Vlasov simulations of collisionless self-gravitating systems and astrophysical plasmas

Kohji Yoshikawa

Division of Astrophysics
Center for Computational Sciences, University of Tsukuba

CCS-EPCC Workshop
December 7 - 8, 2017, CCS, Univ. of Tsukuba

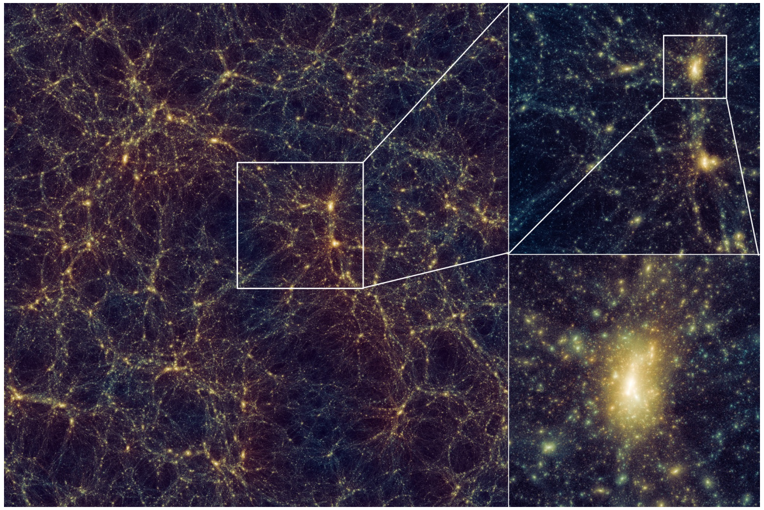
collaborators: Satoshi Tanaka (CCS, Univ. of Tsukuba)
Takashi Minoshima (JAMSTEC)
Naoki Yoshida (Univ. of Tokyo)

N-body Simulation of Self-Gravitating Systems

- a standard method for simulations of self-gravitating systems (galaxies, clusters of galaxies, the large-scale structure in the universe) for more than 40 years.

- the mass distribution is sampled by particles in the 6D phase-space volume (**x**, **p**) in a Monte-Carlo manner.

$$\frac{\mathrm{d}\boldsymbol{v}_i}{\mathrm{d}t} = \sum_j \frac{m_j(\boldsymbol{r}_j - \boldsymbol{r}_i)}{|\boldsymbol{r}_j - \boldsymbol{r}_i|^3}$$



Ishiyama 2013

- sophisticated algorithms to treat large number of particles such as Tree and TreePM methods developed

Particle-In-Cell (PIC) Simulation of Plasmas

- Particle-based approach to solve collisionless (astrophysical) plasma

$$\frac{\mathrm{d}\boldsymbol{v}_i}{\mathrm{d}t} = \frac{q_i}{m_i}\left(\boldsymbol{E} + \frac{\boldsymbol{v}_i \times \boldsymbol{B}}{c}\right)$$

- *E*- and *B*-fields are computed in the finite difference manner

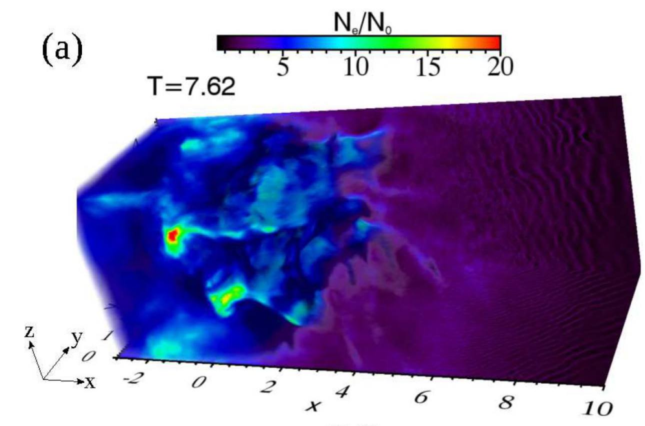
$$\boldsymbol{x}_i, \boldsymbol{v}_i \Rightarrow \boldsymbol{J}$$

$$\frac{\partial \boldsymbol{E}}{\partial t} = c\nabla \times \boldsymbol{B} - 4\pi \boldsymbol{J}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = -c\nabla \times \boldsymbol{E}$$

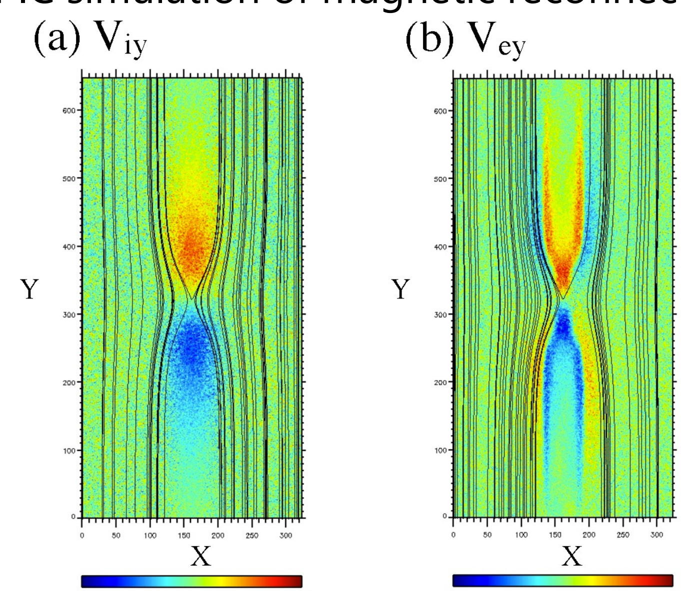
- particle acceleration in collisionless shock
- magnetic reconnection

3D PIC simulation of collisionless shock



Matsumoto et al. (2017)

PIC simulation of magnetic reconnection



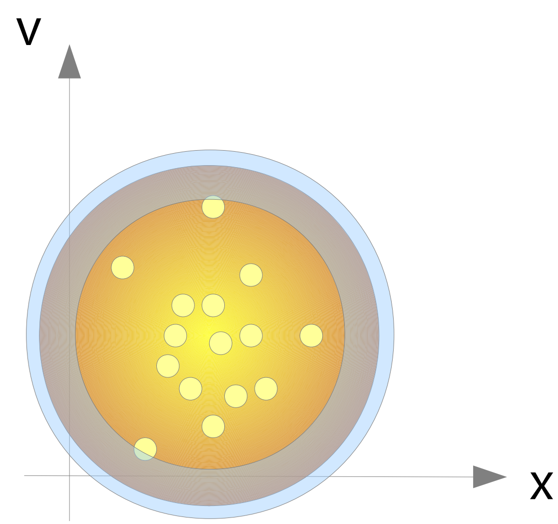
(pCANS web page)

Drawbacks of Particle Simulations

- intrinsic contamination of shot noise in physical quantities

- not good at simulating kinetic physical processes in which the tail of the distribution function plays important roles

  - matter in the tail is not fairly sampled in particle simulations
  - collisionless damping, two-stream instability
  - magneto-rotational instability starting from high-beta plasmas



- The grid spacing of PIC simulations should be less than the Debye length.

  - difficult to simulate phenomena in the macroscopic scale

Vlasov Simulations

- Simulate self-gravitating systems and plasmas by directly solving the Vlasov equation in a finite volume manner

$$\frac{\partial f}{\partial t} + \frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} \cdot \frac{\partial f}{\partial \boldsymbol{x}} + \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} \cdot \frac{\partial f}{\partial \boldsymbol{p}} = 0$$

$f(\boldsymbol{x}, \boldsymbol{p}, t)$: distribution function in six-dimensional phase space

- Vlasov-Poisson simulation
  - self-gravitating system
  - electro-static plasma

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \frac{\partial f}{\partial \boldsymbol{x}} - \nabla \phi \cdot \frac{\partial f}{\partial \boldsymbol{v}} = 0$$

$$\nabla^2 \phi = 4\pi G \rho = 4\pi G \int f \mathrm{d}^3 \boldsymbol{v}$$

- Vlasov-Maxwell simulation
  - magnetized plasma

$$\frac{\partial f_{\mathrm{s}}}{\partial t} + \boldsymbol{v} \cdot \frac{\partial f_{\mathrm{s}}}{\partial \boldsymbol{x}} + \frac{q_{\mathrm{s}}}{m_{\mathrm{s}}} \left(\boldsymbol{E} + \frac{\boldsymbol{v} \times \boldsymbol{B}}{c} \right) \cdot \frac{\partial f_{\mathrm{s}}}{\partial \boldsymbol{v}} = 0$$

$$\frac{\partial \boldsymbol{E}}{\partial t} = c \nabla \times \boldsymbol{B} - 4\pi \boldsymbol{J}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = -c \nabla \times \boldsymbol{E}$$

$$\boldsymbol{J} = \sum_{\mathrm{s}} \int q_{\mathrm{s}} \boldsymbol{v} f_{\mathrm{s}} \mathrm{d}^3 \boldsymbol{v}$$

Vlasov Simulations

- Free from shot noise contamination

- Good at capturing kinematic physical processes

  - collisionless damping, dynamical friction etc in self-gravitating systems

  - Landau damping, weibel instability etc in astrophysical plasma

- Free from mesh size constraint to the Debye length

- requires huge amount of memory and computational costs

  had not been practical for a long time

- solves hyperbolic PDE instead of ODE in particle simulations

  → inevitable numerical diffusion

  → need for sophisticated less diffusive numerical schemes

Vlasov-Poisson Simuation in 6D phase space

Yoshikawa, Yoshida, Umemura (2013)

- directional splitting method

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \frac{\partial f}{\partial \vec{x}} - \nabla \phi \cdot \frac{\partial f}{\partial \vec{v}} = 0 \Rightarrow \left\{ \begin{array}{ll} \dfrac{\partial f}{\partial t} + v_i \dfrac{\partial f}{\partial x_i} = 0 & (i = 1, 2, 3) \\ \dfrac{\partial f}{\partial t} - \dfrac{\partial \phi}{\partial x_i} \dfrac{\partial f}{\partial v_i} = 0 & (i = 1, 2, 3) \end{array} \right.$$

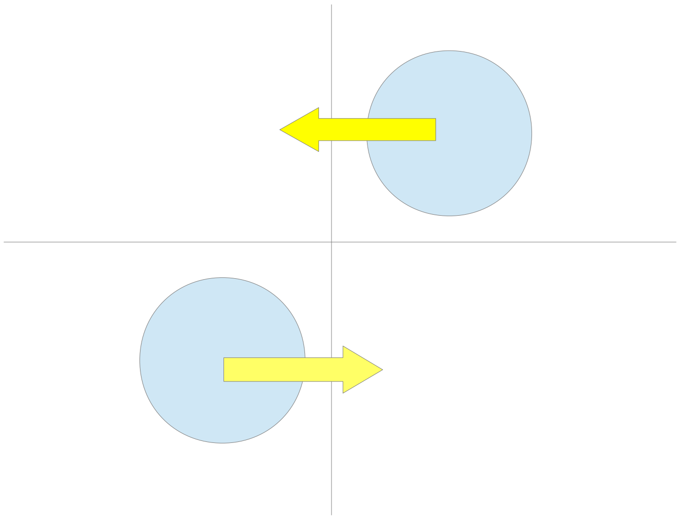
$$\begin{aligned} f(\vec{x}, \vec{v}, t^{n+1}) = {} & T_{v_x}(\Delta t/2) T_{v_y}(\Delta t/2) T_{v_z}(\Delta t/2) \\ & T_x(\Delta t) T_y(\Delta t) T_z(\Delta t) \\ & T_{v_x}(\Delta t/2) T_{v_y}(\Delta t/2) T_{v_z}(\Delta t/2) \; f(\vec{x}, \vec{v}, t^n) \end{aligned}$$

$T_\ell(\Delta t)$: advection along ℓ-direction

- numerical scheme for a one-dimensional advection equation

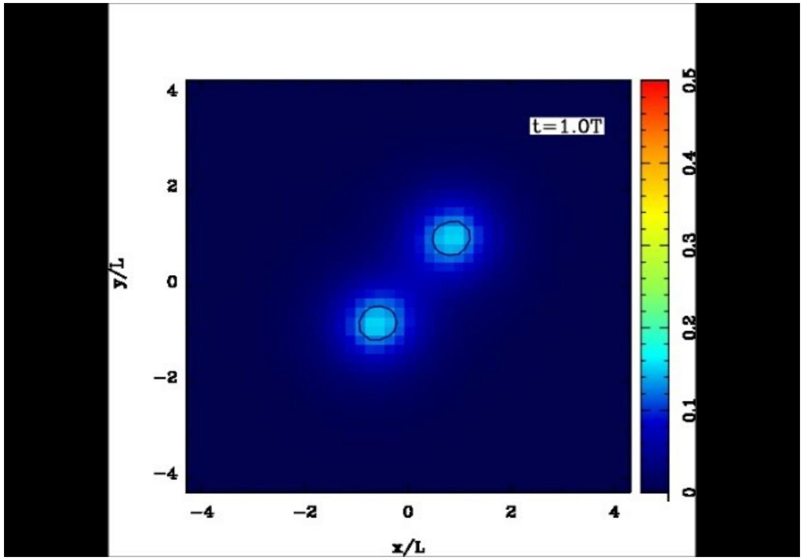
Positive and Flux Conservative (PFC) scheme   Filbet, Sonnendrücker, Bertrand (2001)

Merging of Two Spheres

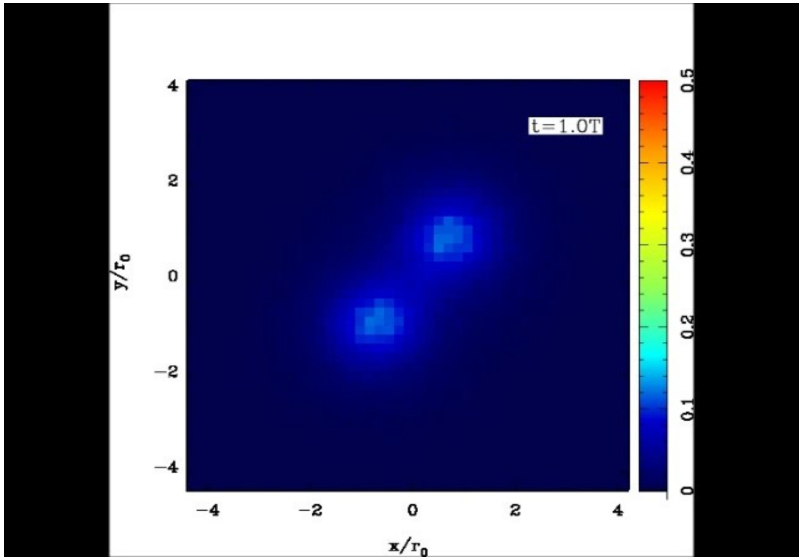


- offset merging of two king self-gravitating spheres
- comparison with a equivalent N-body simulations, in which each King sphere is represented with a million particles

Vlasov simulation



N-body simulation



Grav. Instablity and Collisionless Damping

Initial condition

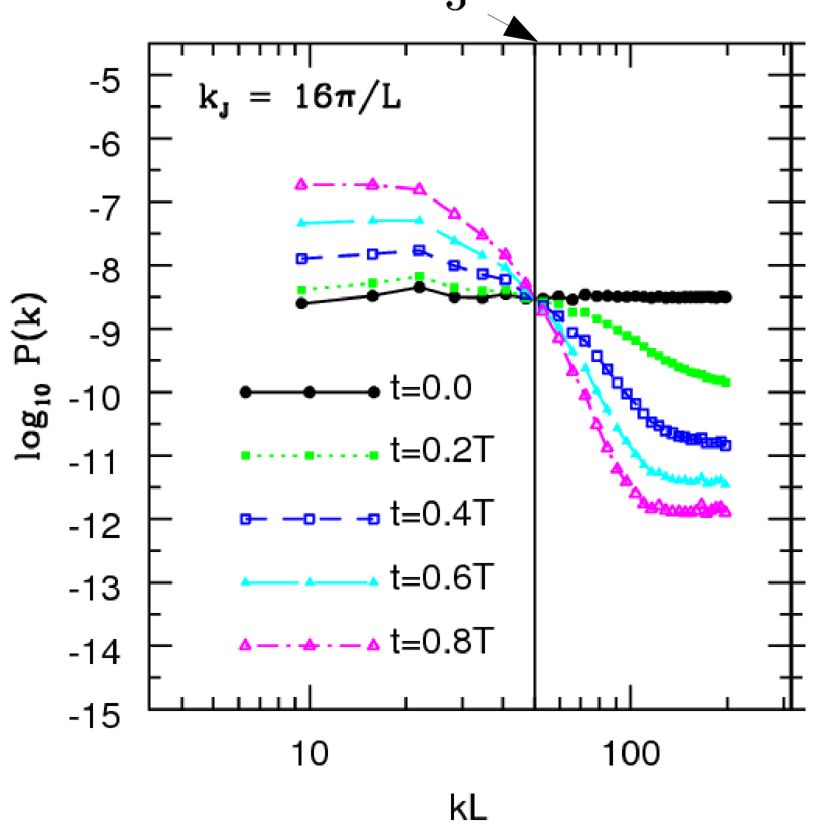
$$\begin{cases} f(\vec{x}, \vec{v}, t=0) = \dfrac{\bar{\rho}(1+\delta(\vec{x}))}{(2\pi\sigma^2)^{3/2}} \exp\left(-\dfrac{|\vec{v}|^2}{2\sigma^2}\right) \\ \rho(x, t=0) = \bar{\rho}(1+\delta(x)) \end{cases}$$

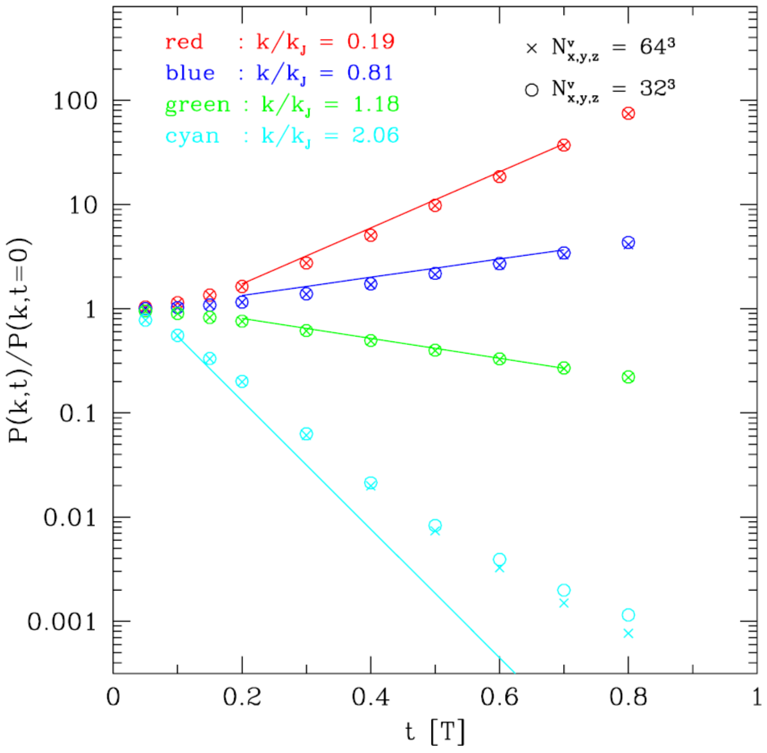
$k < k_{\rm J}$: gravitational instability

$k > k_{\rm J}$: collisionless damping

- The initial density fluctuation $\delta(x)$ is given so that it has a white noise power spectrum.

$k = k_{\rm J}$





lines : theoretical prediction

New High-Order Schemes for Vlasov Simulation

Higher-Order Advection Schemes

- curse of dimensionality in Vlasov simulations

  huge memory consumption due to high dimensionality of phase space

  ⇨ size of numerical simulation is limited by the amount of available memory

- how to overcome
  - adaptive mesh refinement
  - higher-order schemes for the advection equation

$$\frac{\partial f(x,t)}{\partial t} + c\frac{\partial f(x,t)}{\partial x} = 0$$

- mathematical and physical requirements
  - monotonicity

    monotonicity-preserving (MP) constraints (Suresh & Huynh 1997)
  - positivity

    a new positivity-preserving (PP) limiter in the form of a flux limiter

- spatially fifth- and seventh-order schemes with monotonicity- and positivity-preserving features.

(c.f. The PFC scheme has a spatially third-order accuracy.)

Naming Convention of the Schemes

XX-MPP5

RK : 3-stage temporary 3rd order Runge-Kutta time integration

SL : single-stage semi-Lagrangian time integration

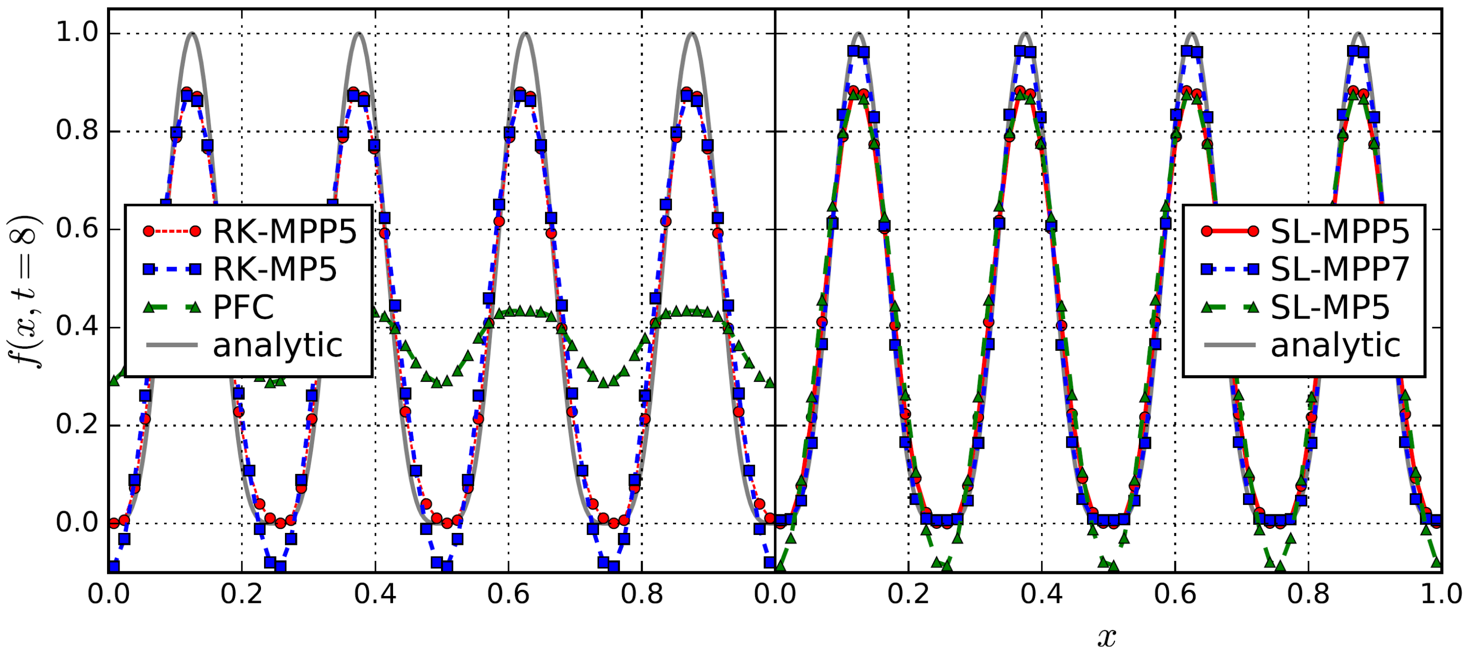
MPP : monotonicity and positivity preserving

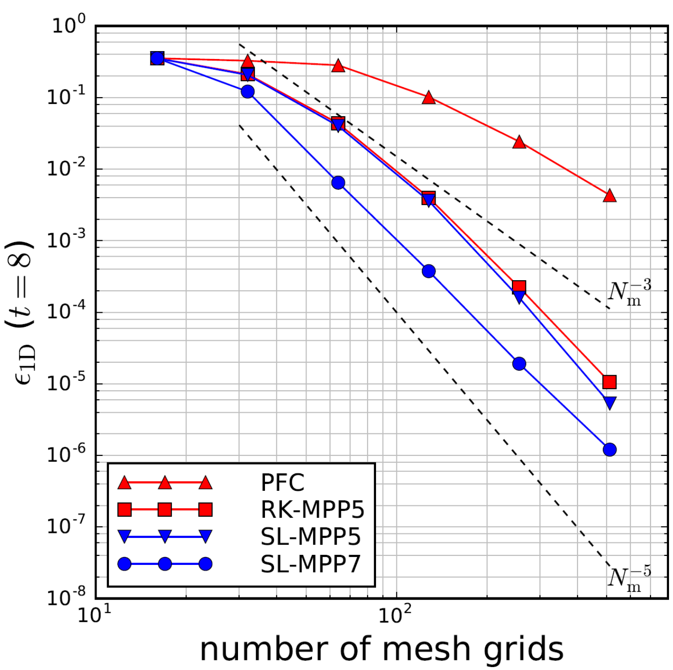
MP : monotonicity preserving

final digit : the spatial order of accuracy

Linear Advection

- linear advection of $f(x, t = 0) = \sin^4(4\pi x)$





- The result of PFC scheme is severely smeared.
- Schemes w/o PP limiter yield significant negative value around the minima.
- Only 7th-order scheme (SL-MPP7) can reproduce the maxima.

1D Self-Gravitating System

- one-dimensional space with periodic boundary condition

- initial condition

$$f(x, v, t=0) = \frac{\bar{\rho}[1 + A\cos(kx)]}{(2\pi\sigma^2)^{1/2}} \exp\left(-\frac{v^2}{2\sigma^2}\right)$$

$$\rho(x, t=0) = \bar{\rho}[1 + A\cos(kx)]$$

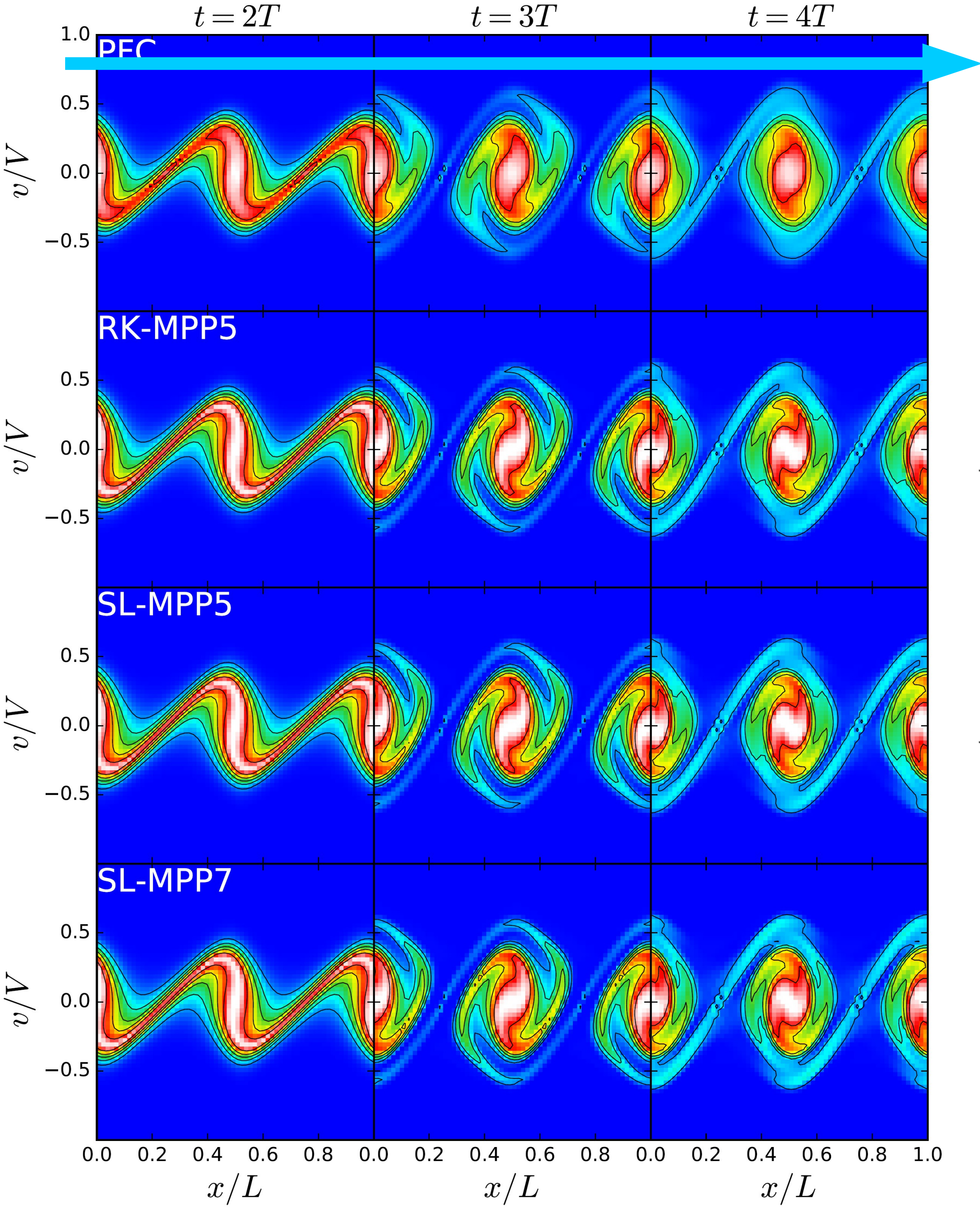
- critical Jeans wave number

$$k_{\mathrm{J}} = \left(\frac{4\pi G\bar{\rho}}{\sigma^2}\right)^{1/2}$$

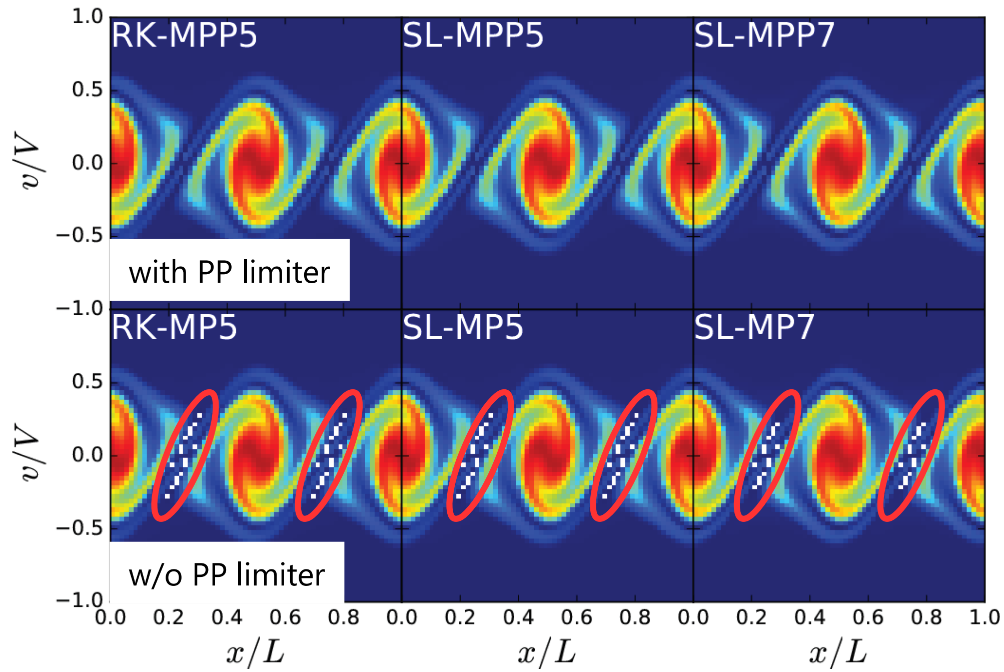
$$\begin{cases} k < k_{\mathrm{J}} \Rightarrow \text{gravitational instability} \\ k > k_{\mathrm{J}} \Rightarrow \text{collisionless damping} \end{cases}$$

1D Self-Gravitating System

$k/k_\mathrm{J} = 0.5 \quad A = 0.01 \quad N_x = N_v = 64$



- As time proceeds, numerical diffusion takes place and smear small structures in the lower-order schemes.



- Negative regions in the lower panels disappear in the results with positivity preserving limiter.

Application To Cosmological Neutrinos

Cosmological Relic Neutrinos

- massive neutrinos in the universe
  - lots of neutrinos in our universe decoupled at early stage of the universe when they are relativistic.
  - Discovery of neutrino oscillation

$$\sum m_\nu > 0.05 \text{ eV}$$

  - currently non-relativistic and gravitationally interacting with cold dark matter (CDM)
  - absolute mass of neutrinos and its hierarchy are still unknown

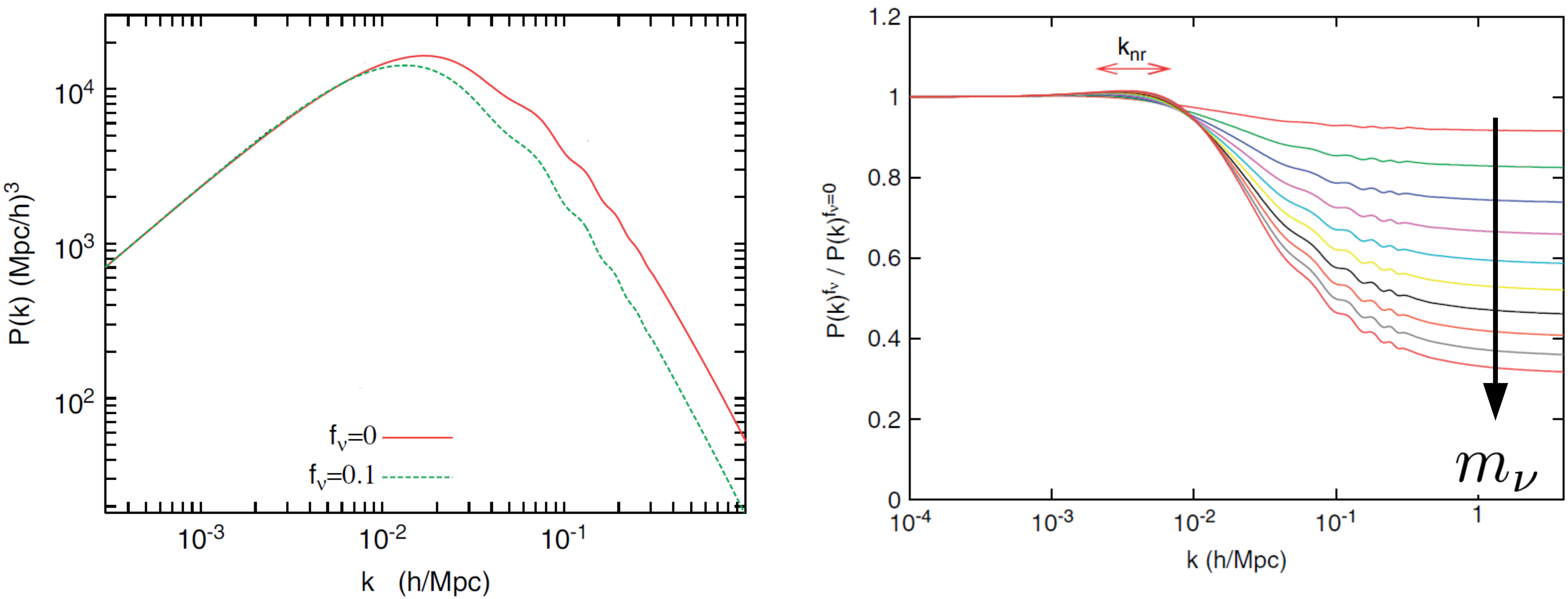
- dynamical effect of massive neutrinos

free streaming (collisionless damping)

  - large velocity dispersion of neutrinos $\sigma \sim 150(1+z)\left(\frac{m_\nu}{1\text{eV}}\right)^{-1} \text{km/s}$
  - growth of density fluctuation suppressed beyond the damping scale

$$k_{\rm FS} = \left(\frac{4\pi G\rho}{\sigma^2}\right)^{1/2} \Rightarrow \lambda_{\rm FS} \sim 640 \left(\frac{\Omega_{\rm m}}{0.3}\right)^{-1/2} \left(\frac{m_\nu}{1\text{eV}}\right)^{-1/2} h^{-1}\text{Mpc}$$

Collisionless Damping on LSS



- density fluctuation suppressed at scales smaller than the damping scale.
- the amount of suppression depends on the mass of neutrinos
- the mass (and its hierarchy) of neutrinos can be estimated with such damping feature.
- non-linear features should be investigated with numerical simulations.

Initial Condition

- cosmological parameters

  PLANCK 2015 results : $\Omega_{\rm m} = 0.308,\ \Omega_\Lambda = 0.692,\ \Omega_{\rm b} = 0.0484,\ h = 0.678$

  curvature fluctuation : $A_{\rm s} = 2.3723 \times 10^{-9}$ (pivot scale : k = 0.002 Mpc^{-1})

- total neutrino mass

$$\sum_i m_i = 0.4\ \mathrm{eV}\ \ (\Omega_\nu = 0.0043 h^{-2})$$

- initial condition created at redshift of z_i = 10

- computaional domain: $L_{\rm box}$ = 20000h^{-1} Mpc, 2000h^{-1} Mpc, 200h^{-1} Mpc

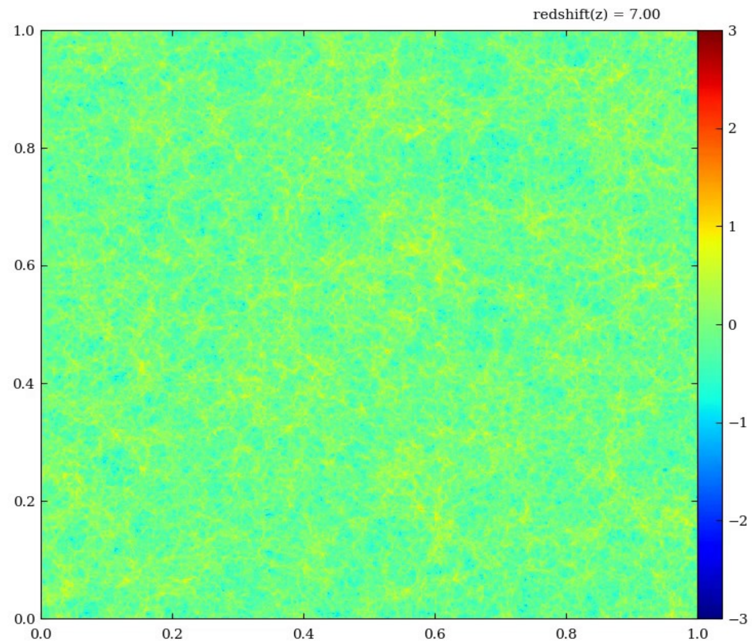
- number of particles / number of mesh grids

  N-body simulation : $N_{\rm p} = 1024^3$

  Vlasov simulation : $N_{\rm x} = 128^3, N_{\rm v} = 32^3$

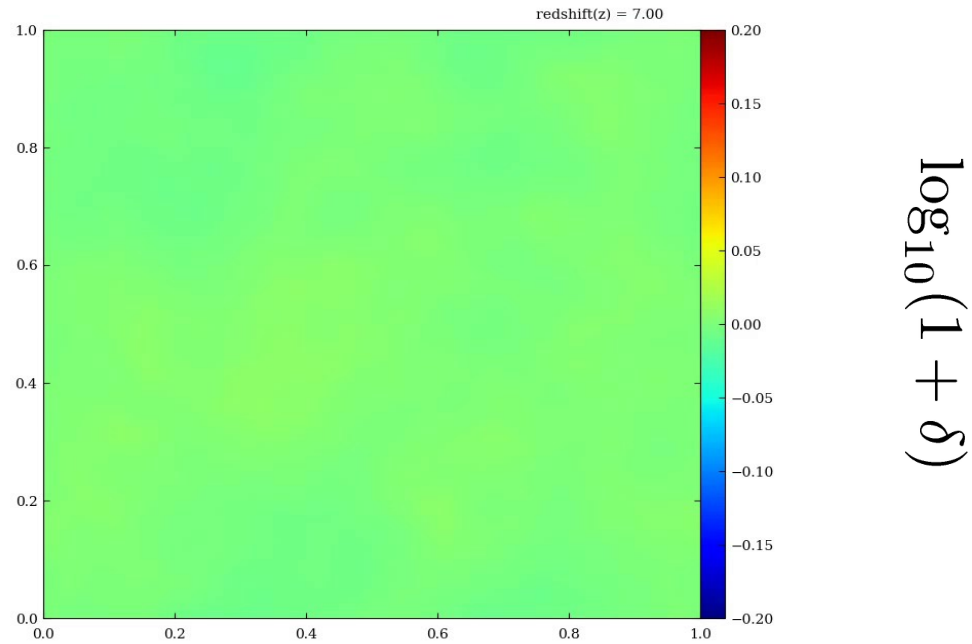
CDM and neutrino distribution

CDM



200 h^{-1} Mpc

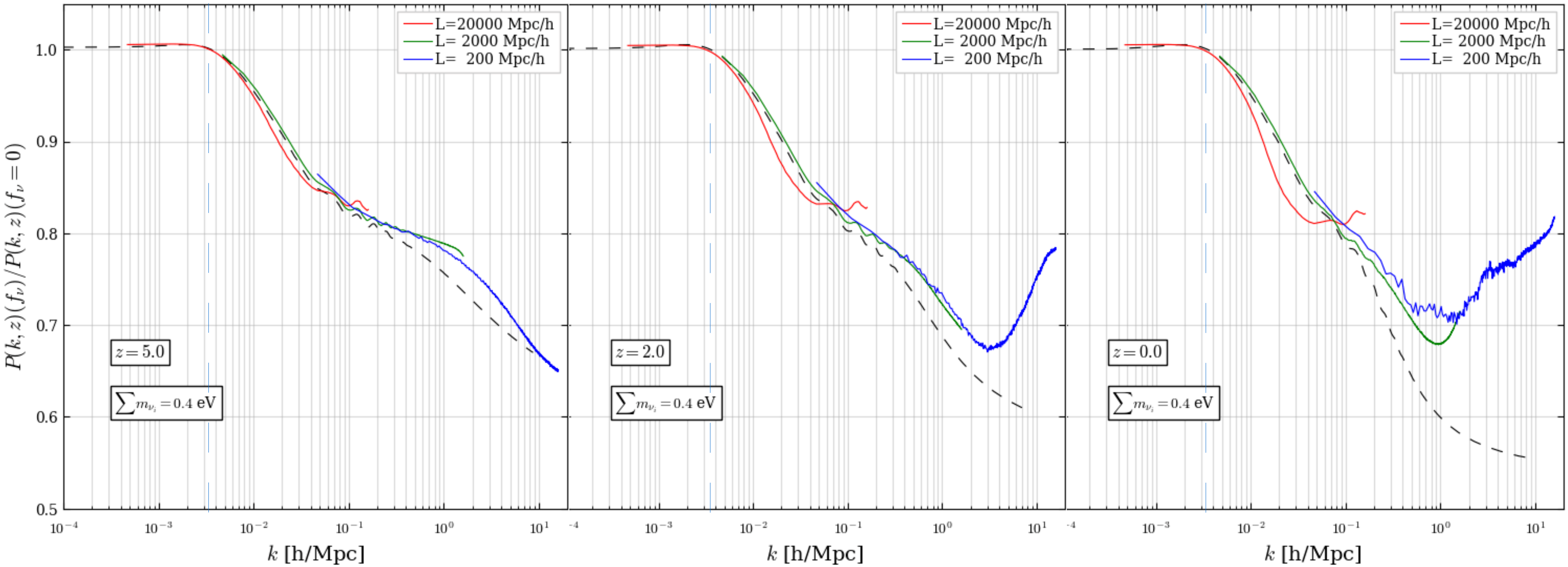
neutrino



200 h^{-1} Mpc

Power Spectrum

$$\frac{P(k, f_\nu)}{P(k, f_\nu = 0)}$$: ratio of power spectra with massive and massless neutrinos

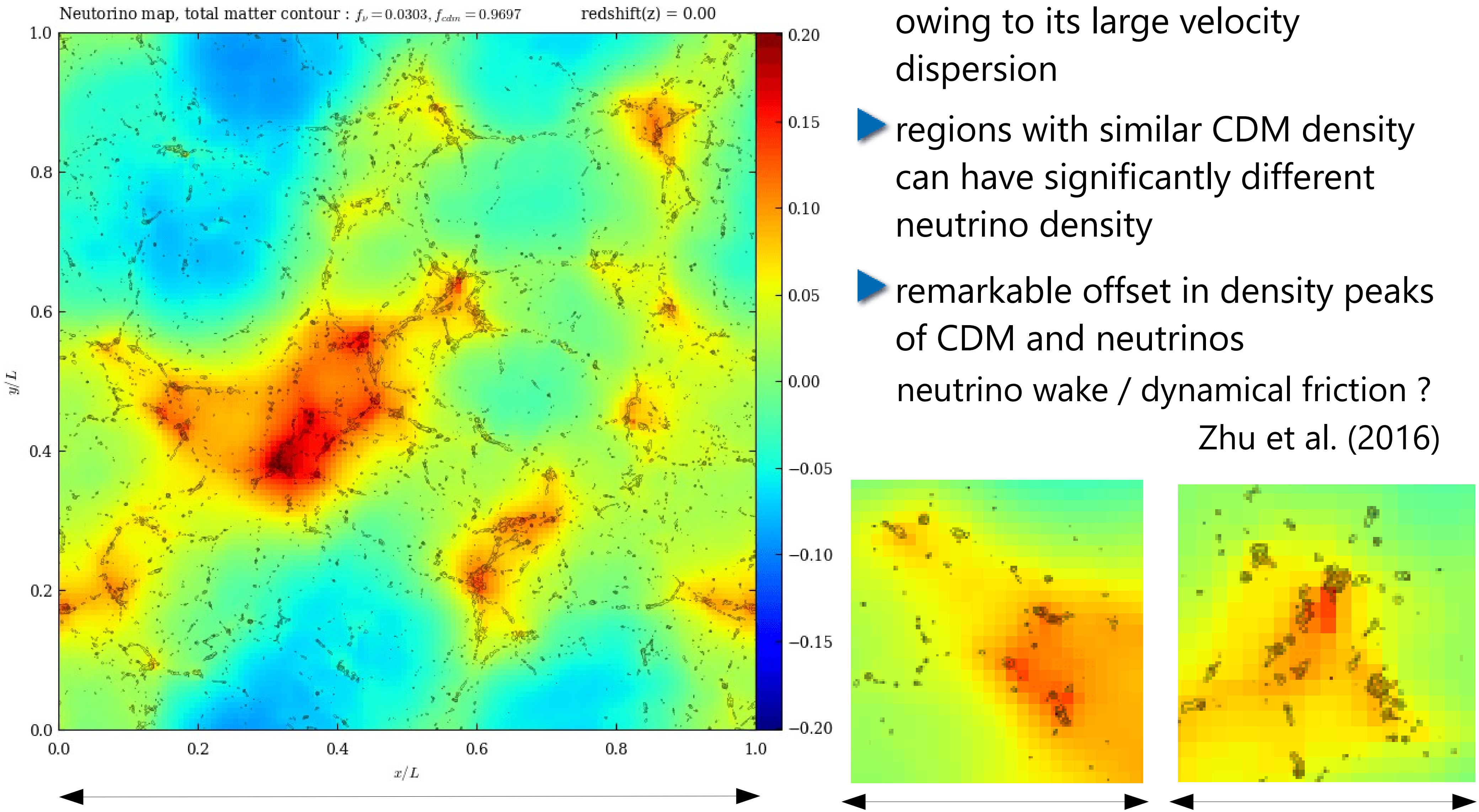


damping scale : $k_{\rm nr} = \sqrt{\frac{4\pi G\bar{\rho}(t_{\rm nr})a(t_{\rm nr})^2}{\sigma_\nu^2(t_{\rm nr})}} \simeq 0.018\Omega_{\rm m}^{1/2}\left(\frac{m}{1\,\rm eV}\right)^{1/2} h\ {\rm Mpc}^{-1} = 3.5\times10^{-3} h\,{\rm Mpc}^{-1}$

- density fluctuation with k > 3x10^{-2} h/Mpc damps owing to collisionless damping
- consistent with the perturbation theory (e.g. Saito et al. 2009) in the early stages
- up turn at k>1 h^{-1}Mpc, probably due to dynamical feedback by CDM

CDM and neutrino distribution

contour : CDM color : neutrino



200 h^{-1} Mpc 40 h^{-1} Mpc 20 h^{-1} Mpc

- diffuse distribution of neutrinos owing to its large velocity dispersion
- regions with similar CDM density can have significantly different neutrino density
- remarkable offset in density peaks of CDM and neutrinos

neutrino wake / dynamical friction ?

Zhu et al. (2016)

Vlasov-Maxwell Simulation

Vlasov-Maxwell Simulation

$$\frac{\partial f_s}{\partial t} + \boldsymbol{v} \cdot \frac{\partial f_s}{\partial \boldsymbol{x}} + \frac{q_s}{m_s}\left(\boldsymbol{E} + \boxed{\frac{\boldsymbol{v} \times \boldsymbol{B}}{c}}\right) \cdot \frac{\partial f_s}{\partial \boldsymbol{v}} = 0$$

$$\frac{\partial \boldsymbol{E}}{\partial t} = c\nabla \times \boldsymbol{B} - 4\pi \boldsymbol{J}$$

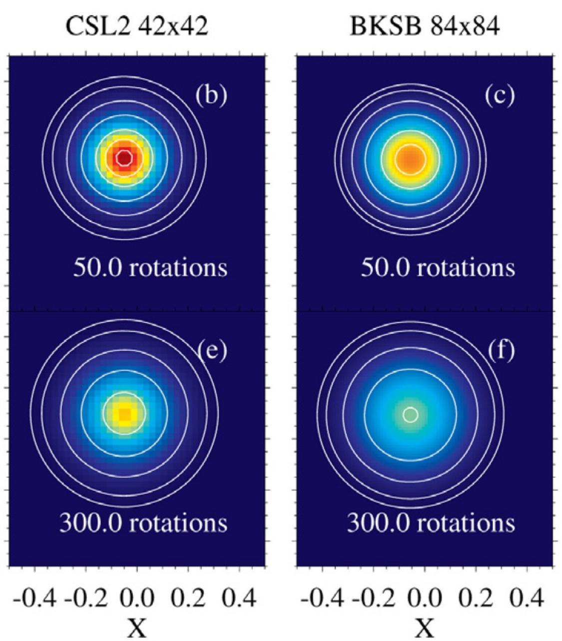
$$\boldsymbol{J} = \sum_{\mathrm{s}} \int q_s \boldsymbol{v} f_s \mathrm{d}^3 \boldsymbol{v}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = -c\nabla \times \boldsymbol{E}$$

- solve the distribution functions for both of ions and electrons
- difficulty to solve gyro-motion in the velocity space corrrectly in a long term
- rigid-body rotation problem

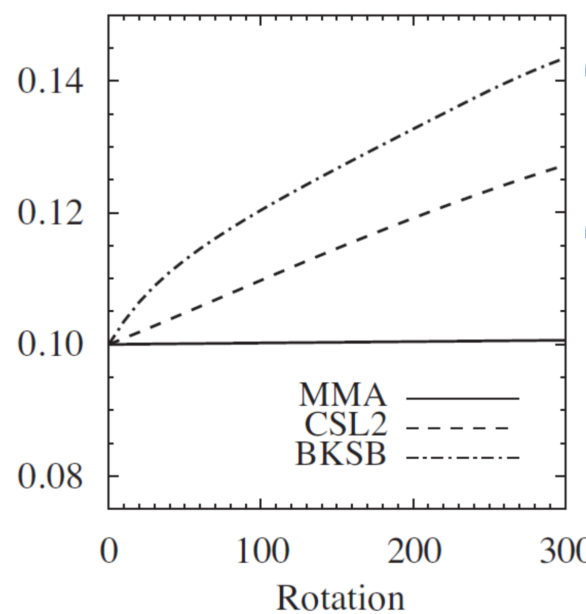
$$\frac{\partial f}{\partial t} + (\boldsymbol{r} \times \boldsymbol{\omega}) \cdot \frac{\partial f}{\partial \boldsymbol{r}} = 0$$

rigid-body rotation of a 2D gaussian profile



Minoshima, Matsumoto, Amano (2011)

temporal variation of dispersion



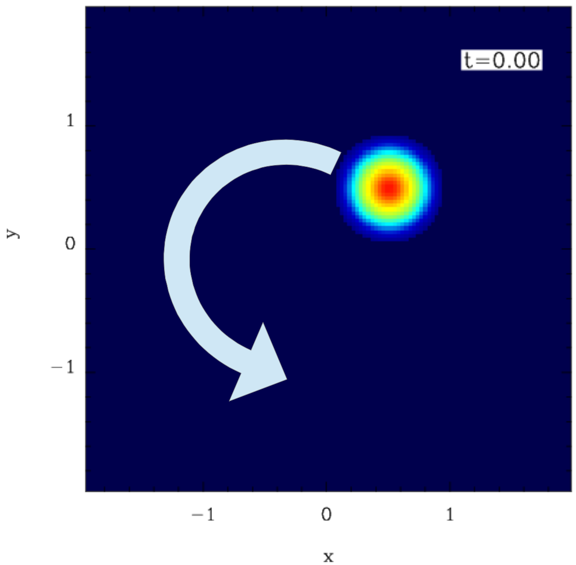
- CSL2 : CIP-CSL2

  Takizawa et al. (2002)
- BKSB : backsubstitution method

  Schmitz & Grauer (2006)

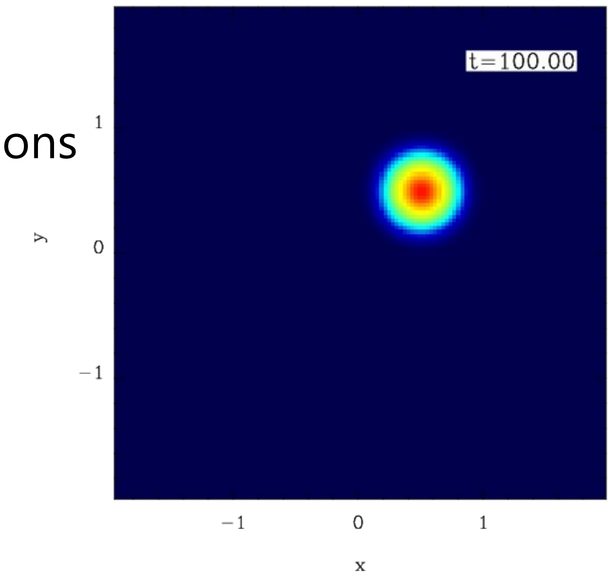
Rigid-Body Rotation with Our Scheme

initial condition

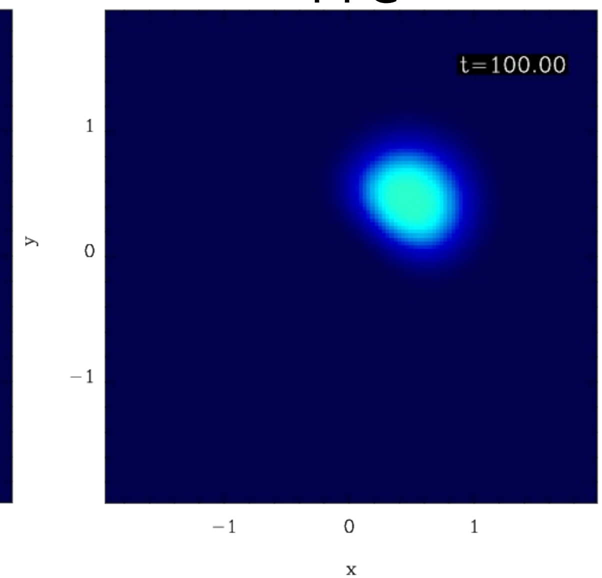


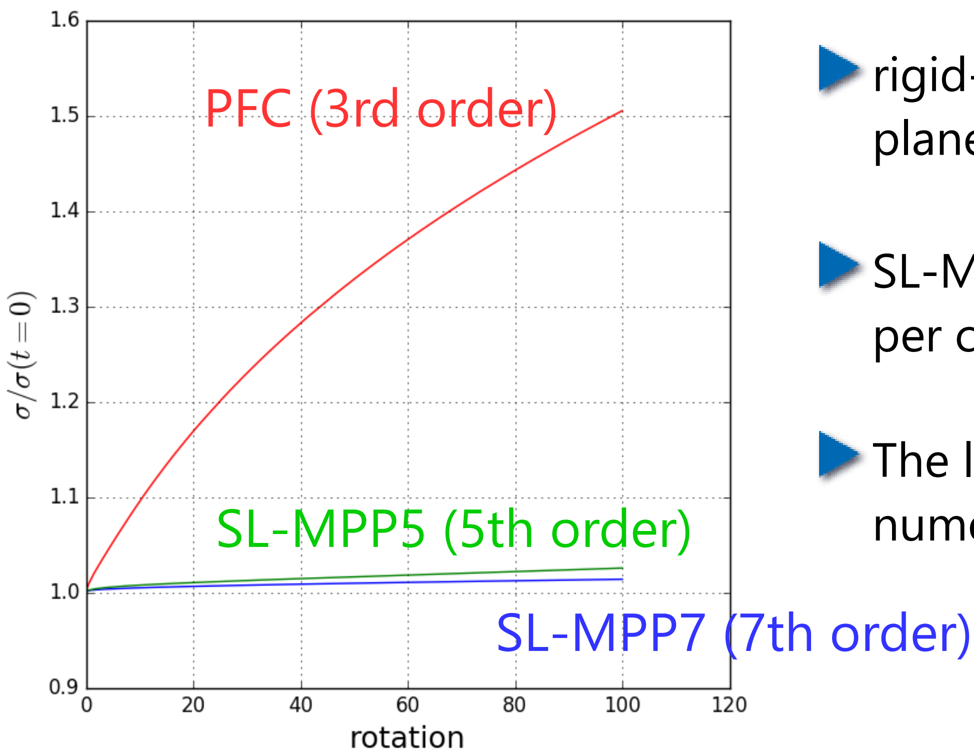
100 rotations

SL-MPP7



PFC



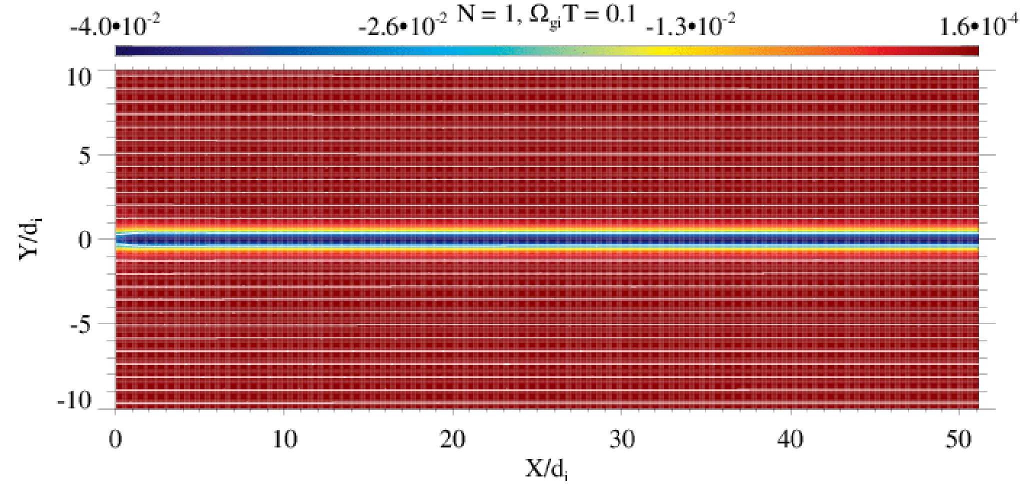


- rigid-body rotation of a gaussian profile in a 2D plane
- SL-MPP5 and SL-MPP7 schemes yields only a few per cent increase in velocity dispersion.
- The lower-order scheme suffers from significant numerical heating.

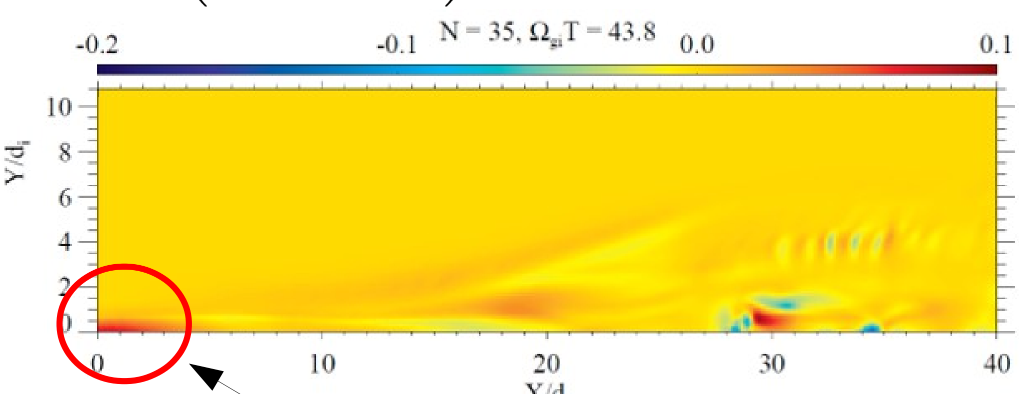
Magnetic Reconnection with Vlasov-Maxwell Simulations

- Vlasov-Maxwell simulation in 5D phase space
  - $N_x = 432$, $N_y = 216$
  - $N_{vx} = N_{vy} = N_{vz} = 32$
  - Hall effect to trigger the fast reconnection

color : $\boldsymbol{J}_z$

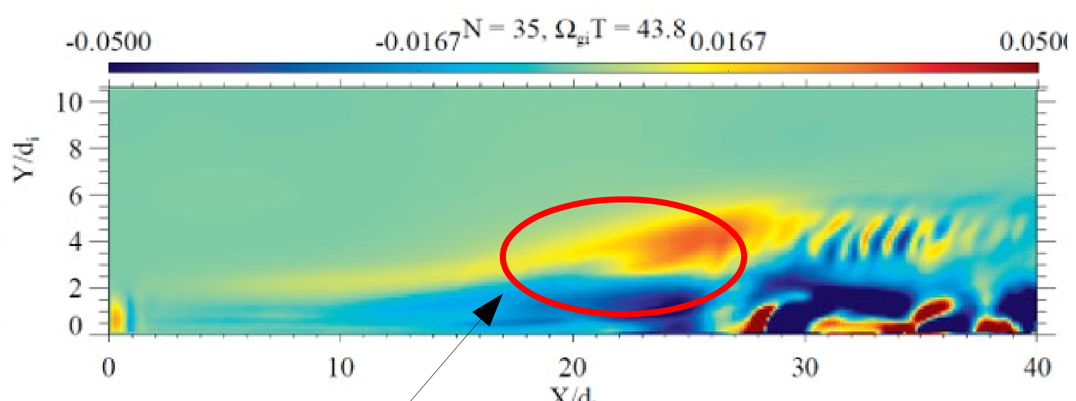


$$\boldsymbol{J} \cdot \left(\boldsymbol{E} + \frac{\boldsymbol{v} \times \boldsymbol{B}}{c} \right)$$



dissipation of magnetic field

$$\frac{\partial \rho v_x}{\partial t} + \nabla \cdot (\rho v_x \boldsymbol{v}) + \frac{\partial p}{\partial x} + \frac{q \rho v_z B_y}{mc}$$

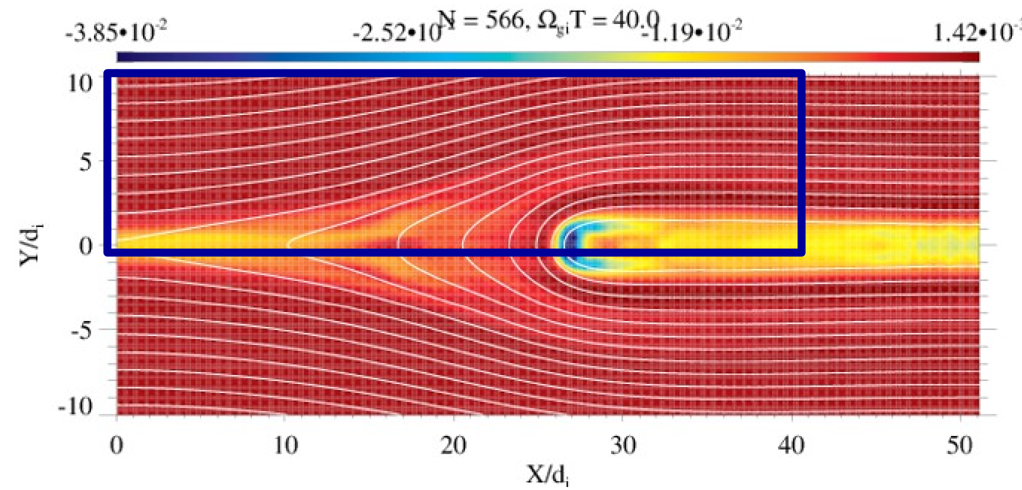


momentum transport to upstream region

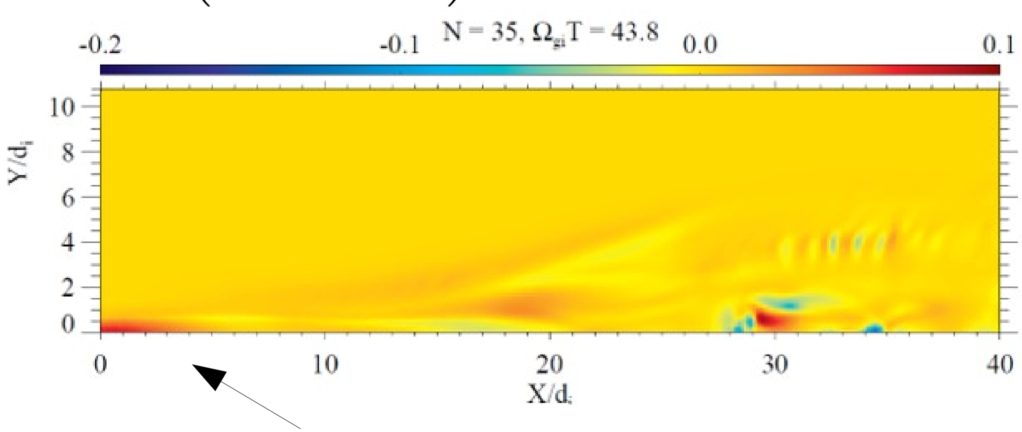
Magnetic Reconnection with Vlasov-Maxwell Simulations

- Vlasov-Maxwell simulation in 5D phase space
  - $N_x = 432$, $N_y = 216$
  - $N_{vx} = N_{vy} = N_{vz} = 32$
  - Hall effect to trigger the fast reconnection
  - Deviation from MHD approximation

color : $\boldsymbol{J}_z$

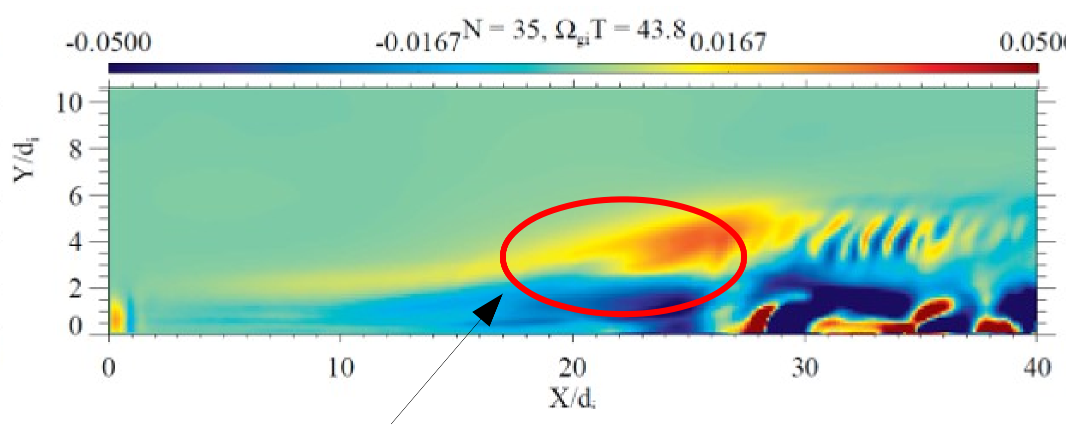


$$\boldsymbol{J}\cdot\left(\boldsymbol{E}+\frac{\boldsymbol{v}\times\boldsymbol{B}}{c}\right)$$



dissipation of magnetic field

$$\frac{\partial \rho v_x}{\partial t}+\nabla\cdot(\rho v_x \boldsymbol{v})+\frac{\partial p}{\partial x}+\frac{q\rho v_z B_y}{mc}$$



momentum transport to upstream region

Summary

- Vlasov simulations in 6-dimensional phase space are now practical.
- New high-order advection schemes with monotonicity- and positivity-preservation and with single-stage time integration
- Vlasov-Poisson simulation of cosmic neutrinos in the large-scale structure in the universe
- Our new schemes are also suitable for Vlasov-Maxwell simulation owing to good accuracy in rigid-body rotation.
- Vlasov-Maxwell simulations of magnetic reconnection in 5D phase space show kinetic features of magnetic recconection in a macroscopic MHD scale.