

Doubly Magic Drip-line Nucleus ^{48}Si — Novel Phenomena & New Physics —

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Collaborators: J. J. Li, N. Van Giai, J. Margueron, et. al.

- 1 Introduction and Motivation**
 - Covariant Density Functional theory
 - Challenges in Nuclear Physics
 - New Magicity and Bubbles in Ca and Si isotopes
- 2 Covariant density functional theory with Fock terms**
 - Relativistic Hartree-Fock (RHF) theory
 - Relativistic Hartree-Fock-Bogoliubov (RHFB) theory
- 3 New Physics in determining the magicity of ^{48}Si**
 - Bubble and magic shells
 - Self-consistent tensor force effects in magicity
 - Neutron and/or proton crossing-shell excitations
- 4 Conclusions and Perspectives**

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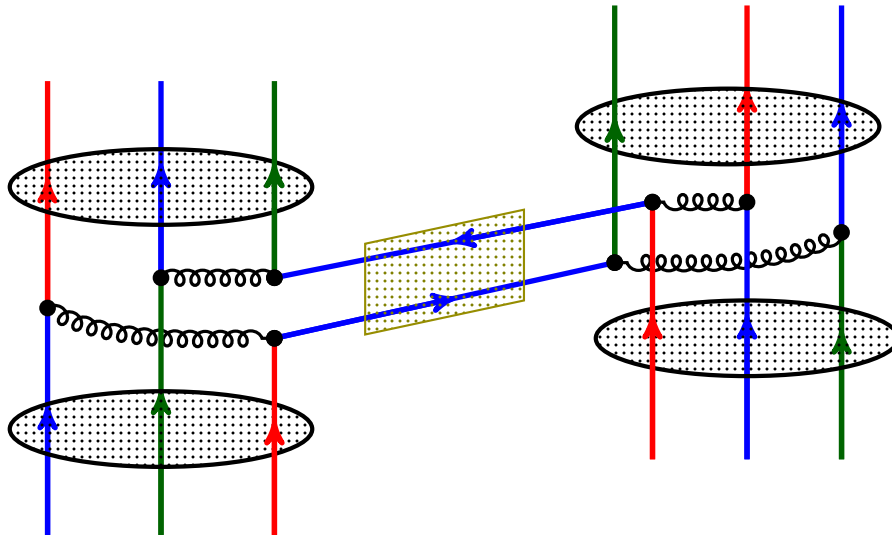
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Nuclear Force: Meson Exchange Diagram

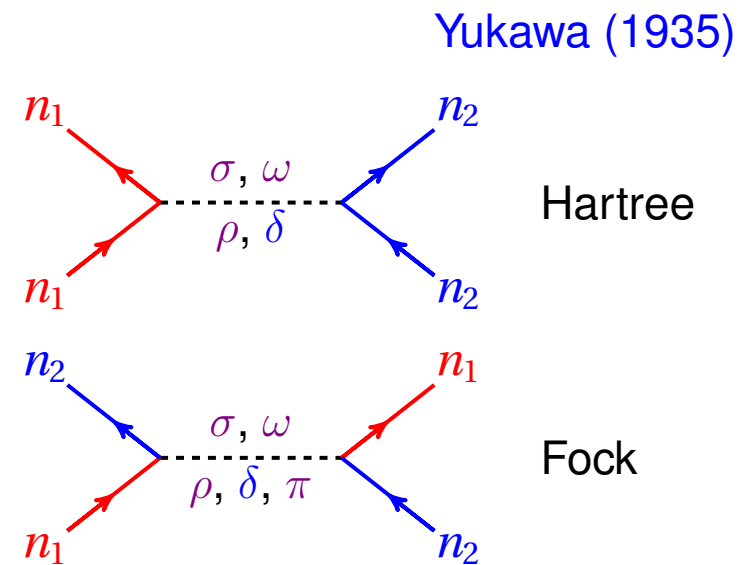
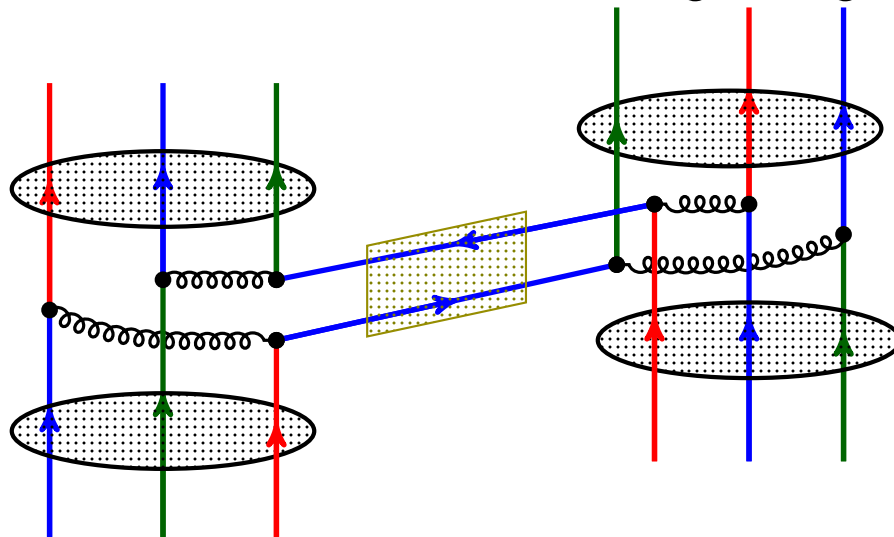
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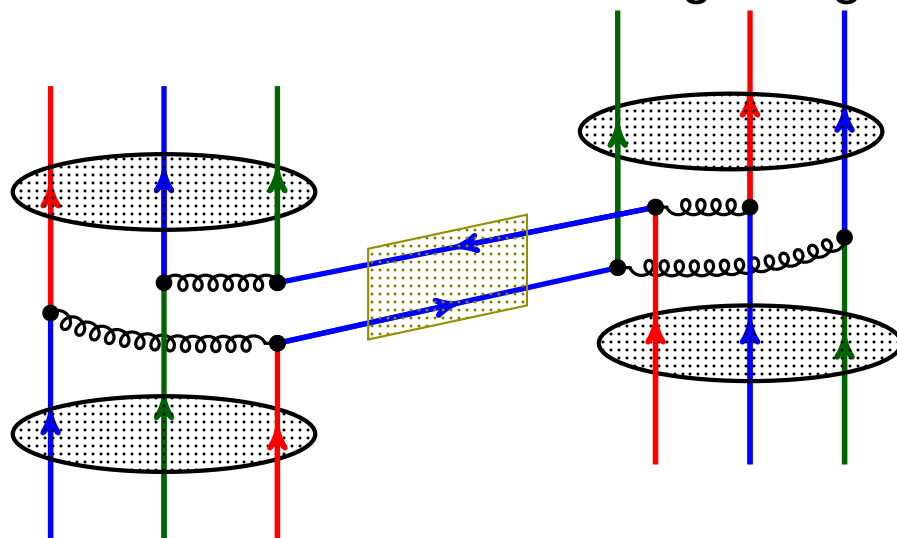
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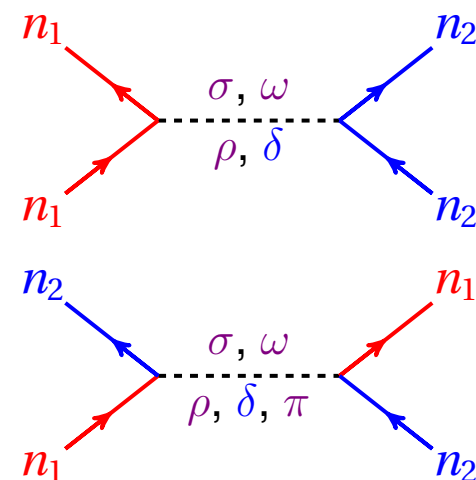


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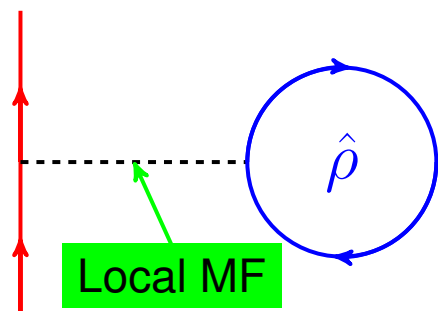
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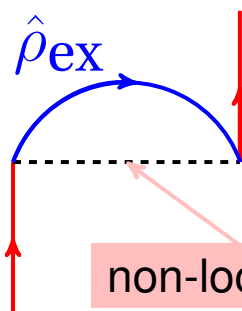
Hartree

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Mean field (MF) approach: nucleon moving in the MF generated by others — Being consistent with principle of density functional theory



Local MF

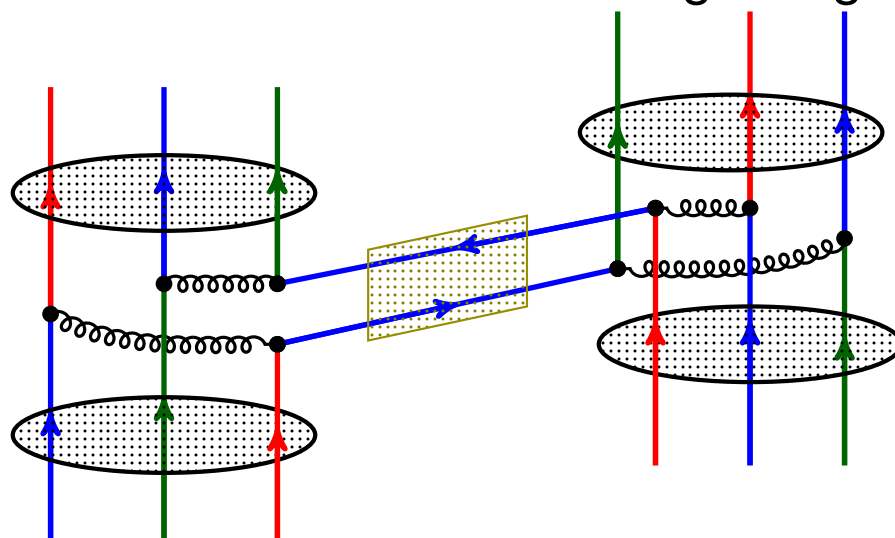


non-local MF

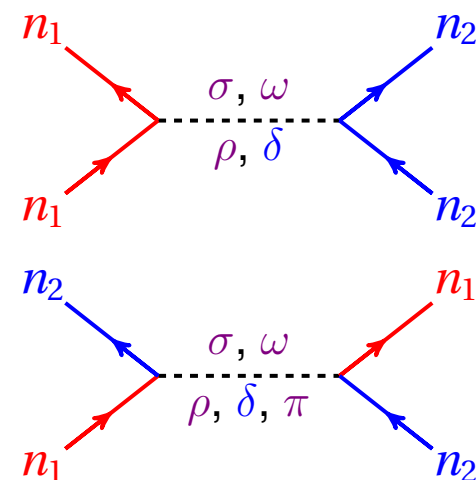
 $\hat{\rho}$: local density $\hat{\rho}_{\text{ex}}$: non-local density

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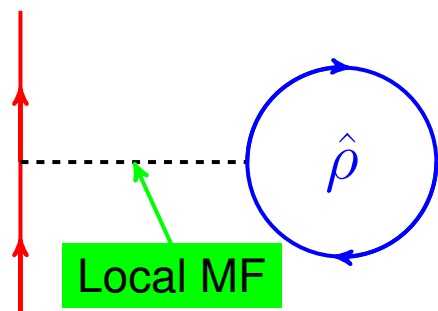
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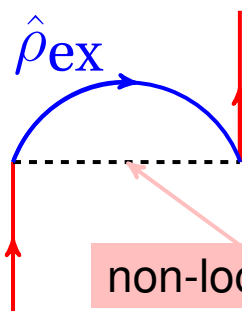
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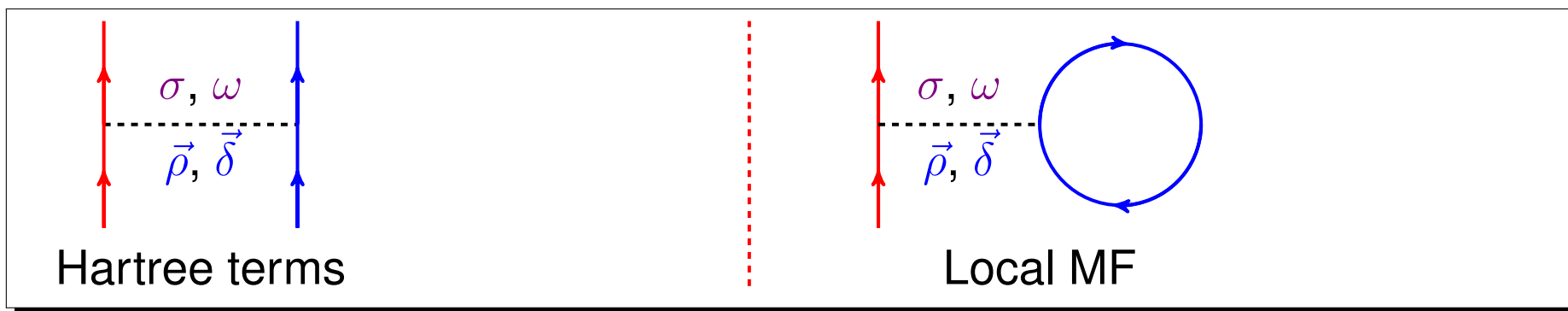
Complicated nuclear in-medium effects: non-perturbative nuclear force?

Covariant Density Functional (CDF) theory

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- CDF theory w/o Fock terms: relativistic mean field (RMF) theory
 - Walecka(1974), Serot(1986), Reihard(1989), Ring(1996), Bender(2003), Meng(2006)
 - Natural treatment of spin-orbit coupling: Covariant framework
 - Limited by Hartree approach, important ingredients, e.g., tensor force, are missing.



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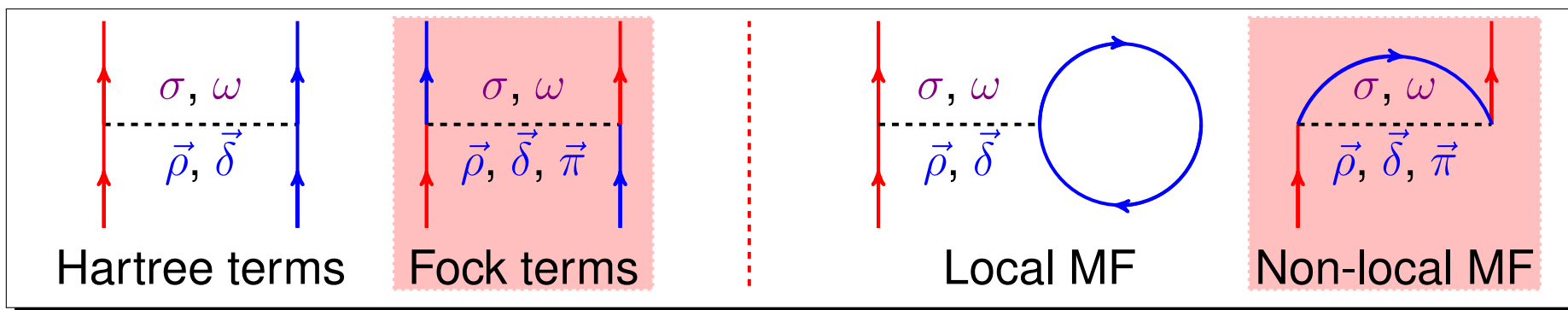
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CDF theory with Fock terms: relativistic Hartree-Fock (RHF) theory

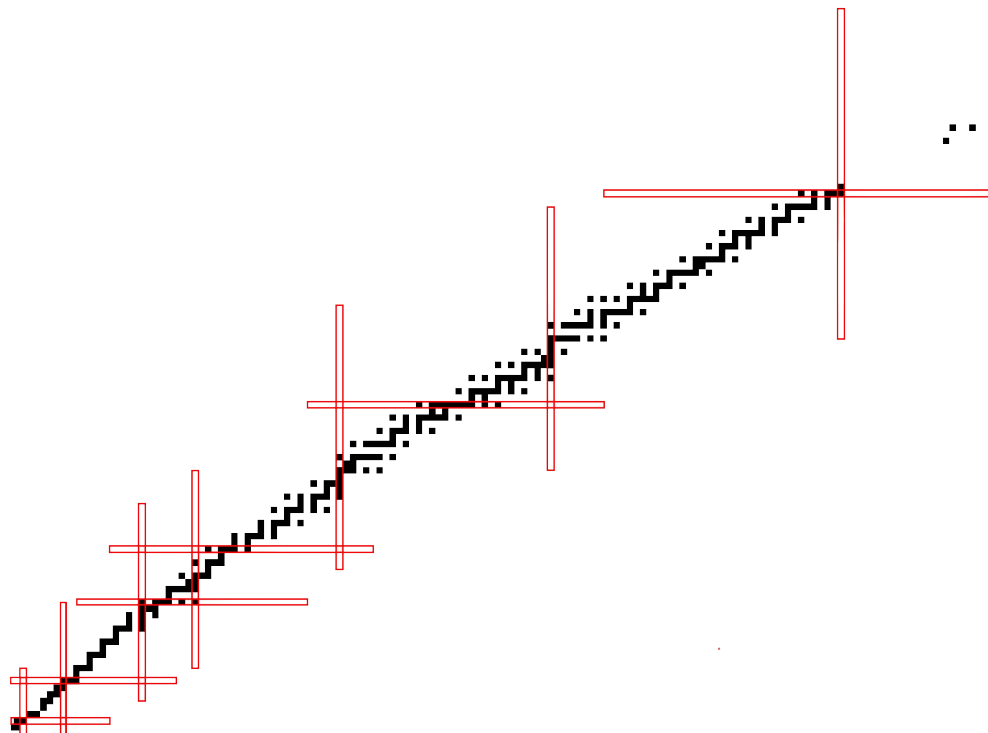
Bouyssy (1987), Bernardos (1993), Shi (1995), Marcos (2004), Long (2006-2010).

- Maintain the advantages of RMF theory, and **include the tensor force naturally**.
- Non-local Fock terms are not easy to handle.**



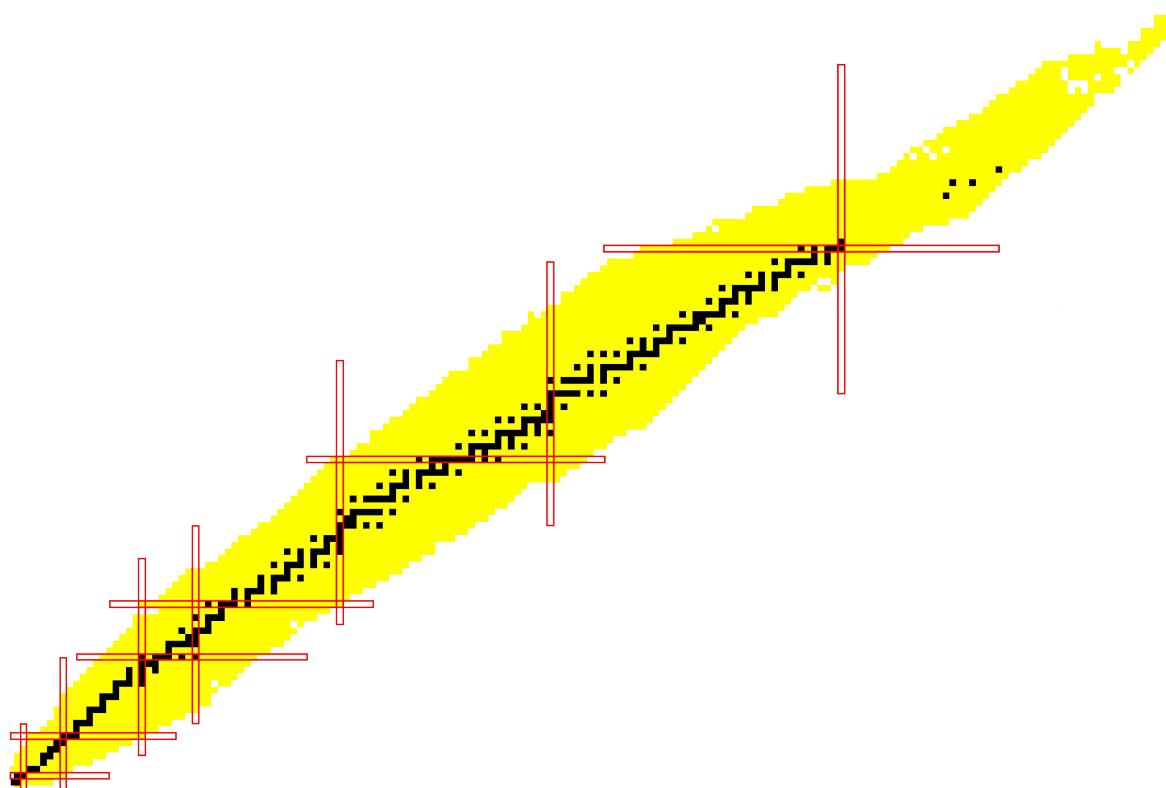
New Challenges and Opportunities

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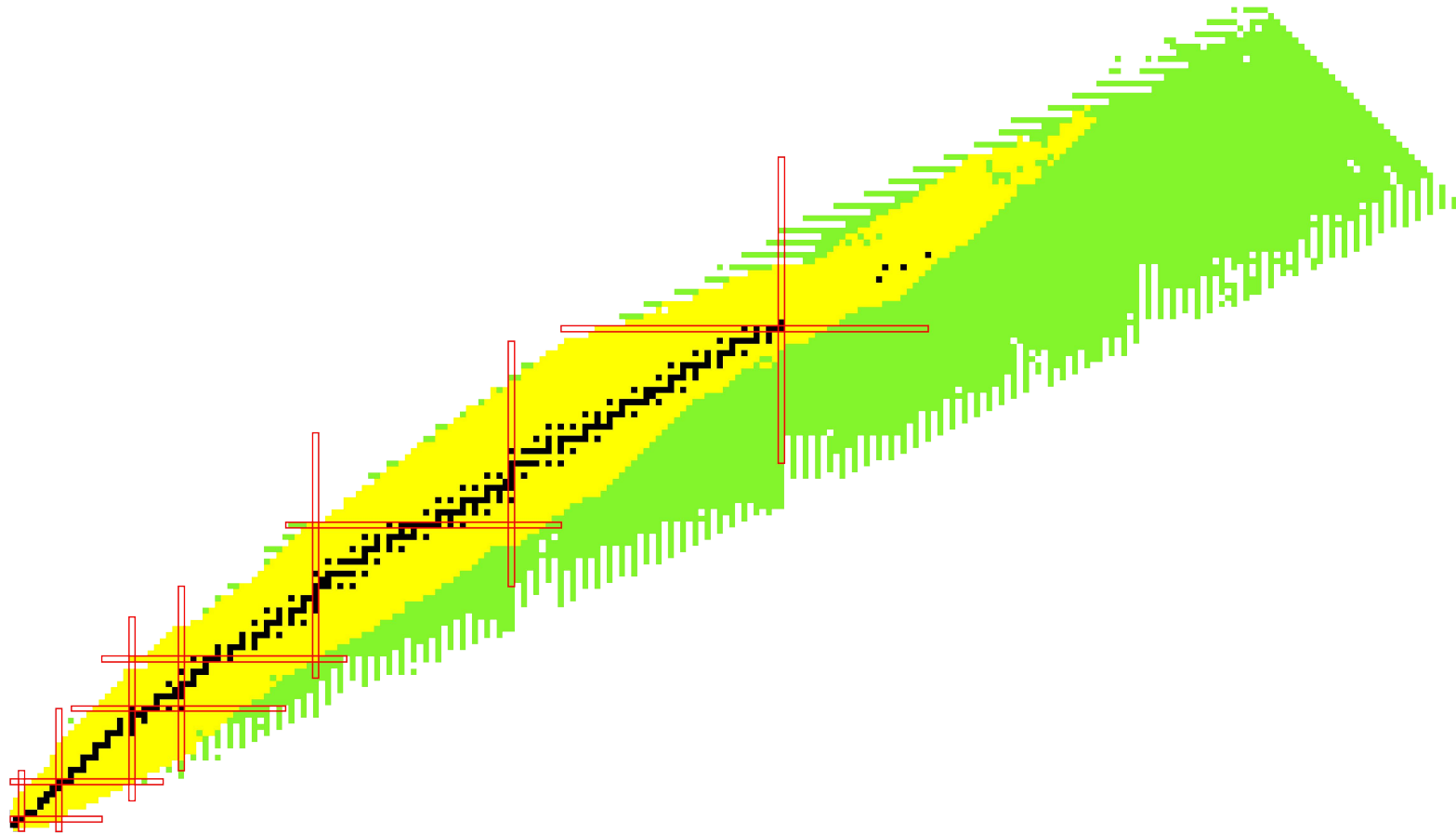
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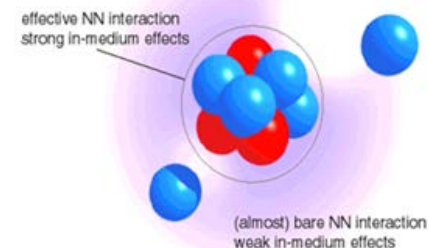
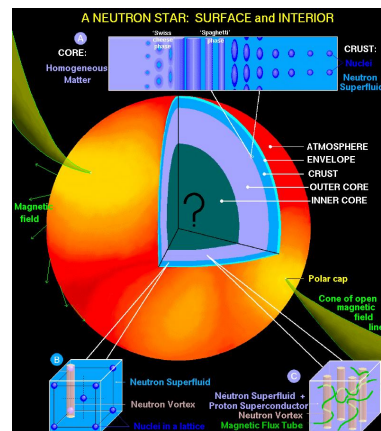
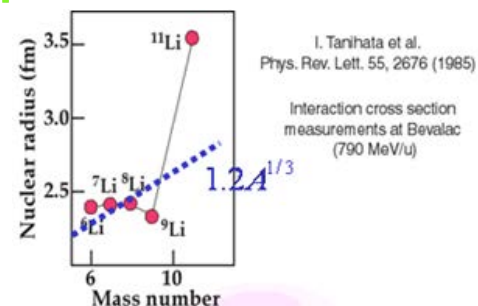
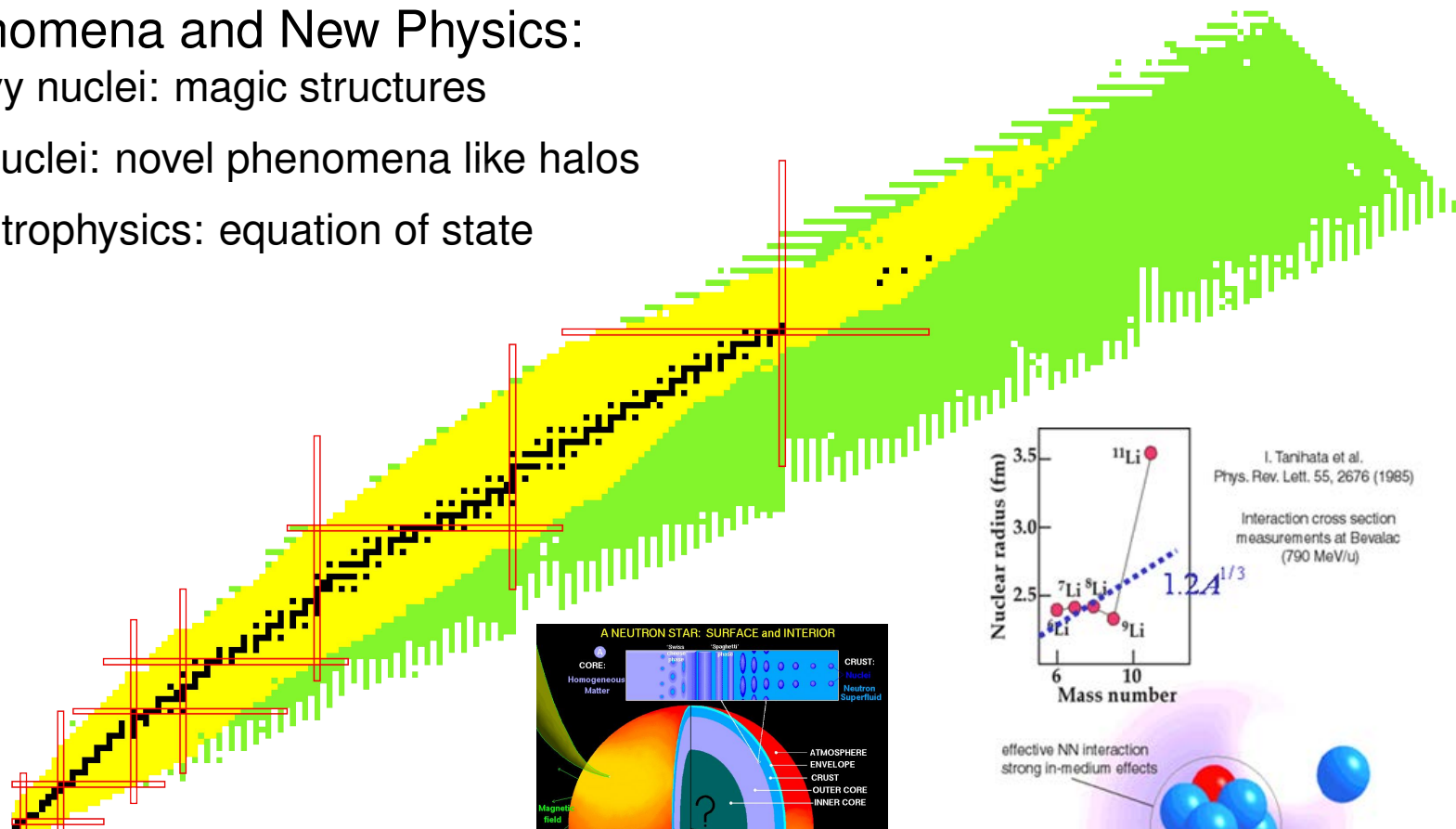


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- 👉 Unstable nuclei: novel phenomena like halos
- 👉 Nuclear astrophysics: equation of state



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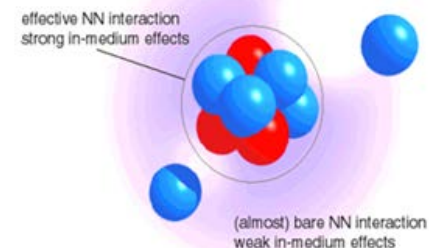
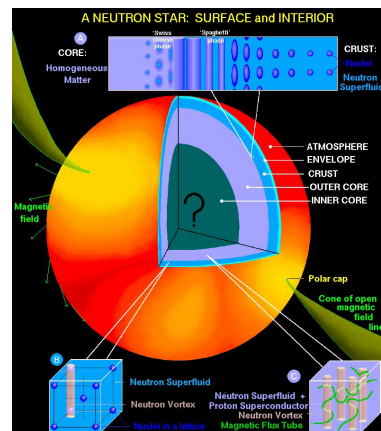
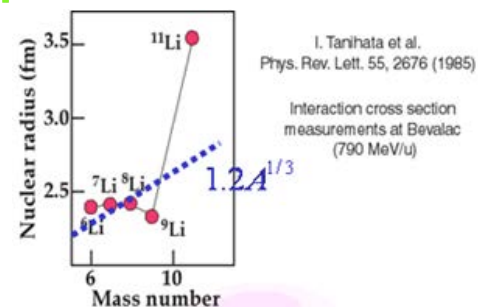
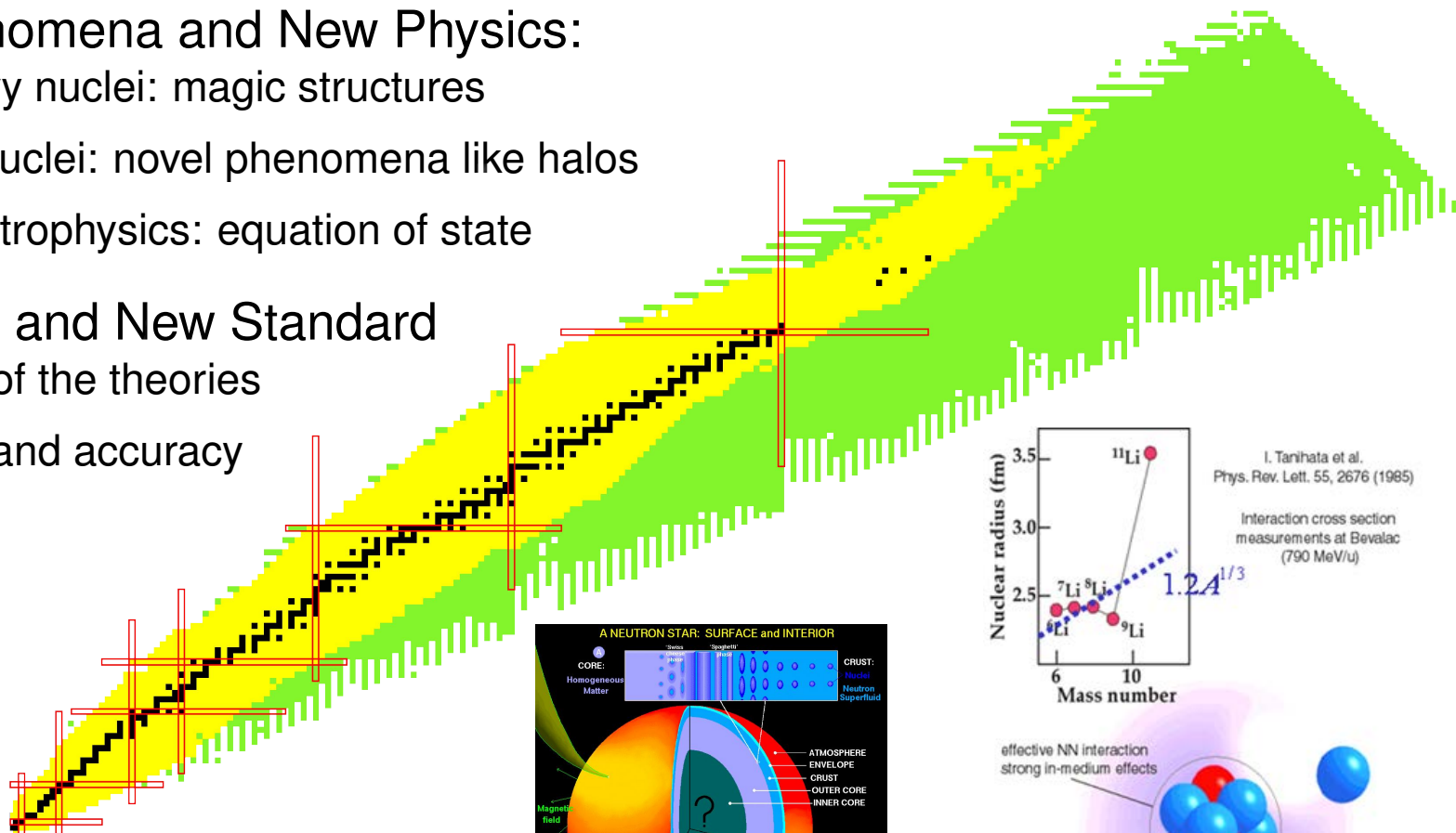
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- ☞ Limitation of the theories
- ☞ Reliability and accuracy



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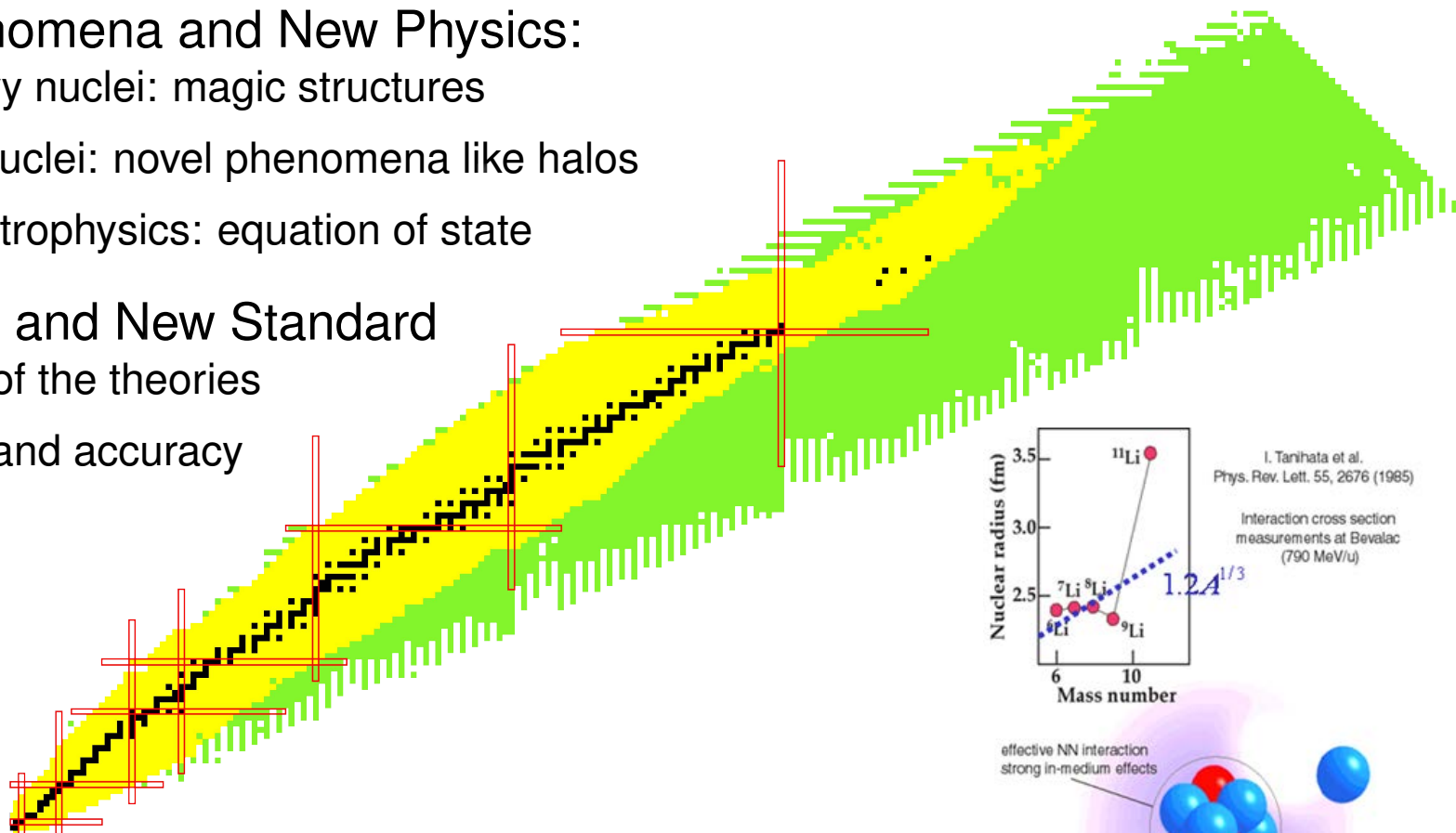
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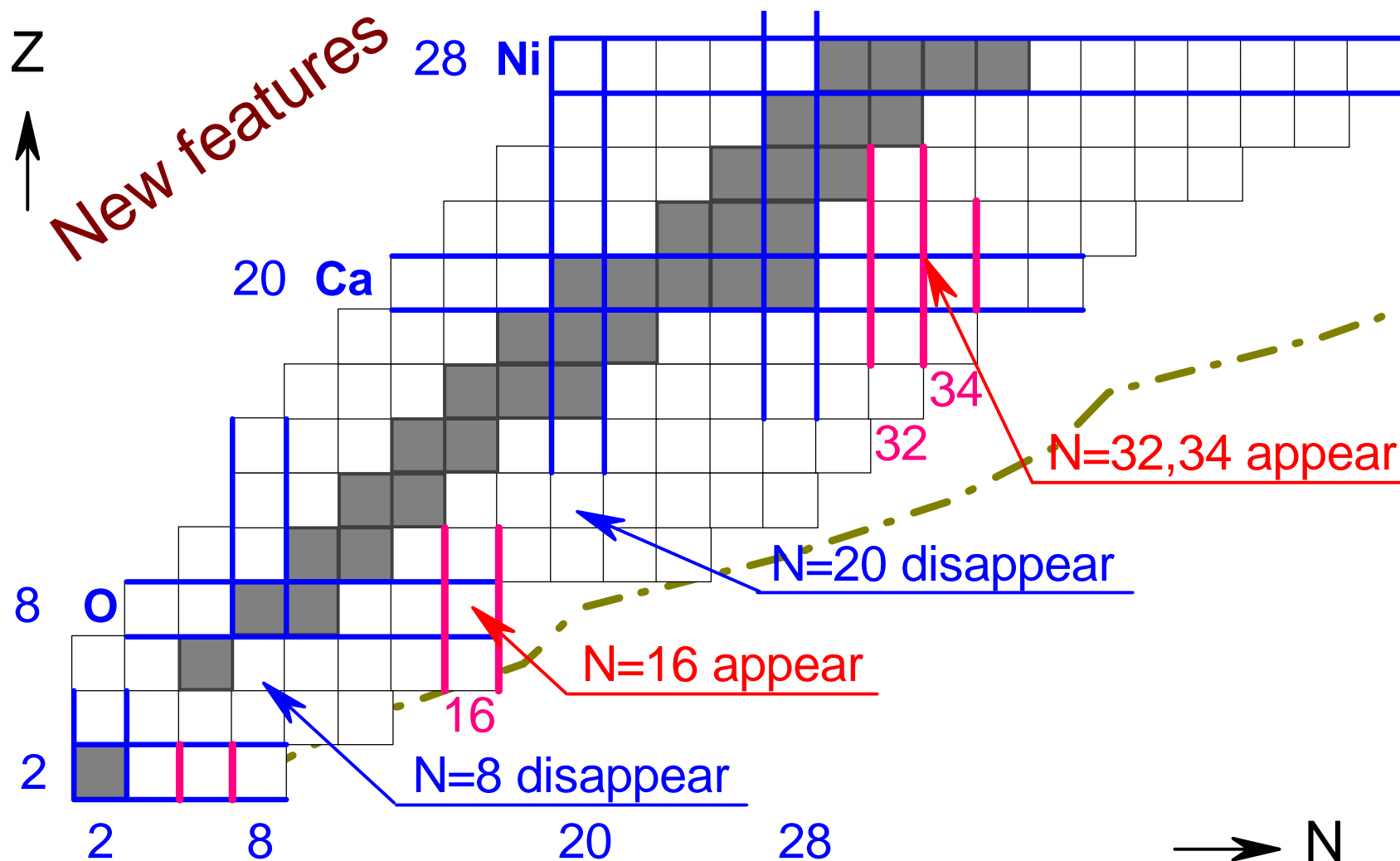
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● Covariant Density Functional theory

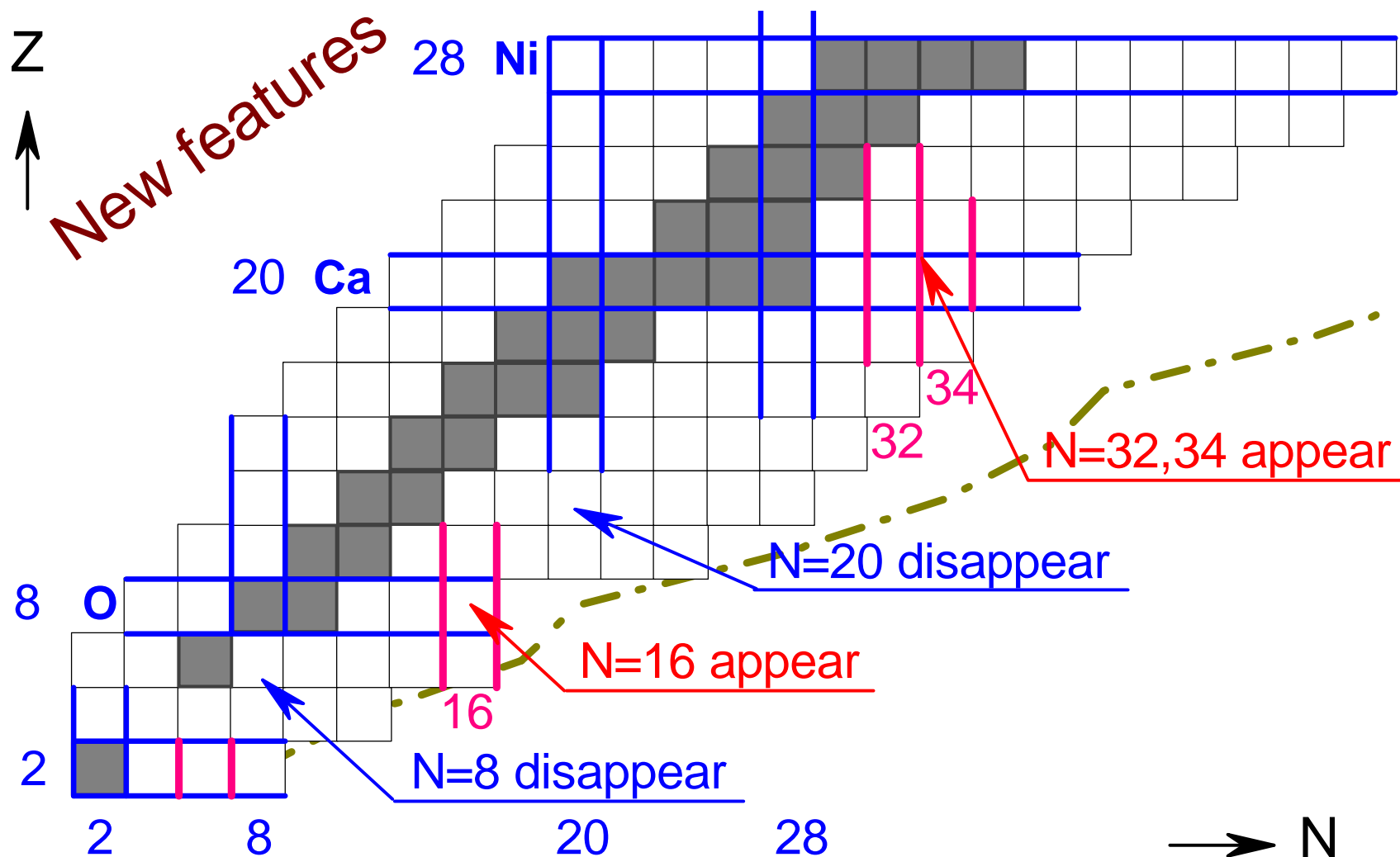
- ☞ Foundation: meson exchange diagram of nuclear force
- ☞ Reliability: relativity and spirit of density functional theory


Magicity in neutron-rich mid-mass nuclei



 Ozawa (2000); Sorlin (2008); Hoffman (2008); Steppenbeck (2013, 2015); Wienholtz (2013).

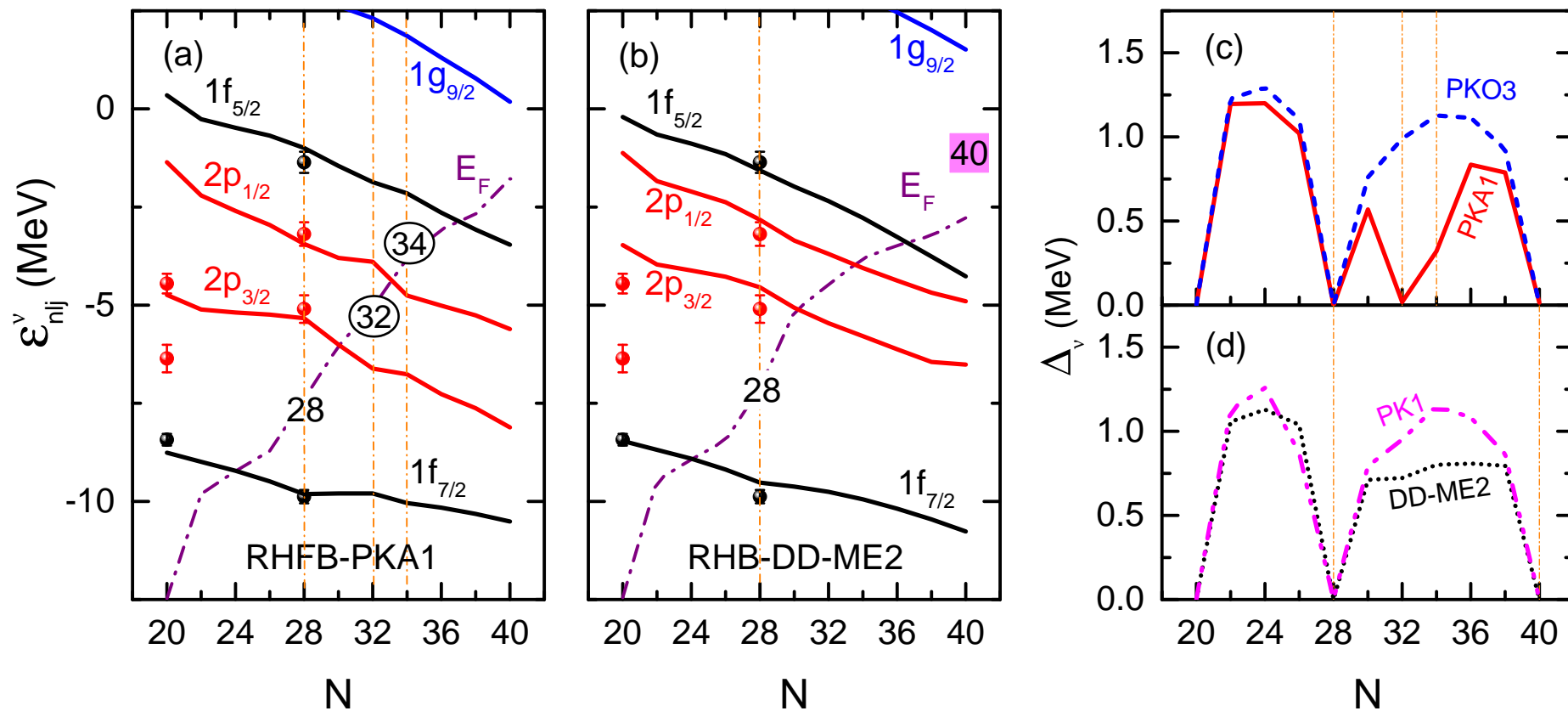
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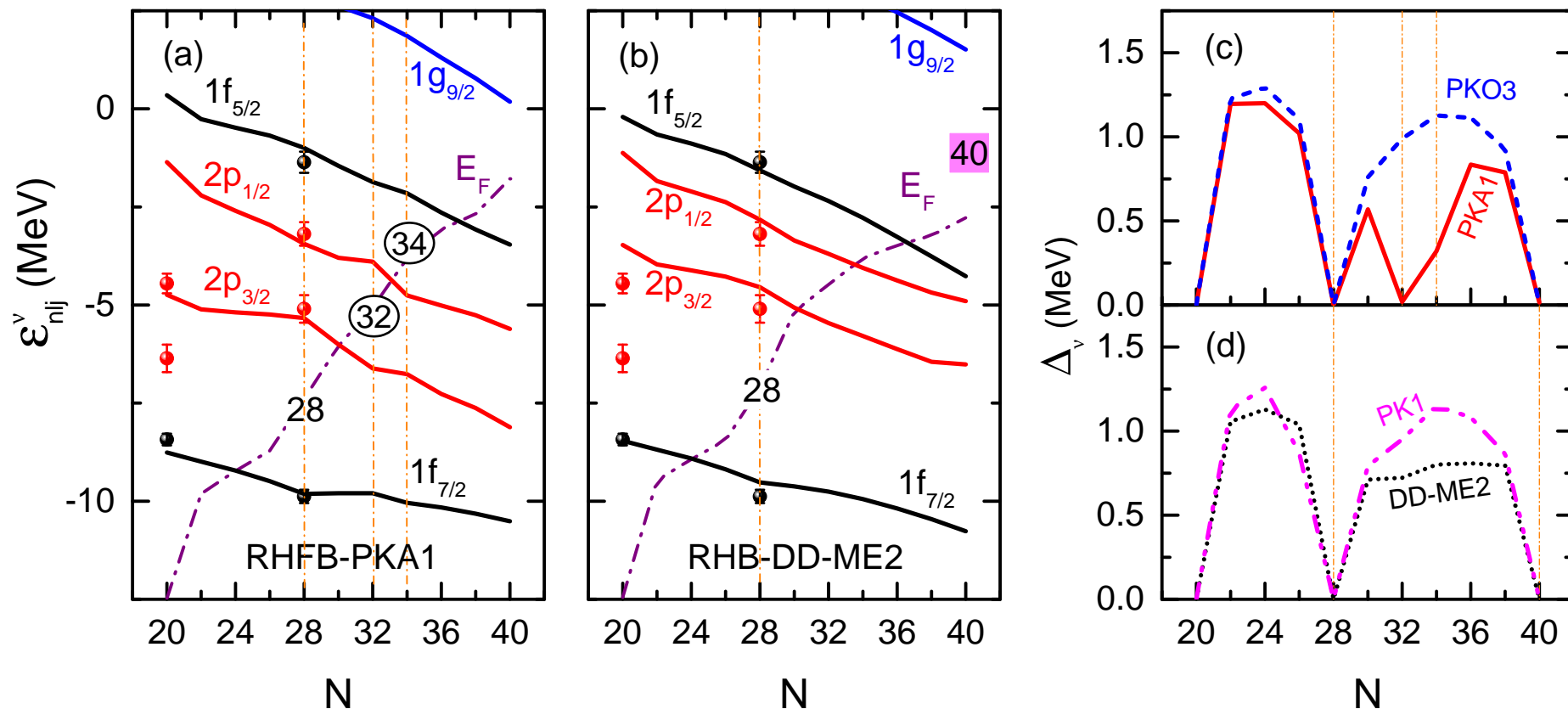
Mechanism in determining magic structures: tensor force?

New Magicity: $N = 32$ & 34



— Li, Margueron, LONG, Giai, PLB 753, 97 (2016) —

New Magicity: $N = 32$ & 34

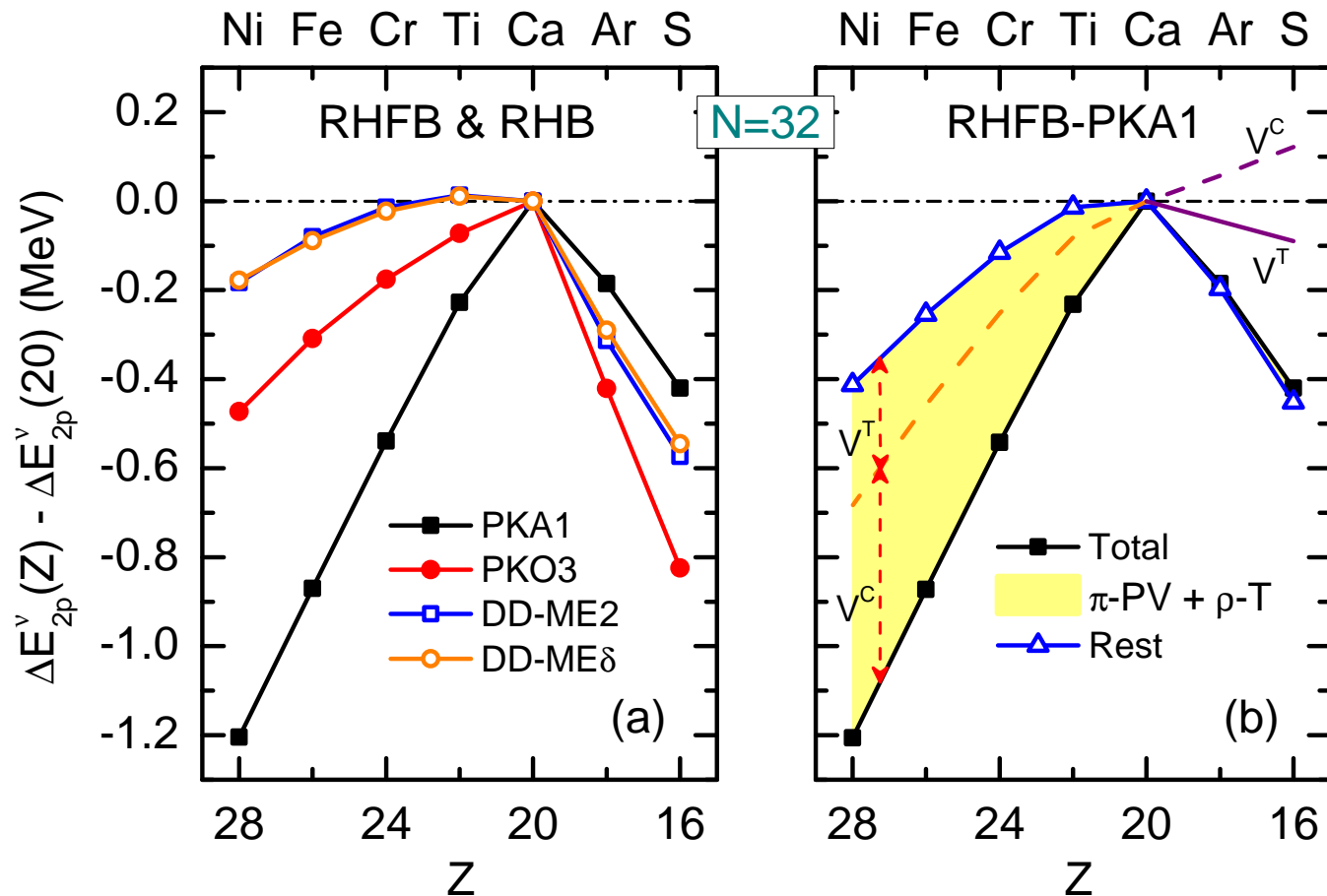


There are four doubly magic nuclei among Ca isotopes: ^{40}Ca , ^{48}Ca , ^{52}Ca , ^{54}Ca . In addition, ^{34}Ca could be also another potential one.

RHFB+PKA1 can well reproduce the magicity at $N = 32$ & 34 , which shows the reliability of the model.

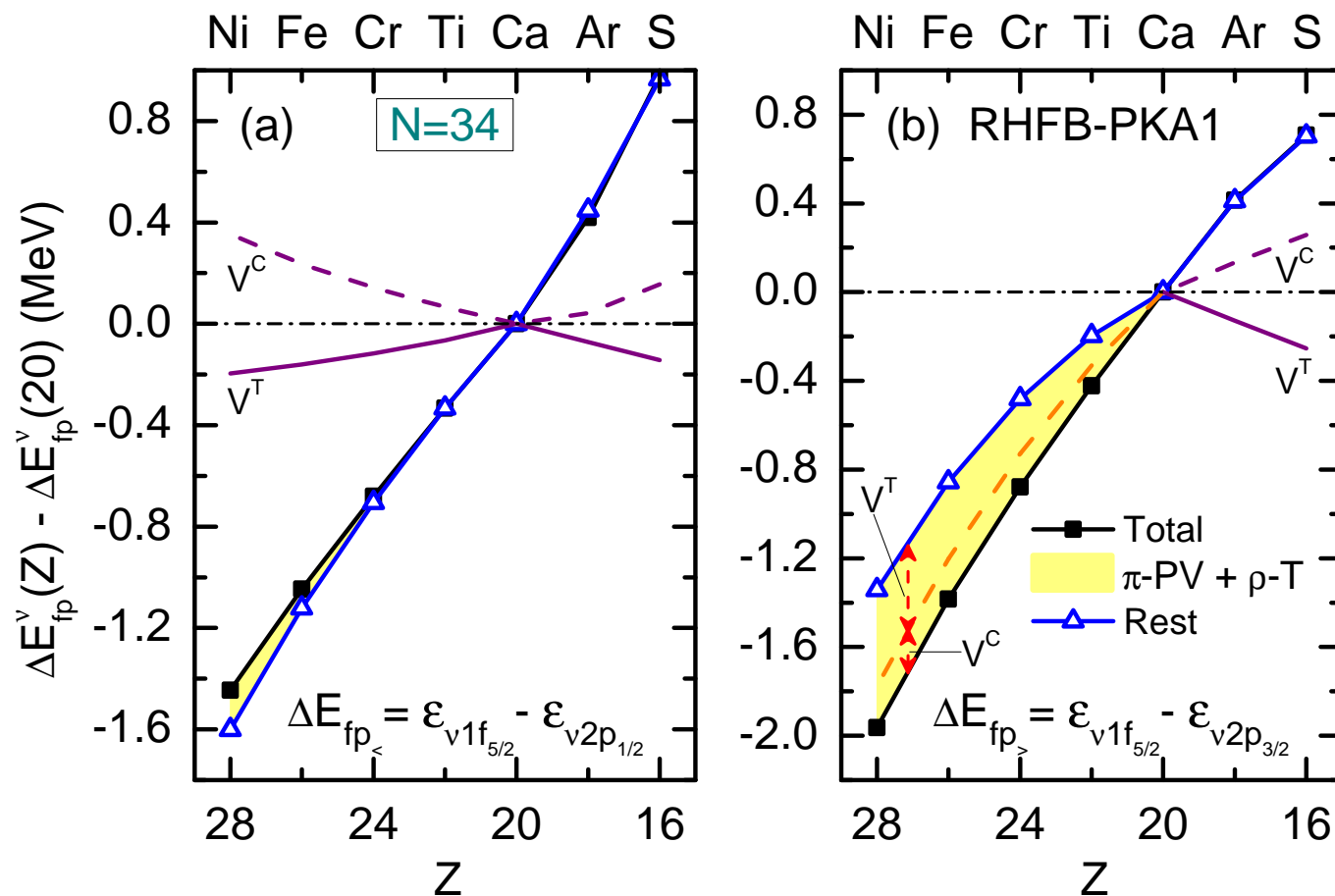
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Physics related to new magicity



$N = 32$: isovector ρ -T & π -PV couplings are the key physics.

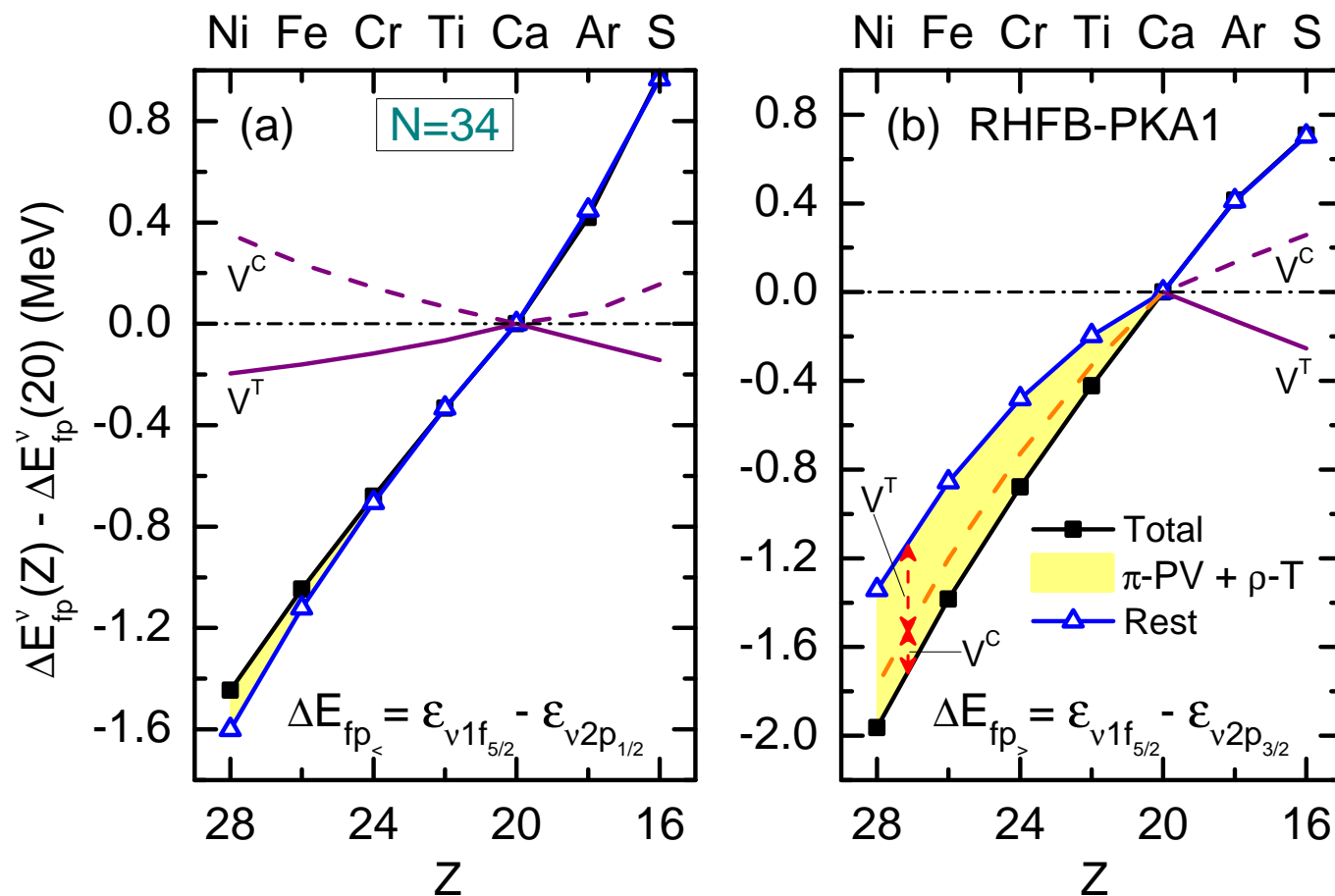
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$N = 32$: isovector ρ -T & π -PV couplings are the key physics.

$N = 34$: isovector ρ -T & π -PV couplings are not so significant any more.

It remains some mystery on the physics that triggers the $N = 34$ shell.

New drip-line magic nucleus: ^{48}Si

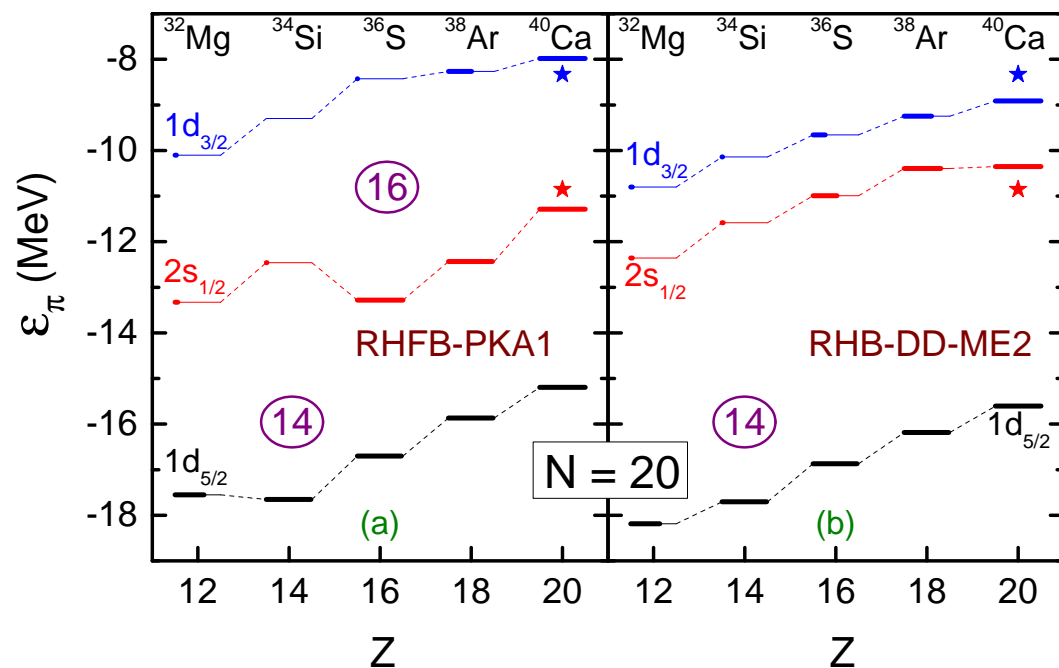
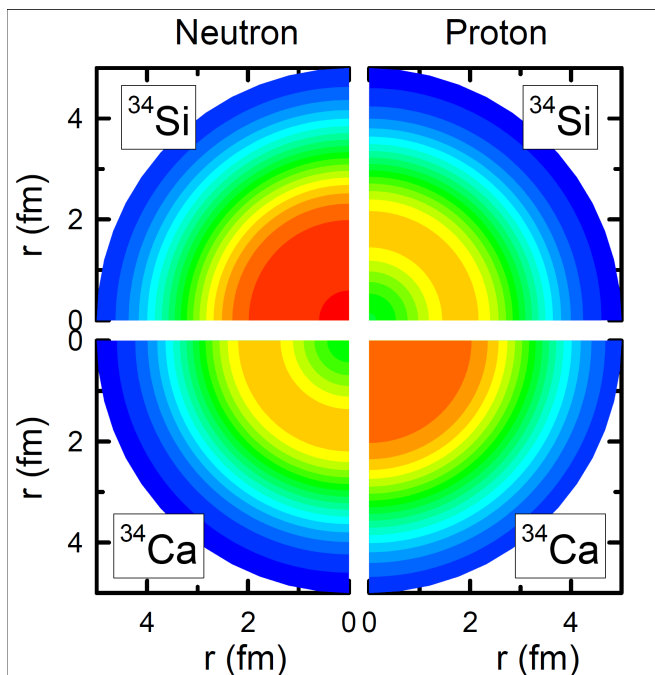
Table 1: Energy gap corresponding to $N = 32$ & 34 in the Ni, Ca and Si isotopes. Results are provided by the calculations with RHFB-PKA1, RHFB-PKO3 and RHB-DD-ME2 models.

Force	$\Delta E(i, i')$	N	Ni	Ca	Si
PKA1	$(\nu 2p_{1/2}, \nu 2p_{3/2})$	32	1.51	2.72	0.81
	$(\nu 1f_{5/2}, \nu 2p_{1/2})$	34	1.04	2.60	4.3
PKO3	$(\nu 2p_{1/2}, \nu 2p_{3/2})$	32	1.22	1.69	0.68
	$(\nu 1f_{5/2}, \nu 2p_{1/2})$	34	-1.72	0.77	2.72
DD-ME2	$(\nu 2p_{1/2}, \nu 2p_{3/2})$	32	1.58	1.76	0.92
	$(\nu 1f_{5/2}, \nu 2p_{1/2})$	34	-1.23	1.21	3.18

For $^{52,54}\text{Ca}$, only PKA1 shows distinct shells $N = 32$ & 34 , whereas for ^{60}Ni and ^{46}Si the models present similar values of $\Delta_{\nu 2p}$.

For ^{48}Si : all the models shows distinct shell $N = 34$, and therefore ^{48}Si can be referred as the new magic drip-line nucleus.

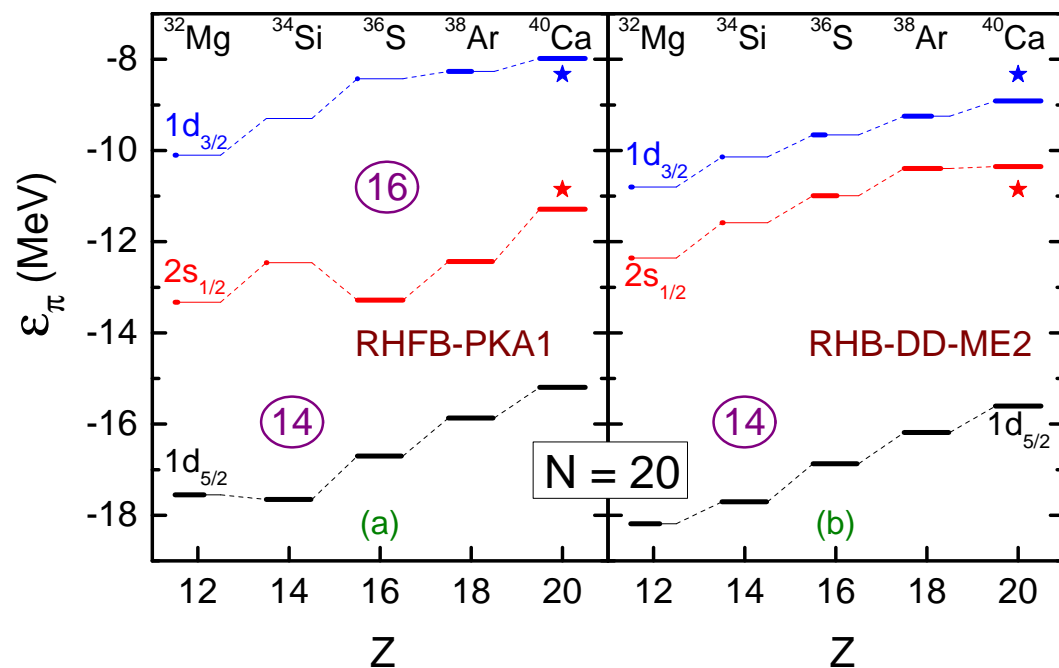
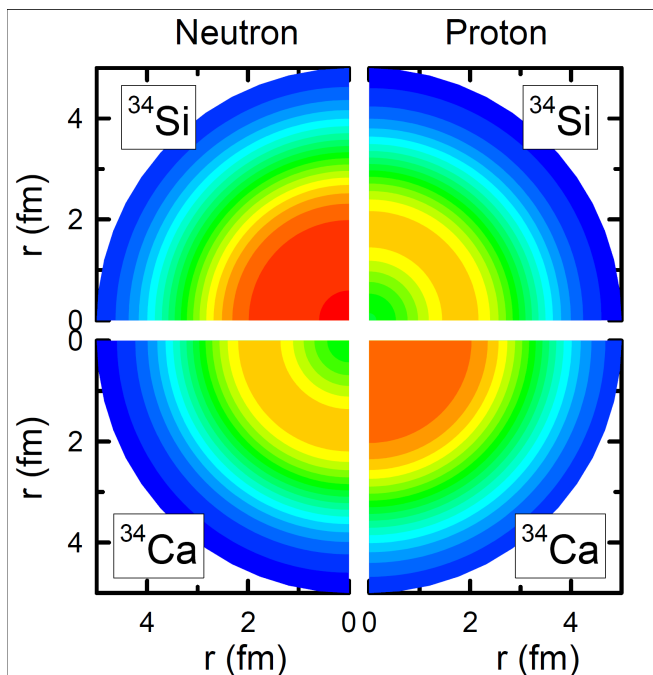
Bubble structure predicted in ^{34}Si & ^{34}Ca



Proton/neutron semi-bubbles occur in mirror systems $^{34}\text{Si}/^{34}\text{Ca}$, since $N/Z = 14$ shells prevent valence protons or neutrons to occupy $2s_{1/2}$ orbit.

—J. J. Li, W. H. Long, J. L. Song, and Q. Zhao, Phys. Rev. C 93, 054312 (2016)

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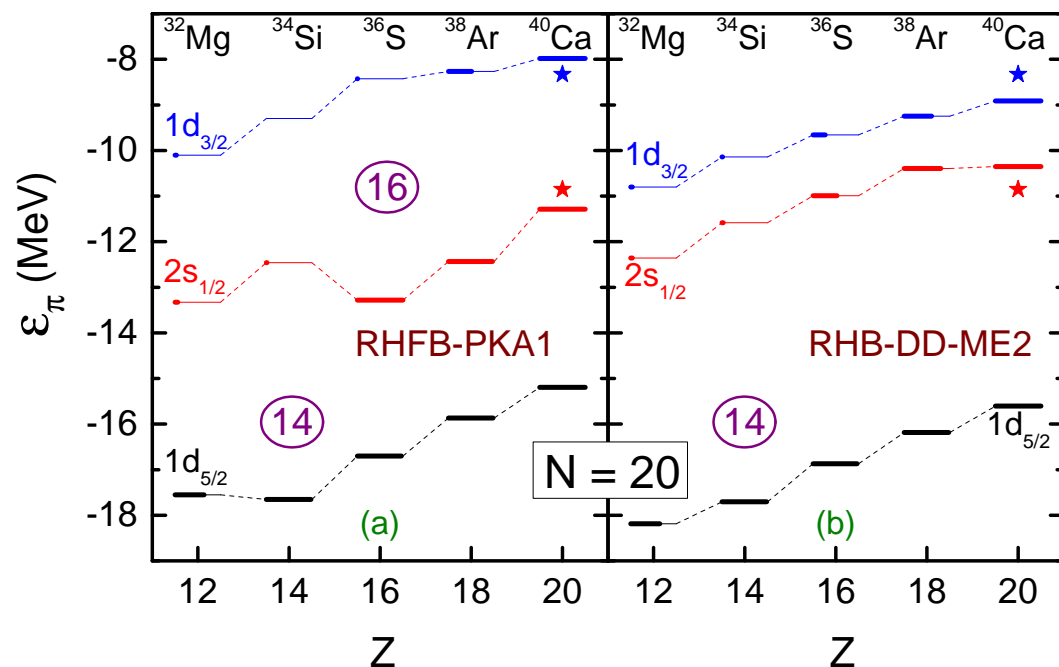
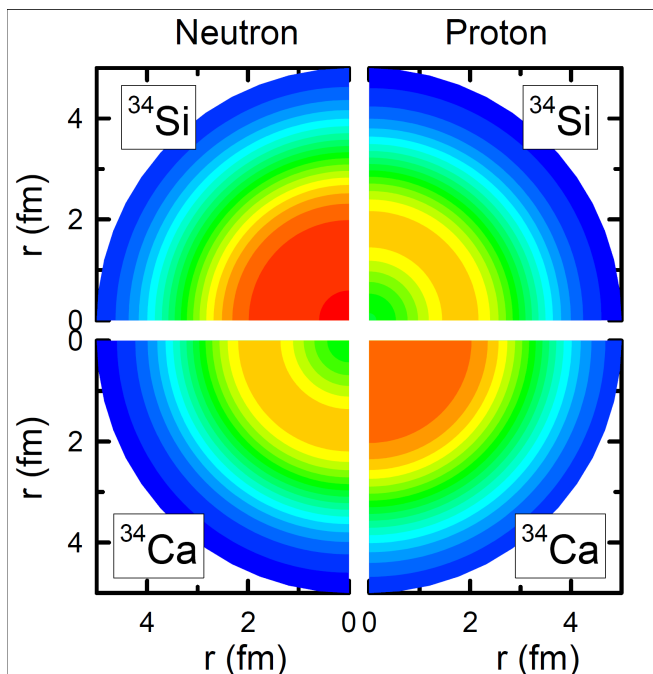
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Experiments in GANIL shows that ^{34}Si is a doubly magic nucleus with proton bubble, and magic shell $Z = 14$ prevents protons to occupy $2s_{1/2}$ orbit.

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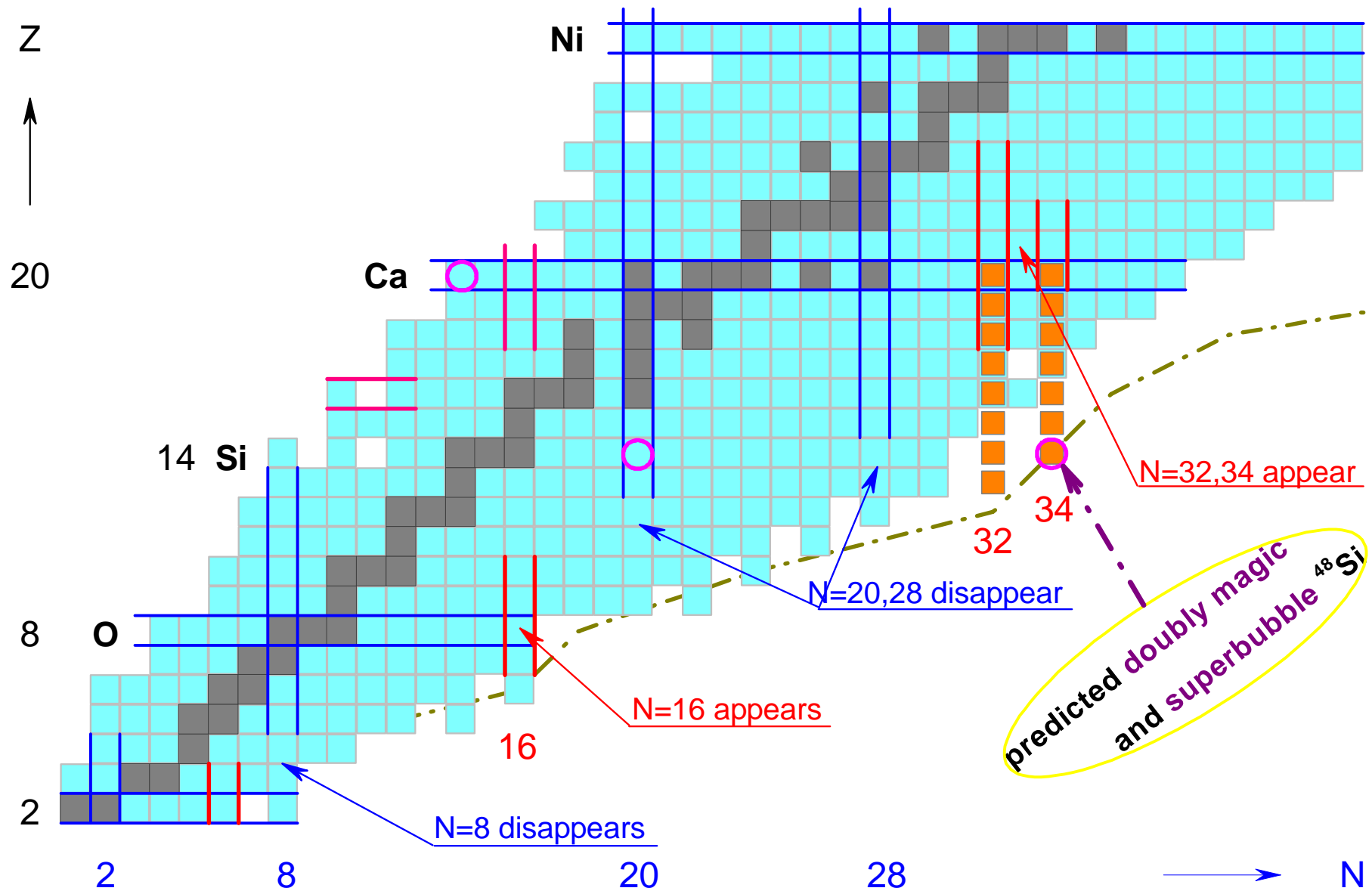
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Does the proton bubble structure still remain in ^{48}Si ?

Studying Object: ^{48}Si



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RHF Hamiltonian

A. Bouyssy (1987)

Effective Hamiltonian for nuclei: $\phi = \sigma\text{-S}; \omega\text{-V}, A\text{-V}, \rho\text{-V}; \rho\text{-VT}, \rho\text{-T}; \pi\text{-PV}$

$$H = \int d\mathbf{x} \bar{\psi} (-i\boldsymbol{\gamma} \cdot \nabla + M) \psi + \frac{1}{2} \int d\mathbf{x} d\mathbf{x}' \bar{\psi}(\mathbf{x}) \bar{\psi}(\mathbf{x}') \Gamma_{\phi} D^{\phi} \psi(\mathbf{x}') \psi(\mathbf{x})$$

Types of the interaction: $\Gamma_{\phi}(\mathbf{x}, \mathbf{x}')$

$$\Gamma_{\sigma\text{-S}} \equiv -\mathbf{g}_{\sigma}(\mathbf{x}) \mathbf{g}_{\sigma}(\mathbf{x}'), \quad \Gamma_{A\text{-V}} \equiv \frac{e^2}{4} [\gamma_{\mu} (1 - \tau_3)]_x [\gamma^{\mu} (1 - \tau_3)]_{x'}, \quad (1)$$

$$\Gamma_{\omega\text{-V}} \equiv (\mathbf{g}_{\omega} \gamma_{\mu})_x (\mathbf{g}_{\omega} \gamma^{\mu})_{x'}, \quad \Gamma_{\pi\text{-PV}} \equiv \frac{-1}{m_{\pi}^2} (f_{\pi} \vec{\tau} \gamma_5 \gamma_{\mu} \partial^{\mu})_x \cdot (f_{\pi} \vec{\tau} \gamma_5 \gamma_{\nu} \partial^{\nu})_{x'}, \quad (2)$$

$$\Gamma_{\rho\text{-V}} \equiv (\mathbf{g}_{\rho} \gamma_{\mu} \vec{\tau})_x \cdot (\mathbf{g}_{\rho} \gamma^{\mu} \vec{\tau})_{x'}, \quad \Gamma_{\rho\text{-T}} \equiv \frac{1}{4M^2} (f_{\rho} \sigma_{\nu k} \vec{\tau} \partial^k)_x \cdot (f_{\rho} \sigma^{\nu l} \vec{\tau} \partial_l)_{x'}, \quad (3)$$

$$\Gamma_{\rho\text{-VT}} \equiv \frac{1}{2M} (f_{\rho} \sigma^{k\nu} \vec{\tau} \partial_k)_x \cdot (\mathbf{g}_{\rho} \gamma_{\nu} \vec{\tau})_{x'} + (\mathbf{g}_{\rho} \gamma_{\nu} \vec{\tau})_x \cdot \frac{1}{2M} (f_{\rho} \sigma^{k\nu} \vec{\tau} \partial_k)_{x'} \quad (4)$$

Yukawa propagator $D_{\phi}(\mathbf{x}, \mathbf{x}')$

neglecting retardation effects

$$D_{\phi}(\mathbf{x}, \mathbf{x}') = \frac{1}{4\pi} \frac{e^{-m_{\phi} |\mathbf{x} - \mathbf{x}'|}}{|\mathbf{x} - \mathbf{x}'|}, \quad D_A(\mathbf{x}, \mathbf{x}') = \frac{1}{4\pi} \frac{1}{|\mathbf{x} - \mathbf{x}'|} \quad (5)$$

RHF energy density functional (EDF)

A. Bouyssy(1987)

• Solutions of Dirac Eq.: $\left\{ \varepsilon_k > 0, c_k, c_k^\dagger \text{ (Fermi sea)}; \varepsilon_l < 0, c_l, c_l^\dagger \text{ (Dirac sea)} \right\}$

• Quantizing nucleon spinor:

$$\psi = \sum_k \psi_k(\mathbf{x}) e^{-i\varepsilon_k t} c_k + \sum_l \psi_l(\mathbf{x}) e^{-i\varepsilon_l t} d_l^\dagger,$$

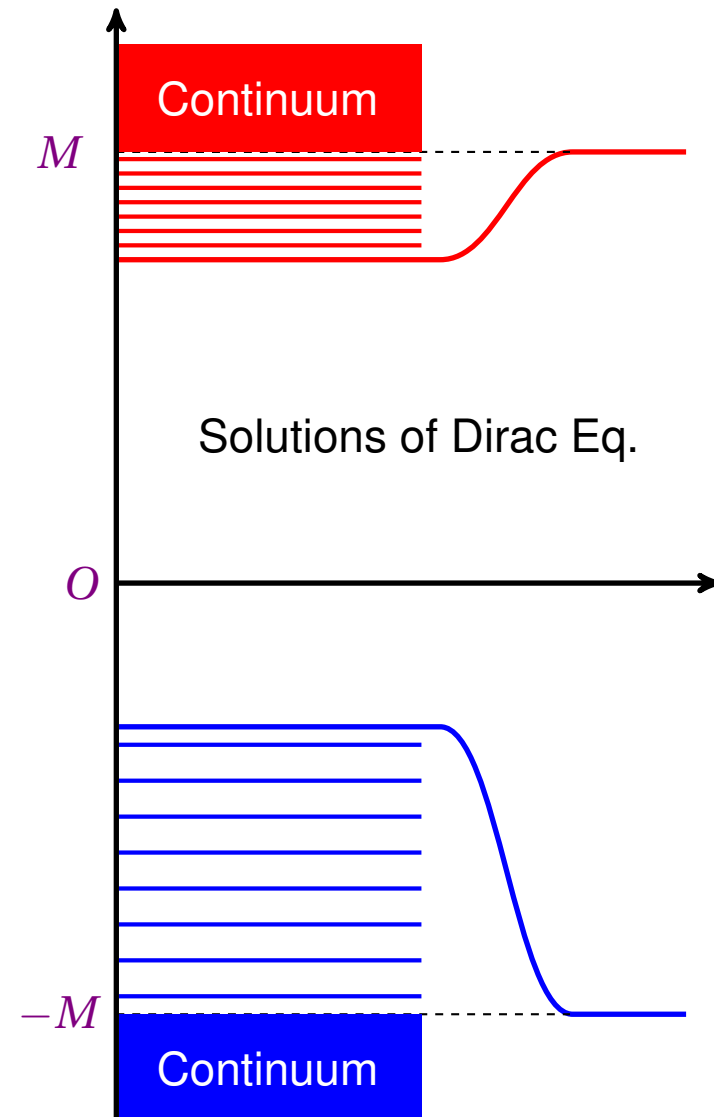
Ground state with **no-sea approximation**

$$|\Phi_0\rangle = \prod_{i=1}^A c_i^\dagger |0\rangle,$$

RHF EDF: expectation of H referring to $|\Phi_0\rangle$

$$E = \langle \Phi_0 | H | \Phi_0 \rangle = \langle \Phi_0 | T | \Phi_0 \rangle + \sum_{\phi} \langle \Phi_0 | V_{\phi} | \Phi_0 \rangle$$

T and V_{ϕ} are the kinetic energy and potential energy terms, respectively.



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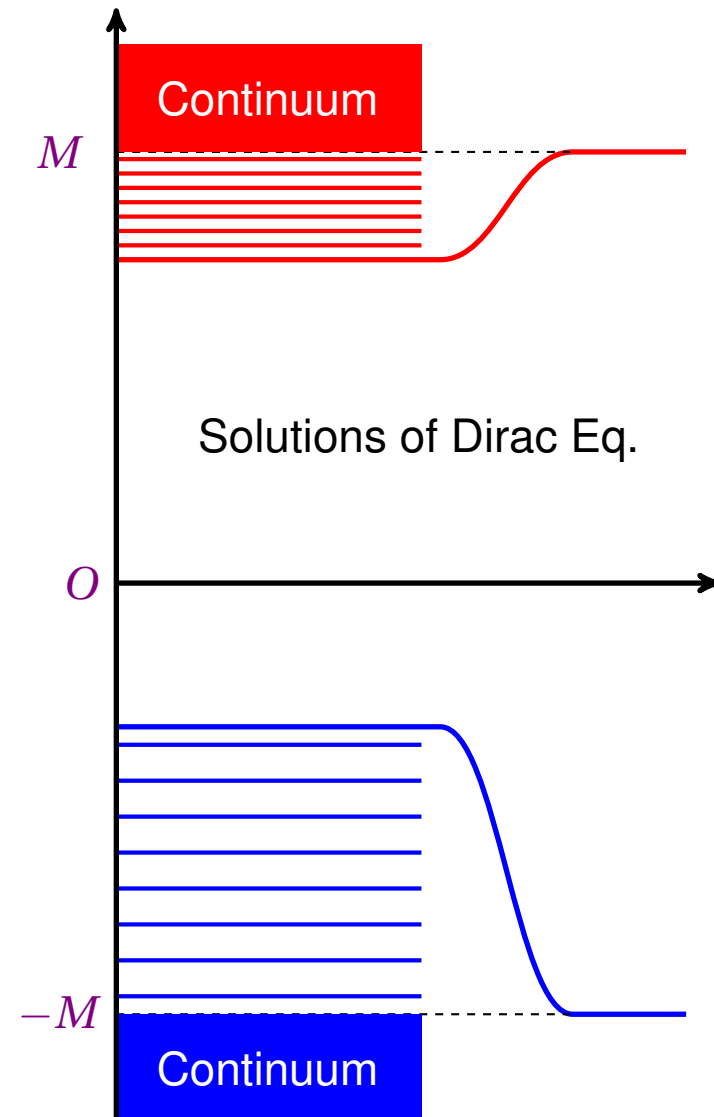
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$$V_\phi = \frac{1}{2} \int d\mathbf{x} d\mathbf{x}' \sum_{\alpha\beta;\alpha'\beta'} \overbrace{c_\alpha^\dagger c_\beta^\dagger c_{\beta'} c_{\alpha'}}^{\text{Fock}} \underbrace{c_\alpha^\dagger c_\beta^\dagger c_{\beta'} c_{\alpha'}}_{\text{Hartree}} \times \bar{\psi}_\alpha(\mathbf{x}) \bar{\psi}_\beta(\mathbf{x}') \Gamma_\phi D_\phi \psi_{\beta'}(\mathbf{x}') \psi_{\alpha'}(\mathbf{x}),$$



Spherical RHF equation

W.H. Long (2010)

● Variation of RHF energy functional E : integro-differential Dirac Eq.

$$\int d\mathbf{r}' h(\mathbf{r}, \mathbf{r}') \psi_\alpha(\mathbf{r}') = \varepsilon_a \psi_\alpha(\mathbf{r}), \quad \psi_\alpha(\mathbf{r}) = \frac{1}{r} \begin{pmatrix} iG_a \mathcal{Y}_{j_a m_a}^{l_a}(\hat{\mathbf{r}}) \\ -F_a \mathcal{Y}_{j_a m_a}^{l_a}(\hat{\mathbf{r}}) \end{pmatrix} \quad (6)$$

where $\mathcal{Y}_{jm}^l = \sum_{\mu\sigma} C_{l\mu\frac{1}{2}\sigma}^{jm} Y_{l\mu} \chi_{\frac{1}{2}\sigma}$, single-particle Hamiltonian $h = h^{\text{kin}} + h^{\text{D}} + h^{\text{E}}$:

$$h^{\text{kin}}(\mathbf{r}, \mathbf{r}') = [\boldsymbol{\alpha} \cdot \mathbf{p} + \beta M] \delta(\mathbf{r} - \mathbf{r}'), \quad (7a)$$

$$h^{\text{D}}(\mathbf{r}, \mathbf{r}') = [\Sigma_T(\mathbf{r}) \gamma_5 + \Sigma_0(\mathbf{r}) + \beta \Sigma_S(\mathbf{r})] \delta(\mathbf{r} - \mathbf{r}'), \quad (7b)$$

$$h^{\text{E}}(\mathbf{r}, \mathbf{r}') = \begin{pmatrix} Y_G(\mathbf{r}, \mathbf{r}') & Y_F(\mathbf{r}, \mathbf{r}') \\ X_G(\mathbf{r}, \mathbf{r}') & X_F(\mathbf{r}, \mathbf{r}') \end{pmatrix} \quad (7c)$$

● Local mean fields Σ_S , Σ_0 , and Σ_T : functionals of local densities

$$\Sigma_S = \mathbf{g}_\sigma \sigma, \quad \Sigma_T = \frac{f_\rho}{2M} (\rho^{\text{VT}} + \rho^{\text{T}}) \tau_3, \quad \Sigma_0 = \mathbf{g}_\omega \omega + \mathbf{g}_\rho (\rho^{\text{V}} + \rho^{\text{TV}}) \tau_3 + e \frac{1 - \tau_3}{2} A + \Sigma_R$$

Hartree mean fields: σ , ω , ρ^{V} , A , ρ^{TV} and ρ^{VT} , ρ^{T} ; rearrangement term Σ_R .

Exchange (Fock) Potentials

W.H. Long (2010)

Non-local MFs: functionals of the non-local densities

$$X_{G_a}^{(\phi)}(r, r') = \sum_b \mathcal{T}_{ab}^\phi \hat{j}_b^2 \mathbf{g}_\phi(r) \mathbf{g}_\phi(r') \underline{F_b(r) G_b(r')} \mathcal{R}_{ab}^{X_G}(m_\phi; r, r'),$$

$$X_{F_a}^{(\phi)}(r, r') = \sum_b \mathcal{T}_{ab}^\phi \hat{j}_b^2 \mathbf{g}_\phi(r) \mathbf{g}_\phi(r') \underline{F_b(r) F_b(r')} \mathcal{R}_{ab}^{X_F}(m_\phi; r, r'),$$

$$Y_{G_a}^{(\phi)}(r, r') = \sum_b \mathcal{T}_{ab}^\phi \hat{j}_b^2 \mathbf{g}_\phi(r) \mathbf{g}_\phi(r') \underline{G_b(r) G_b(r')} \mathcal{R}_{ab}^{Y_G}(m_\phi; r, r'),$$

$$Y_{F_a}^{(\phi)}(r, r') = \sum_b \mathcal{T}_{ab}^\phi \hat{j}_b^2 \mathbf{g}_\phi(r) \mathbf{g}_\phi(r') \underline{G_b(r) F_b(r')} \mathcal{R}_{ab}^{Y_F}(m_\phi; r, r'),$$

\mathcal{T}_{ab}^ϕ : $\delta_{\tau_a \tau_b}$ (isoscalar) and $2 - \delta_{\tau_a \tau_b}$ (isovector).

The underlined terms can be taken as the **non-local density** component.

σ -scalar coupling:

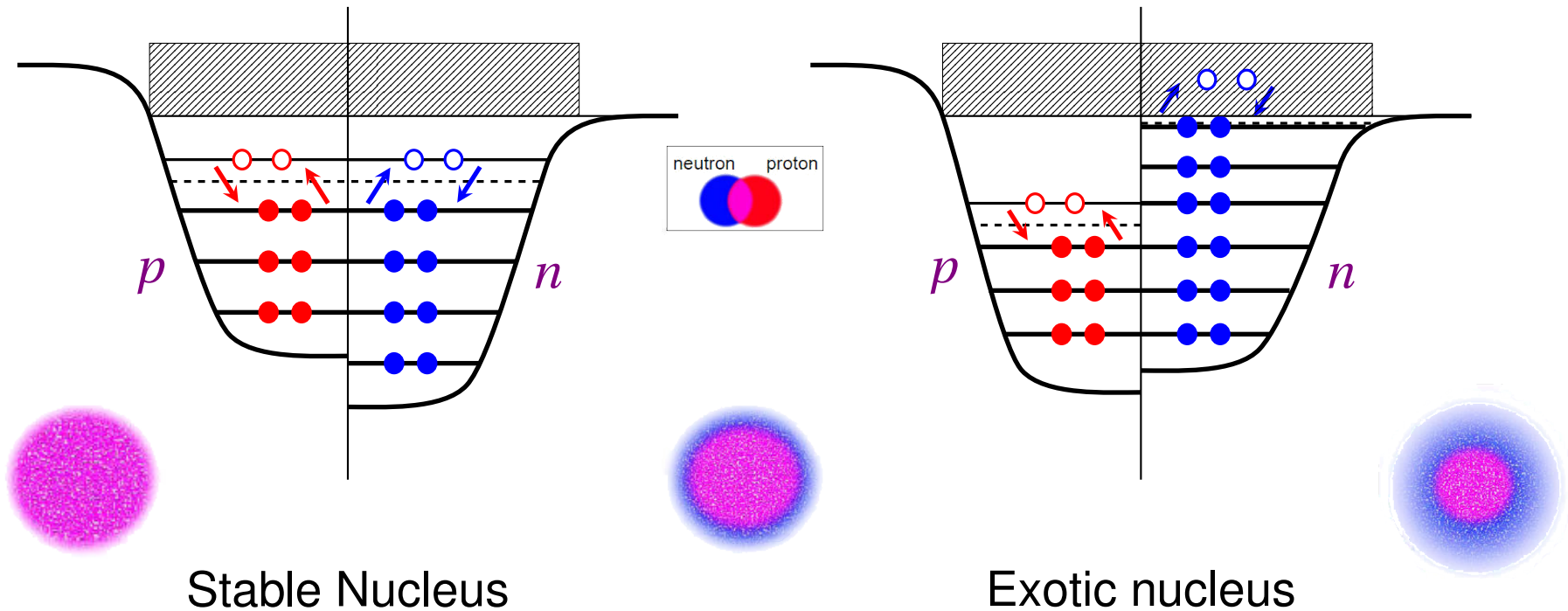
$$\mathcal{R}^{Y_G} = \mathcal{R}^{X_F} = -\mathcal{R}^{Y_F} = -\mathcal{R}^{X_G} = \mathcal{R}^{(\sigma)}$$

$$\mathcal{R}_{ab}^{(\sigma)}(m_\sigma; r, r') = \frac{1}{4\pi} \sum_L' C_{j_a \frac{1}{2} j_b - \frac{1}{2}}^{L0} C_{j_a \frac{1}{2} j_b - \frac{1}{2}}^{L0} R_{LL}(m_\sigma; r, r'). \quad (8)$$

The prime in Eq. (8) requires $L + l_a + l_b$ be even.

Unstable nuclei: continuum effects

- Unstable exotic nuclei reveal lots of new physics: weakly bound mechanism, continuum, halo, etc.



- Bogoliubov scheme: **unified treatment of pairing and mean field effects**

— J. Dobaczewski. NPA 422, 103 (1984); J. Meng, NPA 635, 3 (1998).

- Bogoliubov scheme also has the advantages in exploring superheavy nuclei.

— J.J. Li, W.H. LONG, J. Margueron, N. Van Giai, PLB 732, 169 (2014).

RHFB theory

W.H. Long (2010)

● Bogoliubov transformation: particle $\{c_\alpha, c_\alpha^\dagger\}$ \longrightarrow quasi-particle $\{\beta_\alpha, \beta_\alpha^\dagger\}$

$$\begin{pmatrix} c_\alpha \\ c_\alpha^\dagger \end{pmatrix} = \mathcal{W} \begin{pmatrix} \beta_\alpha \\ \beta_\alpha^\dagger \end{pmatrix} = \begin{pmatrix} \psi_U & \psi_V^* \\ \psi_V & \psi_U^* \end{pmatrix} \begin{pmatrix} \beta_\alpha \\ \beta_\alpha^\dagger \end{pmatrix}, \quad \begin{aligned} \beta_\alpha &= \psi_U^\dagger c_\alpha + \psi_V^\dagger c_\alpha^\dagger \\ \beta_\alpha^\dagger &= \psi_V^T c_\alpha + \psi_U^T c_\alpha^\dagger \end{aligned} \quad (9)$$

where ψ_U and ψ_V quasi-particle spinors, and $\mathcal{W}^\dagger \mathcal{W} = 1$.

● RHFB Equation: chemical potential λ for preserving the particle number

$$\int d\mathbf{r}' \begin{pmatrix} h(\mathbf{r}, \mathbf{r}') & \Delta(\mathbf{r}, \mathbf{r}') \\ -\Delta(\mathbf{r}, \mathbf{r}') & h(\mathbf{r}, \mathbf{r}') \end{pmatrix} \begin{pmatrix} \psi_U(\mathbf{r}') \\ \psi_V(\mathbf{r}') \end{pmatrix} = \begin{pmatrix} \lambda + E & 0 \\ 0 & \lambda - E \end{pmatrix} \begin{pmatrix} \psi_U(\mathbf{r}) \\ \psi_V(\mathbf{r}) \end{pmatrix} \quad (10)$$

where h is RHF single-particle Hamiltonian and pairing potential Δ reads,

$$\Delta_\alpha(\mathbf{r}, \mathbf{r}') = -\frac{1}{2} \sum_\beta V_{\alpha\beta}^{pp}(\mathbf{r}, \mathbf{r}') \kappa_\beta(\mathbf{r}, \mathbf{r}'), \quad \kappa_\alpha(\mathbf{r}, \mathbf{r}') = \psi_{V_\alpha}^*(\mathbf{r}) \psi_{U_\alpha}(\mathbf{r}') \quad (11)$$

● In practice, such integral-differential equation is more convenient to be solved with the help of Dirac Woods-Saxon Basis.

—S.-G. Zhou, J. Meng, P. Ring, PRC 68, 034323 (2003).

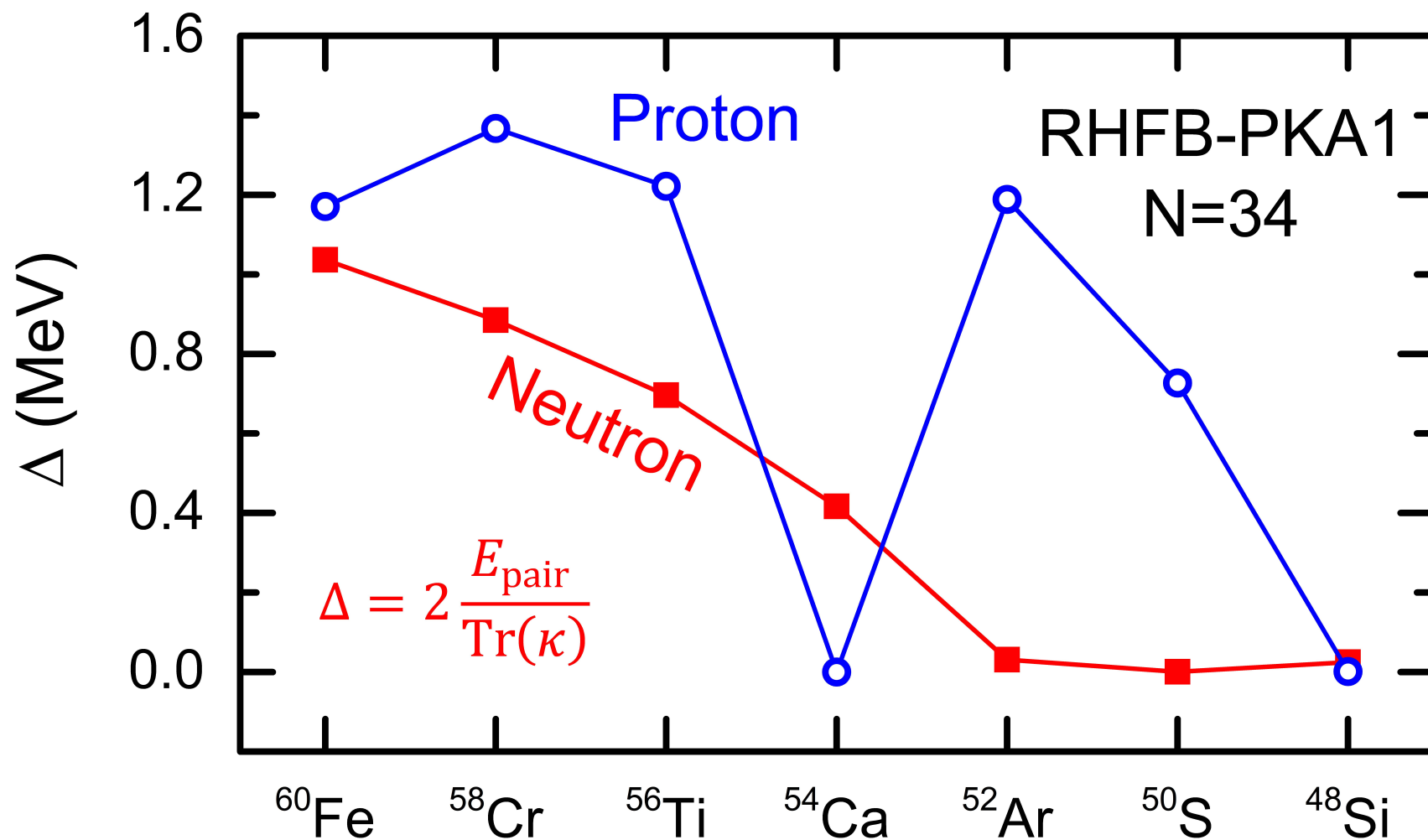
OUTLINE

- 1 **Introduction and Motivation**
 - Covariant Density Functional theory
 - Challenges in Nuclear Physics
 - New Magicity and Bubbles in Ca and Si isotopes

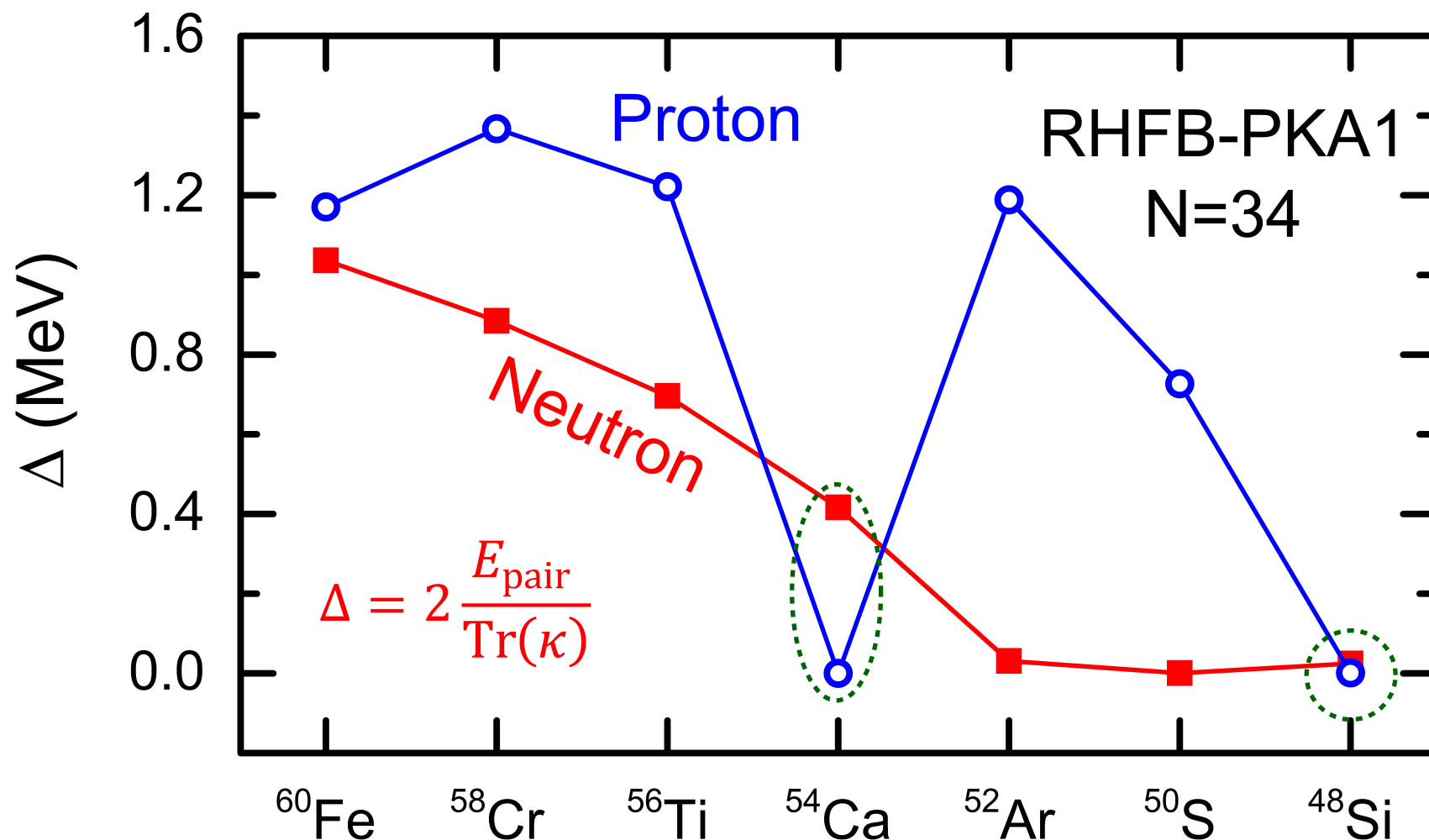
- 2 **Covariant density functional theory with Fock terms**
 - Relativistic Hartree-Fock (RHF) theory
 - Relativistic Hartree-Fock-Bogoliubov (RHFB) theory

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 - Neutron and/or proton crossing-shell excitations

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Pairing gaps along isotonic chain of $N = 34$ 

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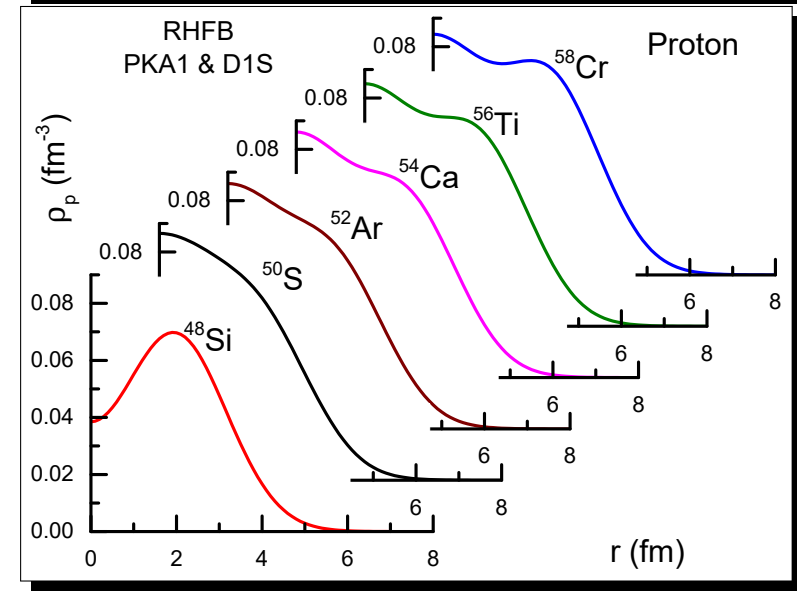
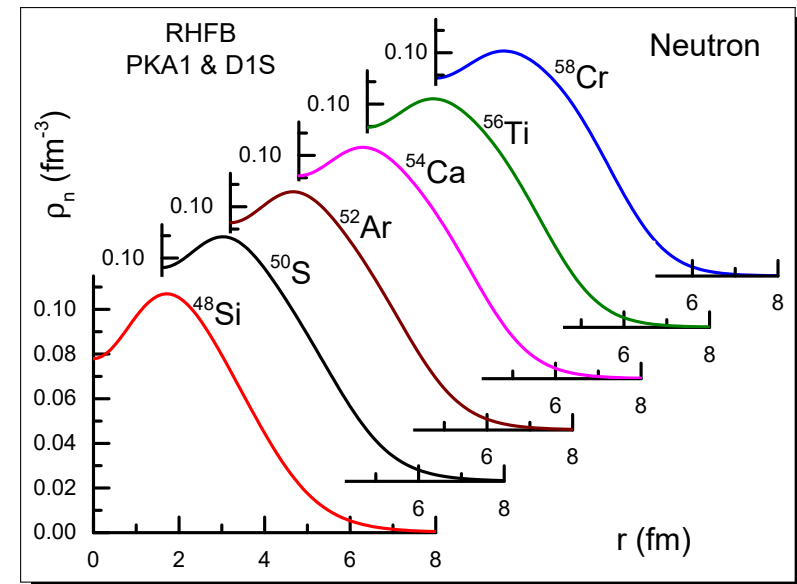
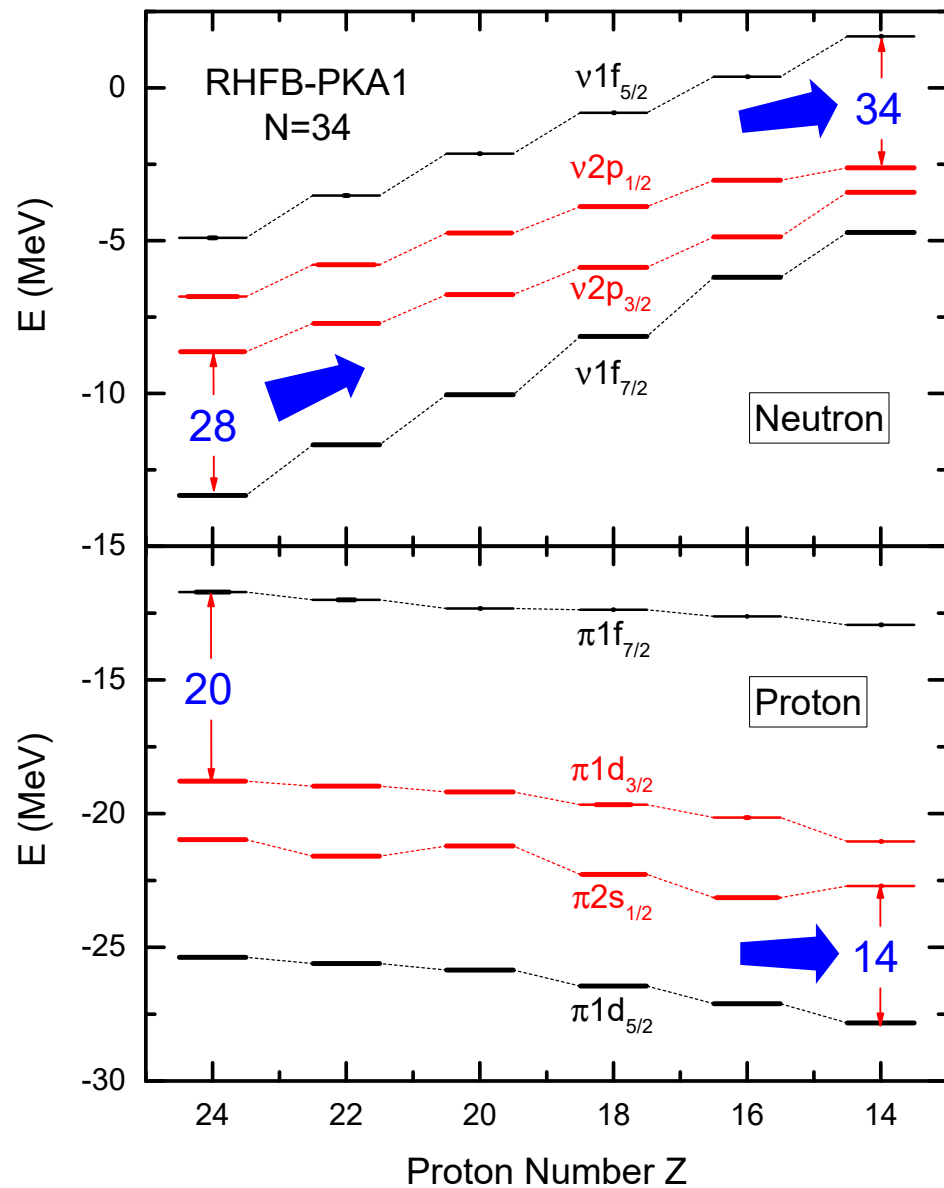


The prediction is consistent with other model calculations, like shell model.

— D. Steppenbeck, S. Takeuchi, N. Aoi, P. Doornenbal, M. Matsushita, et al., PRL 114, 252501 (2015).

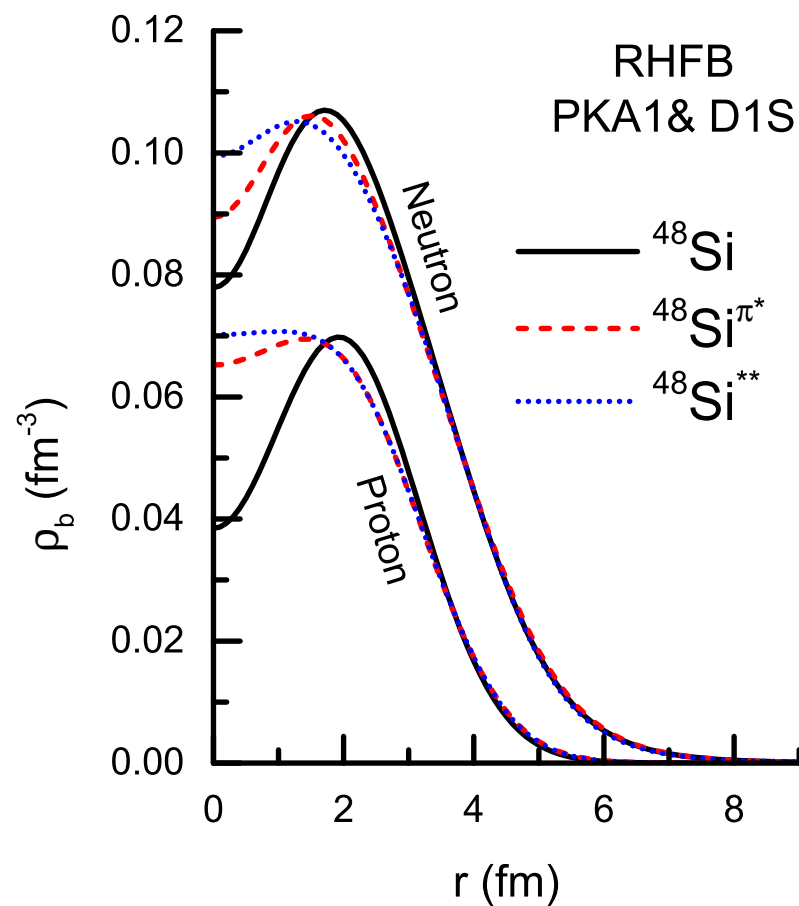
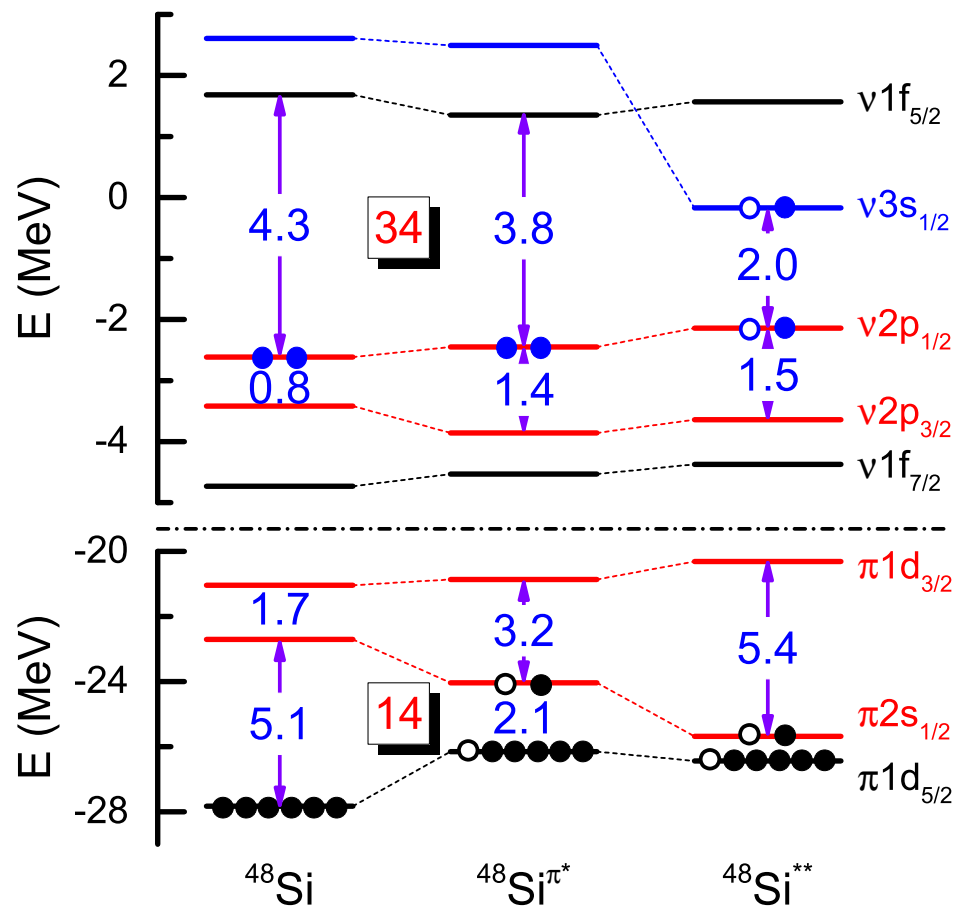
— Y. Utsuno, T. Otsuka, Y. Tsunoda, N. Shimizu, M. Honma, et al., JPS Conf. Proc. 6, 010007 (2015).

Magicity and Bubbles



Bubble structure $\xrightarrow{\text{quench } \Delta_{SO} \text{ of } p \text{ orbits}}$ enhanced shells

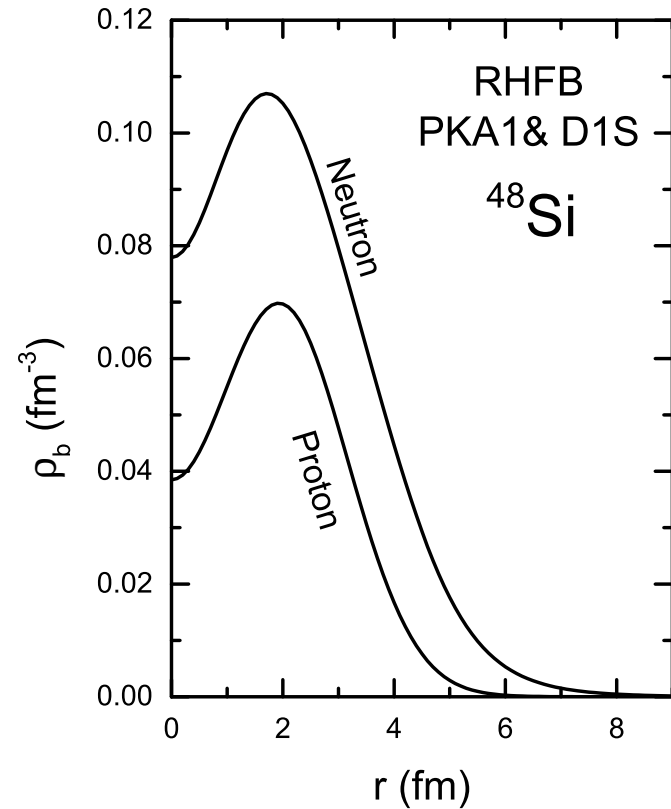
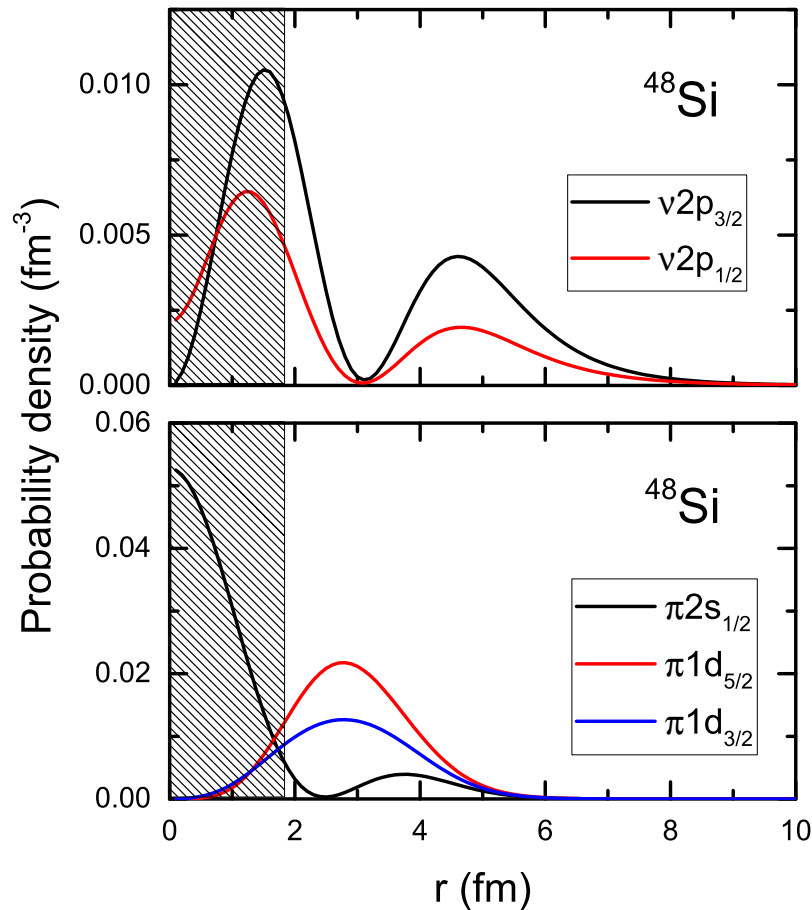
Shells enhanced by bubble structure



$^{48}\text{Si}^{**} \longrightarrow ^{48}\text{Si}\pi^* \longrightarrow ^{48}\text{Si}$ (*super bubble candidate*)

Bubbles: SO splitting $\Delta_{\nu 2p}$ is quenched distinctly, leading to enhanced neutron shell $N = 34$. Similarly, the pseudo-spin splitting of $\Delta_{\pi 1\tilde{p}} = E_{\pi 2s_{1/2}} - E_{1d_{3/2}}$ is also compressed much to give the proton shell $Z = 14$.

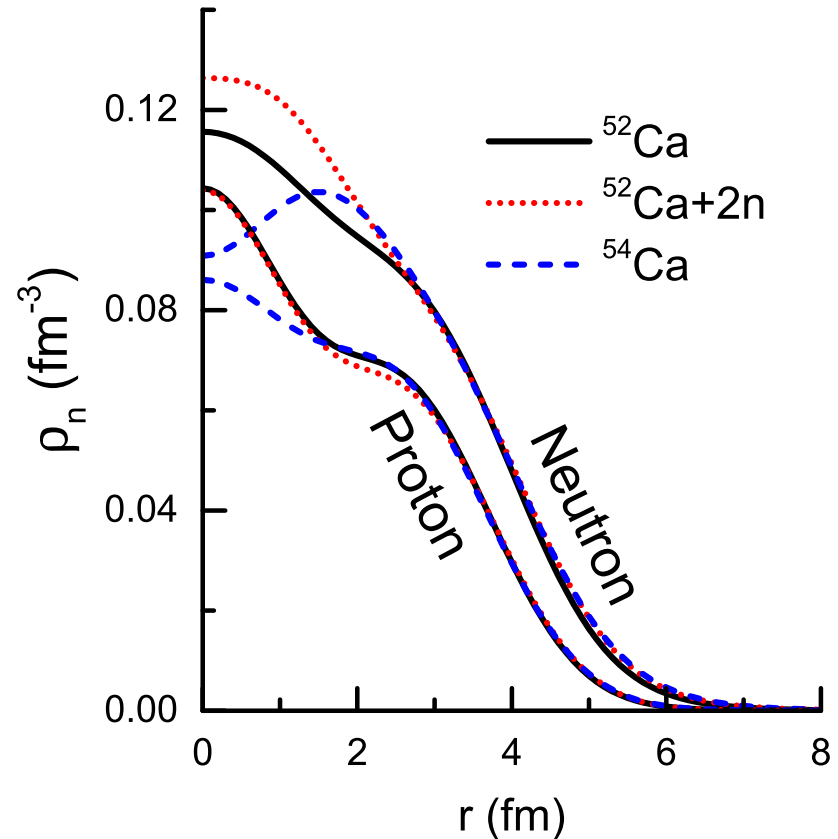
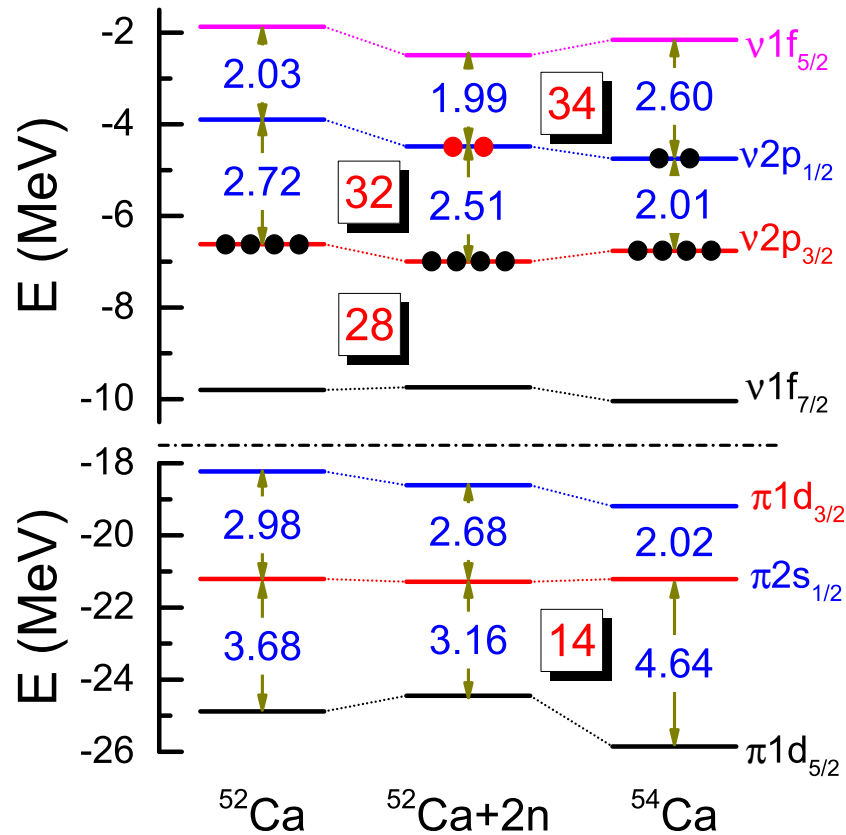
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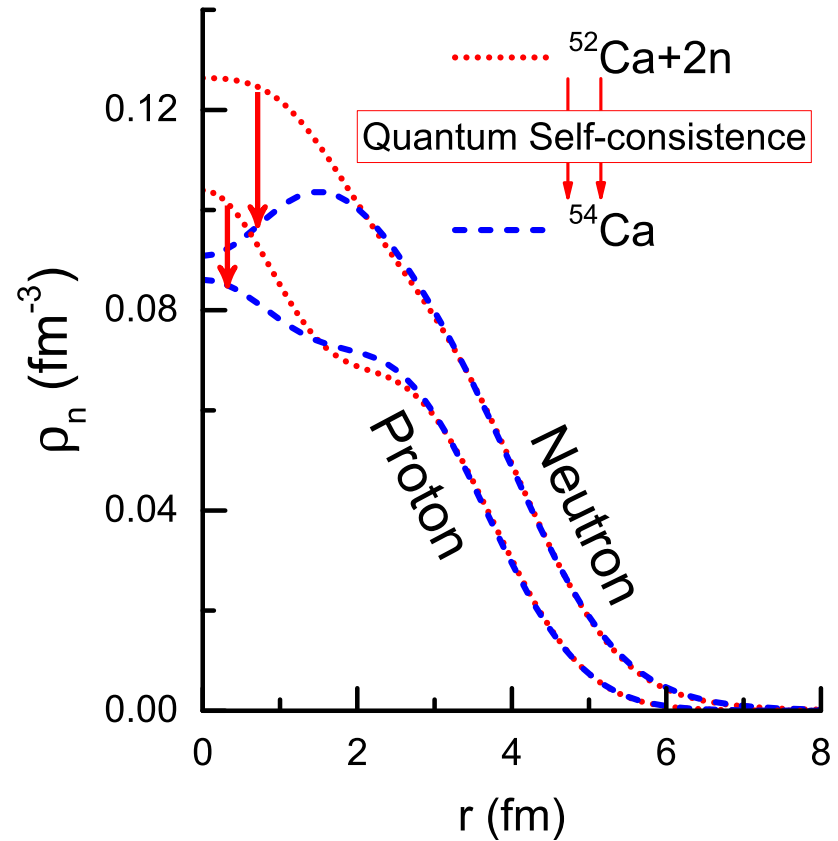
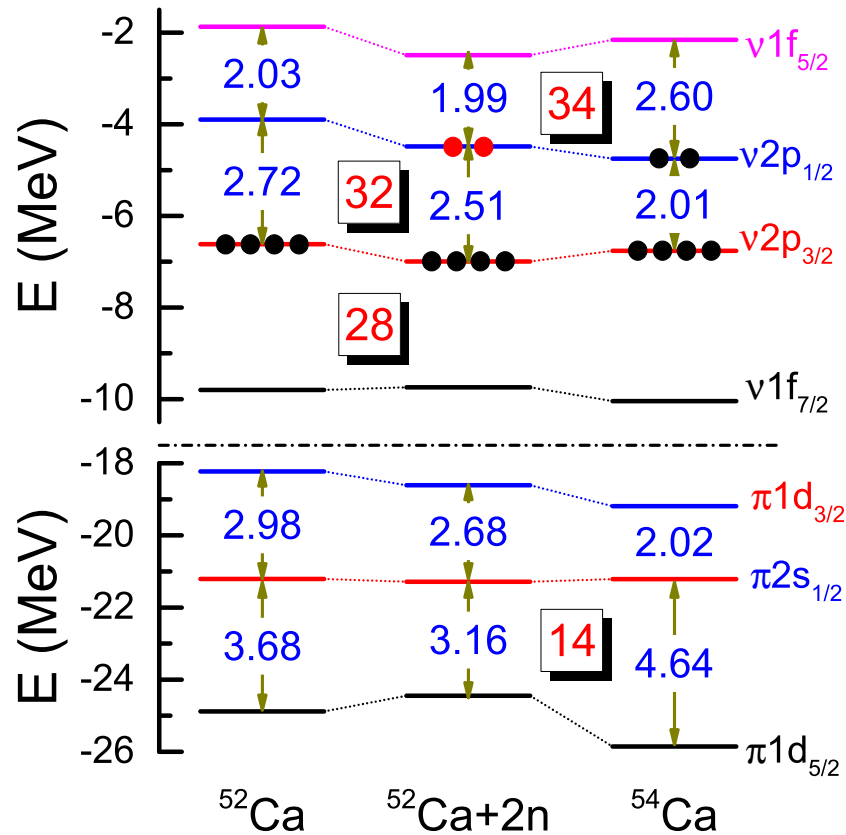
From ^{52}Ca to ^{54}Ca : self-consistent re-equilibrium





In comparison with ^{52}Ca

- In ^{54}Ca , the density profiles is dramatically central-depressed, and consistently the spectra are changed distinctly.*
- In system $^{52}\text{Ca} + 2n$, the spectra are only slightly changed, and therefore the modification on density profiles only reflects the effects of two neutrons.*

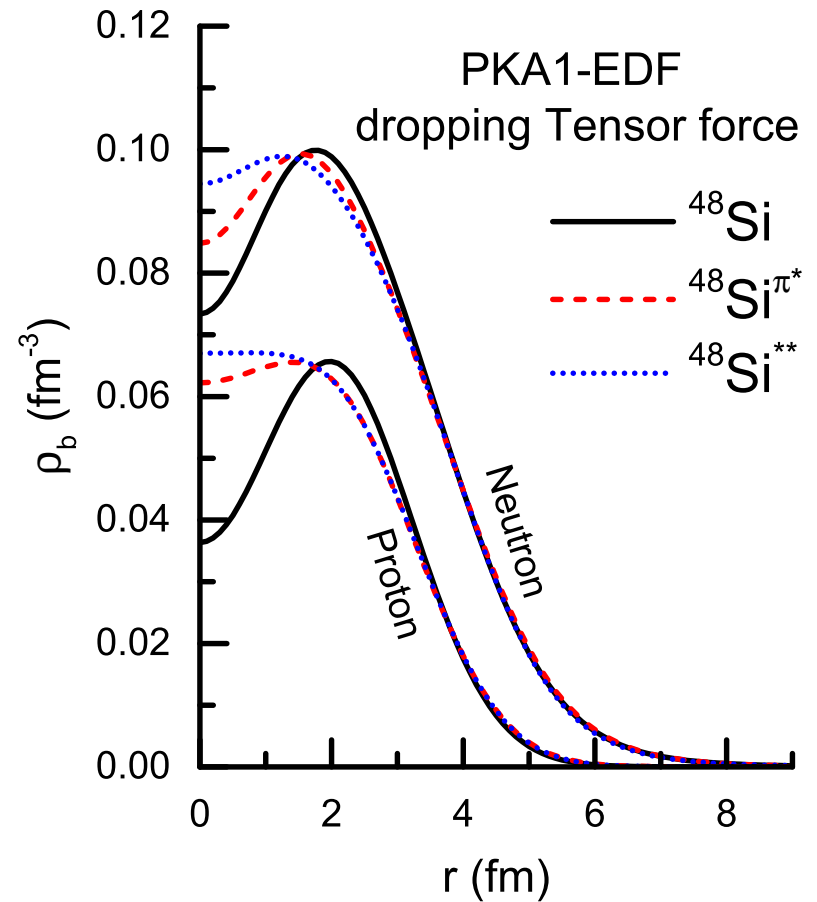
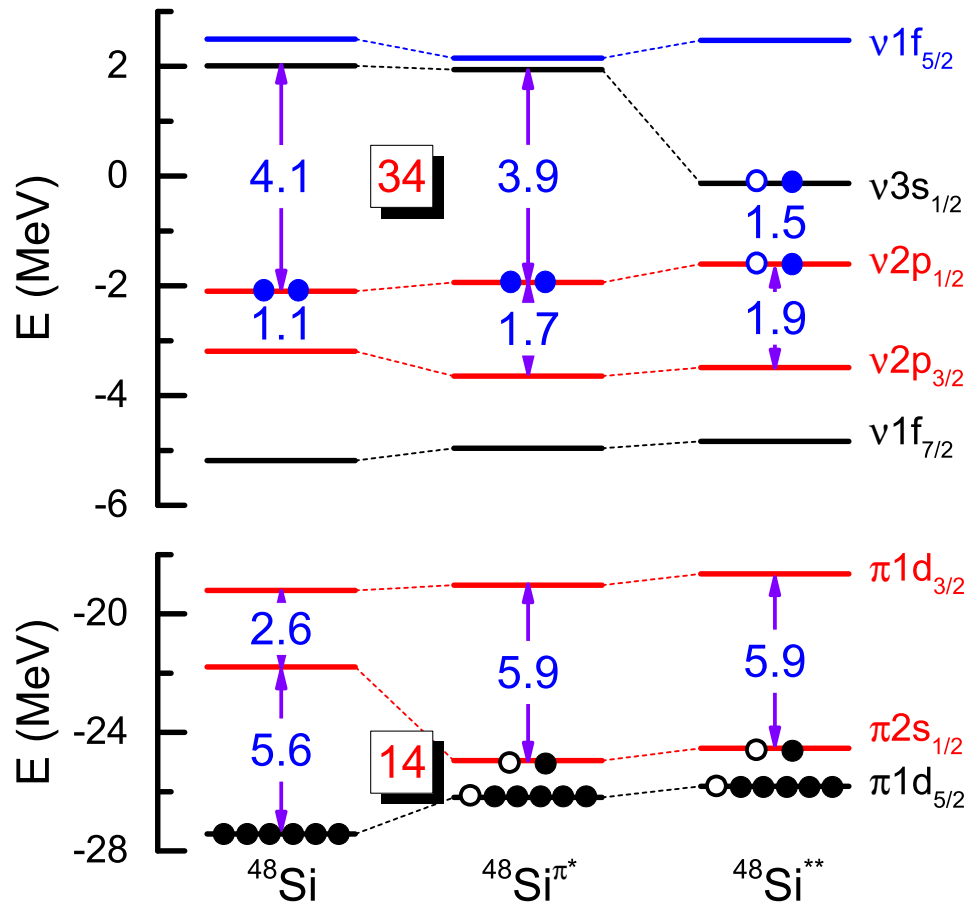
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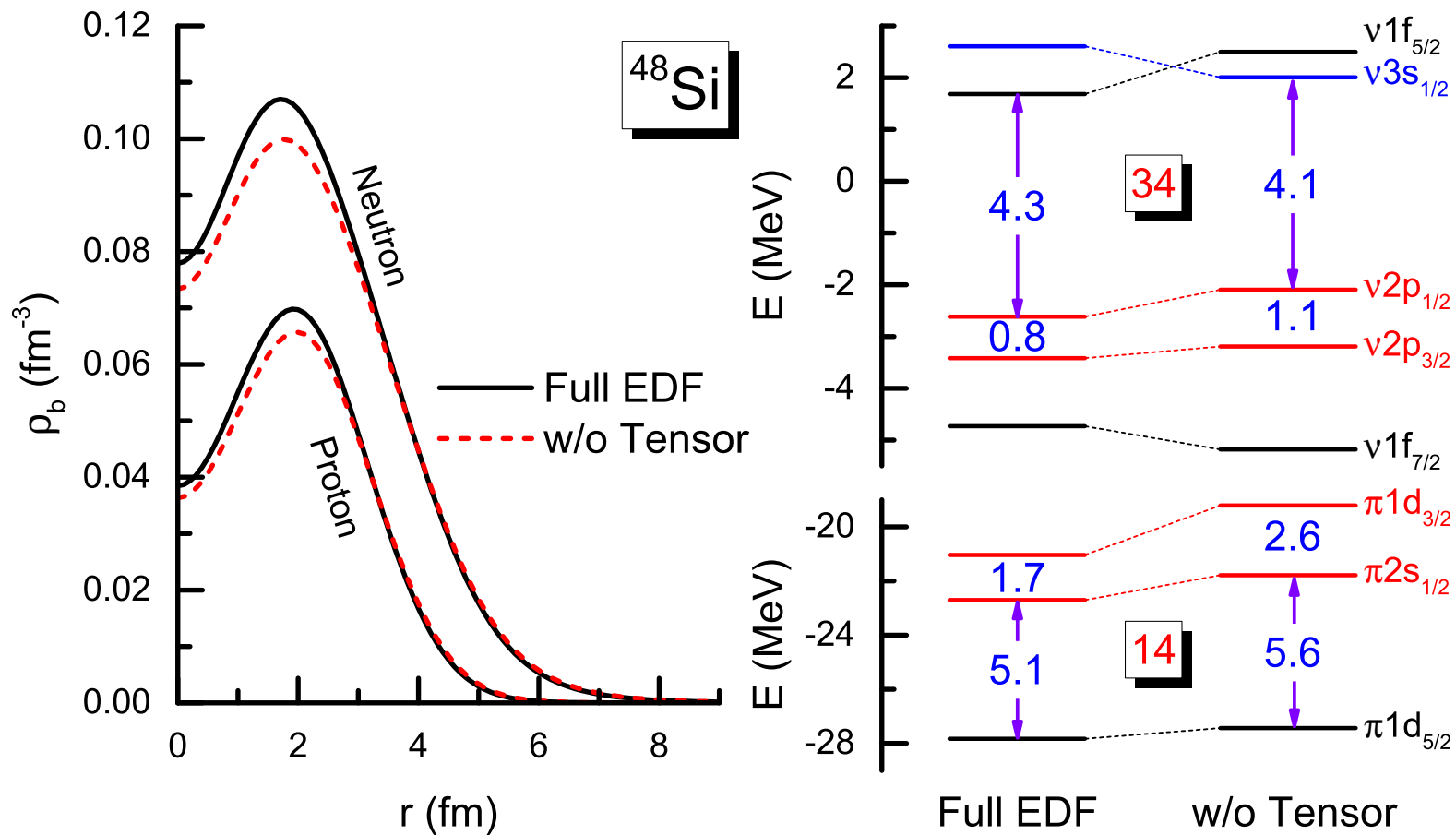
Dropping tensor force terms in Fock diagram



Similar systematics is found as the calculations with full EDF.

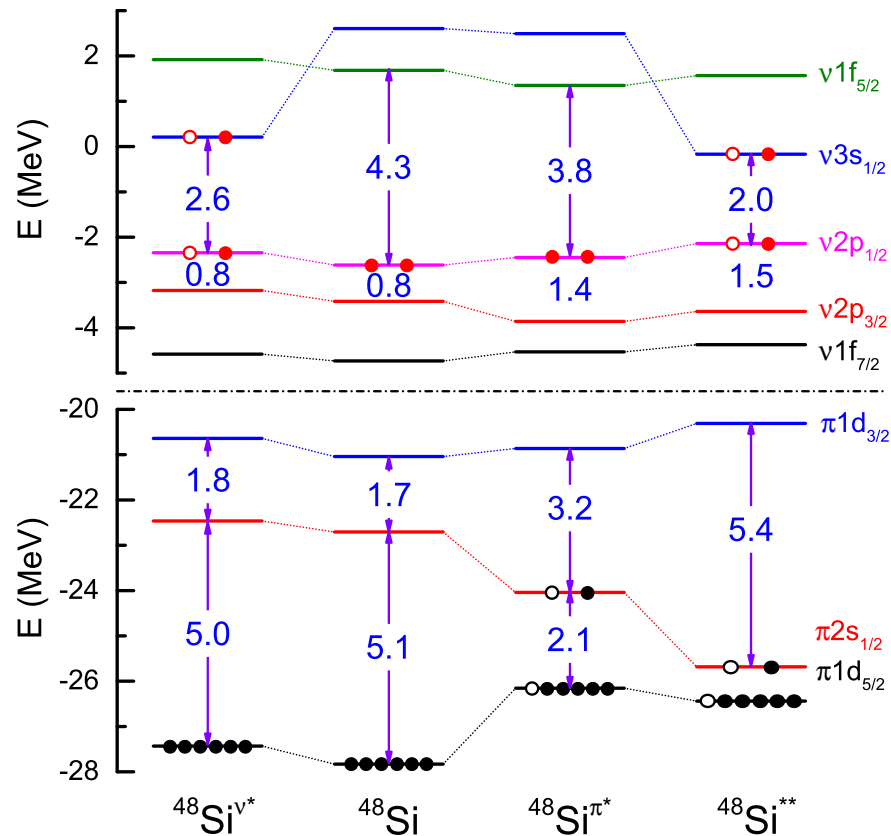
Bubble structure, instead of tensor force, is the key physics in determining the magicity.

Tensor effects in ground states



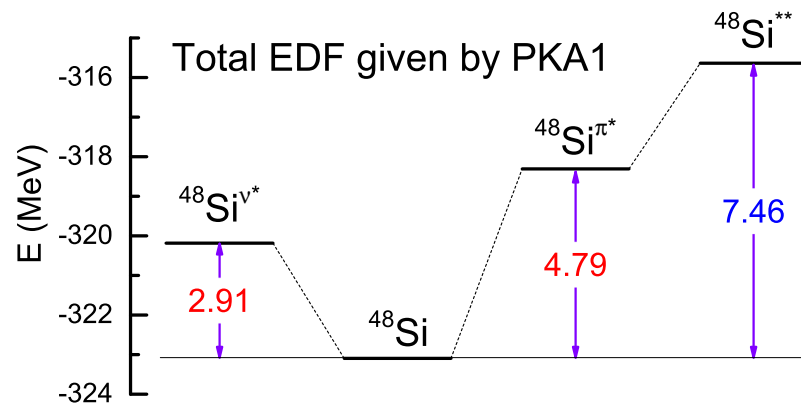
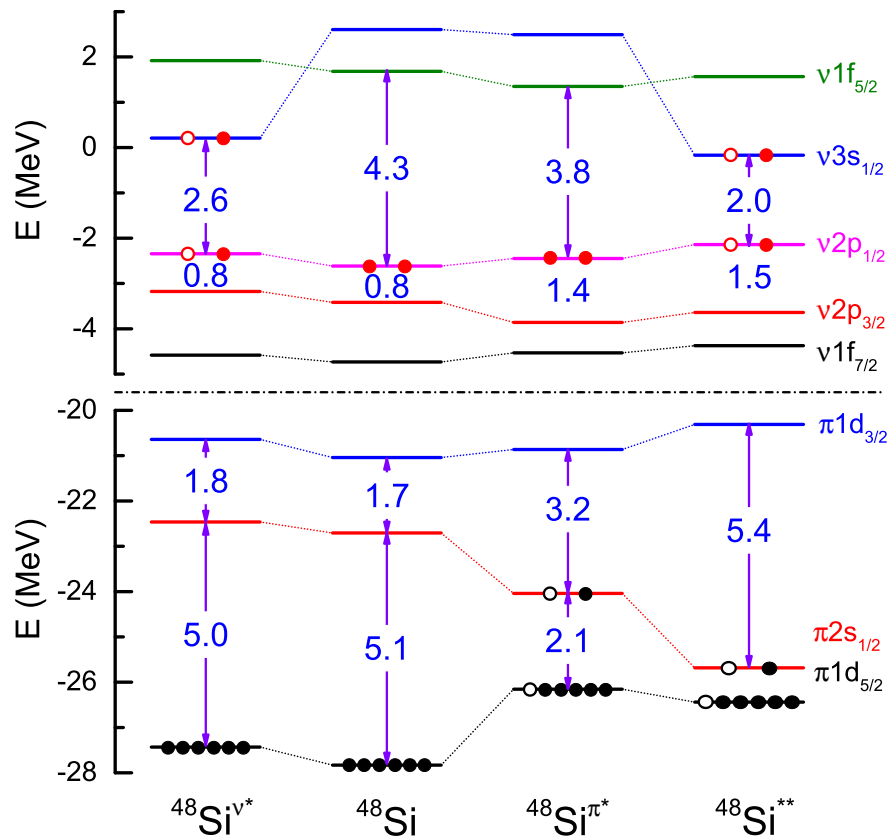
- The proton and neutron bubbles remain as well even after removing the tensor force component in Fock terms.
- Nuclear tensor force tends to quench the proton shell $Z = 14$ while enlarge the neutron one $N = 34$ with a few percent.

Crossing-shell excitations of ^{48}Si



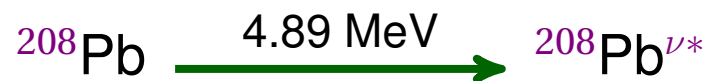
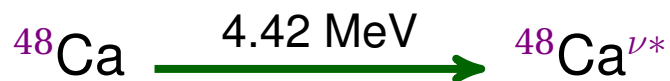
 Neutron (proton) **crossing-shell excitations** reduce the shell itself distinctly.

Crossing-shell excitations of ^{48}Si

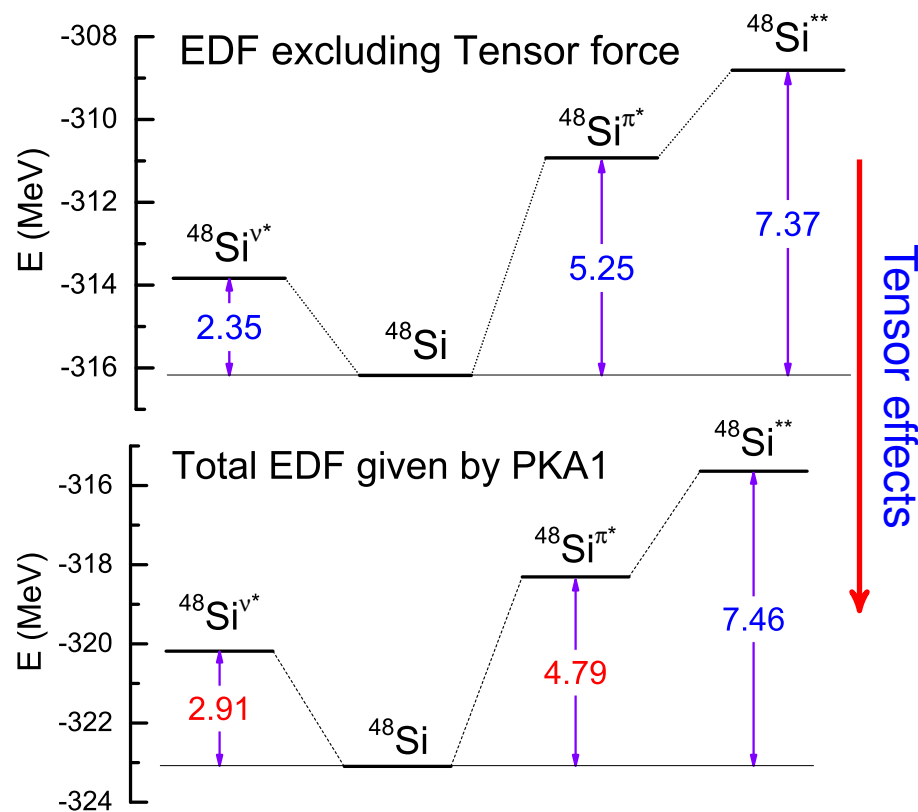
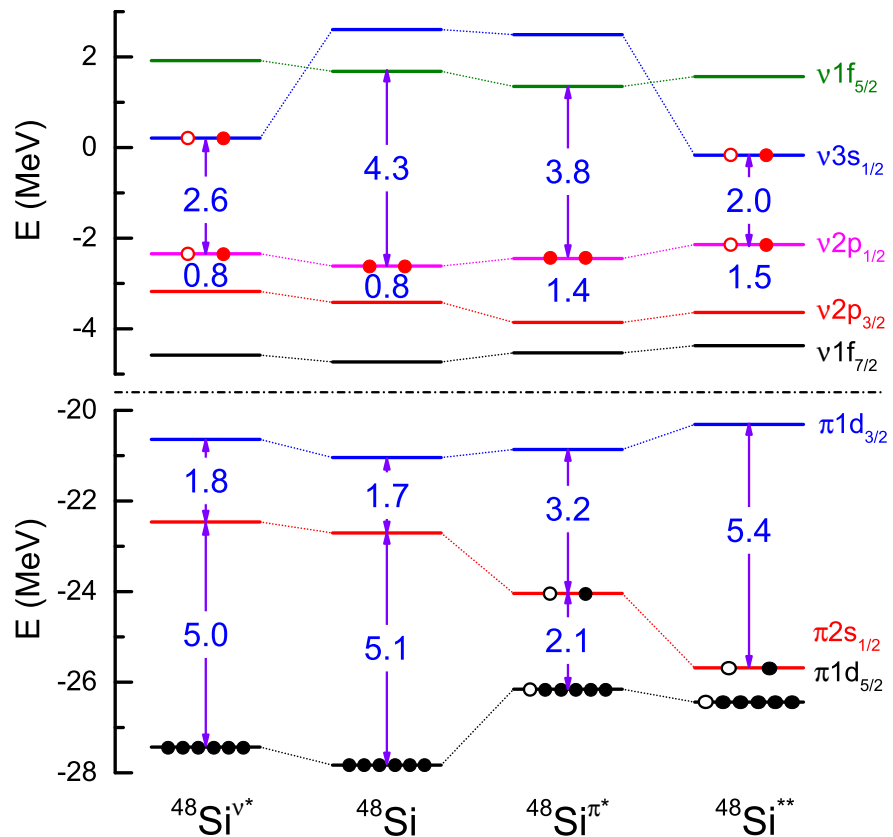


Neutron (proton) **crossing-shell excitations** reduce the shell itself distinctly.

Neutron and proton crossing-shell excitation energies are soundable.

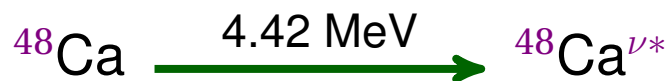


Crossing-shell excitations of ^{48}Si



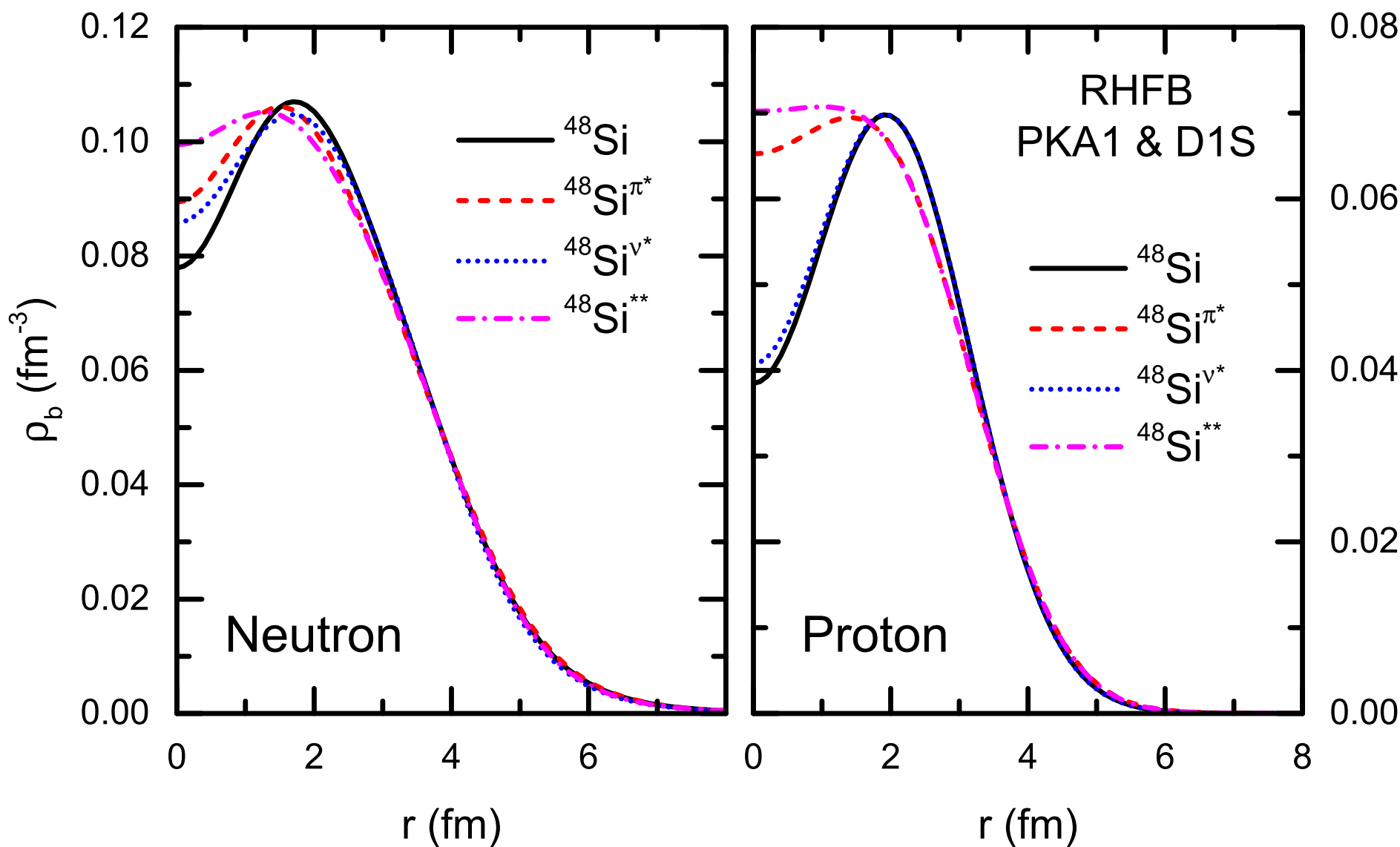
Neutron (proton) **crossing-shell excitations** reduce the shell itself distinctly.

Neutron and proton crossing-shell excitation energies are soundable.



Tensor force plays opposite roles in neutron and proton excitations.

Bubble structure in excited states



Proton bubble is sensitive to proton crossing-shell excitation.

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Conclusions and Perspectives

-  New magicities $N = 32, 34$ and $Z = 14$ and relevant physical mechanism are discussed by using RHFB-PKA1 model.
 -  Magicity $N = 32$ can be well reproduced by PKA1, in which isovector π -PV and ρ -T couplings are crucial and the **tensor force components** are also certainly significant.
 -  Magicity $N = 34$, arising just after $N = 32$, results from the quench of SO splitting of $\nu 2p$ orbits with the emergence of **neutron semi-bubble** in ^{54}Ca .
 -  $^{34}\text{Si}/^{34}\text{Ca}$ is predicted as the proton/neutron bubble nuclide, and ^{34}Si is identified as doubly magic proton bubble nuclide by experiments.

-  Eventually from ^{34}Si and ^{54}Ca towards neutron drip line, a new doubly magic nucleus ^{48}Si is predicted at neutron drip line, by the RHFB-PKA1 model.
 -  Neutron shell $N = 34$ is enhanced distinctly due to the neutron bubble structure along isotonic chain to ^{48}Si .
 -  Both neutron shell $N = 34$ and proton one $Z = 14$ become more distinct with the occurrence of dual bubble structure in ^{48}Si which certainly weaken the coupling with central distributed orbits, like s and p orbits.
 -  Doubly magicity in ^{48}Si is also supported by the evident crossing-shell energy.

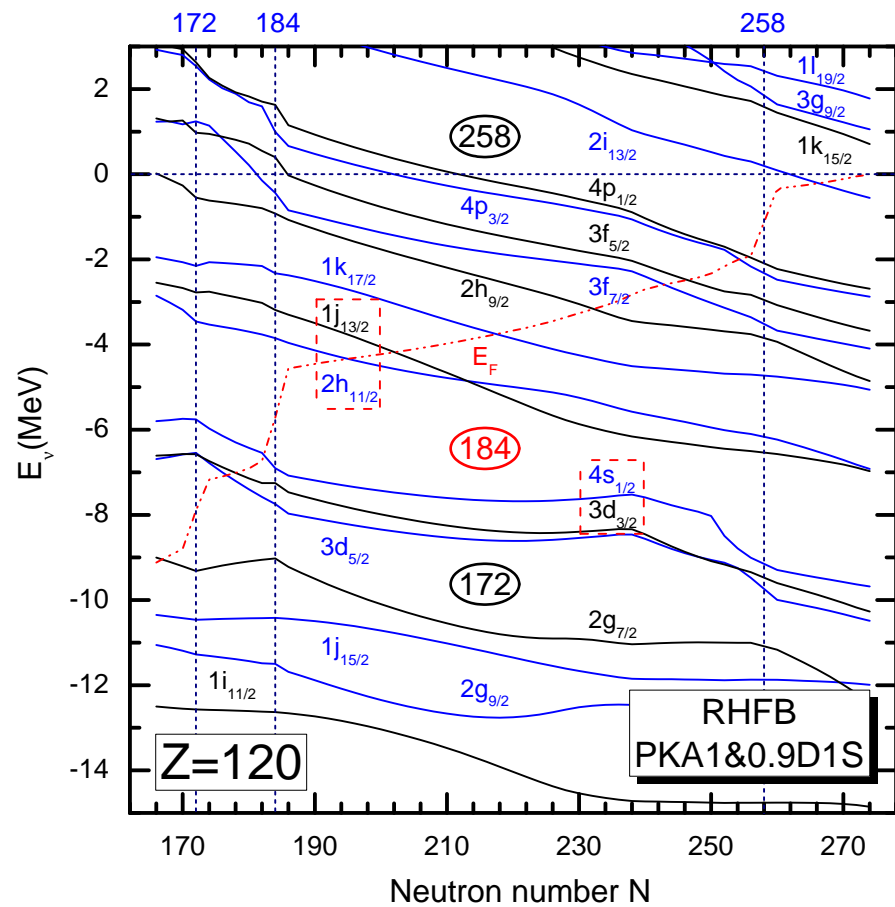
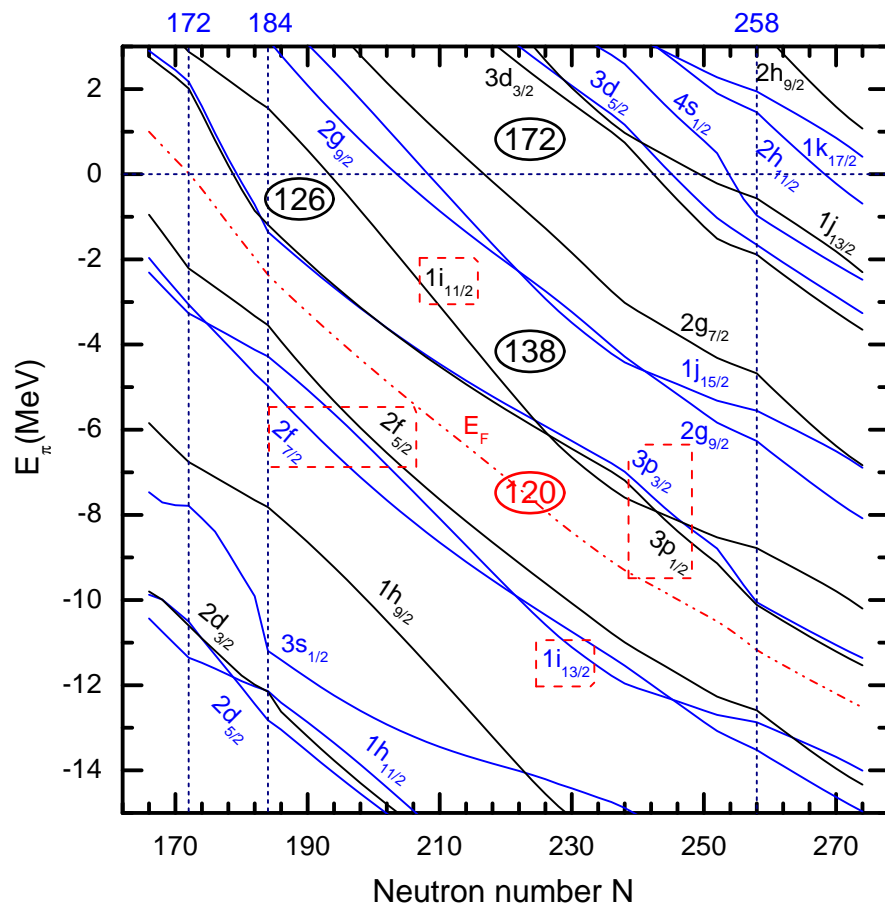
-  Perspective: does the pairing reentrance appear in ^{48}Si ?

Group Photo



Thank you for your attention!

Similar mechanism in Superheavy magicity



Similar mechanism in Superheavy magicity

