Beyond mean-field description of Gamow-Teller resonance and β-decay half-lives

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Gamow-Teller Resonance and $\beta$ decay

- **Gamow-Teller Resonance**

  \[ \sigma(0^\circ) = \delta F(q, \omega) B(GT) \]

  - Gamow-Teller transition operator
    \[ \hat{\sigma}_{GT} = \sum_{i=1}^{A} \tilde{\sigma}(i) \cdot \tau_-(i) \]
    - Transition probability
      \[ B(GT^-) = \sum_{\nu} |\langle \nu | \hat{O} | 0 \rangle|^2 \]
    - Transition strength $S = \text{smoothed } B$

- **$\beta$-decay** dominated by low-energy GT transition

  - $\beta$ decay
    - $Q_\beta = M_i \text{ (atomic)} - M_f \text{ (atomic)}$

  - Gamow-Teller strength distribution
    - $\beta$ decay window
How Were the Heavy Elements Made?

The 11 greatest unanswered questions of physics

Question 3
How were the heavy elements from iron to uranium made?

The r-process

The series of the maximum abundances in each Z-chain is called r-process path.

Rapid neutron capture process (r-process)

nuclear physics inputs: mass, β-decay half-lives, ...

setting the time-scale of r-process
• Development of radioactive ion-beam facilities => important advances
  • $^{78}$Ni and around Hosmer, et al., PRL 94, 112501, 2005; Xu, et al., PRL 113, 032505, 2014
  • very neutron rich Kr to Tc isotopes Nishimura, et al., PRL 106, 052502, 2011
  • Zn Ga isotopes Madurga et al., PRL 109, 112501, 2012
  • 110 neutron-rich nuclei across N=82 shell gap Lorusso, et al., PRL 114, 192501, 2015
  • 94 neutron-rich nuclei from Z=55-67 Wu, et al., PRL 118, 072701, 2017
• provide a good test ground for theoretical models
Lifetime Is a Hard Problem ...

**• RHB+QRPA**

![Graph showing comparison between experimental data and theoretical predictions for 78Ni]

- EXP
- \( V_0 = 0 \) MeV
- \( V_0 = 115 \) MeV
- \( V_0 = 330 \) MeV

**N=50**

**• Skyrme HFB+QRPA**

![Graph showing comparison between theoretical predictions and experimental data for 78Ni]

- HFB+QRPA+SkO’
- HFB+QRPA+SkO’, \( V_0=0 \)
- FRDM+QRPA
- EFTSI+QRPA
- Expt.

Niksic, et al., PRC 71, 014308 (2005)

Engel, et al., PRC 60, 014302 (1999)

- Half-lives are **overestimated**
- Due to the nuclear structure part – Gamow-Teller transition
Density Functional Theory

- Nucleus: quantum many-body system
- Density Functional Theory (DFT)

\[ E = \left\langle \Psi \right| \hat{H} \left| \Psi \right\rangle = \left\langle \Phi \right| \hat{H}_\text{eff} \left| \Phi \right\rangle = E[\hat{\rho}] \]

\( \left| \Phi \right\rangle \) Slater determinant \( \Leftrightarrow \hat{\rho} \) 1-body density matrix

\[ |\Phi\rangle = \prod_{i=1}^{A} a_i^\dagger |-\rangle \quad \hat{h} = \delta E/\delta \hat{\rho} \quad \hat{h}|\varphi_i\rangle = \epsilon_i|\varphi_i\rangle \]

many-body problem
\[ \downarrow \]
one-body problem
Skyrme Density Functional

- Effective Hamiltonian
  \[ \hat{H}_{\text{eff}} = \hat{T} + \hat{V}_{\text{eff}} \]

- Skyrme Effective Interaction

\[
V(r_1, r_2) = t_0(1 + x_0 P_\sigma) \delta(r) \quad \text{central term}
+ \frac{1}{2} t_1 (1 + x_1 P_\sigma) [P'^2 \delta(r) + \delta(r) P^2]
+ t_2 (1 + x_2 P_\sigma) P' \cdot \delta(r) P \quad \text{non-local term}
+ \frac{1}{6} t_3 (1 + x_3 P_\sigma) [\rho(R)]^{\alpha} \delta(r) \quad \text{density-dependent term}
+ i W_0 (\sigma_1 + \sigma_2) \cdot [P' \times \delta(r) P] \quad \text{spin-orbit term}
\]

\[
\begin{align*}
\text{r} &= r_1 - r_2, \\
\text{R} &= \frac{1}{2} (r_1 + r_2), \\
\text{P} &= \frac{1}{2i} (\nabla_1 - \nabla_2), \\
\text{P'} &= \text{cc of } \text{P} \text{ acting on the left} \\
P_\sigma &= (1 + \sigma_1 \cdot \sigma_2)/2.
\end{align*}
\]

- Around 11 parameters to fit the nuclear observables: \(t_i, x_i, W_0, \alpha\) (SkM*, SLy5, ...)
- Successful to describe almost the whole nuclear chart: g.s. and excited states
Random Phase Approximation (RPA)

- RPA: widely used model for the description of collective vibration
  
  - Small oscillation: linear limit of time dependent DFT theory
    
    \[ i\hbar \dot{\rho} = [\hbar[\rho] + f(t), \rho] \quad \rho(t) = \rho^{(0)} + \delta \rho(t) \]

    \[
    \begin{pmatrix}
    A_{mi,nj} & B_{mi,nj} \\
    -B_{mi,nj} & -A_{mi,nj}
    \end{pmatrix}
    = \Omega(v)
    \begin{pmatrix}
    X_{nj} \\
    Y_{nj}
    \end{pmatrix}
    \quad \delta \rho = \rho^{(1)} e^{-i\omega t} + \rho^{(1)} e^{i\omega t}
    \]

    \[ X e^{-i\omega t} - Y e^{i\omega t} \]

    Harmonic oscillation

  - Solution on basis

    The RPA excited state (collective vibration state) is generated by

    \[ Q^\dagger = \sum_{mi} X_{mi}^\nu a_m^\dagger a_i - \sum_{mi} Y_{mi} a_i^\dagger a_m \]

- RPA: magic nuclei
- Quasiparticle RPA (QRPA): superfluid nuclei
Limits of (Q)RPA Description

• (Q)RPA cannot describe the spreading width

- Spreading Width (Damping Width)
  - energy and angular momentum of coherent vibrations
  - more complicated states of 2p-2h, 3p-3h, ... character

• Correlations beyond RPA

- HF
- RPA
- 1p1h + ... + 2p2h + ... + ...
- 1p1h
- 2p2h
Solution: RPA + PVC

\[ HF \]

1p1h \[ \text{Low-lying vibration phonons } |N> \]

2p2h

RPA

- Second RPA drozdz et al., PR 197, 1 (1990)
  Gambacurta et al., PRC 81, 054312 (2010)
- RPA + PVC (particle vibration coupling)

\[ W_{\downarrow \, ph, p'h'}(\omega) = \sum_N \frac{\langle ph | V | N \rangle \langle N | V | p'h' \rangle}{\omega - \omega_N} \]

- RPA+PVC model based on Skyrme DFT
  Colo et al., PRC 50, 1496 (1994); Niu et al., PRC 85, 034314 (2012)
- RPA+PVC model based on relativistic DFT Litvinova et al., PRC 75, 064308 (2007)
RPA+PVC: Gamow-Teller Resonance

• Improved description of GT resonance in $^{208}$Pb


- Develop a spreading width
- Reproduce resonance lineshape
RPA+PVC: $\beta$-Decay Half-Lives

• Improved description of $\beta$-decay half-lives

![Graph showing $T_{1/2}$ for different isotopes with experimental and theoretical data.

- Reduce half-lives systematically
- Reproduce $\beta$-decay half-lives

How PVC reduces half-lives?

\[ T_{1/2} = \frac{D}{g_A^2 \int_{Q_\beta}^0 S(E)f(Z, \omega)dE}, \]  

\[ f(Z, \omega_0) = \frac{1}{(m_e c^2)^5} \int_{m_e c^2}^{\omega_0} p_e E_e (\omega_0 - E_e)^2 F_0(Z + 1, E_e)dE_e. \]

Exp.: Xu, et al., PRL 113, 032505, 2014
How PVC reduces half-lives?

- With the inclusion of PVC, the RPA energy is shifted downwards by about 2 MeV.
- The strength of each peak is basically kept conserved as the RPA case.
- Due to the big increase in phase volume, the contribution to the half-life also changes a lot from RPA to PVC.

Although the PVC doesn’t change the strength of each peak, it reduces the half-life dramatically by shifting downwards the excitation energy.
RPA+PVC: only for magic nuclei...

To include pairing correlations for superfluid nuclei

Quasiparticle RPA + quasiparticle vibration coupling (QRPA) + (QPVC)
✓ for the study of Gamow-Teller resonance in superfluid nuclei
✓ for the study of β-decay half-lives of the whole isotopic chain
The QRPA+QPVC equation reads

\[
\begin{pmatrix}
\mathcal{D} + \mathcal{A}_1(\omega) & \mathcal{A}_2(\omega) \\
-\mathcal{A}_3(\omega) & -\mathcal{D} - \mathcal{A}_4(\omega)
\end{pmatrix}
\begin{pmatrix}
F(\nu) \\
\bar{F}(\nu)
\end{pmatrix}
= (\Omega_\nu - i \frac{\Gamma_\nu}{2})
\begin{pmatrix}
F(\nu) \\
\bar{F}(\nu)
\end{pmatrix},
\]

where \( \mathcal{D} = E_{n} \), and the \( \mathcal{A}_i \) matrices contain the spreading contributions, e.g.,

\[
(\mathcal{A}_1)_{mn} = \sum_{a,b,a',b'} W^{\dagger}_{ab,a'b'}(E) X^{(m)}_{ab} X^{(n)}_{a'b'} + W^{\dagger,*}_{ab,a'b'}(-E) Y^{(m)}_{ab} Y^{(n)}_{a'b'},
\]

The GT strength function

\[
S(E) = -\frac{1}{\pi} \text{Im} \sum_{\nu} \langle 0 | \hat{O}_{\text{GT}} | \nu \rangle^2 \frac{1}{E - \Omega_\nu + i \left( \frac{\Gamma_\nu}{2} + \Delta \right)}
\]
The matrix elements of the spreading term in the quasiparticle basis

\[
W_{ab,a'b'}^\downarrow = \langle ab|V \frac{1}{E - \hat{H}} V |a'b'\rangle = \sum_{NN'} \langle ab|V |N\rangle \langle N| \frac{1}{E - \hat{H}} |N'\rangle \langle N'|V |a'b'\rangle,
\]

where \(|N\rangle = |a''b''\rangle \otimes |nL\rangle\) represents a doorway state, and \(a, b\) are quasi-particle states.

The vertex reduced matrix element:

\[
\langle a||V||a'', nL\rangle = \frac{\hat{L}}{\sqrt{1 + \delta_{cd}}} \sum_{cd} \left[ V(c d L a''; a) X_{cd}^{nL} + (-1)^{j_a - j_{a''} + nL} V(c d L a; a'') Y_{cd}^{nL} \right],
\]

\[
\tilde{V}(c d L a''; a) = V_{ada''c}^{Lph}(u_a u_{a''} u_c v_d - v_a v_{a''} v_c u_d)
\]

\[
+ V_{aca''d}^{Lph}(u_a u_{a''} v_c u_d - v_a v_{a''} u_c v_d)(-1)^{j_c - j_a + nL}
\]

\[
- V_{ad'cd}^{Lpp}(u_a v_{a''} u_c u_d - v_a u_{a''} v_c v_d)
\]
Isoscalar Pairing

Particle-particle interaction

\[ V_{T=1}(r_1, r_2) = V_0 \frac{1 - P_\sigma}{2} \left( 1 - \frac{\rho(r)}{\rho_0} \right) \delta(r_1 - r_2), \]

\[ V_{T=0}(r_1, r_2) = f V_0 \frac{1 + P_\sigma}{2} \left( 1 - \frac{\rho(r)}{\rho_0} \right) \delta(r_1 - r_2), \]

- \( f = 0 \) : without isoscalar pairing
- \( f = 1 \) : with isoscalar pairing

Effect of isoscalar pairing

- QRPA level
  - Increase the low-lying strength
  - Decrease the splitting between two high-lying states

- QRPA+QPVC level
  - Increase the low-lying strength
  - Similar profile

Niu, Colo, Vigezzi, Bai, Sagawa, PRC 94, 064328 (2016)
GT Strength Distribution

- ($^3\text{He},t$) data: cross section $\times 1.6$ so that the main GTR strength exhausts 65% sum rule
  Pham, et al., PRC 51, 526 (1995)
  overestimate the low-lying strength

- ($p,n$) data: normalized by unit cross section
  Sasano, et al., PRC 79, 024602 (2009)

- QRPA + QPVC
  - Develop a width of 5.3 MeV (6.4 MeV from exp.), reproduce exp. profile in GTR
  - Overestimate the low-lying strength

$$\sigma(0^\circ) = \hat{\sigma} F(q,\omega) B(\text{GT})$$
• $f=0 \rightarrow f=1$
  ✓ low-lying strength is increased for both QRPA and QRPA + QPVC

• QRPA $\rightarrow$ QRPA+QPVC:
  ✓ better reproduces the exp. profile
  ✓ cumulative strength is quenched by 10% at $E=25$ MeV
  ✓ QRPA+QPVC strength $\times 0.75 = \text{exp. strength ((p,n) data)}$ at $E=25$ MeV

Niu, Colo, Vigezzi, Bai, Sagawa, PRC 94, 064328 (2016)
β-Decay Half-Lives in Ni isotopes

- Isoscalar Pairing: not so effective for Ni isotopes (nuclei before N=50 closed shell)
- QPVC: reduce the half-lives
β-Decay Half-Lives in Sn isotopes

- Isoscalar Pairing: effective for Sn isotopes (nuclei above N=82 closed shell)
- Isoscalar Pairing + QPVC: reduce the half-lives
Summary and Perspectives

Summary
• Gamow-Teller transition is an important spin-isospin mode of nucleus; β-decay is mainly determined by low-energy GT transition
• Going beyond RPA: RPA+PVC
  ✓ Spreading width
  ✓ Reduce the half-lives
• Going beyond QRPA + pairing correlations: QRPA+QPVC
  ✓ Effect of QPVC
  ✓ Effect of isoscalar pairing

Perspectives
• Deformation Effect
• Forbidden transitions
Thank you!