(−4) |
| 7.4108 | 7.4084 | 0.4900 | 0.4848 | 0.4848 |

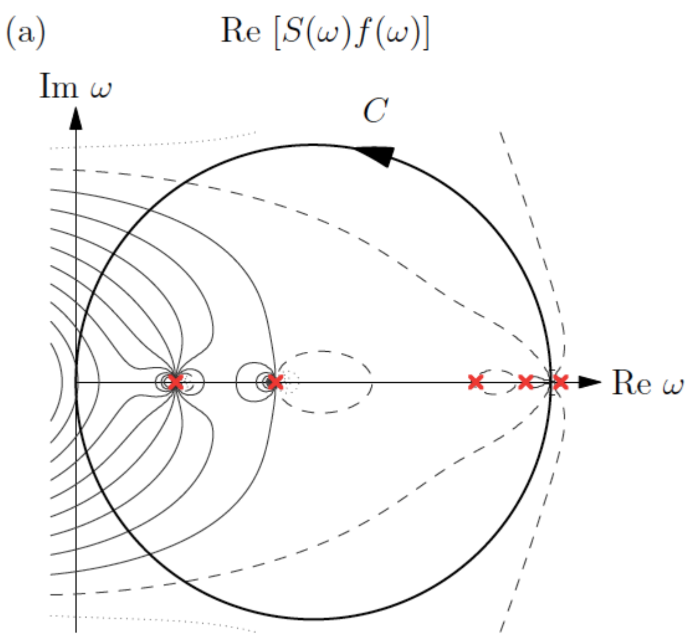
NH, Kortelainen, Nazarewicz, Phys. Rev. C **87**, 064309 (2013)

# FAM for pnQRPA

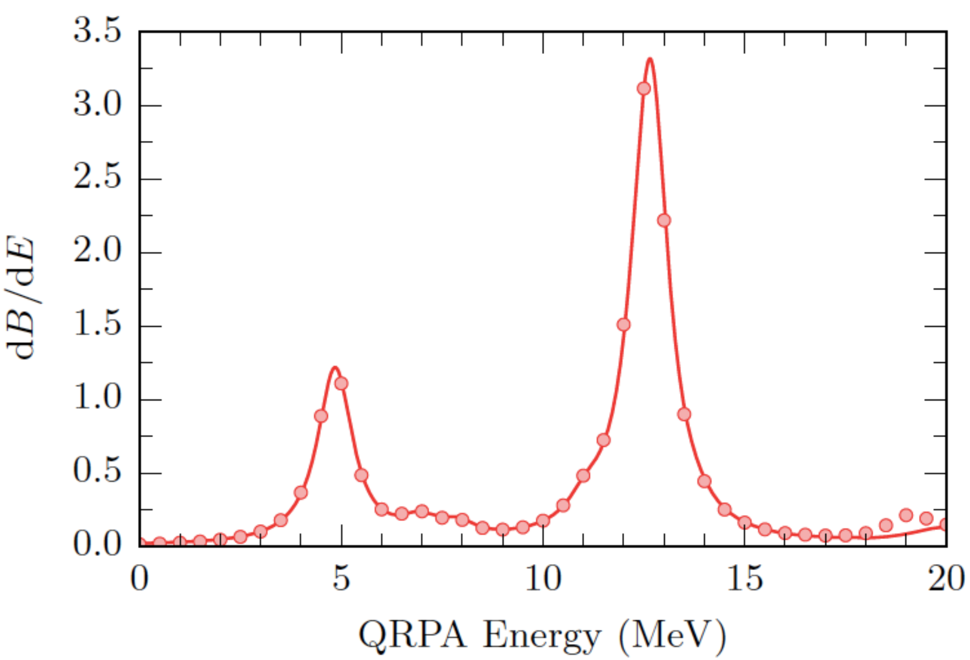
Mustonen Shafer, Zenginerler, Engel, Phys. Rev. C **90**, 024308 (2014)

FAM code development for pnQRPA (Skyrme-EDF HFBTHO)

β decay strength




Gamow-Teller resonance($^{22}$Ne)




Global calculations, new Skyrme parametrizations fitted to β decay and GT
Mustonen and Engel, Phys. Rev. C **93**, 014304 (2016)

# 2νββ decay NME from the FAM

external fields $\hat{F}_{\mathrm{F}}^{K=0} = \sum_a \tau_a^-$ $\hat{F}_{\mathrm{GT}}^{K} = \sum_a \sigma_a^K \tau_a^-$

FAM amplitude

$$X_{pn}^{i}(\omega_i, \hat{F}_{\mathrm{F/GT}}^{K}) = -\sum_{n_i>0}\left\{\frac{X_{pn}^{n_i,K}\langle n_i, K|\hat{F}_{\mathrm{F/GT}}^{K}|0_i^+\rangle}{\Omega_{n_i,K}-\omega_i} + \frac{Y_{pn}^{n_i,K*}\langle 0_i^+|\hat{F}_{\mathrm{F/GT}}^{K\dagger}|n_i, K\rangle}{\Omega_{n_i,K}+\omega_i}\right\}$$

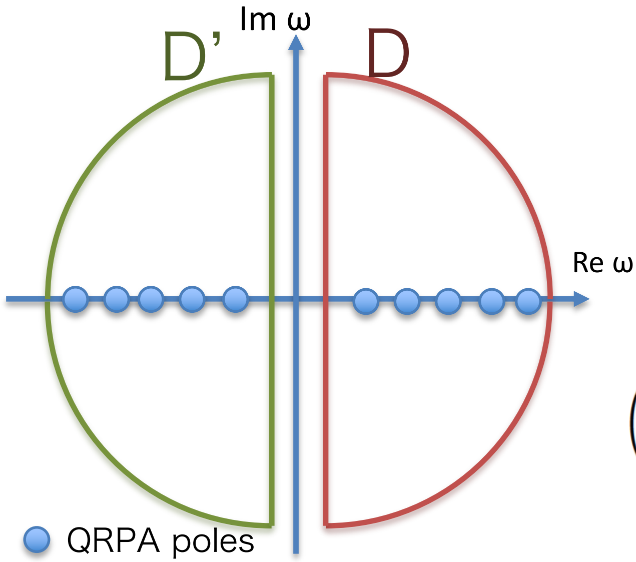
$$Y_{pn}^{i}(\omega_i, \hat{F}_{\mathrm{F/GT}}^{K}) = -\sum_{n_i>0}\left\{\frac{Y_{pn}^{n_i,K}\langle n_i, K|\hat{F}_{\mathrm{F/GT}}^{K}|0_i^+\rangle}{\Omega_{n_i,K}-\omega_i} + \frac{X_{pn}^{n_i,K*}\langle 0_i^+|\hat{F}_{\mathrm{F/GT}}^{K\dagger}|n_i, K\rangle}{\Omega_{n_i,K}+\omega_i}\right\}$$

$$\mathcal{T}_{pn,p'n'}(\alpha, \omega_i, \omega_f, \hat{F}_{\mathrm{F/GT}}^{K}) = Y_{pn}^{f}(\omega_f, \hat{F}_{\mathrm{F/GT}}^{K\dagger})X_{p'n'}^{i}(\omega_i, \hat{F}_{\mathrm{F/GT}}^{K}) - \alpha X_{pn}^{f}(\omega_f, \hat{F}_{\mathrm{F/GT}}^{K\dagger})Y_{p'n'}^{i}(\omega_i, \hat{F}_{\mathrm{F/GT}}^{K})$$

NME related part

$$\begin{aligned}\mathcal{T}_{pn,p'n'}(\alpha, \omega_i, \omega_f, \hat{F}_{\mathrm{F/GT}}^{K}) = \sum_{K}\sum_{n_i>0,n_f>0} & \frac{(X_{pn}^{n_fK*}X_{p'n'}^{n_iK} - \alpha Y_{pn}^{n_fK*}Y_{pn}^{n_iK})\langle 0_f^+|\hat{F}_{\mathrm{F/GT}}^{K}|n_f, K\rangle\langle n_i, K|\hat{F}_{\mathrm{F/GT}}^{K}|0_i^+\rangle}{(\Omega_{n_f,K}+\omega_f)(\Omega_{n_i,K}-\omega_i)} \\ & + \frac{(Y_{pn}^{n_fK*}X_{p'n'}^{n_iK} - \alpha X_{pn}^{n_fK*}Y_{p'n'}^{n_iK})\langle n_f, K|\hat{F}_{\mathrm{F/GT}}^{K}|0_f^+\rangle\langle n_i, K|\hat{F}_{\mathrm{F/GT}}^{K}|0_i^+\rangle}{(\Omega_{n_f,K}-\omega_f)(\Omega_{n_i,K}-\omega_i)} \\ & + \frac{(X_{pn}^{n_fK*}Y_{p'n'}^{n_iK} - \alpha Y_{pn}^{n_fK*}X_{p'n'}^{n_iK})\langle 0_f^+|\hat{F}_{\mathrm{F/GT}}^{K}|n_f, K\rangle\langle 0_i^+|\hat{F}_{\mathrm{F/GT}}^{K}|n_i, K\rangle}{(\Omega_{n_f,K}+\omega_f)(\Omega_{n_i,K}+\omega_i)} \\ & + \frac{(Y_{pn}^{n_fK*}Y_{p'n'}^{n_iK} - \alpha X_{pn}^{n_fK*}X_{p'n'}^{n_iK})\langle n_f, K|\hat{F}_{\mathrm{F/GT}}^{K}|0_f^+\rangle\langle 0_i^+|\hat{F}_{\mathrm{F/GT}}^{K}|n_i, K\rangle}{(\Omega_{n_f,K}-\omega_f)(\Omega_{n_i,K}+\omega_i)}.\end{aligned}$$

# 2νββ decay NME from the FAM




first order poles at

$$\omega_i = \pm\Omega_{n_i,K} \text{ and } \omega_f = \pm\Omega_{n_f,K}$$

$$\mathcal{M}(\alpha, \hat{F}^K_{\mathrm{F/GT}}; f_{p'n',pn}(\omega_i,\omega_f)) = \left(\frac{1}{2\pi i}\right)^2 \int_D d\omega_i \int_{D'} d\omega_f \sum_{pn}\sum_{p'n'} \mathcal{T}_{pn,p'n'}(\alpha,\omega_i,\omega_f,\hat{F}^K_{\mathrm{F/GT}}) f_{p'n',pn}(\omega_i,\omega_f)$$

contour D encircles all the positive energy poles
contour D' encircles all the negative energy poles

double GT transition

$$M_{\mathrm{DGT}} = \sum_{K=-1}^{1} \mathcal{M}(\alpha, \hat{F}^K_{\mathrm{GT}}; f_{p'n',pn}(\omega_i,\omega_f) = \mathcal{O}_{p'p}(\alpha)\mathcal{O}_{n'n}(\alpha))$$

2νββ NME

$$M^{2\nu}_{\mathrm{F}} = \mathcal{M}(\alpha, \hat{F}^{K=0}_{\mathrm{F}}; f_{p'n',pn}(\omega_i,\omega_f) = \frac{2\mathcal{O}_{p'p}(\alpha)\mathcal{O}_{n'n}(\alpha)}{\omega_i - \omega_f})$$

$$M^{2\nu}_{\mathrm{GT}} = \sum_{K=-1}^{1} \mathcal{M}(\alpha, \hat{F}^K_{\mathrm{GT}}; f_{p'n',pn}(\omega_i,\omega_f) = \frac{2\mathcal{O}_{p'p}(\alpha)\mathcal{O}_{n'n}(\alpha)}{\omega_i - \omega_f})$$

complex-energy FAM allows us to compute DGT and 2νββ NME very efficiently

# Summary

- Finite-amplitude method for QRPA calculations of $2\nu\beta\beta$ nuclear matrix element has been formulated.
  - significant reduction of the computational time is expected with parallelization for $2\nu\beta\beta$ NME and DGT transition
  - this technique is useful for determining the neutron-proton pairing using available $2\nu\beta\beta$ half lives.
- Implementation based on pnFAM code (HFBTHO) is in progress
- Collaborator: Jon Engel (University of North Carolina, Chapel Hill)