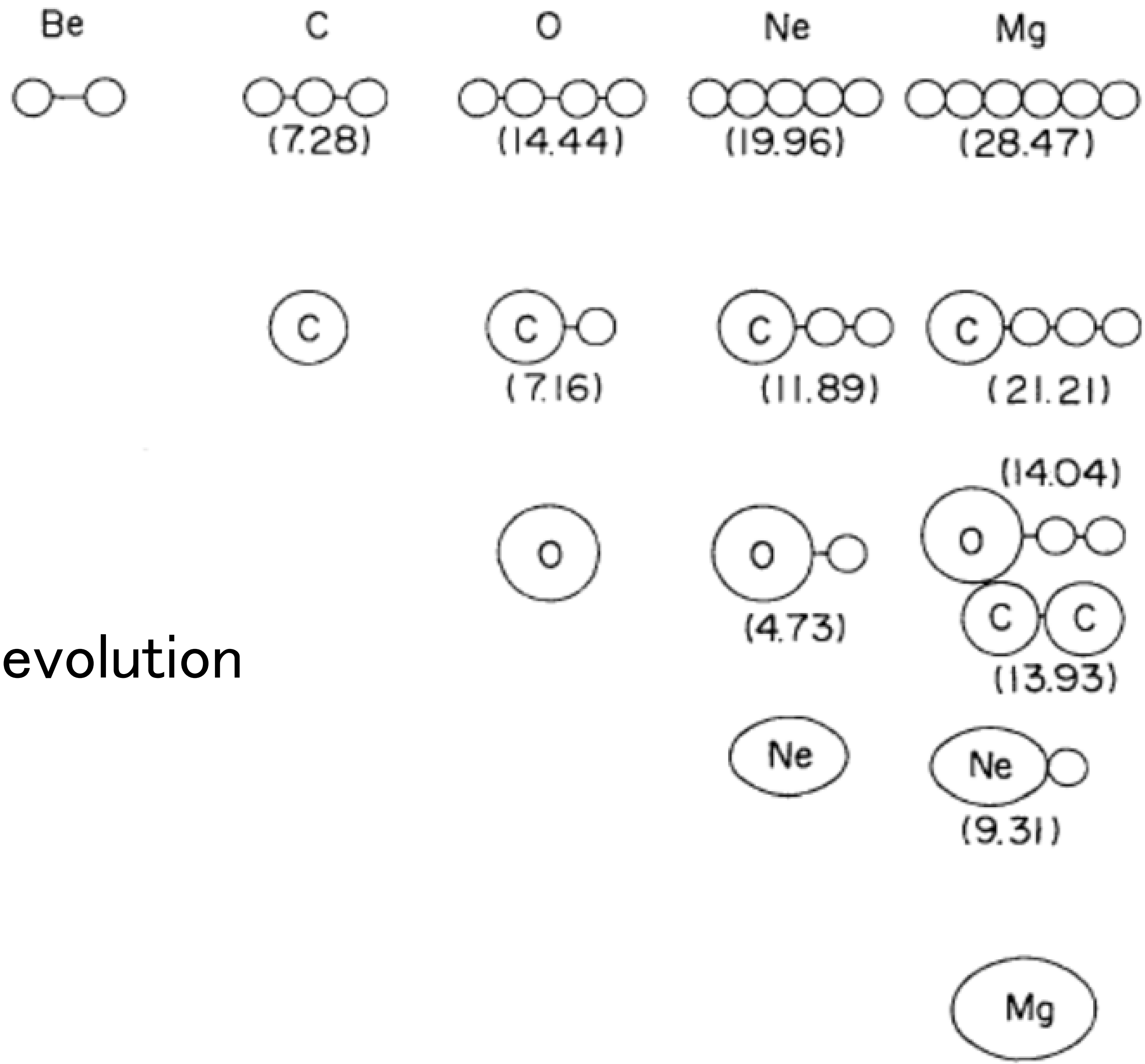
Alpha condensates and dynamics of cluster formation

Yasuro Funaki

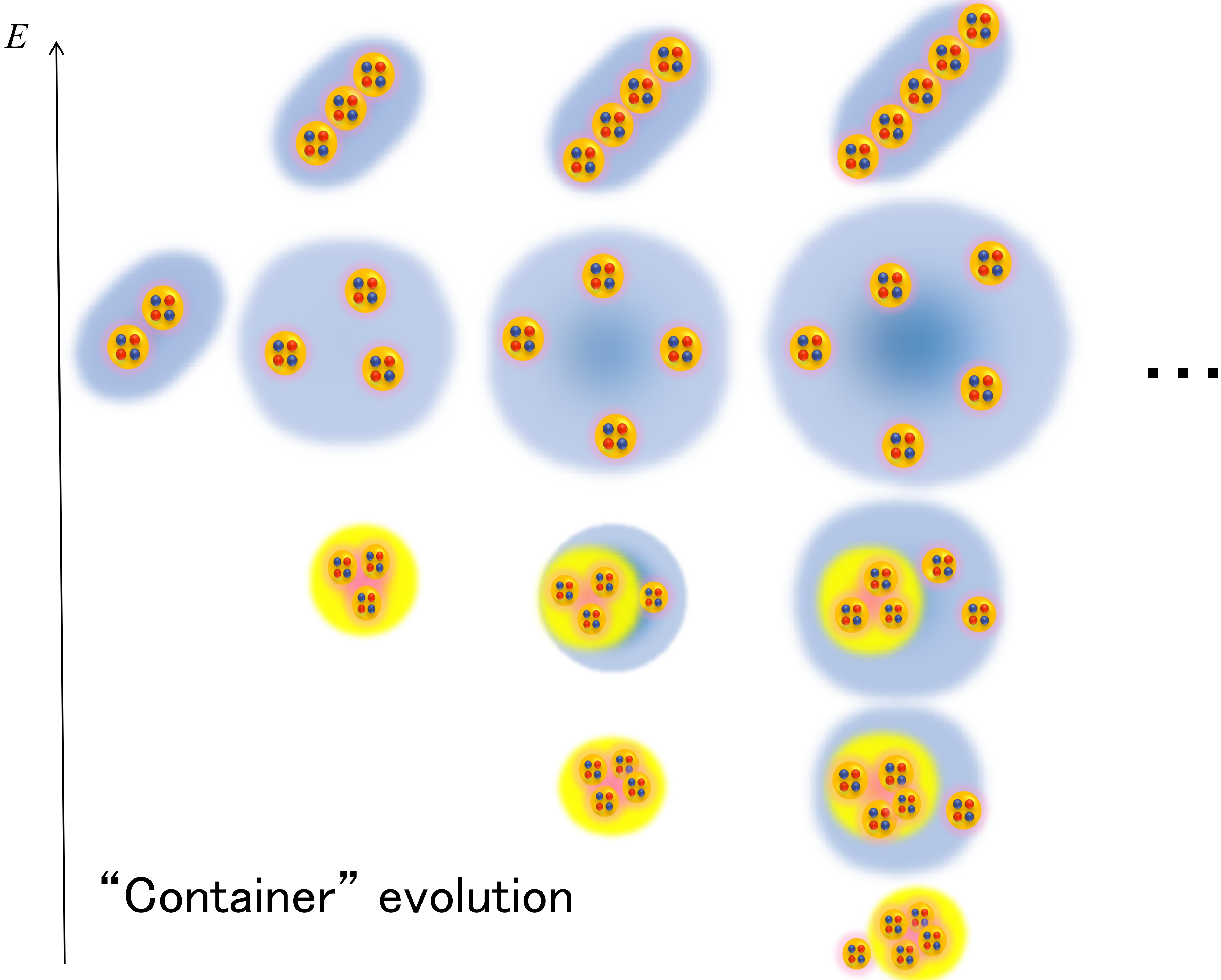
(Kanto Gakuin U.)

Tsukuba-CCS workshop on ``microscopic theories of nuclear structure and dynamics''@U. of Tsukuba, Japan, December 10-12, 2018.

50th anniversary of Ikeda diagram (1968)



Cluster evolution


E
“Container” evolution

Nuclear clusters and alpha condensation physics with THSR ansatz

Container picture with THSR ansatz

Container evolution for ^{16}O

Nuclear clusters and alpha condensation physics with THSR ansatz

Container picture with THSR ansatz

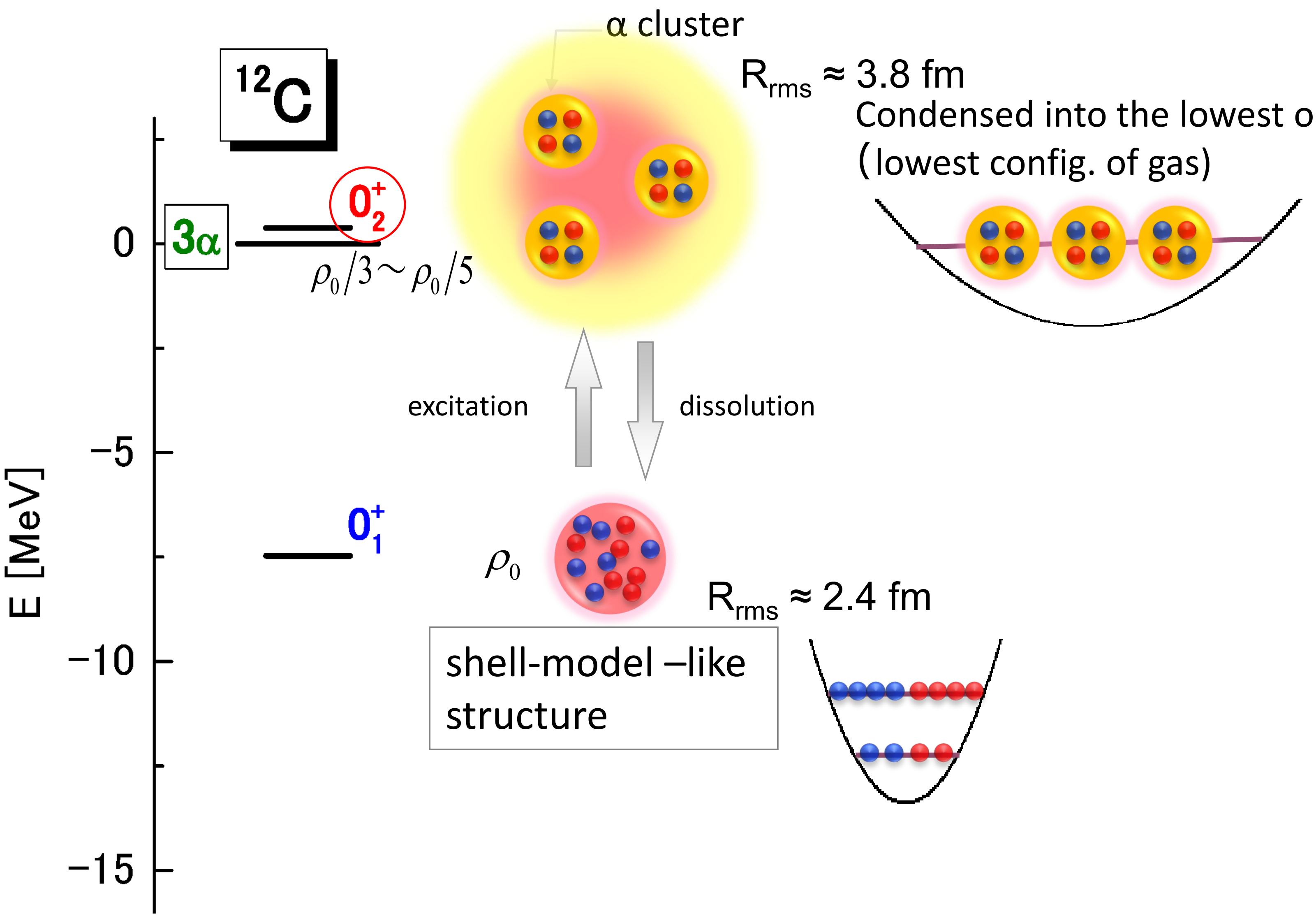
Container evolution for ^{16}O

Alpha condensate state

A. Tohsaki,et al., PRL 87, 192501(2001),

The Hoyle state (0_2^+ state of ^{12}C)

Cluster gas



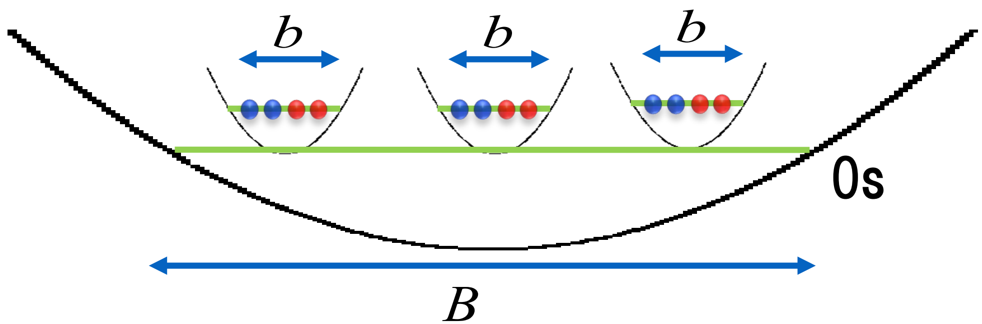
So-called ``THSR'' wave function (or condensate w.f.)

A. Tohsaki,et al., PRL 87, 192501(2001), Y. F. et al., PTP 108, 297 (2002).

$$\Phi_\alpha(\boldsymbol{\beta}, b) = \exp\left(-2\sum_k^{x,y,z} \frac{r_k^2}{b^2 + 2\beta_k^2}\right)\phi_\alpha(b)$$

$$\Phi_{^{12}\mathrm{C}}^{\mathrm{eTHSR}}(\boldsymbol{\beta}, b) = \Psi_G^{-1}\mathcal{A}\{\Phi_\alpha(\boldsymbol{\beta}, b)\Phi_\alpha(\boldsymbol{\beta}, b)\Phi_\alpha(\boldsymbol{\beta}, b)\}$$

$$= \mathcal{A}\Big\{$$



$$\Big\}$$

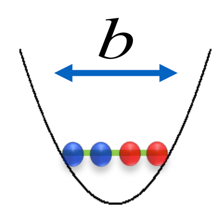
$$\boxed{B_k^2 = b^2 + 2\beta_k^2 \ (k = x, y, z)}$$

Ψ_G: Total center-of-mass w.f. to be eliminated

Internal w.f. of α particle

b=1.35 fm: fixed

$\phi_\alpha(b) :=$



Two limits

B =b: Shell model w.f.

B >>b: Gas of independent α-particles

Why the Hoyle state is the 3α condensate ?

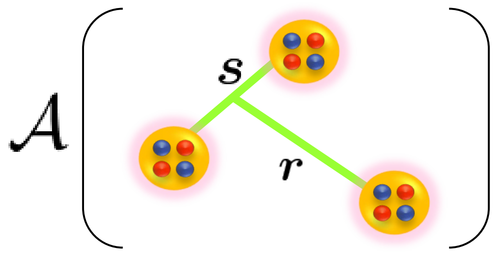
The 3αRGM/GCM eq. of motion gives the solution, which is very 3α cond. w.f.

The 3α RGM/GCM: to give solutions of full 3α problem in a microscopic way

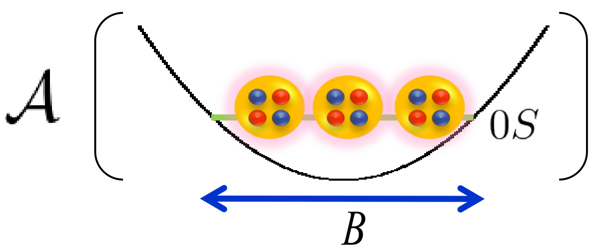
RGM w.f.

$$\langle \phi^3(\alpha) | H - E | \mathcal{A}[\chi(\boldsymbol{s}, \boldsymbol{r}) \phi^3(\alpha)] \rangle = 0$$

M. Kamimura, NPA 351, 456 (1981).



3α cond. w.f. (THSR w.f.)

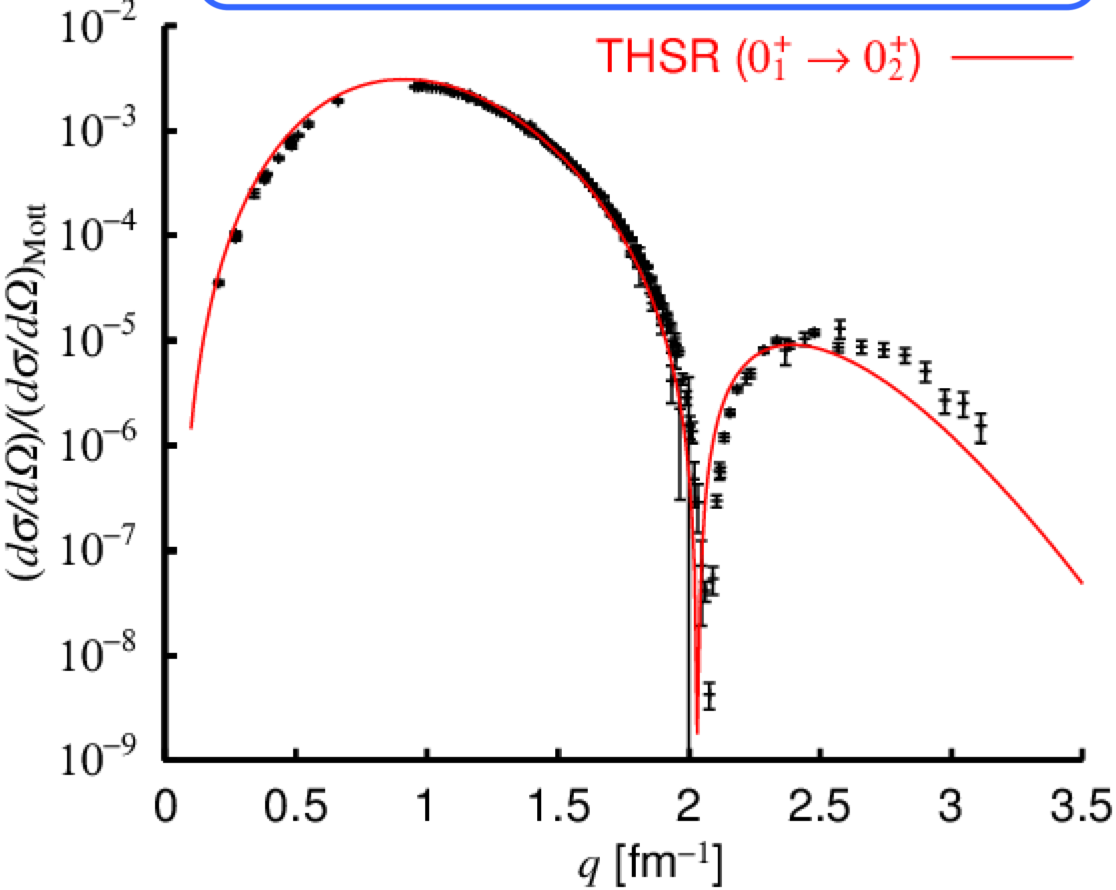


Y. F. et al., PRC 67, 051306(R) (2003).

Comparison with exp. data for the Hoyle state (0_2^+ of ^{12}C) (THSR w.f.)

Inelastic electron scattering ($0_1^+ \to 0_2^+$)

Data from T. Neff and
M. Chernykh et al., PRL ***98****, 032501 (2007).*
Y. F. et al., EPJA ***28****, 259 (2006).*



Very nicely reproduced by THSR w.f.

Energy (MeV)

$$E_{\mathrm{cal}} - E_{3\alpha}^{th} = 0.38$$
$$E_{\mathrm{exp}} - E_{3\alpha}^{th} = 0.23$$

α-decay width (MeV)

$$\Gamma_{\mathrm{cal}} = 7.7\times10^{-6}$$
$$\Gamma_{\mathrm{exp}} = 8.5(10)\times10^{-6}$$

Monopole M.E. (fm^2)

$$M(E0;\ 0_2^+ \to 0_1^+) = 6.4 \quad (\text{Exp: } 5.4(2))$$

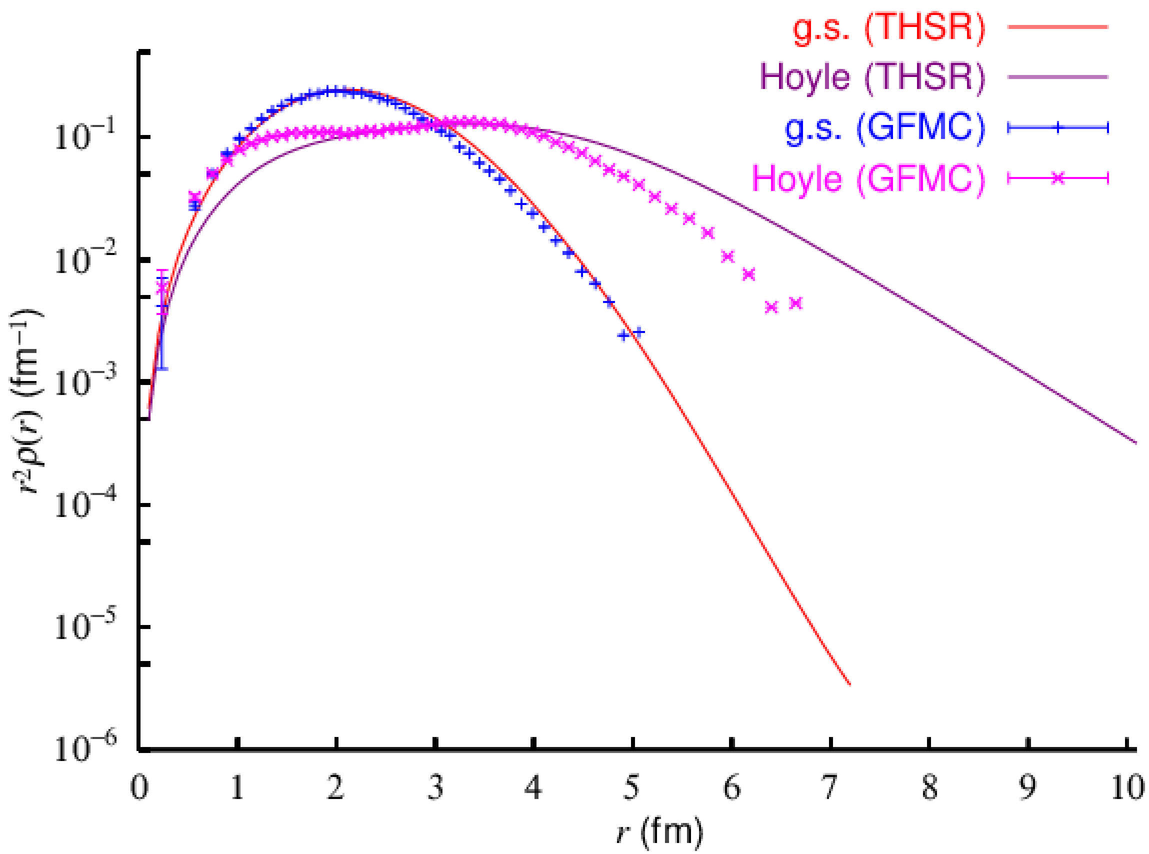
B(E2) (e^{2}fm^4)

$$B(E2;\ 2_1^+ \to 0_2^+) = 2.4 \quad (\text{Exp: } 0.73(13))$$

Y. F. et al., PRC **67**, 051306(R)(2003).
Y. F. et al., EPJA **24**, 321(2005).

GFMC (data from Wringa) and Comparison with THSR

One-body density distribution



THSR, for the Hoyle
R_{rms}=3.8 fm

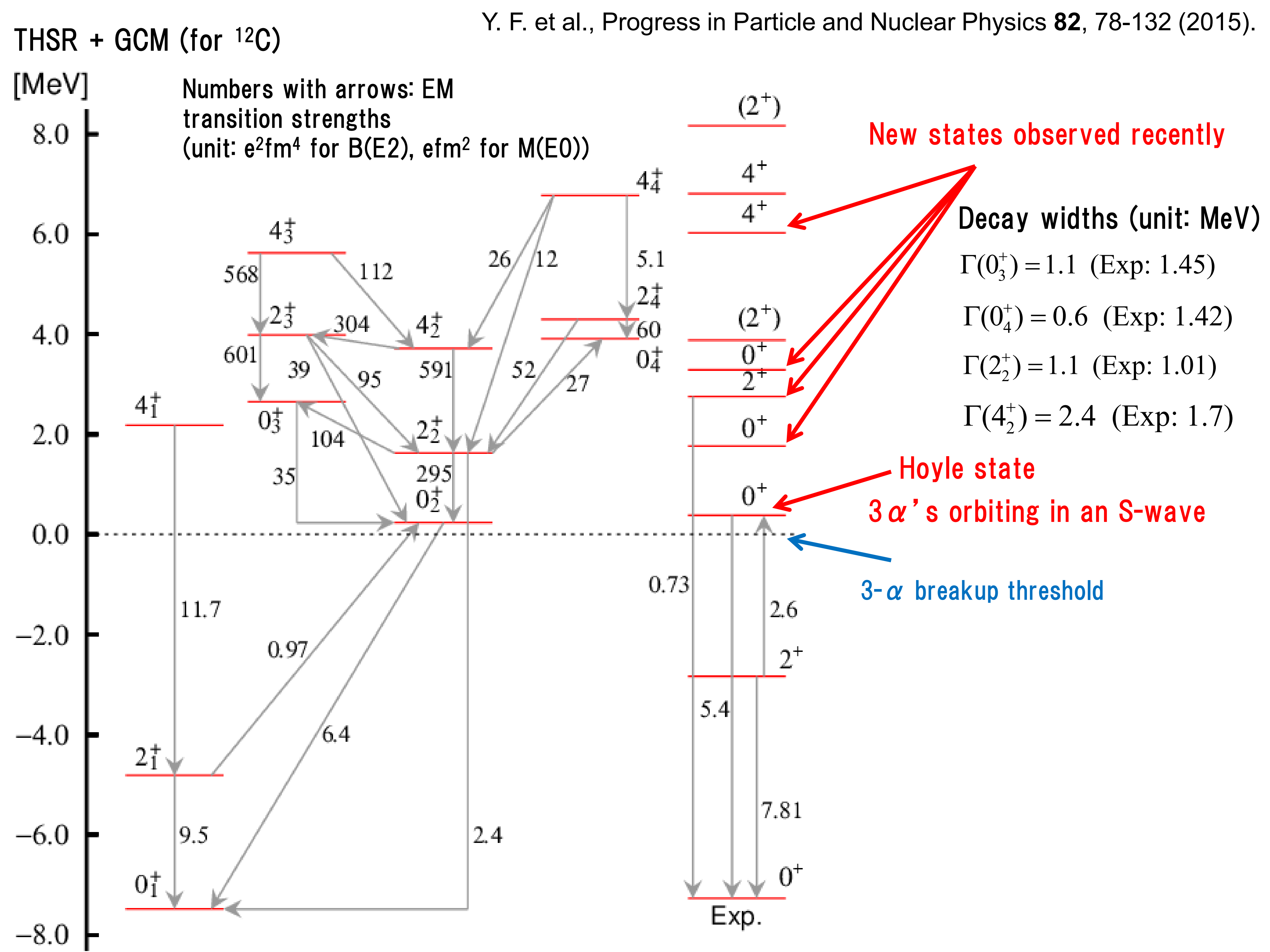
GFMC

For Hoyle state

R_{rms}=3.0 fm ~ 3.5 fm

For Hoyle state

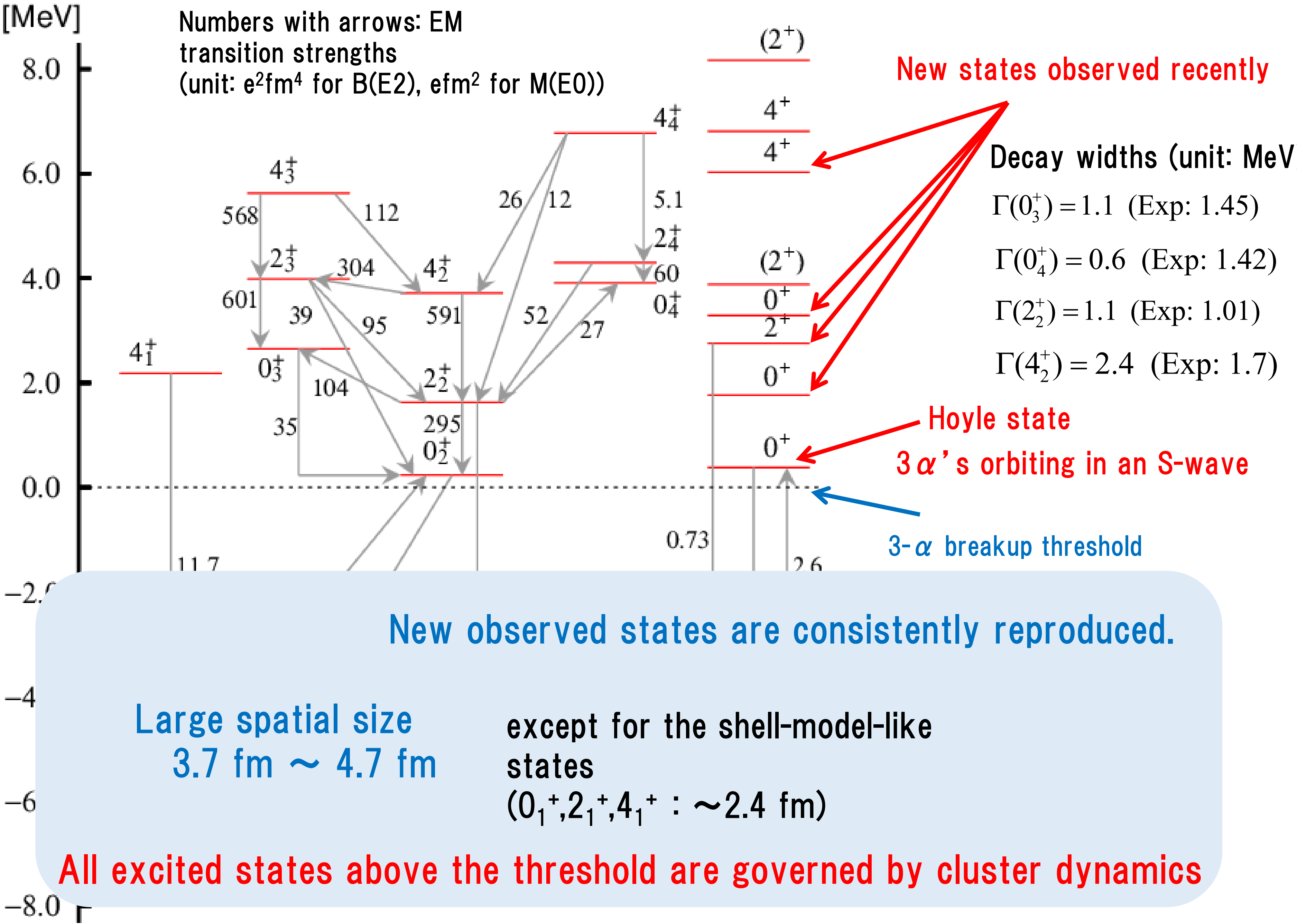
AV18+IL7: 10.4 MeV
Exp: 7.65 MeV



THSR + GCM (for ^{12}C)
Y. F. et al., Progress in Particle and Nuclear Physics **82**, 78-132 (2015).
[MeV]
Numbers with arrows: EM transition strengths
(unit: e^2fm^4 for B(E2), efm^2 for M(E0))
New states observed recently
Decay widths (unit: MeV)
$\Gamma(0_3^+) = 1.1$ (Exp: 1.45)
$\Gamma(0_4^+) = 0.6$ (Exp: 1.42)
$\Gamma(2_2^+) = 1.1$ (Exp: 1.01)
$\Gamma(4_2^+) = 2.4$ (Exp: 1.7)
Hoyle state
3α's orbiting in an S-wave
3-α breakup threshold
Exp.

THSR + GCM (for ^{12}C)

Y. F. et al., Progress in Particle and Nuclear Physics **82**, 78-132 (2015).



New observed states are consistently reproduced.

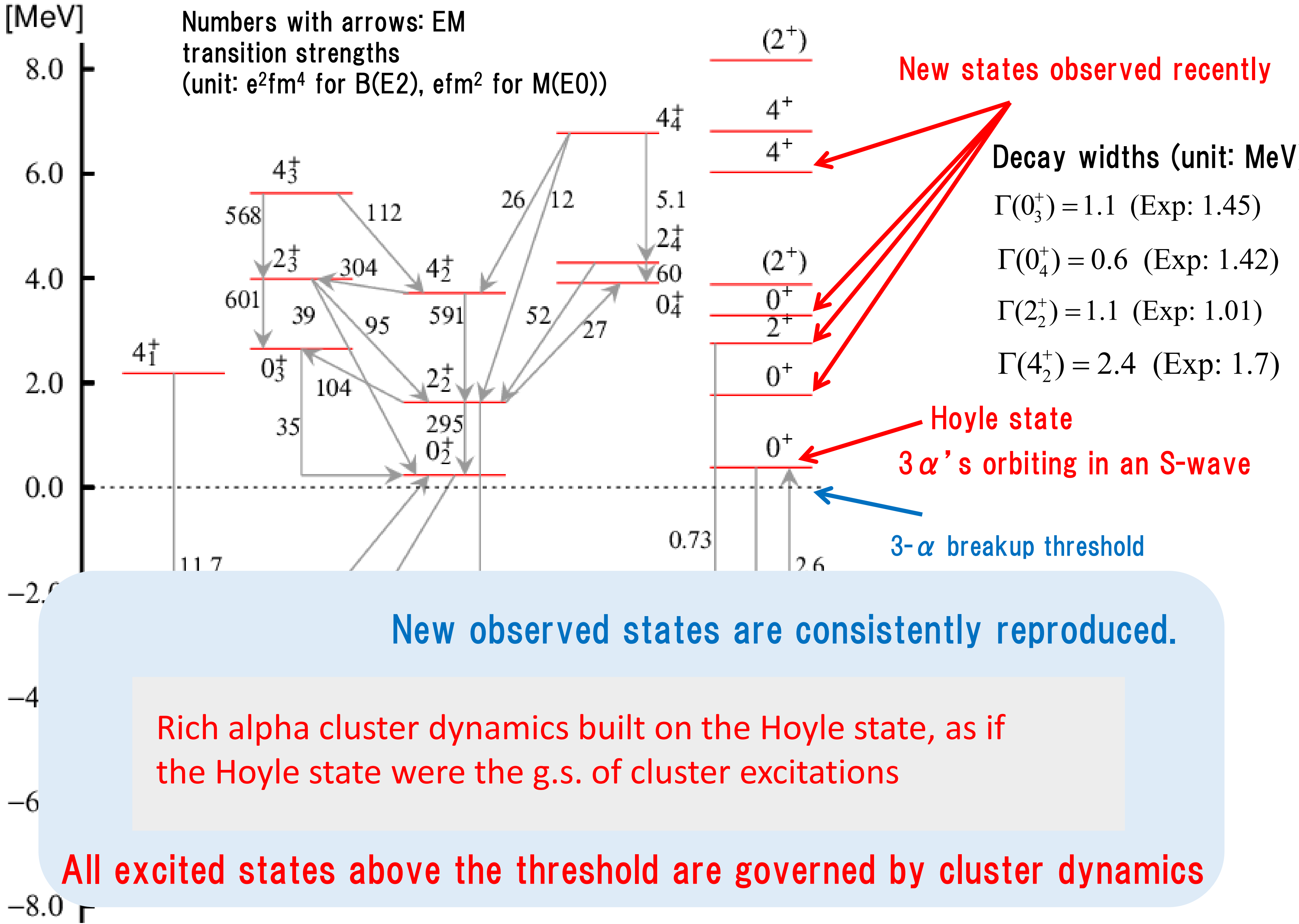
Large spatial size
3.7 fm ～ 4.7 fm

except for the shell-model-like states
(0_1^+, 2_1^+, 4_1^+ : ～2.4 fm)

All excited states above the threshold are governed by cluster dynamics

Y. F. et al., Progress in Particle and Nuclear Physics **82**, 78-132 (2015).

THSR + GCM (for ^{12}C)



New observed states are consistently reproduced.

Rich alpha cluster dynamics built on the Hoyle state, as if the Hoyle state were the g.s. of cluster excitations

All excited states above the threshold are governed by cluster dynamics

Nuclear clusters and alpha condensation physics with THSR ansatz

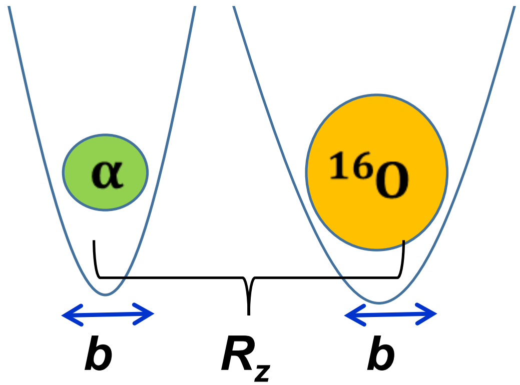
Container picture with THSR ansatz

Container evolution for ^{16}O

First success of container picture for ordinary cluster state

Characterized by rel. distance parameter R_z .

Localized clustering.

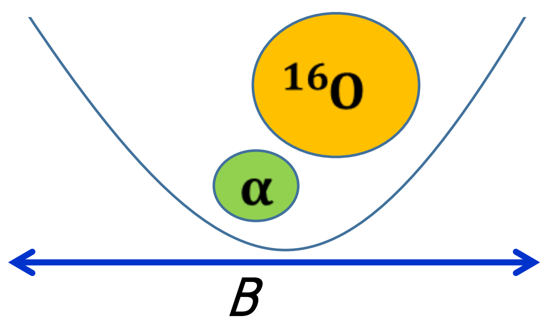


$$\Psi^{\mathrm{Brink}}_{^{20}\mathrm{Ne}}(R_z, b) = \mathcal{A}\left\{\exp\left(\frac{8(r_z - R_z\ )^2}{5b^2}\right)\phi_\alpha(b)\phi_{^{16}\mathrm{O}}(b)\right\}$$

VS

Characterized by the size of container *B.*

Non-localized clustering.



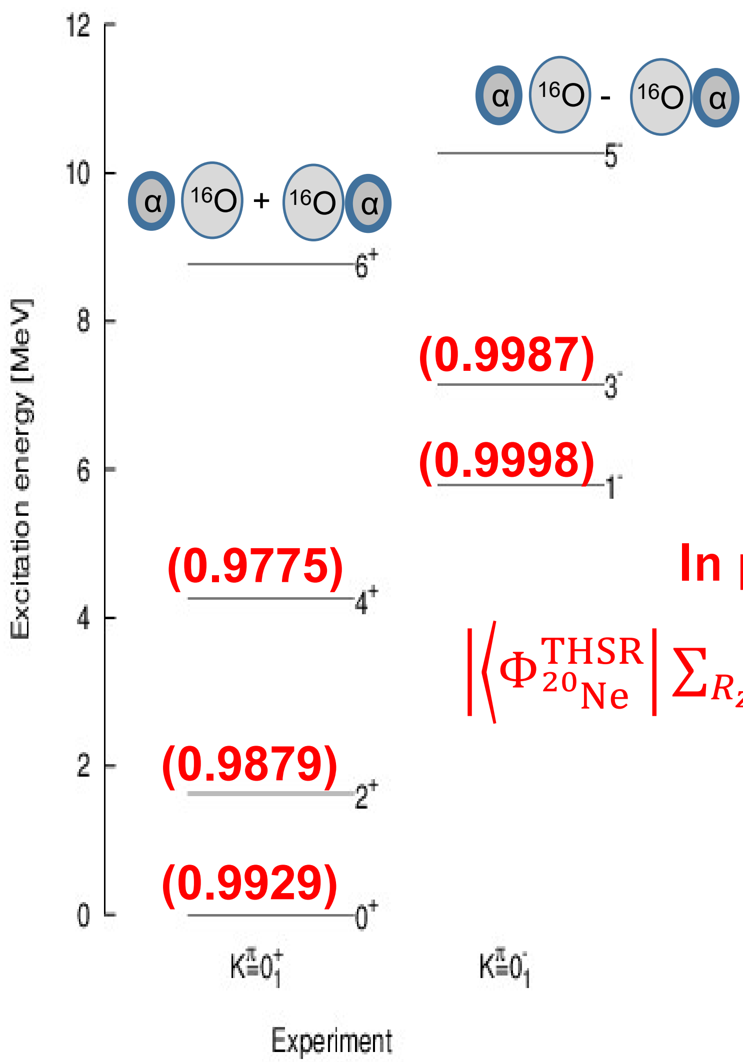
$$\Phi^{\mathrm{THSR}}_{^{20}\mathrm{Ne}}(\beta, b) = \mathcal{A}\left\{\exp\left(\sum_{k}^{x,y,z}\frac{8r_k^2}{5(b^2 + 2\beta_k^2)}\right)\phi_\alpha(b)\phi_{^{16}\mathrm{O}}(b)\right\}$$

$$B_k^2 = b^2 + 2\beta_k^2 \ \ (k = x, y, z)$$

B. Zhou, Y. F. et al., PRC86, 014301 (2012); PRL 110, 262501(2013); PRC 89, 3319 (2014).

First success of container picture for ordinary cluster state

The energy levels of α+^{16}O inversion doublet bands in ^{20}Ne

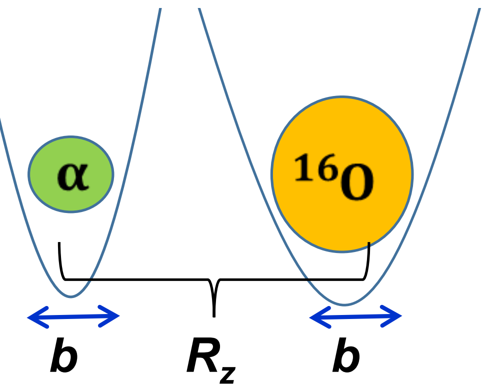


In parentheses

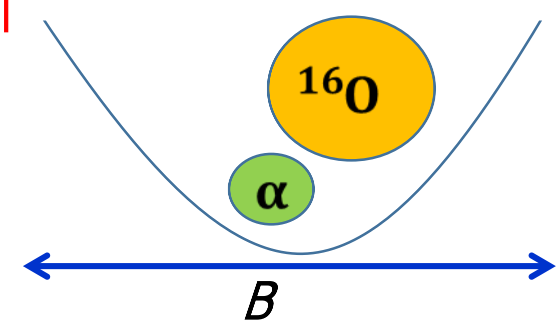
$$\left|\left\langle \Phi^{\mathrm{THSR}}_{^{20}\mathrm{Ne}} \middle| \sum_{R_z} f(R_z)\Psi^{\mathrm{Brink}}_{^{20}\mathrm{Ne}}(R_z,b)\right\rangle\right|^2$$

Brink GCM

Characterized by rel. distance parameter R_z. Localized clustering.



...ed by the size ...er *B*. ...n-localized clustering.



B. Zhou, Y. F. et al., PRC86, 014301 (2012); PRL 110, 262501(2013); PRC 89, 3319 (2014).

Container picture succeeds in describing other non-gaslike cluster states.

T. Suhara, Y. F. et al., PRL112, 062501 (2014)

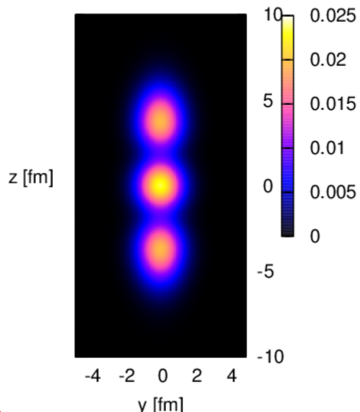
α Linear chain states

Overlap between cond. w.f. and full sol.

J=0: 0.9873 $(\beta_x=\beta_y,\beta_z)=(0.1,5.1)$

J=2: 0.9887 $(\beta_x=\beta_y,\beta_z)=(0.1,5.4)$

J=4: 0.9806 $(\beta_x=\beta_y,\beta_z)=(0.1,6.6)$



Strongly prolate

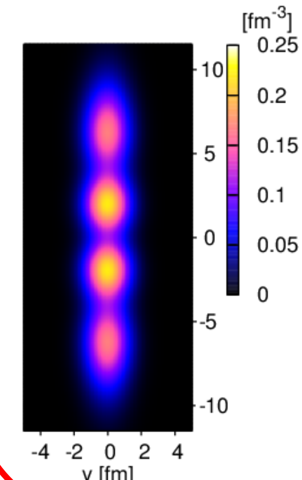
1 config. of THSR w.f. = full sol. of one-dim. Brink-GCM (100 configs. superposed)

Overlap between cond. w.f. and full sol.

J=0: 0.9440 $(\beta_x=\beta_y,\beta_z)=(0.1, 8.2)$

J=2: 0.9417 $(\beta_x=\beta_y,\beta_z)=(0.1,8.4)$

J=4: 0.9307 $(\beta_x=\beta_y,\beta_z)=(0.1,9.0)$



Strongly prolate

1 config. of THSR w.f. = full sol. of one-dim. Brink-GCM (300 configs. superposed)

${}^{9}_{\Lambda}$Be

Overlap between cont. w.f. and full sol.

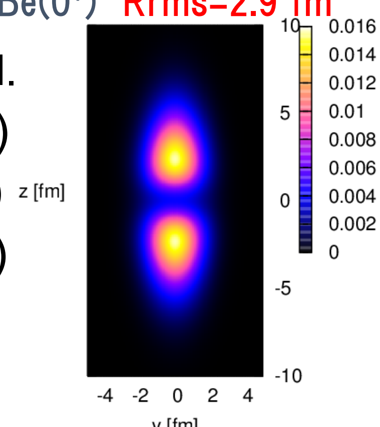
J=0: 0.995 $(\beta_x=\beta_y,\beta_z)=(1.6,3.0)$

J=2: 0.994 $(\beta_x=\beta_y,\beta_z)=(0.1,3.0)$

J=4: 0.977 $(\beta_x=\beta_y,\beta_z)=(0.1,2.1)$

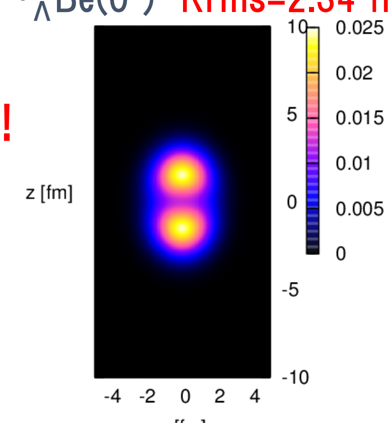
Small size

^{8}Be(0⁺) Rrms=2.9 fm



Adding Λ !

${}^{9}_{\Lambda}$Be(0⁺) Rrms=2.34 fm



Y. F. et al., PTEP (2014) 113D01.

Nuclear clusters and alpha condensation physics with THSR ansatz

Container picture with THSR ansatz

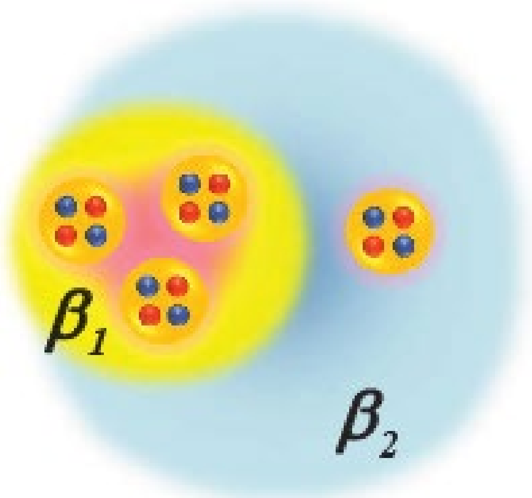
Container evolution for ^{16}O

4α extended THSR wave function

$$\Phi_\alpha(\boldsymbol{\beta}, b) = \exp\left(-2\sum_{k}^{x,y,z} \frac{r_k^2}{b^2 + 2\beta_k^2}\right)\phi_\alpha(b)$$

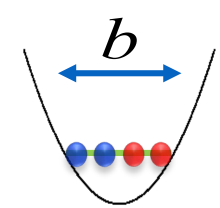
$$\Phi_{^{16}\mathrm{O}}^{\mathrm{eTHSR}}(\boldsymbol{\beta}_1, \boldsymbol{\beta}_2, b) = \Psi_G^{-1}\boldsymbol{\mathcal{A}}\{\Phi_\alpha(\boldsymbol{\beta}_1, b)\Phi_\alpha(\boldsymbol{\beta}_1, b)\Phi_\alpha(\boldsymbol{\beta}_1, b)\Phi_\alpha(\boldsymbol{\beta}_2, b)\}$$

Ψ_G: Total center-of-mass w.f. to be eliminated



Internal w.f. of α particle

b=1.44 fm: fixed

$\phi_\alpha(b) :=$ 

4α extended THSR wave function

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Ψ_G: Total center-of-mass w.f. to be eliminated

Hill-Wheeler eq. or GCM (generator coordinate method)

$$\sum_{\boldsymbol{\beta'}_1, \boldsymbol{\beta'}_2} \left\langle \hat{P}_{MK}^J \Phi_{^{16}\mathrm{O}}^{\mathrm{eTHSR}}(\boldsymbol{\beta}_1, \boldsymbol{\beta}_2, b) \middle| \hat{H} - E \middle| \hat{P}_{MK}^J \Phi_{^{16}\mathrm{O}}^{\mathrm{eTHSR}}(\boldsymbol{\beta'}_1, \boldsymbol{\beta'}_2, b) \right\rangle f(\boldsymbol{\beta'}_1, \boldsymbol{\beta'}_2) = 0$$

$\hat{P}_{MK}^J$: Angular momentum projection operator

Hamiltonian (NN force: F1 force)

A. Tohsaki, PRC **49**, 1814 (1994).

$$\hat{H} = -\frac{\hbar^2}{2m}\sum_{i}^{16} \nabla_i^2 - T_G + \sum_{i<j}^{16}(V_{ij}^{(N)} + V_{ij}^{(C)}) + \sum_{i<j<k}^{16} V_{ijk}^{(N)}$$

$\boldsymbol{\beta}_i = (\beta_{ix} = \beta_{iy}, \beta_{iz})$

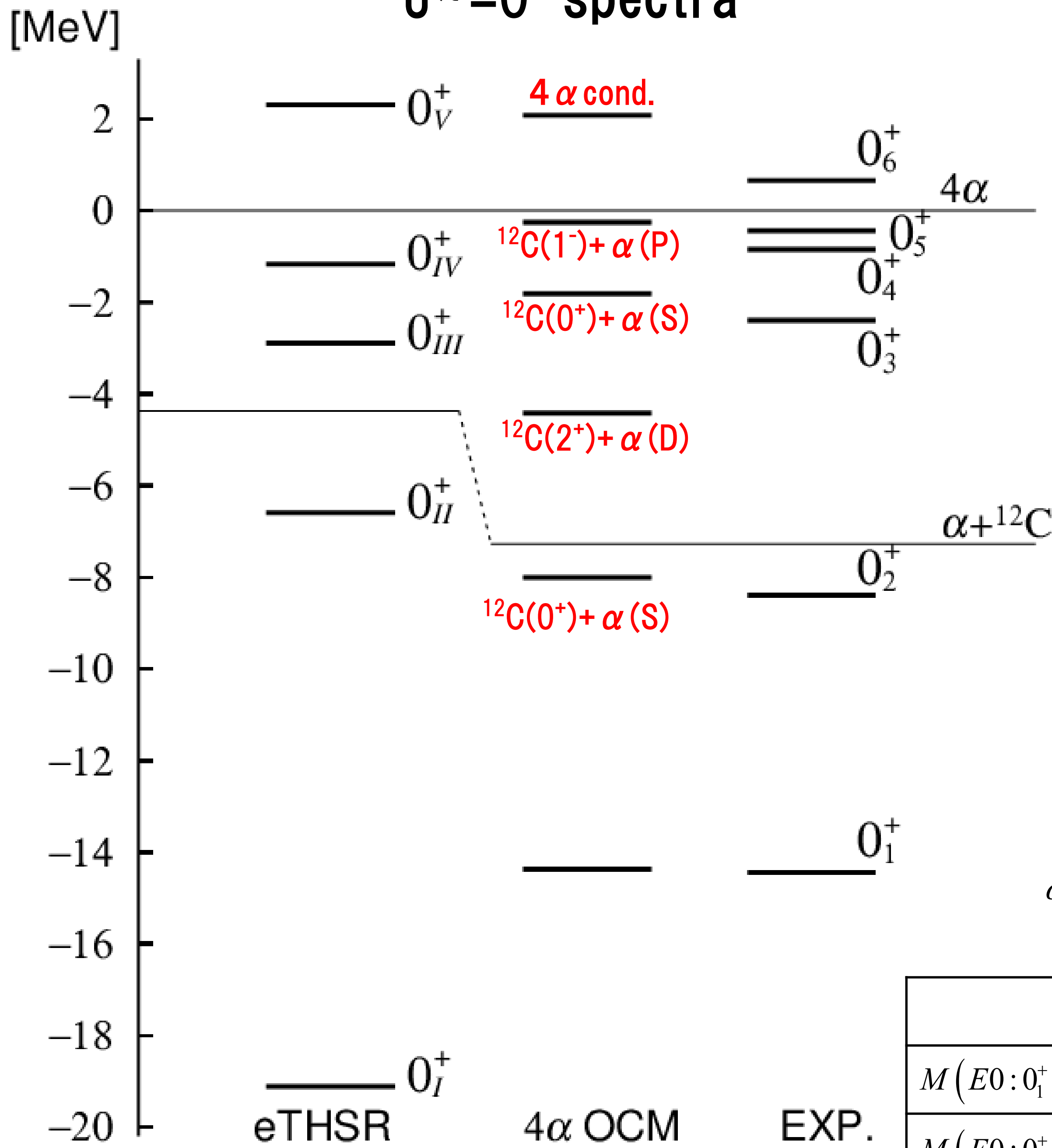
With (axially symmetric) deformation

Spurious continuum components are effectively eliminated by r^2 constraint method.

See Y. F. et al., PTP **115,** 115 (2006).

$J^{\pi}=0^{+}$ spectra

Y. F, PRC 97, 021304(R)(2018)



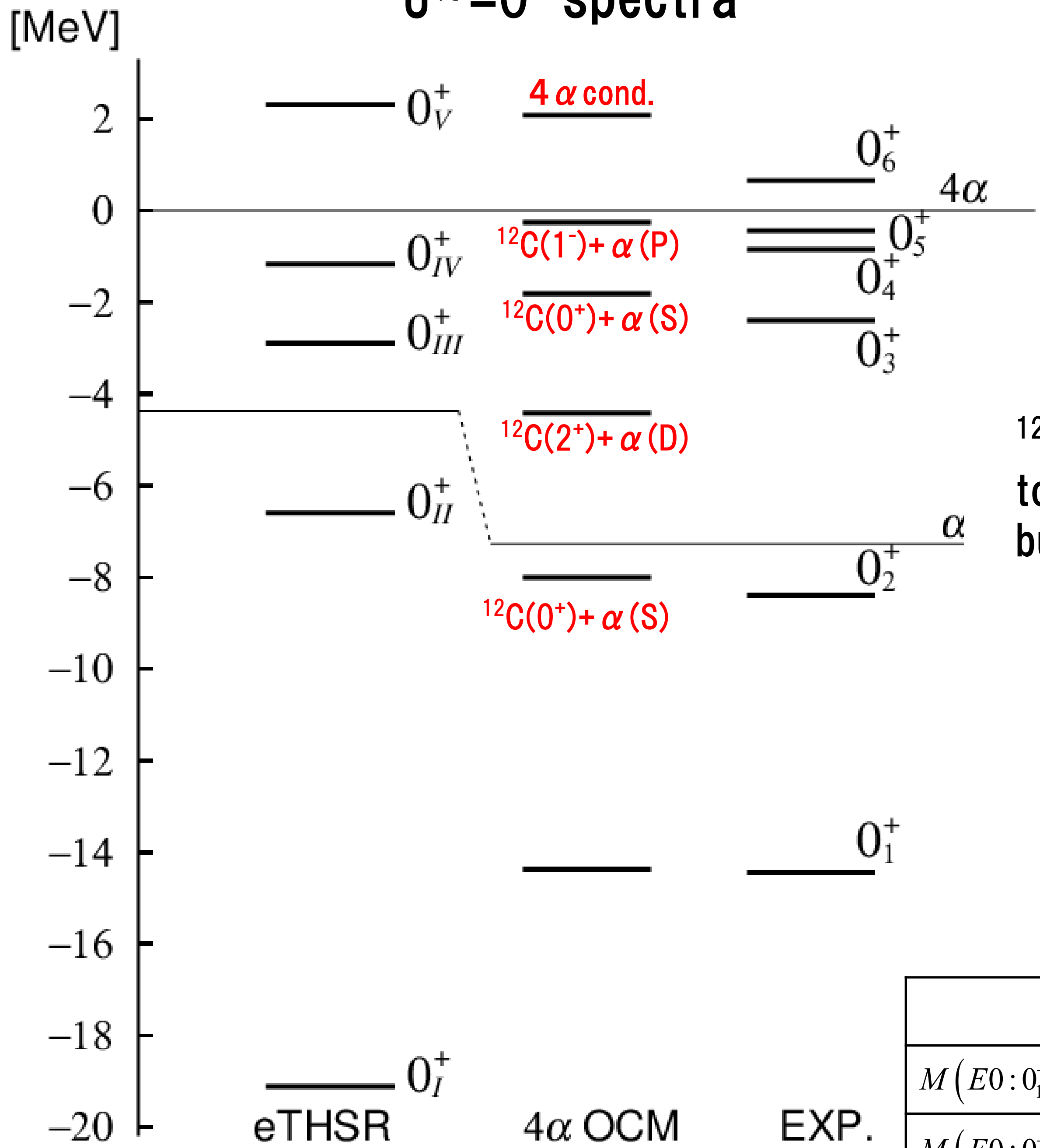
$\alpha+{}^{12}$C OCM

Y. Suzuki, PTP **55**, 1751 (1976); **56**, 111 (1976).

	eTHSR	$\alpha+{}^{12}$C OCM	EXP.
$M\left(E0:0_1^+\to 0_2^+\right)$	5.9	3.88	3.66 ± 0.55
$M\left(E0:0_1^+\to 0_3^+\right)$	5.7	3.50	4.40 ± 0.44

$J^\pi=0^+$ spectra

Y. F, PRC 97, 021304(R)(2018)



^{12}C(1$^-$)+α(P) structure is difficult to describe by the present eTHSR but **extension is possible**.

$\alpha+^{12}$C OCM

Y. Suzuki, PTP **55**, 1751 (1976); **56**, 111 (1976).

	eTHSR	$\alpha+^{12}$C OCM	EXP.
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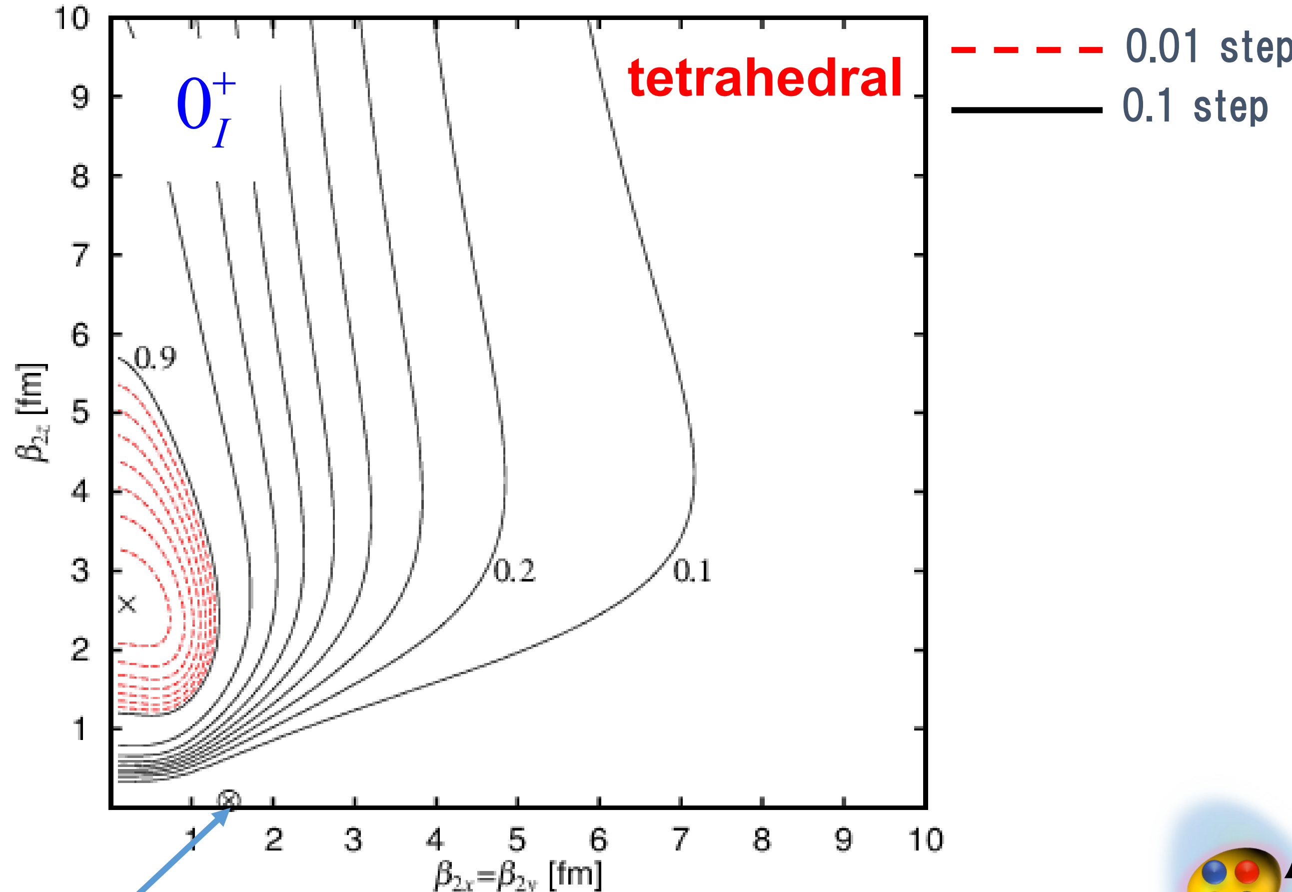
Squared overlap surface with single config. of eTHSR

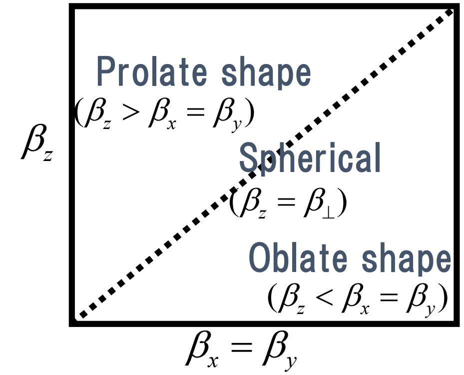


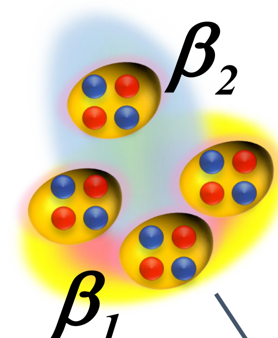
$(\beta_{1x} = \beta_{1y}, \beta_{1z})$: fixed at $\otimes$

Container for 3 α

× : maximum

For the fourth α

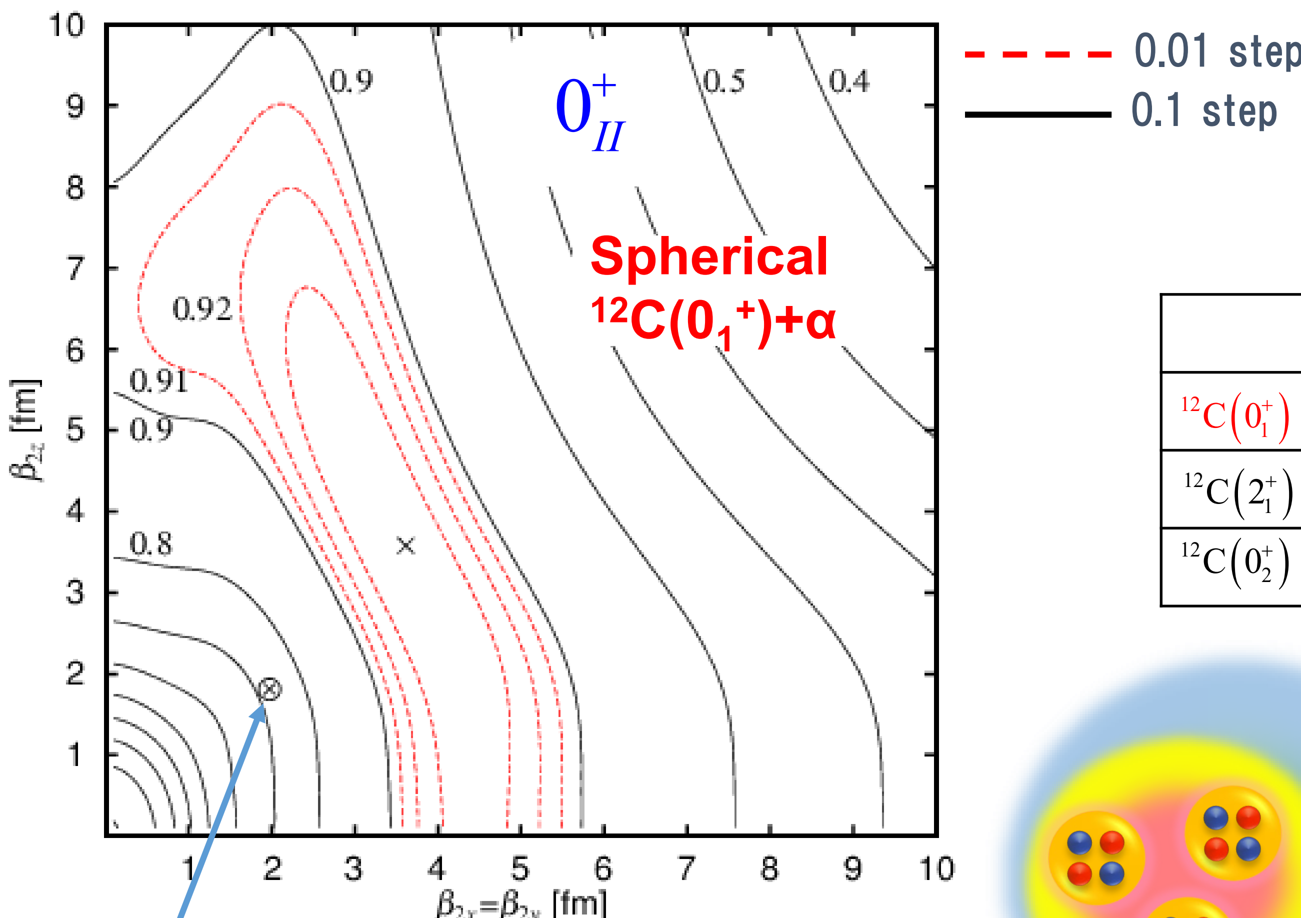


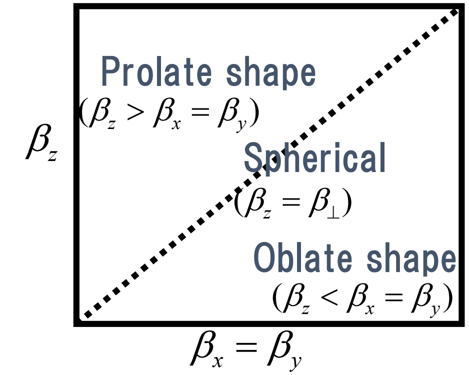


THSR+GCM

$|\langle \Phi(\boldsymbol{\beta}_1, \boldsymbol{\beta}_2) | 0_I^+ \rangle|^2$

Squared overlap surface with single config. of eTHSR





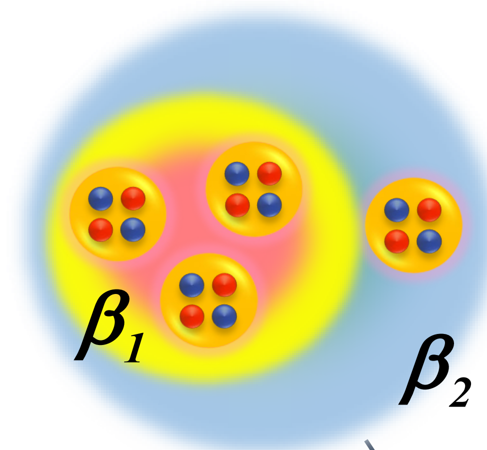
	Sq. overlap	$(\beta_{1x} = \beta_{1y}, \beta_{1z})$
$^{12}C(0_1^+)$	0.93	(1.9,1.8fm)
$^{12}C(2_1^+)$	0.90	(1.9,0.5fm)
$^{12}C(0_2^+)$	0.99	(5.6,1.4fm)

$(\beta_{1x} = \beta_{1y}, \beta_{1z})$: fixed at ⊗

Container for 3α

× : maximum

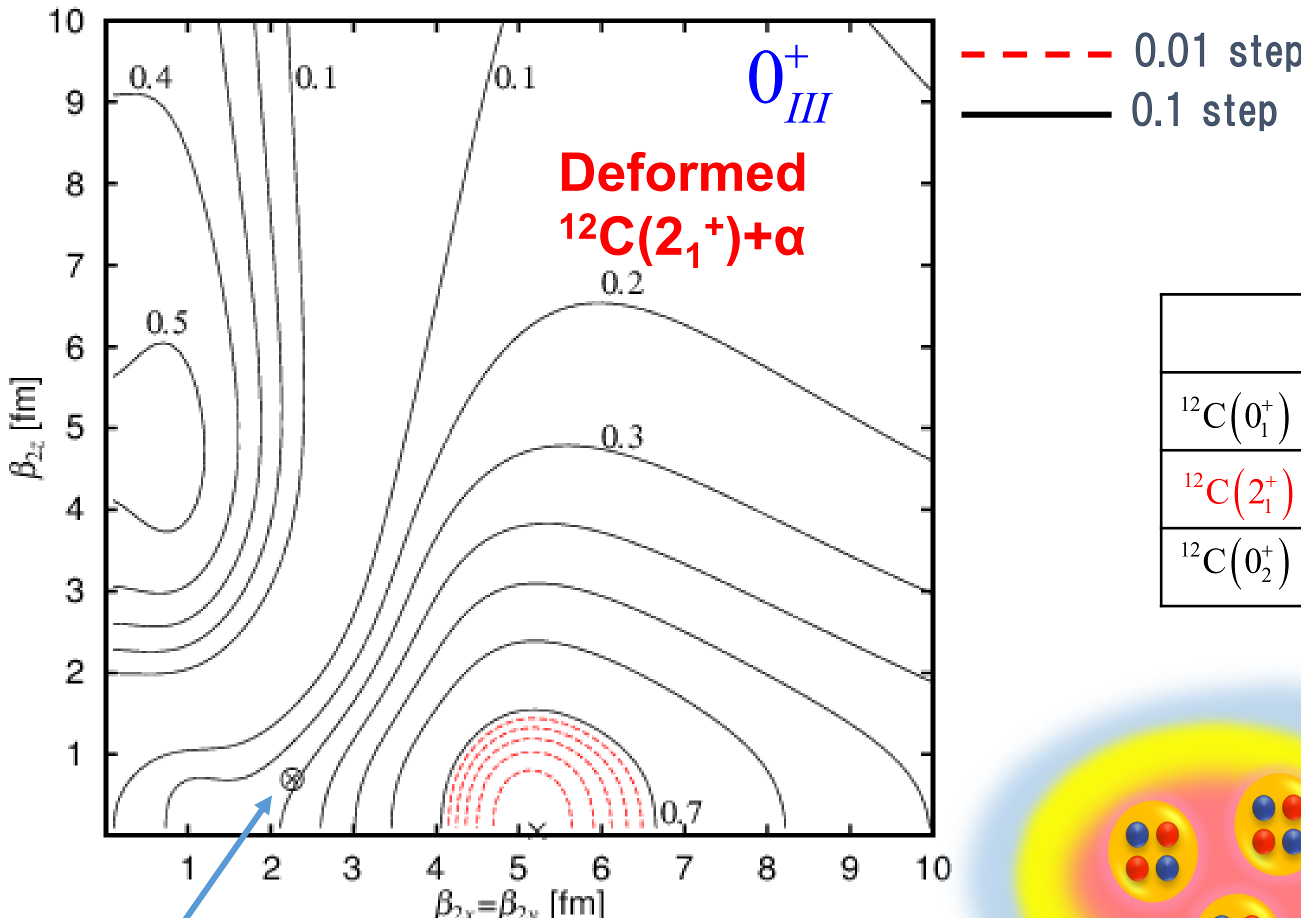
For the fourth α

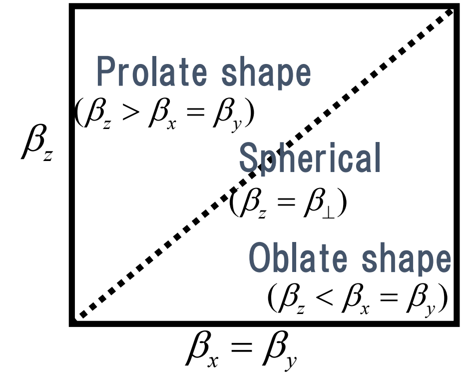


THSR+GCM

$$|\langle \Phi(\boldsymbol{\beta}_1, \boldsymbol{\beta}_2) | 0^+_{II} \rangle|^2$$

Squared overlap surface with single config. of eTHSR





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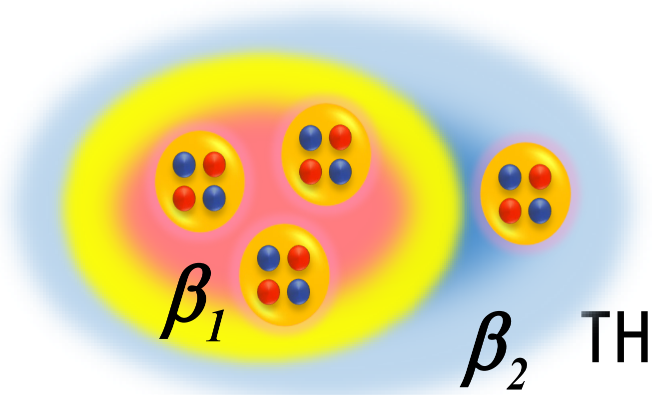
Container for 3α

× : maximum

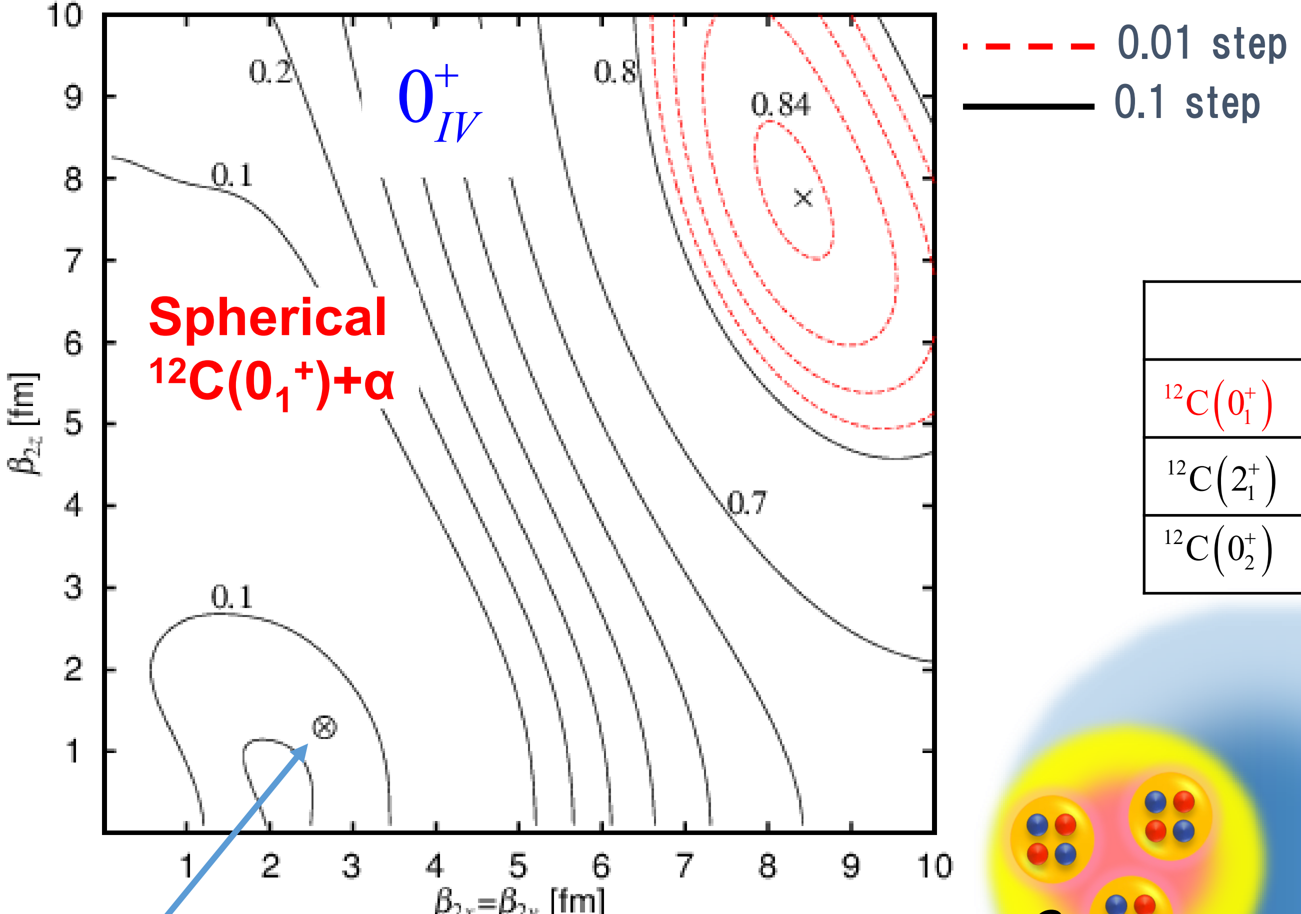
For the fourth α

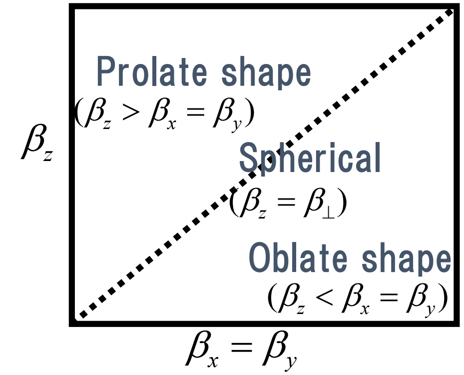


THSR+GCM

$$|\langle \Phi(\boldsymbol{\beta}_1, \boldsymbol{\beta}_2) | 0^+_{III} \rangle|^2$$

Squared overlap surface with single config. of eTHSR





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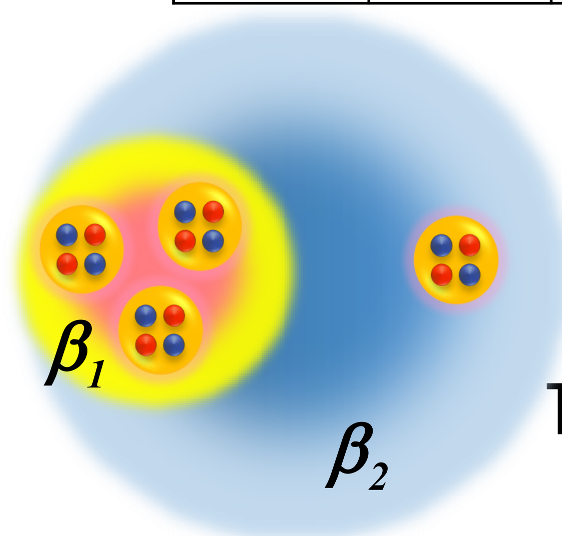
$^{12}C(2_1^+)$	0.90	(1.9,0.5fm)
$^{12}C(0_2^+)$	0.99	(5.6,1.4fm)



$(\beta_{1x}=\beta_{1y},\beta_{1z})$: fixed at ⊗

Container for 3α

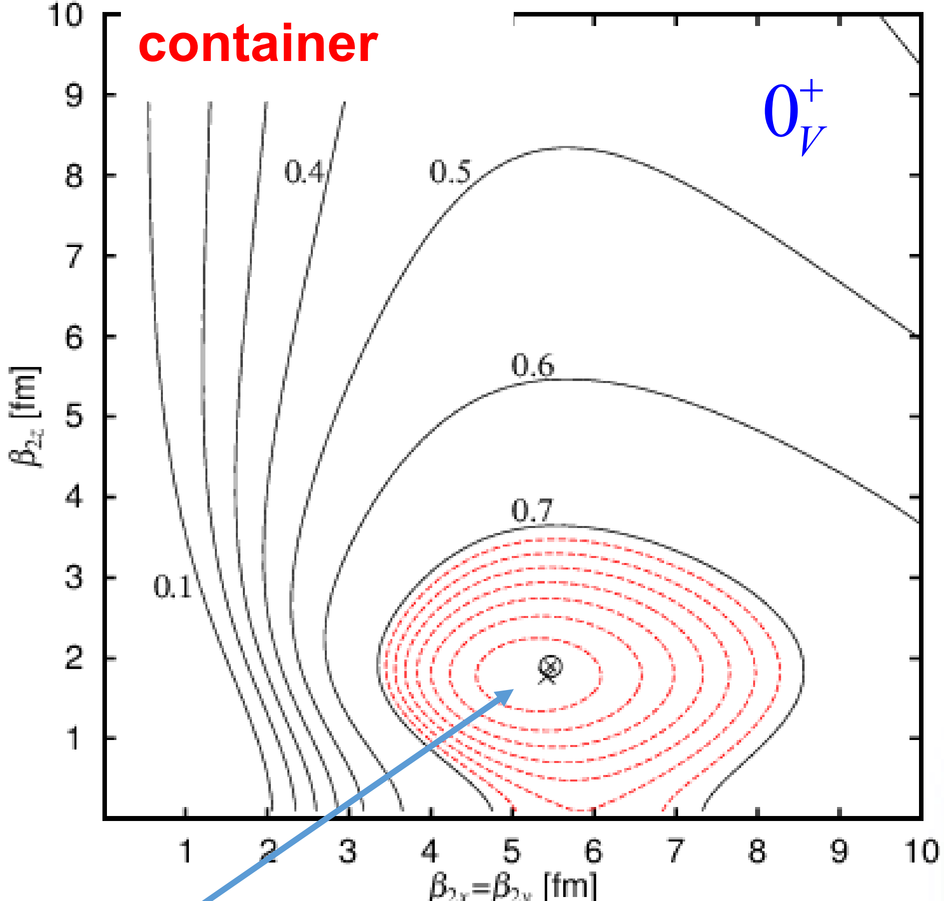
× : maximum

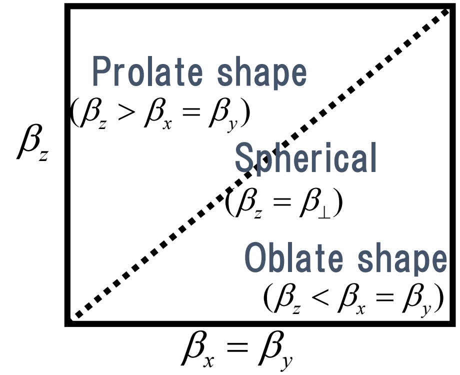
For the fourth α

THSR+GCM

$$|\langle\Phi(\boldsymbol{\beta}_1,\boldsymbol{\beta}_2)|0^+_{IV}\rangle|^2$$

Squared overlap surface with single config. of eTHSR





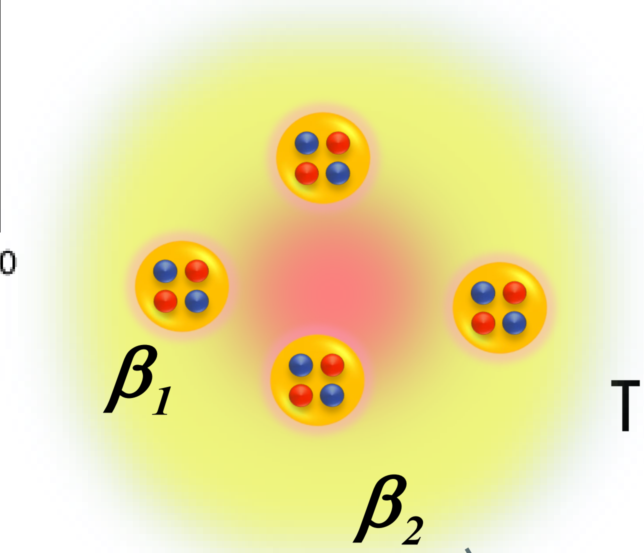
	Sq. overlap	$(\beta_{1x} = \beta_{1y}, \beta_{1z})$
$^{12}C(0_1^+)$	0.93	(1.9,1.8fm)
$^{12}C(2_1^+)$	0.90	(1.9, 0.5fm)
$^{12}C(0_2^+)$	0.99	(5.6,1.4fm)

$(\beta_{1x} = \beta_{1y}, \beta_{1z})$: fixed at $\otimes$

Container for 3α

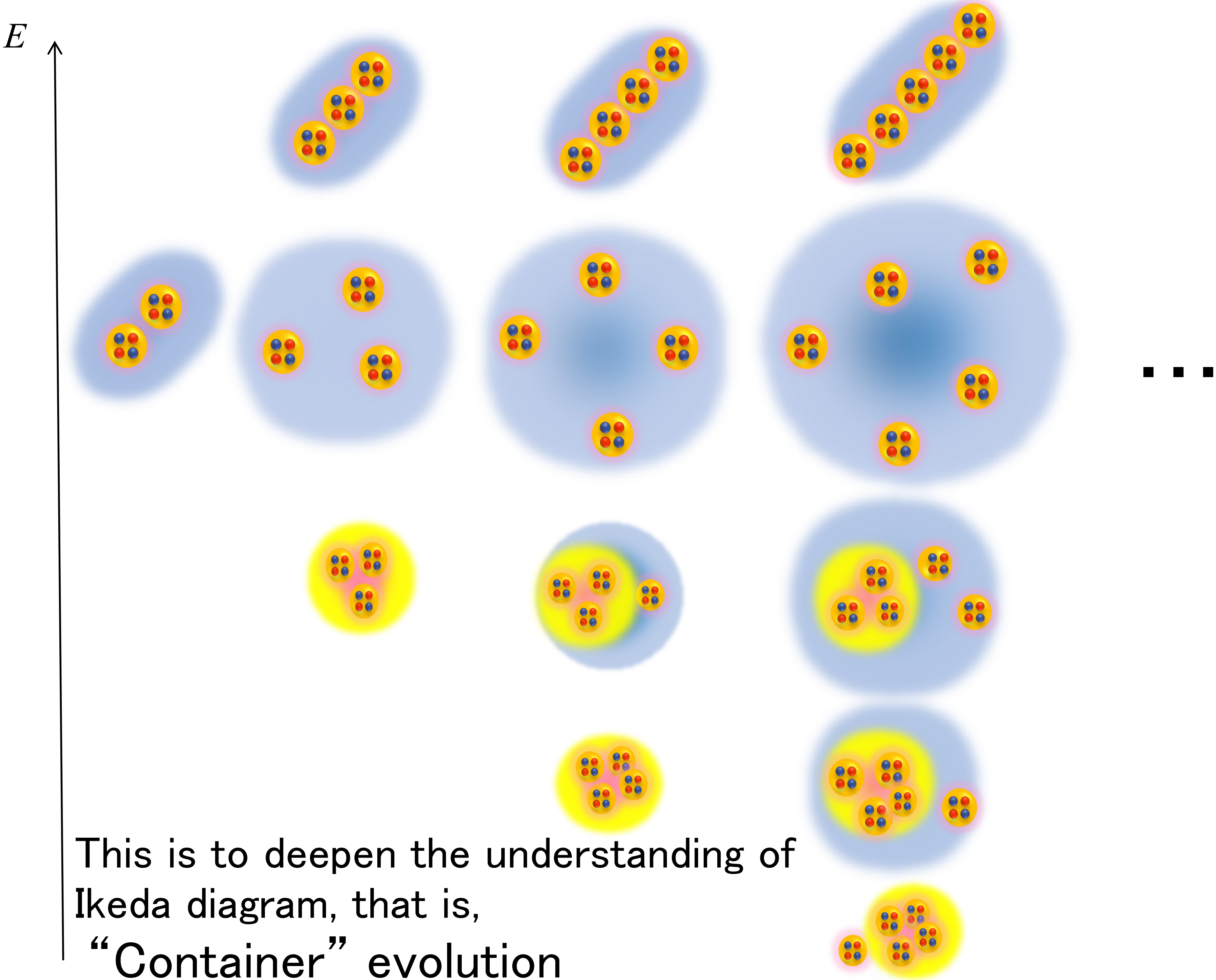
× : maximum

For the fourth α



THSR+GCM

$$|\langle \Phi(\boldsymbol{\beta}_1, \boldsymbol{\beta}_2) | 0_V^+ \rangle|^2$$



This is to deepen the understanding of
Ikeda diagram, that is,
"Container" evolution

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