Properties of nuclear masses for heavy and neutron-rich nuclei

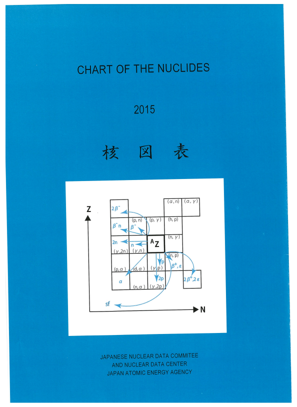
Hiroyuki KOURA(小浦寛之)

Advanced Science Research Center
Japan Atomic Energy Agency (JAEA)

I. Introduction

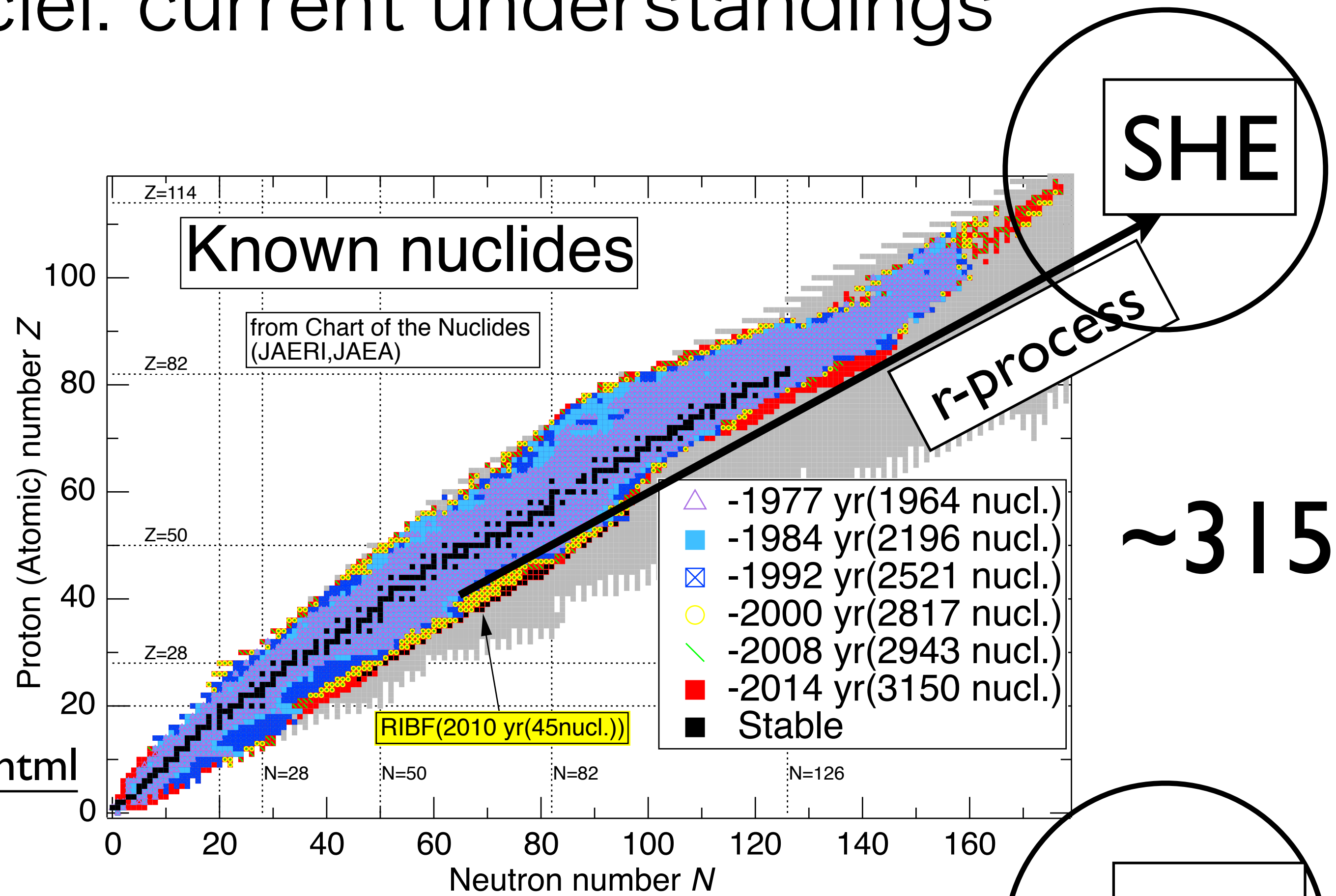
Search of nuclei: current understandings

Identified



taken from Chart of the nuclides by JAERI and JAEA (HK, et al., 2015)

wwwndc.jaea.go.jp/CN14/index.html



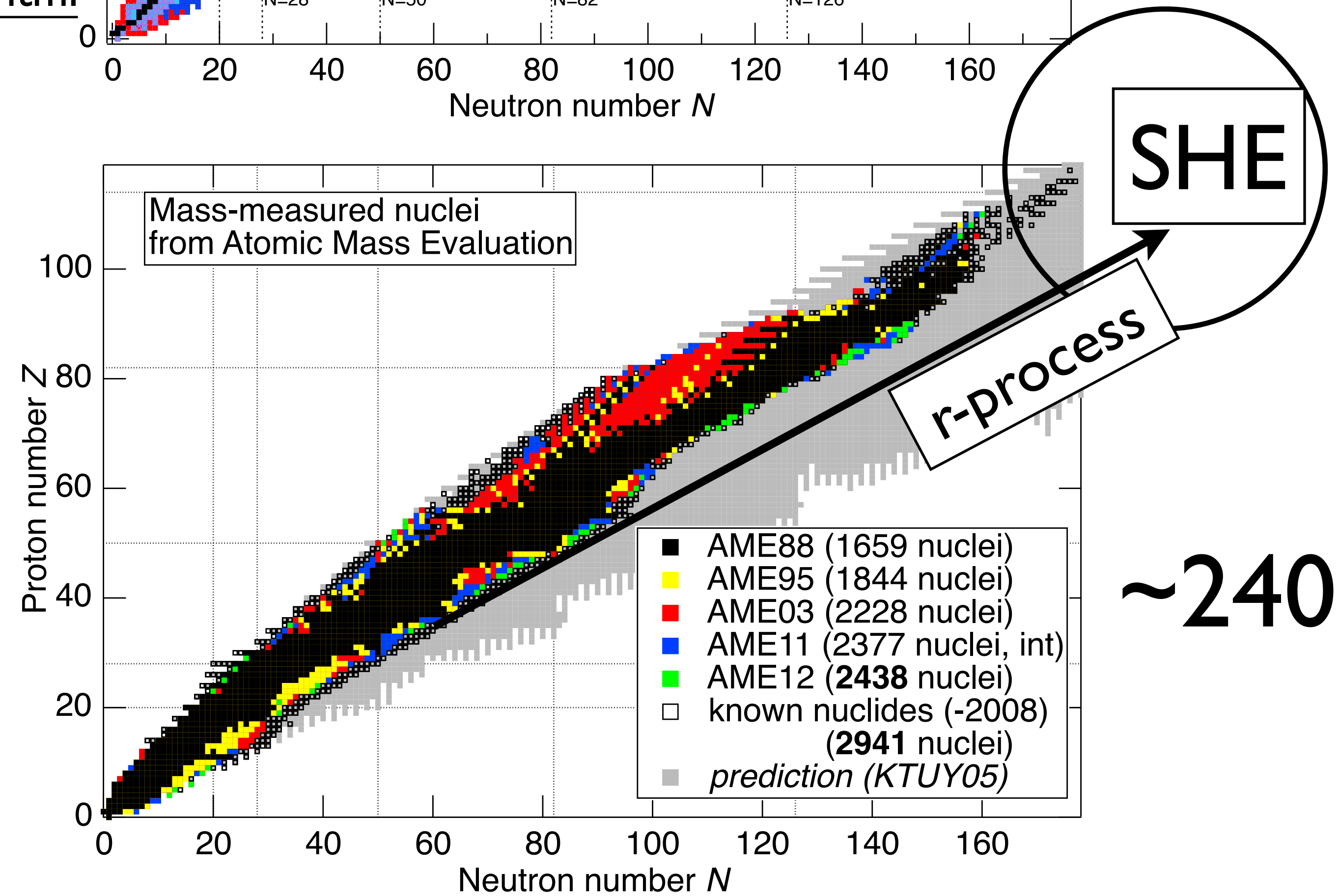
~3150 nuclei

Mass-measured

AMDC
Atomic Mass Data Center

Atomic Mass Evaluation is updated as AME2016

amdc.impcas.ac.cn/



~2400 nuclei

II. Bulk properties of nuclear mass

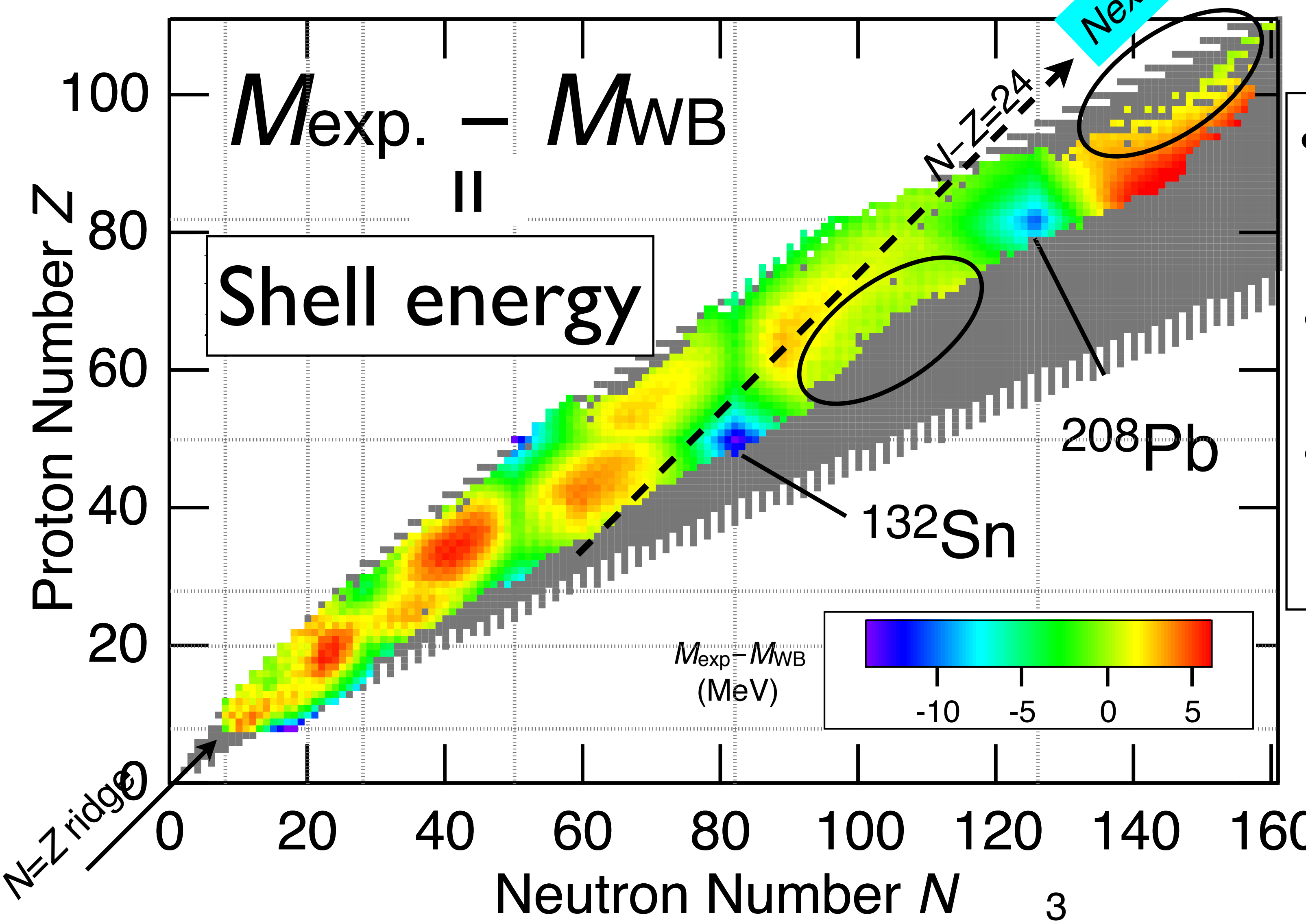
Weizsäcker-Bethe semi-empirical atomic mass formula

$$M_{\mathrm{WB}}(Z, N) = Z\,m_{\mathrm{H}} + N\,m_{\mathrm{n}} - B(Z,N)$$

$$= Z\,m_{\mathrm{H}} + N\,m_{\mathrm{n}} - a_{\mathrm{V}}A + a_{\mathrm{s}}A^{2/3} + a_{\mathrm{I}}(N-Z)^2/A + a_{\mathrm{C}}Z(Z-1)/A^{1/3} + \delta_{\mathrm{eo}}$$

a_{V}	a_{s}	a_{I}	a_{C}	a_{eo}	
15.604	17.472	22.99	0.7073	12.338	(MeV)

$$\delta_{\mathrm{eo}} = \begin{cases} -a_{\mathrm{eo}}/A^{1/2} & \text{for even-}Z\text{ and even-}N \\ 0 & \text{for odd-}A \\ +a_{\mathrm{eo}}/A^{1/2} & \text{for odd-}Z\text{ and odd-}N \end{cases}$$



From the 'exp.' shell energy:

- Existence of magic number
  N=28,50,82,126
  Z=28,50,82
- Wigner energy
  N=*Z* ridge
- Depression due to the deform.
  rare-earth, actinide

Mass data : 2012 Atomic mass evaluation (M. Wang, G. Audi, A.H. Wapstra *et al.*)

<u>Trend in MeV-order</u>

Shape transition and shell energies

- Schematic -

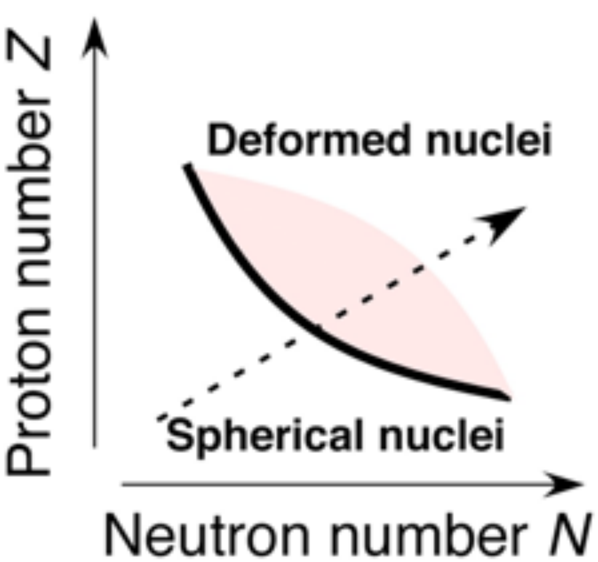


Fig. (a) N-Z plane

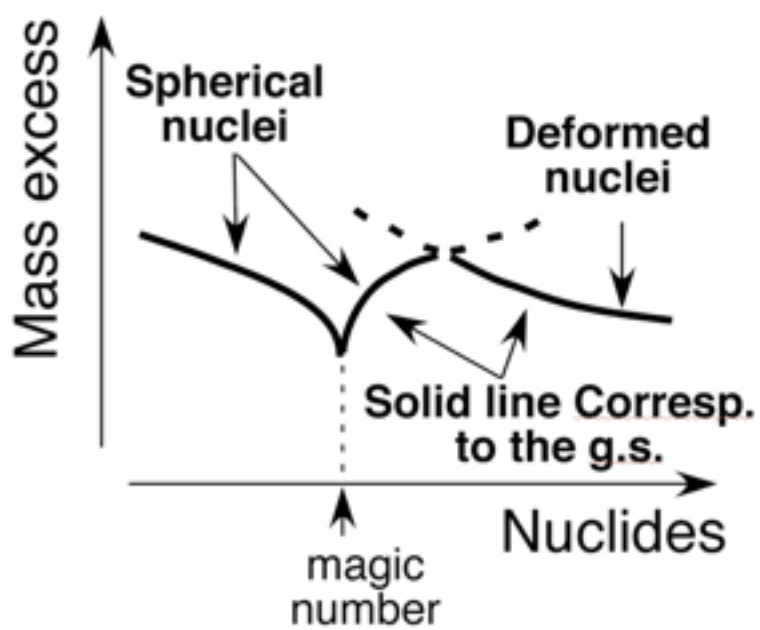
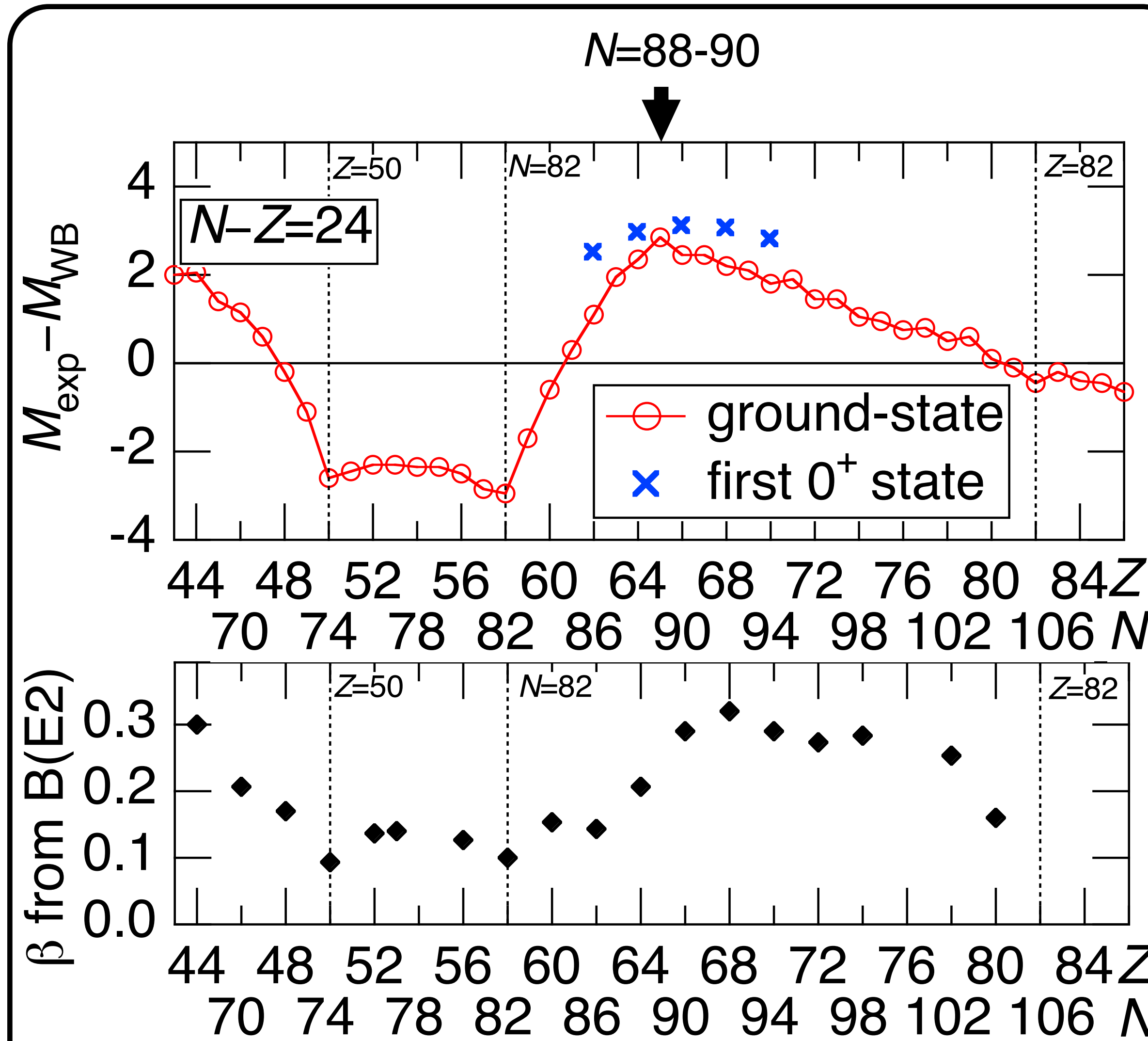


Fig. (b) cross section along dashed line in (a)

- Experiment -

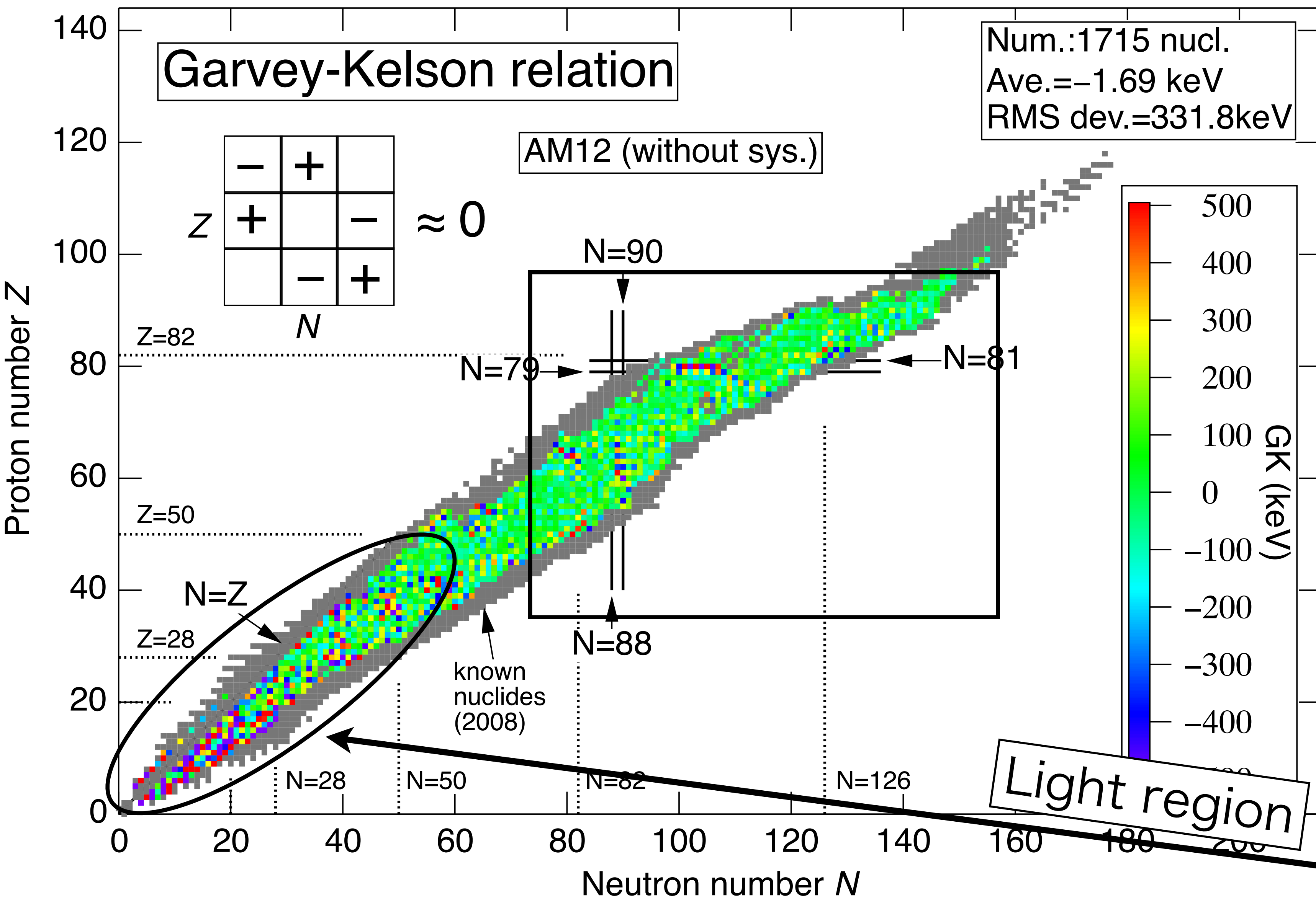


β from B(E2) : S. Raman et al., ADNDT78 (2001)

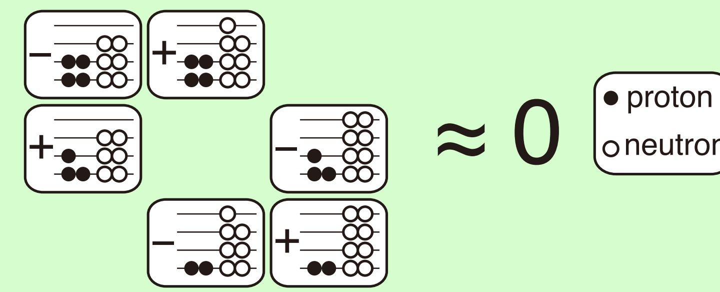
Notable feature on discontinuity of derivative of mass values

- Z=50, N=82 and Z=82 discontinuity of derivative: Spherical single-particle shell closure
- N=88-90 discontinuity: Shape transition

Mass relation: Garvey-Kelson systematics



A consideration of cancellation of core + valence nucleons (based on the shell model)

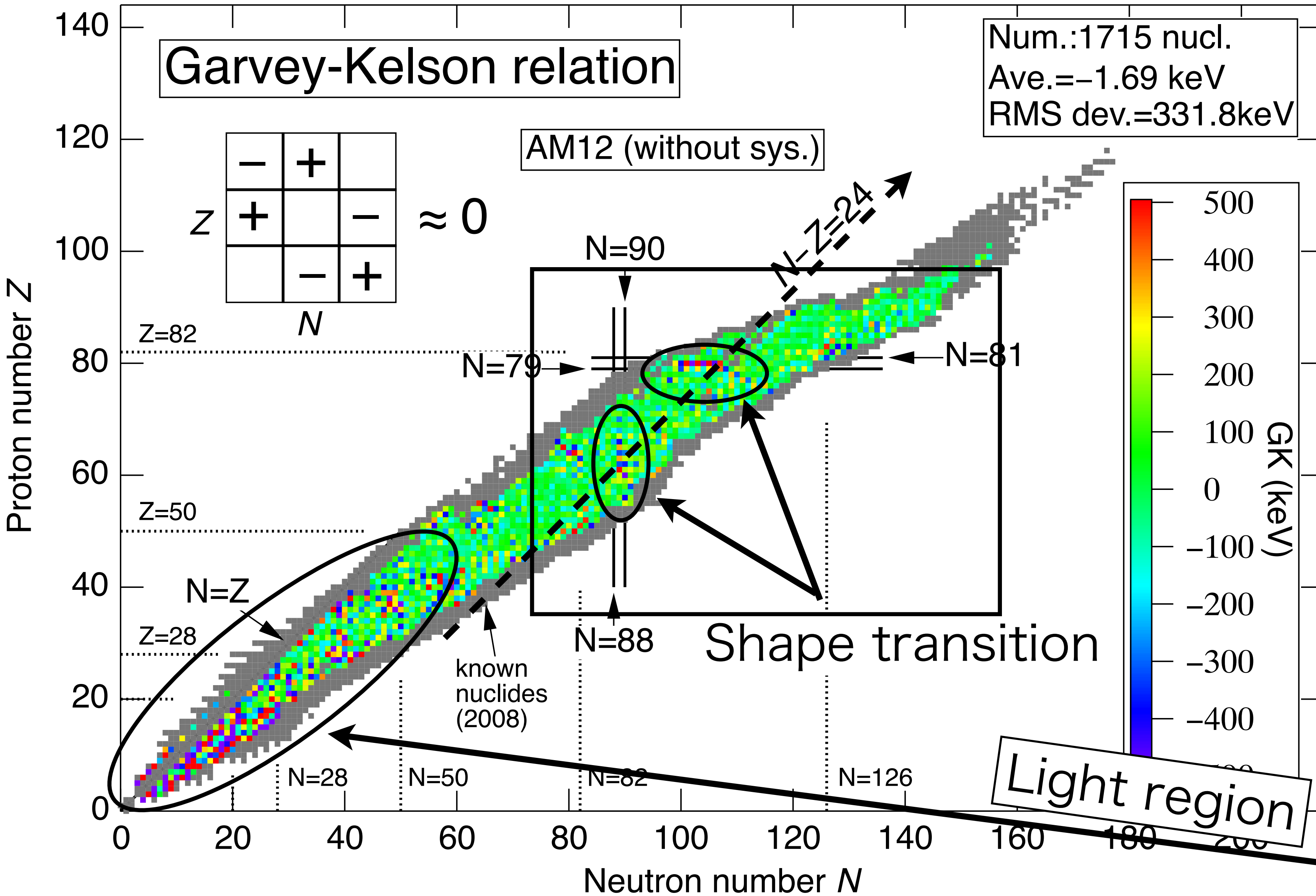


- Assumption: Cores among related six nuclei are the same.

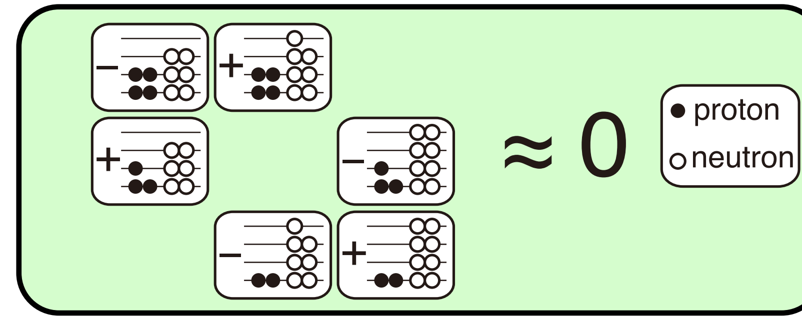
Region	num.	Average	RMS dev.
All	**1715**	**-1.7 (keV)**	**331.8(keV)**
$A>100$	1202	-0.03	161.2
$A\leq100$	513	-29.3	554.1

larger

Mass relation: Garvey-Kelson systematics



A consideration of cancellation of core + valence nucleons (based on the shell model)



- Assumption: Cores among related six nuclei are the same.

Region	num.	Average	RMS dev.
All	**1715**	**-1.7 (keV)**	**331.8(keV)**
A>100	1202	-0.03	161.2
A≤100	513	-29.3	554.1

larger

GK type mass formula:
Comay-Kelson-Zidon (CKZ1988)
Jänecke-Masson (JM1988)
Masson-Jänecke(MJ1988)
(ADNDT39, 1988)

In heavy region, the GK sys. gives some transition as changes of structures, especially nuclear shape.

Trend in 100 keV-order

III. Mass formulae

KTUY mass model

KUTY (KTUY) mass formula (Koura,Uno,Tachibana,Yamada (00,05)

$$M(Z, N)=M_{\mathrm{gross}}(Z, N)+M_{\mathrm{eo}}(Z, N)+M_{\mathrm{shell}}(Z, N)$$

H. K, et al., Nucl. Phys. A **674**, 47 (2000)
H. K, et al., Prog. Theor. Phys. **113**, 305 (2005)

$M_{\mathrm{gross}}(Z, N)$: gross term

$$M_{\mathrm{gross}}(Z,N)=M_{\mathrm{H}}Z+M_{\mathrm{n}}N+a(A)A+b(A)\left|N-Z\right|+c(A)(N-Z)^2/A+E_{\mathrm{C}}(Z,N)-k_{\mathrm{el}}Z^{2.39}$$

$$a(A)=a_1+a_2A^{-1/3}+a_3A^{-2/3}+a_4(A+\alpha_{\mathrm{a}})^{-1}$$

$$b(A)=\qquad +b_4(A+\alpha_{\mathrm{b}})^{-1}$$

$$c(A)=c_1+c_2A^{-1/3}+c_3A^{-2/3}+c_4(A+\alpha_{\mathrm{c}})^{-1}$$

$M_{\mathrm{shell}}(Z,N)$: shell term

Spherical nuclei

Calculated from **Spherical single-particle potential** for any nuclei (includes the BCS paring, reduction)

Deformed nuclei (Spherical-basis condieration)

Obtained by **an appropriate mixture** of the above spherical shell energies + liquid-drop deform. energies

KUTY mass formula (Koura, Uno, Tachibana, Yamada (2000))

$$M(Z, N) = M_{\mathrm{gross}}(Z, N) + M_{\mathrm{eo}}(Z, N) + M_{\mathrm{shell}}(Z, N)$$

- $M_{\mathrm{shell}}(Z,N)$: shell term

H. K. and M. Yamada, Nucl. Phys. A **671**, 96 (2000)

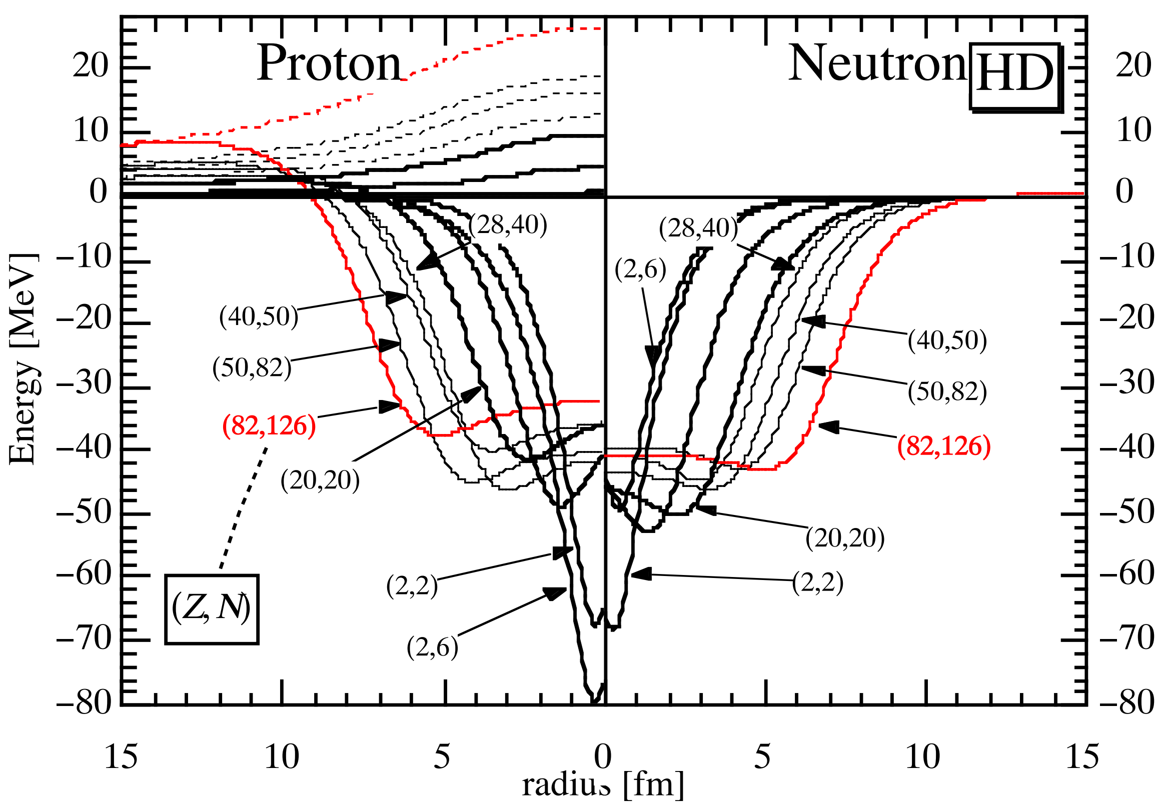
Spherical nuclei

calculated from **Spherical single-particle potential**

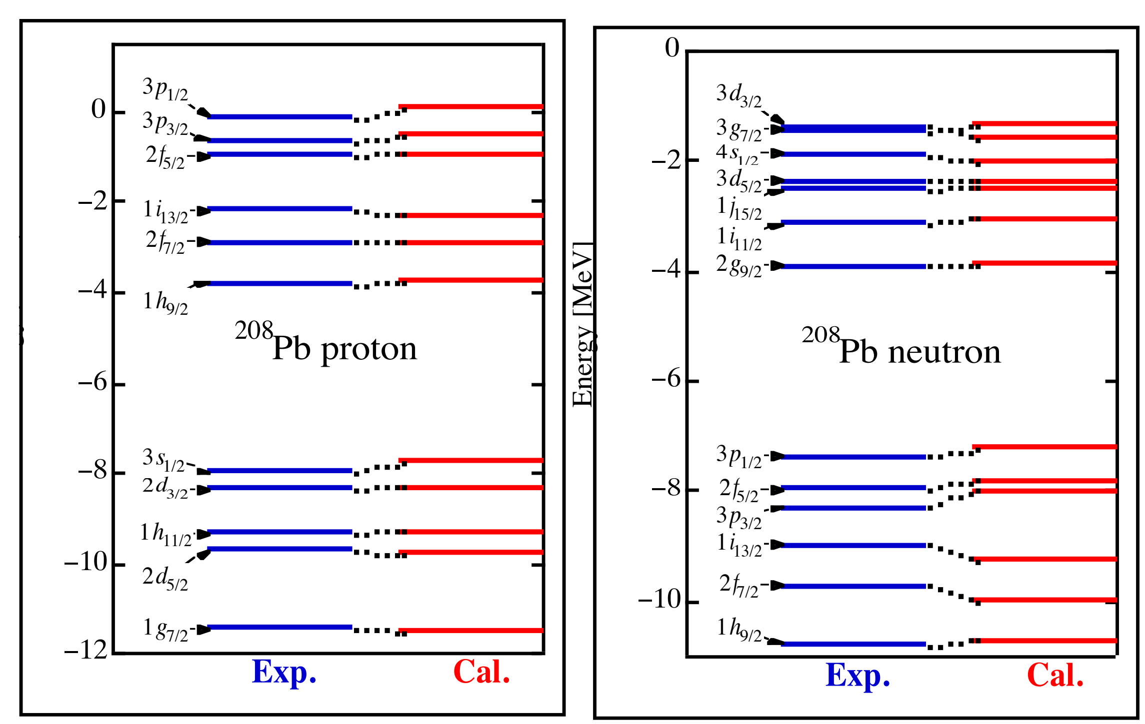
for any nuclei (includes the BCS paring, reduction)

$$V_{\mathrm{cen}}(r) = V_0 \frac{1}{\left\{1 + \exp\left[(r - R_v)/a_v\right]\right\}^{a_v/\kappa}} \left\{1 + V_{\mathrm{dp}} \frac{1}{1 + \exp\left[-(r - R_v)/a_v\right]}\right\}$$

(Central component)



Single-particle levels of ^{208}Pb



Differences between calc. and exp. levels: **within 330 keV for 132Sn, 208Pb**

Nuclear shell energy with deformation

(Spherical-basis consideration)

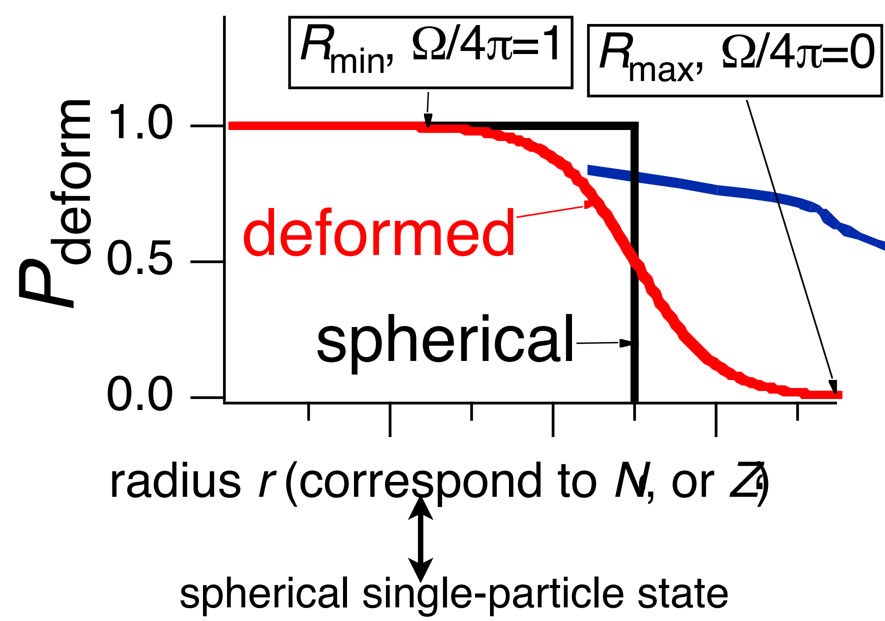
H. K, et al., Nucl. Phys. A **674**, 47 (2000)

$$E_{\rm sh}(Z,N) = \sum_{\rm def.} \left(\underbrace{< E^{\rm sph}_{\rm sh}(Z,N) >_{\rm def.}}_{\text{micro.}} + \underbrace{\Delta E_{\rm S}(Z,N) - \Delta E_{\rm C}(Z,N)}_{\text{Liquid-drop}} - \Delta E_{\rm pro}(Z,N) \right)$$

$$\underbrace{< E^{\rm sph}_{\rm sh}(Z,N) >_{\rm def}}_{\text{micro.}} = \sum_{Z'} W_{\rm pdef}(Z';Z,N) E_{\rm psph}(Z'.N'') + \sum_{N'} W_{\rm ndef}(N';Z,N) E_{\rm nsph}(Z''.N')$$

$E_{\rm psph}$, $E_{\rm nsph}$: Spherical shell energy

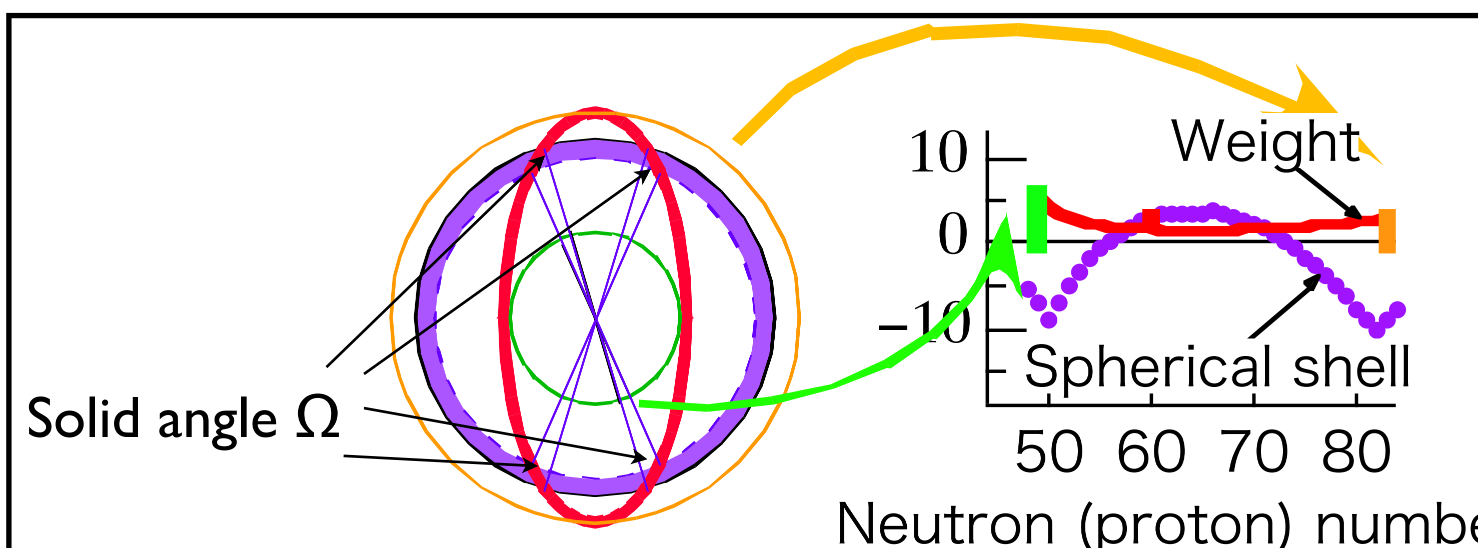
Mixing weight $W_{\rm def}$



$P_{\rm deform}$: occupation probability of deformation state on the spherical basis

Differential

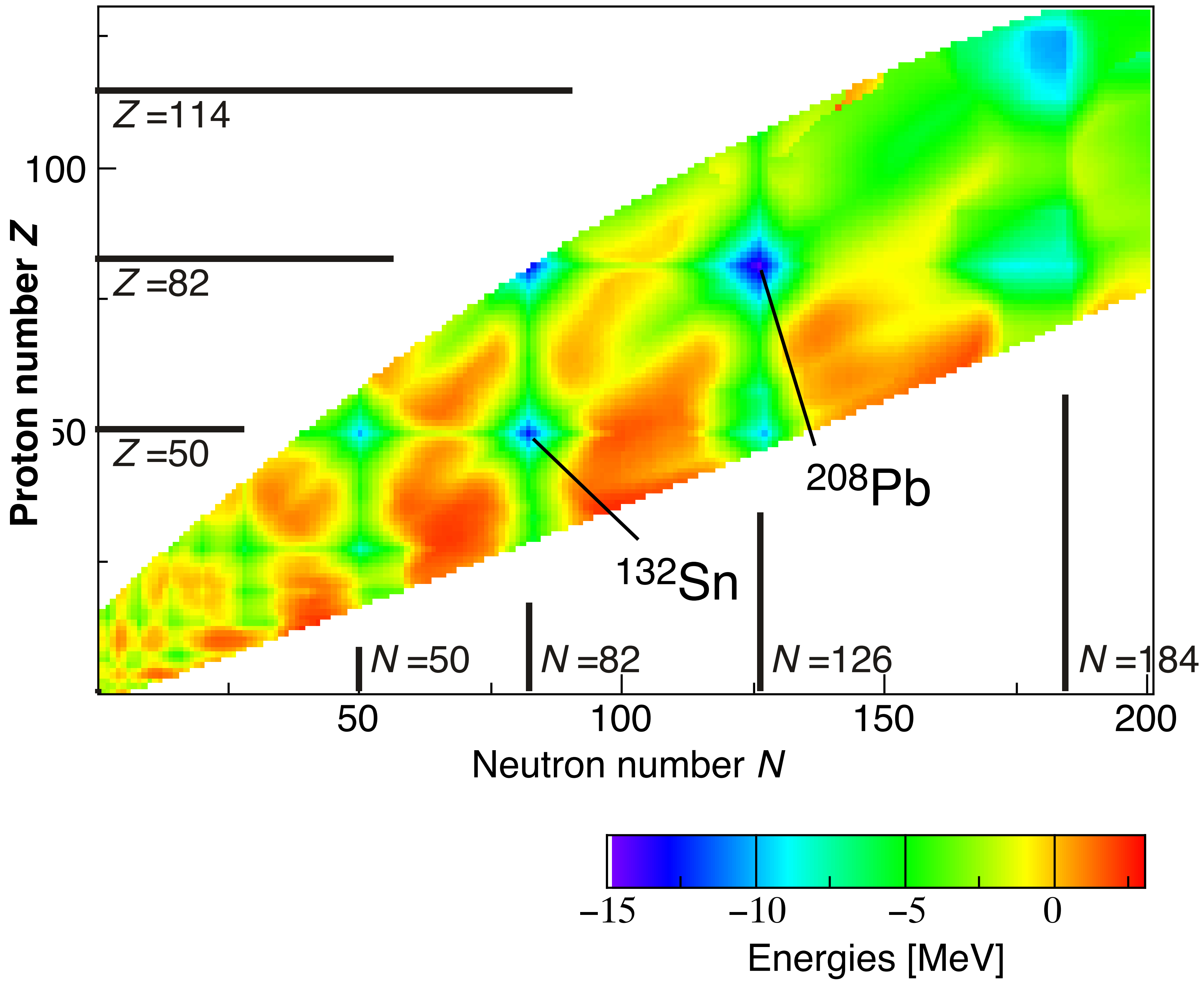
$$W_{\rm ndef}(N';Z,N) = -\frac{1}{4\pi}\frac{d\Omega(r(N'))}{dN'}$$



Current calc.:
$\alpha_{2,4,6}$ deformation
(not asym. shape)

Nuclear shell energy of KUTY

Nuclear shell energies $E_{sh}(Z, N)$



Other mass models and comparison

Skyrme-Hartree-Fock-Bogoliubov mass formula (2002-)

by S. Goriely et al.

$$E_{tot} = E_{HFB} + E_{wigner}$$

BSk21 force parameter set:

t_0=-3961.39 MeV fm^3, t_1=396.131 MeV fm^5
t_2=0 MeV fm^5, t_3=22588.2 MeV fm$^{3+3\alpha}$
t_4=-100.000 MeV fm$^{5+3\beta}$, t_5=-150.000 MeVfm$^{5+3\gamma}$
x_0=0.885231, x_1=-0.0648452, t_2x_2=1390.38 MeV fm^5
x_3=1.03928, x_4=2.00000, x_5=-11.0000
W_0=109.622 MeV fm^5, α=1/12, β=1/2, γ=1/12
f^+_n=1.00, f^+_p=1.07, f^-_n=1.05, f_p=1.13
V_W=-1.80 MeV, λ=280, V'_W=0.96, A_0=24

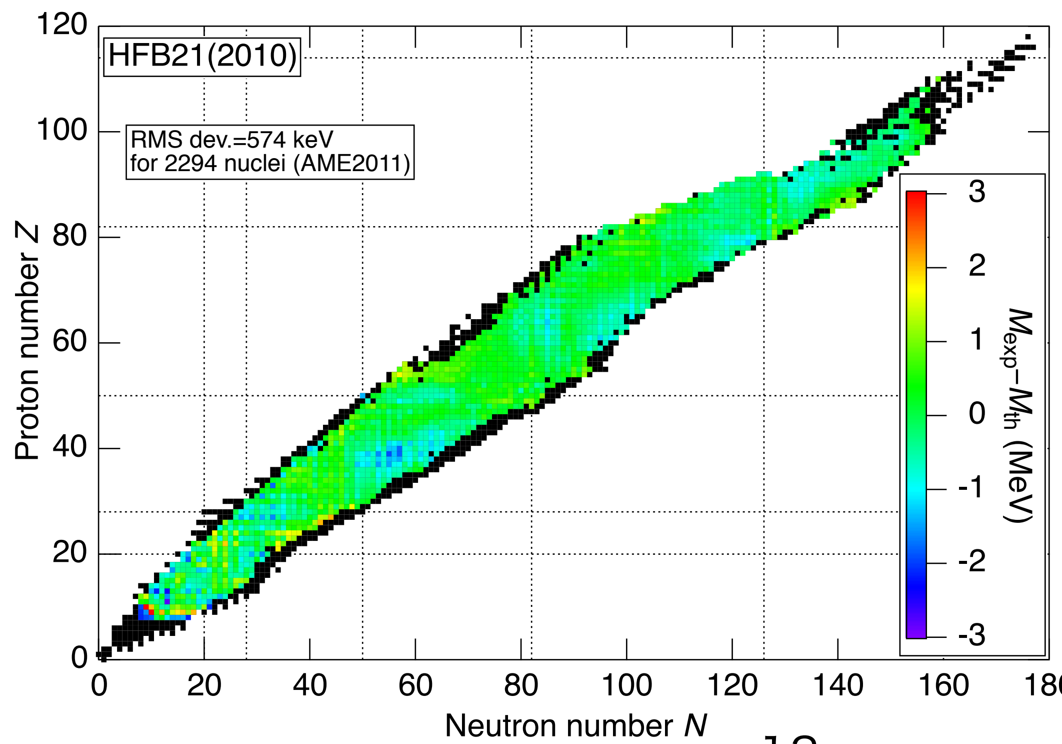
The long road in the HFB mass model development

		Accuracy σ_{rms} (2149 nuc)
HFB-2 :	Possible to fit all 2149 exp masses Z≥8	**659 keV**
HFB-3:	Volume versus surface pairing	**635 keV**
HFB-4-5:	Nuclear matter EoS: M^*_s=0.92	**660 keV**
HFB-6-7:	Nuclear matter EoS: M^*_s=0.80	**657 keV**
HFB-8:	Particle-number projection	**635 keV**
HFB-9:	Neutron matter EoS	**733 keV**
HFB-10-13:	Low pairing & NLD	**717 keV**
HFB-14:	Collective correction and Fission B_f	**729 keV**
HFB-15:	Coulomb correlations / CSB	**678 keV**
HFB-16:	**Pairing constrained to NM**	**632 keV**

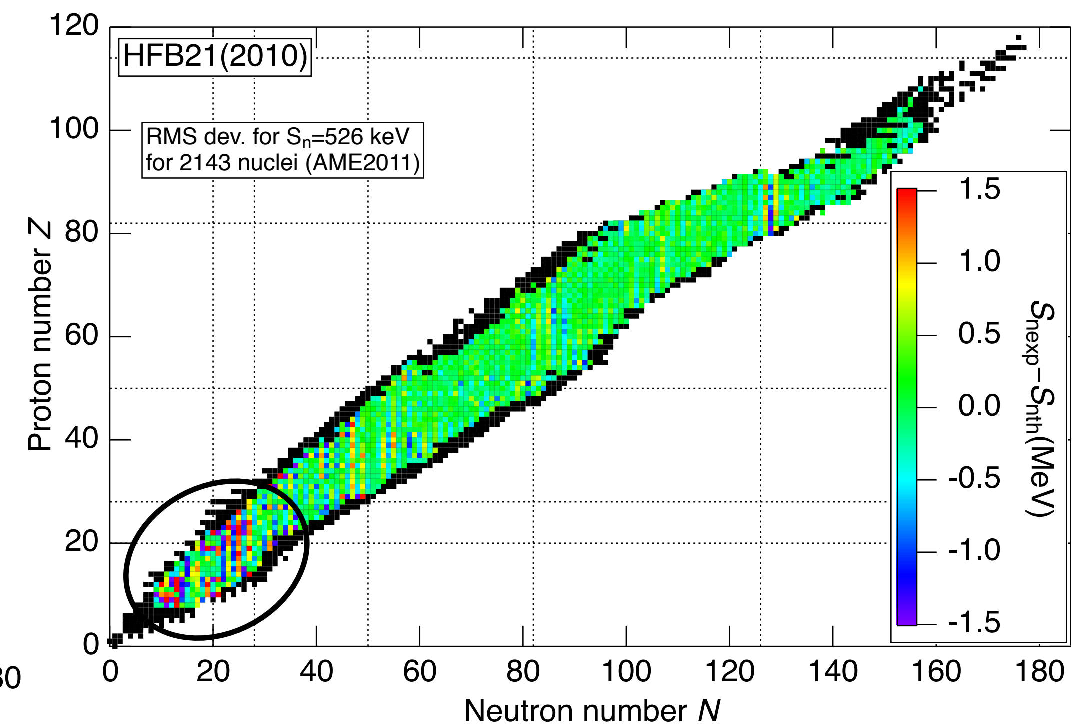
HFB21 gives a less than 600 keV of the RMS dev. In the light region there is some discrepancy in derivatives as S_n.

Referred mass data:AME03

Mass

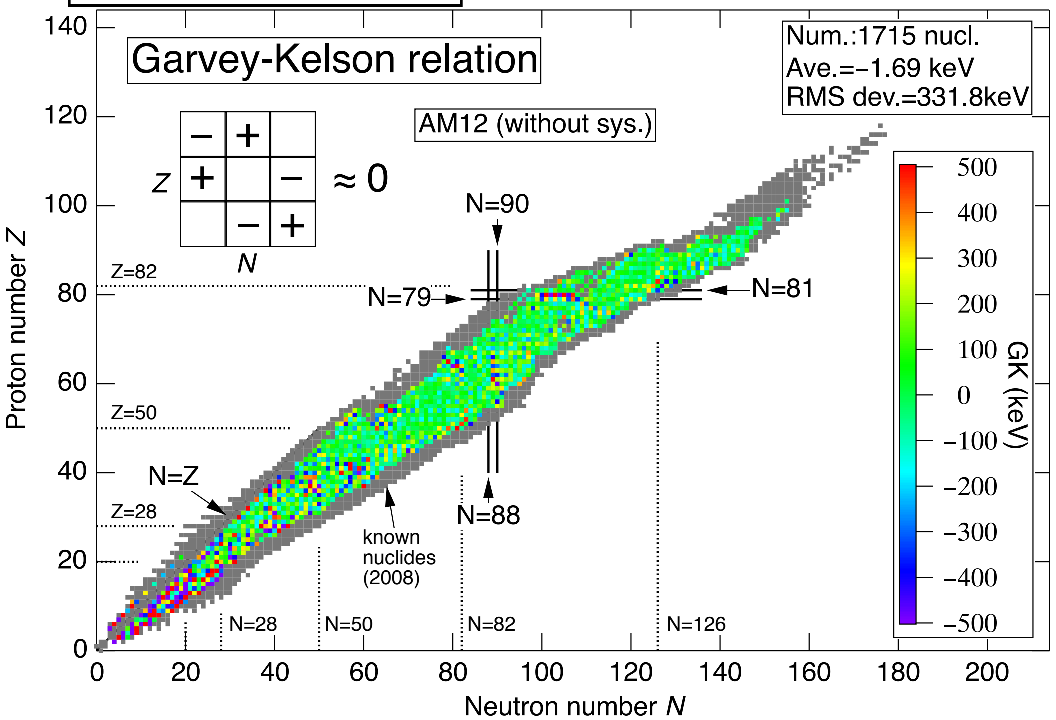


Sn

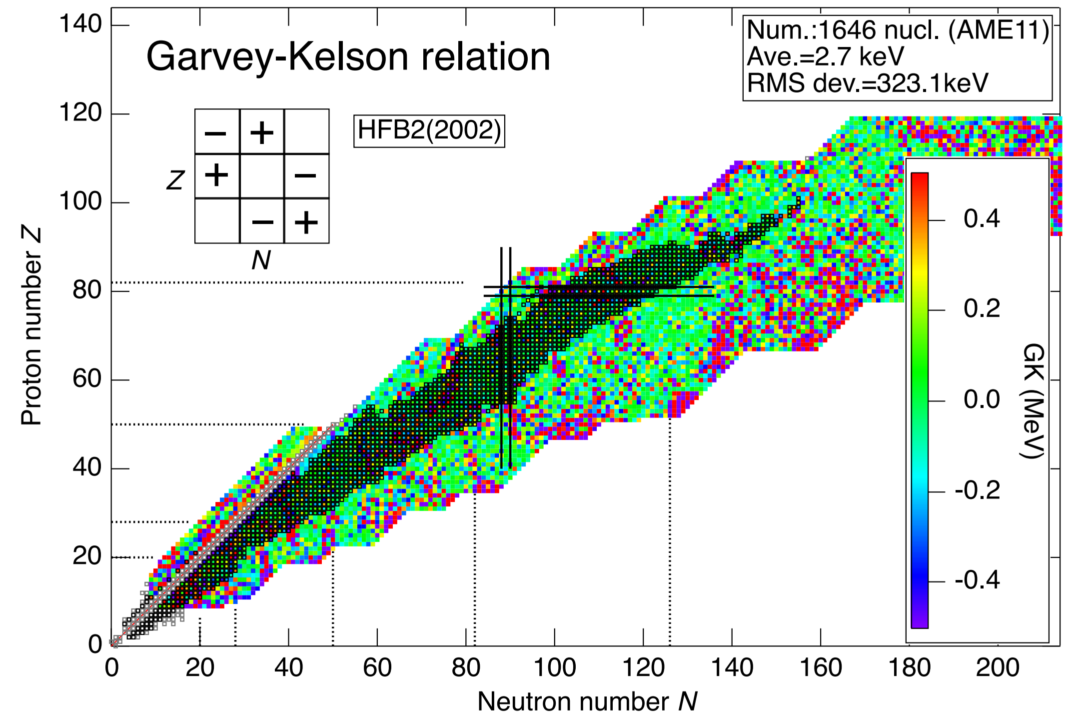


G-K sys. check for HFB formulae

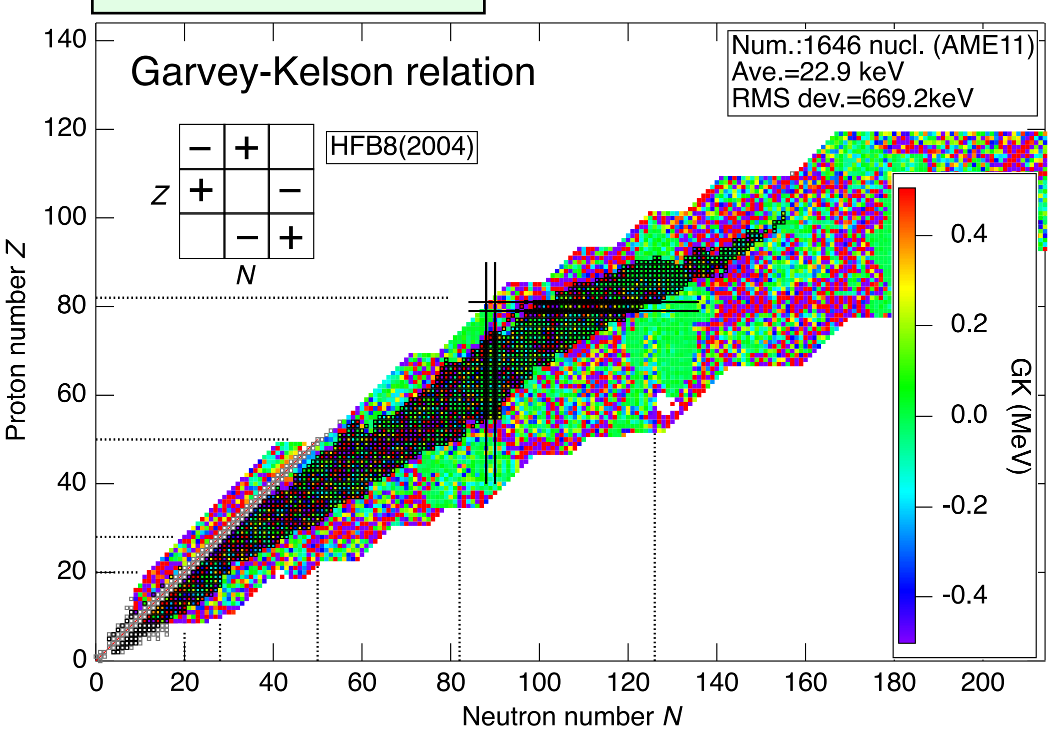
AME11(exp)



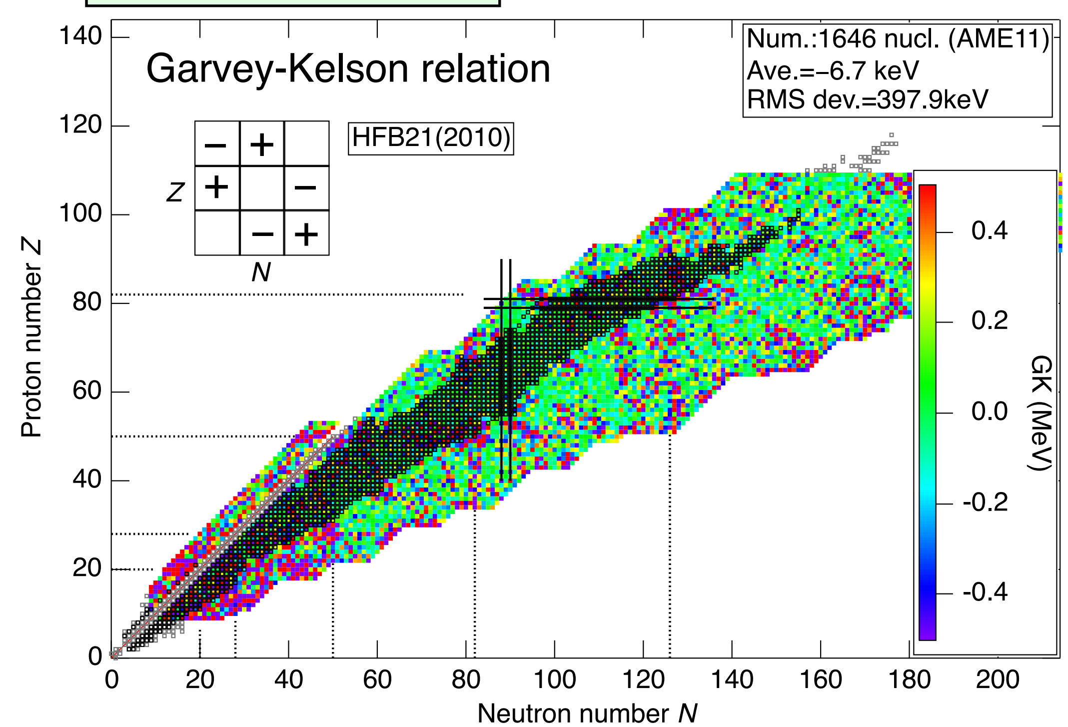
HFB2(2002)



HFB8(2004)



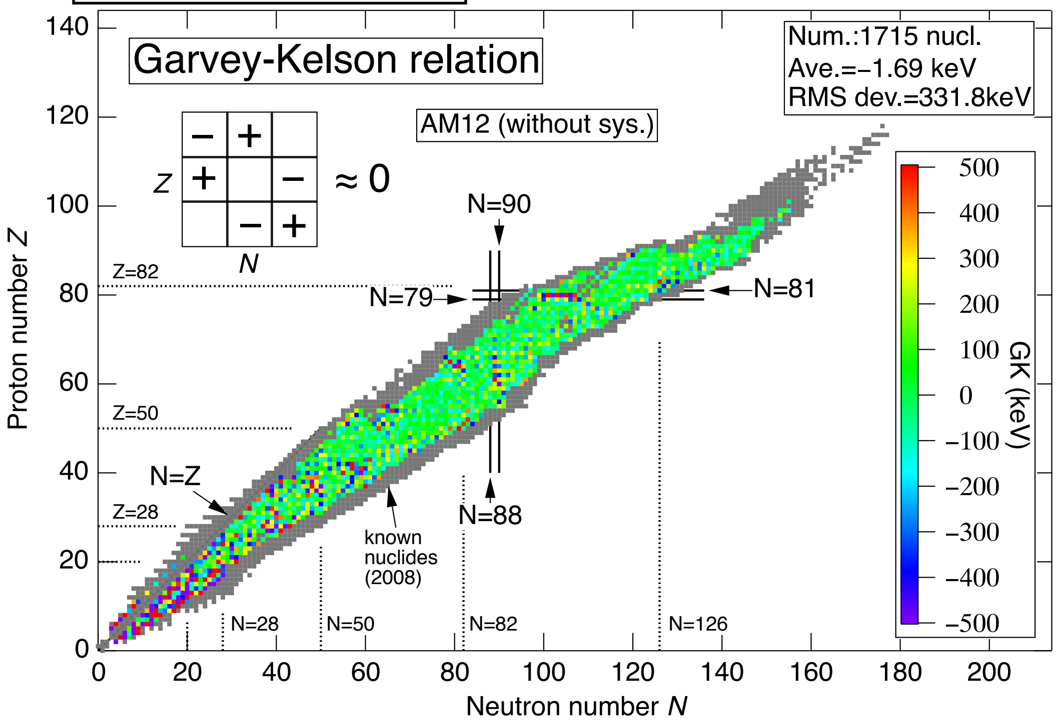
HFB21(2010)



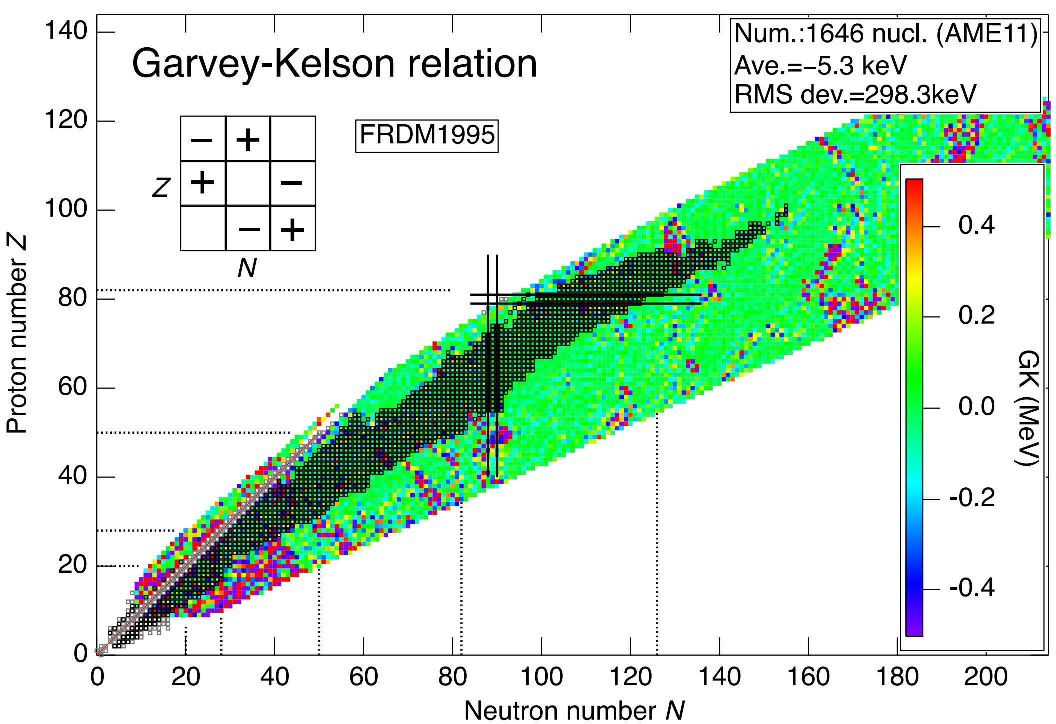
- The HFB calc. seems to violate the GK summation.

G-K sys. check for FRDM and KTUY

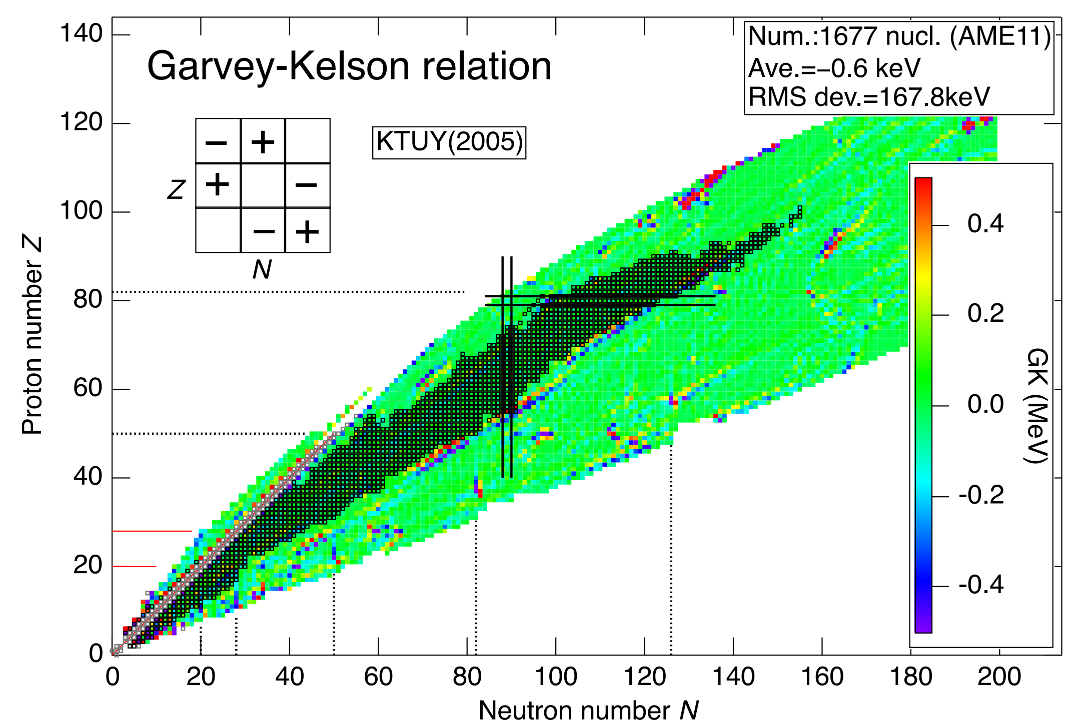
AME11(exp)



FRDM1995 : gives a smooth trends



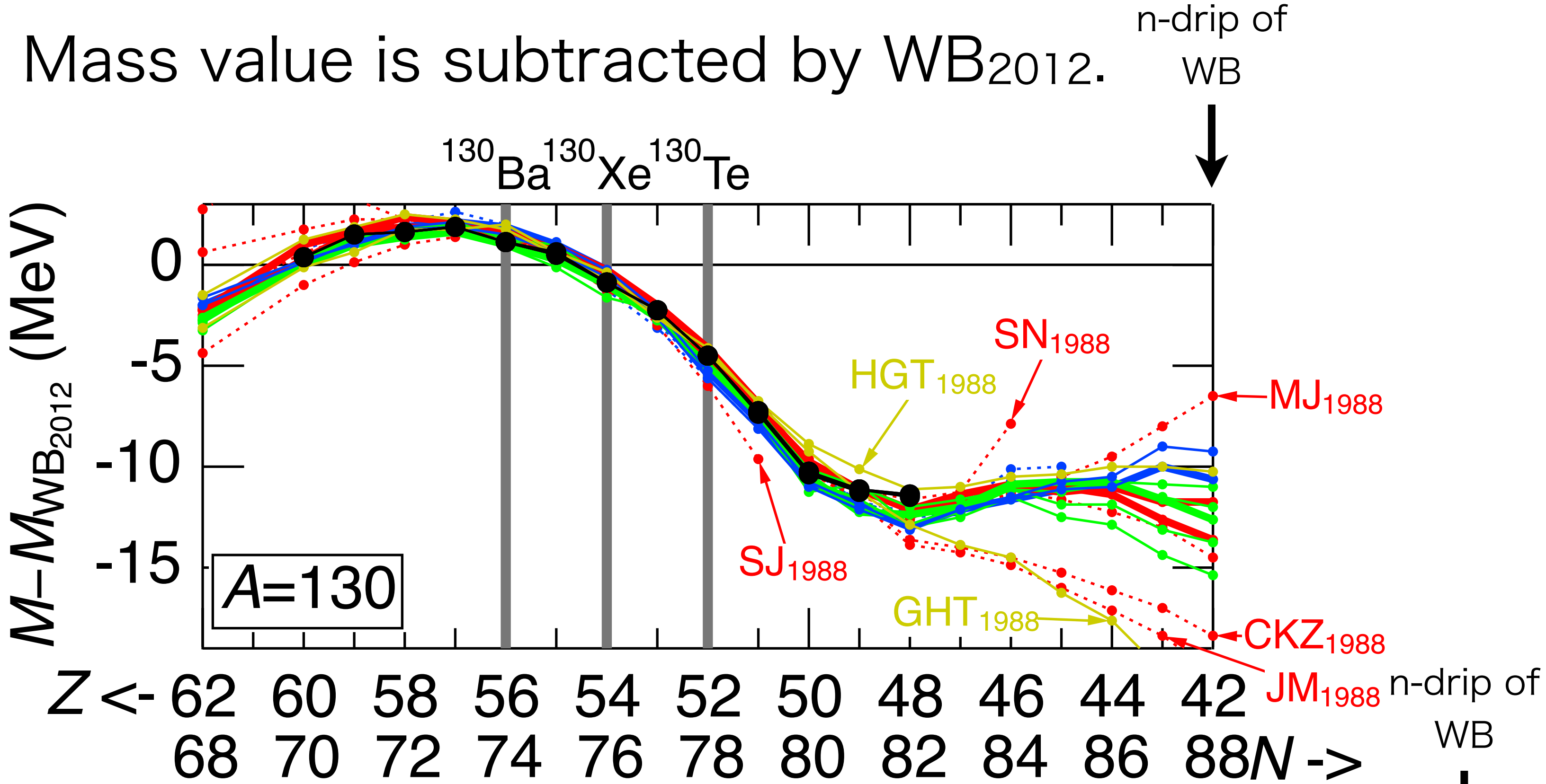
KTUY2005 : gives a smooth trends

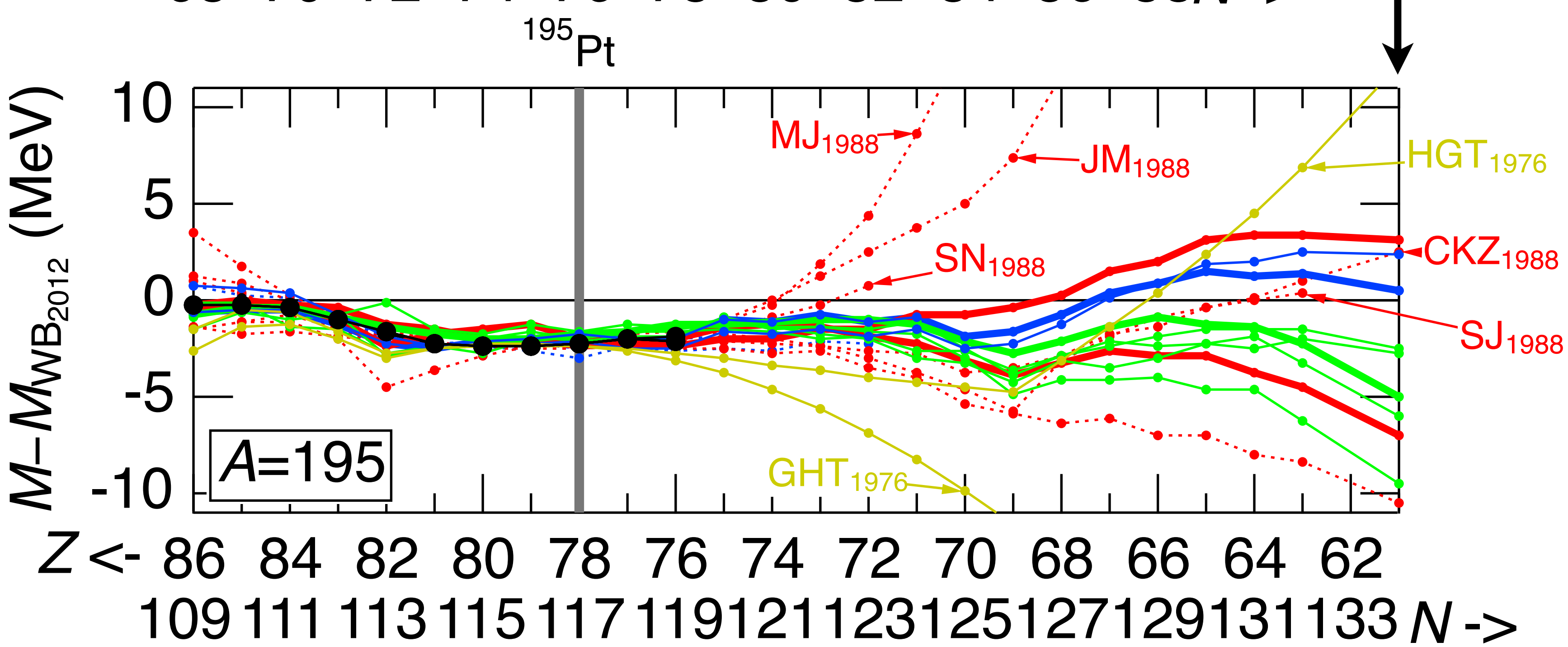


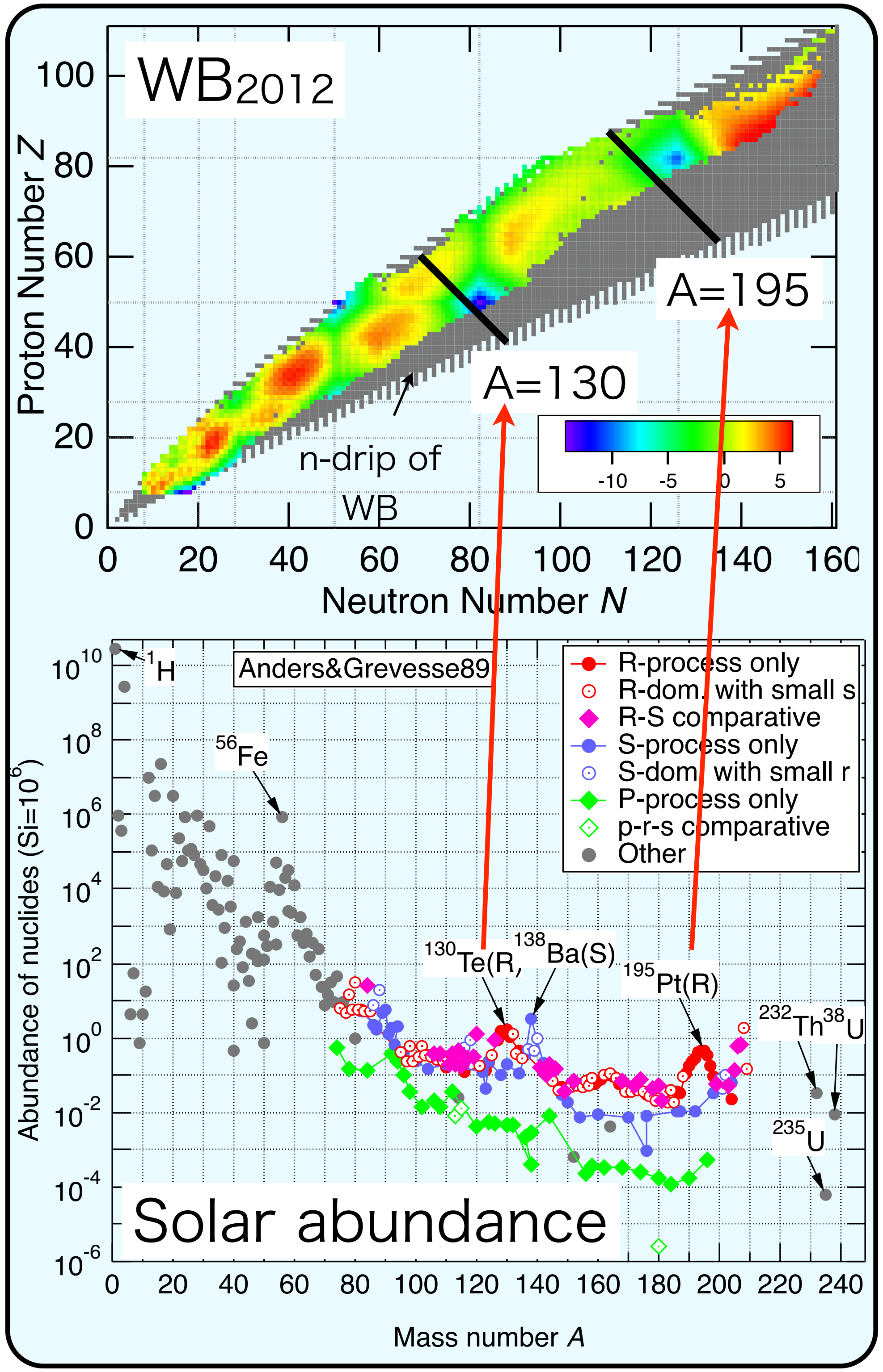
- FRDM and KTUY give a smooth trends.

Extrapolation to the n-rich nuclei

Mass value is subtracted by WB2012.



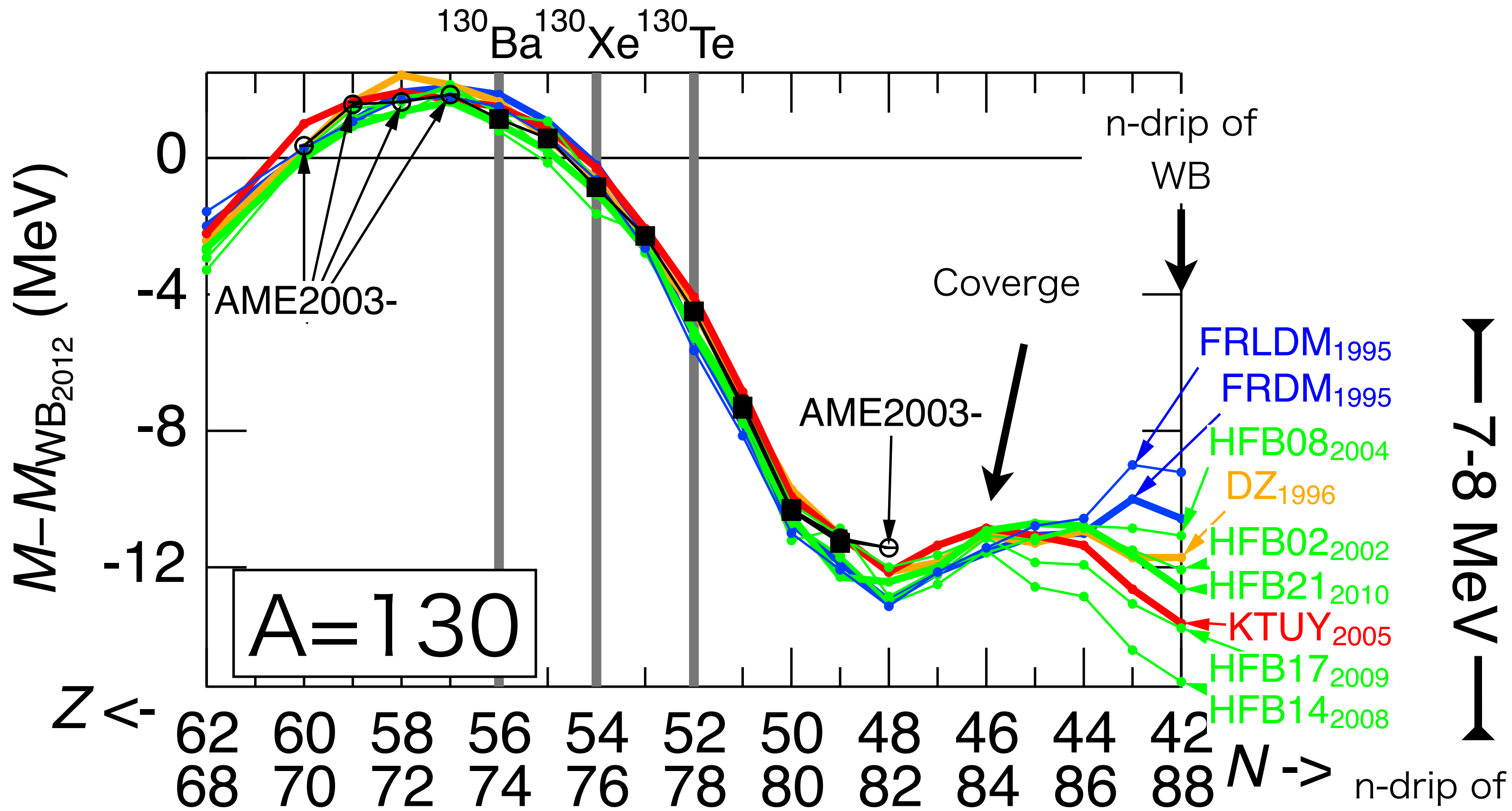




- In old-type mass formulae (-1988), mass values extremely diverge in the very neutron-rich region

Extrapolation to the n-rich nuclei

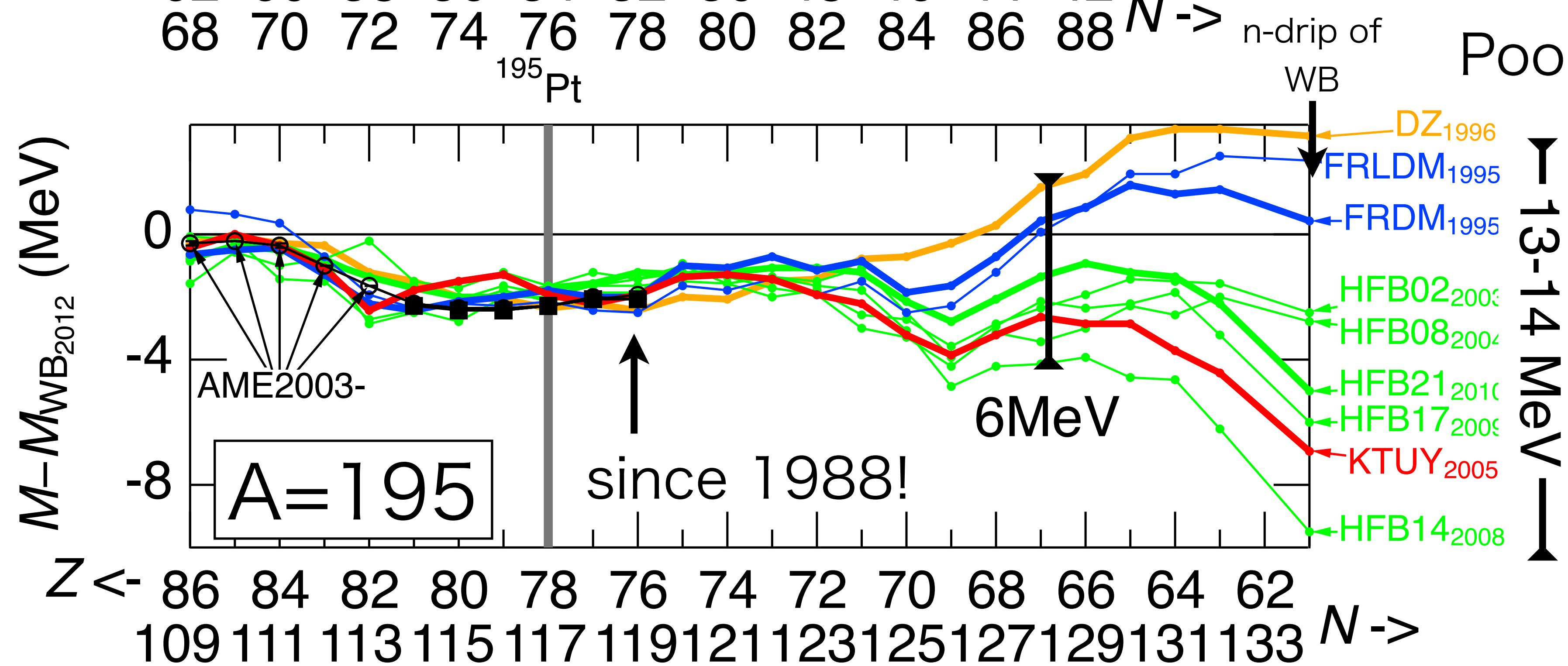
(Only since 1990-)



Predicted mass values still diverge.

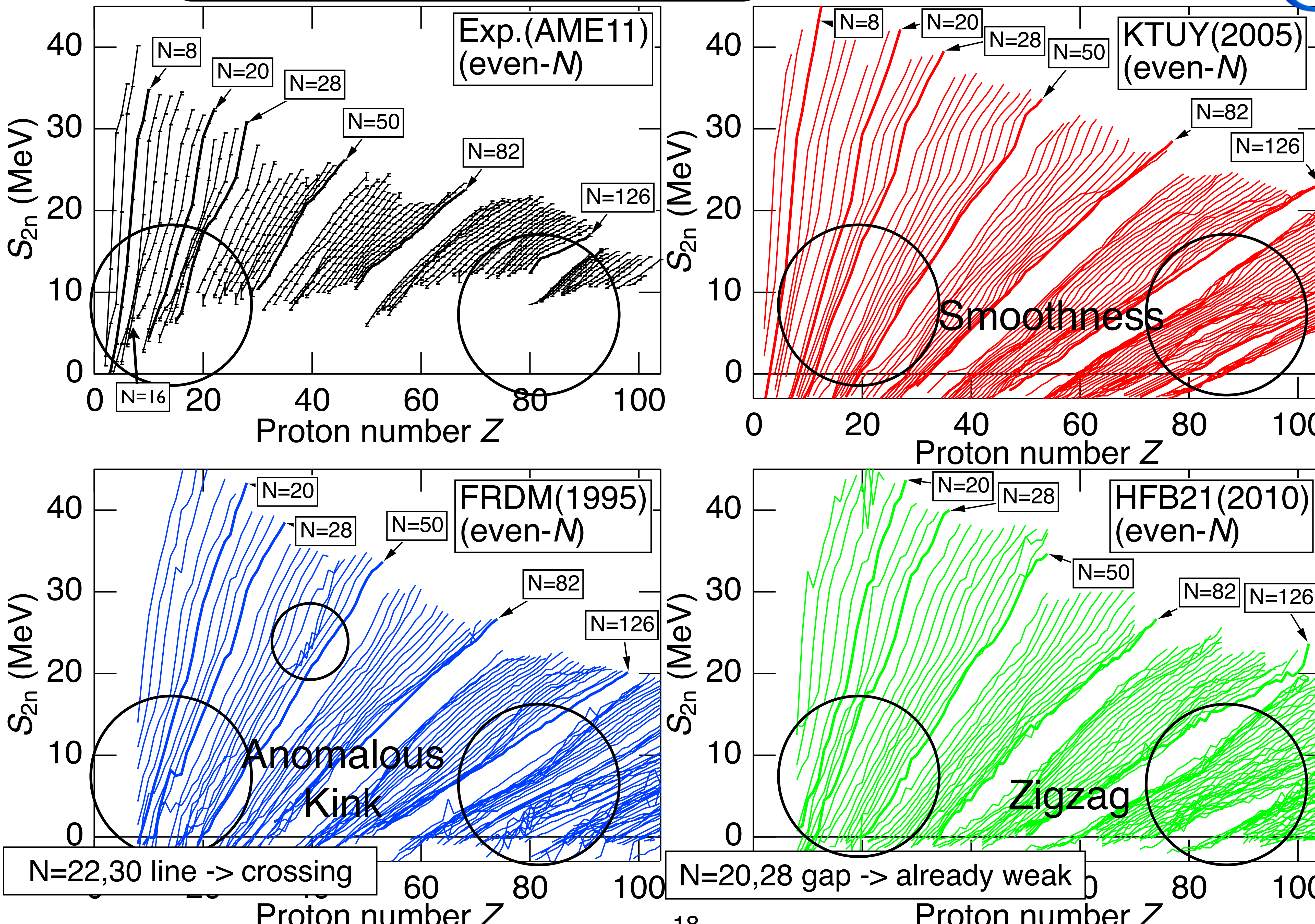
Even among HFB's, mass values diverge in the n-rich region. (several MeV)

Poor experimental mass data.



S_{2n} systematics

shell gap and smoothness



Mass-model dependency

R-process nucleosynthesis

-Check the mass formulae as astrophysical data-

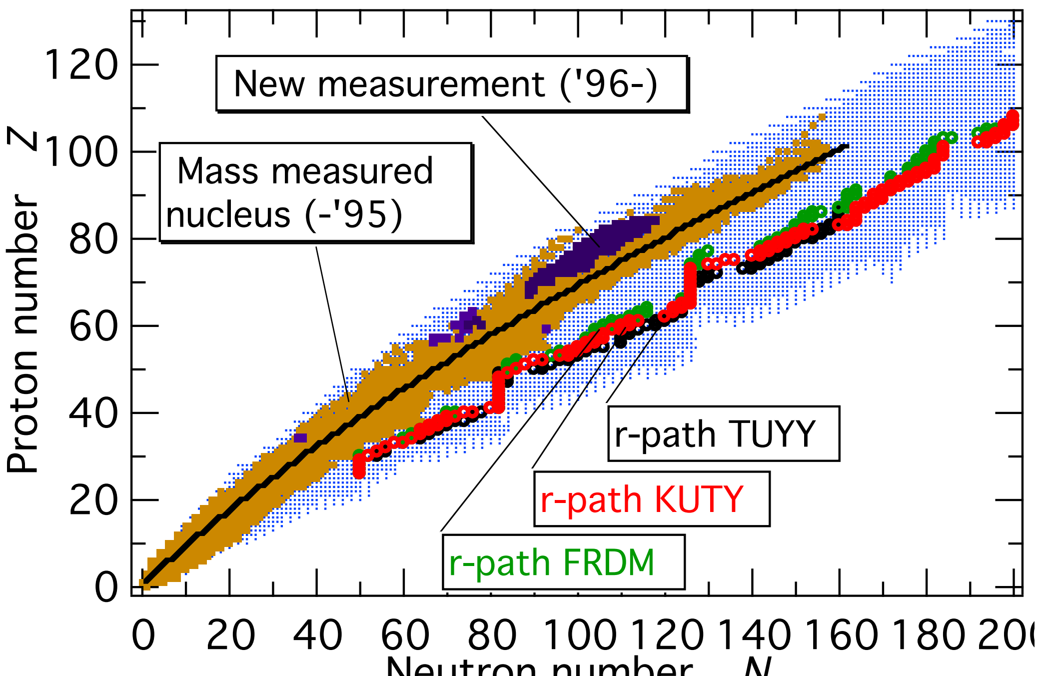
- **Canonical model** Steady flow +Waiting point Approximation

Neutron-number density (N_n) and temperature (T_9) are constants

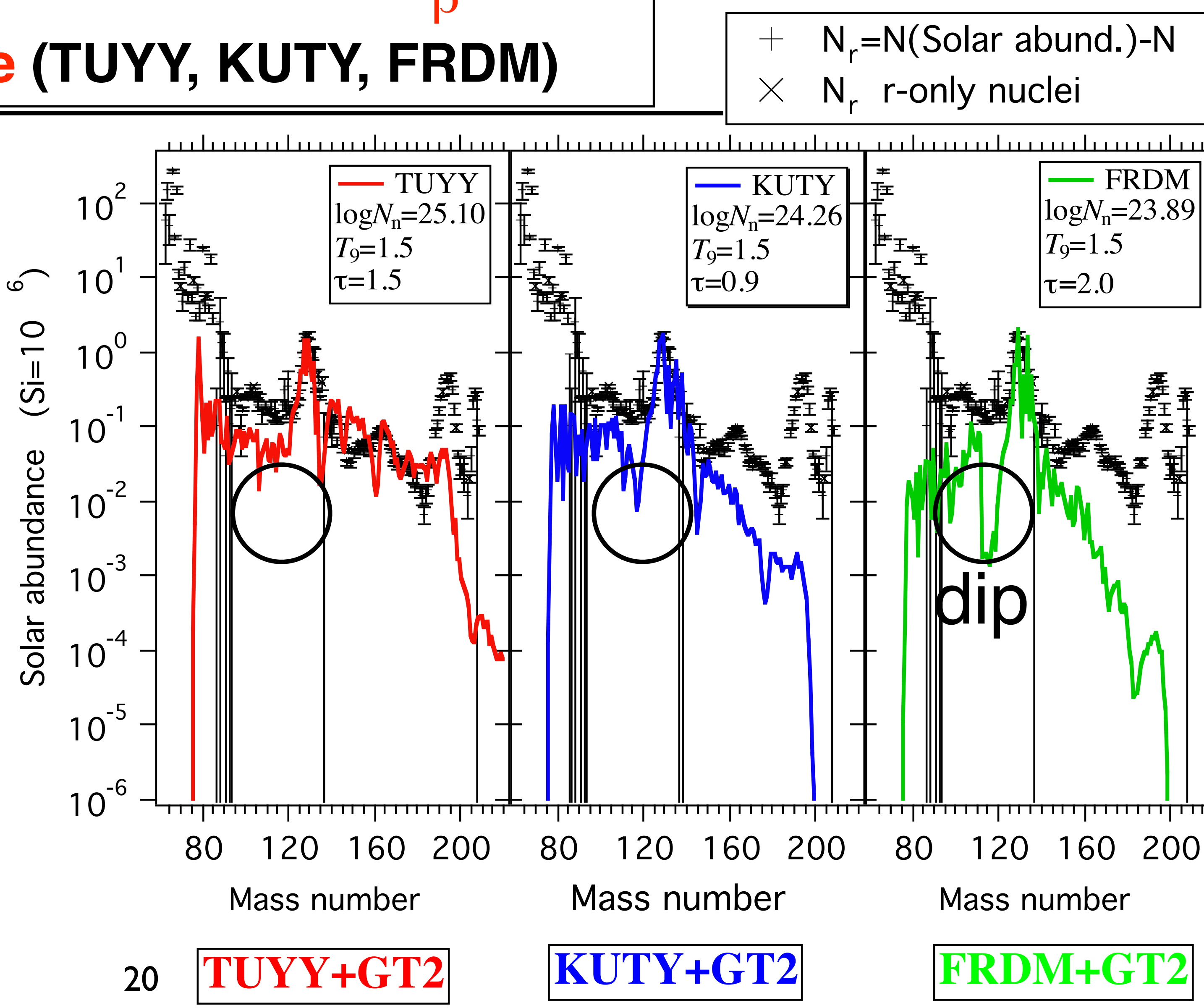
(n,γ)-(γ,n) equilibrium is established over an irradiation time τ

N_n,T_9,τ: chosen to reproduce the abundance peak at *A*=130 (obs.)

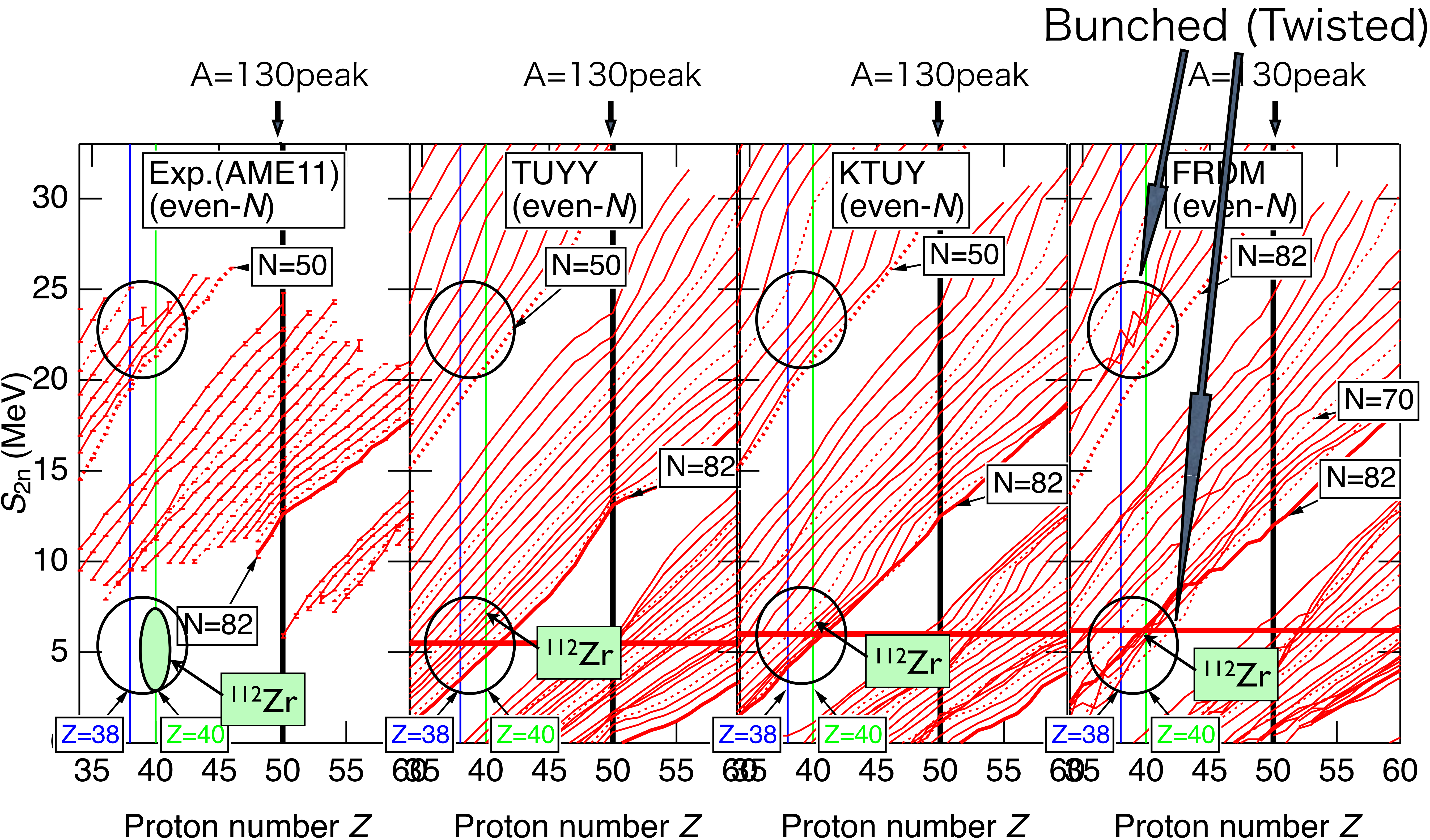
S_{2n} for equilibrium eq. (determine the path) and Q_β for λ_β:
estimated from mass formulae (TUYY, KUTY, FRDM)



- **TUYY**: gross term (WB-like with higher expansion) + empirical shell term.
- **KTUY**: TUYY gross term + deformed shell with a modified Woods-Saxon pot.
- **FRDM**: Macroscopic Droplet + microscopic deformed shell with a folded Yukawa pot.

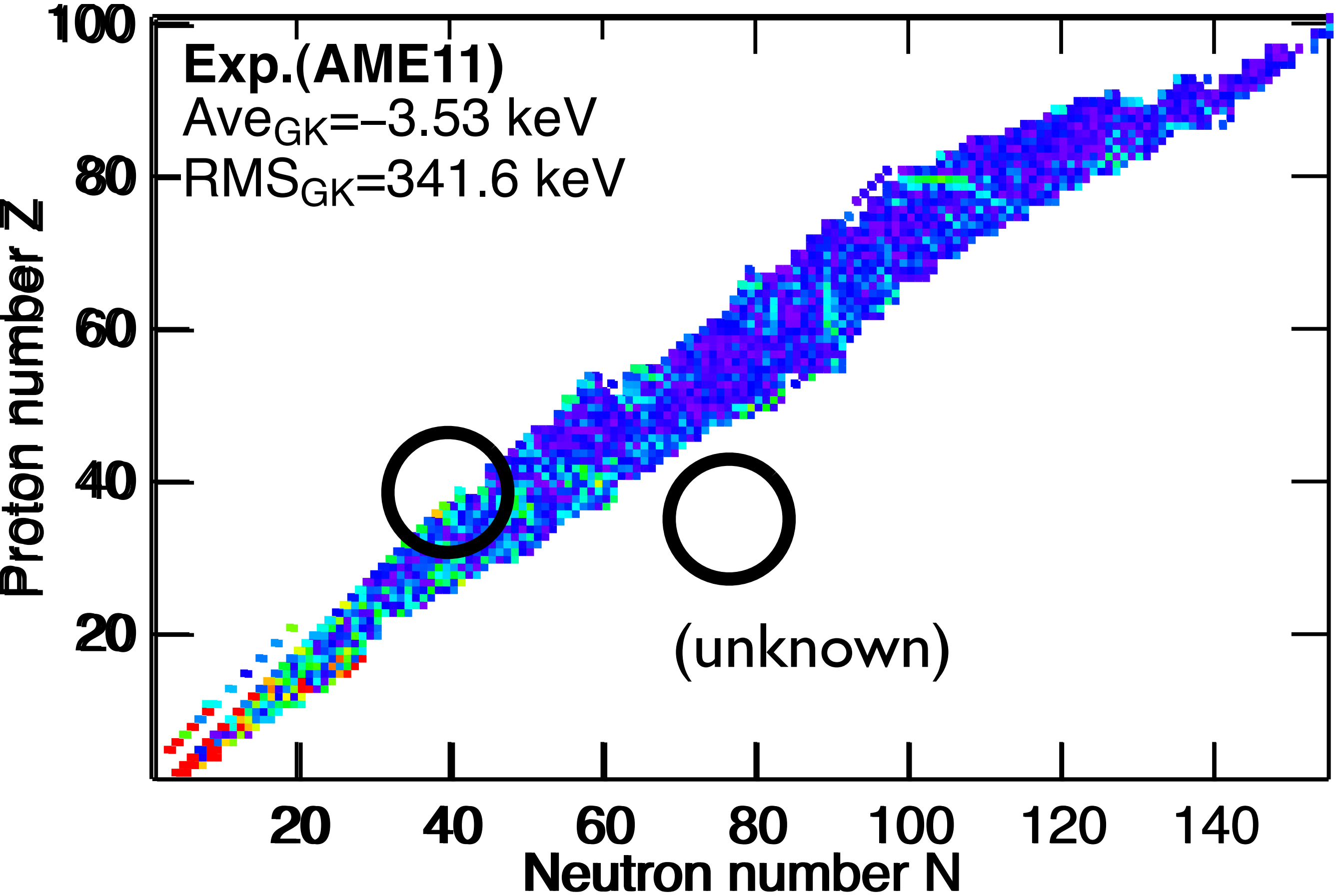


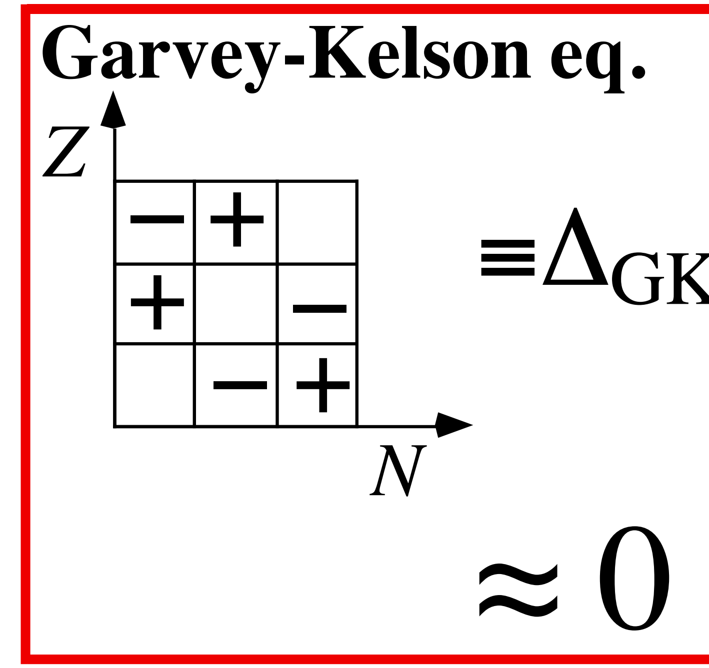
S_{2n} systematics

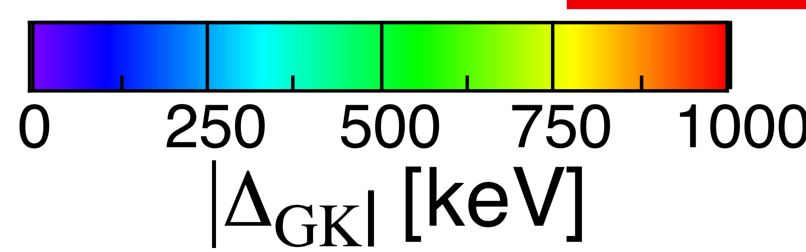


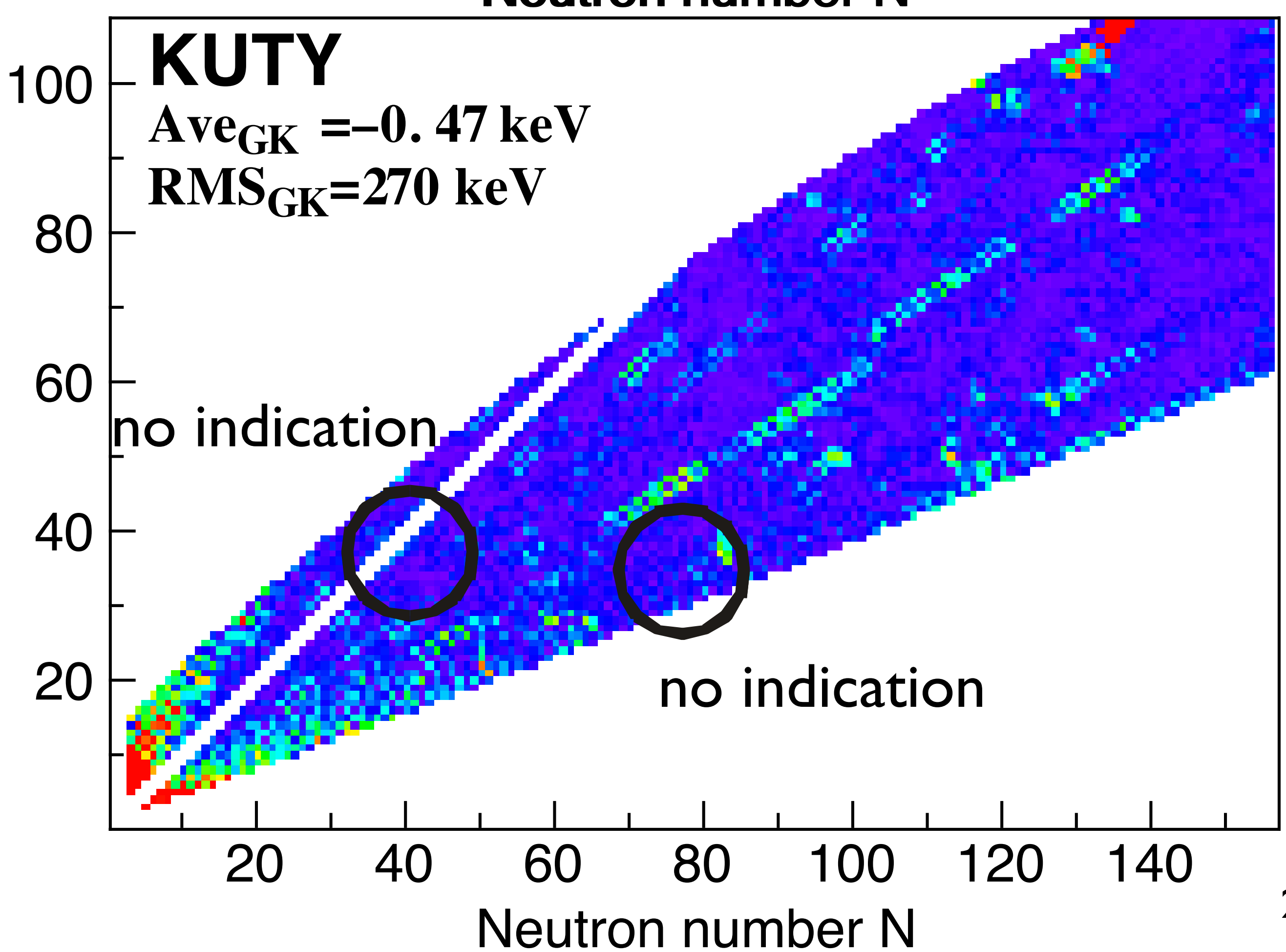
Experiment **TUYY** **KUTY** **FRDM**

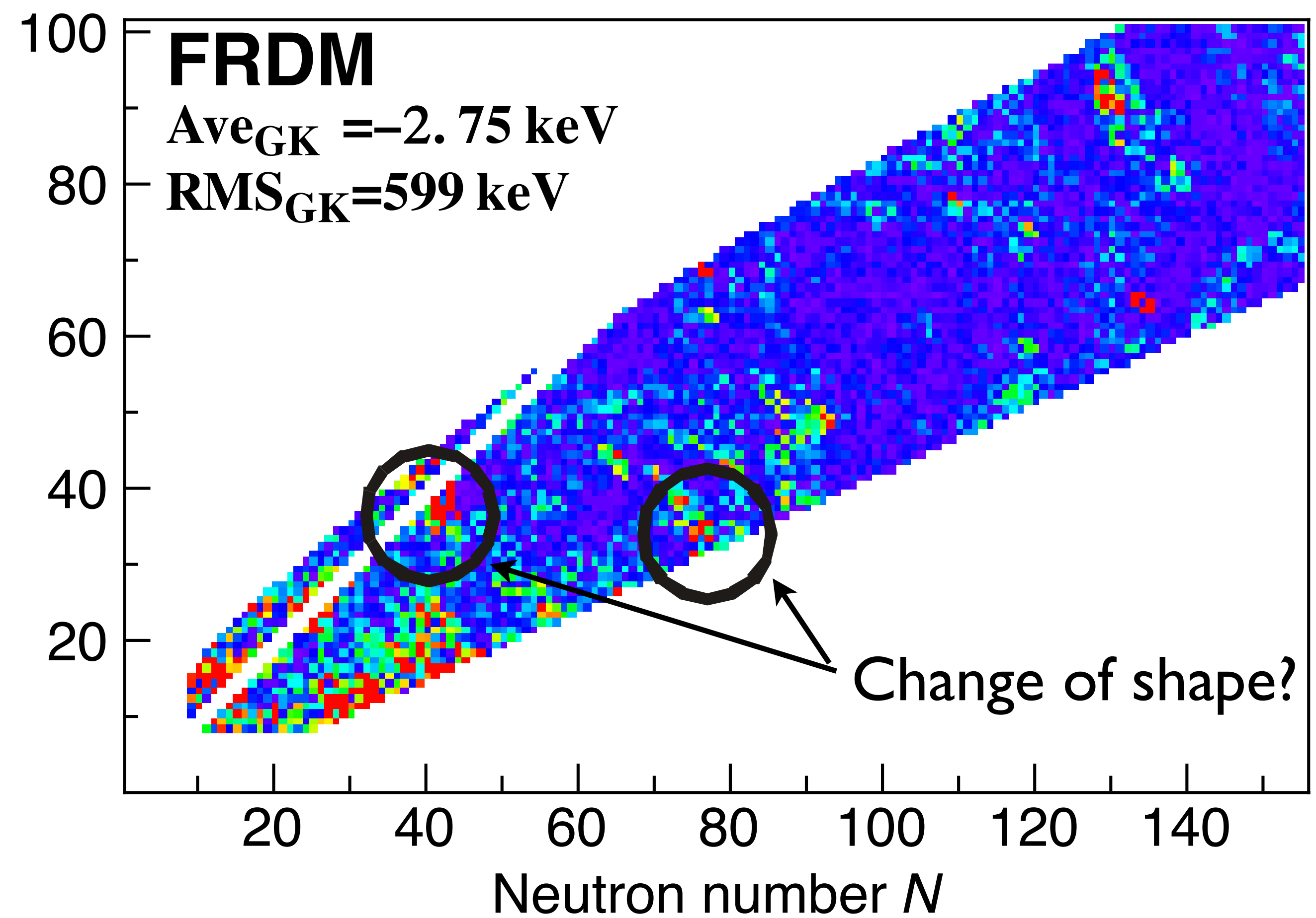
Garvey-Kelson mass relationship





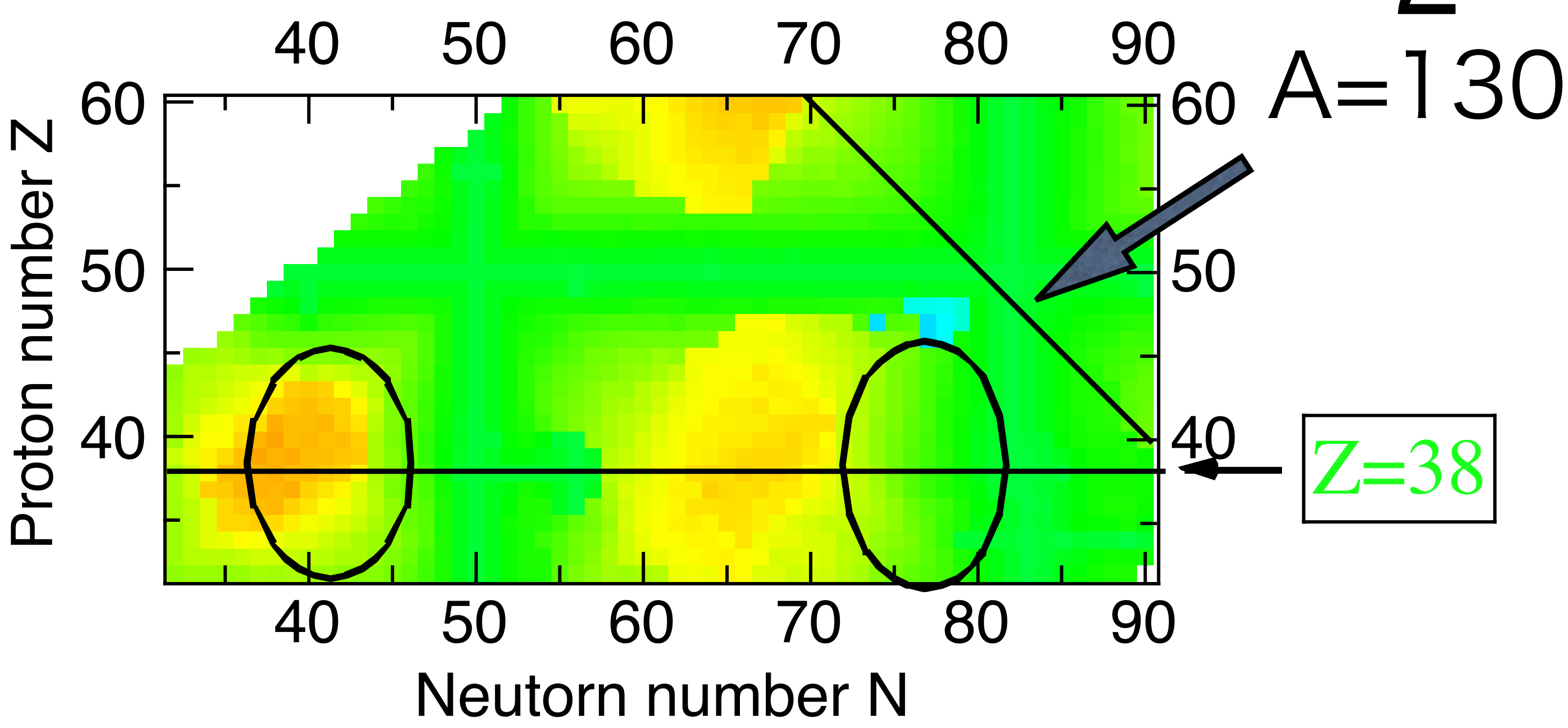




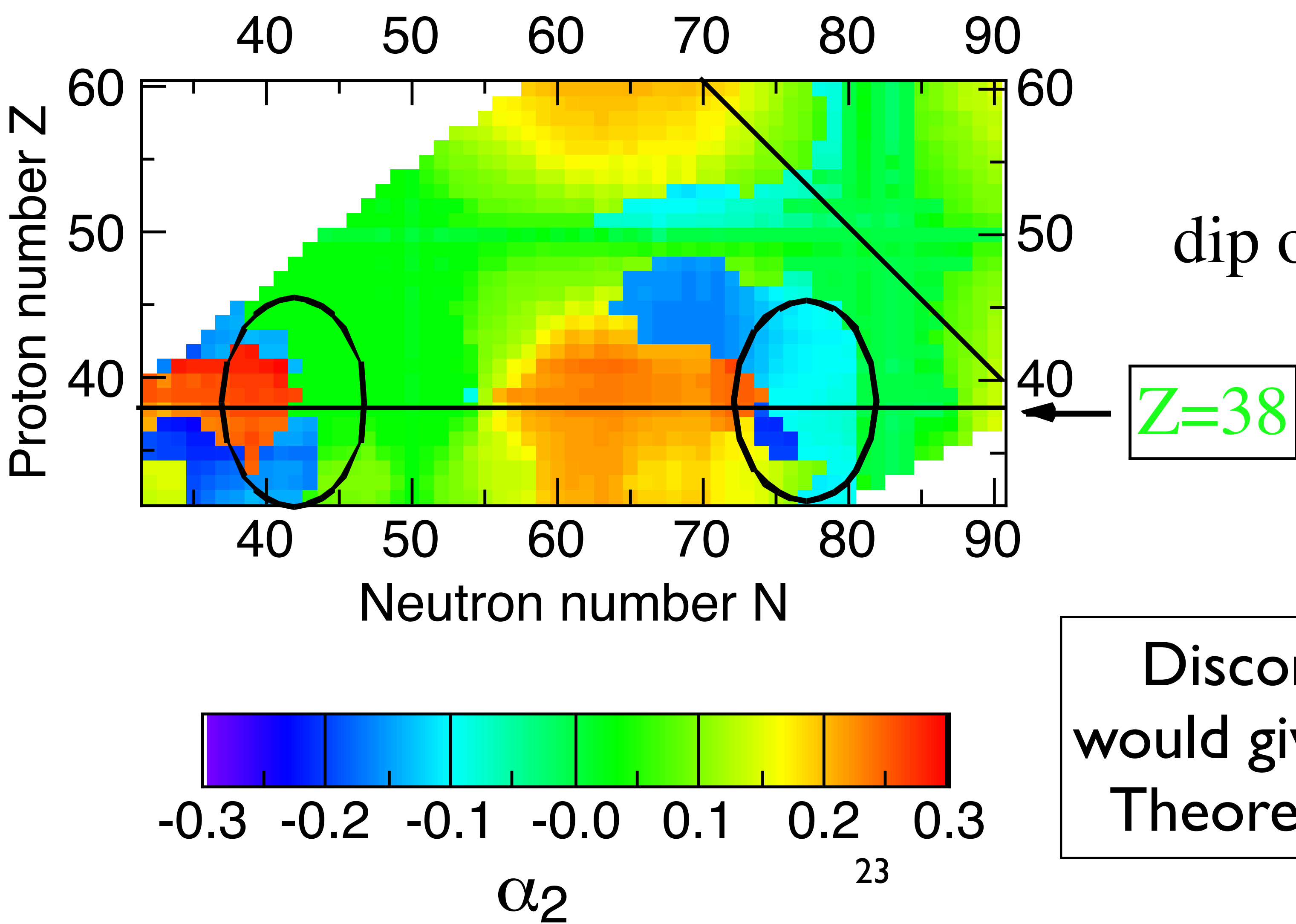


Deformation parameter α_2

KUTY



FRDM



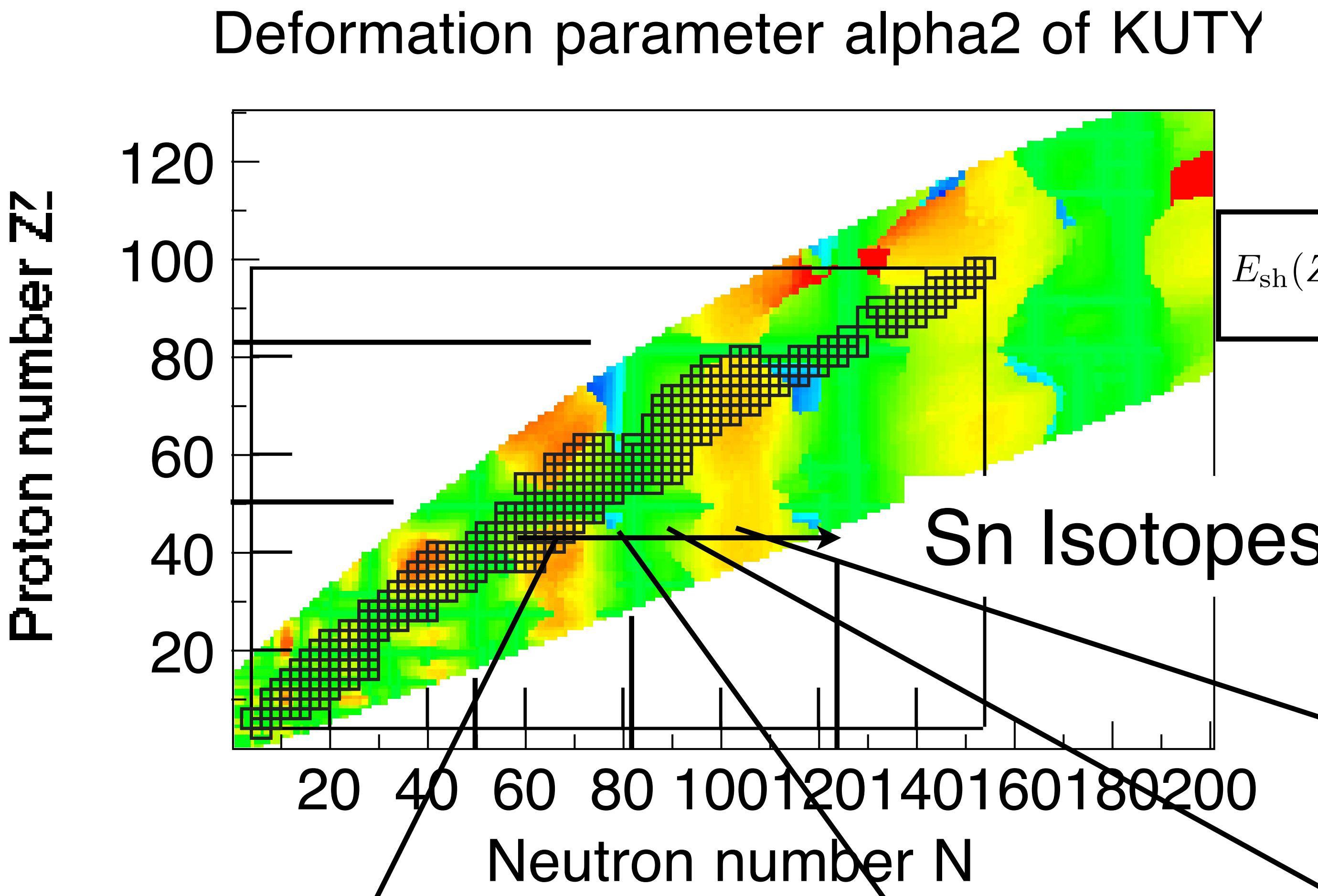
dip of S_{2n} <=> change of α_2

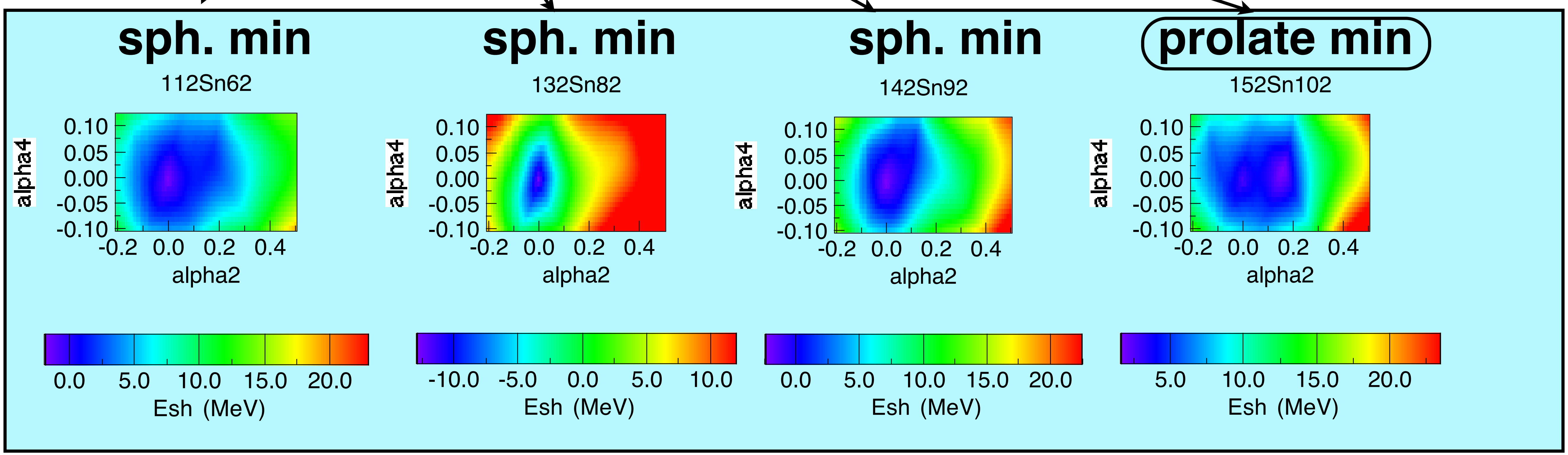
(FRDM)

Discontinuous change of shape would gives kink of Sn. (FRDM case) Theoretical (numerical) problem?

Potential energy surface

$$E_{\rm sh}(Z,N) = \sum_{\rm def.} (< E_{\rm sh}^{\rm sph}(Z,N) >_{\rm def.} + \Delta E_{\rm S}(Z,N) - \Delta E_{\rm C}(Z,N) - \Delta E_{\rm pro}(Z,N))$$





$^{112}Sn_{62}$ $^{132}Sn_{82}$ $^{142}Sn_{92}$ $^{152}Sn_{102}$

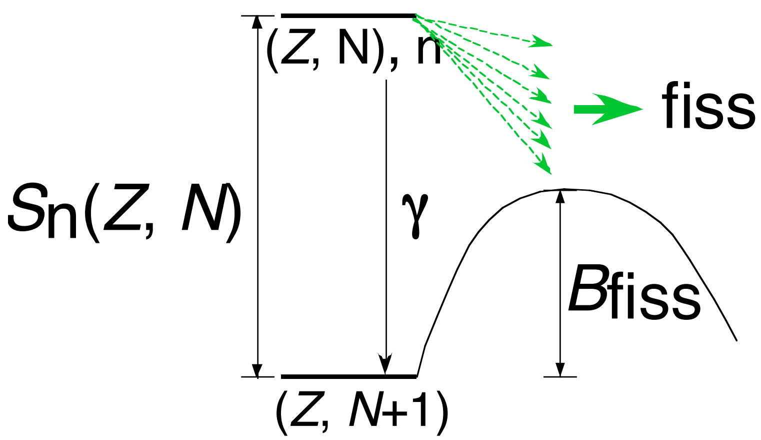
Systematical Study of Tin deformation (Shape coexistence)

Effect of fission

Neutron-induced fission and β-delayed fission

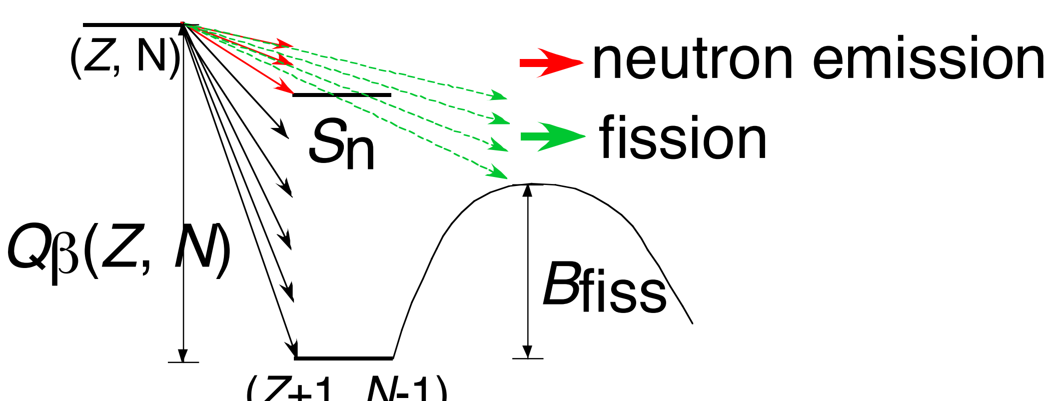
Nuclear masses and fission barrier:
KTUY (Koura-Tachibana-Uno-Yamada) mass formula

- n-induced fission

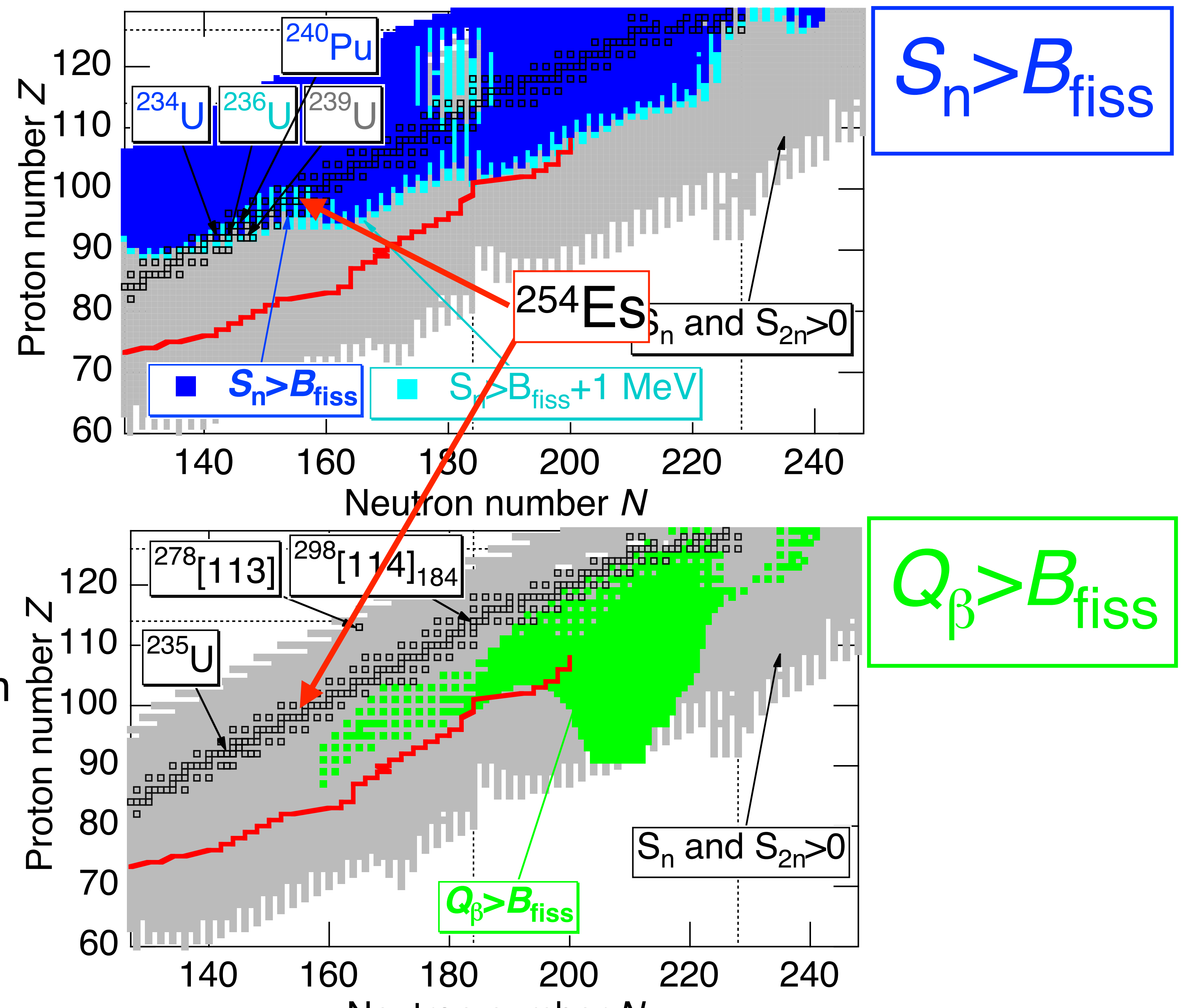


neutron-induced fission

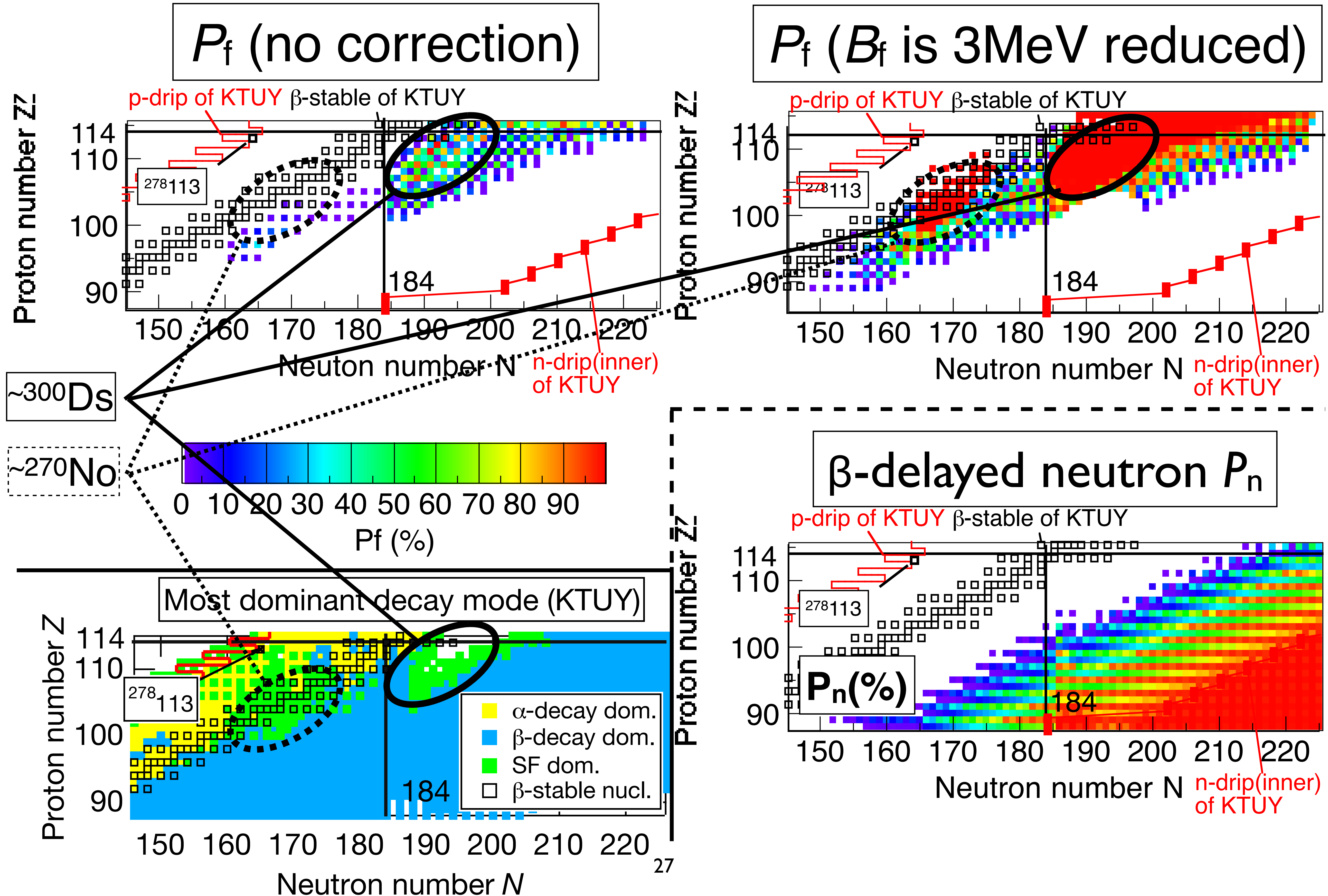
- β-delayed fission



β-delayed neutron emission and β-delayed fission

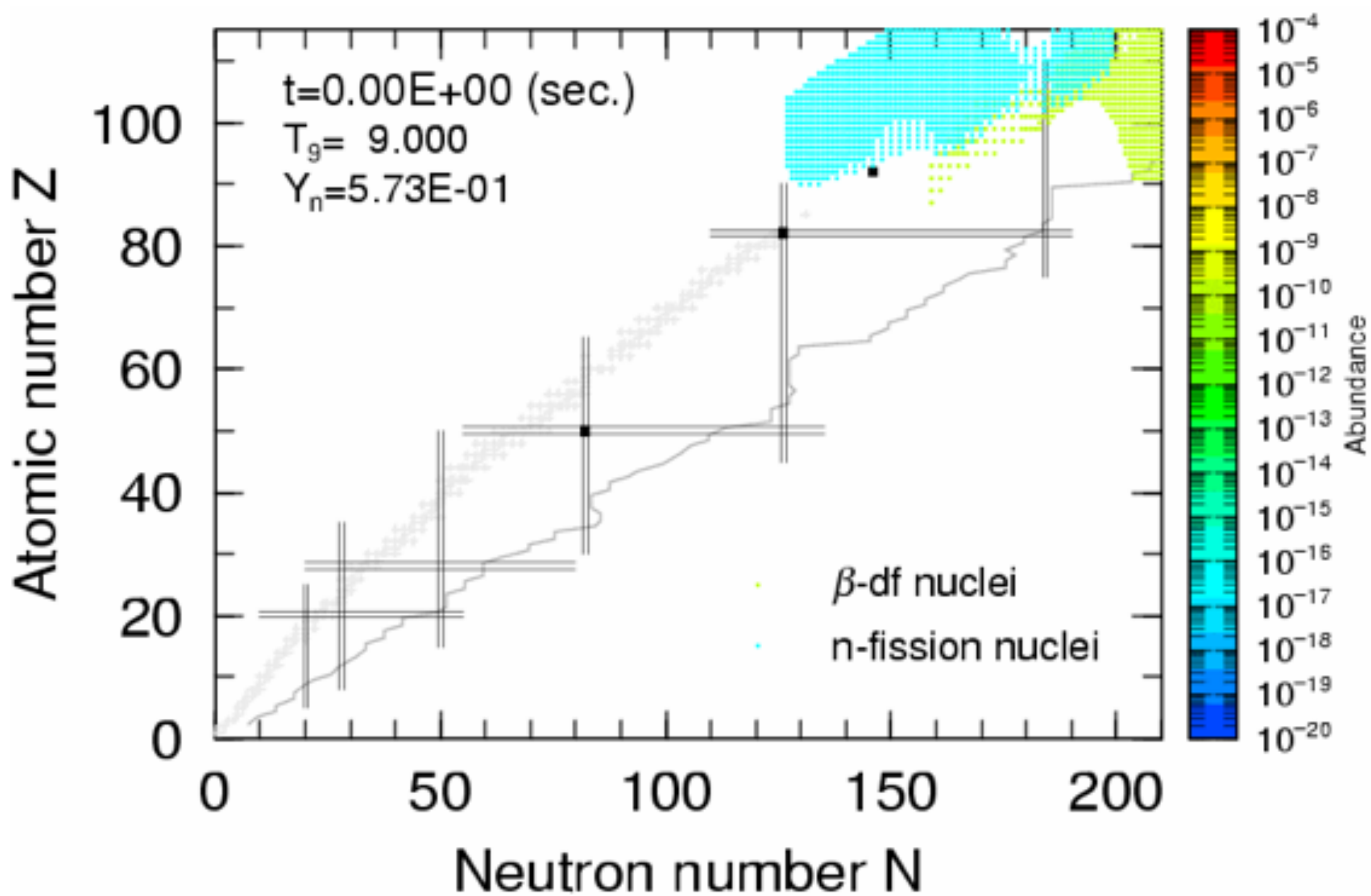


β-delayed fission probability P_f : KTUY model

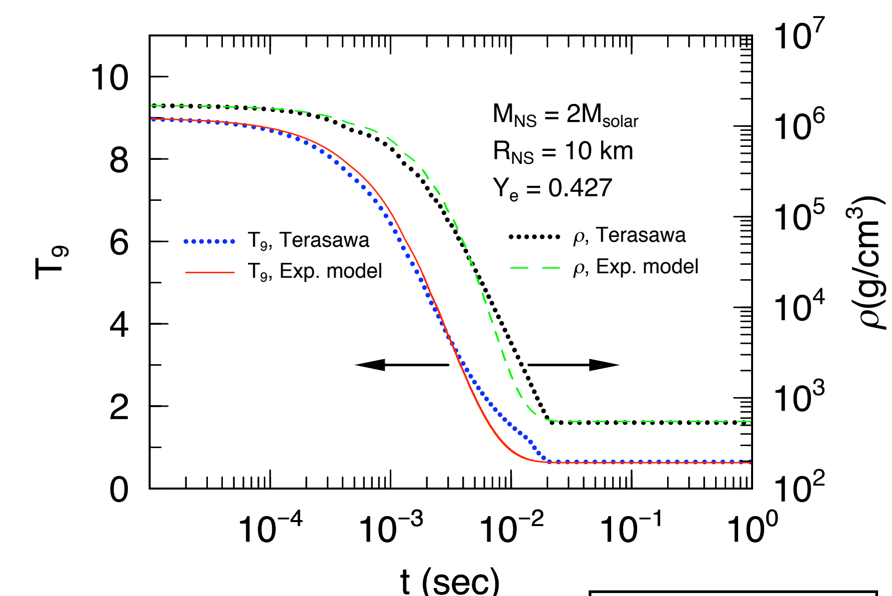


Time evolution of the r- process

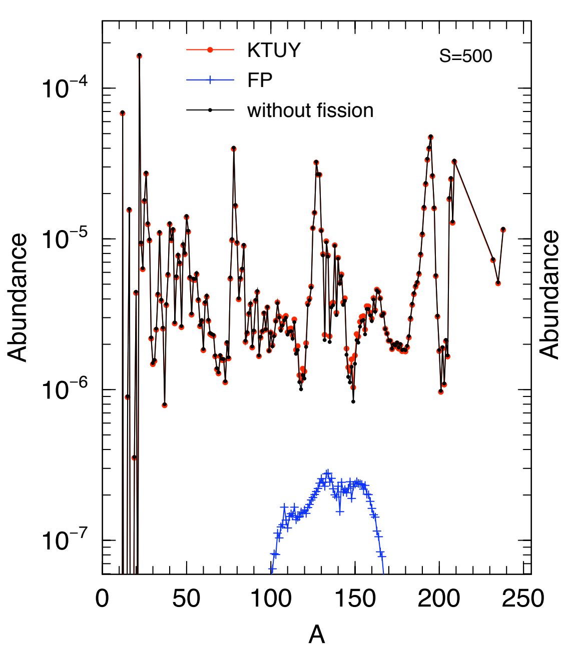
S=200, B_f -> B_{fKTUY}−3MeV

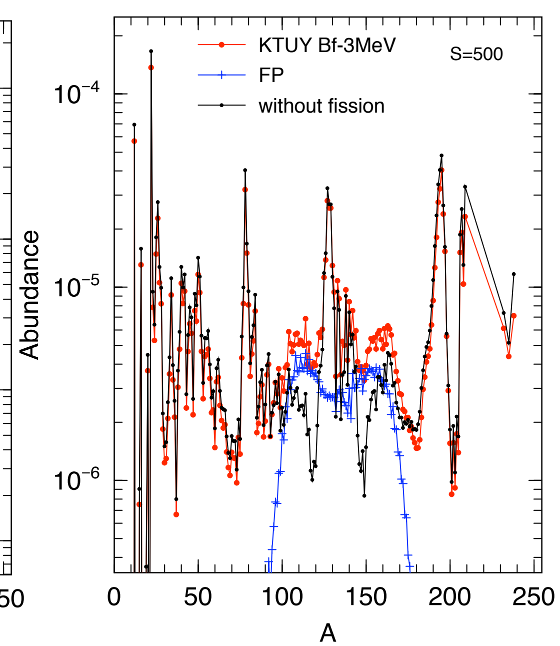


Trajectory of supernova matter



Sumiyoshi, Kajino





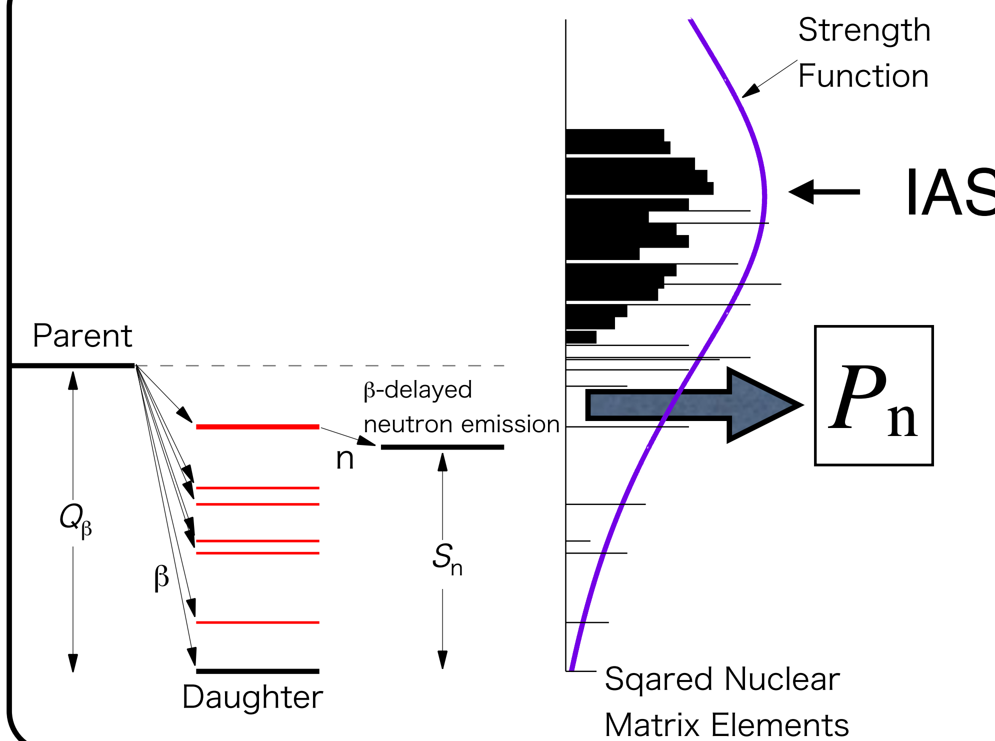
Fission cycle from superheavy region affects the r-process abundance.

S. Chiba, H. Koura,… (2007)

V. Recent study of beta-decay

Recent study: Improvement of the gross theory

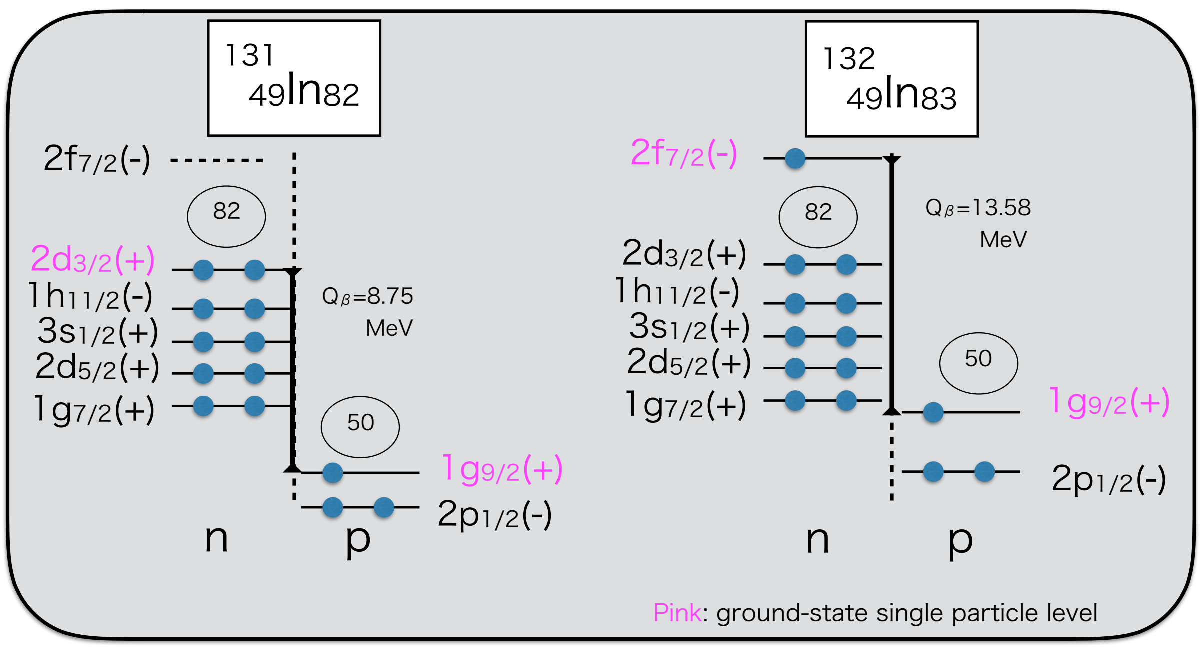
Gross theory



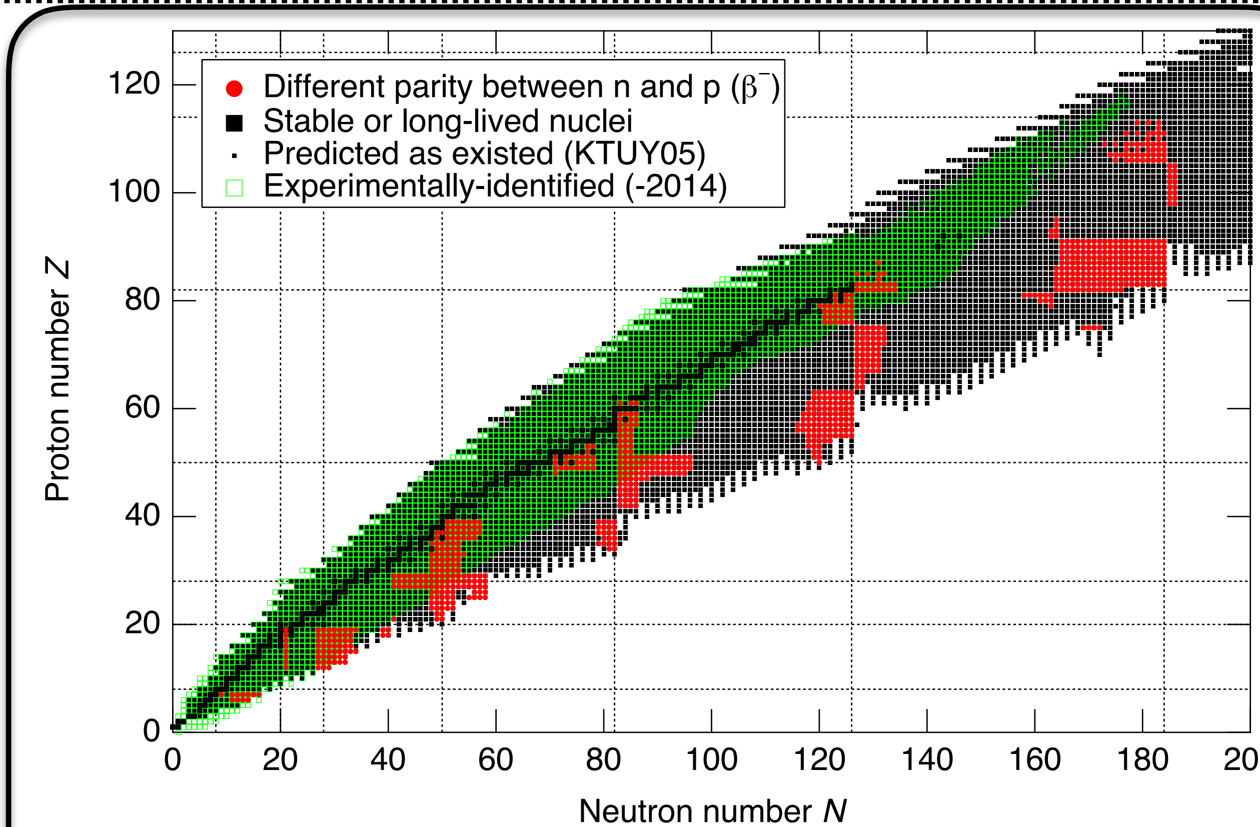
Macroscopic treatment

K. Takahashi et al., PTP **41** (1969)
S. Koyama et al., PTP **44** (1970)
K. Takahashi et al., ADNDT **12** (1973)
T. Kondoh et al., PTP **74** (1985)
T. Tachibana et al.. PTP **84** (1990)
T. Tachibana et al., Proc. ENAM95 (1995)

Improvement of the gross theory : Hindrance factor is introduced for allowed transition on change in parity obtained from calculated SPL.



No parity change Parity change

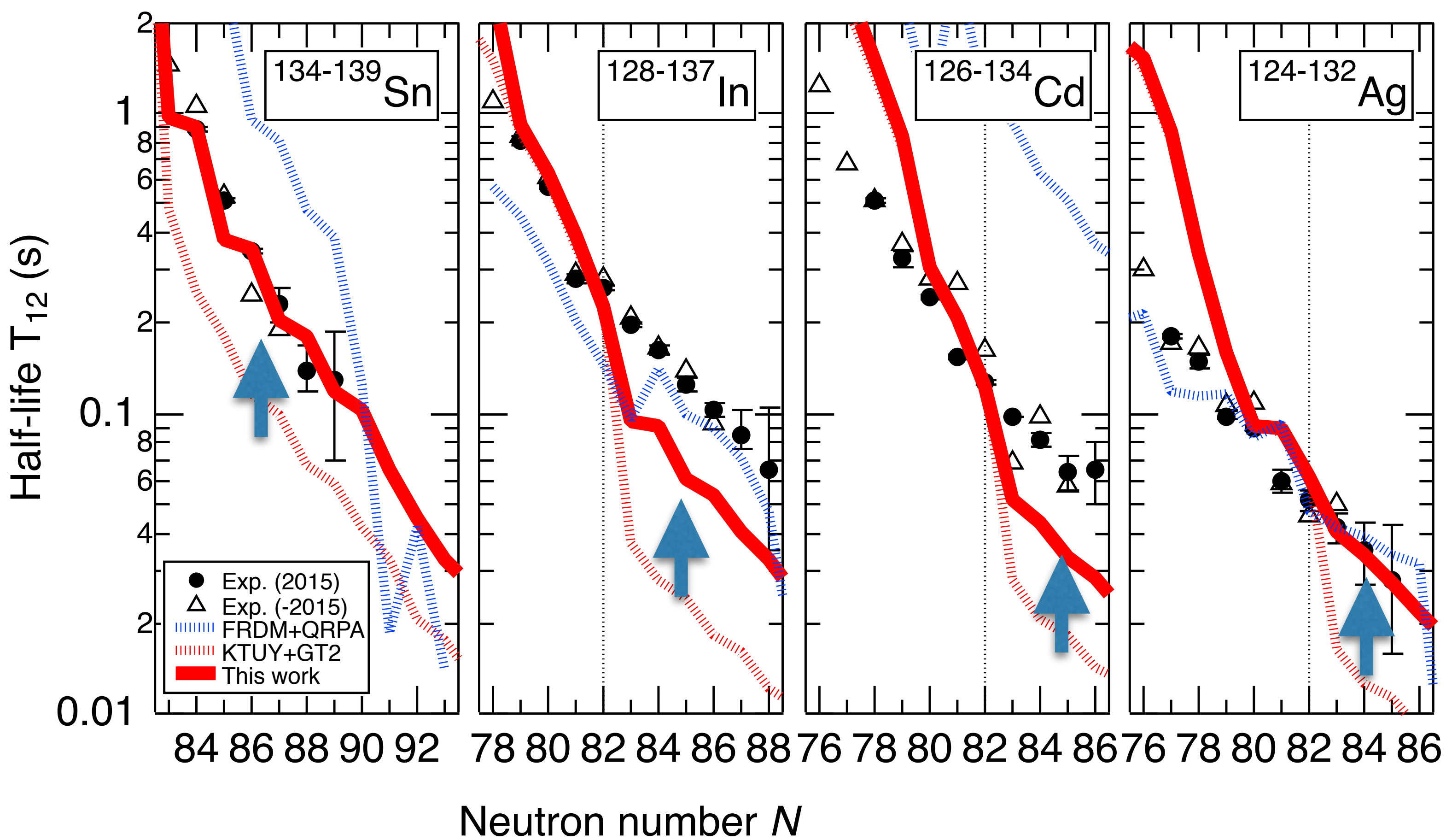


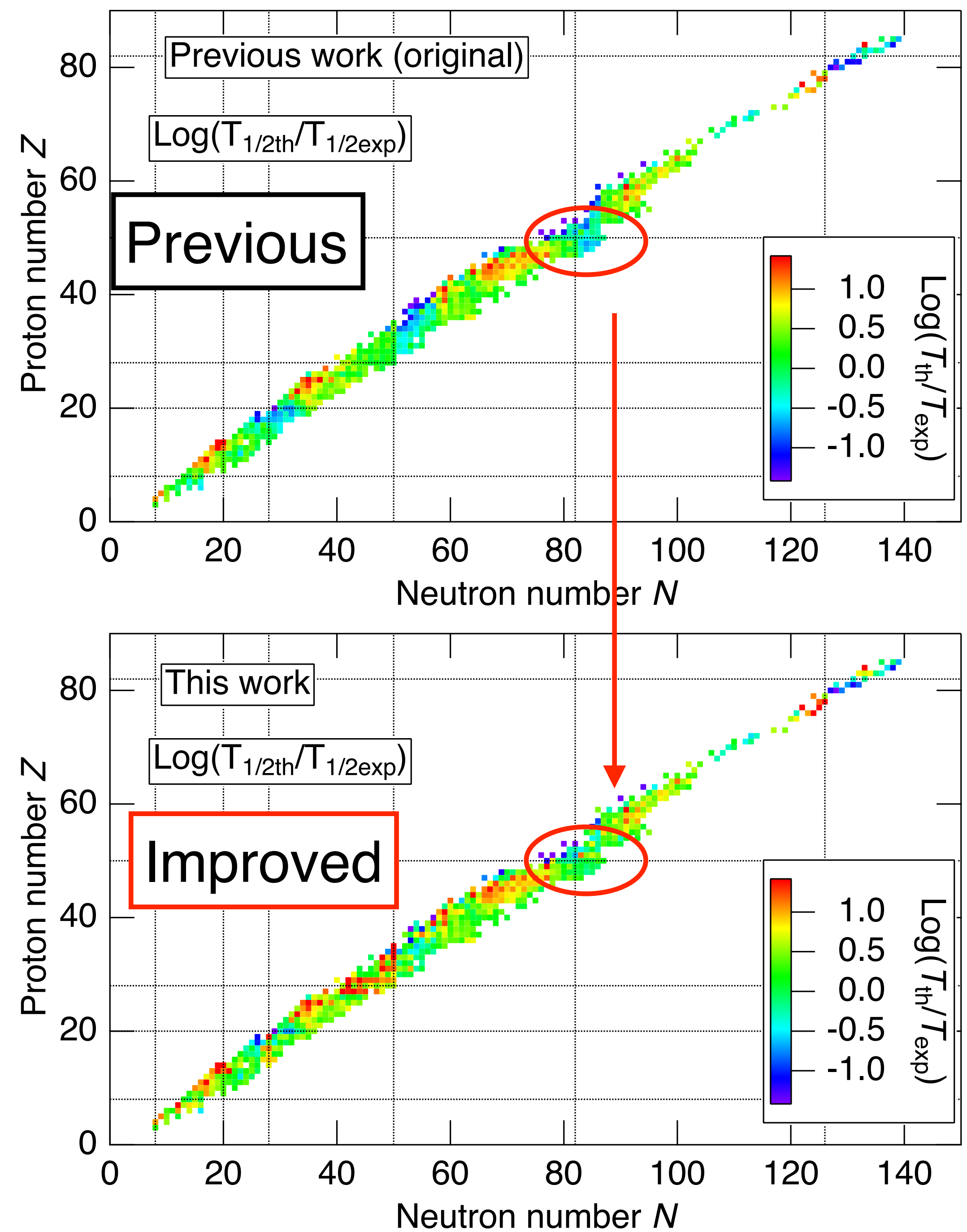
Estimated region of parity-mismatching by a modified Woods-Saxon pot.
(H.K. and M. Yamada, NPA 671 2000)

Recent study: Improvement of the gross theory

Results

H. K. and S. Chiba, PRC **95** 064304 (2017)





KTUY+improved gross theory

Steep changes at N=82(closed shell) disappear.

VI. Conclusion

- A short review of systematical properties of experimental nuclear masses is given.
    - Mass-systematics like G-K is a good tool to check mass values.
- Some mass formulae are reviewed and compared:
    - Old-parametrized mass formulae (in 1976, 1988) generally fail to extrapolation.
    - HFB-type mass formulae give anomaly on GK-sys.
- A mass-model dependency to the r-process is discussed.
- Fission cycle would affect the r-process abundance.