Identifying resonances with wave-packet dynamics

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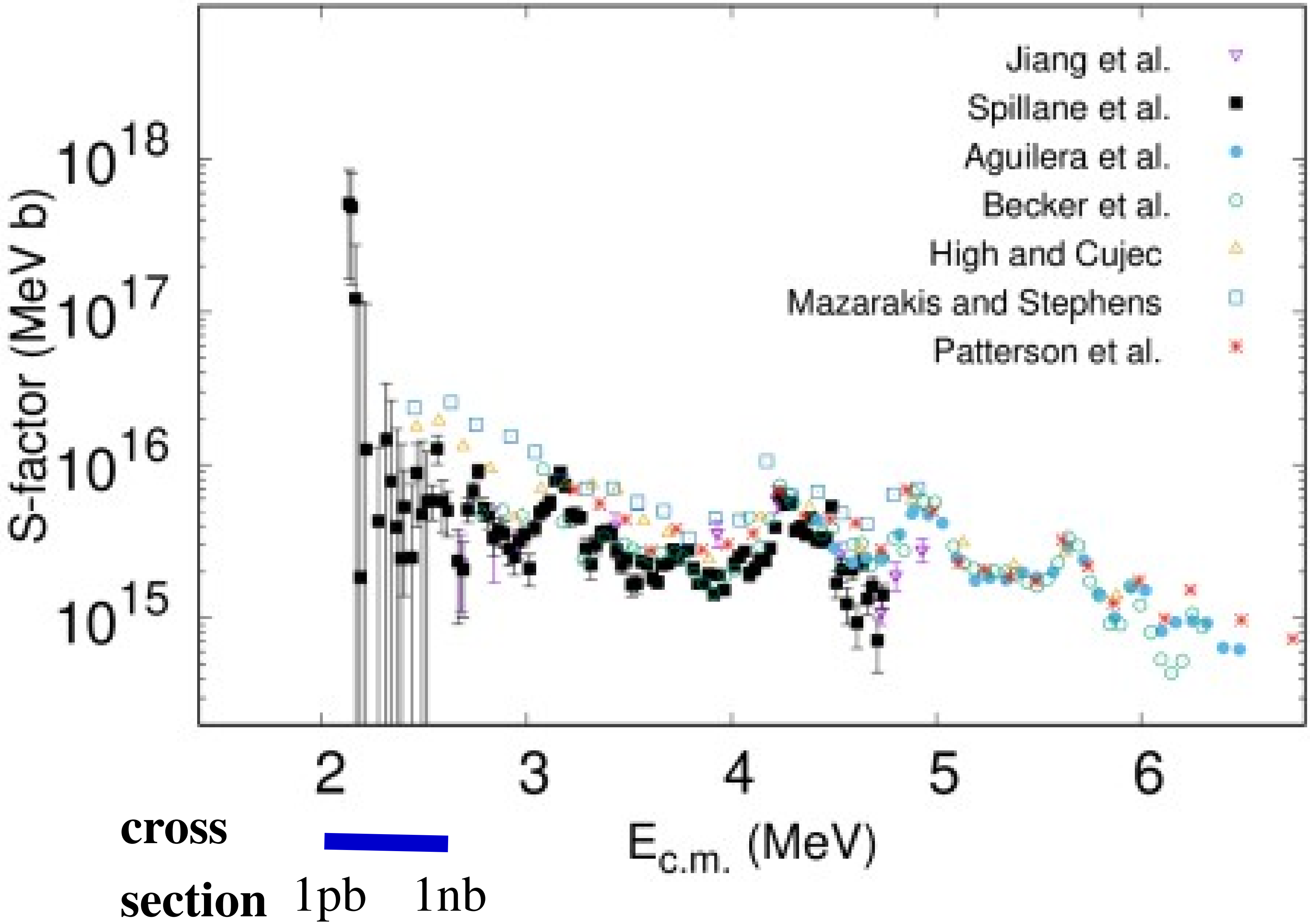
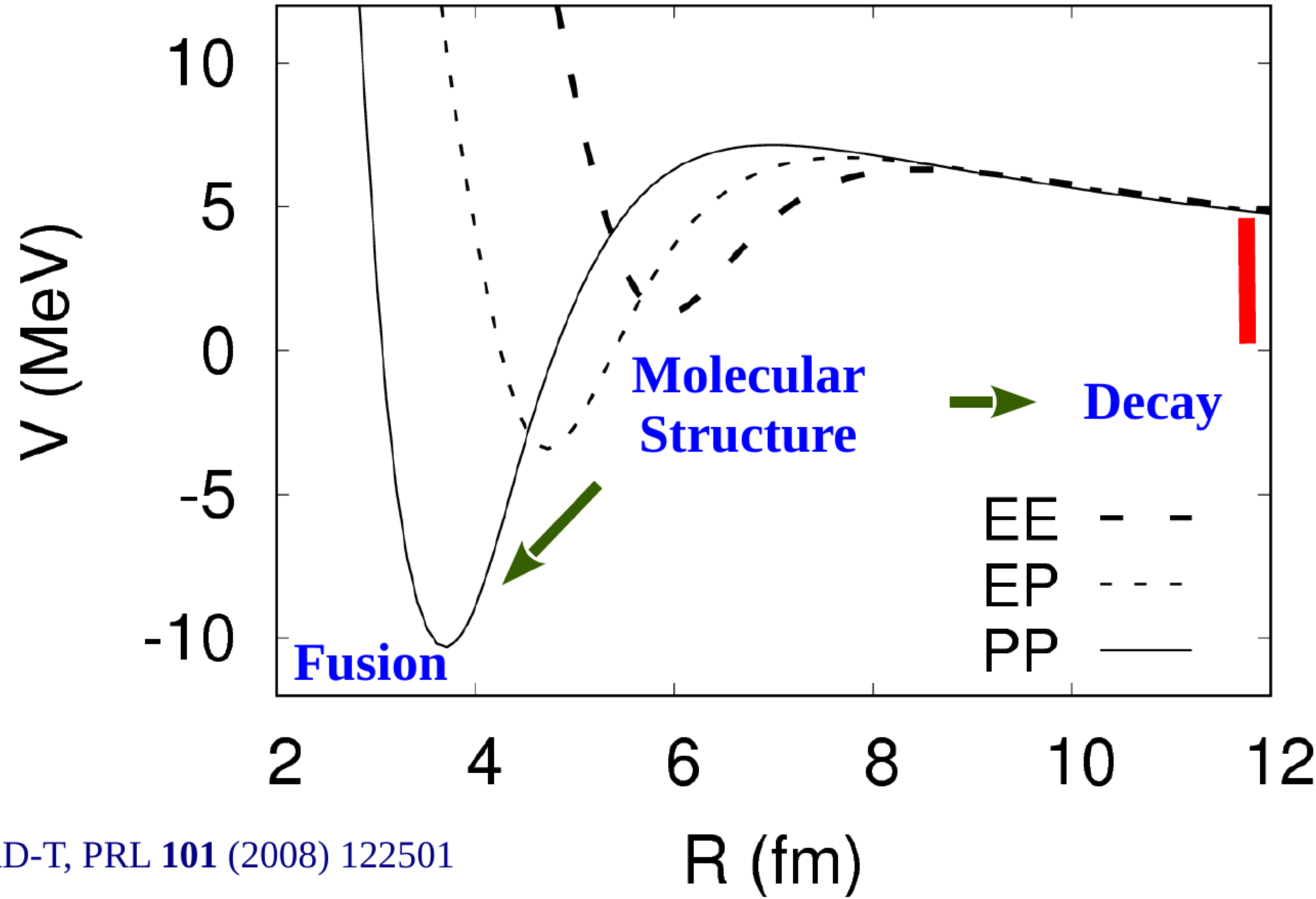
Astrophysical S-Factor for $^{12}C + ^{12}C$ Fusion

AD-T & Wiescher, PRC **97**, 055802 (2018)



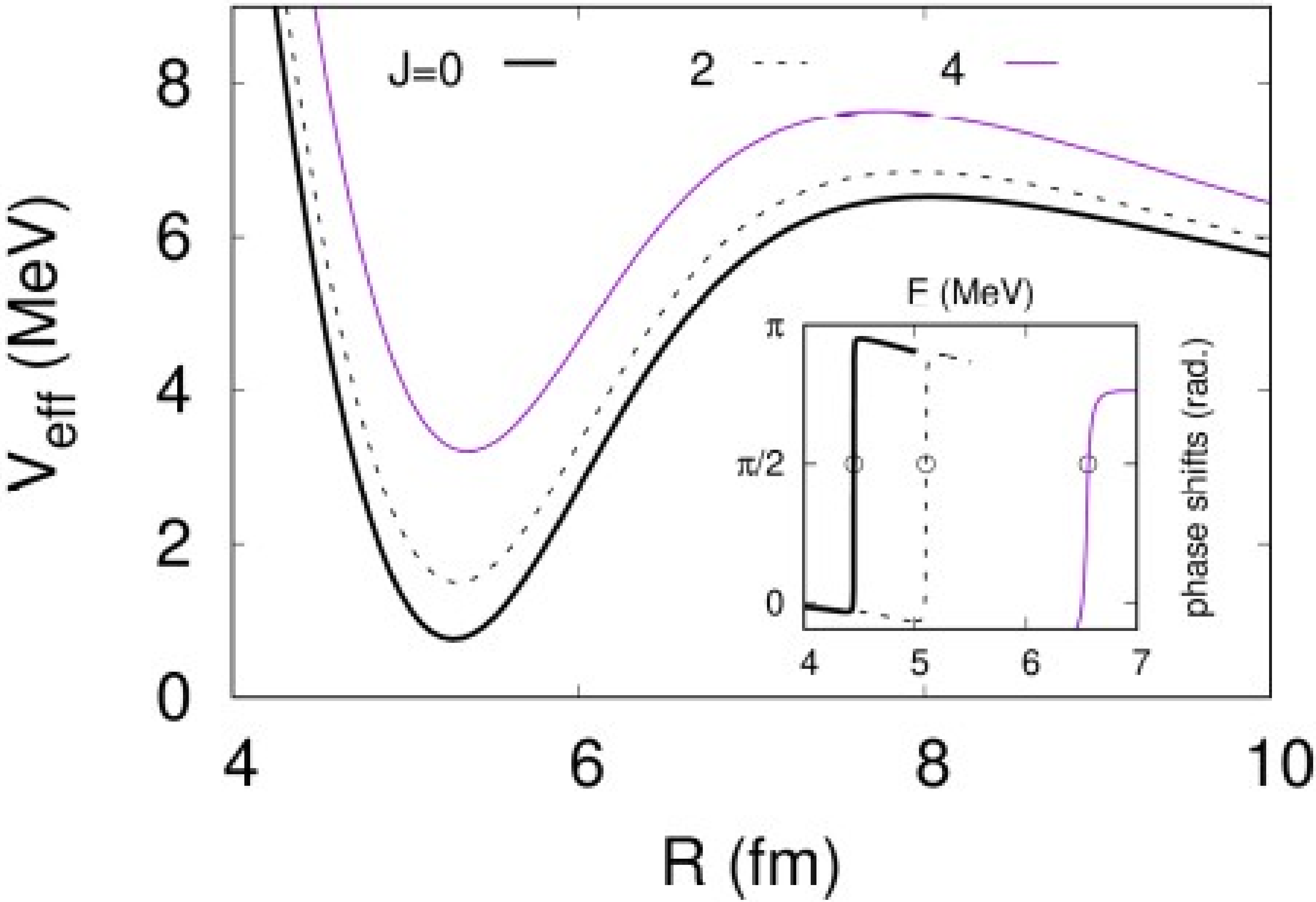
Collective Potential-Energy Landscape for $^{12}C + ^{12}C$



AD-T, PRL **101** (2008) 122501

Moeller & Iwamoto, NPA **575** (1994) 381

Phase shift analysis of the effective potentials for $^{12}C + ^{12}C$



A NEW METHOD FOR POTENTIAL RESONANCES FROM WAVE-PACKET DYNAMICS

Wave-Packet Dynamics

D.J. Tannor, Quantum Mechanics: a Time-Dependent Perspective, USB, 2007

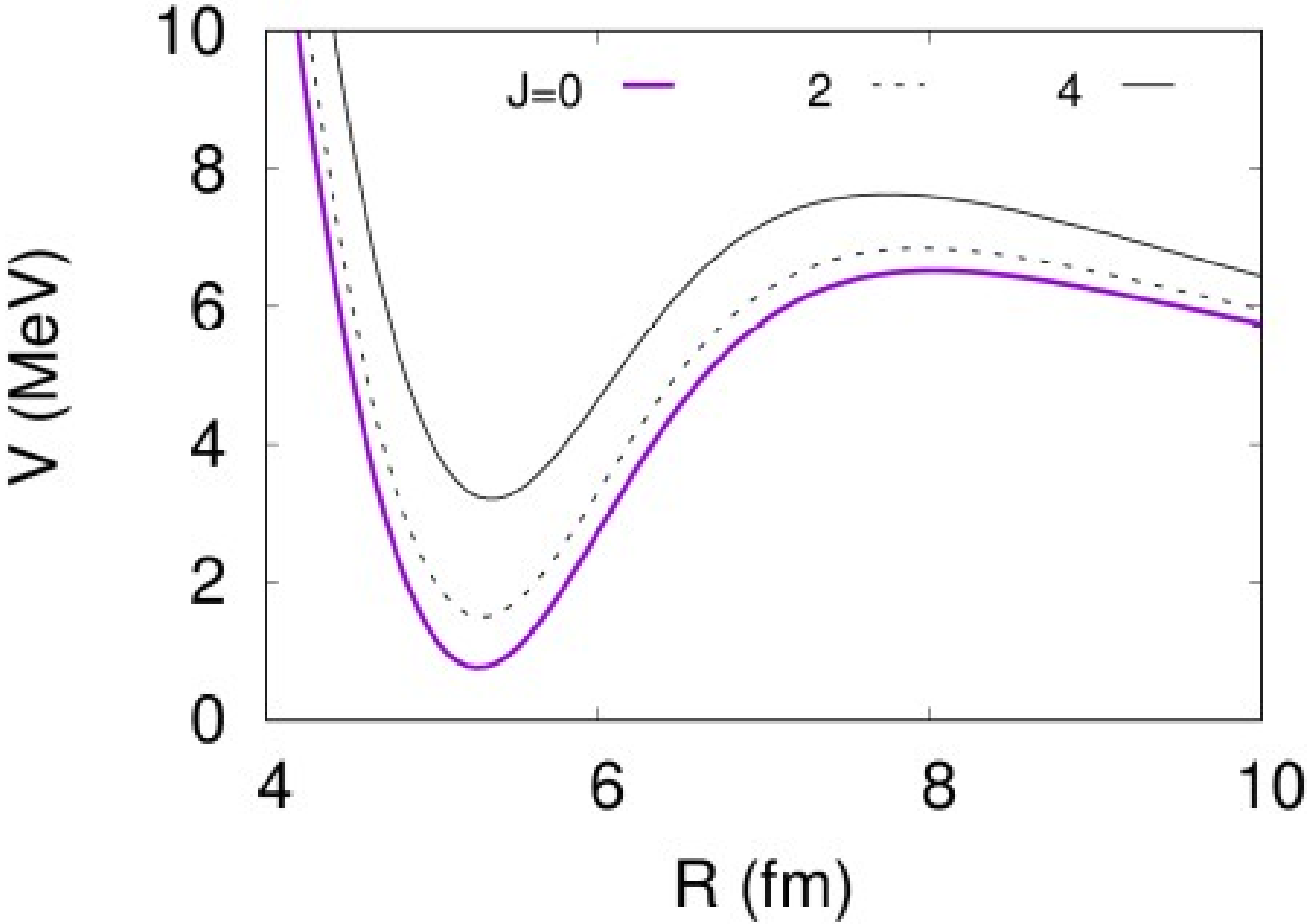
- **Preparation:** the initial state $\Psi(t=0)$

- **Time propagation:** $\Psi(0) \rightarrow \Psi(t)$,
  guided by the operator, $\exp(-i\,\hat{H}\,t/\hbar)$

  $\hat{H}$ is the model Hamiltonian

- **Analysis:** extraction of probabilities from the time-dependent wave function

Real $^{12}C + {}^{12}C$ total potentials for the J = 0, 2, 4 partial waves



Initial Wave-Packet

$$\Psi_0(R) = \frac{1}{\pi^{1/4}\sqrt{\sigma}}\, exp\Big[-\frac{(R-R_0)^2}{2\sigma^2} \Big]\, e^{iK_0(R-R_0)}$$

$R = 0 - 1000\,\mathrm{fm}$ with 2048 evenly spaced points

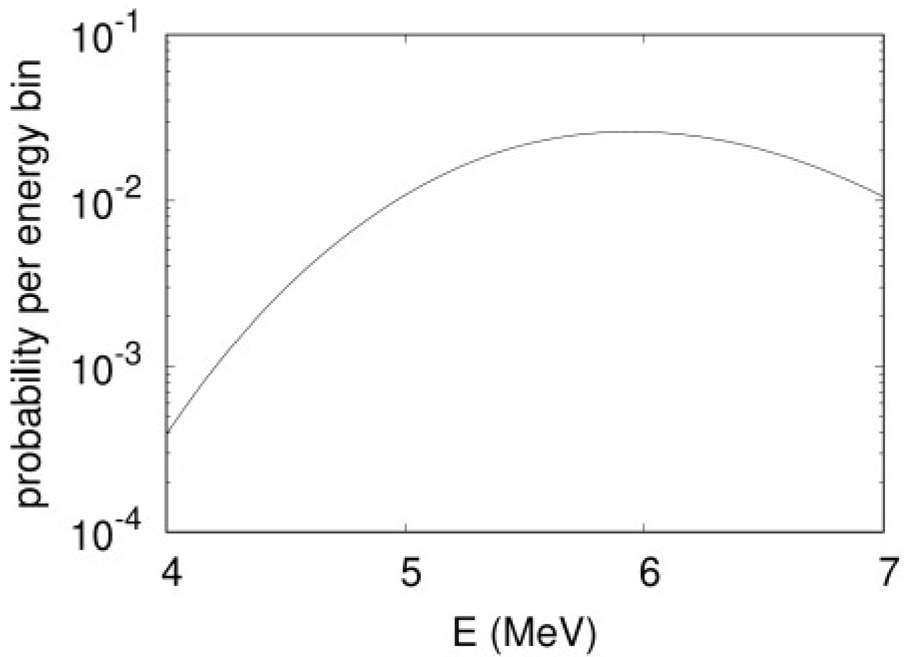
$$R_0 = 400\,\mathrm{fm}$$

$$\sigma = 10\,\mathrm{fm}$$

$$E_0 = 6\,\mathrm{MeV}$$

Energy Projection of the Wave Function

Schafer & Kulander,
PRA **42** (1990) 5794



- **Energy spectra** of $\Psi(t)$ as expectation values of **the window operator**

$$\hat{\boldsymbol{\Delta}}(E_k, n, \epsilon) \equiv \frac{\epsilon^{2^n}}{(\hat{\mathcal{H}} - E_k)^{2^n} + \epsilon^{2^n}}$$

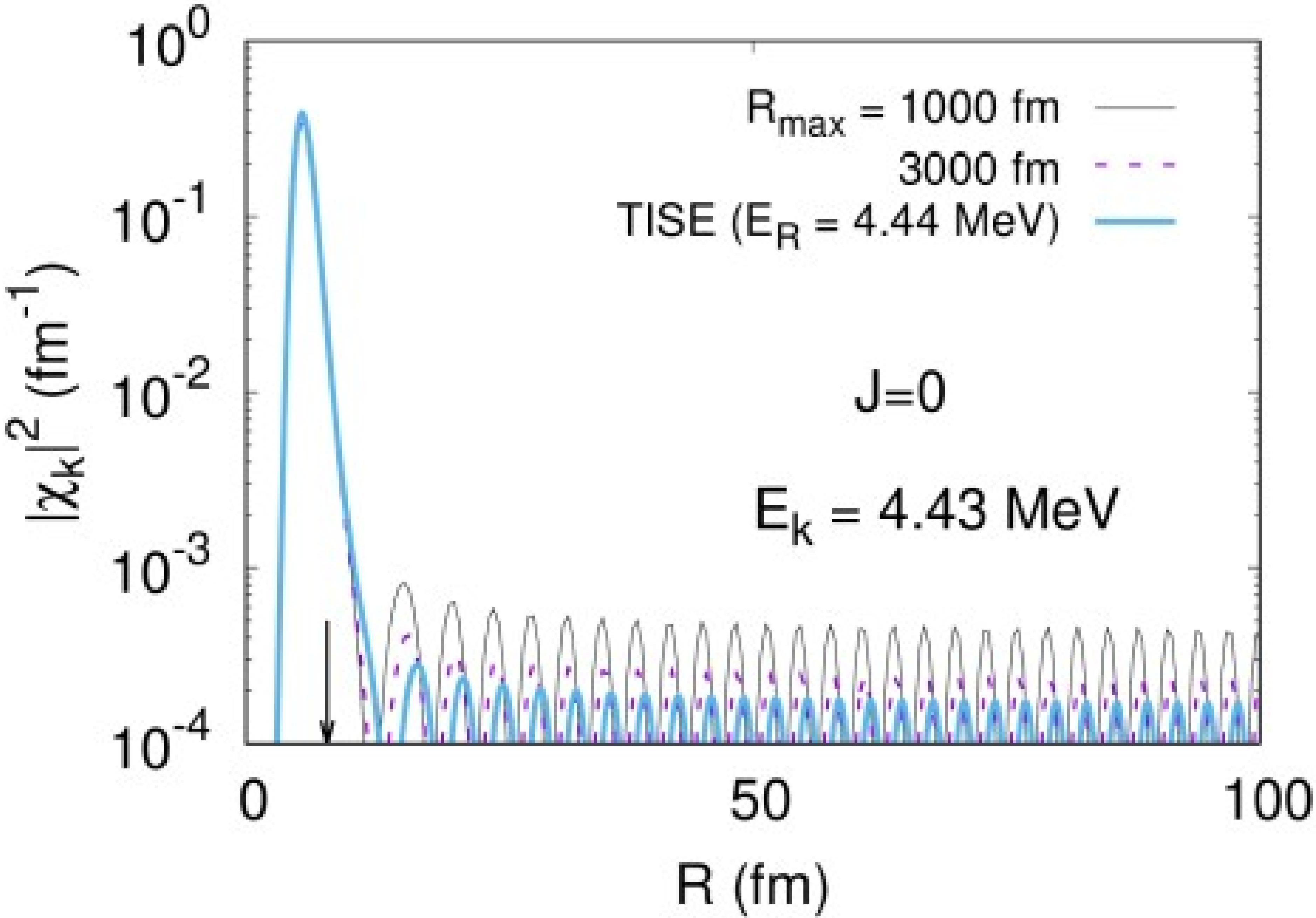
$$E_{k+1} = E_k + 2\epsilon$$

- $\mathcal{P}(E_k) = \langle \Psi | \hat{\Delta} | \Psi \rangle$, for instance, **n = 2** :

$$\boxed{(\hat{H} - E_k + \sqrt{i}\,\epsilon)(\hat{H} - E_k - \sqrt{i}\,\epsilon)\,|\chi_k\rangle = \epsilon^2\,|\Psi\rangle}$$

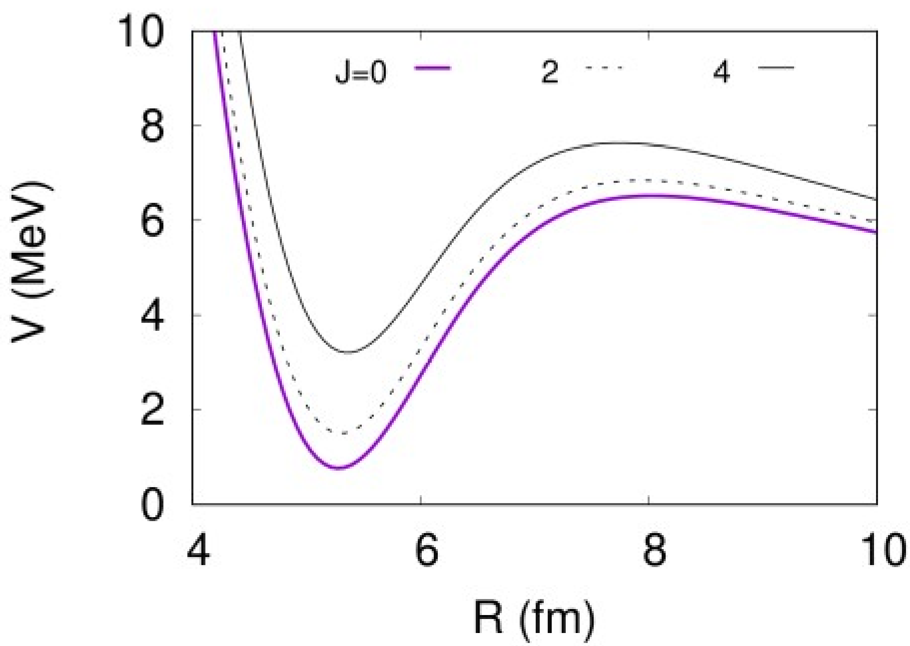
$\rightarrow$ $\boxed{\mathcal{P}(E_k) = \langle \chi_k | \chi_k \rangle}$

Radial probability density for a resonant bin state with J=0

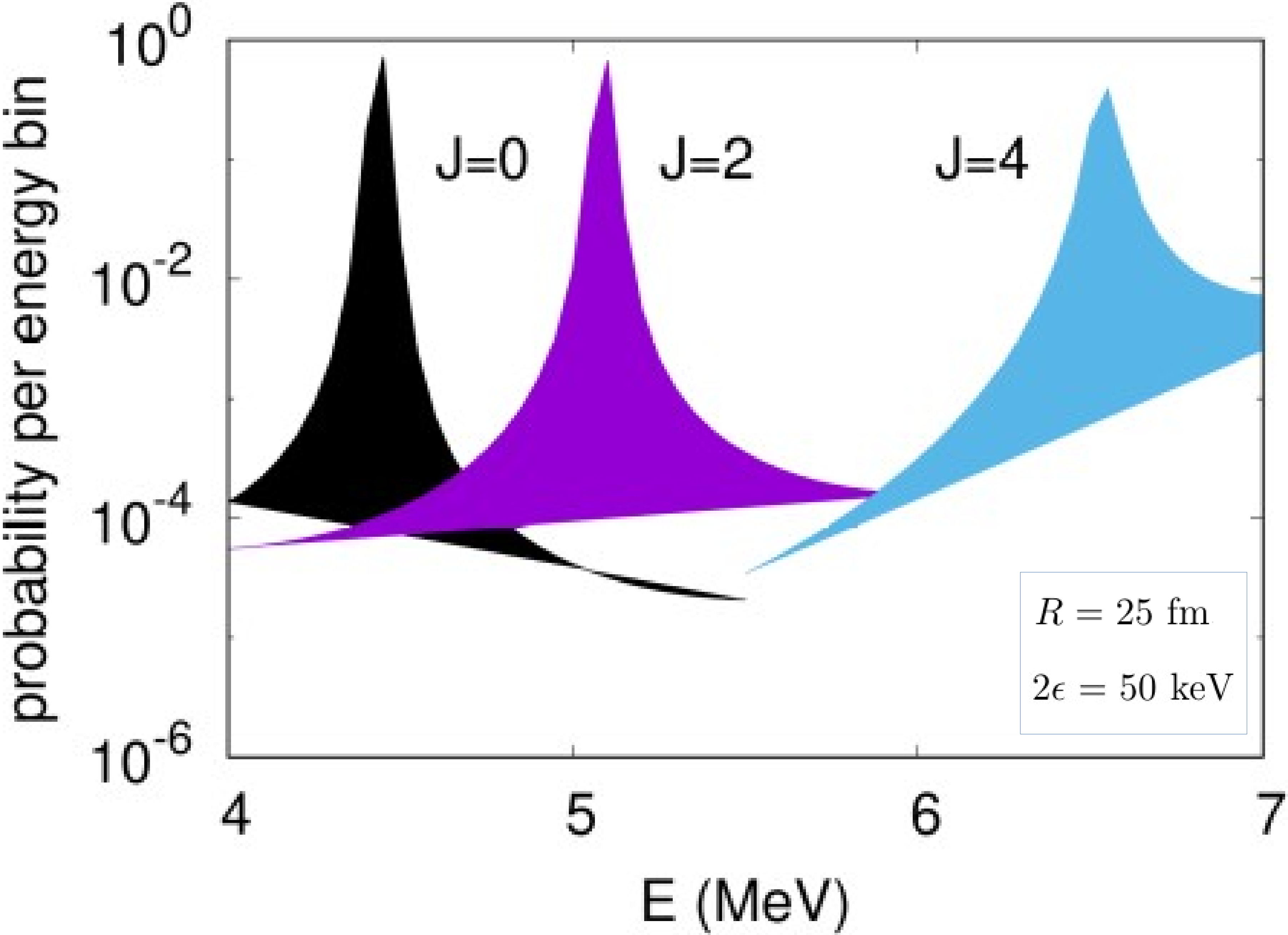


Resonances from wave-packet dynamics:
Effective Energy Spectra

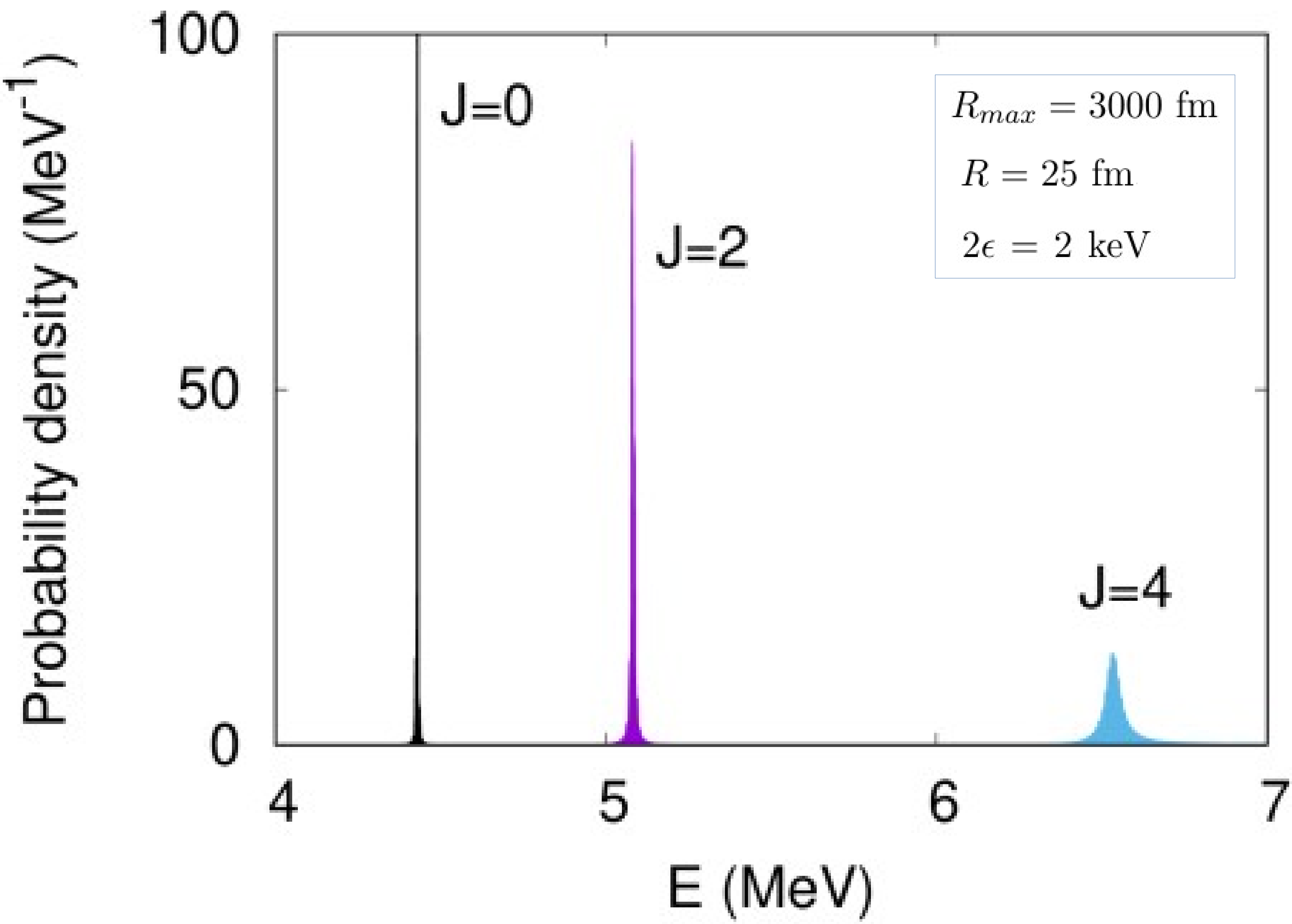
$$\mathcal{P}(E_k) \equiv \frac{\langle\tilde{\Psi}|\hat{\Delta}|\tilde{\Psi}\rangle}{\langle\Psi_0|\hat{\Delta}|\Psi_0\rangle}$$



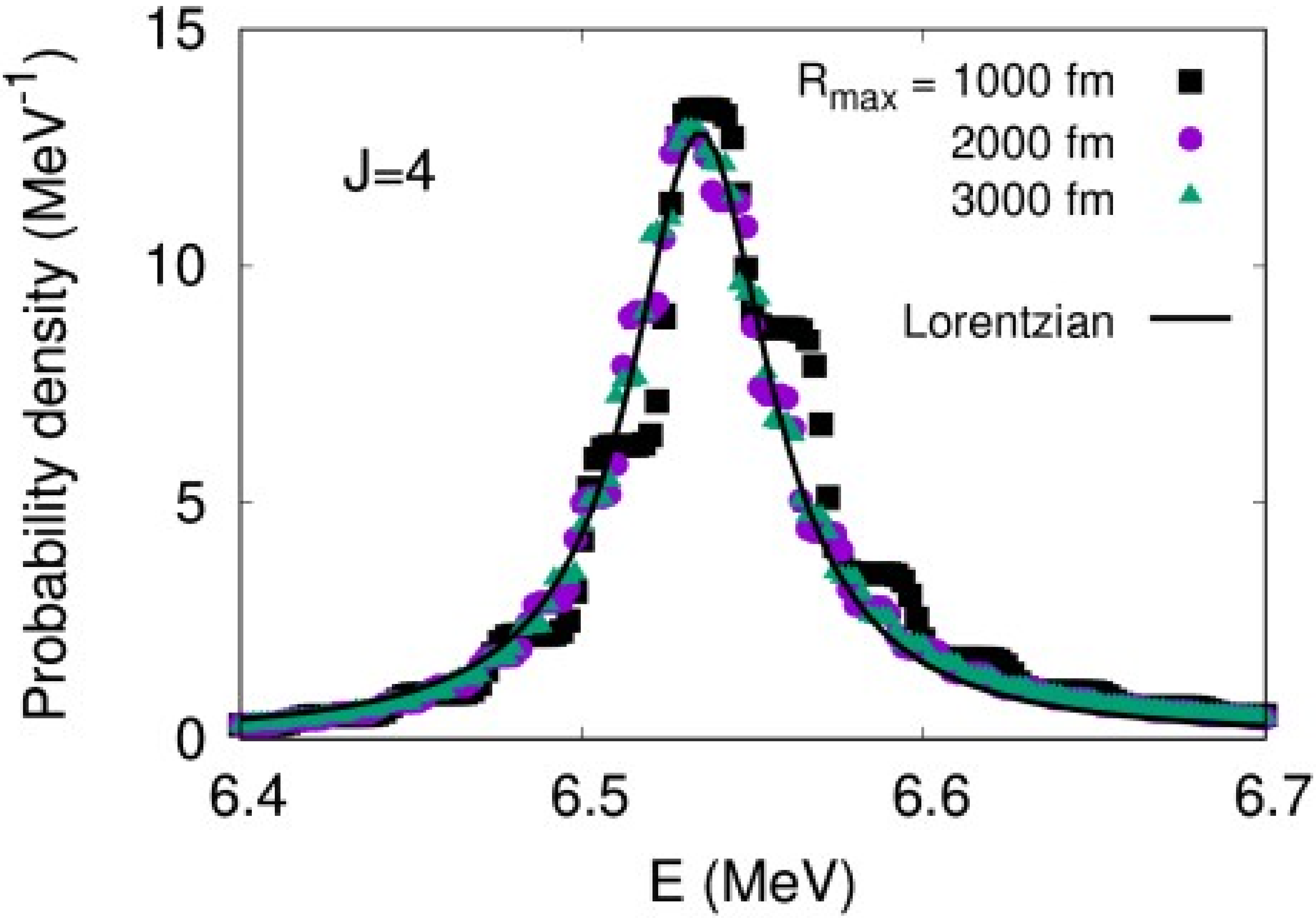
Effective Energy Spectra



Probability Density Function $\mathcal{P}(E_k)/2\epsilon$



Probability Density Function for J=4



Present method *versus* conventional approaches

TABLE I: Resonance energies (MeV) and their widths (keV), extracted from Fig. 6 for R_{max} = 3000 and 7000 fm, are compared to those from the S-matrix pole search and scattering phase shift methods. While the absolute errors of all E_R from the present method are $< 0.01\%$, those for Γ_R are between 0.5 and 6%.

	present method $R_{max} = 3000$ (7000) fm		S-matrix poles		phase shifts	
J	E_R	Γ_R	E_R	Γ_R	E_R	Γ_R
0	4.430 (4.431)	2.75 ± 0.09 (1.10 ± 0.07)	4.437	1.55	4.437	1.58
2	5.081 (5.082)	6.62 ± 0.15 (5.91 ± 0.29)	5.088	5.70	5.089	6.14
4	6.535 (6.535)	49.62 ± 0.23 (49.72 ± 0.30)	6.538	49.50	6.555	73.85

- The new method calculates resonance energies and widths in agreement with alternative, established methods.

- The new method is applicable and useful for the quantitative study of resonance phenomena in different fields, where particles are temporarily trapped by attractive potential pockets.

AD-T & Tostevin, arXiv: 1809.10517v1