Binding energy differences of even-even nuclei as pairing indicators

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“Nuclear mass and life for unravelling mysteries of r-process”

Outline

- Introduction
  - Binding energy differences
  - gauge symmetry breaking and pairing rotation

- Theoretical framework

- Pairing rotations in single-closed nuclei
- Pairing rotations in doubly-open-shell nuclei

- Constraining pairing EDF using pairing rotational MOI

- Summary

Binding energy differences

What does binding energy systematics tell us about?

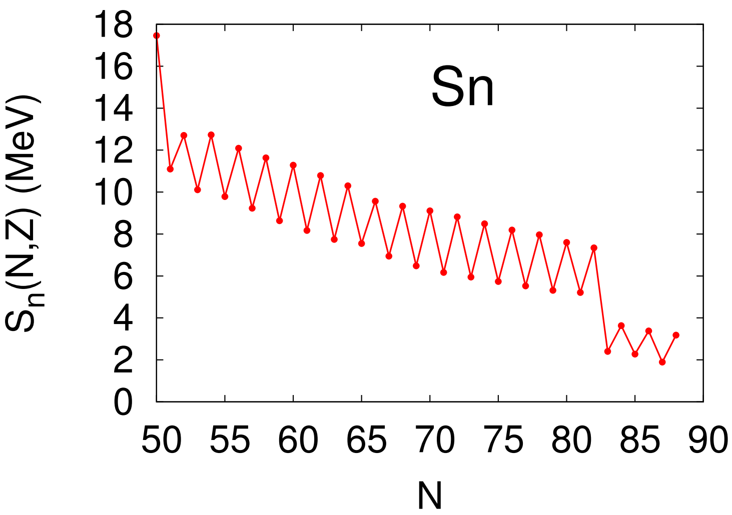
one-neutron/proton separation energy

$$S_n(N,Z) = B(N,Z) - B(N-1,Z)$$
$$S_p(N,Z) = B(N,Z) - B(N,Z-1)$$

odd-even staggering (OES)

$$\Delta_n^{(3)}(N,Z) = -\frac{(-1)^N}{2}[S_n(N,Z) - S_n(N+1,Z)]$$
$$= \frac{(-1)^N}{2}[B(N-1,Z) - 2B(N,Z) + B(N+1,Z)]$$

$$\Delta_p^{(3)}(N,Z) = -\frac{(-1)^Z}{2}[S_p(N,Z) - S_p(N,Z+1)]$$
$$= \frac{(-1)^Z}{2}[B(N,Z-1) - 2B(N,Z) + B(N,Z+1)]$$



$\Delta^{(3)}_{n,p}$: size of the pairing

Binding energy differences

What does binding energy systematics tell us about?

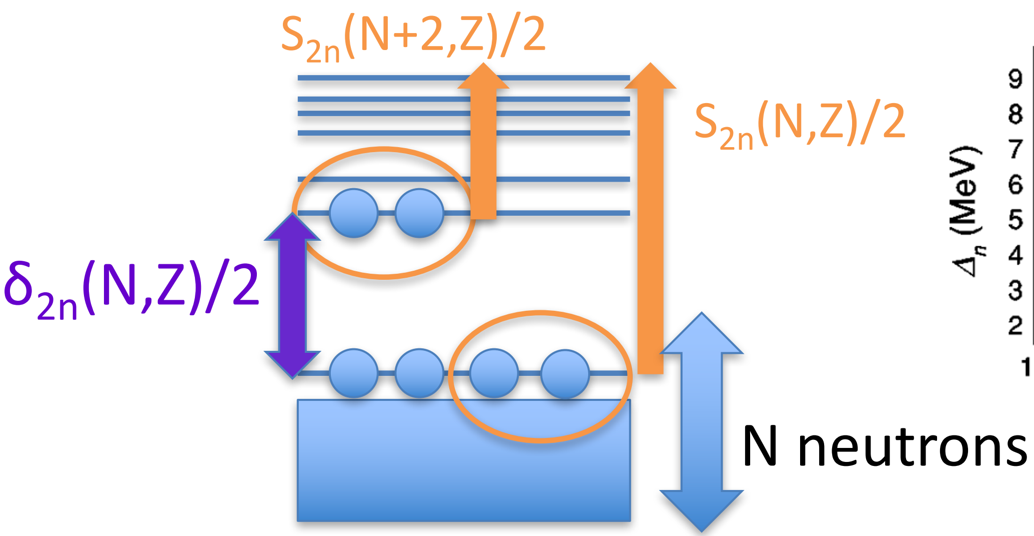
two-neutron/proton separation energy

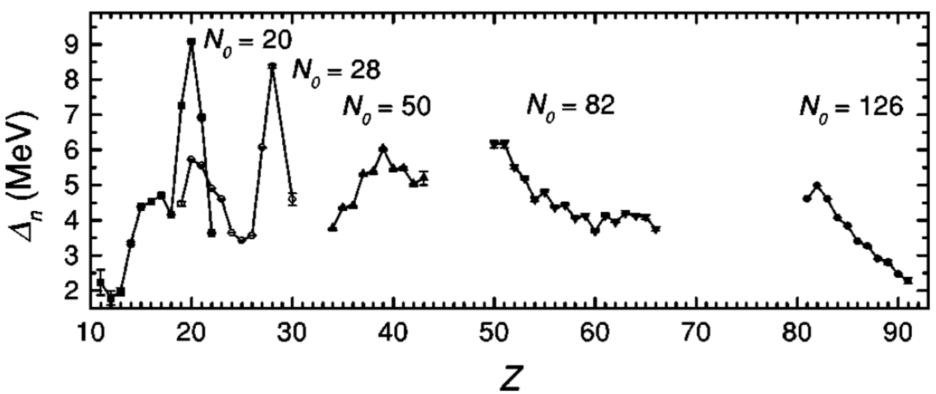
$$S_{2n}(N,Z) = B(N,Z) - B(N-2,Z) \qquad S_{2p}(N,Z) = B(N,Z) - B(N,Z-2)$$

empirical shell gaps

$$\begin{aligned}\delta_{2n}(N,Z) =& S_{2n}(N,Z) - S_{2n}(N+2,Z) \\ =& - B(N-2,Z) + 2B(N,Z) - B(N+2,Z)\end{aligned}$$

$$\begin{aligned}\delta_{2p}(N,Z) =& S_{2p}(N,Z) - S_{2p}(N,Z+2) \\ =& - B(N,Z-2) + 2B(N,Z) - B(N,Z+2)\end{aligned}$$





Lunney et al. Rev. Mod. Phys. **75**, 1021 (2003)

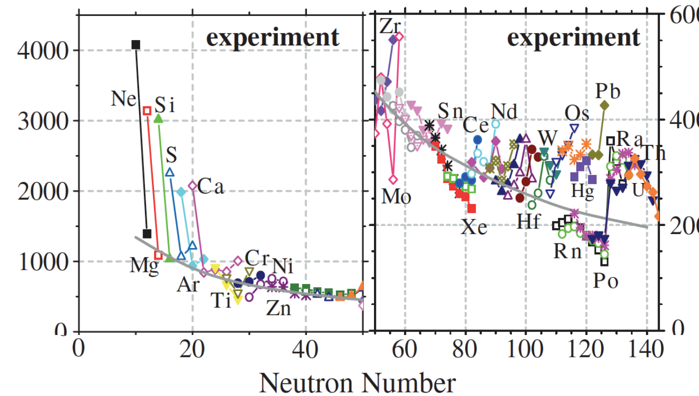
$\delta_{2n,2p}$: size of the magic shell gap

Binding energy differences

proton-neutron interaction energy

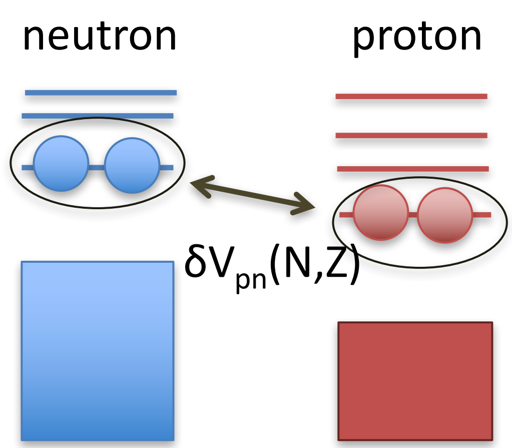
Zhang et al., Phys. Lett. B **227**, 1 (1989)

$$\delta V_{pn}(N,Z) = \frac{1}{4}[S_{2n}(N,Z) - S_{2n}(N,Z-2)]$$
$$= \frac{1}{4}[S_{2p}(N,Z) - S_{2p}(N-2,Z)]$$
$$= \frac{1}{4}[B(N,Z) - B(N-2,Z) - B(N,Z-2) + B(N-2,Z-2)]$$



Stoitsov et al., Phys. Rev. C **98**,132502 (2007)

$$-B(N-2,Z-2) = E_{\rm ncore} + E_{\rm pcore} + V_{\rm ncore-pcore}$$
$$-B(N,Z-2) = -B(N-2,Z-2) + E_{\rm 2n} + V_{\rm 2n-ncore} + V_{\rm 2n-pcore}$$
$$-B(N-2,Z) = -B(N-2,Z-2) + E_{\rm 2p} + V_{\rm 2p-ncore} + V_{\rm 2p-pcore}$$
$$-B(N,Z) = -B(N-2,Z-2) + E_{\rm 2n} + E_{\rm 2p} + V_{\rm 2n-2p} + V_{\rm 2n-ncore} + V_{\rm 2n-pcore} + V_{\rm 2p-ncore} + V_{\rm 2p-pcore}$$



$$\delta V_{pn}(N,Z) = -\frac{1}{4}V_{\rm 2n-2p}$$

N-2 neutrons
Z-2 protons in the core

δV_{pn}: interaction energy of two neutrons and two protons in the valence space

Pairing in mean-field theory

pairing (superconductivity) : nucleon pairs get additional binding

odd-even staggering (OES)

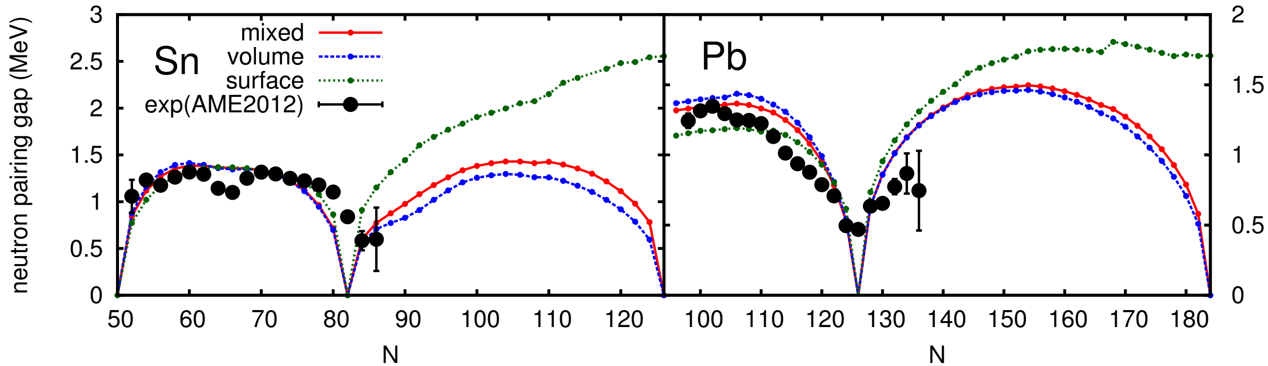
$$\Delta_n^{(3)}(N,Z) = \frac{(-1)^N}{2}[B(N-1,Z) - 2B(N,Z) + B(N+1,Z)]$$

$$\Delta_n^{\exp}(N,Z) = \frac{1}{2}[\Delta_n^{(3)}(N+1,Z) + \Delta_n^{(3)}(N-1,Z)]$$

Kortelainen et al., Phys. Rev. C **82**, 024313 (2010)

theoretical pairing gap (average pairing gap)

$$\Delta_n(\rho) = \frac{\mathrm{Tr}\tilde{h}_n[\rho,\tilde{\rho}_n]\rho_n}{\mathrm{Tr}\rho_n} \qquad \Delta_n(\tilde{\rho}) = \frac{\mathrm{Tr}\tilde{h}_n[\rho,\tilde{\rho}_n]\tilde{\rho}_n}{\mathrm{Tr}\tilde{\rho}_n}$$

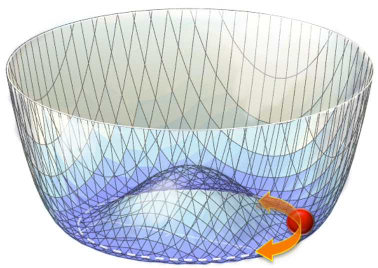


Problems

- ☐ pairing gap: not an experimental observable
- ☐ multiple definitions both in OES and pairing gaps
- ☐ time-reversal symmetry: broken in OES, pairing gap is from even-system

Spontaneous symmetry breaking

continuous symmetry of the system S: $[\hat{H}, \hat{S}] = 0$



SSB: ground state does not have the symmetry of the system

infinite degeneracy of ground state

zero-energy mode of excitation (Nambu-Goldstone modes/bosons) appears as a transition between degenerated ground states.

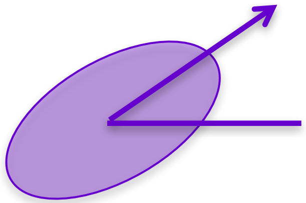
Nambu (1960), Goldstone (1961)

system	broken symmetry	ground state	Nambu-Goldstone mode (boson)
magnet	rotation	spin alignment	spin-wave (magnons)
crystal	translation/rotation	crystal	vibration (phonons)
superconductor	gauge symmetry	BCS state	(massive gauge field due to Anderson-Higgs mechanism)
QCD	chiral symmetry (approximate)	quark-antiquark condensation	pions (with small mass)

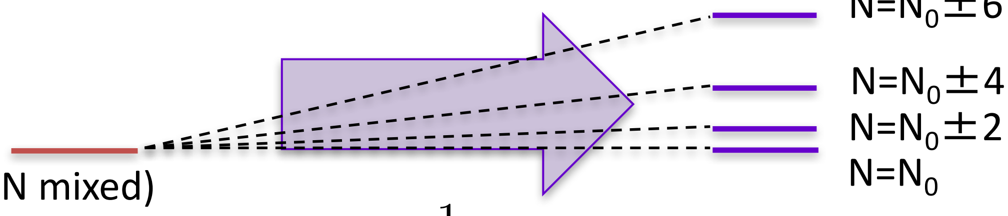
Gauge symmetry breaking

pairing (superconductivity) : gauge symmetry breaking, NG mode(pairing rotation)

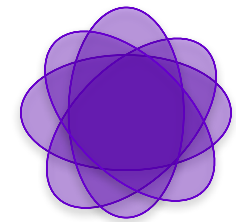
superconducting state
in gauge space



NG mode excitation (pairing rotation)
(particle number projection)



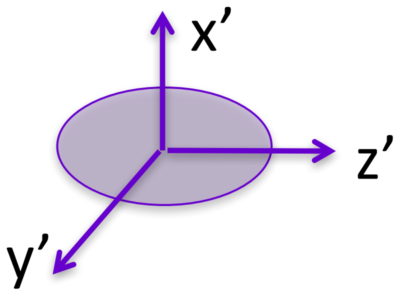
symmetry-restored state
(Laboratory frame)



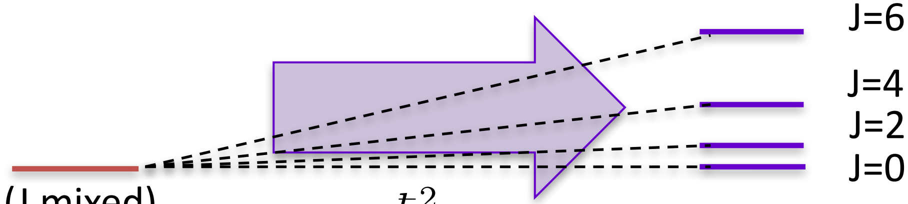
$$E(N) = \frac{1}{2\mathcal{J}_{\mathrm{N}}}(N - N_0)^2$$

<u>pairing rotational moment of inertia: magnitude of pairing collectivity</u>

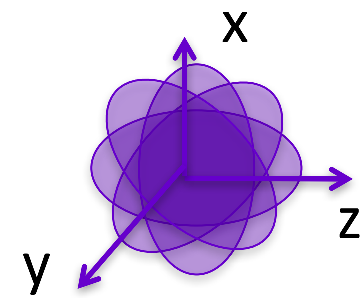
deformed state
in coordinate space



NG mode excitation (rotation)
(angular momentum projection)



symmetry-restored state
(Laboratory frame)



$$E(J) = \frac{\hbar^2}{2\mathcal{J}_{\mathrm{rot}}}J(J+1)$$

moment of inertia (or E(2_1^+)): magnitude of quadrupole collectivity

Pairing observables: pairing rotational MOI

Pairing rotation in isotopes

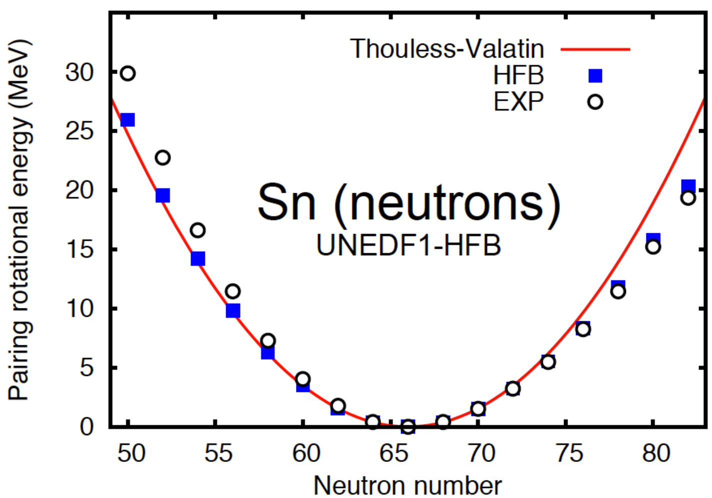
pairing rotational energy

$$E(N+\Delta N,Z)=E(N,Z)+\lambda_n(N,Z)\Delta N+\boxed{\frac{(\Delta N)^2}{2\mathcal{J}_{nn}(N,Z)}}$$

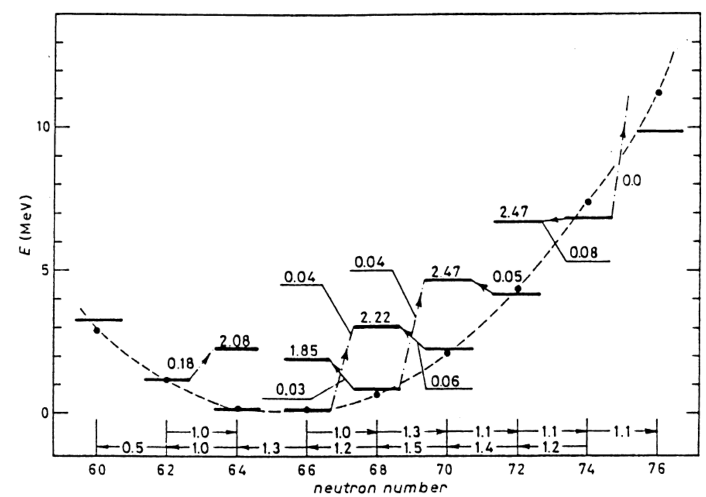
moment of inertia

$$\mathcal{J}_{nn}^{-1}(N,Z)=\frac{E(N+2,Z)-2E(N,Z)+E(N-2,Z)}{4}=\frac{\delta_{2n}(N,Z)}{4}$$

pairing rotation measured from ^{116}Sn



Brink and Broglia "Nuclear Superconductivity"
review: Broglia et al., Phys. Rep. **335**, 1(2000)



pairing rotational moment of inertia

- quantity from gauge symmetry breaking
- double binding-energy differences (inverse of empirical shell gaps($\delta_{2n,2p}$))
- involves even-even systems only
- change with particle number

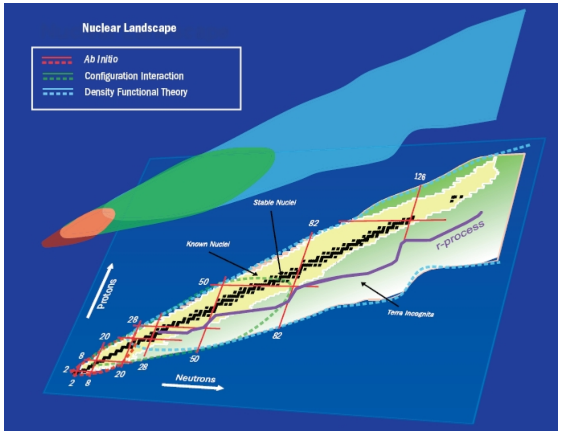
Theoretical description of NG mode

Symmetry-broken state

Density Functional Theory (DFT)

Skyrme HFB (HFBTHO, UNEDF1-HFB)

- 20 harmonic oscillator basis
- axial deformation
- pairing superconductivity
- UNEDF1-HFB: Schunck, J. Phys. G **42**,034024 (2015)



NG mode excitation

Quasiparticle Random-Phase Approximation (time-dependent DFT)

$$[\hat{H}_{\rm QRPA}, \hat{\mathcal{P}}_{\rm NG}] = i\hbar^2\omega_{\rm NG}^2 \mathcal{J}_{\rm TV}\hat{Q}_{\rm NG} = 0 \quad [\hat{H}_{\rm QRPA}, \hat{\mathcal{Q}}_{\rm NG}] = -\frac{i}{\mathcal{J}_{\rm TV}}\hat{\mathcal{P}}_{\rm NG}$$

$P_{\rm NG}$: particle number operator

moment of inertia from QRPA: Thouless-Valatin inertia

Thouless and Valatin, Nucl. Phys. **31** (1962)211

$$\mathcal{J}_{\rm TV} = 2P_{\rm NG}^R(A+B)^{-1}P_{\rm NG}^R + 2P_{\rm NG}^I(A-B)^{-1}P_{\rm NG}^I$$

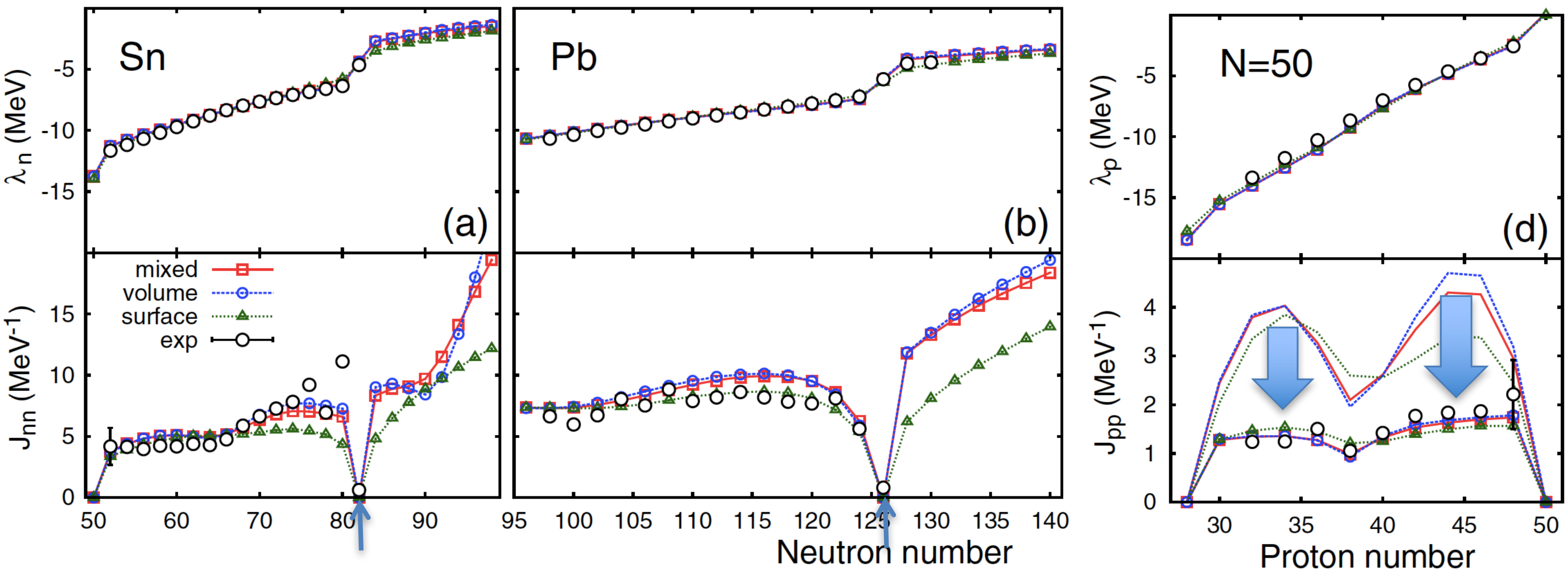
efficient solution based on linear response theory: Finite-amplitude method (FAM)

Nakatsukasa et al., Phys. Rev. C **76**, 024318 (2007)

NH, Phys. Rev. C **92**, 034321 (2015)

Pairing rotation in single-closed shell

NH and Nazarewicz, Phys. Rev. Lett. **116**, 152502 (2016)



broken symmetry: neutron U(1), Δn≠0 or proton U(1) Δp≠0

pairing density functional (strength fitted to ^{120}Sn and ^{92}Mo gaps)

η=0 (**volume pairing**) 0.5(**mixed pairing**) and 1 (**surface pairing**)

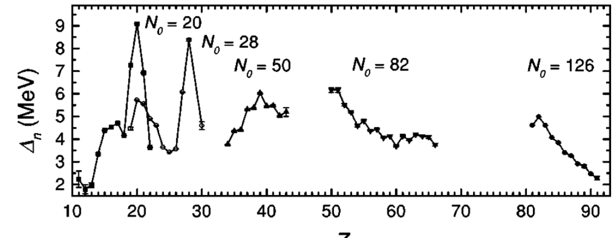
$$\chi_\tau(\boldsymbol{r}) = \frac{1}{2} V_0^{(\tau)} \left[1 - \eta \frac{\rho_0(\boldsymbol{r})}{\rho_c} \right] |\tilde{\rho}_\tau(\boldsymbol{r})|^2$$

- experimental MOI: sensitive to the symmetry breaking
- pairing rotational assumption is not valid at next to magic numbers (experimental values)
- contribution from residual Coulomb suppresses the MOI (ph Coulomb force ~ Z^2)

Shell gap and pairing rotational moment of inertia

Experimental pairing rotational moment of inertia

$$\mathcal{J}_{\rm nn}(N) = \frac{4}{E(N+2) - 2E(N) + E(N-2)}$$



Lunney et al., Phys. Rep. **75**, 1021(2003)

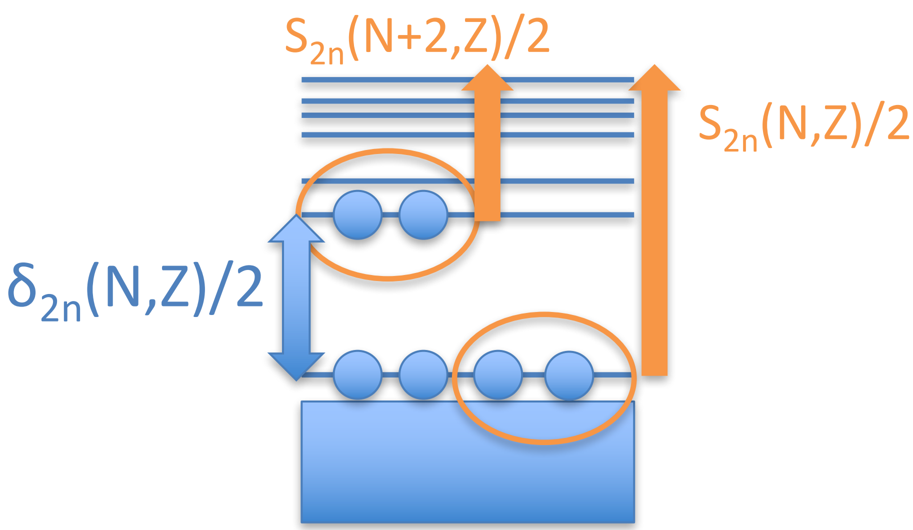
empirical shell gap: difference of 2n/2p separation energies

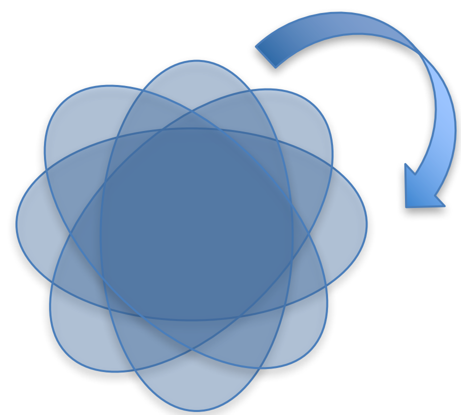
$$\delta_{\rm 2n}(N,Z) = E(N+2,Z) - 2E(N,Z) + E(N-2,Z) = S_{\rm 2n}(N,Z) - S_{\rm 2n}(N+2,Z)$$
$$\delta_{\rm 2p}(N,Z) = E(N,Z+2) - 2E(N,Z) + E(N,Z-2) = S_{\rm 2p}(N,Z) - S_{\rm 2p}(N,Z+2)$$

$$\mathcal{J}^{-1}_{\rm nn}(N) = \frac{1}{4}\delta_{\rm 2n}(N) \qquad \mathcal{J}^{-1}_{\rm pp}(N,Z) = \frac{1}{4}\delta_{\rm 2p}(N,Z)$$

shell model picture: weak pairing
(before symmetry breaking)

collective pairing picture: strong pairing
(after symmetry breaking)





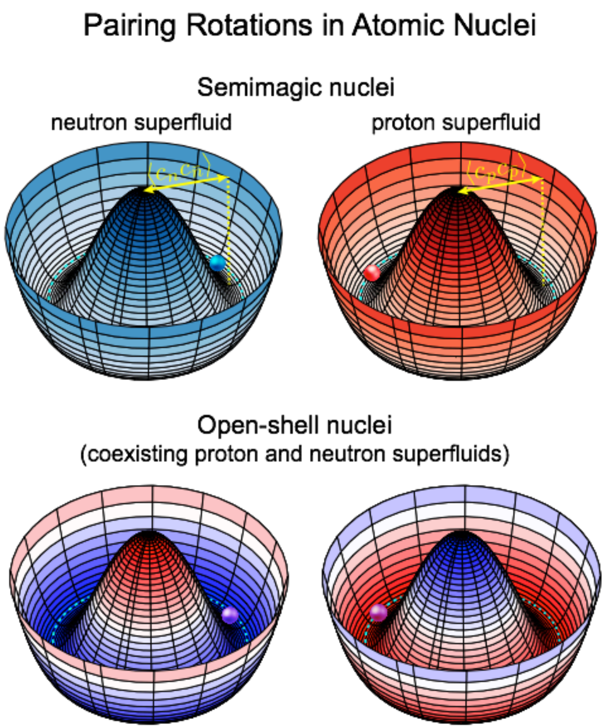
$$E_{\rm pairrot}(N) = \frac{1}{2\mathcal{J}_{\rm nn}(N)}(\Delta N)^2$$

size of magic shell gap

measure of gauge symmetry breaking

Pairing NG modes in doubly open-shell nuclei

broken symmetry: neutron and proton U(1), Δn≠0 and Δp≠0



$$[\hat{H}_{\rm HFB}, \hat{N}_{\rm n}] \neq 0 \qquad [\hat{H}_{\rm HFB}, \hat{N}_{\rm p}] \neq 0$$

NG modes as QRPA eigenmodes are neutron-proton mixed

formal theory: Marshalek, Nucl. Phys. A **275**,416 (1977)
first calculation: NH, Phys. Rev. C **92**,034321 (2015)

$$[\hat{H}_{\rm HFB}, \hat{N}_1] \neq 0 \qquad [\hat{H}_{\rm HFB}, \hat{N}_2] \neq 0$$

$$\hat{N}_1 = \hat{N}_{\rm n} \cos\theta + \alpha \hat{N}_{\rm p} \sin\theta$$
$$\hat{N}_2 = -\hat{N}_{\rm n} \sin\theta + \alpha \hat{N}_{\rm p} \cos\theta$$

$$E(N,Z) = E(N_0,Z_0) + \lambda_1(N_0,Z_0)\Delta N_1 + \lambda_2(N_0,Z_0)\Delta N_2 + \frac{(\Delta N_1)^2}{2\mathcal{J}_1(N_0,Z_0)} + \frac{(\Delta N_2)^2}{2\mathcal{J}_2(N_0,Z_0)}$$

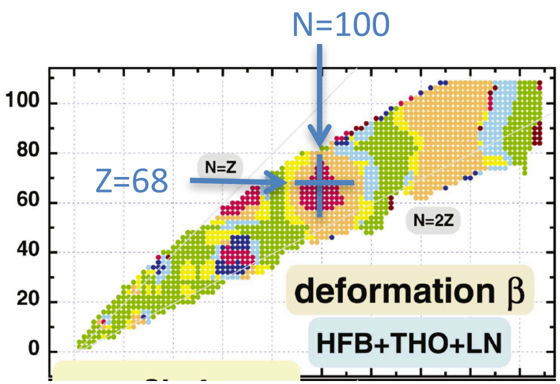
$$E_{\rm pairrot}(N+\Delta N, Z+\Delta Z) = \frac{(\Delta N)^2}{2\mathcal{J}_{nn}(N,Z)} + \frac{2(\Delta N)(\Delta Z)}{2\mathcal{J}_{np}(N,Z)} + \frac{(\Delta Z)^2}{2\mathcal{J}_{pp}(N,Z)}$$

$$\mathcal{J}_{np}^{-1}(N,Z) = \frac{E(N+2,Z+2) - E(N+2,Z) - E(N,Z+2) + E(N,Z)}{4}$$
$$= -\delta V_{pn}(N+2, Z+2)$$

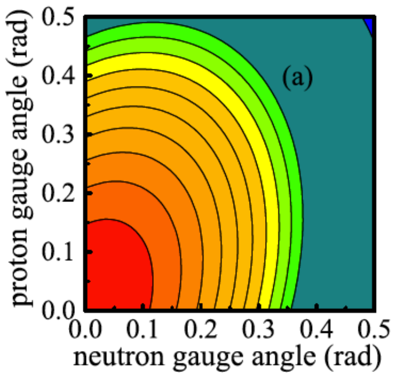
note: no neutron-proton pairing

Pairing rotation in doubly open-shell nuclei

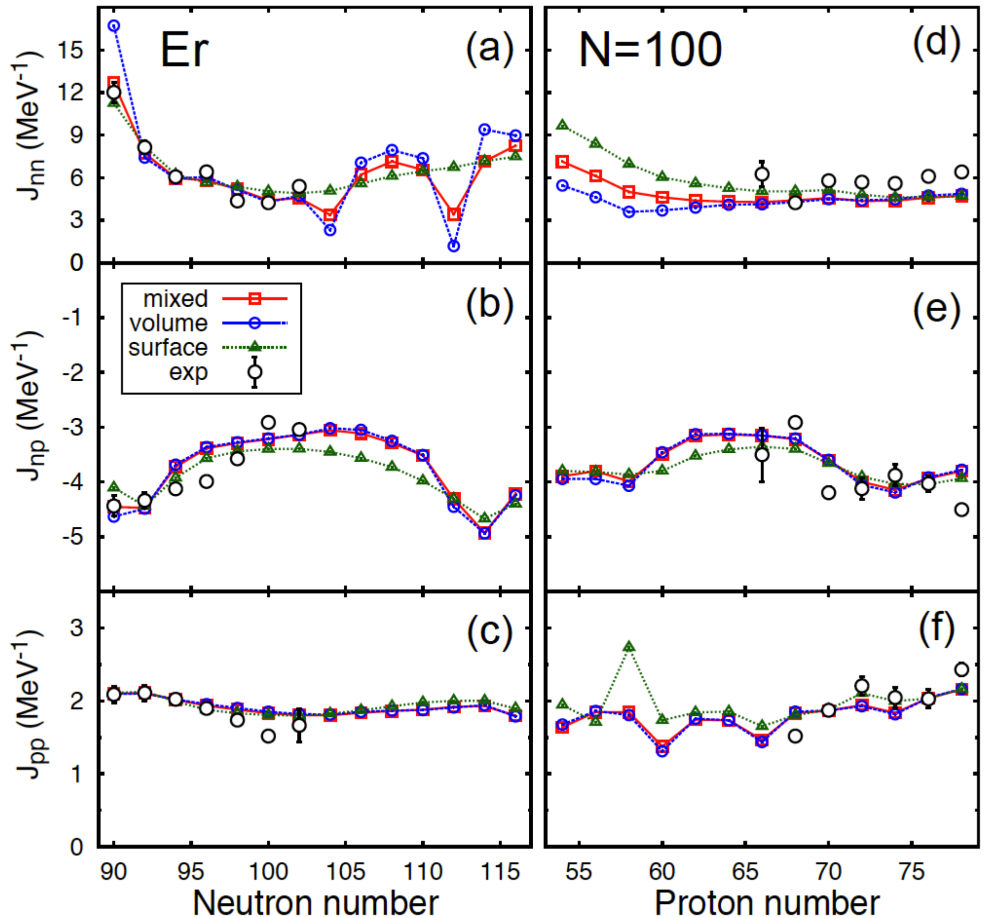
NH and Nazarewicz, Phys. Rev. Lett. **116**, 152502 (2016)



Stoitsov et al (2003)



Wang et al., PRC90,014312(2014)



- ☐ simultaneous agreement of three pairing rotational moments of inertia in prolately deformed open-shell nuclei

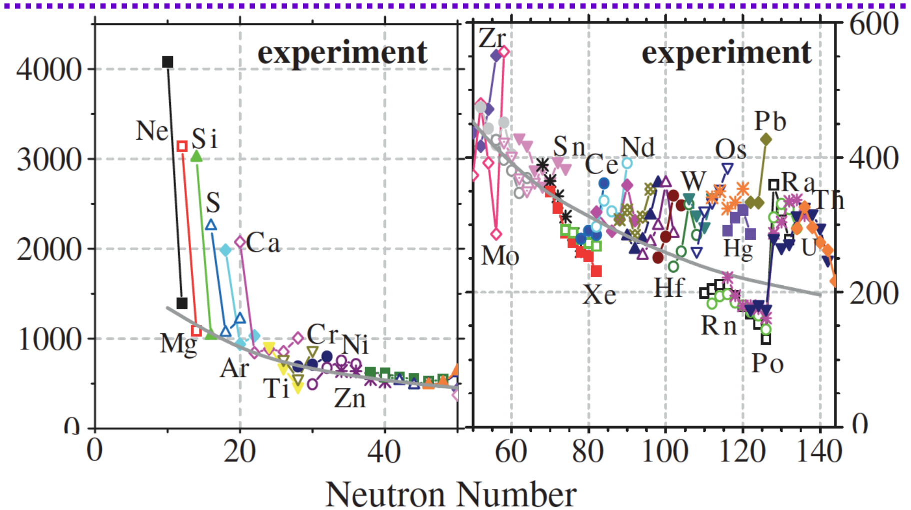
  evidence for mixing of neutron and proton NG modes
- ☐ some sensitivity to density dependence in neutron-rich (proton-deficient) nuclei

δVpn and pairing rotational moment of inertia

Experimental pairing rotational moment of inertia

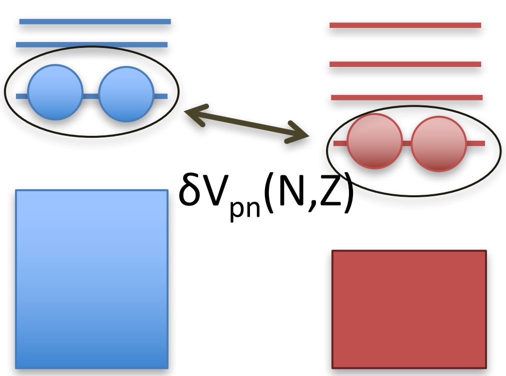
$$\mathcal{J}_{np}^{-1}(N,Z) = \frac{E(N+2,Z+2) - E(N+2,Z) - E(N,Z+2) + E(N,Z)}{4}$$

$$= -\delta V_{pn}(N+2,Z+2)$$



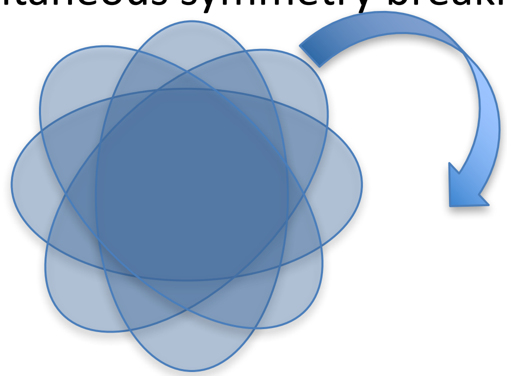
Stoitsov et al., Phys. Rev. C **98**,132502 (2007)

shell model picture
(before symmetry breaking)



interaction energy of
valence 2n and 2p

collective pairing picture
(after simultaneous symmetry breaking)



$$E_{\rm pairrot}(N,Z) = \frac{(\Delta N)^2}{2\mathcal{J}_{\rm nn}(N_0,Z_0)} + \frac{2(\Delta N)(\Delta Z)}{2\mathcal{J}_{\rm np}(N_0,Z_0)} + \frac{(\Delta Z)^2}{2\mathcal{J}_{\rm pp}(N_0,Z_0)}$$

measure of simultaneous
gauge symmetry breaking of neutron and proton

Pairing picture of binding energy differences

NH and Nazarewicz, Phys. Rev. Lett. **116**, 152502 (2016)

pairing collectivity

quadrupole collectivity

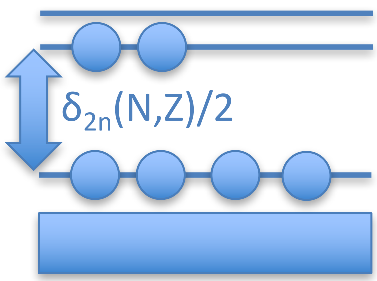
empirical shell gap

δVpn

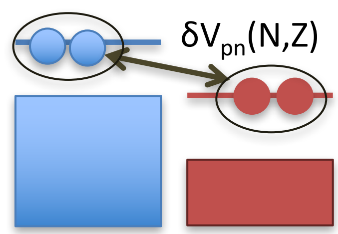
$E(2_1^+)$

magic nuclei

signature of magic number

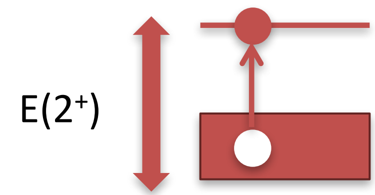


valence proton-neutron interaction energy



doubly magic nuclei

^{208}Pb: $E(2^+)$=4.085 MeV
proton h9/2-h11/2
ph excitation



superconducting nuclei

excitation of two NG modes

$$E_{\rm pairrot}(N,Z) = \frac{(\Delta N)^2}{2\mathcal{J}_{\rm nn}(N_0,Z_0)} + \frac{2(\Delta N)(\Delta Z)}{2\mathcal{J}_{\rm np}(N_0,Z_0)} + \frac{(\Delta Z)^2}{2\mathcal{J}_{\rm pp}(N_0,Z_0)}$$

$$\mathcal{J}_{\rm nn}^{-1}(N) = \frac{1}{4}\delta_{2n}(N) \qquad \mathcal{J}_{\rm np}^{-1}(N,Z) = -\delta V_{\rm pn}(N+2,Z+2)$$

$$\mathcal{J}_{\rm pp}^{-1}(N,Z) = \frac{1}{4}\delta_{2p}(N,Z)$$

magnitude of pairing collectivity
(gauge symmetry breaking)

deformed nuclei

excitation of NG mode

$$E(2_1^+) \sim \frac{3}{\mathcal{J}_{\rm rot}}$$

magnitude of quadrupole collectivity
(deformation)

Extending pairing functional

NH, arXiv:1706.06547

we have used simple pairing functionals because of lack of observables

conventional pairing functional $\tilde{\chi}_t(\boldsymbol{r}) = \tilde{C}_t^\rho(\rho_0)|\tilde{\rho}_t|^2$ $\qquad \tilde{C}_t^\rho[\rho_0] = \frac{1}{4}V_t\left(1 - \eta_t \frac{\rho_0(\boldsymbol{r})}{\rho_c}\right)$

general pair-density-bilinear form of the isovector pairing functional

Perlinska et al., Phys. Rev. C **69**, 014316 (2004)

$$\tilde{\chi}_t(\boldsymbol{r}) = \tilde{C}_t^\rho(\rho_0)|\tilde{\rho}_t|^2 + \tilde{C}_t^{\Delta\rho}\mathrm{Re}(\tilde{\rho}_t^*\Delta\tilde{\rho}_t) + \tilde{C}_t^\tau\mathrm{Re}(\tilde{\rho}_t^*\tilde{\tau}_t) + \tilde{C}_t^{J0}|\tilde{J}_t|^2 + \tilde{C}_t^{J1}|\tilde{\boldsymbol{J}}_t|^2 + \tilde{C}_t^{J2}|\underline{\tilde{\mathsf{J}}}_t|^2 + \tilde{C}_t^{\nabla J}\mathrm{Re}(\tilde{\rho}_t^*\boldsymbol{\nabla}\cdot\tilde{\boldsymbol{J}}_t)$$

constraints on the coupling constants from local gauge invariance

$$\tilde{C}_t^{\Delta\rho} = -\frac{1}{4}\tilde{C}_t^\tau$$
$$\tilde{C}_t^{\nabla J} = 0$$

omitting the tensor-pairing terms for simplicity,

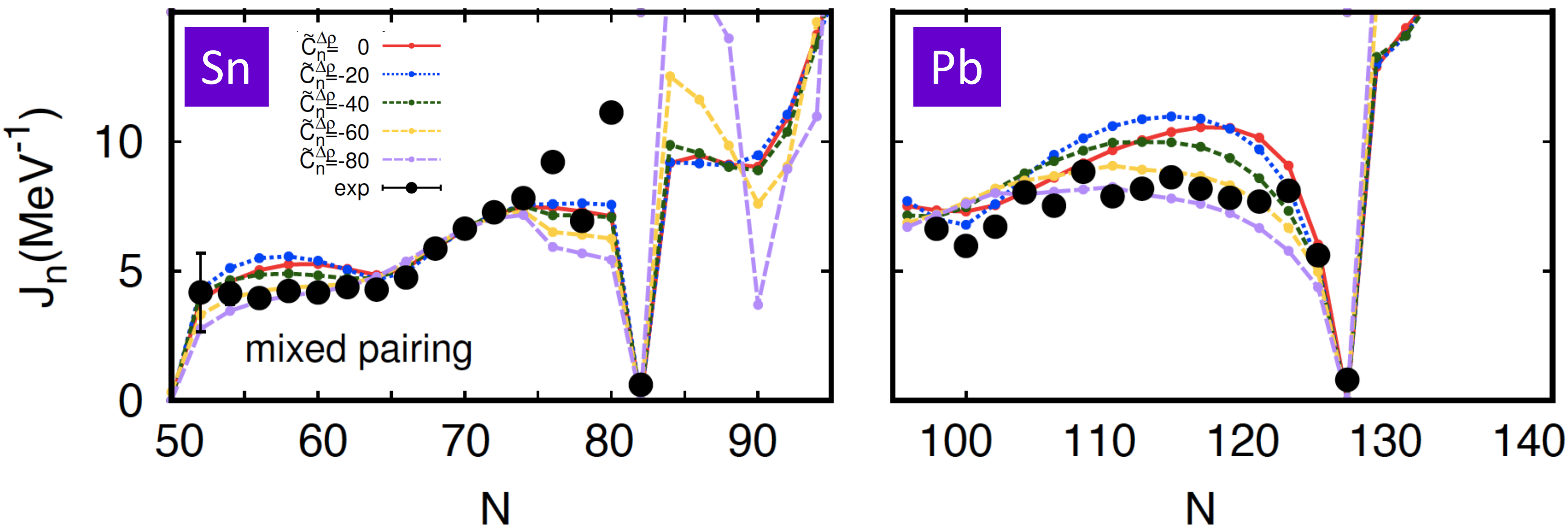
$$\tilde{\chi}_t(\boldsymbol{r}) = \tilde{C}_t^\rho(\rho_0)|\tilde{\rho}_t|^2 + \boxed{\tilde{C}_t^{\Delta\rho}}[\mathrm{Re}(\tilde{\rho}_t^*\Delta\tilde{\rho}_t) - 4\mathrm{Re}(\tilde{\rho}_t^*\tilde{\tau}_t)]$$

one new coupling constant

Extending pairing functional

NH, arXiv:1706.06547

coupling constant (C^{ρ}) fitted to the pairing rotational MOI in ^{120}Sn for selected values of $C^{\Delta\rho}$

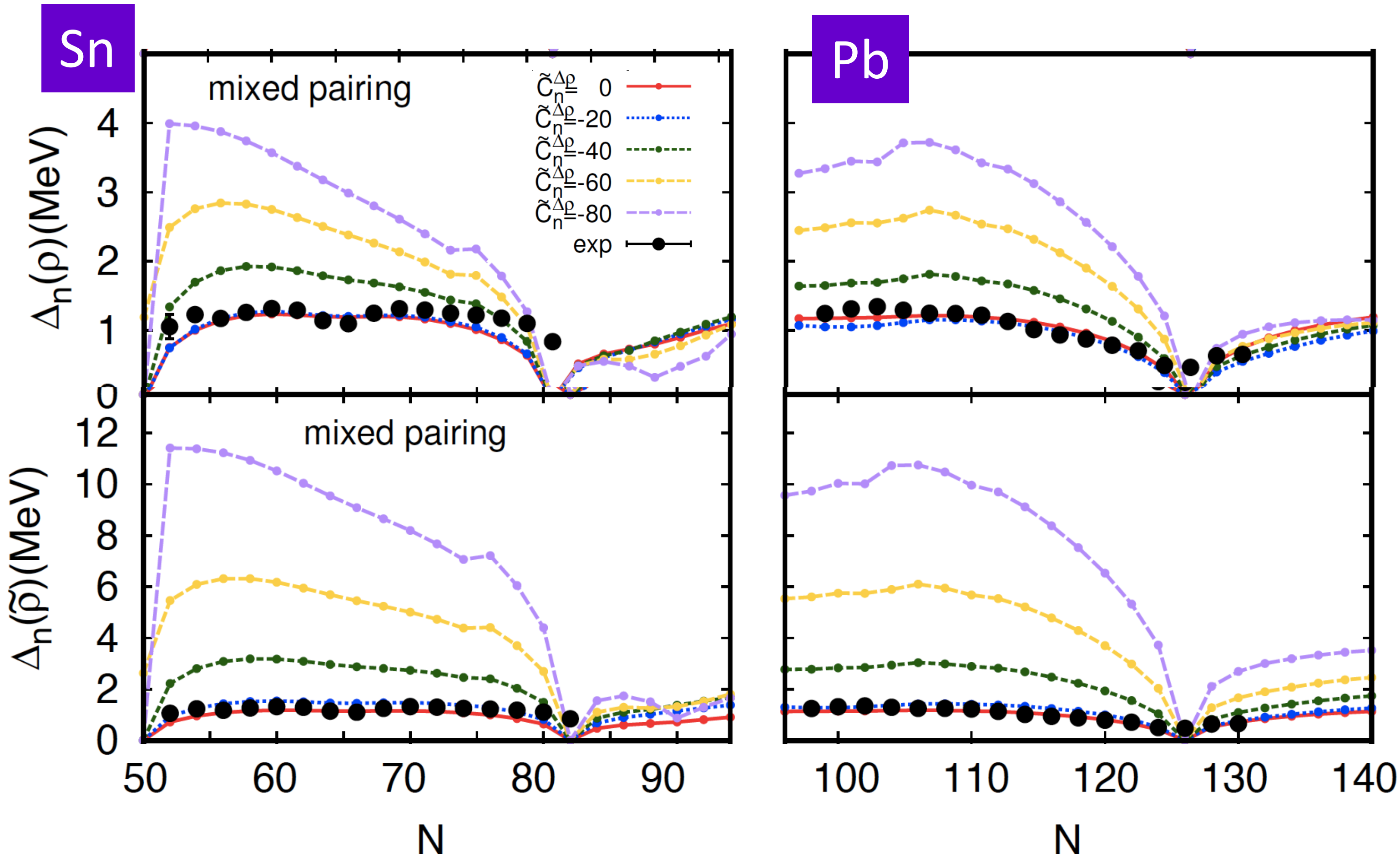


improvement of the pairing rotational MOI with pair-density derivative terms

Pairing gaps

NH, arXiv:1706.06547

$$\Delta_n(\rho) = \frac{\mathrm{Tr}\tilde{h}_n[\rho, \tilde{\rho}_n]\rho_n}{\mathrm{Tr}\rho_n} \qquad \Delta_n(\tilde{\rho}) = \frac{\mathrm{Tr}\tilde{h}_n[\rho, \tilde{\rho}_n]\tilde{\rho}_n}{\mathrm{Tr}\tilde{\rho}_n}$$



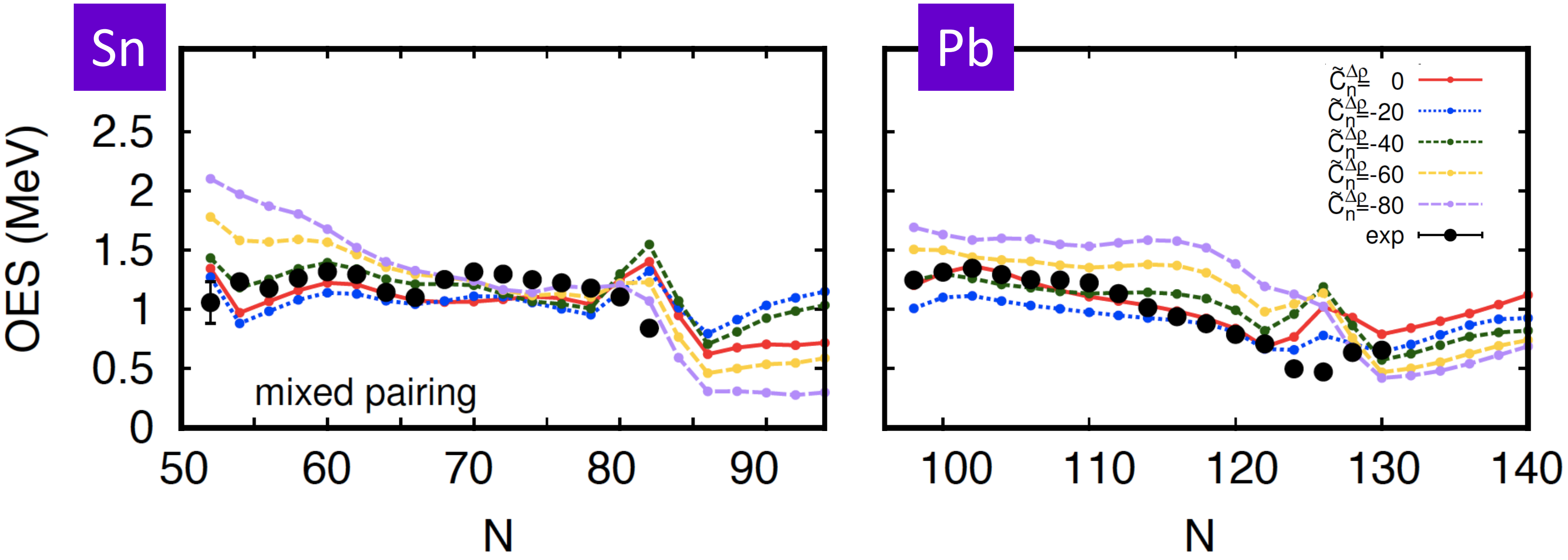
- pairing gaps increase with pair-density derivative terms
- two average pairing gaps are not consistent with pair-density derivative terms

OES

NH, arXiv:1706.06547

$$E(N_{\rm odd}) \approx \frac{1}{2}[E_{\rm HFB}(N_{\rm odd}-1)+E_{\rm qp}(N_{\rm odd}-1)+E_{\rm HFB}(N_{\rm odd}+1)+E_{\rm qp}(N_{\rm odd}+1)]$$

$$\Delta_n^{(3)}(N_{\rm even}) \approx \frac{1}{4}[E_{\rm qp}(N_{\rm even}+2)+2E_{\rm qp}(N_{\rm even})+E_{\rm qp}(N_{\rm even}-2)]$$



- OES from qp energies is consistent with the experimental data

Summary

first systematic calculation of pairing rotational MOI in doubly open-shell nuclei
Evidence of neutron and proton mixed pairing Nambu-Goldstone modes

- Double-binding energy differences of even-even nuclei
  - pairing rotational MOI: double binding-energy differences
  - shell gap δ_{2n} and proton-neutron interaction energy δV_{pn}

- Pairing functionals with derivatives of local pair densities
  - improve the systematic values of pairing rotational MOI
  - pairing gap becomes large and does not correspond to OES

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