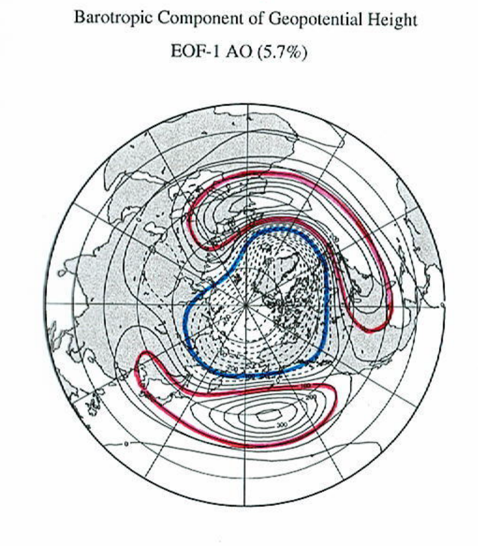
3D Energy Spectrum of the Global Atmosphere



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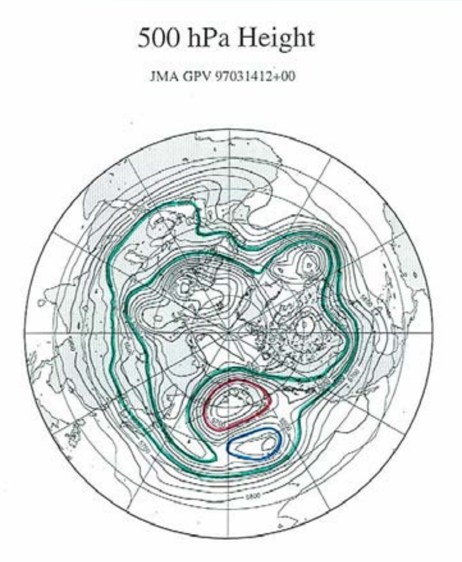
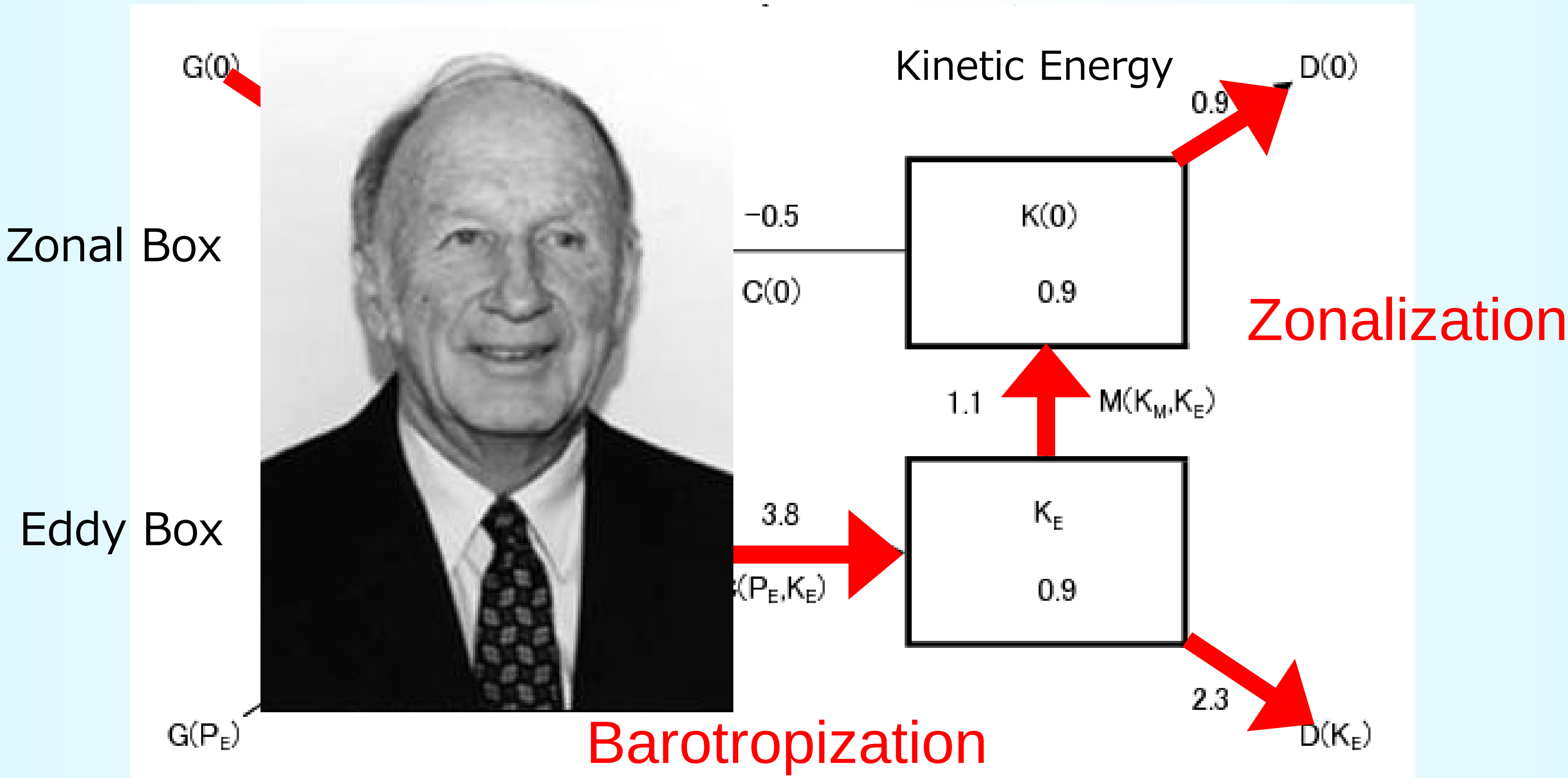


(Presentation at LBNL 2018)

Lorenz Energy Box Diagram

Energy accumulates at zonal wave 0



Energy accumulates at vertical wave 0

(Kung and Tanaka 1983, JAS)

Lorenz cycle, Saltzman cycle

Spectral Energetics

(Saltzman 1957; 1970)

$$p = \sum_{n=-\infty}^{\infty} p_n \exp(inx)$$

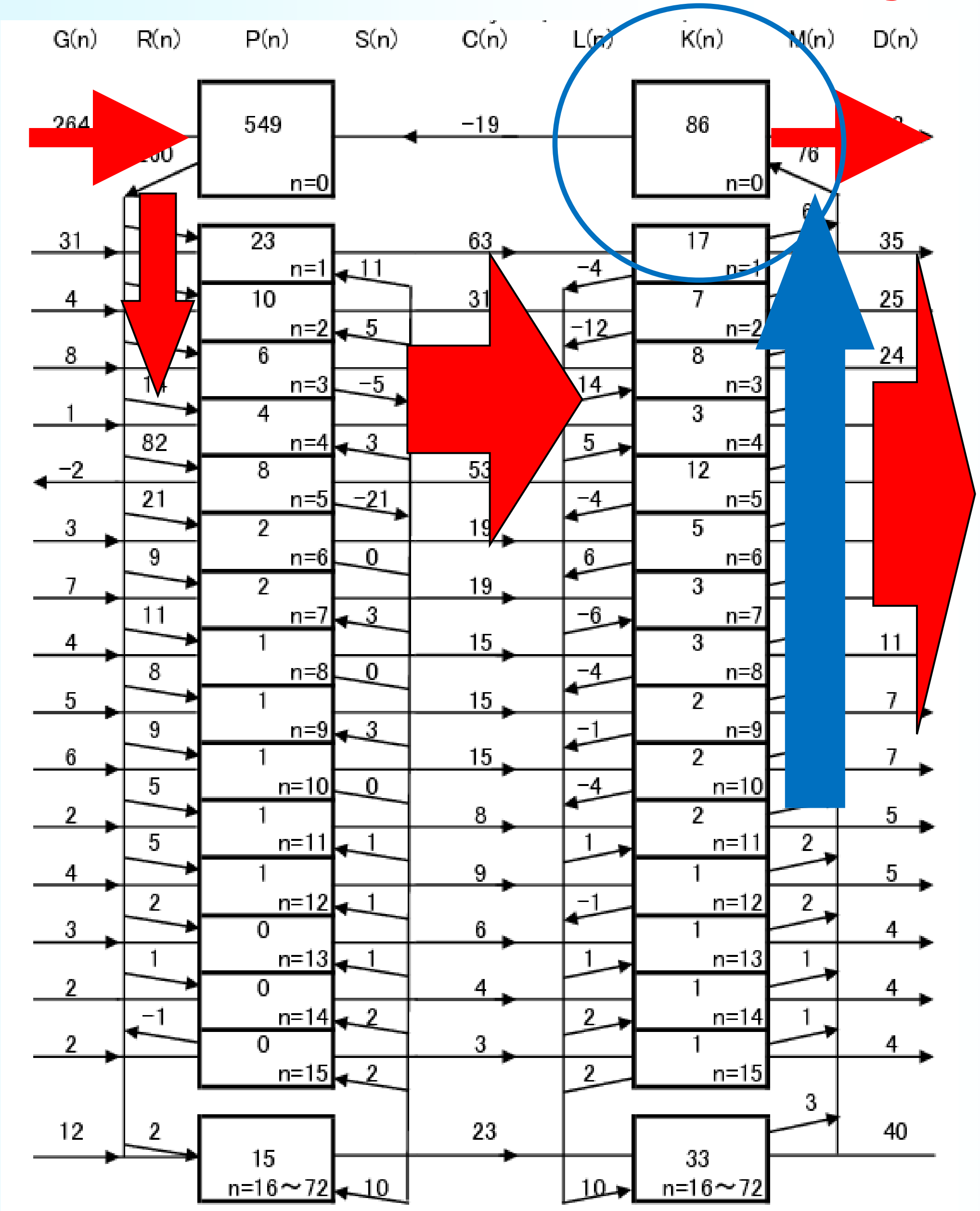
K(0) $$\frac{\partial Kz}{\partial t} = \sum_{n=1}^{N} M(n) + C(0) - D(0),$$

K(n) $$\frac{\partial K(n)}{\partial t} = -M(n) + L($$

P(0) $$\frac{\partial Pz}{\partial t} = -\sum_{n=1}^{N} R(n) -$$

P(n) $$\frac{\partial P(n)}{\partial t} = R(n) + S(n)$$

Saltzman cycle



G: Generation of P(n)

P: Available potential energy

R: zonal-wave interaction of P(n)

S: wave-wave interaction of P(n)

C: Baroclinic conversion

from P(n) to K(n)

K: Kinetic energy

M: zonal-wave interaction of K(n)

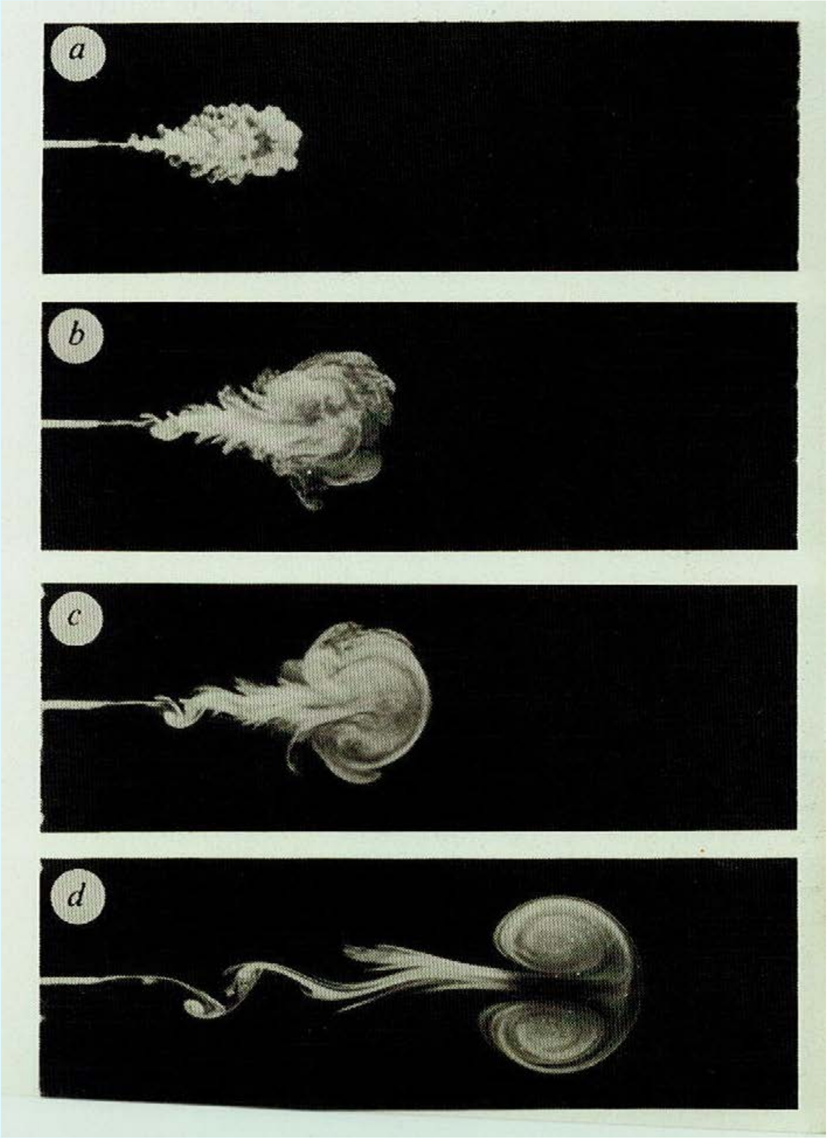
L: wave-wave interaction of K(n)

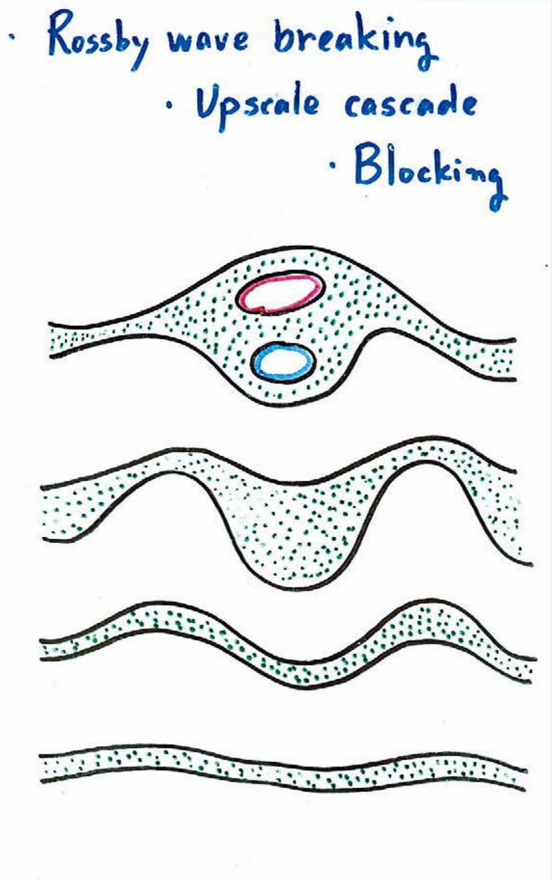
D: Dissipation of K(n)

(Saltzman, 1957 & 1970)

(Kung and Tanaka 1983)

Inverse Energy Cascades





2D Spectral model

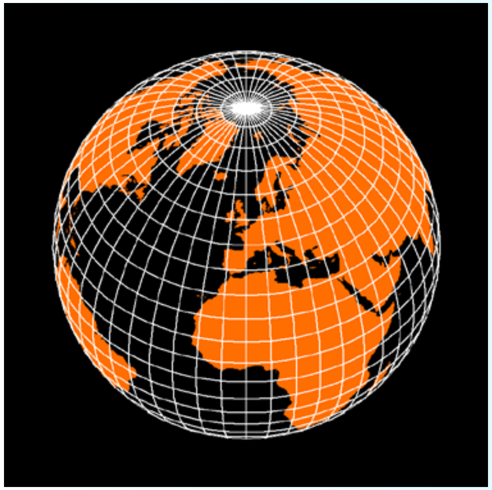
- 1D: Expansion in Fourier harmonics
- 2D: Expansion in spherical harmonics

$$p = \sum_{n=-\infty}^{\infty} p_n \exp(inx)$$

$$\frac{\partial p}{\partial x} = \sum_{n=-\infty}^{\infty} in\, p_n \exp(inx)$$

$$Y_l^n(\lambda, \theta) = P_l^n(\theta) \exp(in\lambda)$$

$$p(\lambda, \theta) = \sum_{n=-N}^{N} \sum_{l=|n|}^{L} p_{nl} Y_l^n(\lambda, \theta)$$



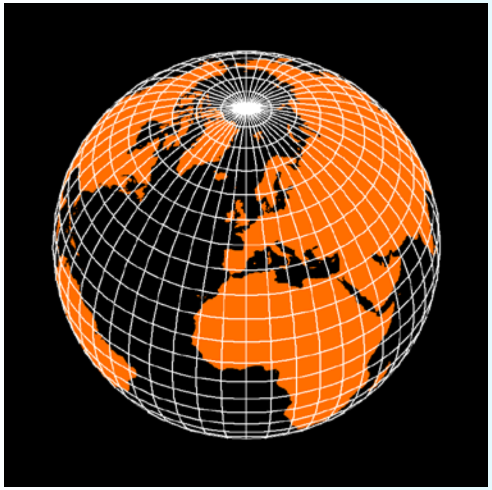
3D Spectral model

- Vertical normal mode
- Horizontal normal mode: Hough harmonics

Expansion in 3D Normal Mode Functions

$$\Pi_{nlm}(\lambda,\theta,\sigma) = \Theta_{nlm}(\theta)\,\mathrm{G}_{\mathrm{m}}(\sigma)\exp(in\lambda)$$

$$U(\lambda,\theta,\sigma) = \sum_{n=-N}^{N}\sum_{l=0}^{L}\sum_{m=0}^{M} w_{nlm} X_m \Pi_{nlm}(\lambda,\theta,\sigma)$$

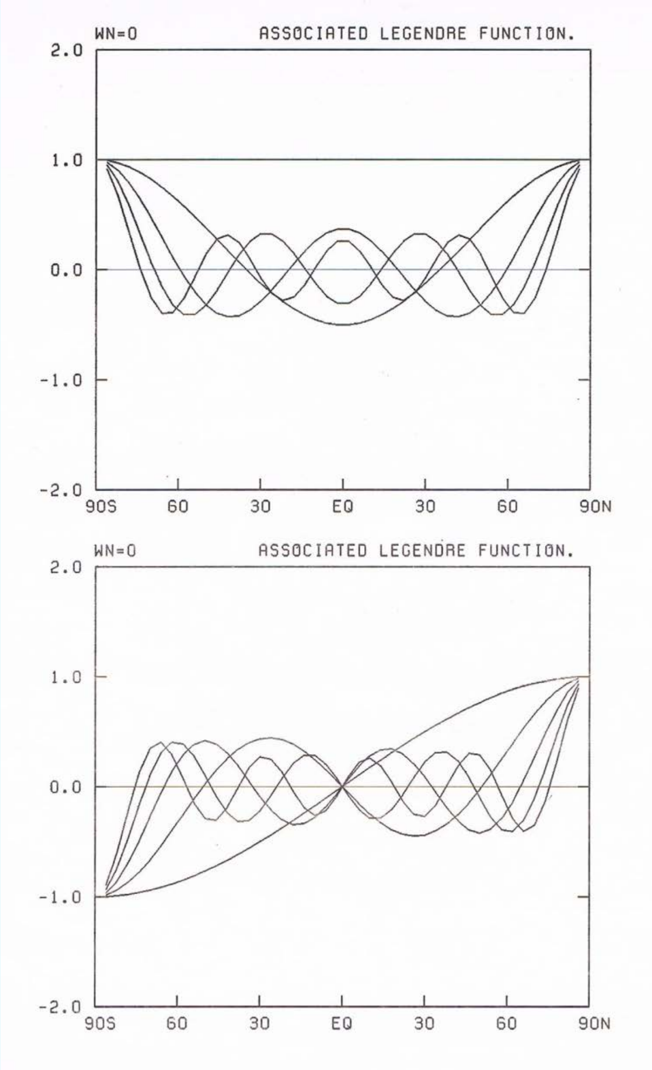


$$\frac{dw_i}{d\tau} = -i\sigma_i w_i - i\sum_{jk} r_{ijk} w_j w_k + f_i, \quad i = 1,2,3,\ldots$$

$$E_i = \frac{1}{2} p_s h_m |w_i|^2, \quad w_{nlm} \rightarrow w_i$$

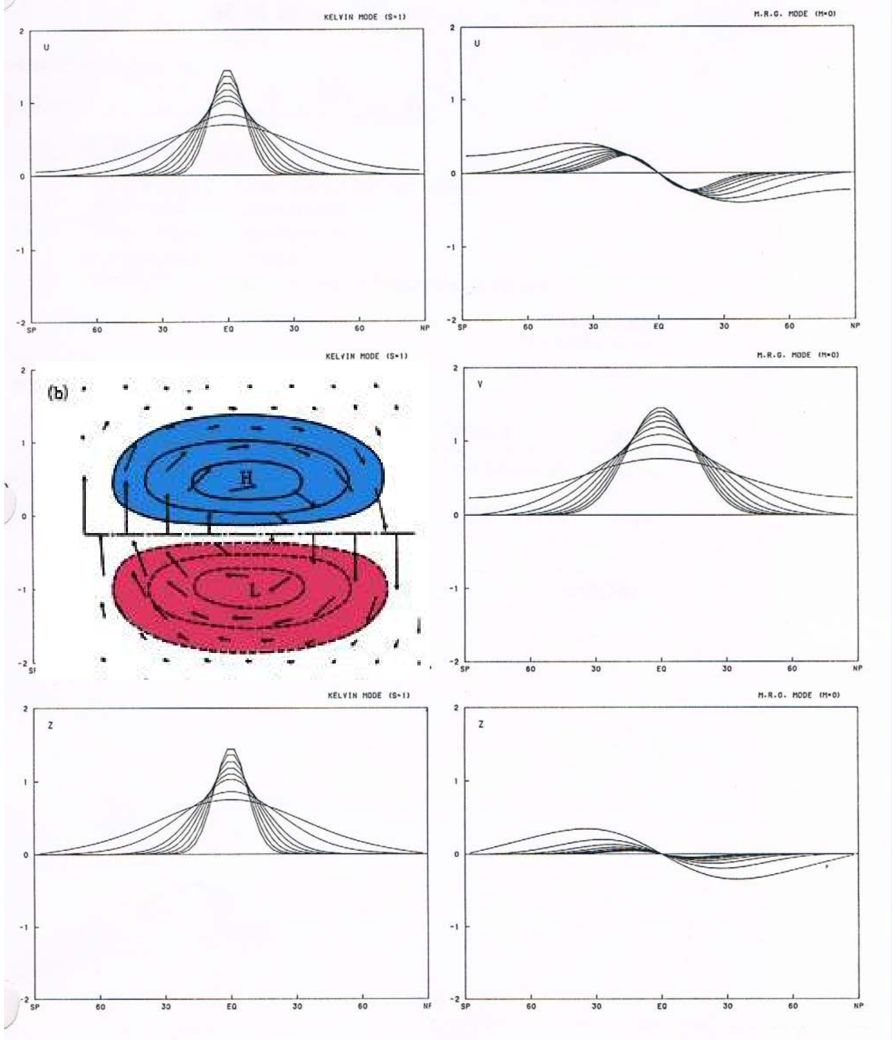
Spherical harmonics (n=0)

Hough harmonics



Kelvin mode

Mixed Rossby-gravity mode



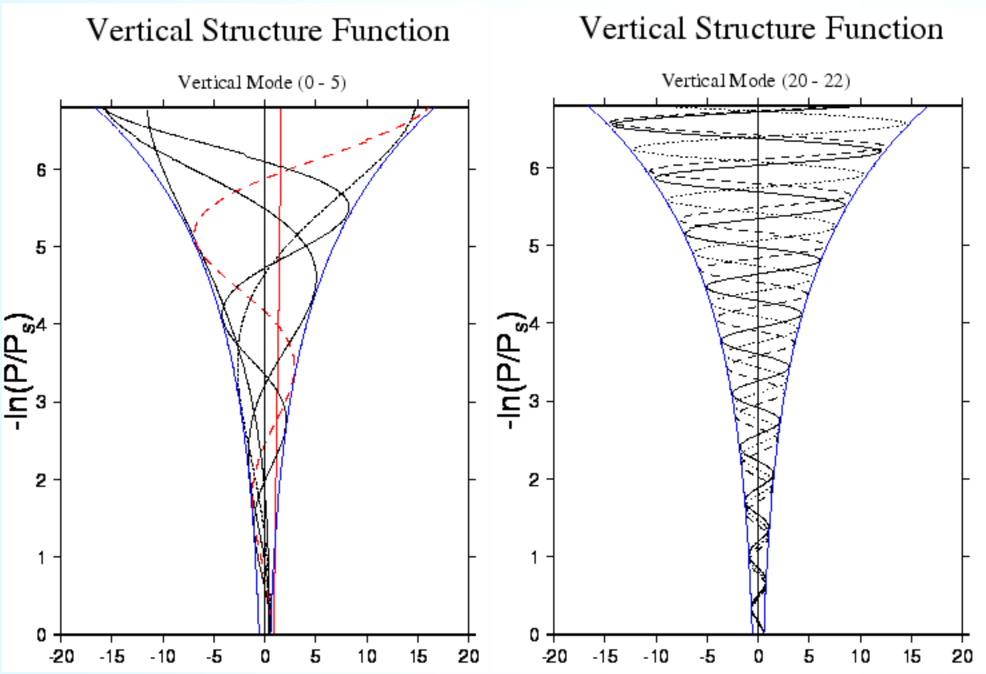
Vertical energy spectrum

Vertical modes $$\frac{\partial}{\partial\sigma}\left(\sigma^2\frac{\partial G_m}{\partial\sigma}\right)+\lambda_m G_m=0,\quad \lambda_m=\frac{R\gamma}{gh_m}$$

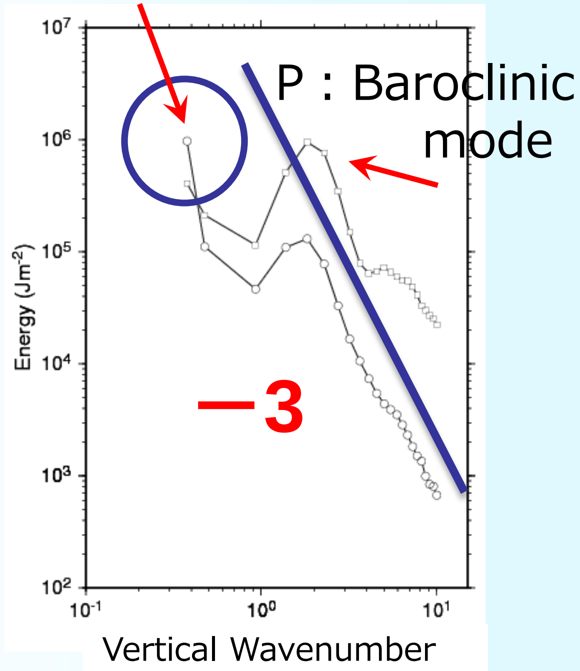
$$G_0(\sigma)=C_1\sigma^{r_1}+C_2\sigma^{r_2}$$

Barotropic and baroclinic modes

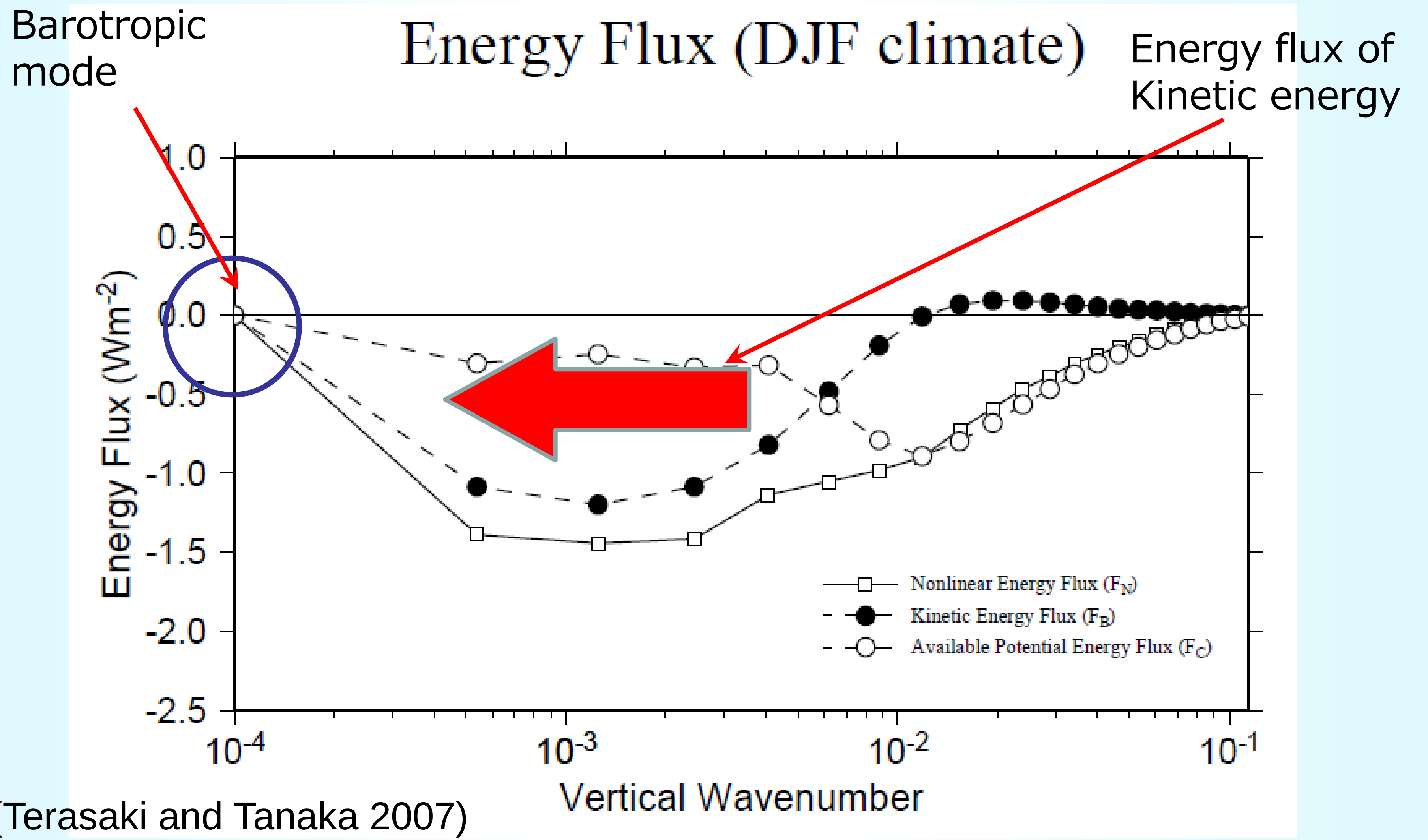
$$G_m(\sigma)=\sigma^{-\frac{1}{2}}\left(C_1\sin(\mu\ln\sigma)+C_2\cos(\mu\ln\sigma)\right)\qquad \mu=\sqrt{\lambda_m-\frac{1}{4}}$$



Terasaki and Tanaka (2007)



Barotropization
by baroclinic instability



(Terasaki and Tanaka 2007)

3D Normal Mode Energetics

Zonal Energy Spectrum *n*

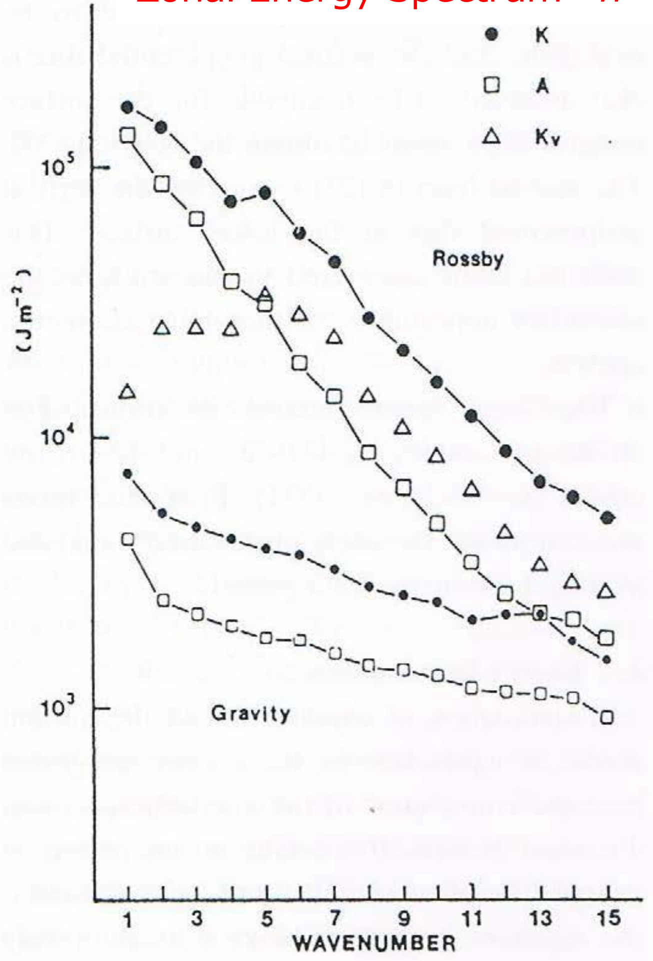


Fig. 2. Energy distributions in the wavenumber domain. *K* : kinetic energy, A : available potential energy, K_v : *v*-component of *K*

Vertical Energy Spectrum *m*

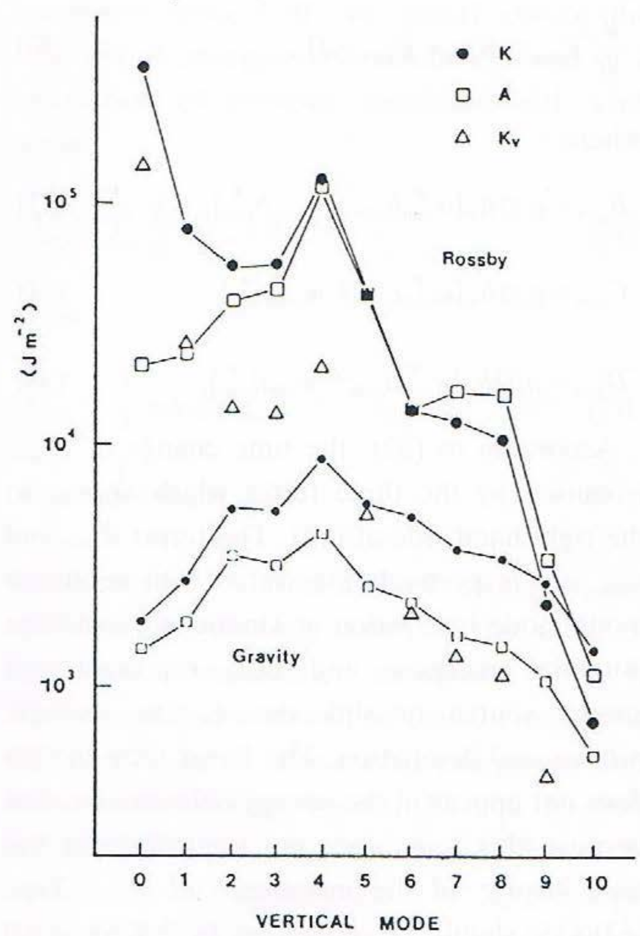
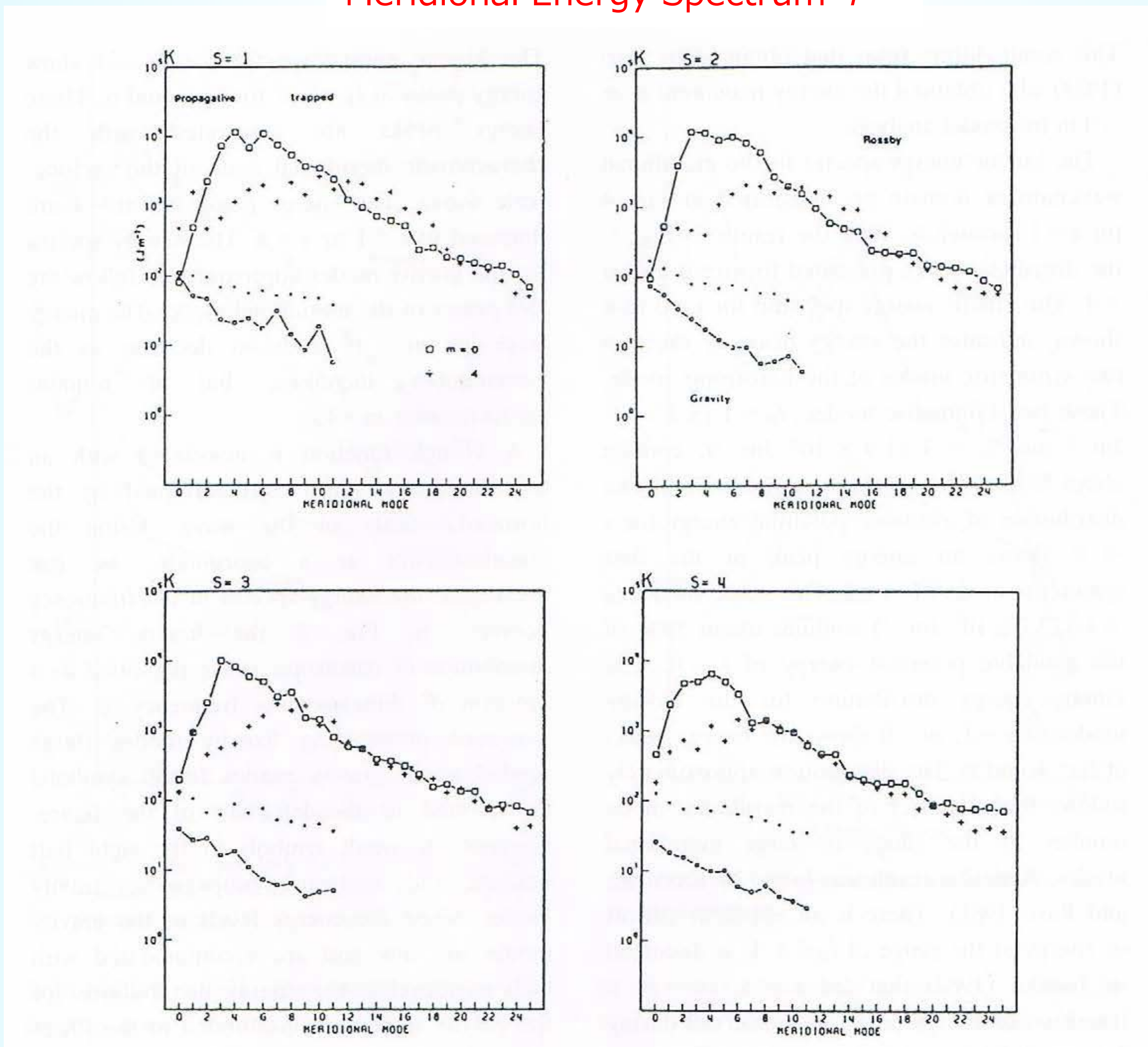


Fig. 3. Eddy energy distributions in the vertical mode domain.

Tanaka (1985)

3D Normal Mode Energetics

Meridional Energy Spectrum /



Tanaka (1985)

Energy spectrum in the 3D wavenumber space

$$c = -\frac{\beta}{n^2 + l^2 + m^2} = -\frac{\beta}{k^2}$$

n, l, m : zonal, meridional and vertical waves

k : total wave

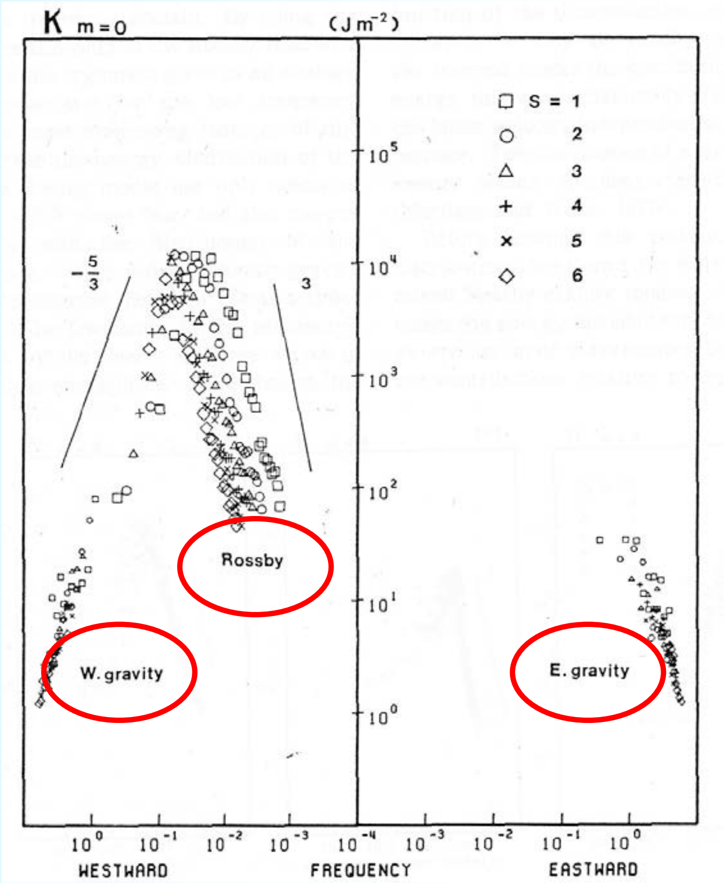
$$c = \sigma / n$$

$$\frac{dw_i}{d\tau} = -i\sigma_i w_i - i\sum_{jk} r_{ijk} w_j w_k + f_i$$

Use c for the scale in place of 3D wavenumber

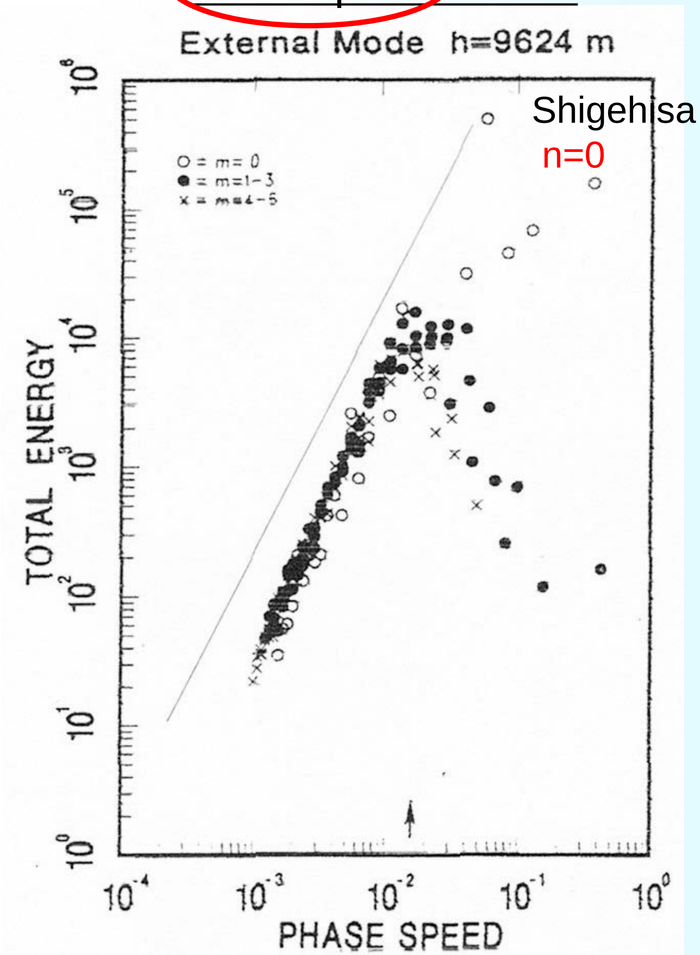
Observed energy spectrum in *c*-domain

Frequency domain



Tanaka (1985)

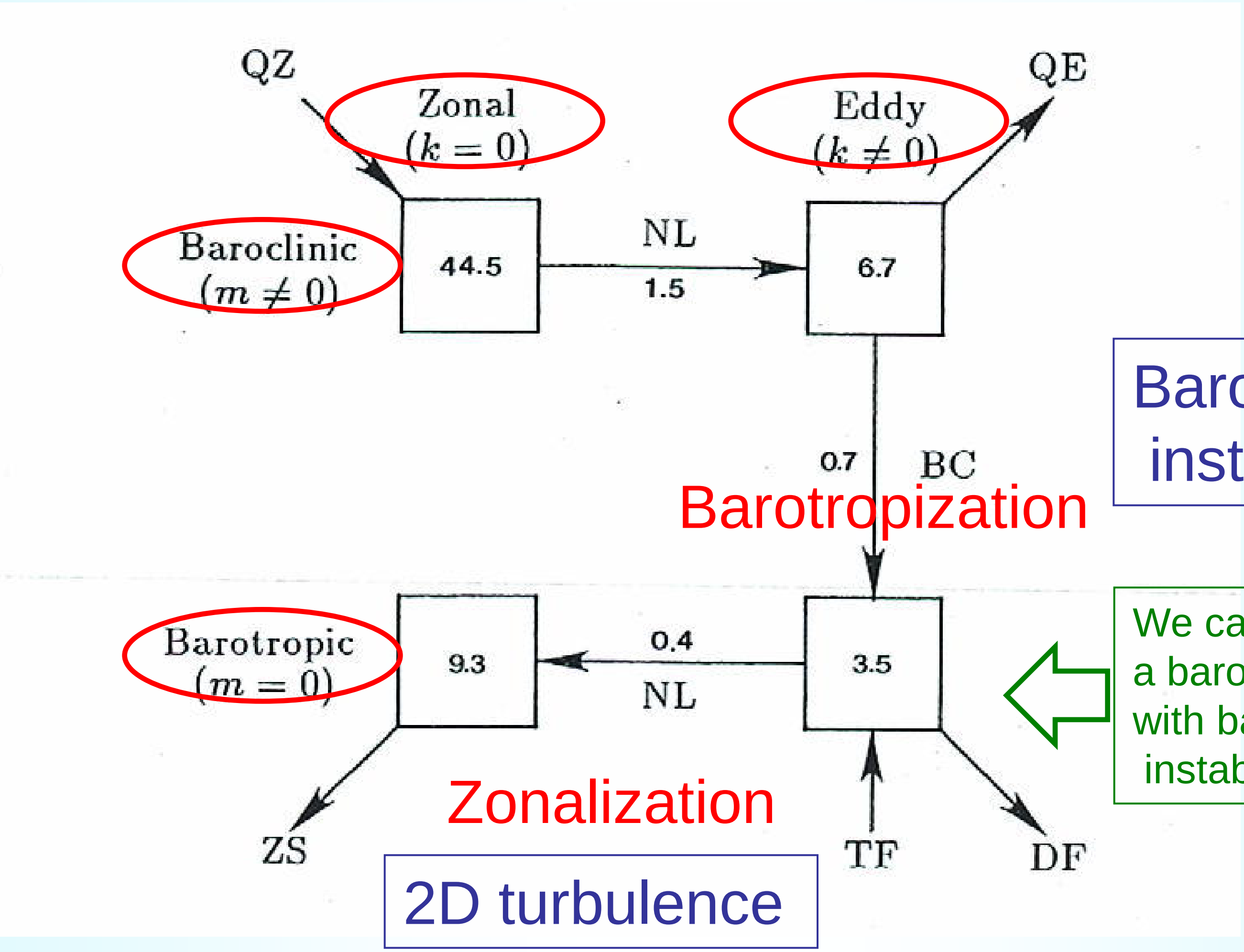
Phase speed domain



Tanaka and Kasahara (1992)

Energy flow box diagram
in barotropic-baroclinic decomposition

Tanaka Cycle



Baroclinic instability

We can construct a barotropic model with baroclinic instability

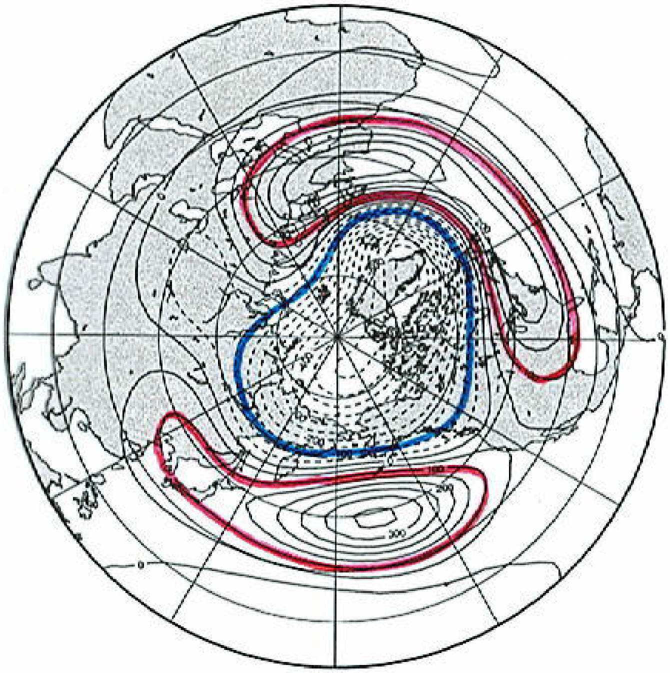
Arctic Oscillation

Barotropic height (EOF-1)

NCEP/NCAR

Barotropic Component of Geopotential Height

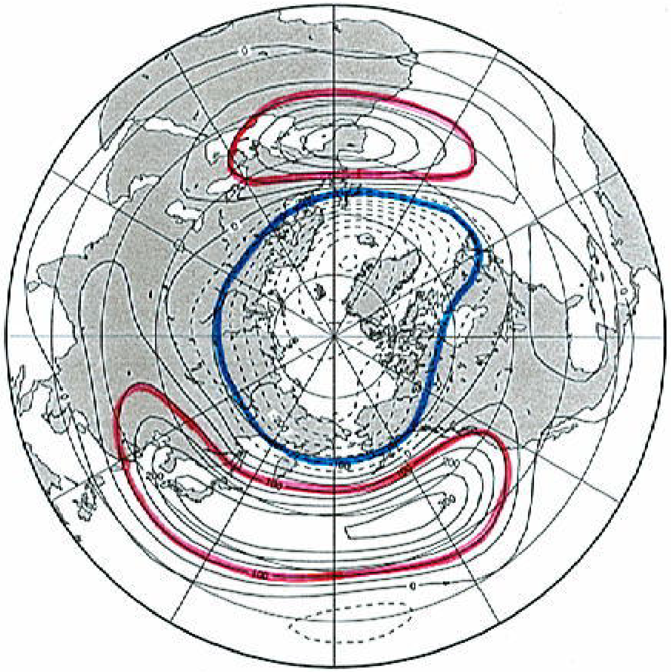
EOF-1 AO (5.7%)



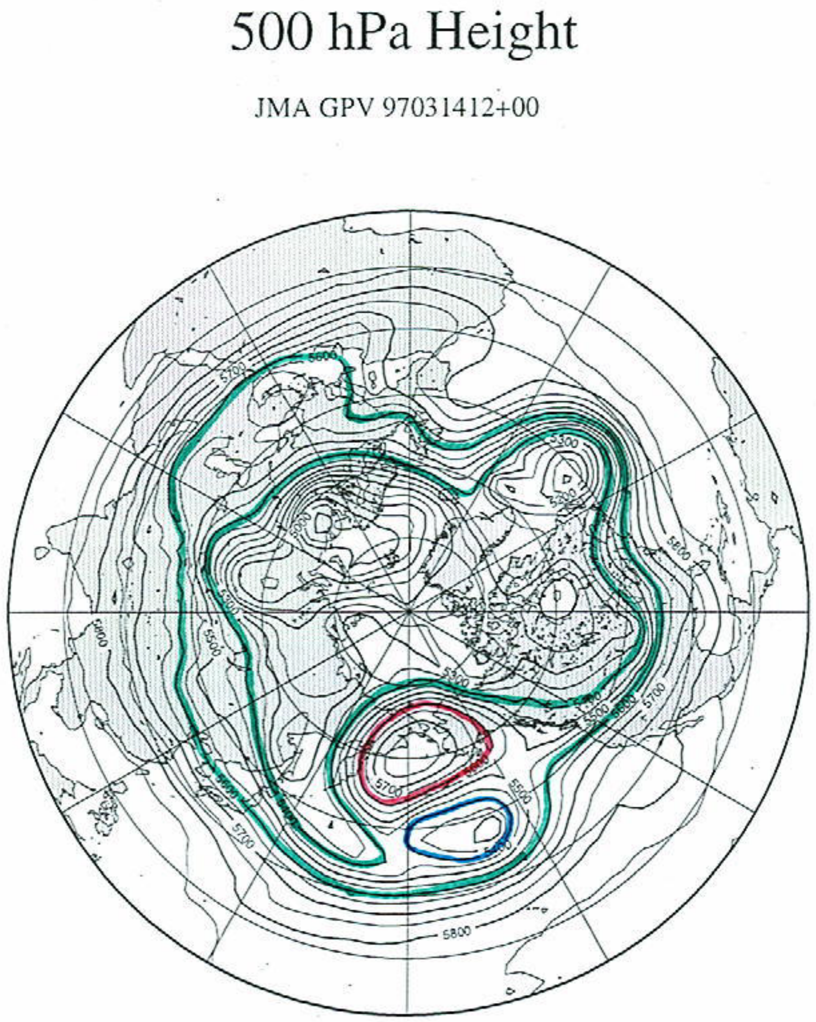
Barotropic S-Model

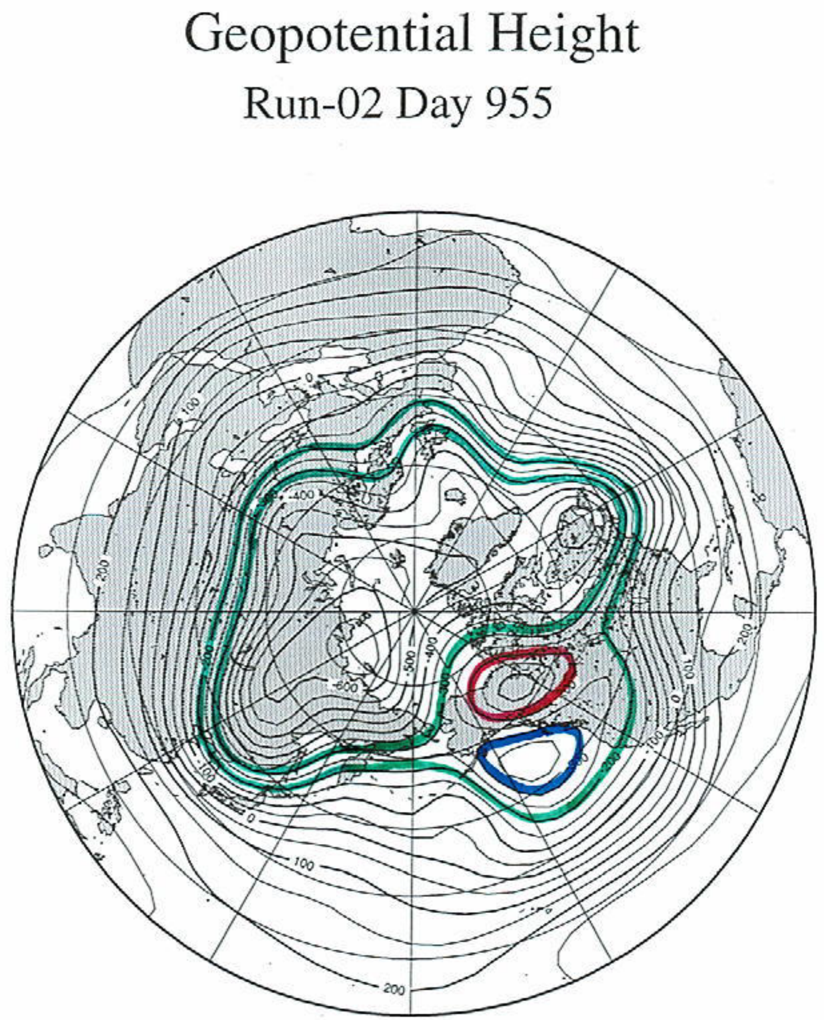
Barotropic Height

EOF-1 (16%)

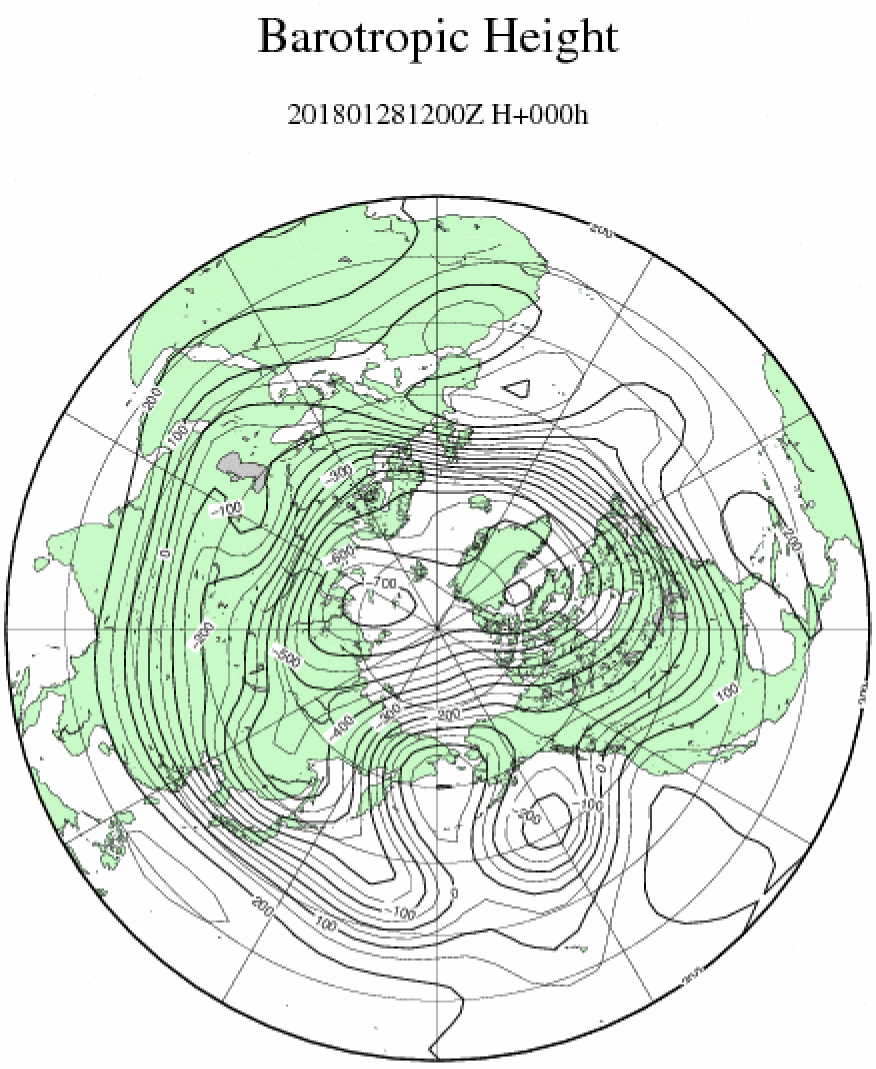


Blocking in the model

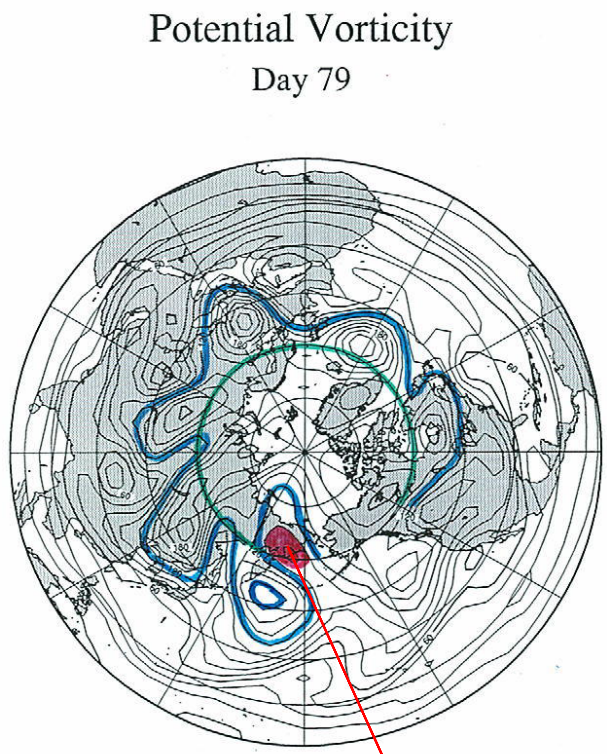


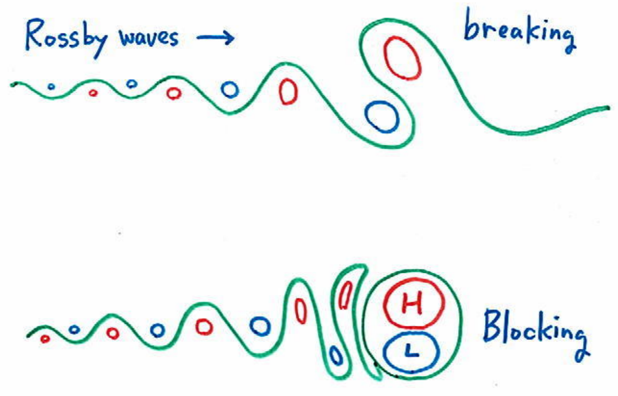


Blocking on 30 January 2018

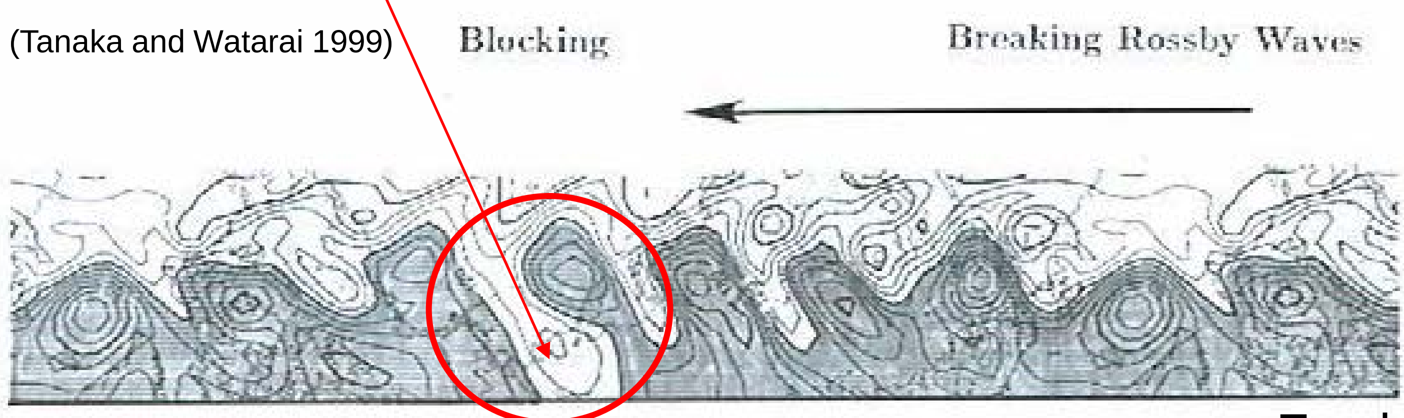


Blocking formation by Rossby wave breaking





(Tanaka and Watarai 1999)

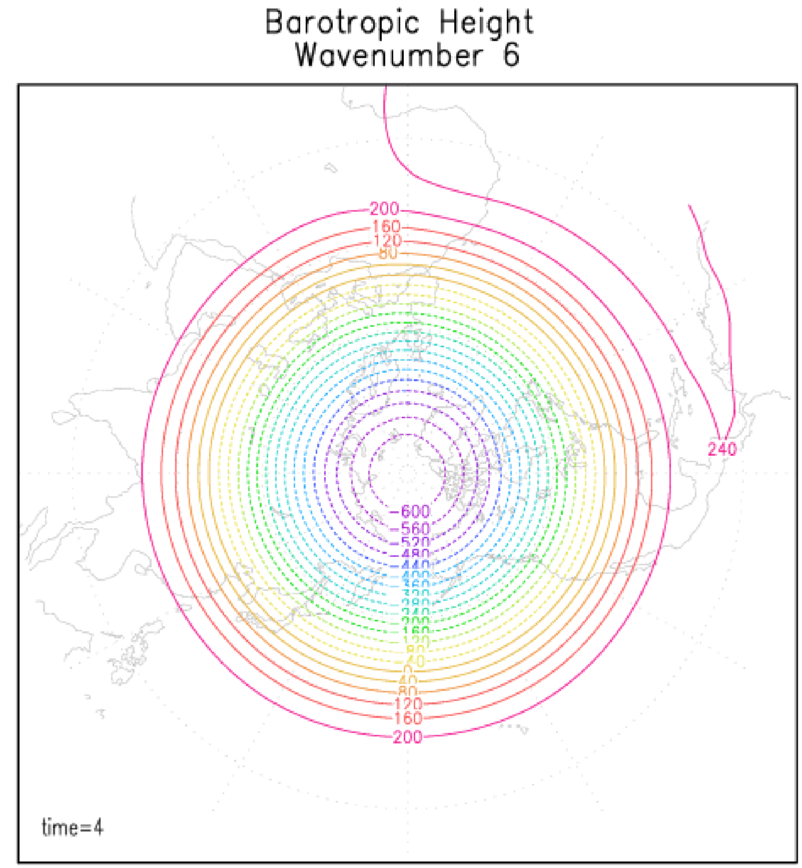


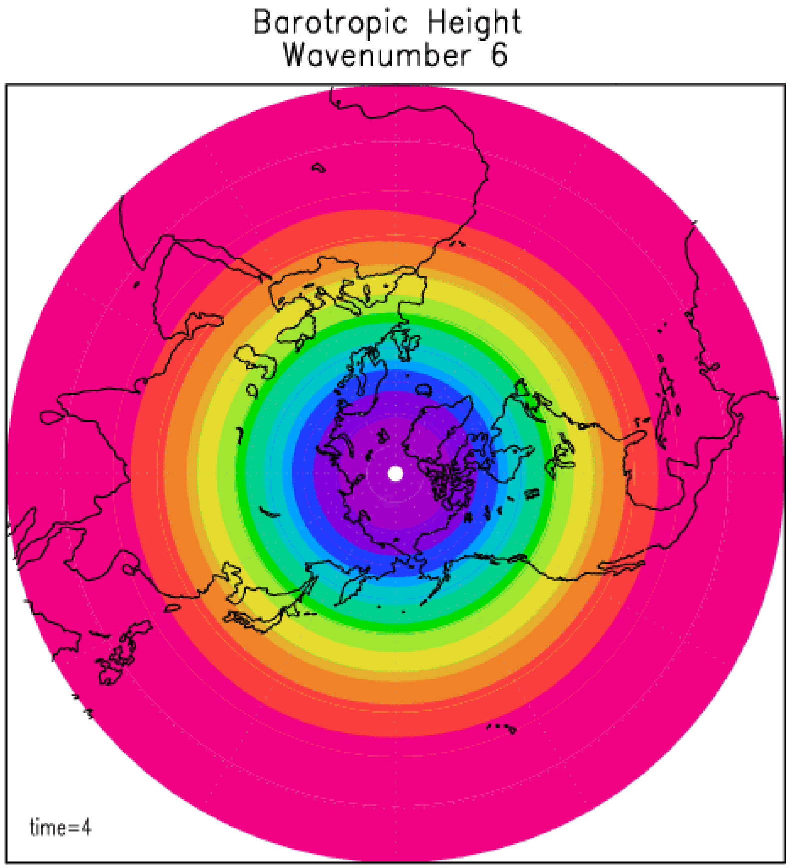
Tanaka (1998)

Zonalization

Rossby wave breaking for n=6

Growthrate × 1.7





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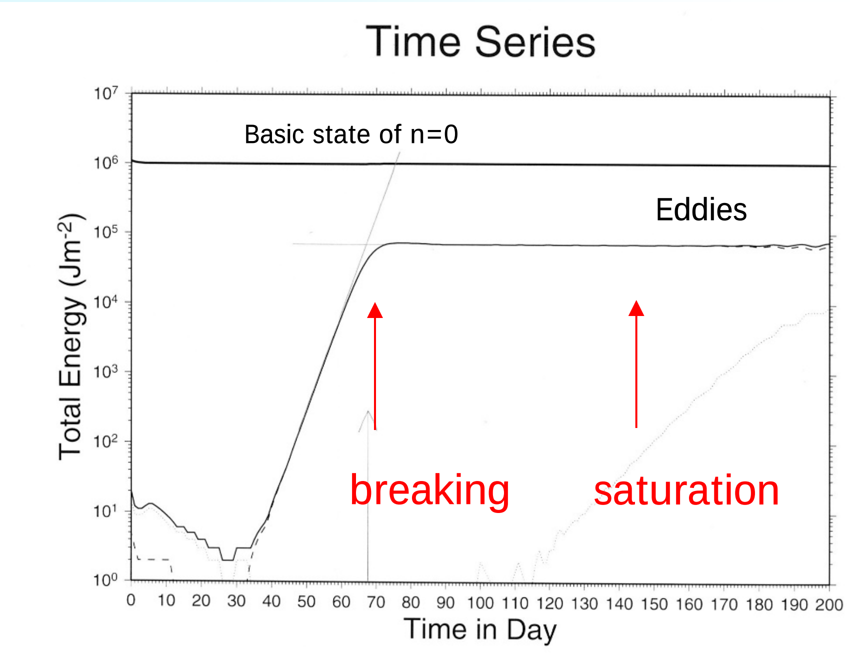
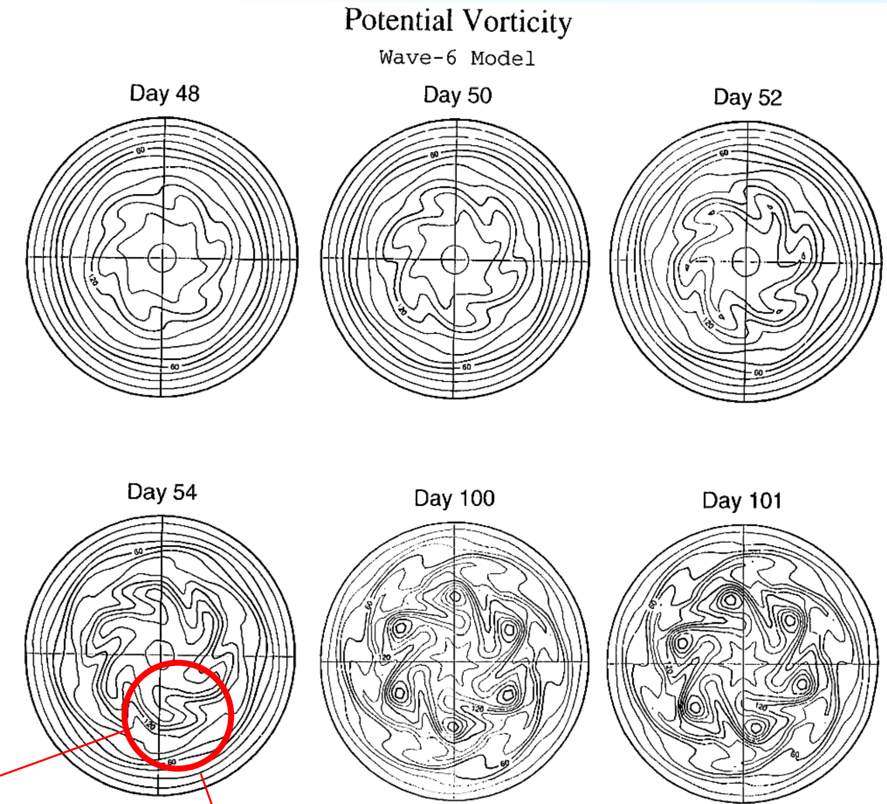
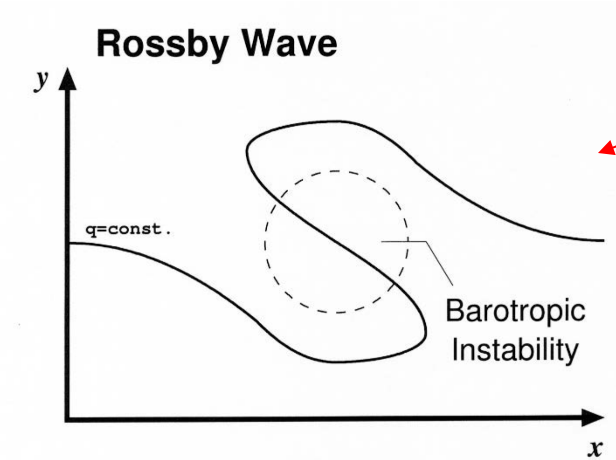
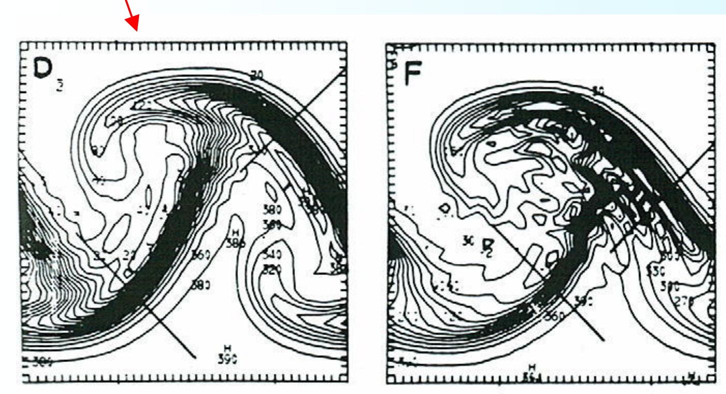
Rossby wave breaking and saturation





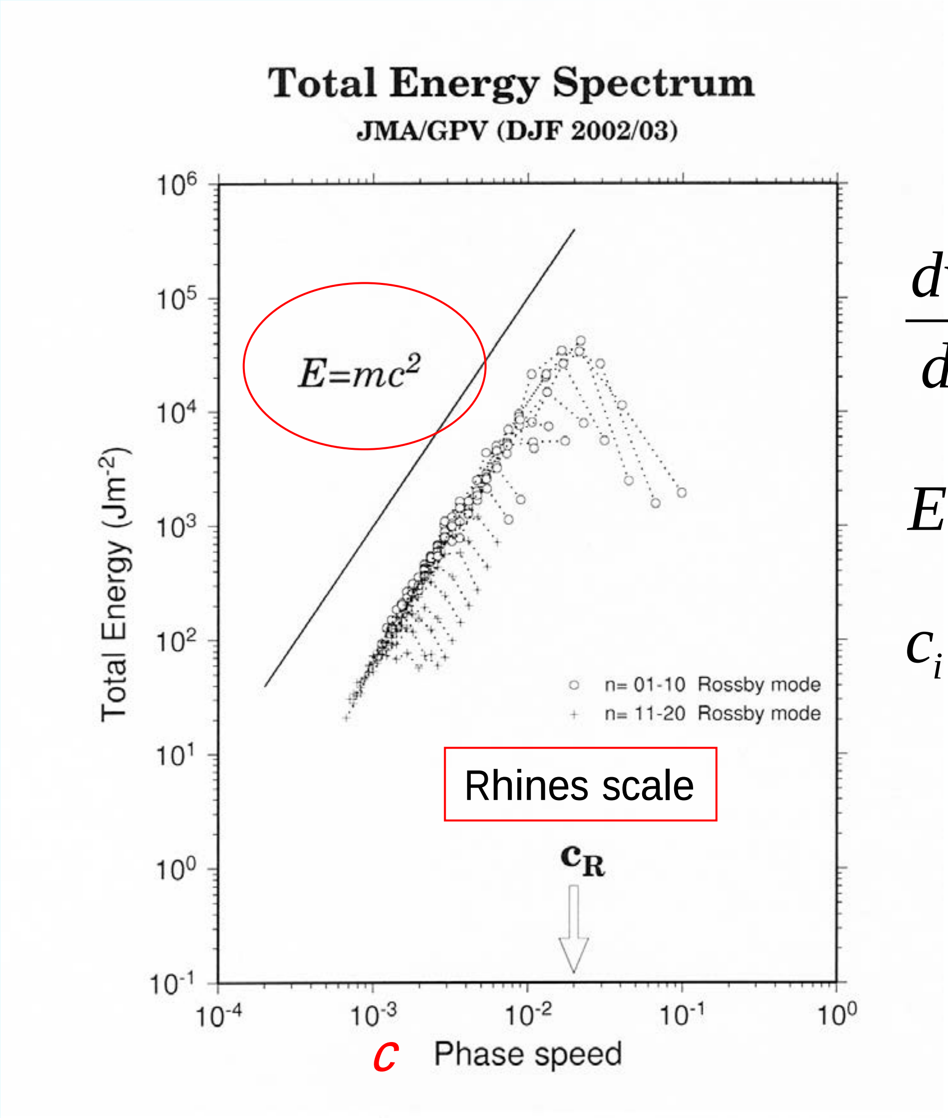


Tanaka and Watarai (1999)



Mudrick (1974)

3D energy spectrum



By 3D normal mode expansion

$$\frac{dw_i}{d\tau} = -i\sigma_i w_i - i\sum_{jk} r_{ijk} w_j w_k + f_i$$

$$E_i = \frac{1}{2} p_s h_0 \, | w_i |^2$$

$$c_i = \sigma_i / n$$ Phase speed

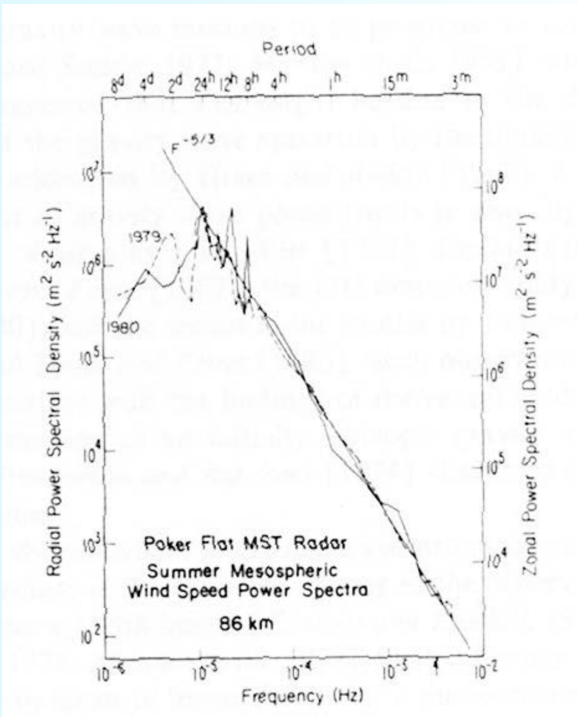
$$E = mc^2$$ **?**

Tanaka et al. (2004 GRL)

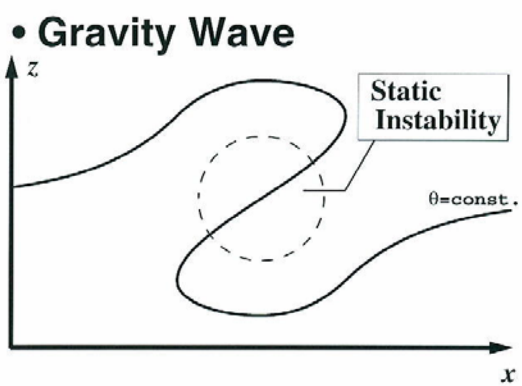
Saturation theory of gravity waves

Saturation spectrum

Breaking gravity waves

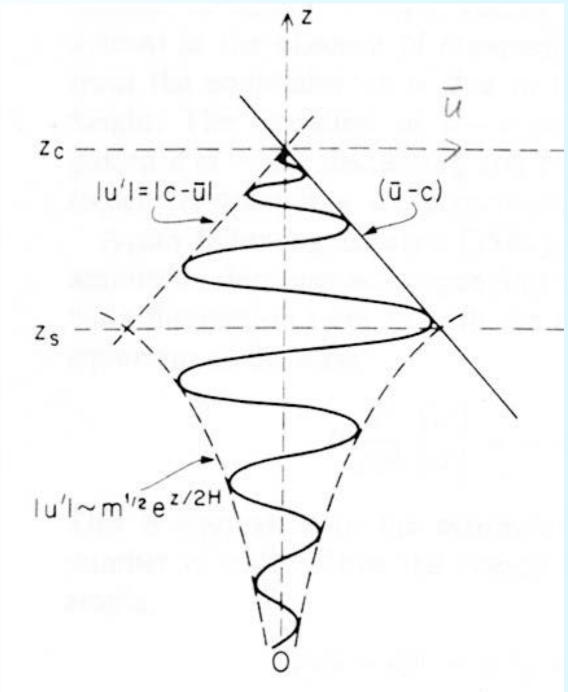


-3/5 power



$$\frac{\partial \theta}{\partial z} < 0$$

Breaking condition



Fritts (1984)

Saturation theory of Rossby waves

$$\frac{\partial q}{\partial y} < 0, \qquad q = \nabla^2 \psi + f$$

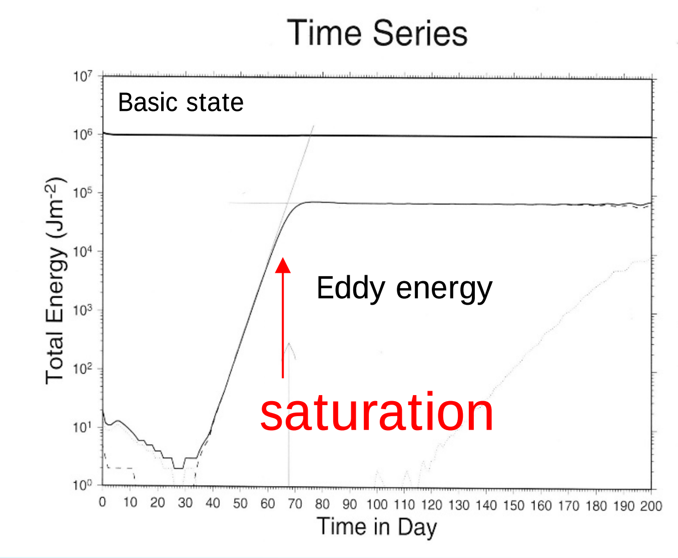
Tanaka and Watarai (1999)

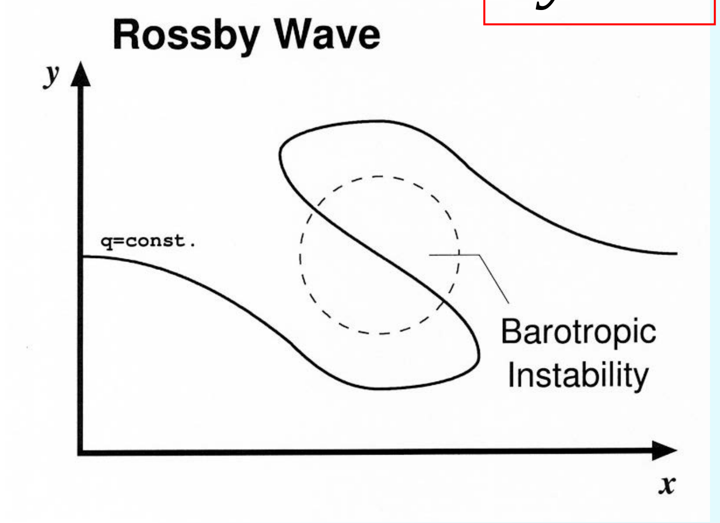
PV in barotropic model

$$\frac{\partial}{\partial y}(\nabla^2 \psi + f) = -\nabla^2 u + \beta \quad < 0$$

$$u < -\frac{\beta}{n^2 + l^2} = c$$

Breaking condition $\frac{\partial q}{\partial y} < 0$





Saturation energy spectrum

$$u < -\frac{\beta}{n^2 + l^2} = c$$

$$|u| \approx |v|$$

(Tanaka and Kasahara 1992)

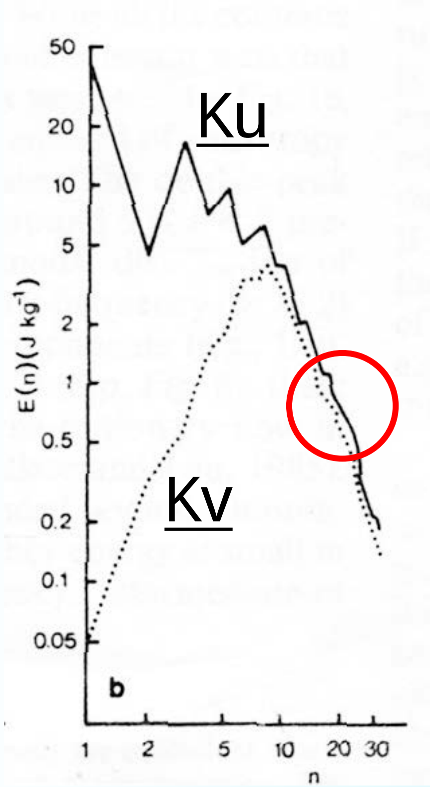
$$E = \frac{1}{g}\int_0^{p_s} \frac{1}{2}(u^2 + v^2)dp$$

$$= \frac{p_s}{g}c^2 = mc^2$$

$$m = p_s / g$$

Mass for unit area

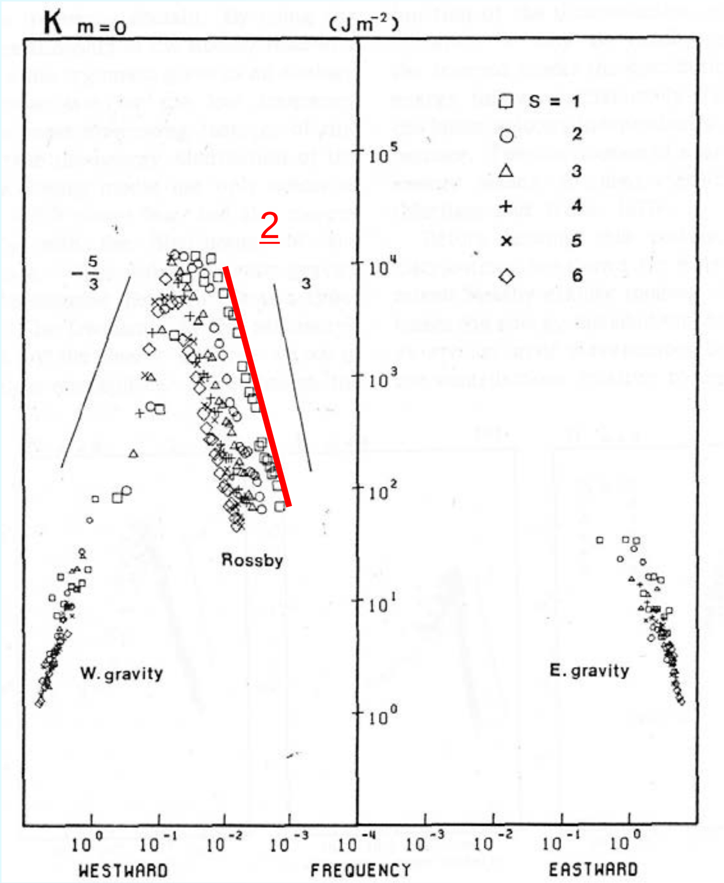
$$\frac{\partial q}{\partial y} < 0 \quad \Rightarrow \quad E = mc^2$$



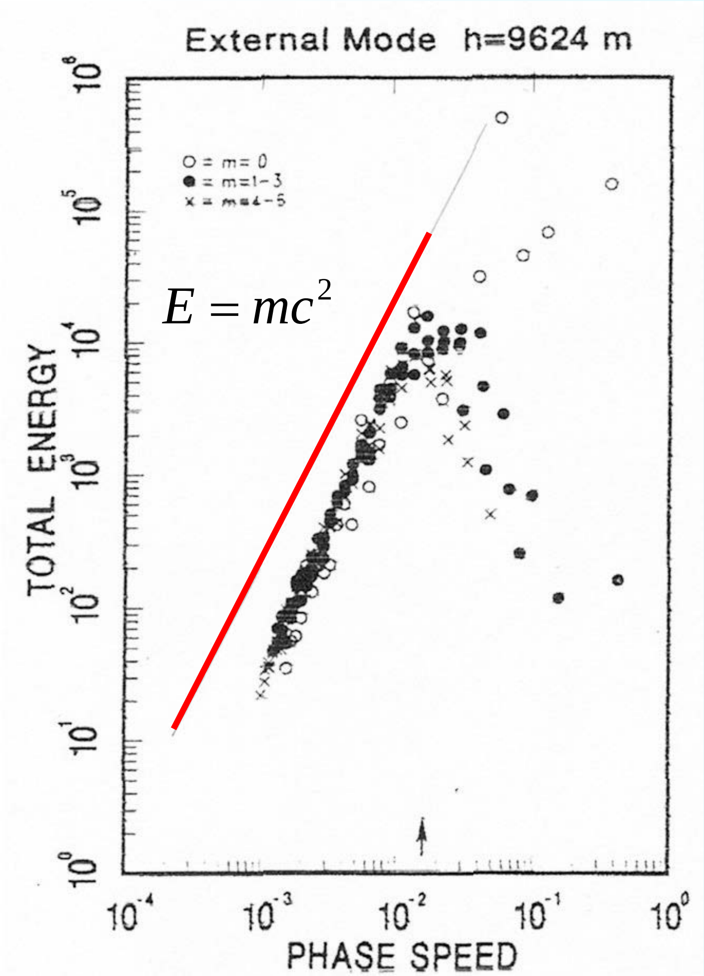
Shepherd (1987)

Observed energy spectrum in *c*-domain

FGGE SOP1



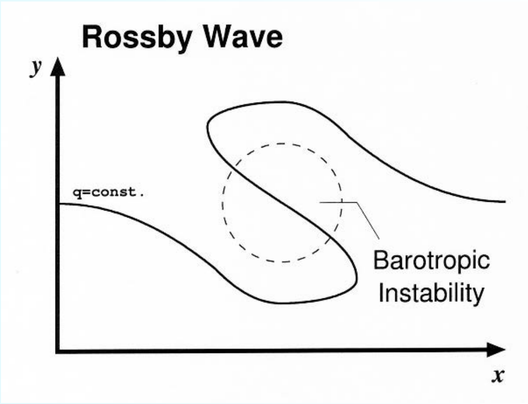
Tanaka (1985)



Tanaka and Kung (1988)

Global energy spectrum of $E = mc^2$

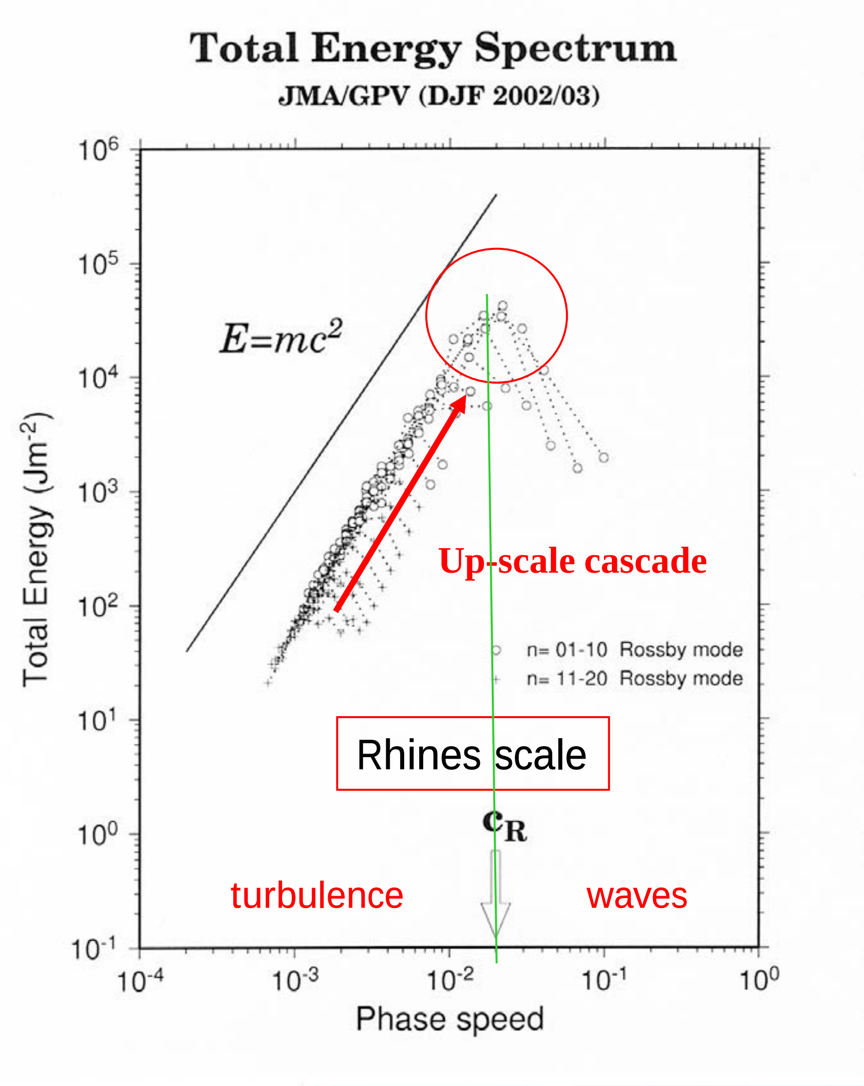
(Tanaka et al. 2004 GRL)



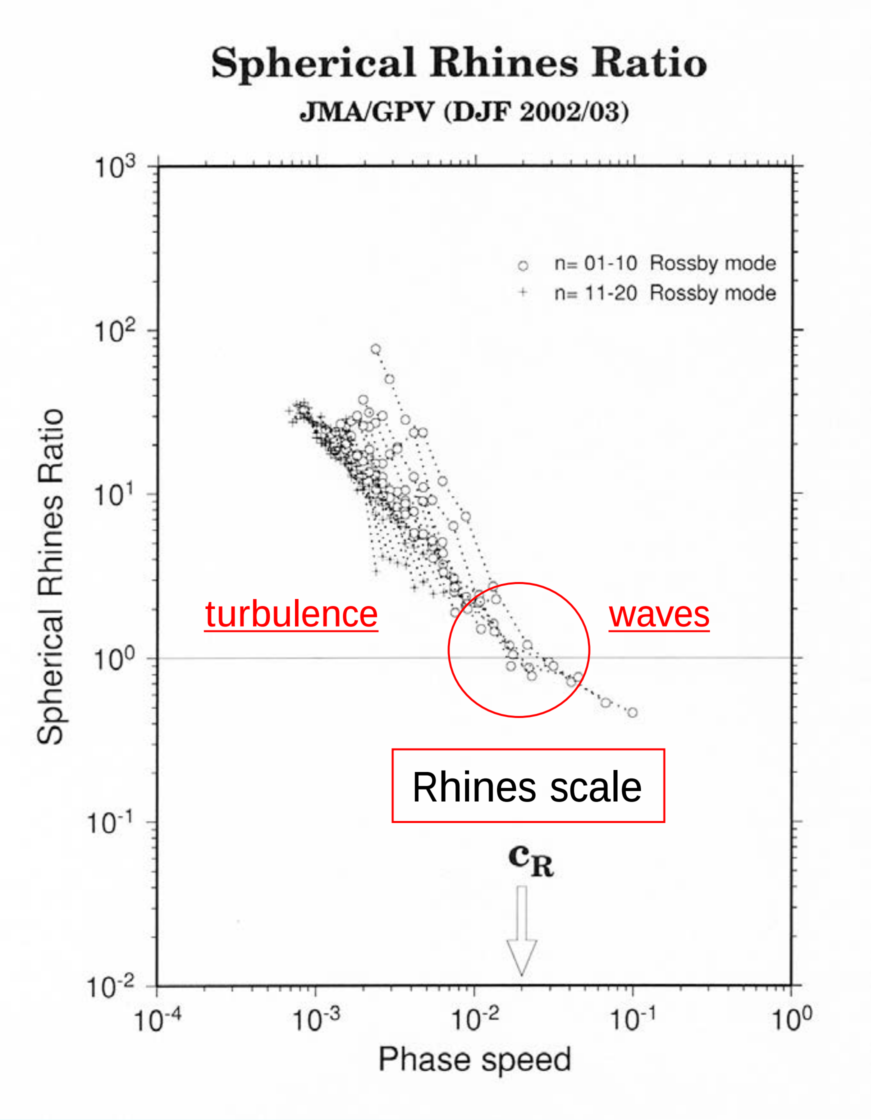
$$\frac{\partial q}{\partial y} < 0 \quad \Rightarrow \quad E = mc^2$$

c Rossby phase speed

$m = p_s / g$ Mass of the air



Rhines sclae on a sphere



$$\frac{dw_i}{d\tau} = -i\sigma_i w_i - i\sum_{jk} r_{ijk} w_j w_k + f_i$$

$$R_i = \frac{|\sum r_{ijk} w_j w_k|}{|\sigma_i w_i|}$$ Rhines ratio

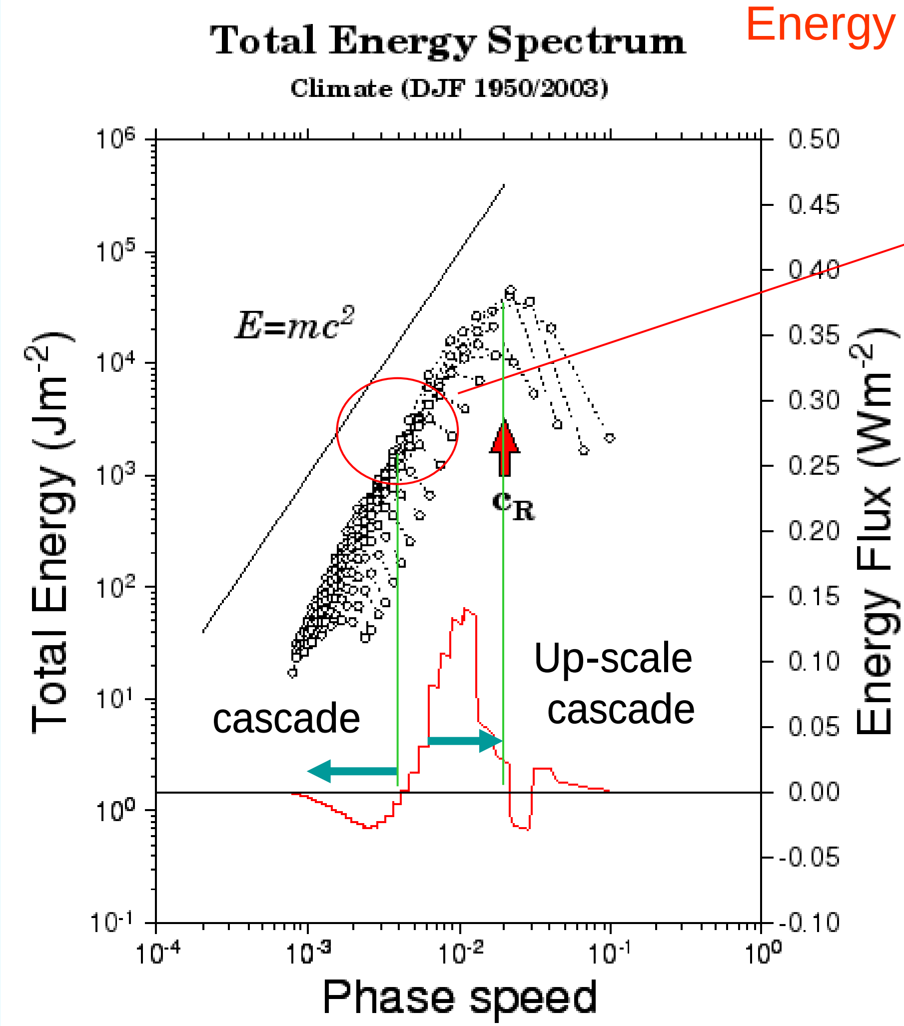
turbulance $R_i > 1$

waves $R_i < 1$

Rhines scale $R_i = 1$

（Tanaka et al. 2004 GRL)

Energy flux in *c*-domain



wave-wave

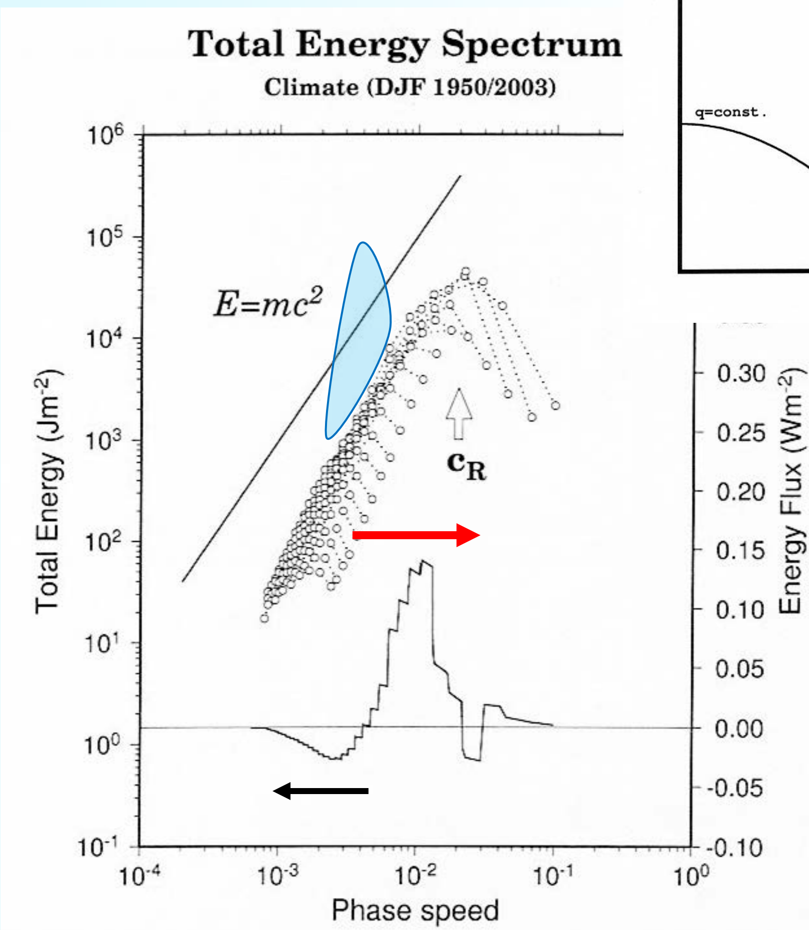
Energy accumulates at phase speed 0

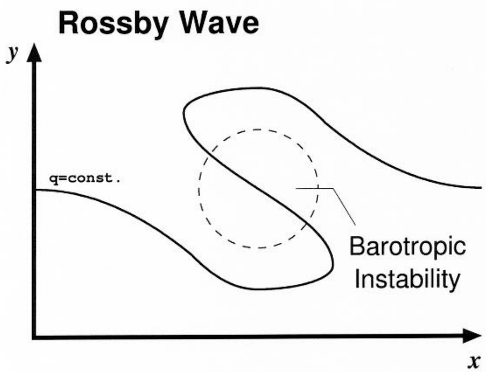
Source of energy

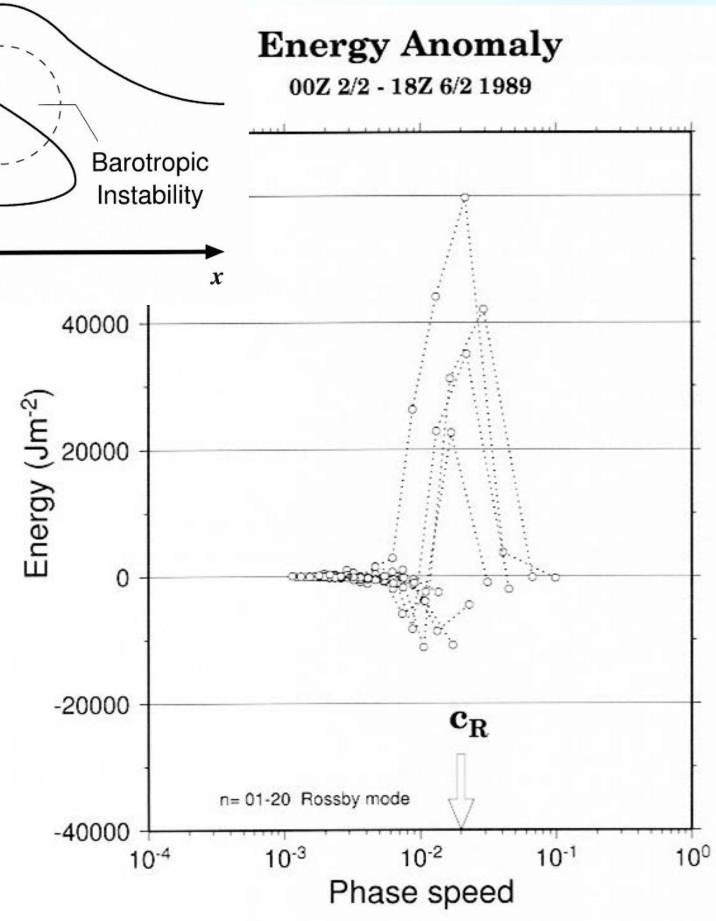
There is an energy source in the middle of the spectrum: inertial subrange theory fails.

Theory of Rossby wave saturation

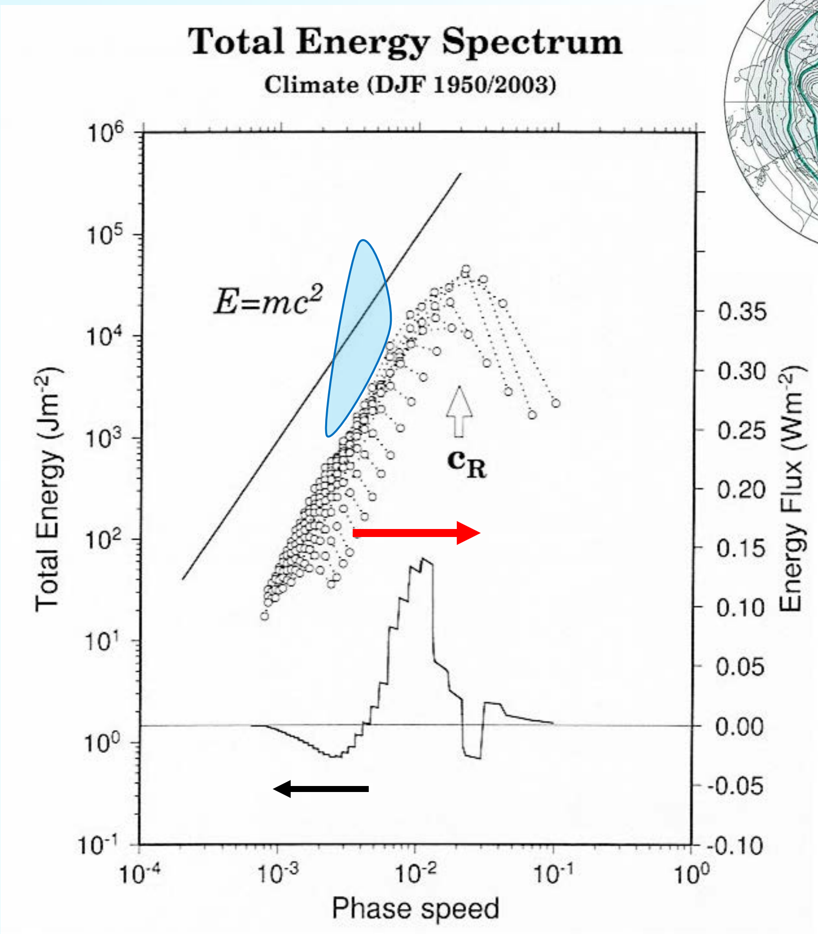
Energy Anomaly during Blocking

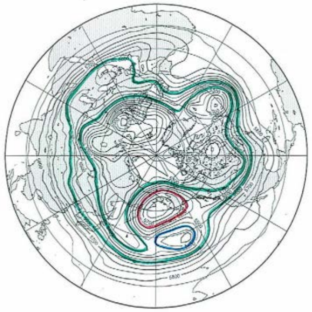


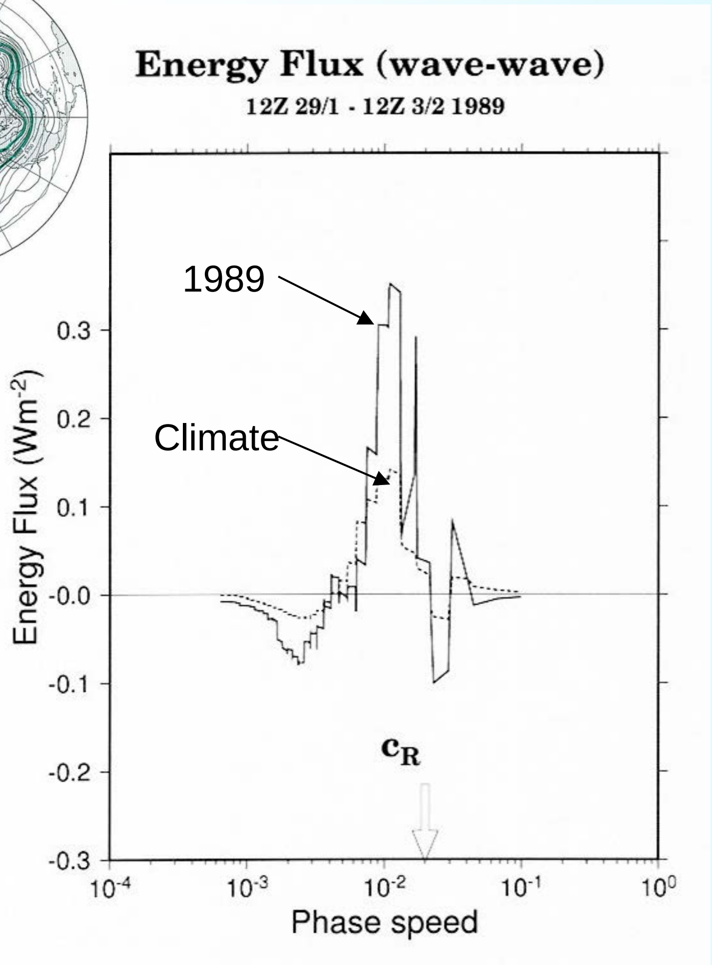




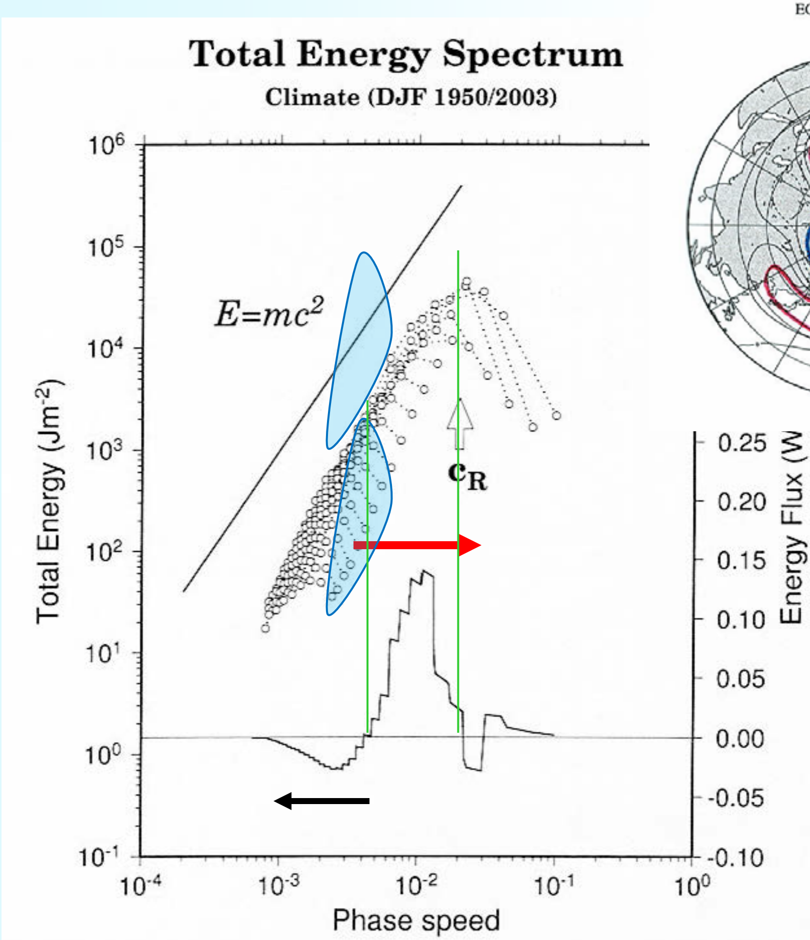
Energy Flux during Blocking

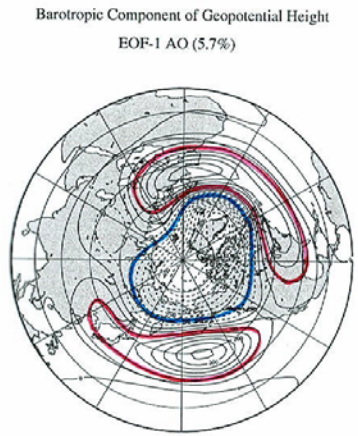


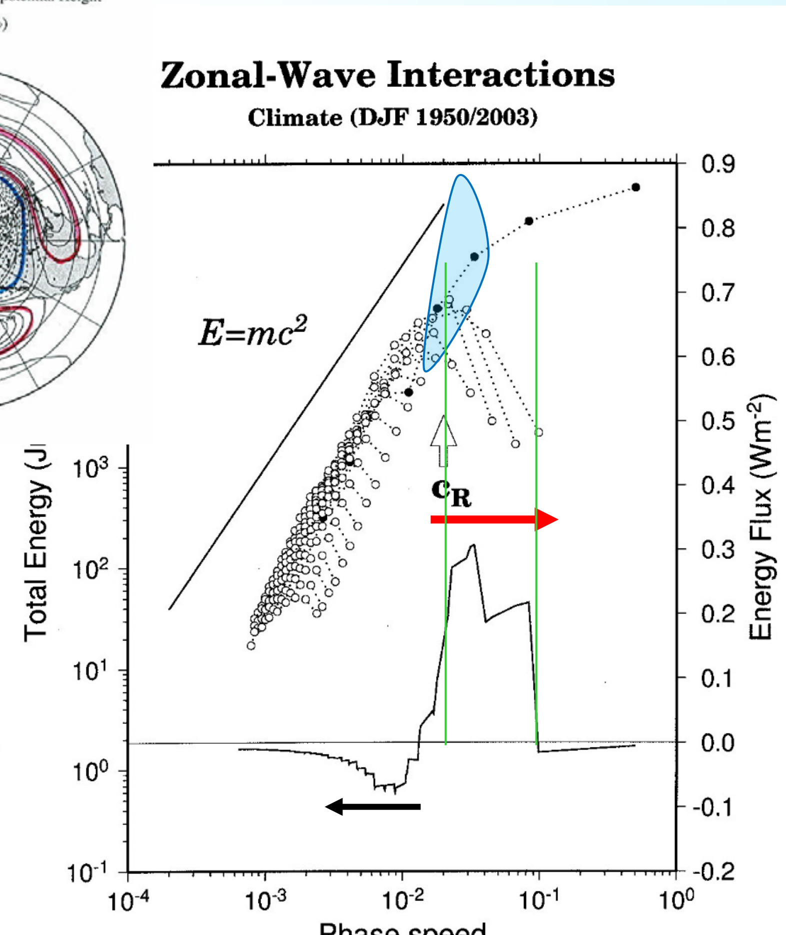




Excitation of blocking and AO by up-scale energy cascade





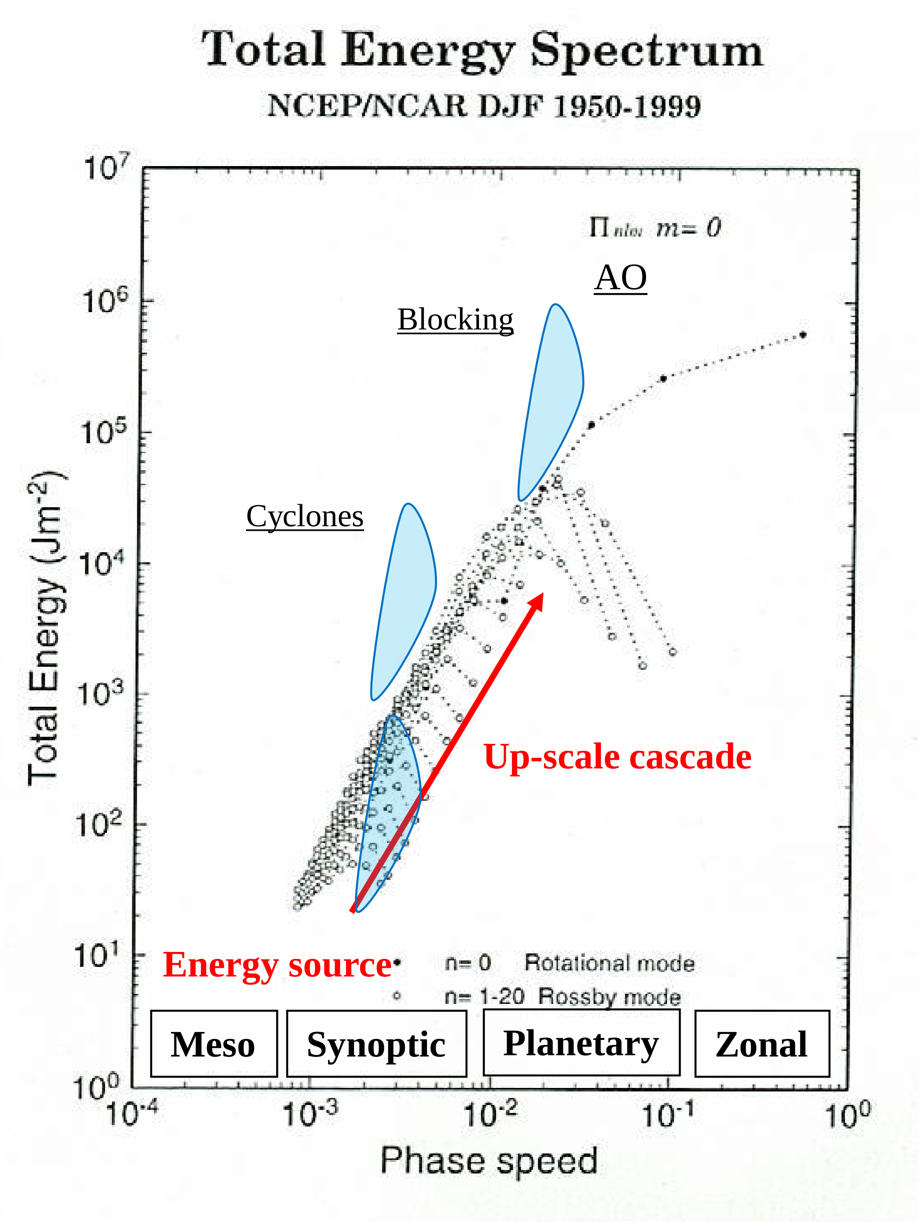


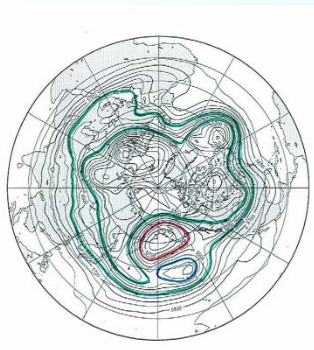
wave-wave

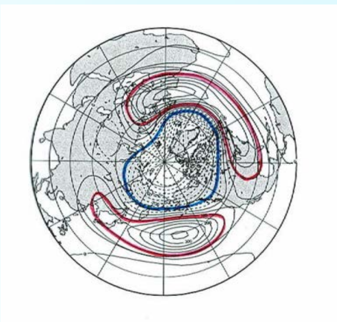
zonal-wave

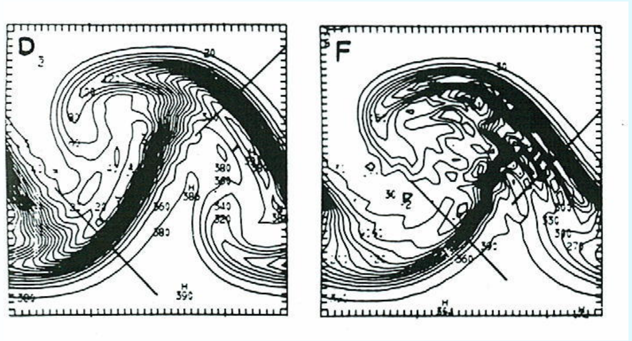
(Tanaka and Terasaki 2004)

Low-frequency variability of the atmosphere









Summary

(1) Energy spectrum is examined in the 3D wavenumber domain using phase speed *c*.

(2) Energy spectrum of $E=mc^2$ is obtained and explained by Rossby wave saturation

(3) Up-scale energy cascade to Rhine's scale forms blocking

(4) Further up-scale cascade to zonal energy forms the Arctic Oscillation