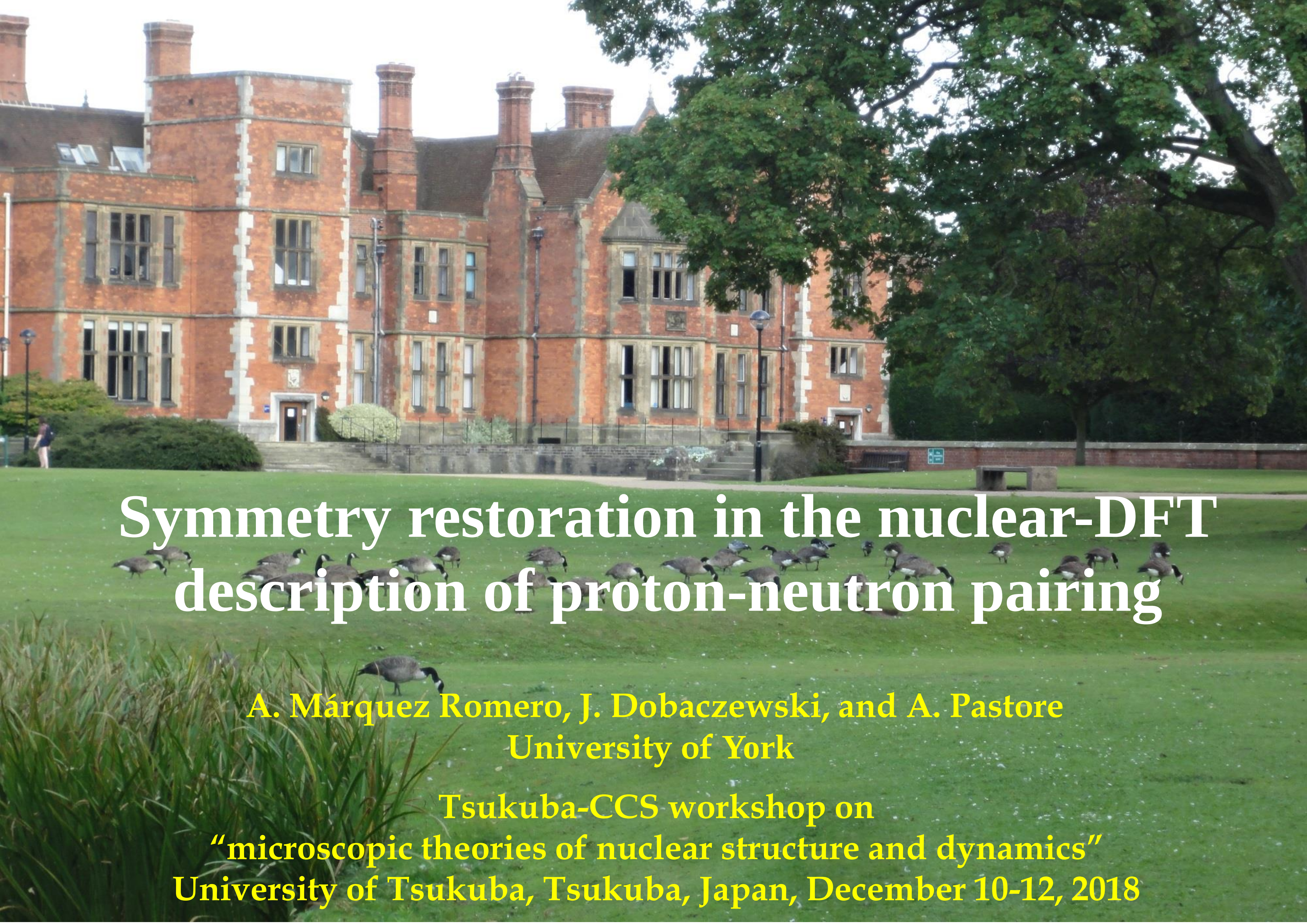

# Symmetry restoration in the nuclear-DFT description of proton-neutron pairing

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Tsukuba-CCS workshop on
“microscopic theories of nuclear structure and dynamics”
University of Tsukuba, Tsukuba, Japan, December 10-12, 2018

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# Outline

1. Pairs of nucleons
2. Pair condensate
3. Symmetries
4. Symmetry breaking
5. Symmetry restoration
6. SO(8) Hamiltonian
7. HFB versus VAP
8. VAP versus exact
    - Energies
    - Deuteron-transfer matrix elements
9. Quartet-condensate versus pair-condensate
10. Conclusions



# Nucleonic pairs I

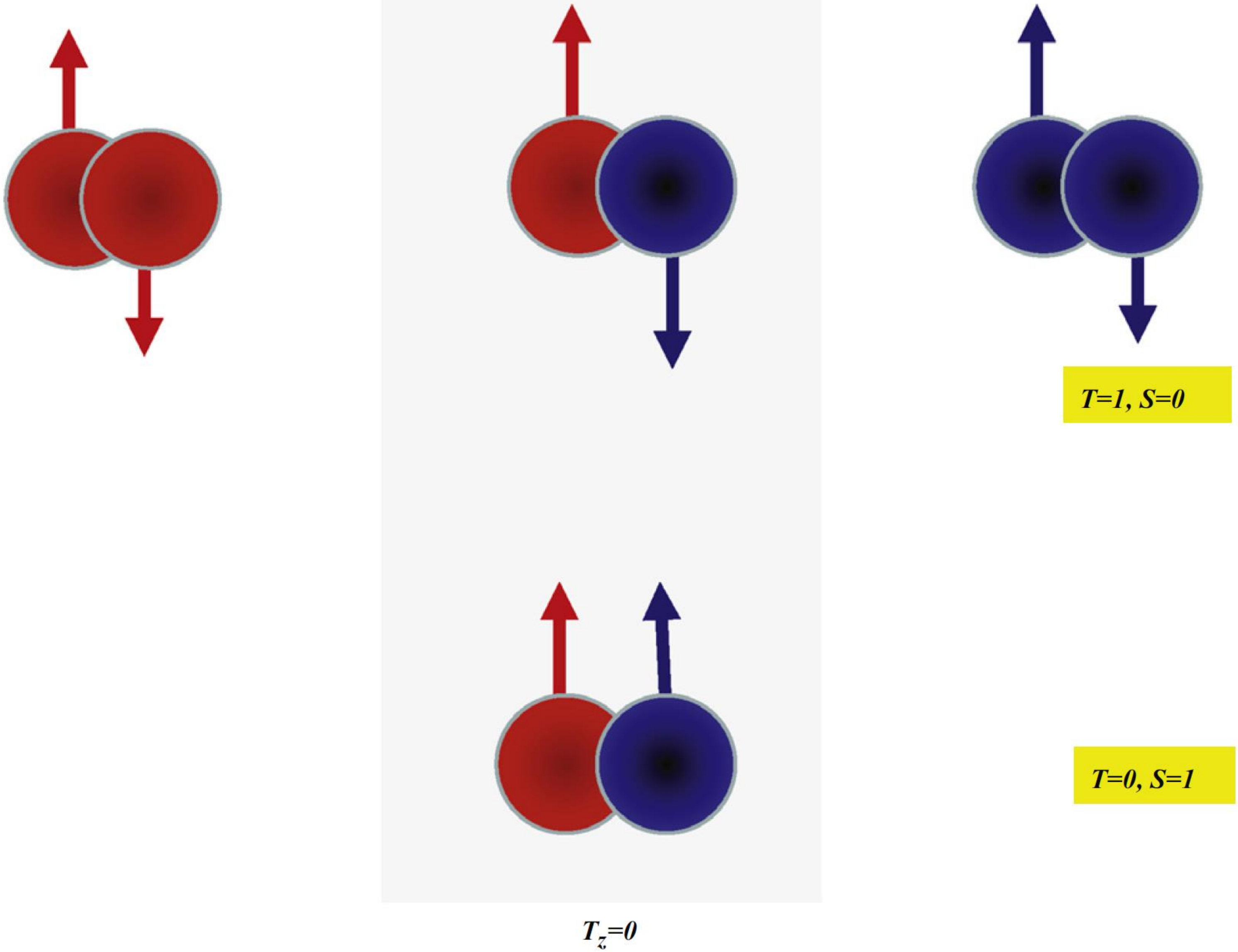
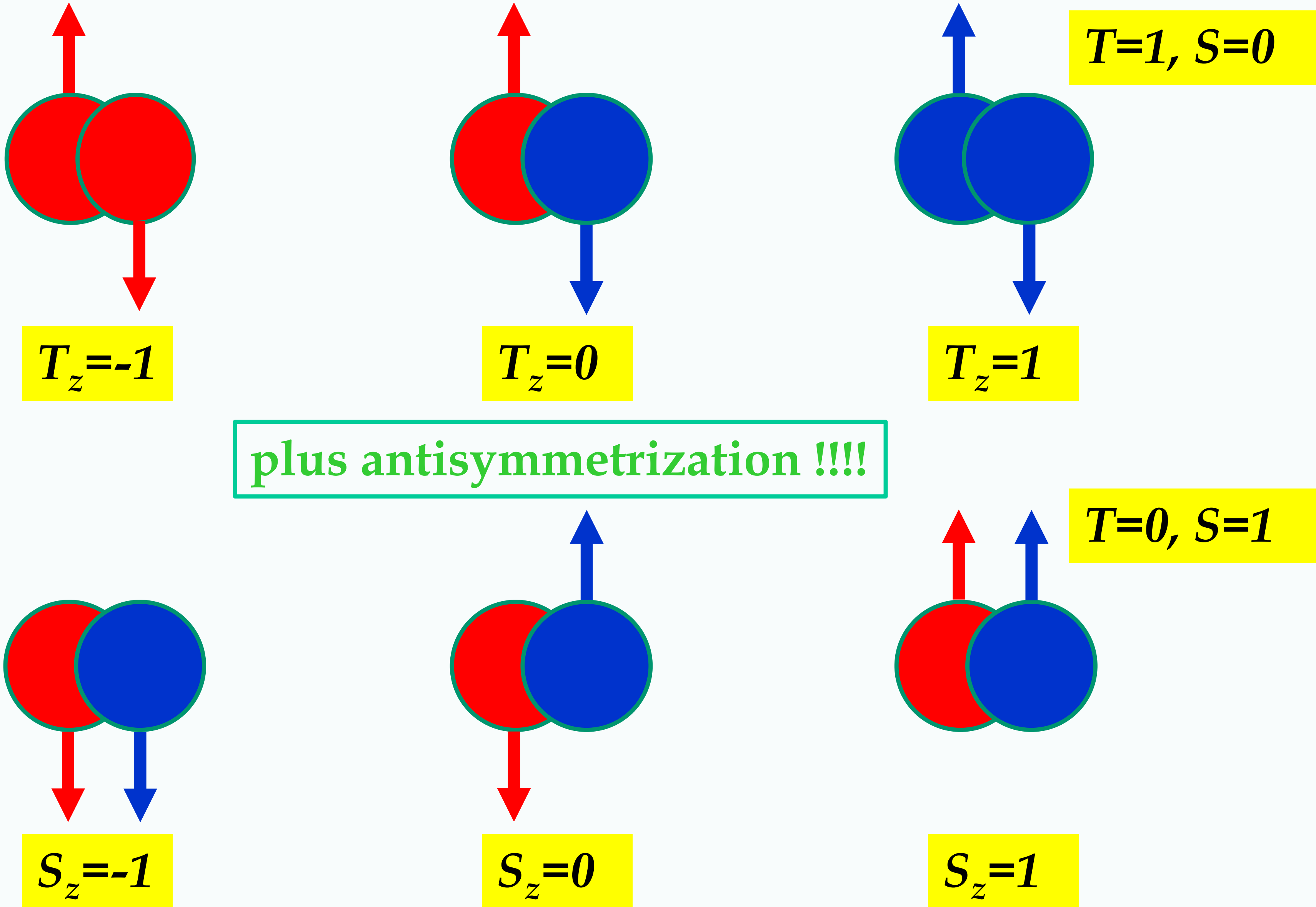



Fig. 1. Schematic picture showing the possible pair arrangements in $N = Z$ nuclei.

S. Frauendorf, A.O. Macchiavelli
Prog. Part. Nucl. Phys. 78 (2014) 24–90

# Nucleonic pairs II



# Scalar-isovector and vector-isoscalar pairs

$$\hat{P}^+_\nu = \sum_\ell \sqrt{\frac{2\ell+1}{2}} \left( a^+_{\ell\frac{1}{2}\frac{1}{2}} a^+_{\ell\frac{1}{2}\frac{1}{2}} \right)^{L=0,S=0,T=1}_{M=0,S_z=0,T_z=\nu}$$

$$\hat{D}^+_\mu = \sum_\ell \sqrt{\frac{2\ell+1}{2}} \left( a^+_{\ell\frac{1}{2}\frac{1}{2}} a^+_{\ell\frac{1}{2}\frac{1}{2}} \right)^{L=0,S=1,T=0}_{M=0,S_z=\mu,T_z=0}$$

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$$\hat{Z}^+ = \sum_{\nu=0,\pm1} p_\nu \hat{P}^+_\nu + \sum_{\mu=0,\pm1} d_\mu \hat{D}^+_\mu$$

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$$|\Phi\rangle = \mathcal{N} \exp\{\hat{Z}^+\}|0\rangle \longrightarrow |\Phi_A\rangle = \mathcal{N}'(\hat{Z}^+)^{A/2}|0\rangle$$

**A. Márquez Romero, J. D., and A. Pastore, arXiv:1812.03927**

# Symmetries

## rotational and isorotational symmetries

$$p_0 = \sin(\tfrac{1}{2}\alpha)e^{-i\varphi} \quad \text{and} \quad d_0 = \cos(\tfrac{1}{2}\alpha)e^{i\varphi}$$

$$\hat{Z}^+ = \sin(\tfrac{1}{2}\alpha)e^{-i\varphi}\hat{P}_0^+ + \cos(\tfrac{1}{2}\alpha)e^{i\varphi}\hat{D}_0^+$$

## signature and isosignature symmetries

$$\hat{\mathcal{S}}\hat{Z}^+(\alpha,\varphi)\hat{\mathcal{S}}^+ = i\hat{Z}^+(\alpha,\varphi+\tfrac{\pi}{2})$$

$$\hat{\mathcal{T}}\hat{Z}^+(\alpha,\varphi)\hat{\mathcal{T}}^+ = i\hat{Z}^+(\alpha,\varphi-\tfrac{\pi}{2})$$

## time-reversal symmetry

$$\hat{T}\hat{Z}^+(\alpha,\varphi)\hat{T}^+ = i\hat{Z}^+(\alpha,\tfrac{\pi}{2}-\varphi)$$

**States periodic with Δφ=π/2 and symmetric wrt π/4**

A. Márquez Romero, J. D., and A. Pastore, arXiv:1812.03927

# Hamiltonian and symmetry restoration

$$\hat{H} = -g(1-x) \sum_{\nu=0,\pm1} \hat{P}^{+}_{\nu} \hat{P}_{\nu} - g(1+x) \sum_{\mu=0,\pm1} \hat{D}^{+}_{\mu} \hat{D}_{\mu}$$

**trial space parametrized by two angles α and φ**

$$\hat{Z}^{+} = \sin(\tfrac{1}{2}\alpha) e^{-i\varphi} \hat{P}^{+}_{0} + \cos(\tfrac{1}{2}\alpha) e^{i\varphi} \hat{D}^{+}_{0}$$

**AST symmetry-breaking state**

$$|\Phi\rangle = \mathcal{N} \exp\{\hat{Z}^{+}\} |0\rangle$$

**ST symmetry-breaking state**

$$|\Phi_A\rangle = \mathcal{N}' (\hat{Z}^{+})^{A/2} |0\rangle$$

**AST symmetry-restored state**

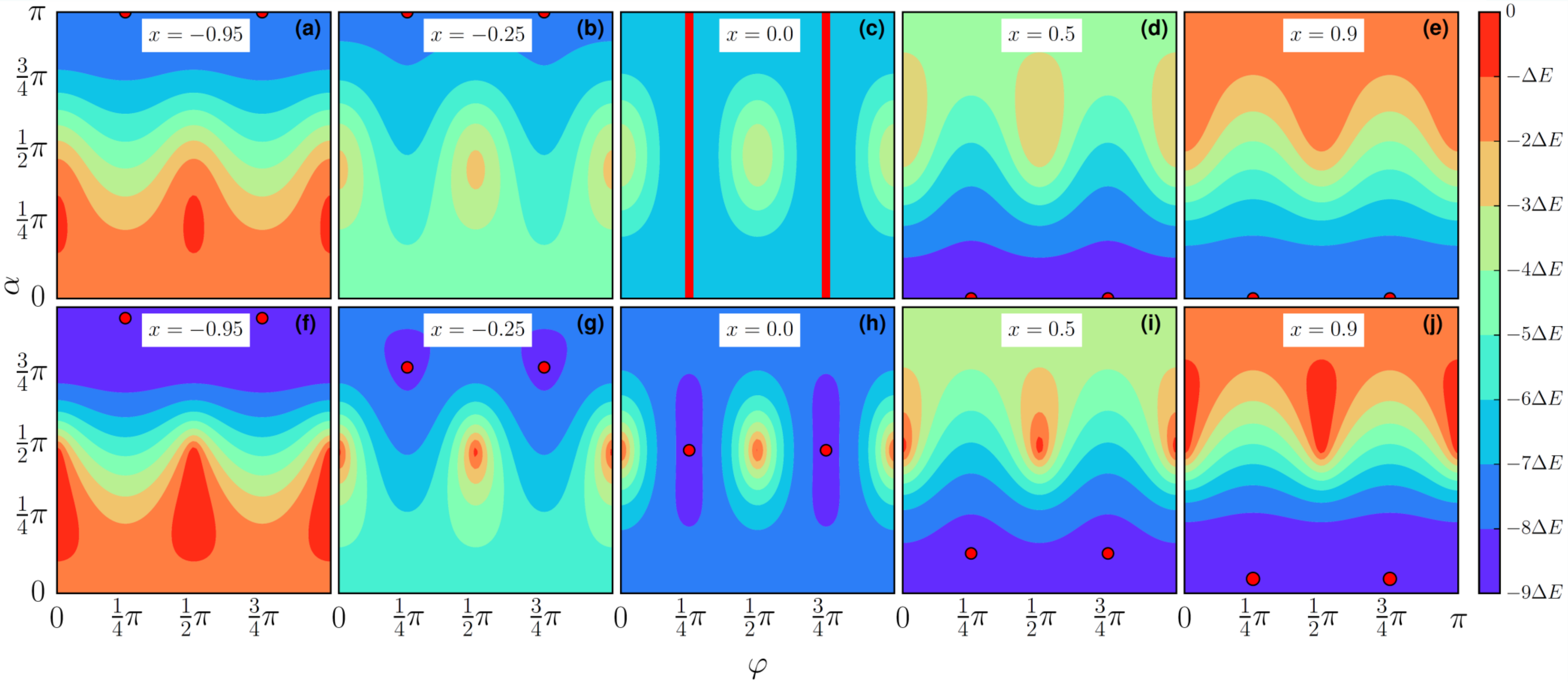
$$|\Phi^{AST}_{MK,NL}\rangle = \hat{P}_A \hat{P}^{S}_{MK} \hat{P}^{T}_{NL} |\Phi\rangle$$ **with** $(-1)^{S+T} = (-1)^{A/2}$

A. Márquez Romero, J. D., and A. Pastore, arXiv:1812.03927



# Hamiltonian and average energies

$$\hat{H} = -g(1-x)\sum_{\nu=0,\pm1}\hat{P}^{+}_{\nu}\hat{P}_{\nu} - g(1+x)\sum_{\mu=0,\pm1}\hat{D}^{+}_{\mu}\hat{D}_{\mu}$$




**Upper panels, before projection = HFB    Lower panels, after projection = VAP**

**A. Márquez Romero, J. D., and A. Pastore, arXiv:1812.03927**

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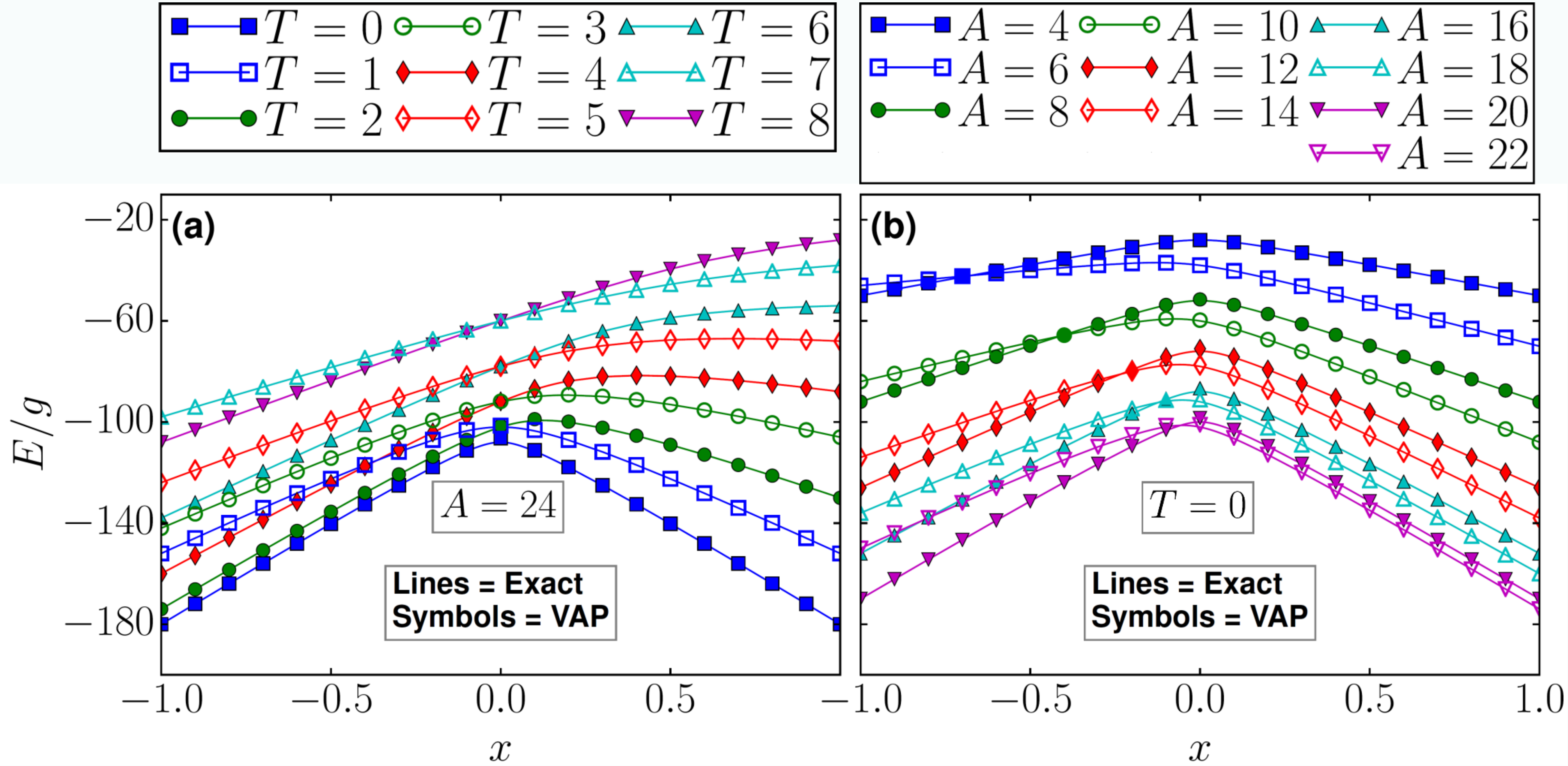
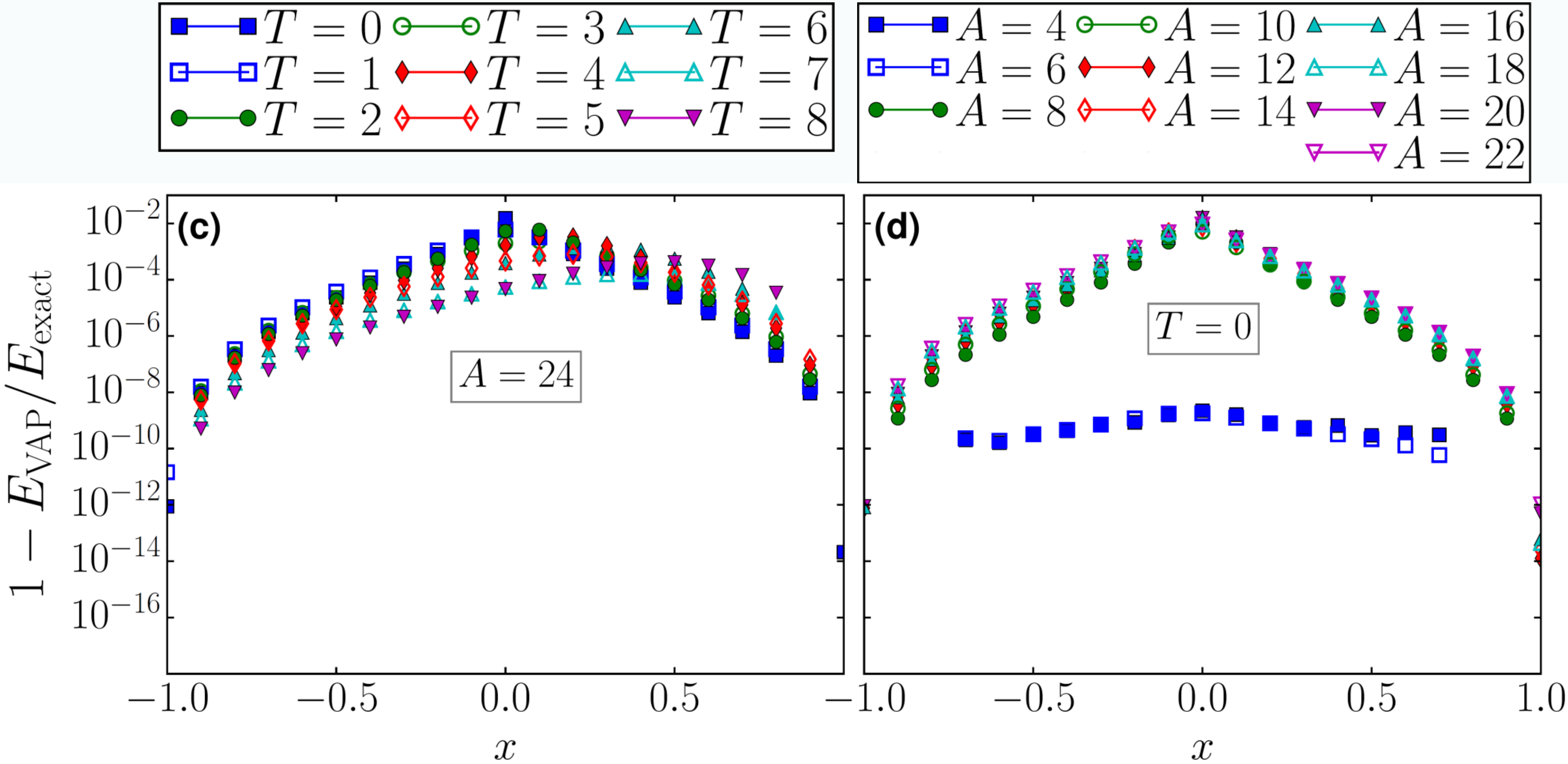
# Exact energies versus VAP




**A. Márquez Romero, J. D., and A. Pastore, arXiv:1812.03927**



# Exact energies versus VAP

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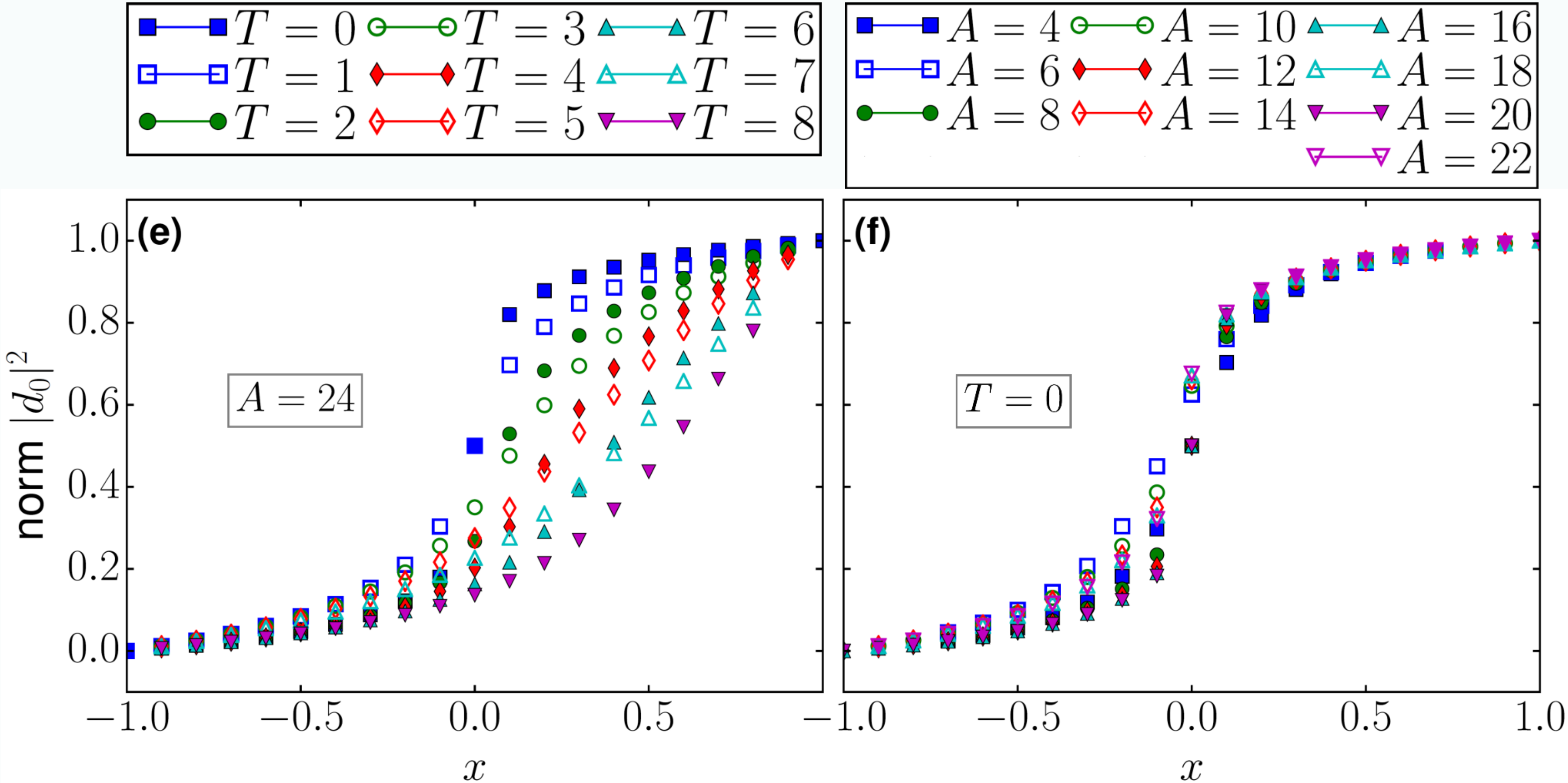
# Norms of VAP isoscalar Thouless pairs

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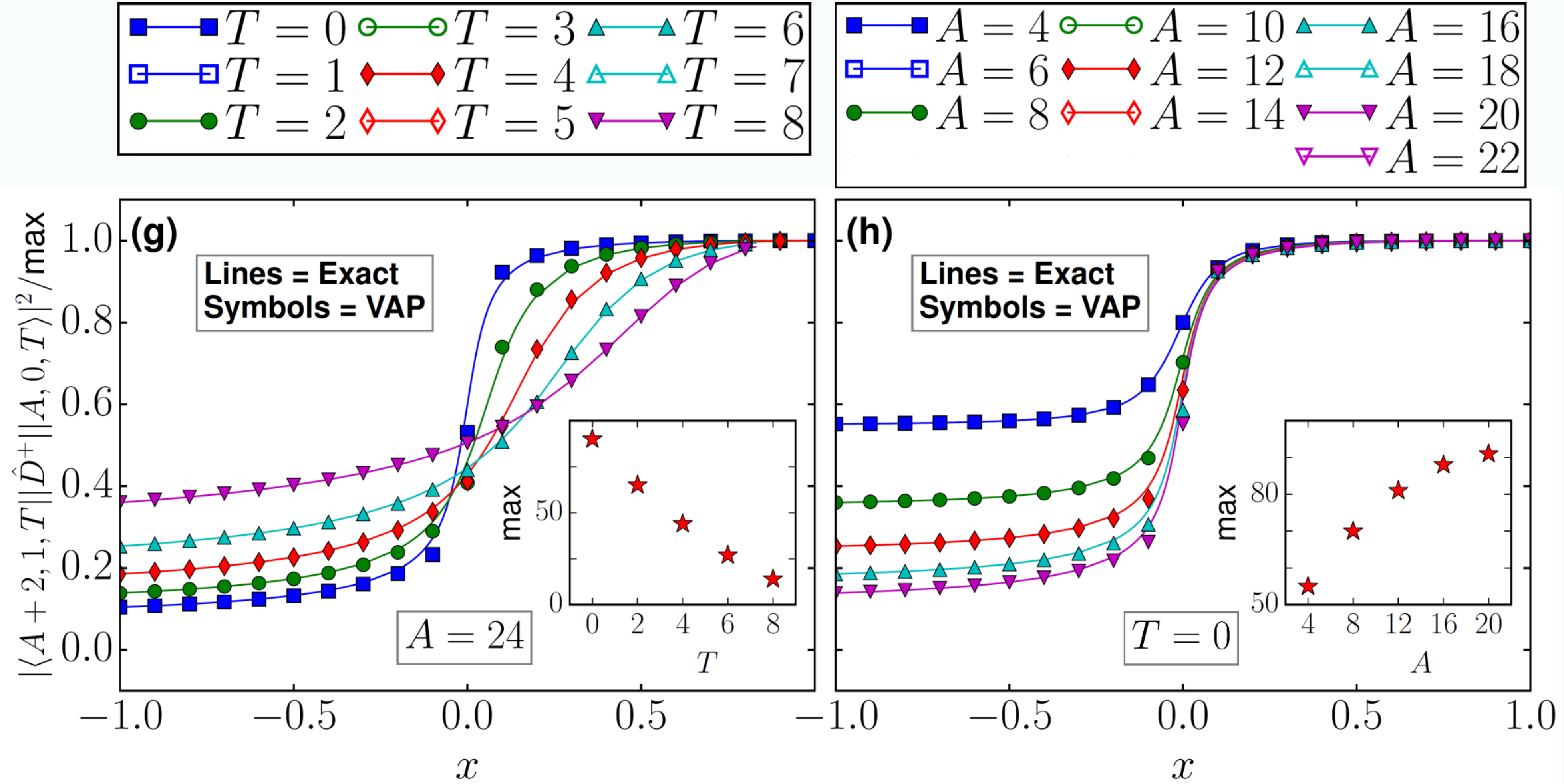
# Exact deuteron transfer m.e.'s versus VAP




A. Márquez Romero, J. D., and A. Pastore, arXiv:1812.03927

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# Quartet-condensate versus pair-condensate

$$\hat{Z}^{+} = \sum_{\nu=0,\pm1} p_\nu \hat{P}^{+}_\nu + \sum_{\mu=0,\pm1} d_\mu \hat{D}^{+}_\mu$$

$$\begin{aligned}
|\Phi_4\rangle &= \mathcal{N}'(\hat{Z}^{+})^2|0\rangle \\
&= \Big\{\mathcal{N}'_1(P^+P^+)^{(00)} + \mathcal{N}'_2(D^+D^+)^{(00)} \\
&\quad +\mathcal{N}'_3(P^+D^+)^{(11)} + \mathcal{N}'_4(P^+P^+)^{(02)} \\
&\quad +\mathcal{N}'_5(D^+D^+)^{(20)}\Big\}|0\rangle
\end{aligned}$$

$$|\Phi_{400}\rangle = \Big\{\mathcal{N}'_1(P^+P^+)^{(00)} + \mathcal{N}'_2(D^+D^+)^{(00)}\Big\}|0\rangle$$

$$|\Phi_{610}\rangle = \Big\{\mathcal{N}'_1 D^+(P^+P^+)^{(00)} + \mathcal{N}'_2 D^+(D^+D^+)^{(00)}\Big\}|0\rangle$$

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# Conclusions

1. Within a simple SO(8) pairing model, the symmetry-projected pair condensates very accurately **describe properties of the exact solutions**, including the coexistence of the isovector and isoscalar pairing.
2. Lack of the symmetry restoration thus explains a limited success in describing such a **coexistence in standard mean-field** approaches.
3. Symmetry restoration is also key to reconciling the **pair-condensation and quartet-condensation** pictures of paired systems.
4. Further work on properties of the proton-neutron nuclear pairing should be, and can be carried out within the **variation-after-projection approach to mean-field** pairing methods.



# Thank you