Energy-Momentum Tensor Correlation function in Nf=2+1 QCD at finite temperature

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for
WHOT QCD collaboration

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Energy-Momentum Tensor

Relativistic extension of energy density, momentum density, energy flow and stress

RHS of Einstein's equation

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Source of gravity

Perfect fluid assumption

$$T_{\mu\nu} = (\rho + p) u^\mu u^\nu + g^{\mu\nu} p$$

Simpler in quantum field theory

EM tensor = conserved current

Noether's theorem

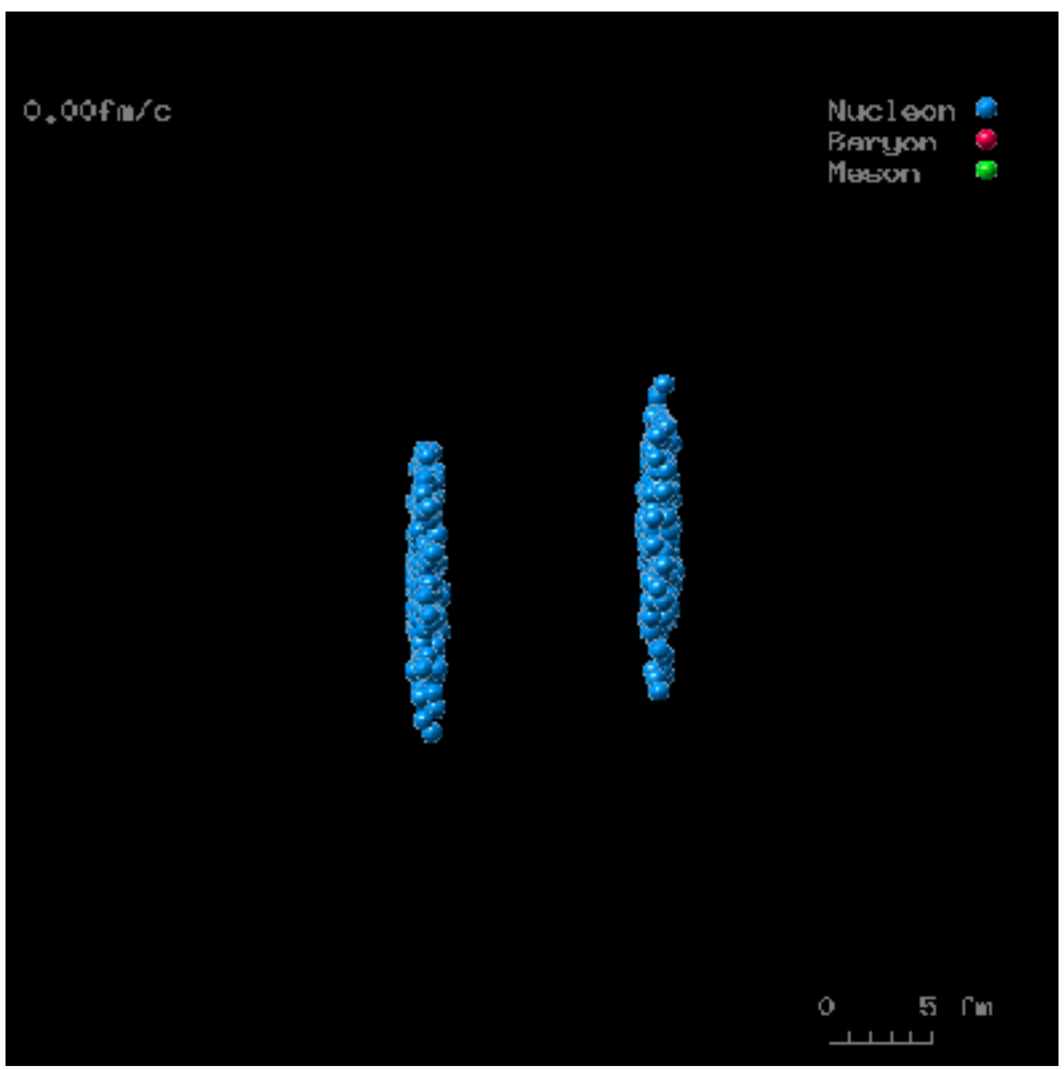
translation invariance

Energy-Momentum Tensor

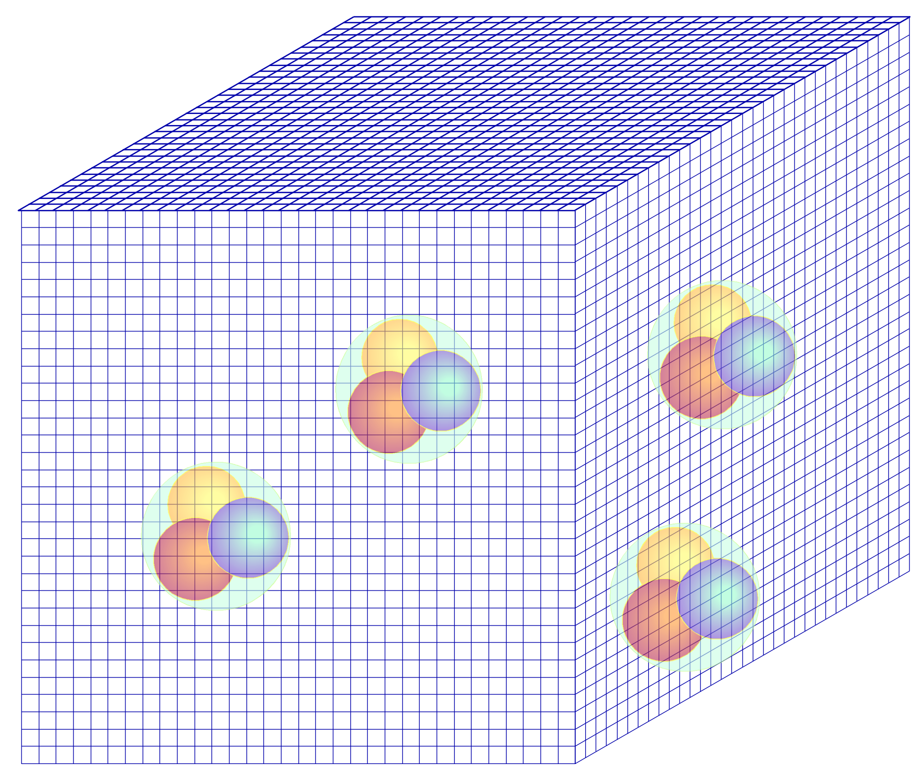
Use for a study of Quark-Gluon Plasma

Very high temperature state of nuclear

Observed in heavy ion collision experiment



Lattice QCD



Energy-Momentum Tensor

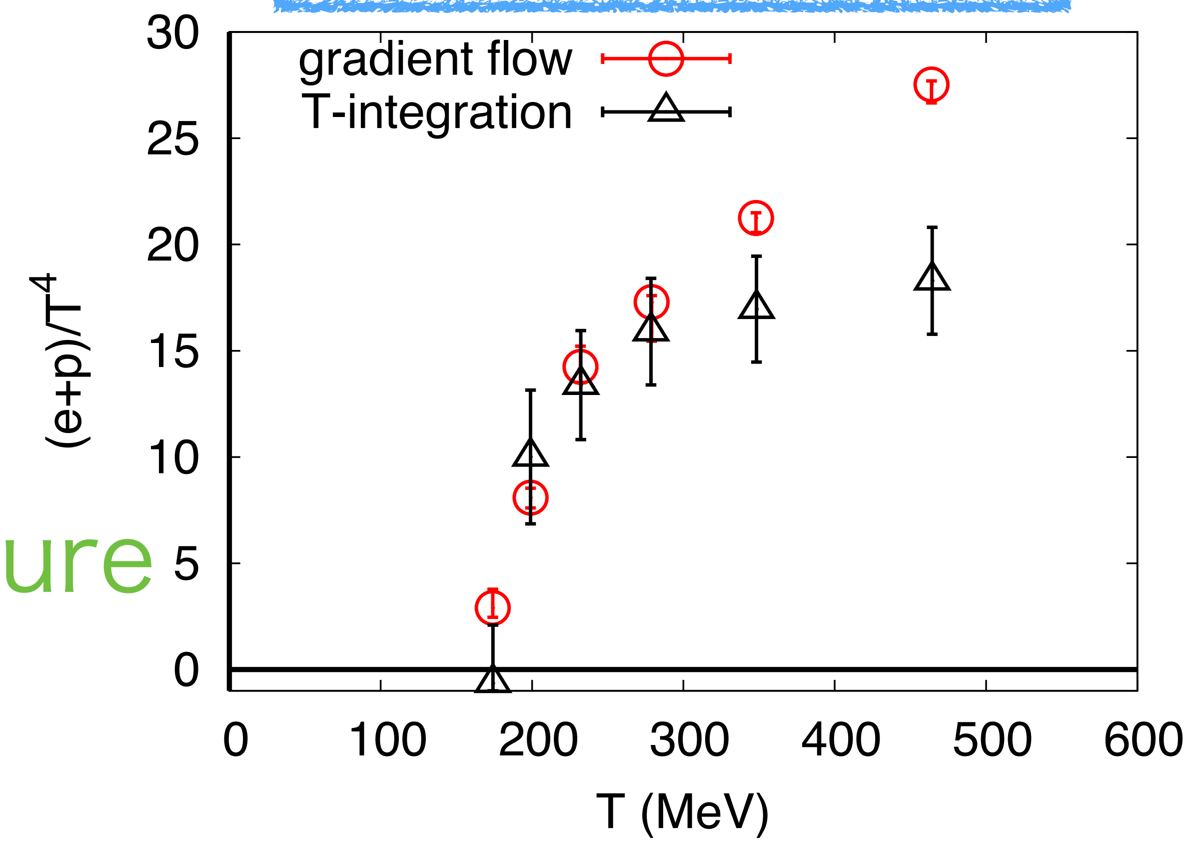
Use for a study of Quark-Gluon Plasma

energy momentum

$$\begin{pmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{pmatrix}$$

stress pressure

entropy density



- If we have $T_{\mu\nu}$

→ direct measurement of thermodynamic quantity

Successful for 1 point function

Today's 1st topic

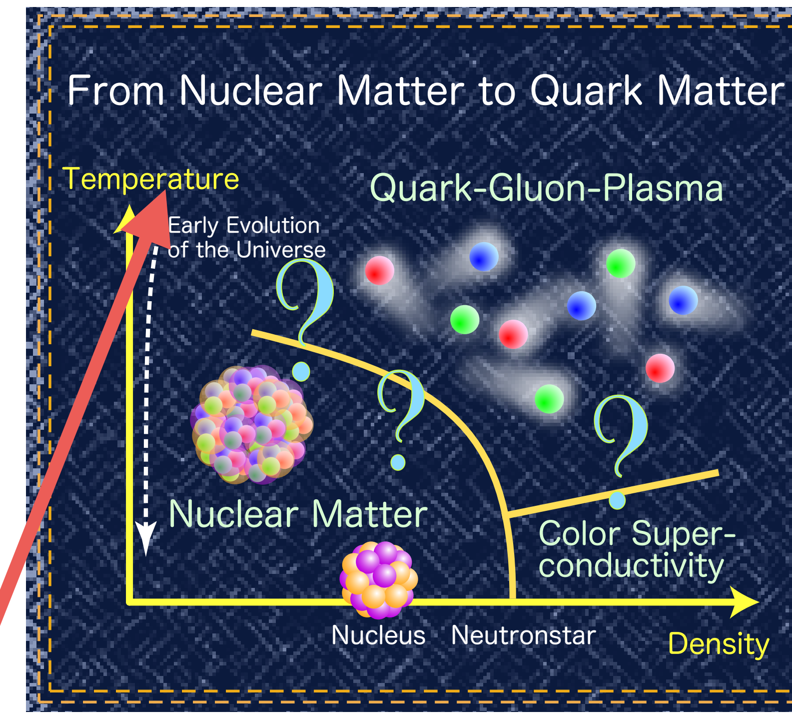
Energy-Momentum Tensor

Use for a study of Quark-Gluon Plasma

energy momentum

$$\begin{pmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{pmatrix}$$

stress pressure



hot topics in QGP

- If we have $T_{\mu\nu}$
  - Fluctuations and correlations of $T_{\mu\nu}$

→ specific heat, viscosity, $\cdots$

Today's 2nd topic

How to calculate $T_{\mu\nu}$ on lattice?

Difficulty: No translation invariance of lattice

- Measure expectation value of operators on lattice

terms in QCD Lagrangian

$$\delta_{\mu\nu} F^a_{\rho\sigma}(x) F^a_{\rho\sigma}(x) \qquad \delta_{\mu\nu} \bar{\psi}(x) \overleftrightarrow{\not{D}} \psi(x) \qquad \delta_{\mu\nu} \bar{\psi}(x) \psi(x)$$

terms in QCD Lagrangian when trace is taken

$$F^a_{\mu\rho}(x) F^a_{\nu\rho}(x) \qquad \bar{\psi}(x) \left(\gamma_\mu \overleftrightarrow{D}_\nu + \gamma_\nu \overleftrightarrow{D}_\mu \right) \psi(x)$$

- Renormalization

Well established for E and P

Karsch coefficients

problems

- non universal (No translation symmetry)
  - depends on: lattice action, operator

How to calculate $T_{\mu\nu}$ on lattice?

Easier method for renormalization?

Gradient Flow

Narayanan-Neuberger(2006)
Lüscher(2009–)

Flow the gauge field

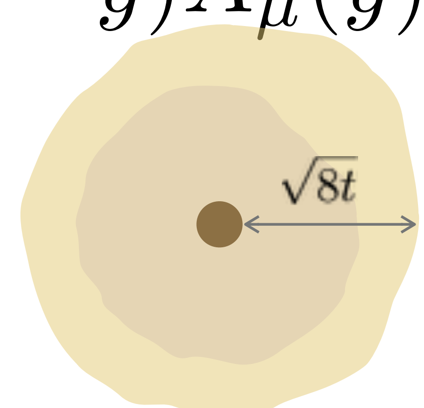
$$\partial_t A_\mu(t,x) = -\frac{\delta S_{\rm YM}}{\delta A_\mu} \qquad A_\mu(t=0,x) = A_\mu(x)$$

t: flow time, dim=[length^2]

A kind of diffusion equation $\partial_t A_\mu(t,x) = D_\nu G_{\nu\mu}$

Solution $A_\mu(t,x) = \int d^4y K_t(x-y) A_\mu(y) + \text{interactions}$

heat kernel $K_t(x) = \dfrac{e^{-x^2/4t}}{(4\pi t)^{D/2}}$



smear field within $\sqrt{8t}$

How to calculate $T_{\mu\nu}$ on lattice?

Easier method for renormalization?

Gradient Flow as a renormalization scheme

Narayanan-Neuberger(2006), Lüscher(2010), Lüscher-Weisz(2011)

Gauge operators with flowed field $A_\mu(t,x)$

- does not have UV divergence
- operators are renormalized

scale: $\mu = \frac{1}{\sqrt{8t}}$

visit $t \ll 1/\Lambda_{QCD}^2$ region non-perturbatively

NP renormalized operator ← gradient flow ← lattice operator

$F^a_{\mu\nu}F^a_{\mu\nu}(x,t)$

Tr<1 − □>

universal

finite ren.

H. Suzuki (2016)

take a→0 limit safely

$\overline{\text{MS}}$ scheme

Matching coefficients are calculable perturbatively

First topic

1 point function of energy-momentum tensor

Highlights?

- Thermodynamical quantity: energy, pressure

energy

$$\begin{pmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{pmatrix}$$

pressure

- Entropy density

$$s = \left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V = \frac{\epsilon + p}{T} = \frac{\langle T_{00} + T_{ii} \rangle}{T}$$

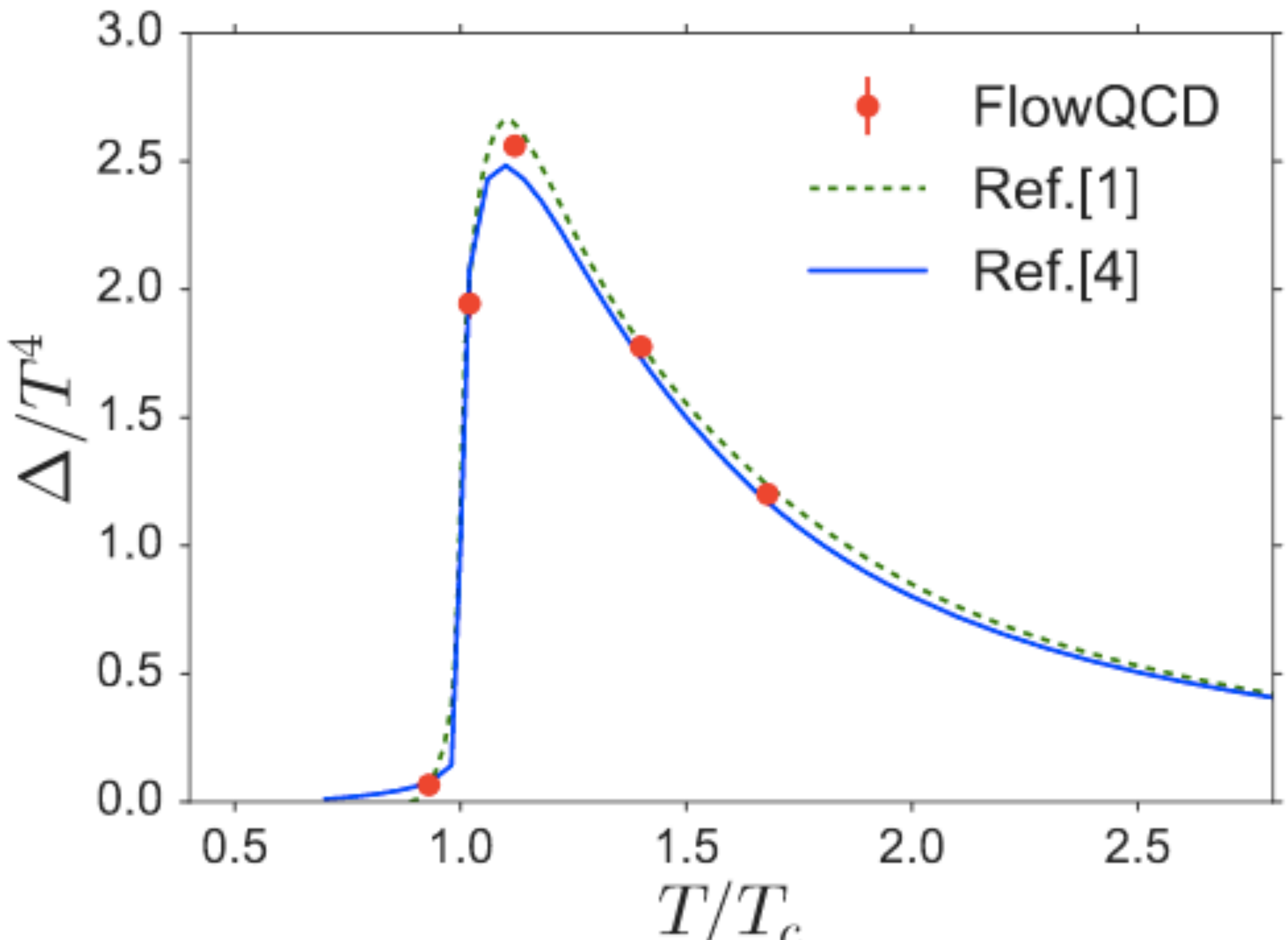
Maxwell's relation   integrable condition of entropy

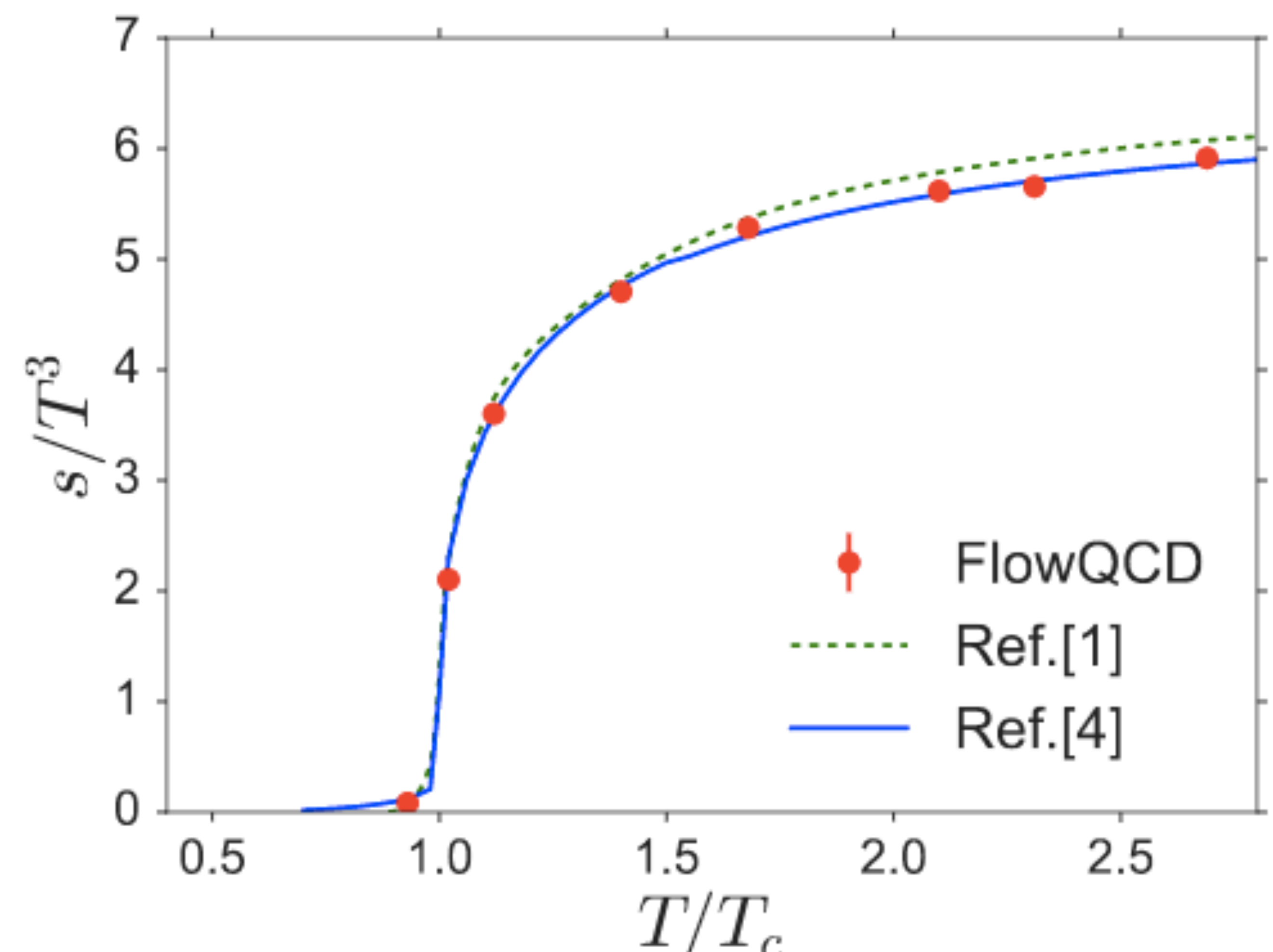
- Comparison with established method

SU(3) Yang-Mills (Quench)

FlowQCD: Kitazawa, Iritani, Asakawa, Hatsuda, Suzuki

a→0 limit done!





- Ref[1]: Boyd et. al., Nucl. Phys. B 469, 419 (1996)
- Ref[4]: Borsanyi et. al., JHEP 1207, 056 (2012)

Nf=2+1 QCD

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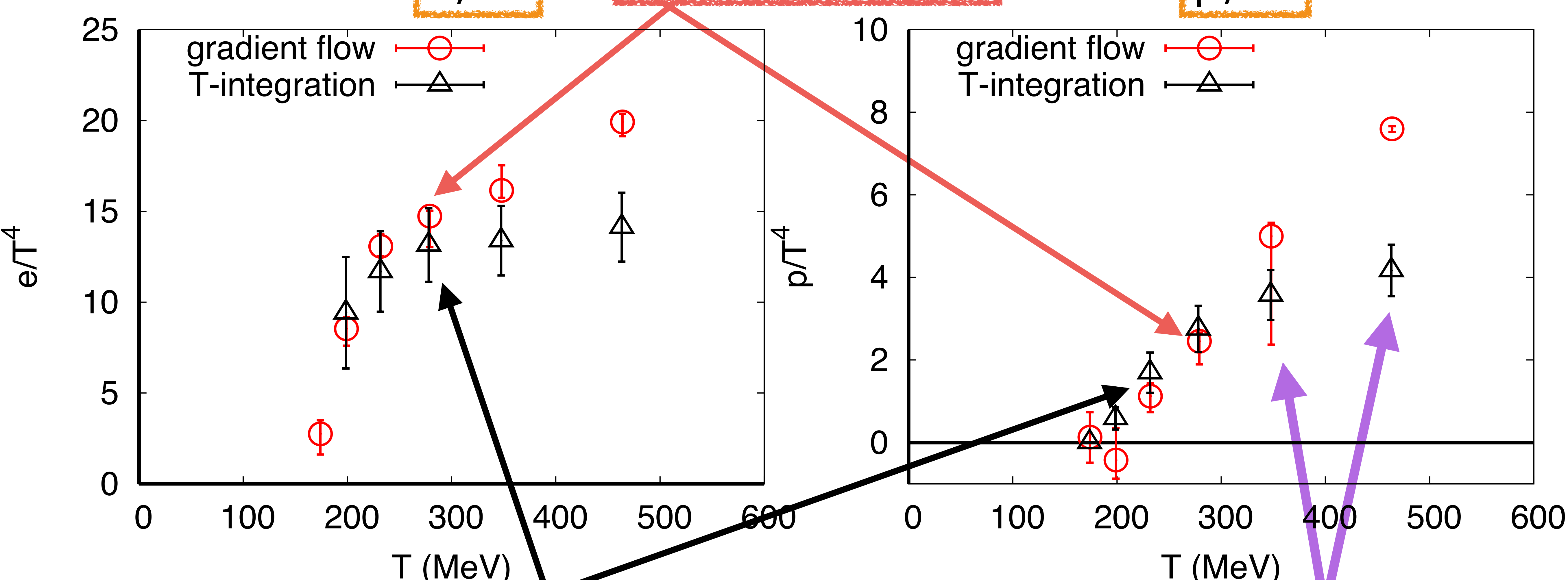
a∼0.07 [fm], heavy ud quark $\frac{m_\pi}{m_\rho} \sim 0.6$

e/T^4 gradient flow p/T^4



WHOT-QCD, Phys. Rev. D 85, 094508 (2012)
integration method

aT=1/Nt artifact is severe

Nf=2+1 QCD

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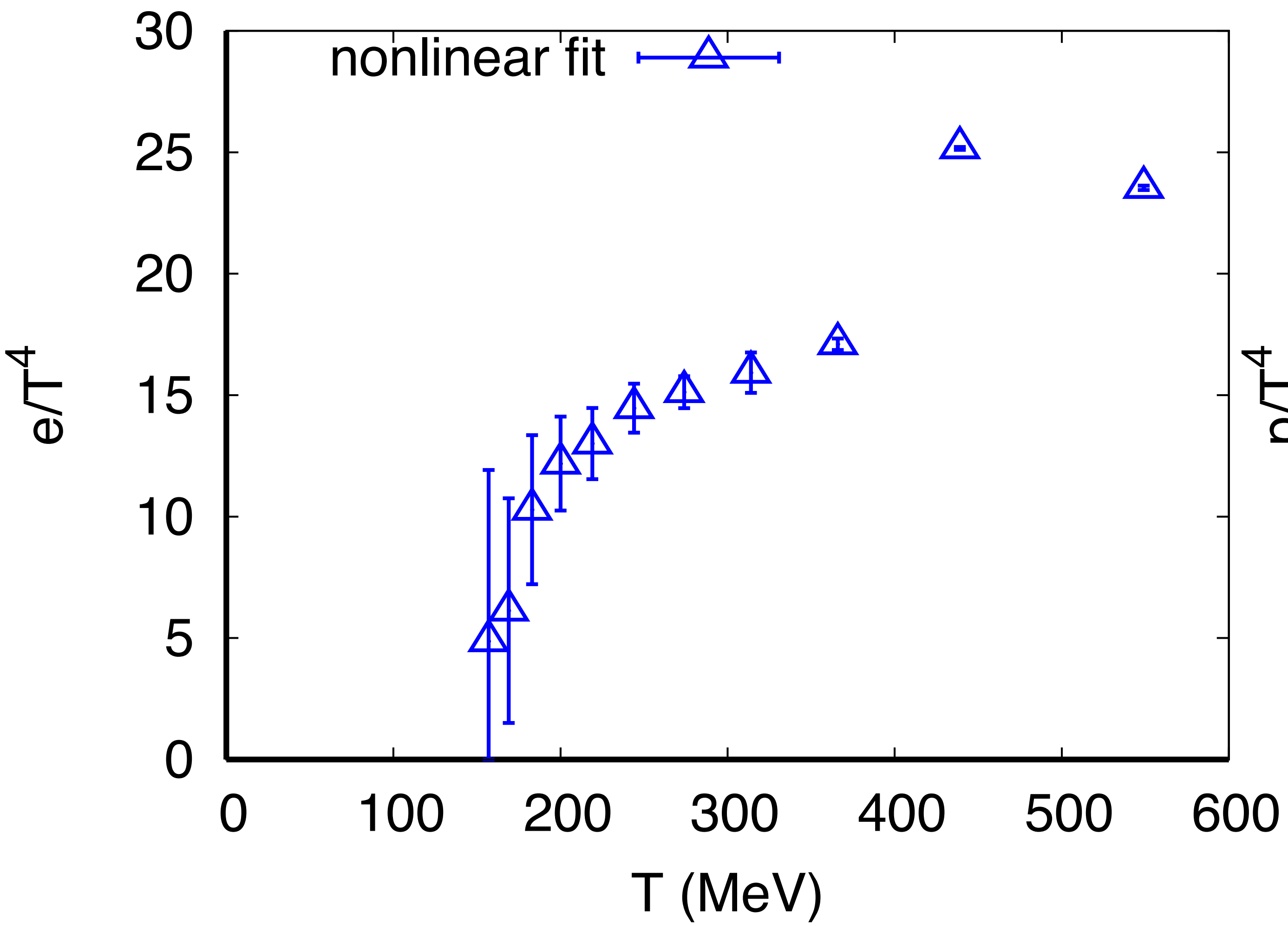
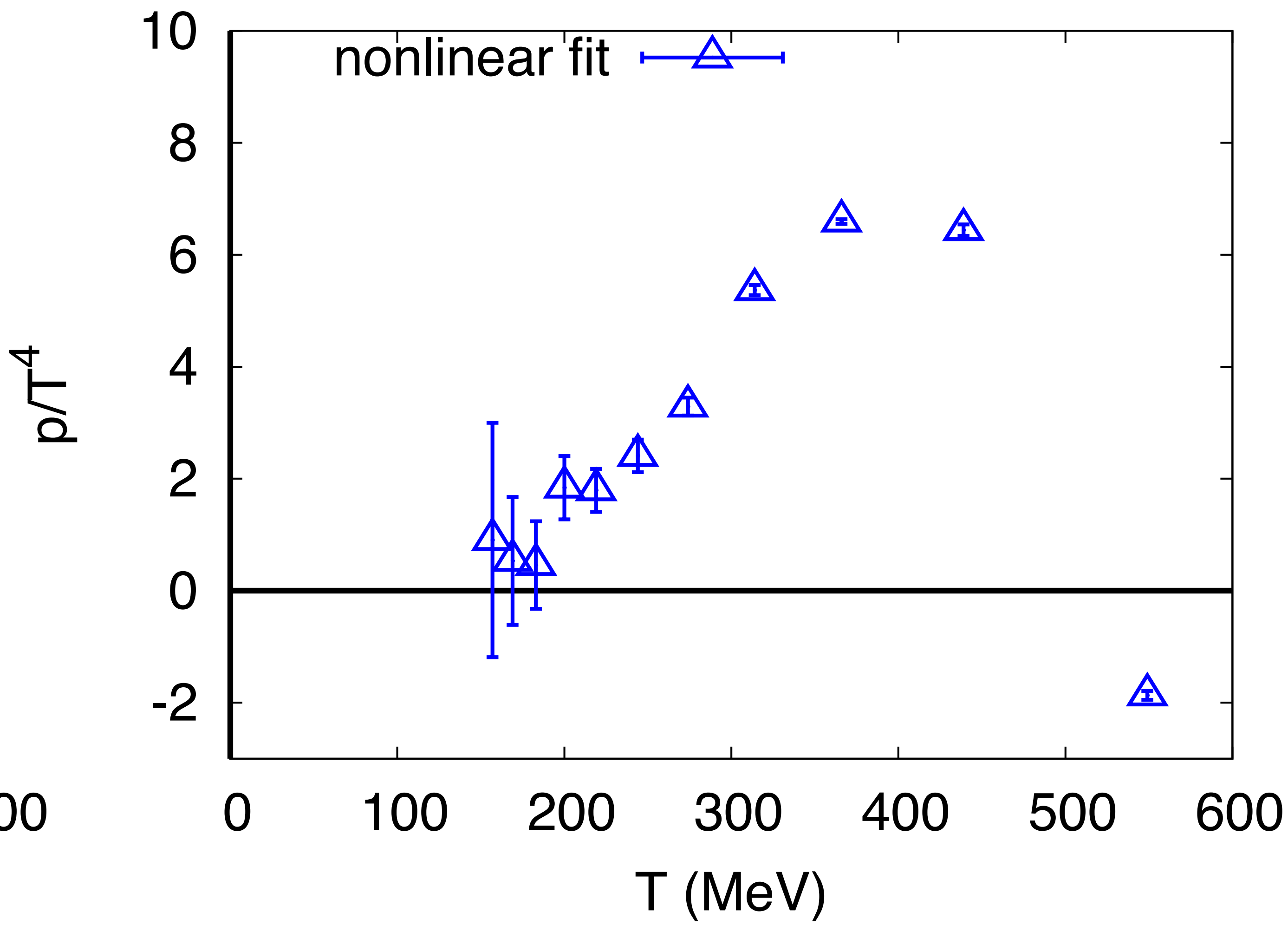
a∼0.090 [fm], physical quark mass

e/T^4



p/T^4



Second topic

2 point correlation function of fluctuation

$$C_{\mu\nu;\rho\sigma}(t;x_0) = \frac{1}{T^5}\int_{V_3} d^3x \, (\langle \delta T_{\mu\nu}(t;x_0,\vec{x})\delta T_{\rho\sigma}(t;0)\rangle)$$

fluctuation: $\delta T_{\mu\nu}(t;x) = T_{\mu\nu}(t;x) - \langle T_{\mu\nu}(t;x)\rangle$

Highlights?

- Conservation law $\quad \dfrac{d}{dx_0} C_{0\nu;\rho\sigma}(x_0) = 0$

- Linear response relations

$$C_{0i;0i} = C_{00;ii} = -\frac{\epsilon + p}{T^4} \qquad C_{00;00} = \frac{c_V}{T^3}$$

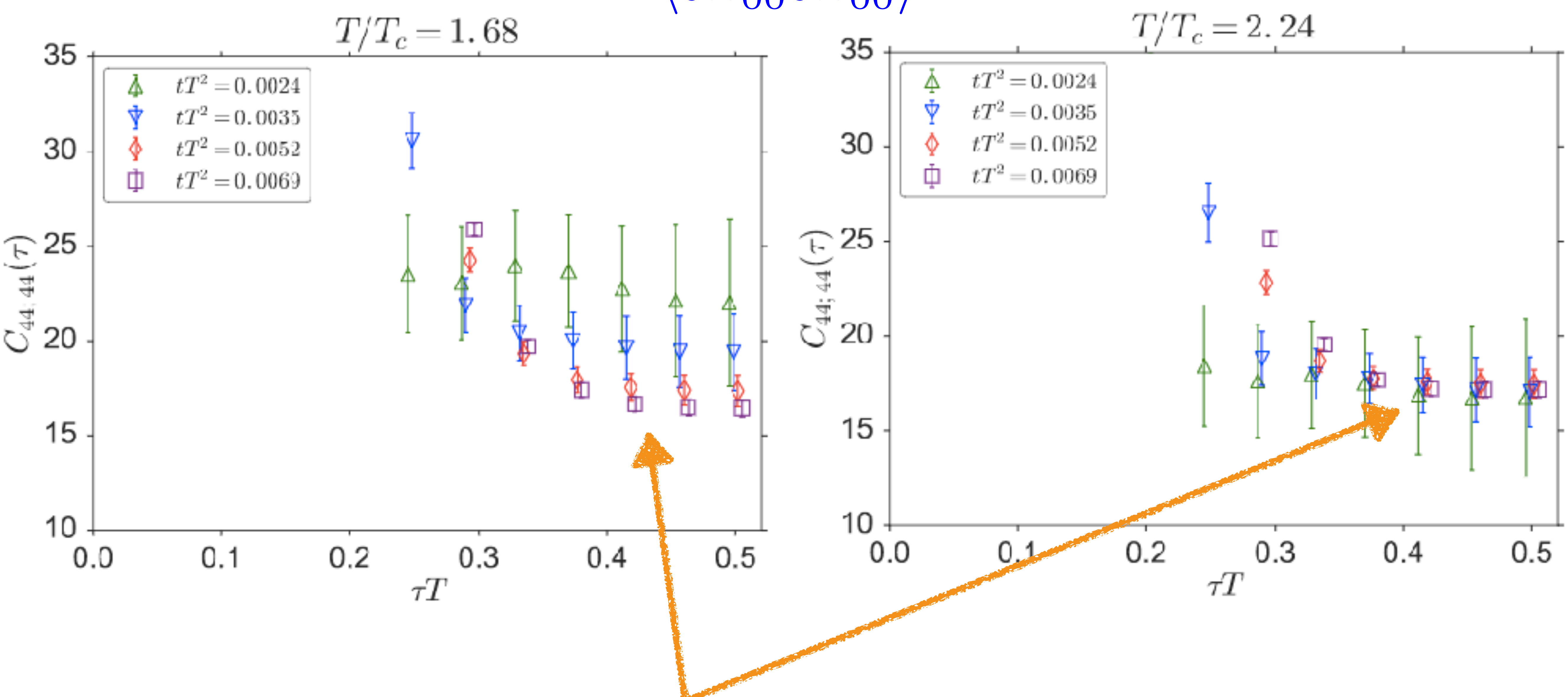
Conservation law

$$\frac{d}{dx_0}\underbrace{\int_{V_3} d^3x\,(\langle\delta T_{0\nu}(t;x_0,\vec{x})}_{P_\mu}\delta T_{\rho\sigma}(t;0)\rangle) = 0$$

SU(3) Yang-Mills (Quench)

FlowQCD: Kitazawa, Iritani, Asakawa, Hatsuda

$\langle \delta T_{00} \delta T_{00} \rangle$



correlation function is very flat in the middle

Nf=2+1 QCD

a∼0.07 [fm], heavy ud quark

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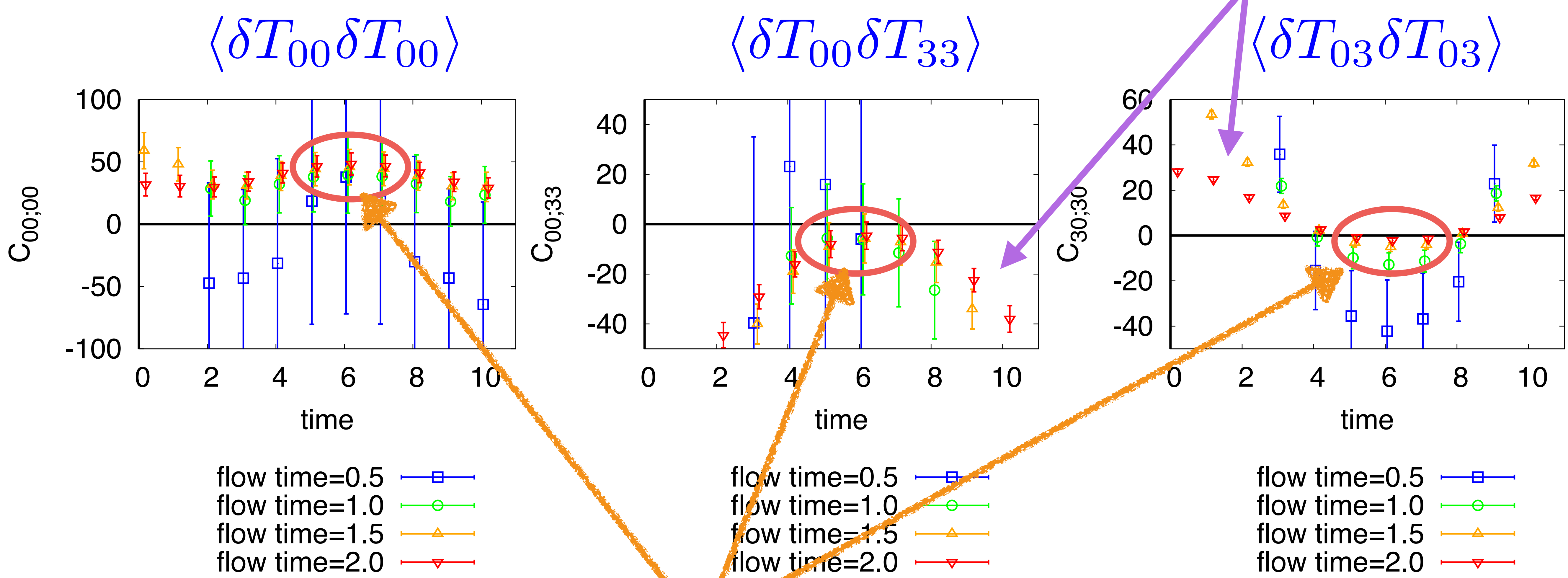
- T=232 MeV, Nt=12

lattice artifact is severe



correlation function is very flat in the middle

Entropy density

Thermodynamical relation

compare

Maxwell's relation

$$s = \left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V = \frac{\epsilon + p}{T} = \frac{\langle T_{00} + T_{ii}\rangle}{T}$$

integrable condition of entropy

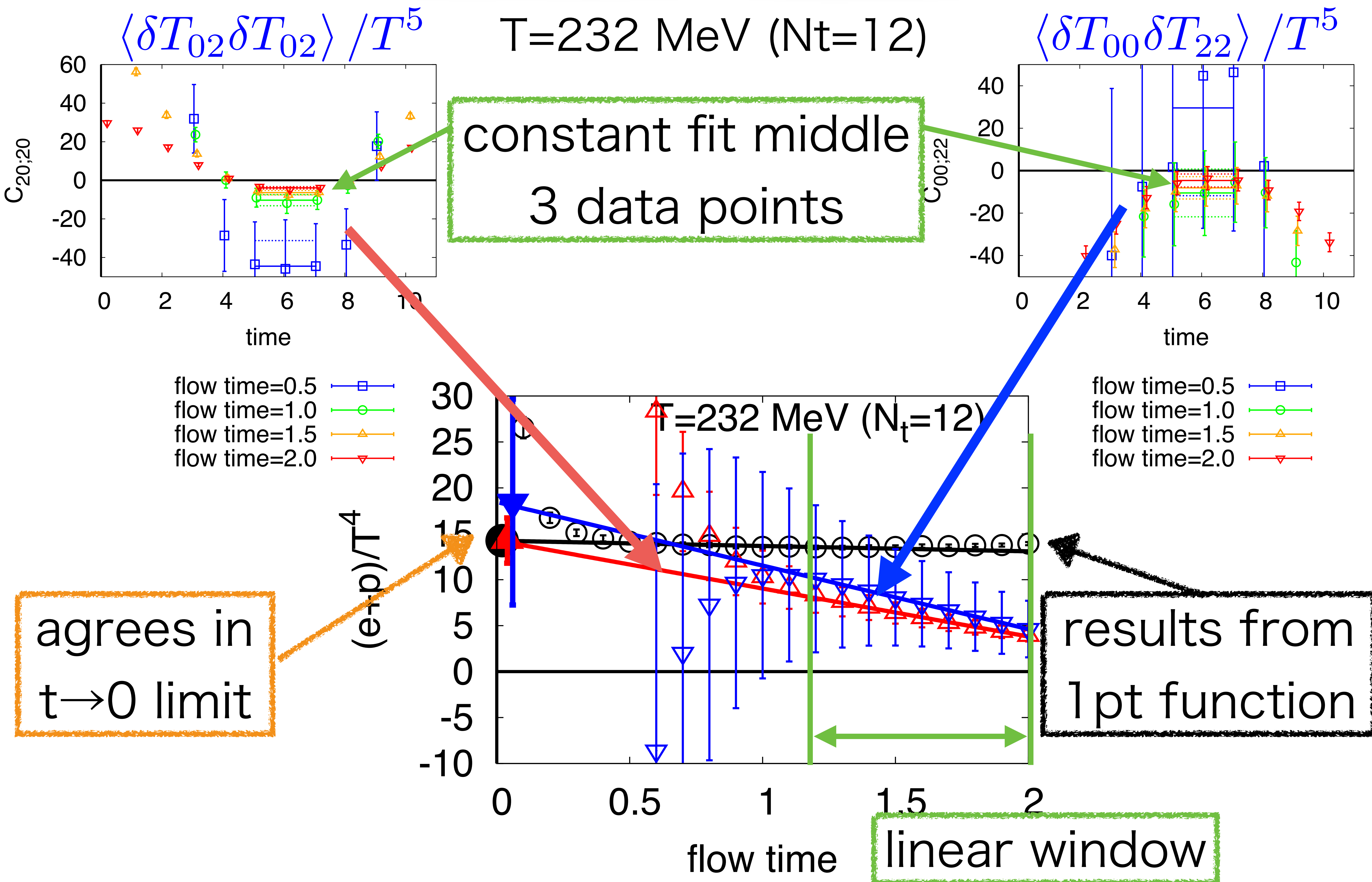
Linear response relation

$$s = \left(\frac{\partial p}{\partial T}\right)_V = \frac{\partial \langle T_{ii}\rangle}{\partial T} \qquad \langle T_{ii}\rangle = \frac{1}{Z}\mathrm{Tr}\left(T_{ii}e^{-H/T}\right)$$

$$s = \frac{1}{T^2}\langle \delta H \delta T_{ii}\rangle = \frac{1}{T^2}\int_{V_3} d^3x\,(\langle \delta T_{00}(t; x_0, \vec{x})\delta T_{ii}(t; 0)\rangle)$$

$$\epsilon + p = \left.\frac{\partial \langle T_{01}\rangle}{\partial v_1}\right|_{\vec{v}=0} = \frac{1}{T}\int_{V_3} d^3x\,(\langle \delta T_{0i}(t; x_0, \vec{x})\delta T_{0i}(t; 0)\rangle)$$

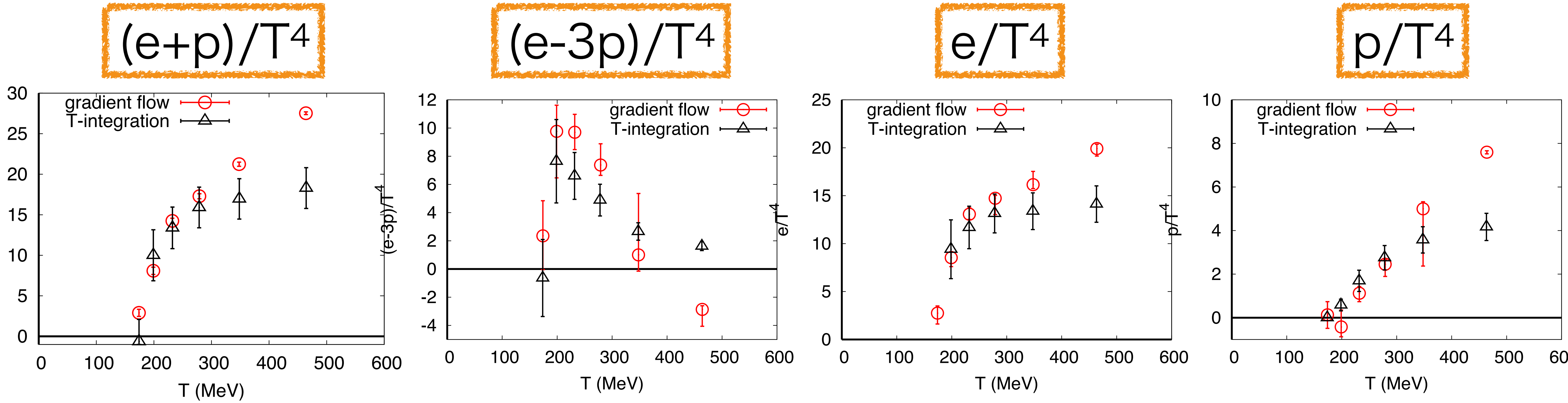
Nf=2+1 QCD



$\langle \delta T_{02} \delta T_{02} \rangle / T^5$
T=232 MeV (Nt=12)
$\langle \delta T_{00} \delta T_{22} \rangle / T^5$
$C_{20;20}$
$C_{00;22}$
time
time
constant fit middle
3 data points
flow time=0.5
flow time=1.0
flow time=1.5
flow time=2.0
flow time=0.5
flow time=1.0
flow time=1.5
flow time=2.0
T=232 MeV (N_t=12)
$(e-p)/T^4$
agrees in
t→0 limit
results from
1pt function
flow time
linear window

Summary

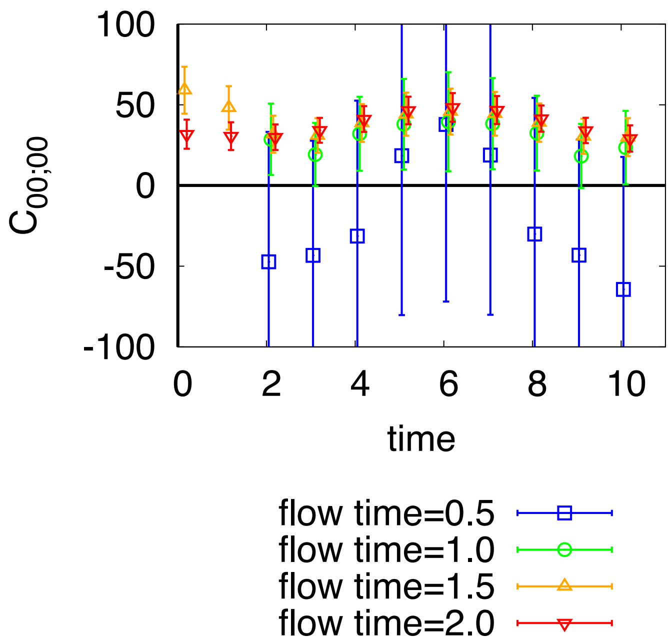
- Flow method works well for EM tensor!
  - as powerful as the derivative method.
- More suitable for Wilson fermion.
- Good agreement with T integration method

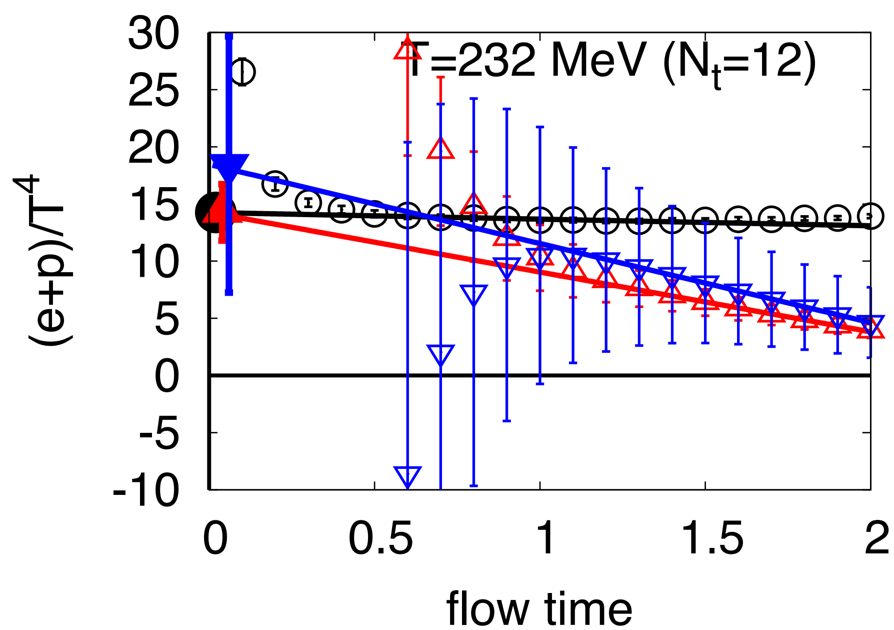


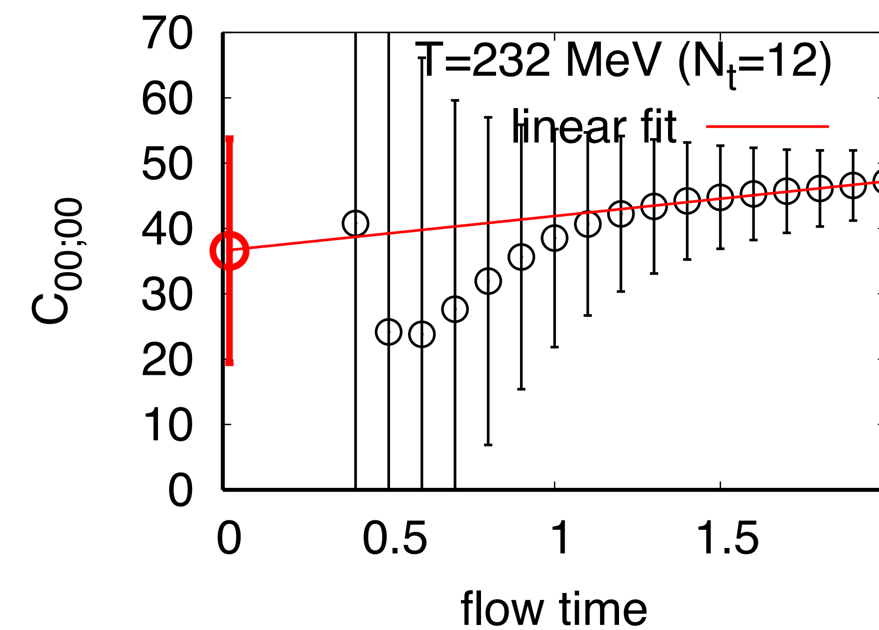
- Lattice artifact is severe for Nt=4, 6, 8

Summary

- Gradient flow works well for EMT correlation function
- We have good results:
  - Conservation law
  - Linear response relation







$$\frac{c_V}{T^3} = 37(17)$$