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Isospin-density Dependence of Pairing in Asymmetric Nuclear Matter and Medium Polarization

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- **Introduction**
- **Isospin-dependent pairing interaction in ANM**

  PRC**81** (2010) 044313
- **New effective pairing interaction for finite nuclei**

  SCEG**54** (2011) 236
- **Medium polarization in ANM** PRC **93** (2016) 044329
- **Summary and perspective**

pairing phenomenon

Pairing is a two particle correlation around the Fermi surface. As such it manifests in all quantum-mechanical many fermion systems:

Microscopically,

- ***metals: superconducting electrons***

Bardeen, Cooper, Schrieffer, Superconductivity in metals, Phys. Rev. 108 (1957) 1175

- ***atoms: superfluidity of ^3He-^3He anisotropic phases***
- ***nuclei: odd-even effect***

Bohr, Mottelson, Excited spectrum in odd-even nuclei, Phys. Rev. 110 (1958) 936

- ***quark phase: color superconductivity***

Macroscopically,

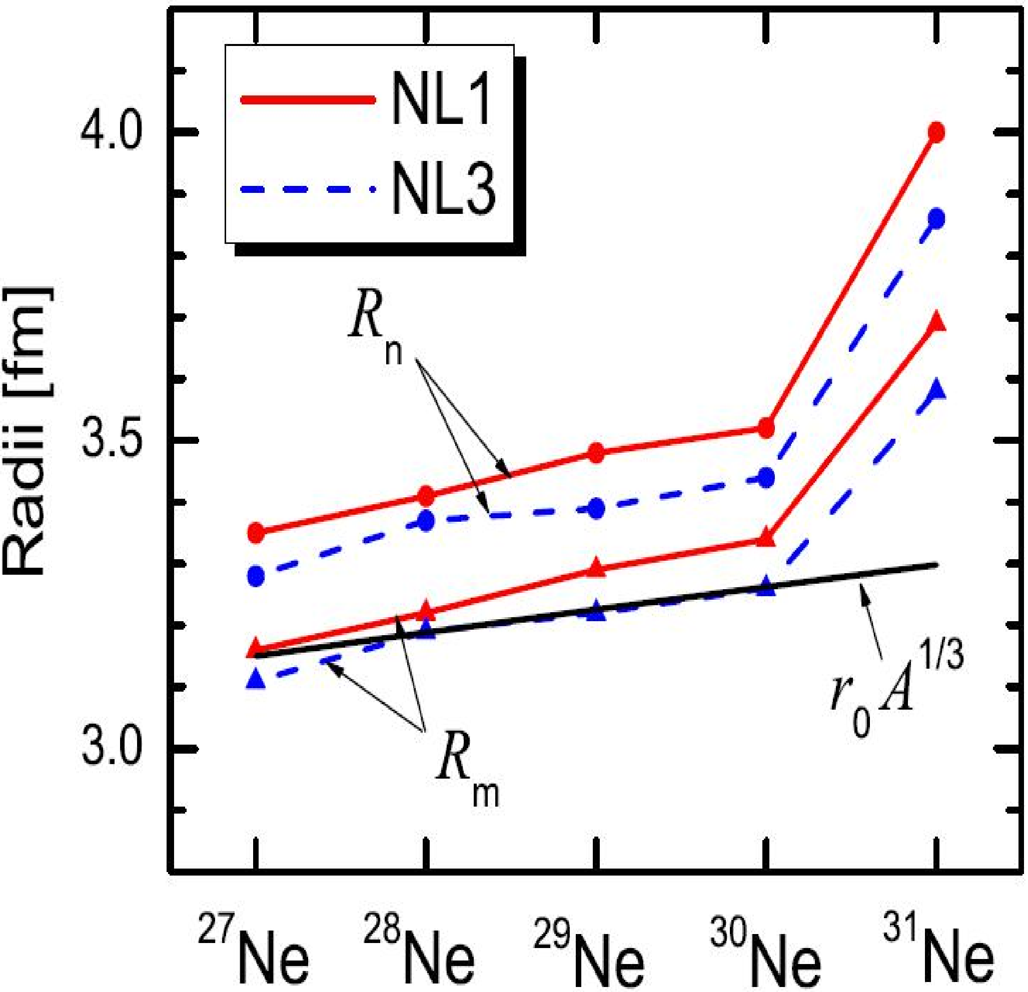
- ***neutron stars: post-glitches and cooling***

Migdal, neutron star, Soviet Physics JETP 10 (1960) 176

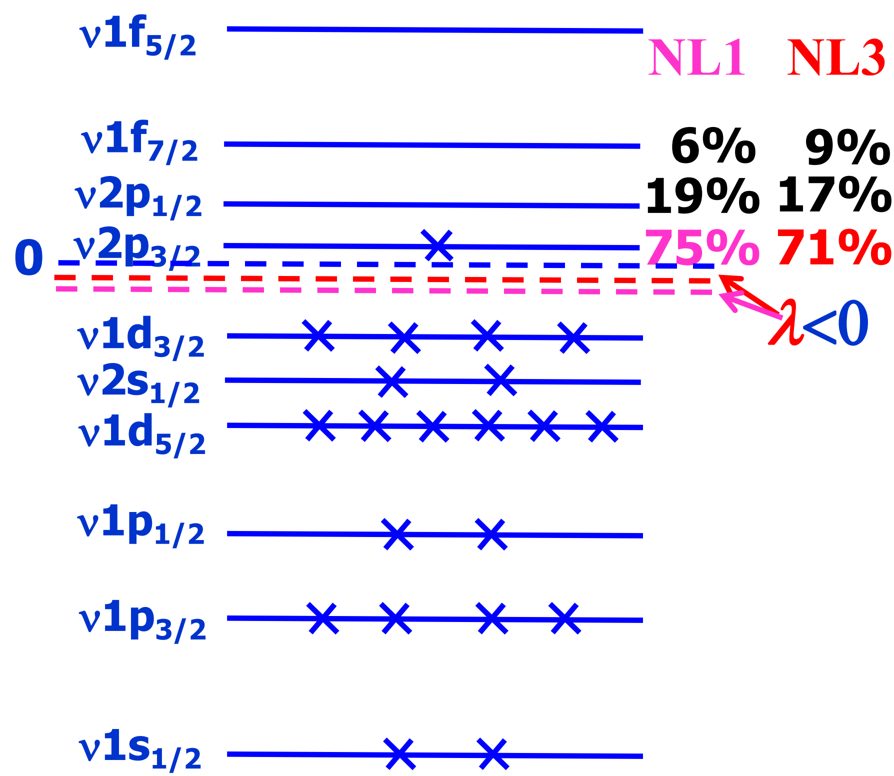
- ***stellar evolution: reaction cross section***

Important for formation of a weekly bound halo nucleus!

S. S. Zhang, M. S. Smith, Z. S. Kang, J. Zhao, Phys. Lett. B 730, 30 (2014).



RMF+ACCC+BCS approach



N=21

^{31}Ne

R_n: Neutron radii

R_m: Matter radii

$$R_m^{def} = R_m^{sph}\sqrt{1+\frac{5}{4\pi}\beta_2^2}$$

Two ways for pairing in nuclei

$$N = \sum_k \left(1 - \frac{\varepsilon(k) - \varepsilon_F}{E(k)} \right)$$

$$\Delta(k) = \sum_{k'} \frac{V(k,k')\Delta(k')}{E(k')}$$

- Pairing gap interpolation

$$\Delta_q(\rho_n, \rho_p) = \Delta_{SM}(\rho)(1 - |\beta|) \pm \Delta_{NM}(\rho_q)\beta \frac{\rho_q}{\rho}$$

S. Goriely, N. Chamel and J. M. Pearson, PRL 102 (2009) 152503

- Parametrize eff. pairing interaction by fitting pairing gap

$$V(\rho, \beta) = \frac{1}{2} v_0 \left[1 - \eta_s \left(\frac{\rho}{\rho_0} \right)^{\alpha_s} (1 \mp \beta) \mp \eta_n \left(\frac{\rho}{\rho_0} \right)^{\alpha_n} \beta \right] \delta(\vec{r} - \vec{r}\,')(1 - P_\sigma)$$

J. Margueron, H. Sagawa and K. Hagino, PRC 77 (2008) 054309

Pairing gaps:
- microscopic calc. $\Delta_{SM}(\rho)$
- empirical odd-even mass staggering

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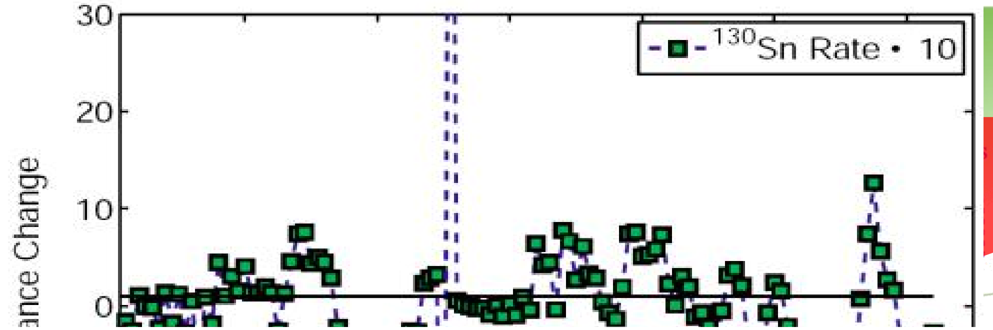
S. Goriely, Nucl. Phys. A **718**, 287c (2003).



bound levels

resonant levels (above neutron capture thresholds)

↓

neutron capture (NC cross section

↓

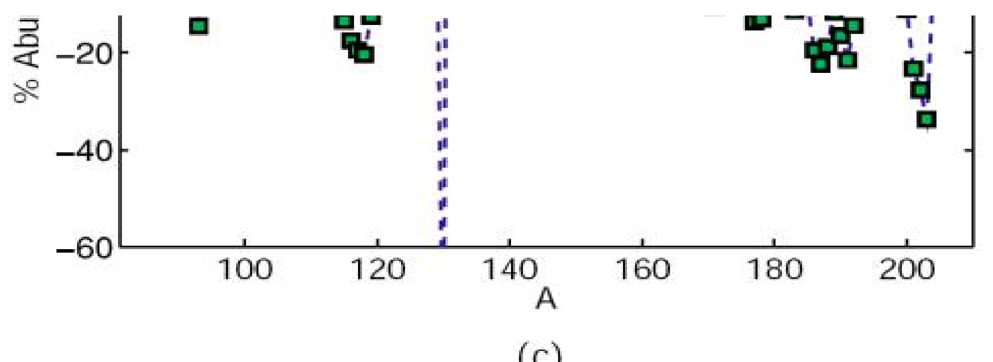
NC reaction rate

↓

synthesis of heavy elem the r-process in super

Z \ N	78	79	80	81	82	83	84	85	86	87
52	^{130}Te	^{131}Te	^{132}Te	^{133}Te	^{134}Te	^{135}Te	^{136}Te	^{137}Te	^{138}Te	^{139}Te
51	^{129}Sb	^{130}Sb	^{131}Sb	^{132}Sb	^{133}Sb	^{134}Sb	^{135}Sb	^{136}Sb	^{137}Sb	^{138}Sb
50	^{128}Sn	^{129}Sn	^{130}Sn	^{131}Sn	^{132}Sn	^{133}Sn	^{134}Sn	^{135}Sn	^{136}Sn	^{137}Sn
49	^{127}In	^{128}In	^{129}In	^{130}In	^{131}In	^{132}In	^{133}In	^{134}In	^{135}In	^{136}In
48	^{126}Cd	^{127}Cd	^{128}Cd	^{129}Cd	^{130}Cd	^{131}Cd	^{132}Cd	^{133}Cd	^{134}Cd	^{135}Cd

Significantly impact !

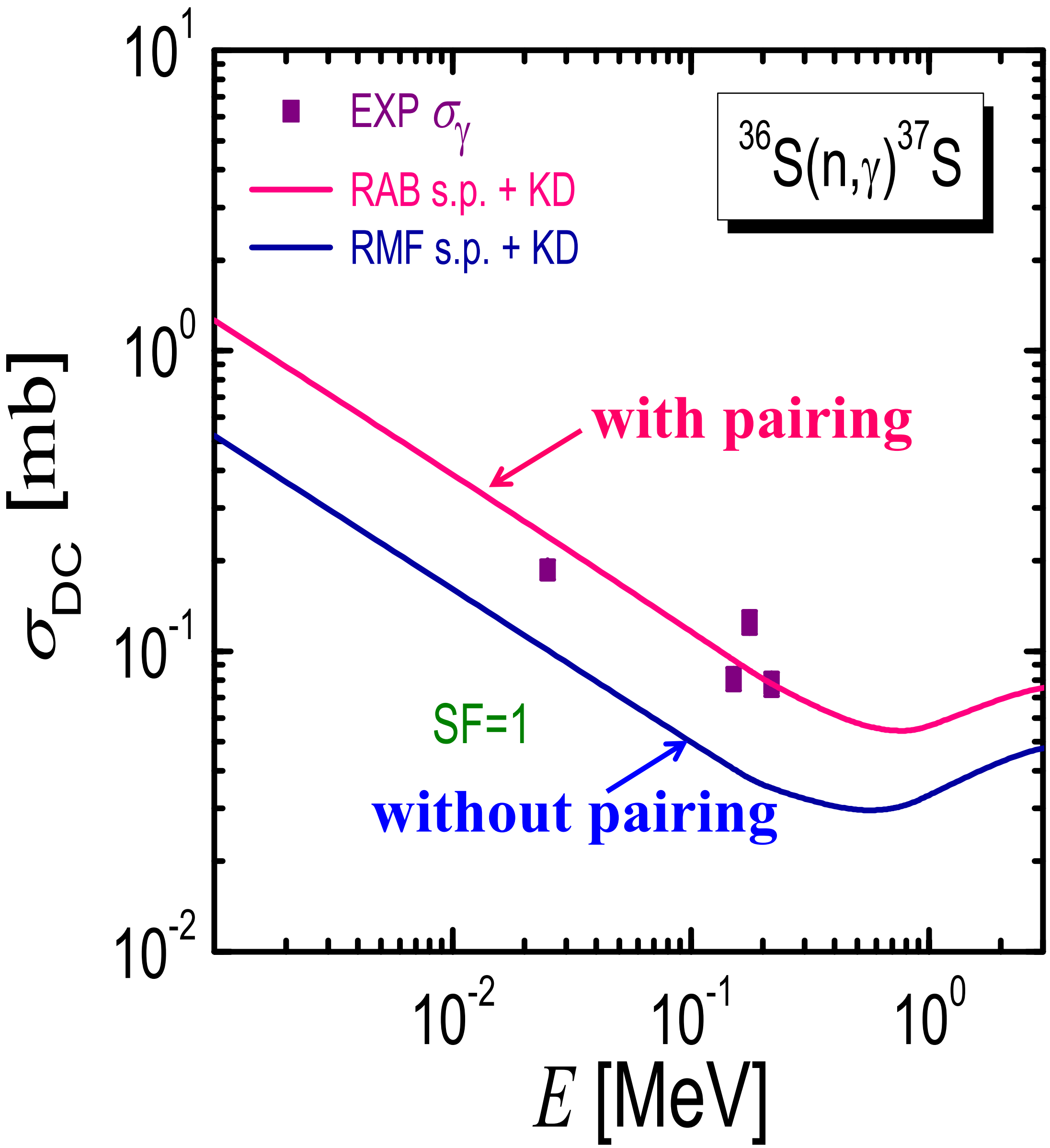


Global impact of neutron capture rate on r-process abundances

R. Surman, etc, PRC **79**, 045809 (2009).

Important for neutron capture on Exotic Nuclei!

S. S. Zhang, J. P. Peng, M. S. Smith, G. Arbanas, R. L. Kozub, Phys. Rev. C 91, 045802 (2015).



RAB : with resonant & pairing

RMF : without pairing

- ✓ **The results with resonant & pairing contributions are closer to the available cross section data**

$$\sigma_{DC}^{RAB} \sim \sigma_\gamma^{EXP}$$

- ✓ **Direct Capture Cross Section: 2~3 times larger than those without pairing consideration**

$$\sigma_{DC}^{RAB} \sim 2\text{-}3\,\sigma_{DC}^{RMF}$$

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Macroscopically,

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Migdal, neutron star, Soviet Physics JETP 10 (1960) 176

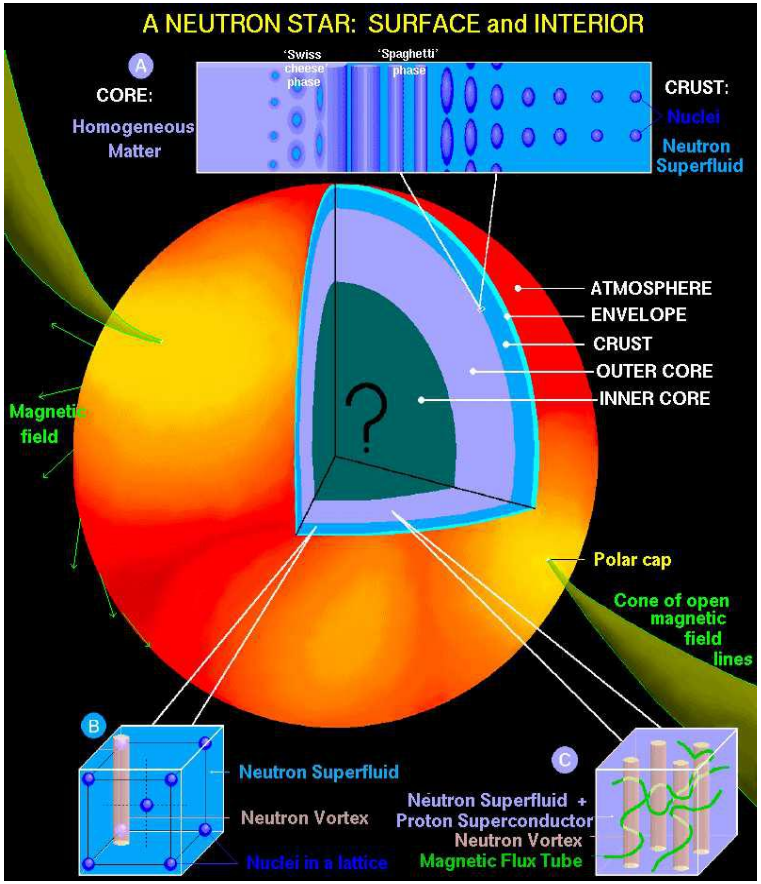
- ***stellar evolution: reaction cross section***

Motivation

Superfluid states of neutron stars (NSs)

has been reviving after the real-time temperature measurements of the CasA remnant

W. C. G. Ho and C. O. Heinke, Nature (London) 462 (2009) 71



- ☐ Cooling of NSs:
  demands pairing gaps for
  - ✓ $^1S_0$ neutron-neutron （crust）
  - ✓ $^1S_0$ proton-proton （inner core）
  - ✓ $^3PF_2$ neutron-neutron （inner core)
- ☐ Superfluid States occurs at
  - ✓ high isospin imbalance $\beta = (N-Z)/(N+Z)$
  - ✓ high baryon density $\rho = \rho_n + \rho_p$

A nuclear system is a strong interacting Fermi system :

Effective interaction:

- *short range correlations*

Since the nuclear force is a short range force, a mean field can be built up to describe g.s. of nuclei and EoS of nuclear matter.

- microscopic BHF, DBHF calcs.
- phenomenological Skyrme, RMF calcs.

- *l ong range correlations (collective excitations)*

- microscopic induced interaction, ….

  GE Brown et al, Phys Rep 124 (1985) 1
- phenomenological extending Skyrme forces

  Milano group, PRC 87 (2013) 054303

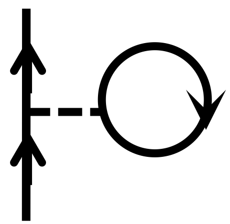
Effective Nuclear Interaction byond BHF

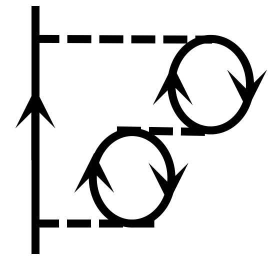
$$N = \sum_k \left(1 - \frac{\varepsilon(k) - \varepsilon_F}{E(k)} \right)$$

$$\Delta(k) = \sum_{k'} \frac{V(k,k')\Delta(k')}{E(k')}$$

- The BHF is a mean field theory

- where the interaction of a particle with surrounding medium is approximated with a potential

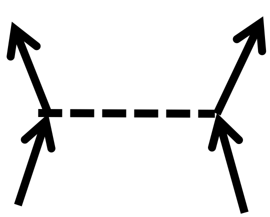


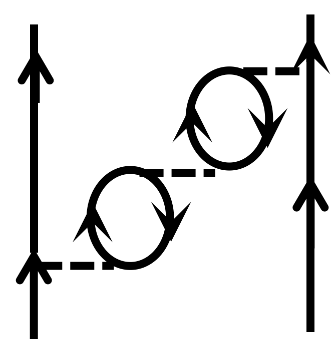


BHF *Self-Energy* : $\Sigma = \Sigma_{BHF} + \quad \Sigma_{coll}$

- G-matrix

- an effective interaction accouting for short range correlations





medium polarization effects

Isospin-dependent pairing interaction in asymmetric nuclear matter

- Evolution of the paring gaps from PNM to SNM

- Effective pairing interaction with the momentum cut-off from infinite nuclear matter to finite nuclei

S. S. Zhang, L. G. Cao, U. Lombardo, E. G. Zhao and S. G. Zhou
PRC**81** (2010) 044313

Theoretical framework

PRC**81** (2010) 044313

Step 1. Solve pure BCS gap equation with *realistic interaction* Argonne AV_{18}

$$N = \sum_k \left(1 - \frac{\varepsilon(k) - \varepsilon_F}{E(k)}\right)$$

$$\Delta_{ex}(k) = -\sum_{|k'-k_F|<\Delta k} \frac{V_{bare}(k,k')\Delta(k')}{2E(k')}$$

$$E^2(k) = (\varepsilon(k) - \varepsilon_F)^2 + \Delta^2(k)$$

$$\varepsilon(k) = k^2/2m + U(k)$$ - **Skyrme- LNS potential**

PRC 73 (2006) 014313

$$= k^2/2m^*$$

m* - **effective mass**

$$\varepsilon_F = k_F^2/2m^*$$

Step 2. Renormalize the pairing interaction (effective int.)

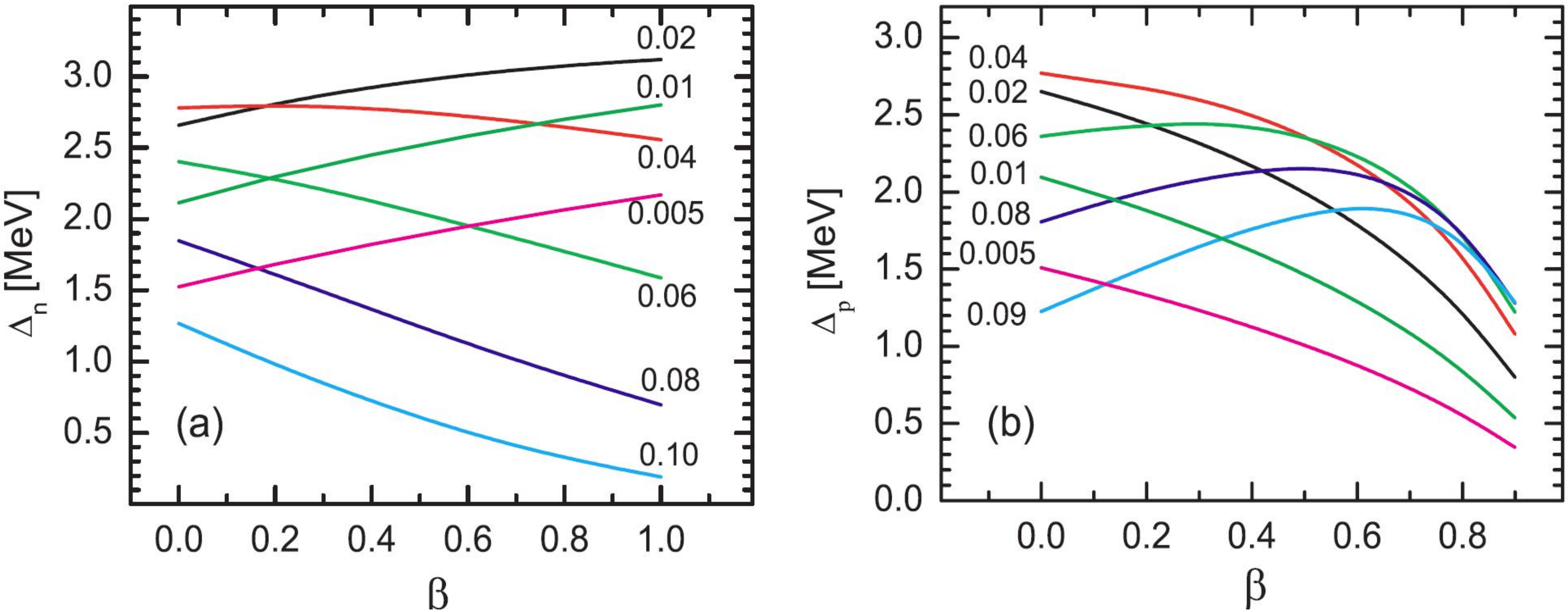
$$\tilde{V}(k,k') = V_{bare}(k,k') - \sum_{|k''-k_F|>\Delta k} \frac{V_{bare}(k,k'')\tilde{V}(k'',k')}{2E(k'')}$$

Step 3. Solve gap equation with *effective pairing interaction*.

$$\Delta(k) = -\sum_{|k'-k_F|<\Delta k} \frac{\tilde{V}(k,k')\Delta(k')}{2E(k')}$$

Neutron and proton gaps in ANM

PRC**81** (2010) 044313



not linearly dependent on β due to nonlinear structure of gap equation

$$\Delta(k) = -\sum_{|k'-k_F|<\Delta k} \frac{\tilde{V}(k,k')\Delta(k')}{2E(k')}$$

Neutron gaps :

- ✓ the β - slope
  - ✓ positive for $\rho < 0.02$ fm^{-3} ;
  - ✓ negative for $\rho > 0.02$ fm^{-3}
- ✓ $\rho = 0.02$ fm^{-3} corresponding to the peak value of Δ(r)

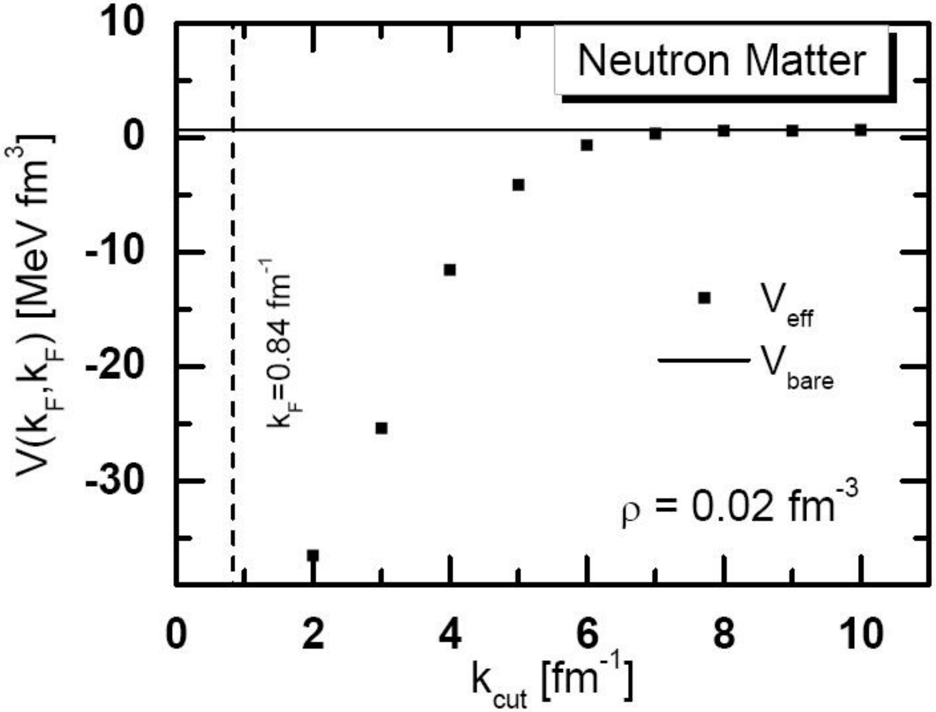
Proton gaps :

- ✓ the β - slope
  - ✓ negative for $\rho < 0.02$ fm^{-3}
  - ✓ In the range of $\rho > 0.02$ fm^{-3},
    - ✓ competition with mean field effect, which becomes more and more attractive for increasing β.

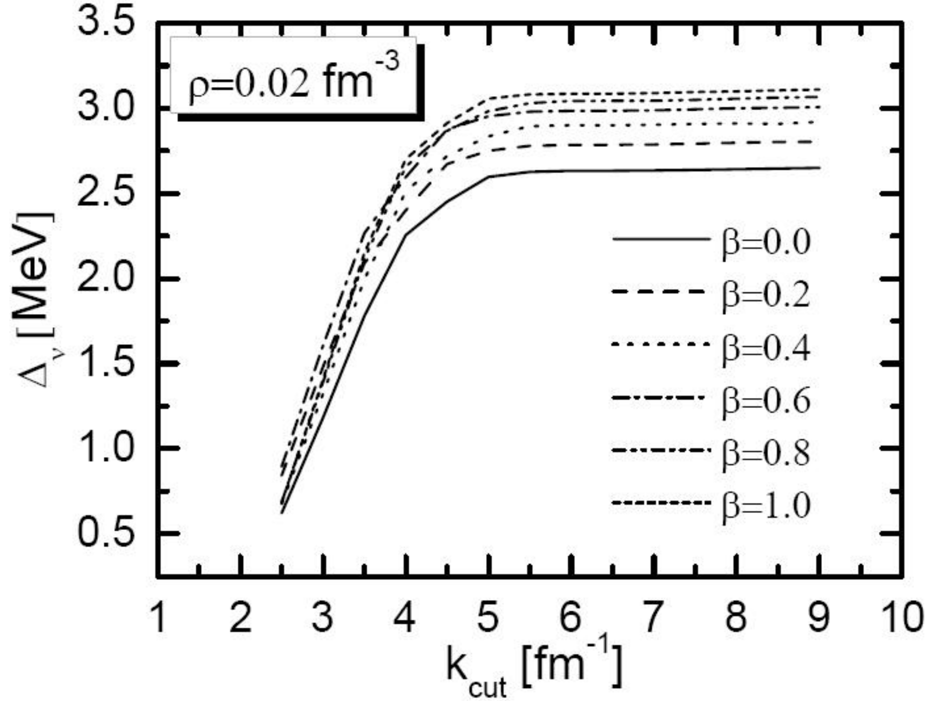
PRC**81** (2010) 044313

Sensitivity to the chosen momentum cut-off $k_{cut} = |k_F - k|$

pairing effective interaction



energy gap



For a cut-off large enough (with Argonne AV_{18} , k_{cut} ~ 5 fm^{-1}),

the eff. int. and the gap saturates to the exact value;

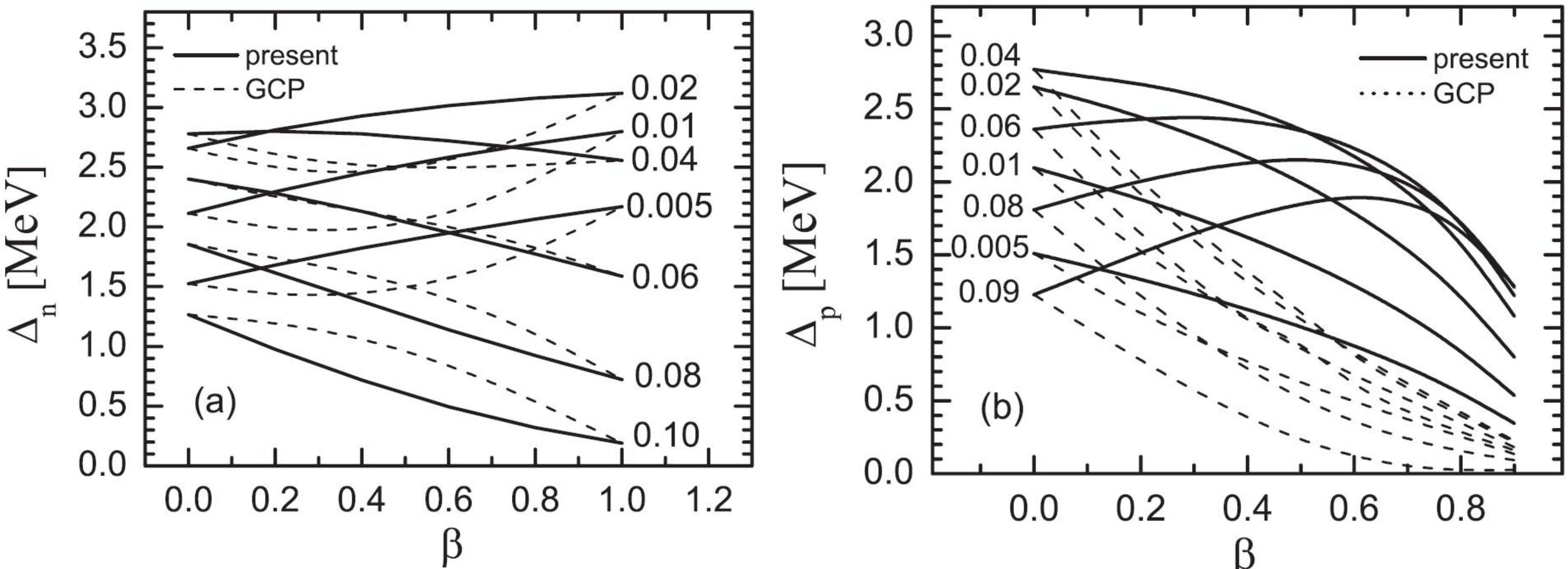
✓ **k_{cut} : range of nonlocality of the chosen potential for the pairing correlations**

✓ **Its large value casts doubt on zero range pairing interactions**

One way: Gap interpolation for ANM

PRC**81** (2010) 044313

$^1S_0$ neutron and proton gaps



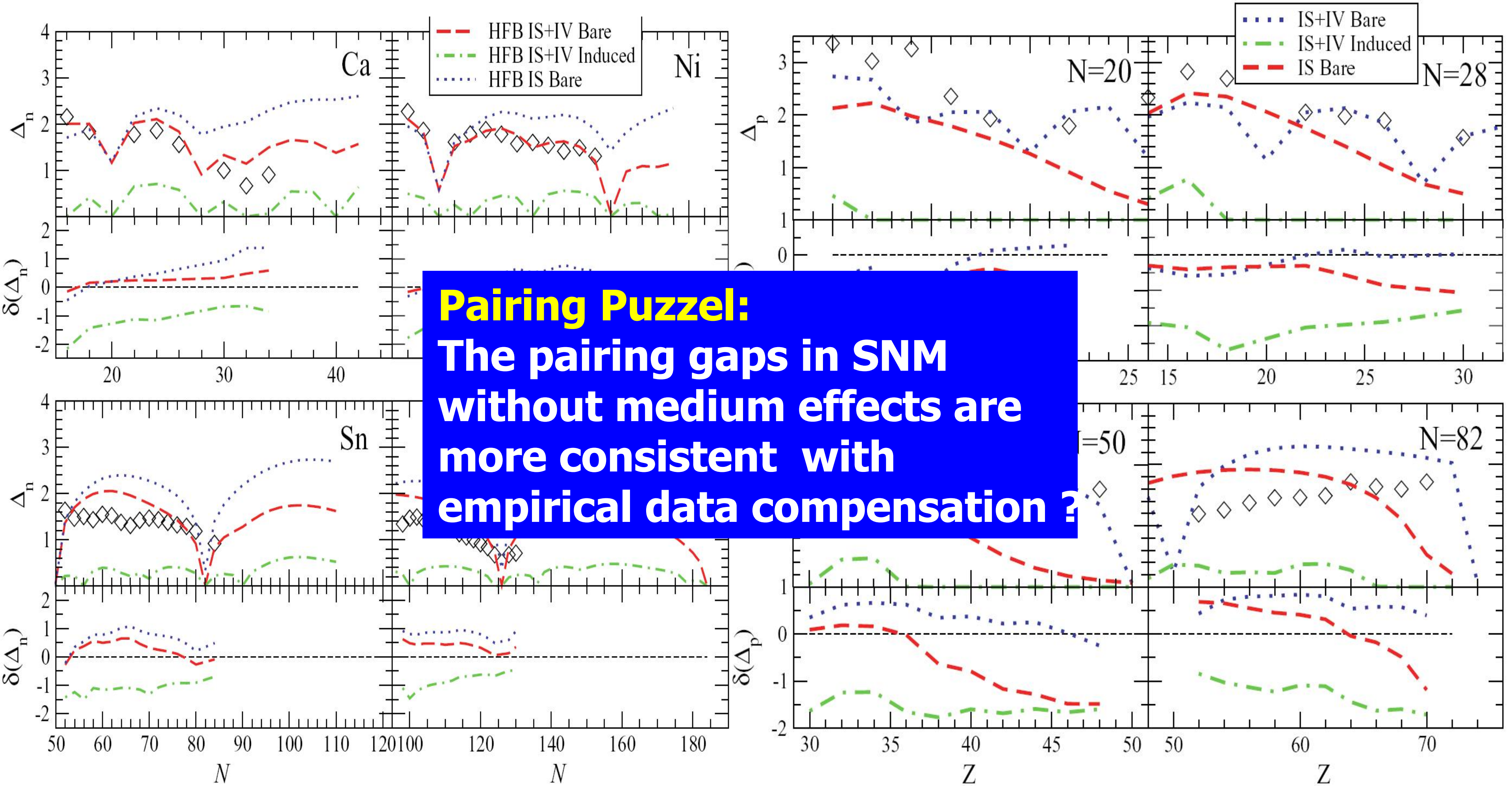
$$\Delta_q(\rho_n,\rho_p) = \Delta_{SM}(\rho)(1-|\beta|) \pm \Delta_{NM}(\rho_q)\beta\frac{\rho_q}{\rho}$$

S. Goriely, N. Chamel and J. M. Pearson, PRL 102 (2009) 152503

The GCP gap interpolation formula seems not to be valid.

The other way: Fit exact gap by effective inter. for ANM

J. Margueron, H. Sagawa and K. Hagino, PRC 77 (2008) 054309



Pairing Puzzel:
The pairing gaps in SNM without medium effects are more consistent with empirical data compensation ?

$$V(\rho,\beta)=\frac{1}{2}v_0\left[1-\eta_s\left(\frac{\rho}{\rho_0}\right)^{\alpha_s}(1\mp\beta)\mp\eta_n\left(\frac{\rho}{\rho_0}\right)^{\alpha_n}\beta\right]\delta(\vec{r}-\vec{r}\,')(1-P_\sigma)$$

$$E^2(k)=(\varepsilon(k)-\varepsilon_F)^2+\Delta^2(k)$$

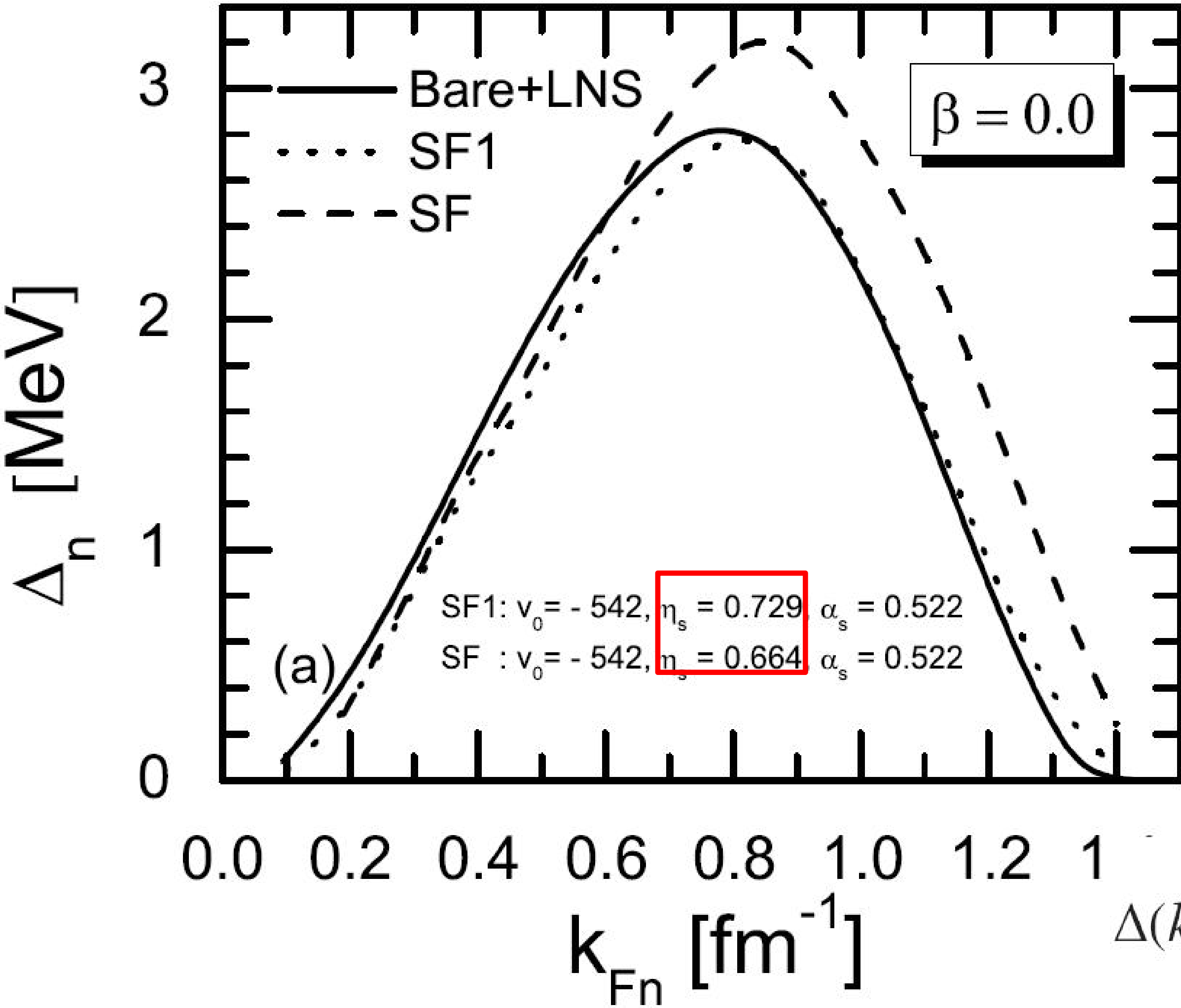
$$\varepsilon(k)=k^2/2m+U(k)$$

$$=k^2/2m^*$$

SF: $v_0=-542,\eta_s=0.664,\alpha_s=0.522,\eta_n=1.01,\alpha_n=0.525$

New eff. pairing inter. for finite nuclei

$^1S_0$ neutron gaps in symmetric nuclear matter



SCEG **54** (2011) 236

Guess:
The reason might be they fit the neutron gaps of ref. [6], which are obtained from the BCS theory with realistic bare NN interaction, without mean field.

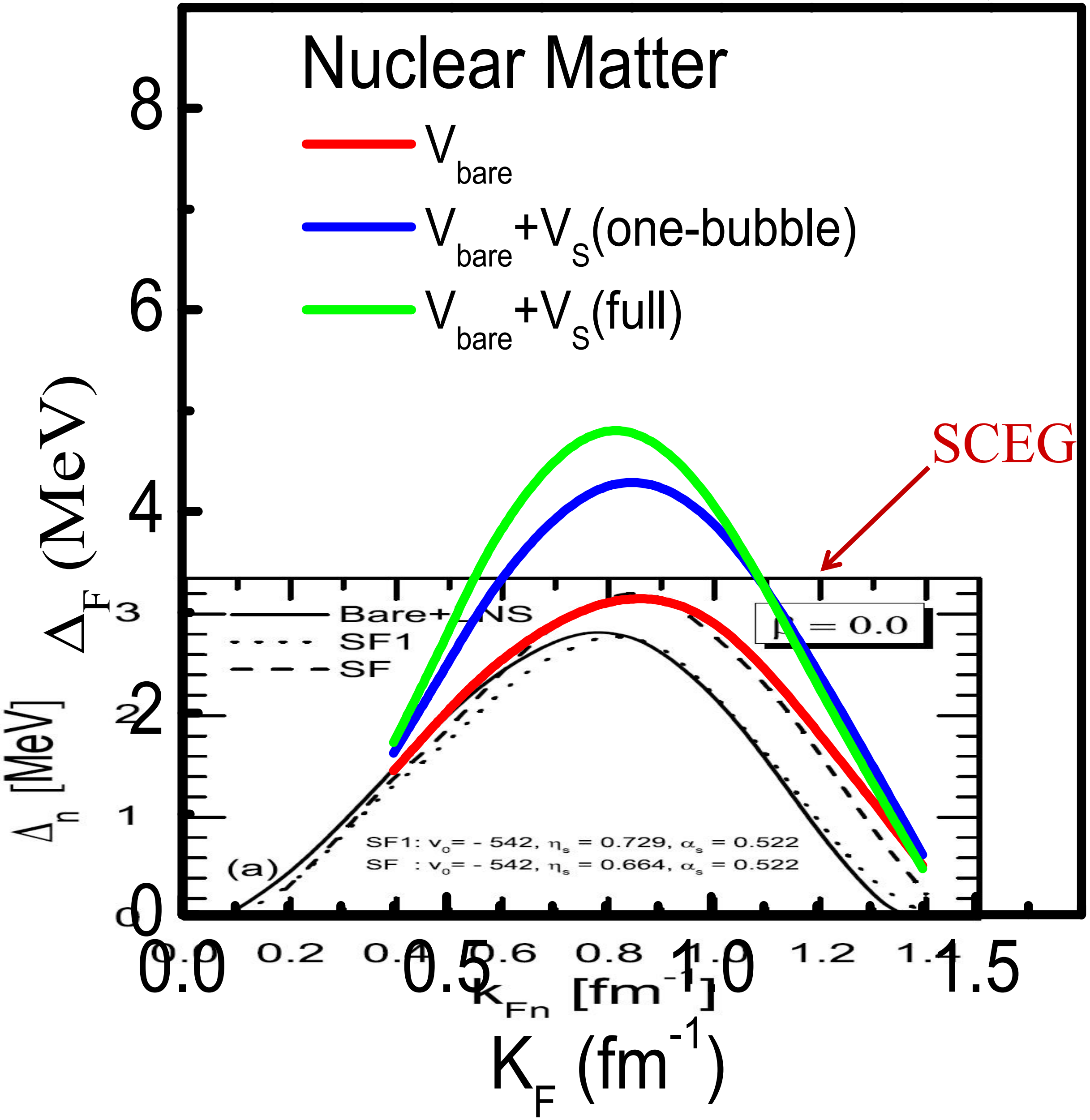
$$\Delta(k) = -\frac{v(\rho,\beta)}{2(2\pi)^3}\int_0^{k_{\rm cut}} d^3k' \frac{\Delta(k')}{E(k')}$$

Bare + LNS : the results of bare NN interaction in the Skyrme-LNS mean field

SF1 (SF) : ZSS (MSH) parameter sets from the fit of isospin-density effective interaction in the SLy4 mean field

$^1S_0$ neutron gaps in symmetric nuclear matter

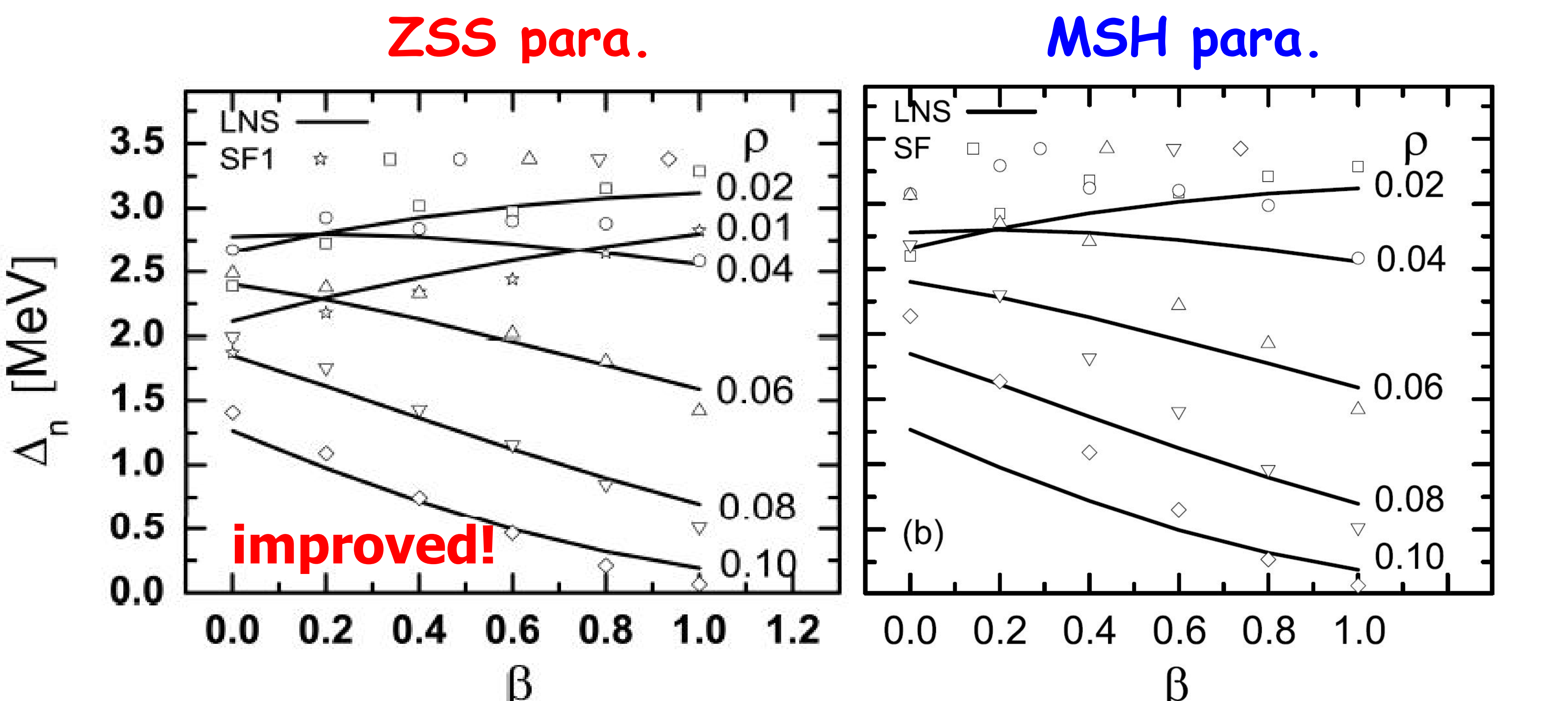
L. G. Cao, U. Lombardo, and P. Schuck, PRC 74 (2006) 064301



SCEG **54** (2011) 236

SCEG **54** (2011) 236

Neutron gap for zero-range effective pairing interaction



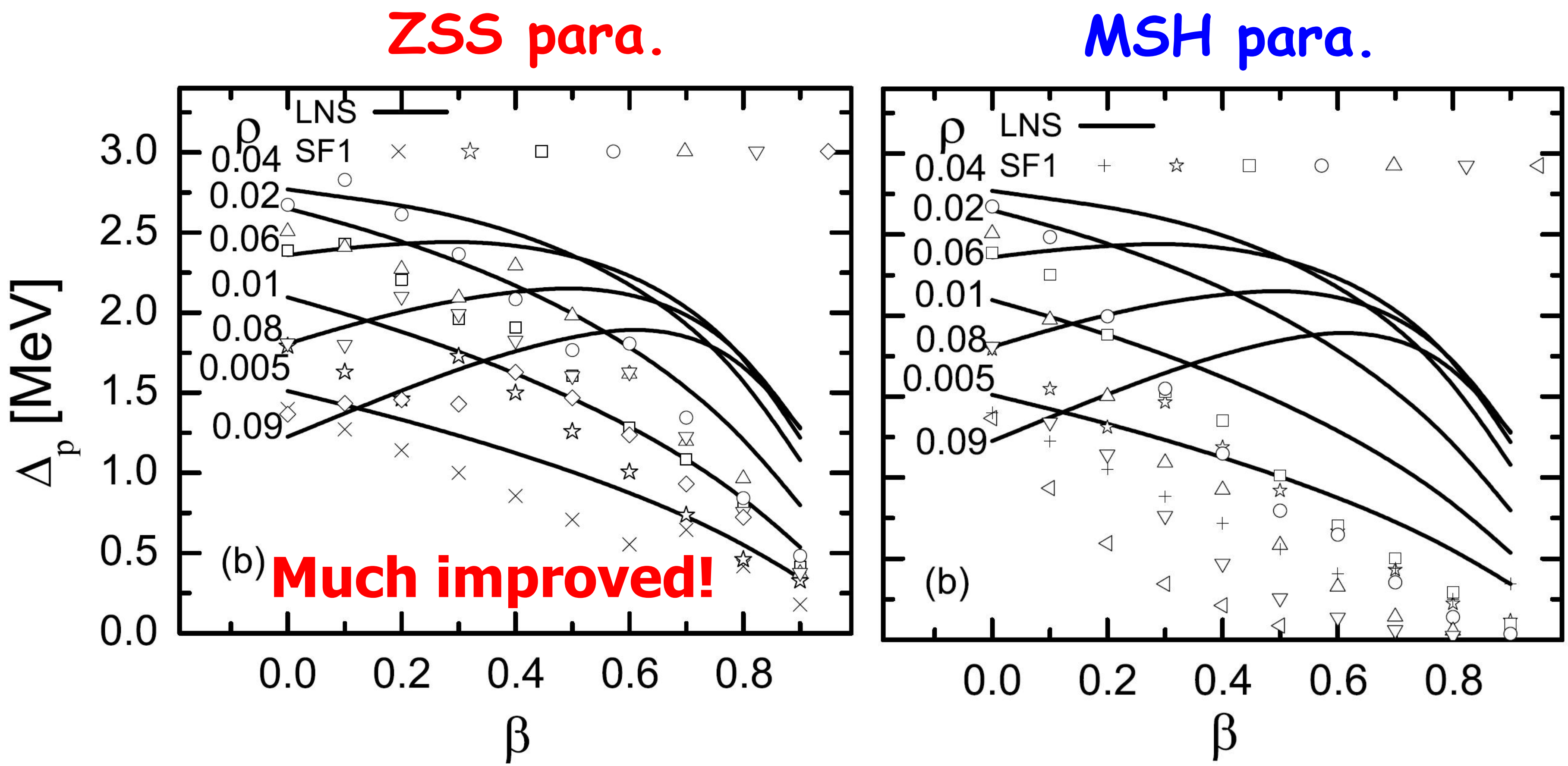
$$V(\rho,\beta)=\frac{1}{2}v_0\left[1-\eta_s\left(\frac{\rho}{\rho_0}\right)^{\alpha_s}(1-\beta)-\eta_n\left(\frac{\rho}{\rho_0}\right)^{\alpha_n}\beta\right]\delta(\vec{r}-\vec{r}\,')(1-P_\sigma)$$

SF1: $v_0=-542, \eta_s=0.729, \alpha_s=0.522, \eta_n=1.01, \alpha_n=0.525$ SCEG**54** (2011) 236

SF: $v_0=-542, \eta_s=0.664, \alpha_s=0.522, \eta_n=1.01, \alpha_n=0.525$ **J. Margueron, H. Sagawa and K. Hagino, PRC 77 (2008) 054309**

unpublished

Proton gap for zero-range effective pairing interaction



$$V(\rho,\beta)=\frac{1}{2}v_0\left[1-\eta_s\left(\frac{\rho}{\rho_0}\right)^{\alpha_s}(1+\beta)+\eta_n\left(\frac{\rho}{\rho_0}\right)^{\alpha_n}\beta\right]\delta(\vec{r}-\vec{r}\,')(1-P_\sigma)$$

SF1: $v_0=-542, \eta_s=0.729, \alpha_s=0.522, \eta_n=1.01, \alpha_n=0.525$ SCEG**54** (2011) 236

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J. Margueron, H. Sagawa and K. Hagino, PRC 77 (2008) 054309

On the basis of pure BCS approx.

- The isospin effect
  - from the isovector component of the nuclear mean field in ANM
- the momentum cutoff window
  - renormalized effective interaction
  - pairing gap
- check the interpolation formulism of the pairing gap (GCP)
  - from that of SNM and PNM
  - not to agree with microscopic results
- improve the isospin and density dependent zero range effective pairing interaction
  - right fitting with the same procedure of MSH

Medium polarization in asymmetric nuclear matter

- induced interaction
  - from symmetric nuclear matter to neutron matter
  - strong isospin dependece of density and spin-density
  - spin-singlet state: crossover appears
  - spin-triplet state: no crossoer; repulsive

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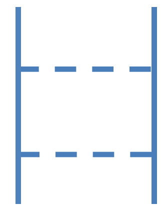
Effective Nuclear Interaction

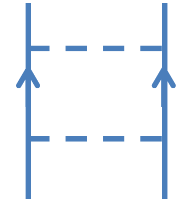
a) In - vacuum Scattering Amplitude

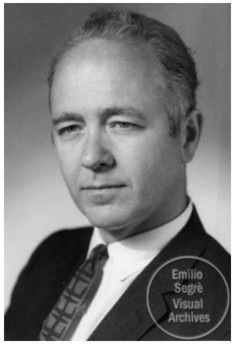
$$T = V + V \frac{1}{\omega - e_{12}} T$$



b) In - medium Scattering Amplitude in BHF

$$G = V + V \frac{Q_{12}}{\omega - e_{12}} G$$

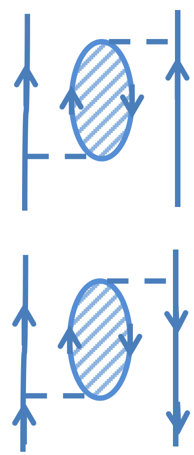


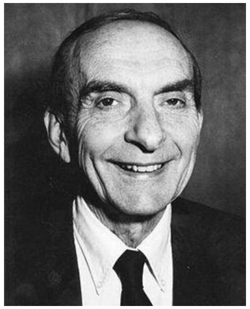


K.A. Brueckner

c) In - medium Scattering Amplitude with Induced Int.

$$\Gamma = G + G \Lambda G$$



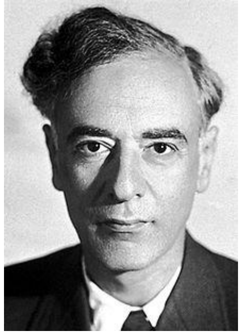


G.E.Brown

courtesy to U. Lombardo

LANDAU theory of quasipaticle

To the limit q→0 (Landau limit)
- the high momentum transitions of the scattering amplitude can be incorporated in an effective interaction Γ^{ω} such that



L. Landau

$$\Gamma(q) = \Gamma^{\omega} + \Gamma^{\omega}\ \Lambda_0(q)\,\Gamma(q) = \Gamma^{\omega} + \Gamma^{\omega}\ \Lambda(q)\,\Gamma^{\omega}$$

where Γ^{ω} is the q=0 limit, and $\Lambda(q)$ is the full polarization propagator.

Landau-Migdal parameters

- The effective interaction describing transitions around the Fermi surface (q=0)
- can be expressed in terms of a few phenomenological parameters

$$\Gamma^{\omega} = F + F'\tau_1\cdot\tau_2 + G\,\sigma_1\cdot\sigma_2 + G'\sigma_1\cdot\sigma_2\ \tau_1\cdot\tau_2$$

A. Migdal

$$F = F_{nn} + F_{np} \qquad F' = F_{nn} - F_{np}$$

$$G = G_{nn} + G_{np} \qquad G' = G_{nn} - G_{np}$$

$$E_{\mathrm{sym}} = \frac{1}{3}\varepsilon_F(1+F') \quad \text{symmetry energy}$$

$$K = 6\varepsilon_F(1+F) \quad \text{compressibility}$$

$$G \quad \rightarrow \quad \text{spin waves}$$

$$G' \quad \rightarrow \quad \text{Gamow - Teller R}$$

courtesy to U. Lombardo

Induced Interaction

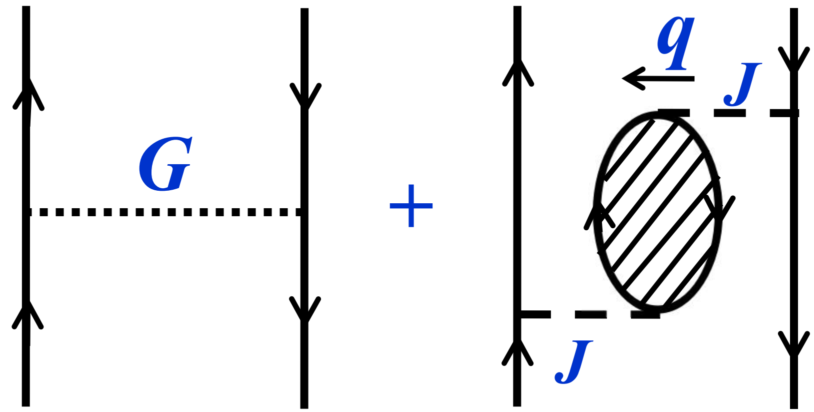
S. S. Zhang, L. G. Cao, U. Lombardo, P. Schuck

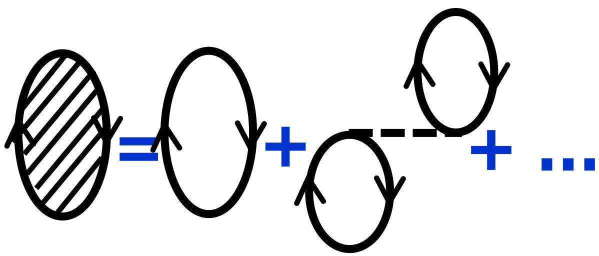
PRC 93, 044329 (2016)

$$\Gamma_{induced} = G + G\Lambda G$$

ph **interaction:**

$$J_{ph}(q) =$$





RPA

$$J^S(q) = G^S(q) + J_i^S(q)$$

$$J_i^{S=0}(q) = \frac{1}{2}\sum_{S'}(2S'+1)\,J^{S'}(q)\,\Lambda^{S'}(q)\,J^{S'}(q)$$

$$J_i^{S=1}(q) = \frac{1}{2}\sum_{S'}(-1)^{S'}J^{S'}(q)\,\Lambda^{S'}(q)J^{S'}(q)$$

the diagonal matrix elements

$$\Lambda^S{}_{\tau,\tau}(q) = \lambda_\tau(q)\frac{[1+\lambda_{\tau'}(q)J^S{}_{\tau',\tau'}(q)]}{D^S(q)}$$

the off-diagonal matrix elements

$$\Lambda^{S}_{\tau,\tau}(q) = -\frac{\lambda_{\tau}(q)\lambda_{\tau'}(q)J^{S}_{\tau,\tau'}(q)}{D^{S}(q)}$$

$$1/D^{S}(q) = (1+\lambda_{\tau}J^{S}_{\tau,\tau})(1+\lambda_{\tau'}J^{S}_{\tau',\tau'}) - \lambda_{\tau}\lambda_{\tau'}(J^{S}_{\tau,\tau'})^{2}$$

free polarization propagator

$$\lambda_{\tau}(q) = Z_{\tau}^{\ 2} N_{\tau} \lambda\left(\frac{k}{k_{\tau}^{\ F}}, \frac{\omega}{\varepsilon_{\tau}^{\ F}}\right) \qquad Z_{\tau}^{\ 2} << 1$$

***pp* coupling:**

$$\left(J_{i}^{{}^{1}S_{0}}\right)_{\tau,\tau} = \frac{1}{2}\left[\left(J_{i}\right)^{0}_{\tau,\tau} - 3\left(J_{i}\right)^{1}_{\tau,\tau}\right]$$

$$\left(J_{i}^{{}^{3}PF_{2}}\right)_{\tau,\tau} = \frac{1}{2}\left[\left(J_{i}\right)^{0}_{\tau,\tau} + \left(J_{i}\right)^{1}_{\tau,\tau}\right]$$

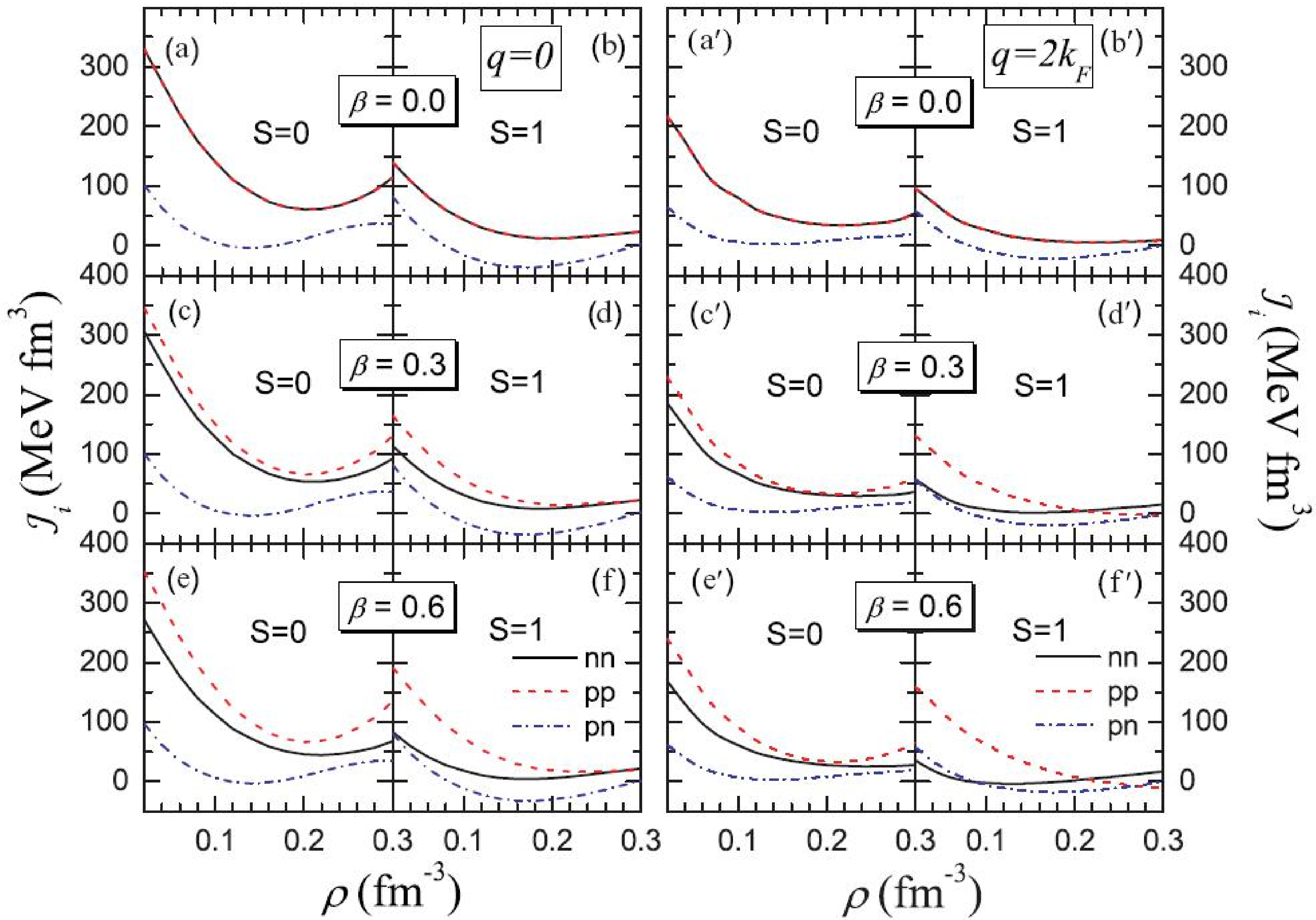
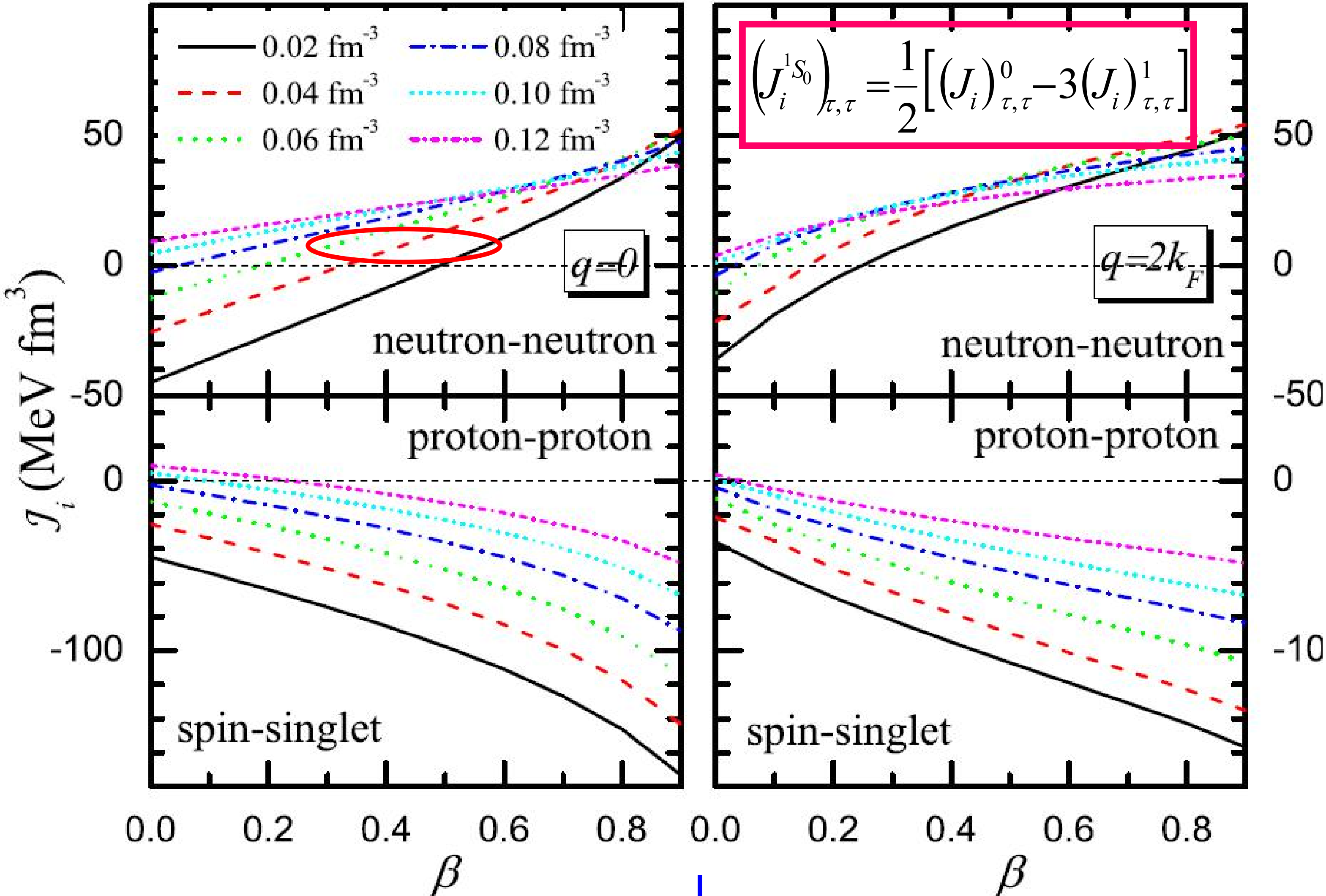


FIG. 2. Particle-hole induced interaction in nuclear matter vs density for three values of β at $q = 0$ (left) and $q = 2k_F$ (right).

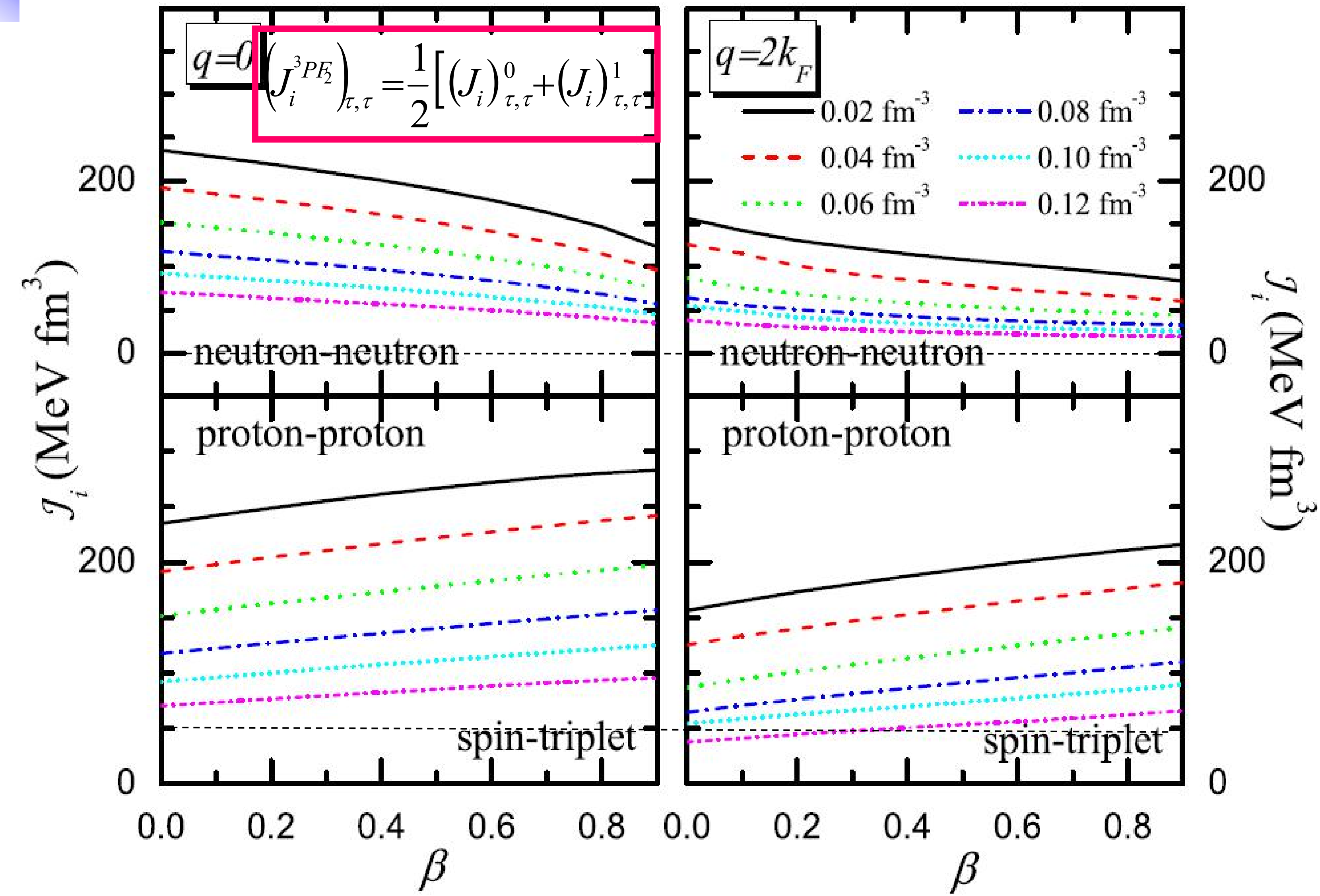


low-density *n-n* $^1S_0$ *pairing* （NS crust）

- ✓ **attractive(SNM); repulsive (PNM)**
- ✓ **gap for neutron matter : reduced by the induced interaction**
- ✓ **crossover: dominance of pure density modes over spin density modes**

low-density *p-p* $^1S_0$ *pairing* (NS inner core)

- ✓ **embedded into high-density neutron matter**
- ✓ **neutron core induces a strong attractive enhancement on the *p-p Cooper pairs***
- ✓ **compete with the self-energy suppression**



high density *n-n* $^3PF_2$ *pairing* (NS inner core)

- ✓ **medium polarization: strong; repulsive;**
- ✓ **suppress completely the high density *n-n pairing***

Outlook

- calculate pairing gap with induced interaction for ANM
- check new effective pairing interaction in finite nuclei
- understand phonon vibration from microscopic point of view

Collaborators

Pairing work

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Structure in Exotic Nuclei

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Stellar evolution and reactions

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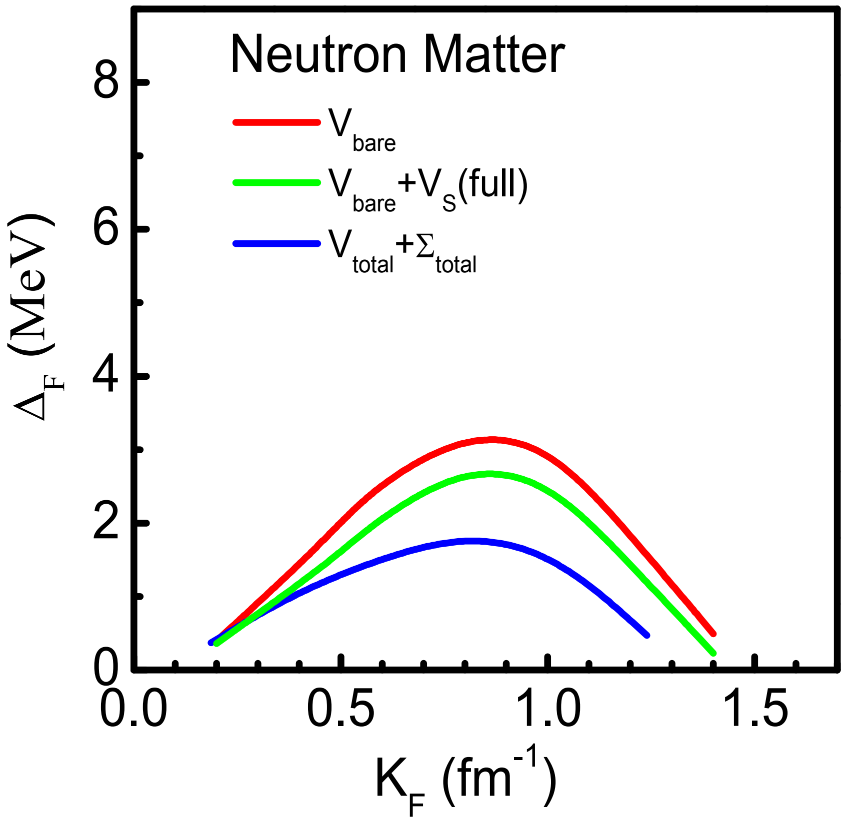
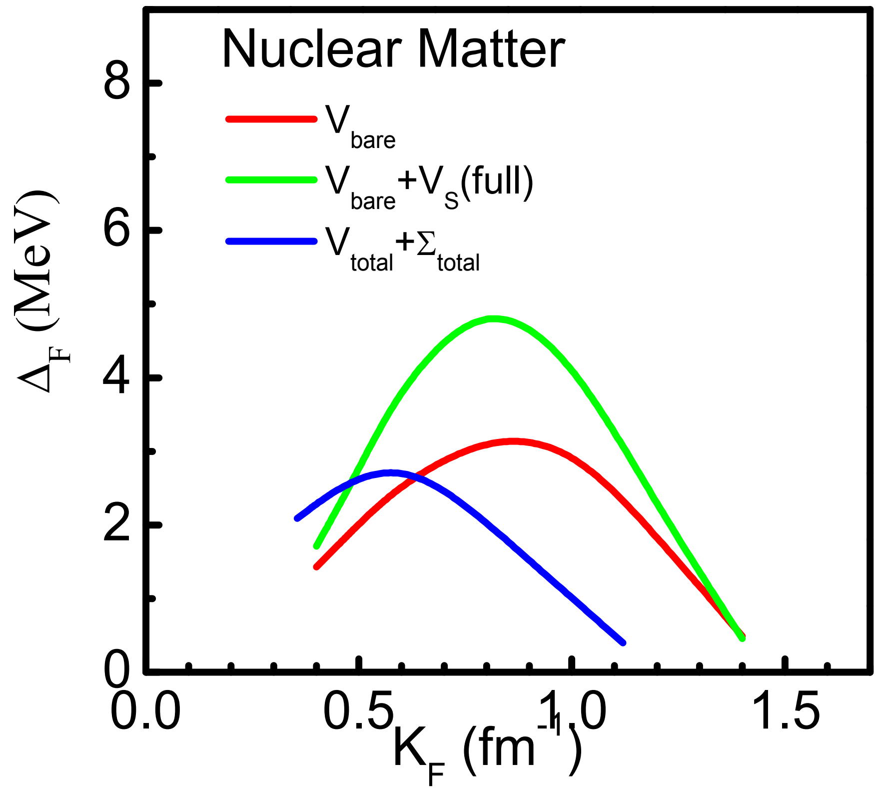
- G. Arbanas, ORNL, USA

Thanks !

$^1S_0$ pairing gaps in neutron and nuclear matter





- **In neutron matter,**

screening reinforces the pairing suppression

- due to the dominance of spin density modes over pure density modes.

- **In nuclear matter,**

- the tensor force enhances pairing (antiscreening)

Self-energy effects renormalize the pairing interaction by a Z^2 damping factor, related to the depletion of the Fermi surface.