Quasiparticle RPA calculation with Skyrme energy density functional for rotating unstable nuclei

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in collaboration with

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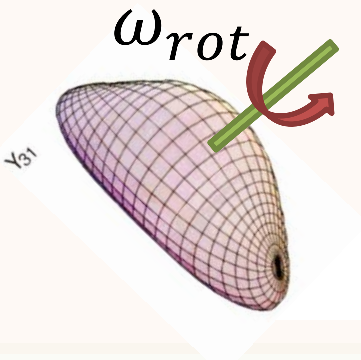
Introduction

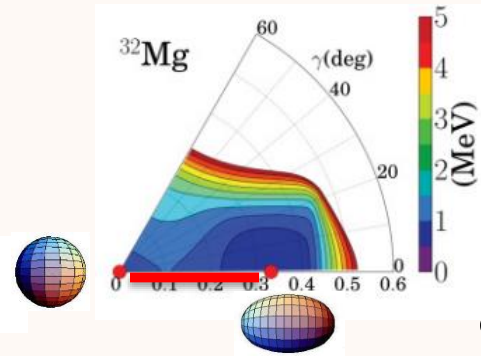
Purpose: Cranked Quasiparticle RPA with Skyrme-EDF

- Elementary excitations from various nuclear states
  (**Triaxial deformation** & **Pairing correlations** & **Rotational effect**)
- For more advanced microscopic collective models



$E_{vib} \searrow 0$

Soft mode (Y_{31})



Large-amp. motions

(Figure: Hinohara et al.)

Method: Fourier-series expansion method (k-space rep.)

Application 1: Cranked RPA with Skyrme-EDF

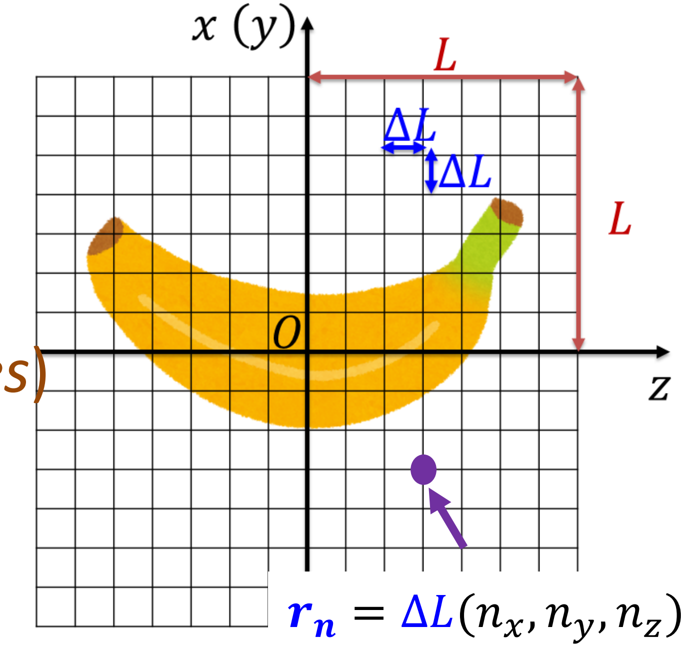
Octupole vibrations of *rotating* Superdeformed state in ^{40}Ca

Application 2: Cranked Quasiparticle RPA with Skyrme-EDF

Quadrupole-pairing vibrations of *rotating* neutron-rich Mg

Real-space representation ($\boldsymbol{r}$-space rep.)

$$\varphi(\boldsymbol{r}_{\boldsymbol{n}}), \quad \boldsymbol{r}_{\boldsymbol{n}} = \Delta L(n_x, n_y, n_z)$$
$$n_x, n_y, n_z = 0, \pm 1, , \cdots, \pm N_{max}$$

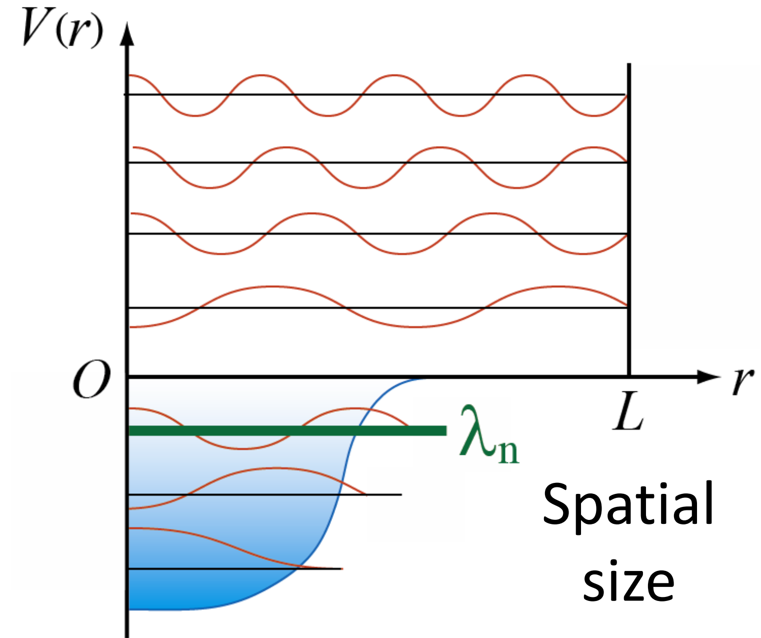


Advantage

- Unstable nuclei (*Weakly, unbound sp states*)
- Exotic shapes (*e.g., octupole shape)*
- Easy coding

Disadvantage

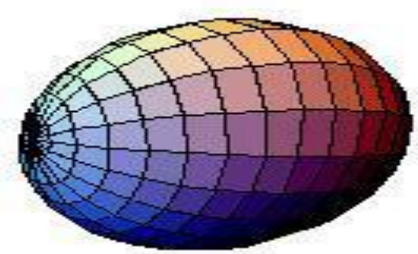
- Large computational effort (cf. Harmonic oscillator basis)



Hard work for performing beyond MF calculations such as QRPA for rotating nuclei and the beyond

Fourier-series expansion method ($\boldsymbol{k}$-space rep.)

$$\varphi(\boldsymbol{r}) = \sum_{\boldsymbol{n}} \hat{\varphi}_{\boldsymbol{k_n}} f_{n_x}^{(\pi_x)}(x) f_{n_y}^{(\pi_y)}(y) f_{n_z}^{(\pi_z)}(z)$$

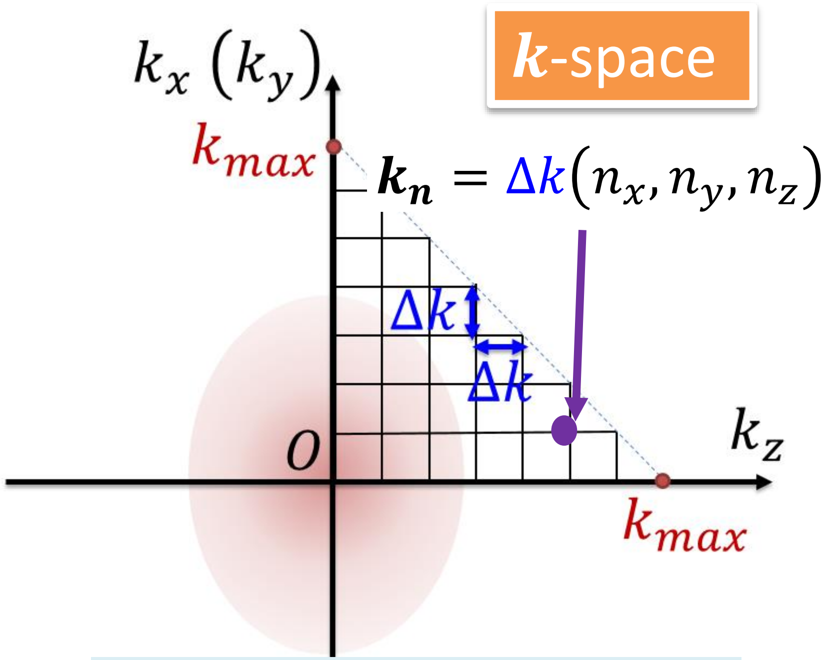


Triaxial shape

$$\boldsymbol{k_n} = \Delta k \boldsymbol{n} = \Delta k (n_x, n_y, n_z)$$

$$f_n^{(+)}(x) = \frac{1}{\sqrt{(1+\delta_{n,0})L}} \cos k_n x$$

$$f_n^{(-)}(x) = \frac{1}{\sqrt{L}} \sin k_n x$$



$$0 \le k_x, k_y, k_z \le k_{max}$$
$$k_x + k_y + k_z \le k_{max}$$

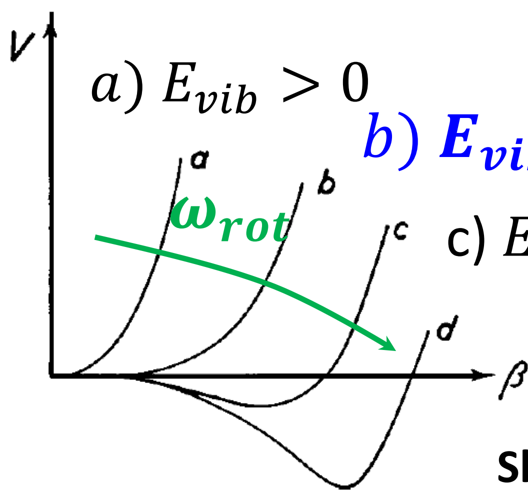
Application 1:

Rotational effect on octupole vibrations of Superdeformed states in ^{40}Ca

Soft banana mode

$$E_{vib} \searrow 0$$
$$\left(\omega_{rot} \nearrow \omega_{rot}^{(critical)}\right)$$

Precursor of *Static* banana shape



a) $E_{vib} > 0$

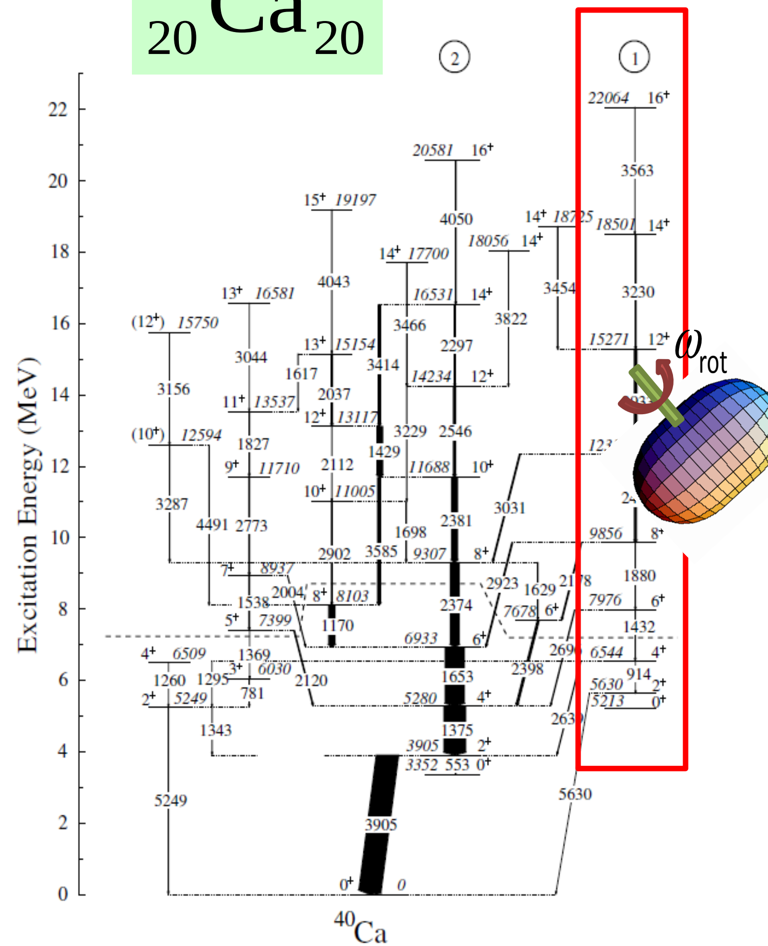
b) $E_{vib} \approx 0$ (Soft mode)

c) $E_{vib} = imaginary$ $(Static\ shape)$

Shape transition

$^{40}_{20}\mathrm{Ca}_{20}$

SD band

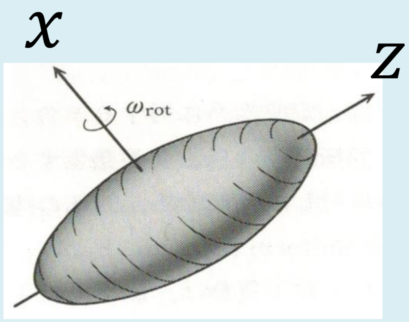


E. Ideguchi *et al.*, PRL 87, 222501 (2001)

Cranked Skyrme-RPA calculation

Microscopic description of coupling between *rotation* and *vibration*

Cranked mean-field calculation



$$h' = h_{Skyrme} - \omega_{rot} j_x$$

- Skyrme-EDF (SLy4)
- Triaxial shape & NO time-reversal symmetry
- Fourier-series expansion method ($\boldsymbol{k}$–space)

RPA matrix equation

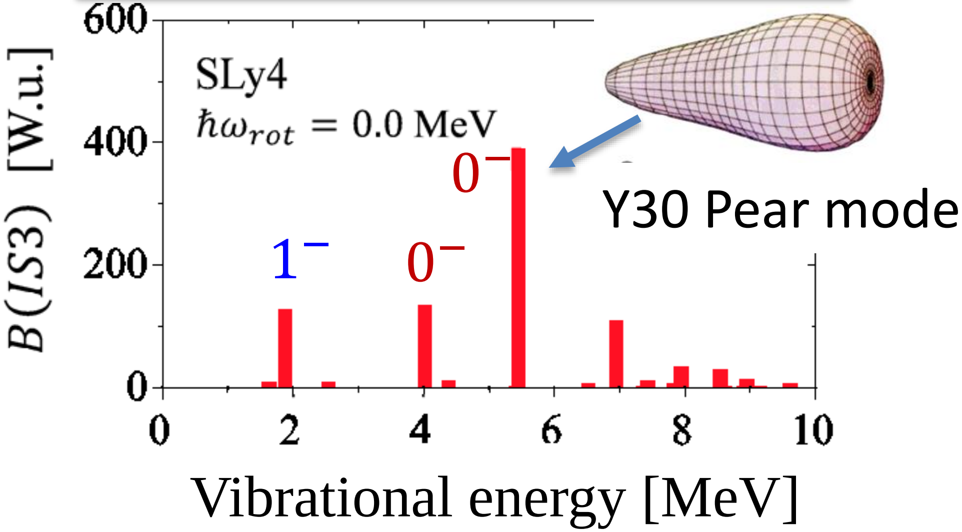
$$\sum_{n,j} \begin{pmatrix} A_{minj} & B_{minj} \\ -B^*_{minj} & -A^*_{minj} \end{pmatrix} \begin{pmatrix} X^{(\lambda)}_{nj} \\ Y^{(\lambda)}_{nj} \end{pmatrix} = E_\lambda \begin{pmatrix} X^{(\lambda)}_{mi} \\ Y^{(\lambda)}_{mi} \end{pmatrix}$$

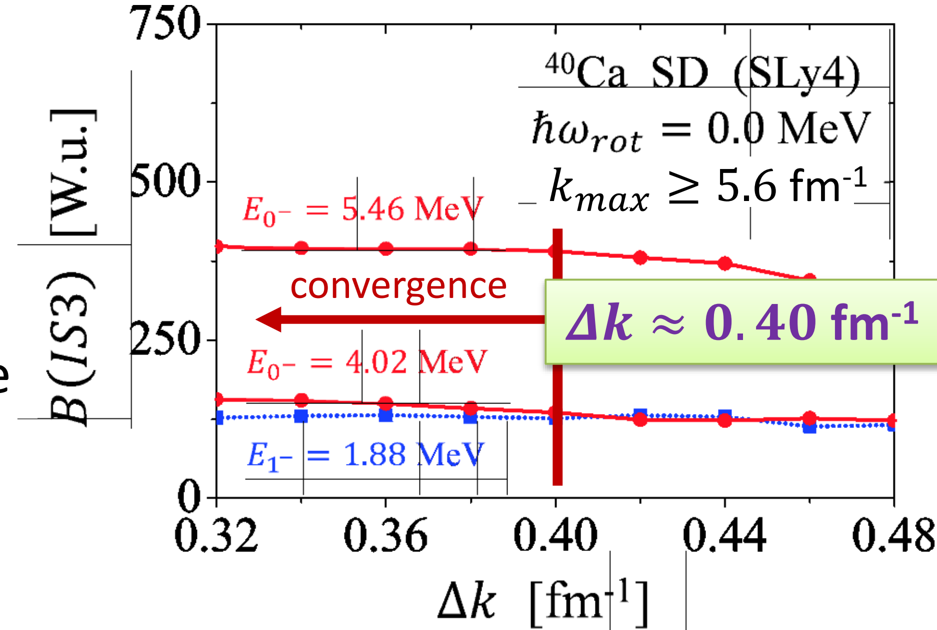
- Residual interaction

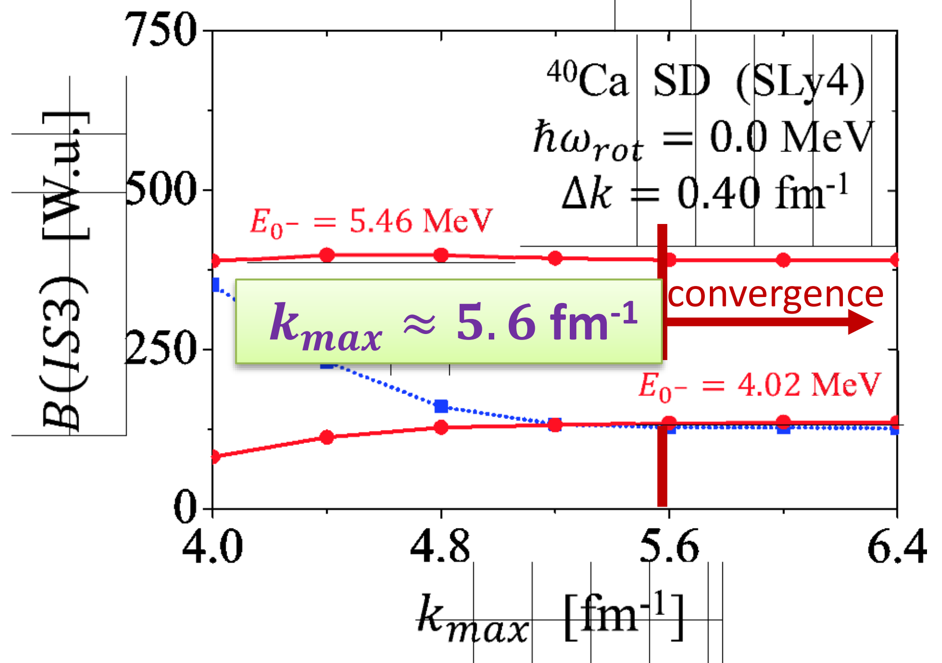
  Landu-Migdal approximation of Skyrme interaction
- Particle-hole Energy cutoff: $\varepsilon_m - \varepsilon_i < 40$ MeV

Convergence check

Band head of SD in ^{40}Ca







For reasonable results

$\Delta k \approx 0.40$ fm^{-1}

$k_{max} \approx 5.6$ fm^{-1}

$\Downarrow$

$N^{(k)}_{grid} \approx$ **560 grid points**

Comparison: $\boldsymbol{k}$-space vs $\boldsymbol{r}$-space rep.

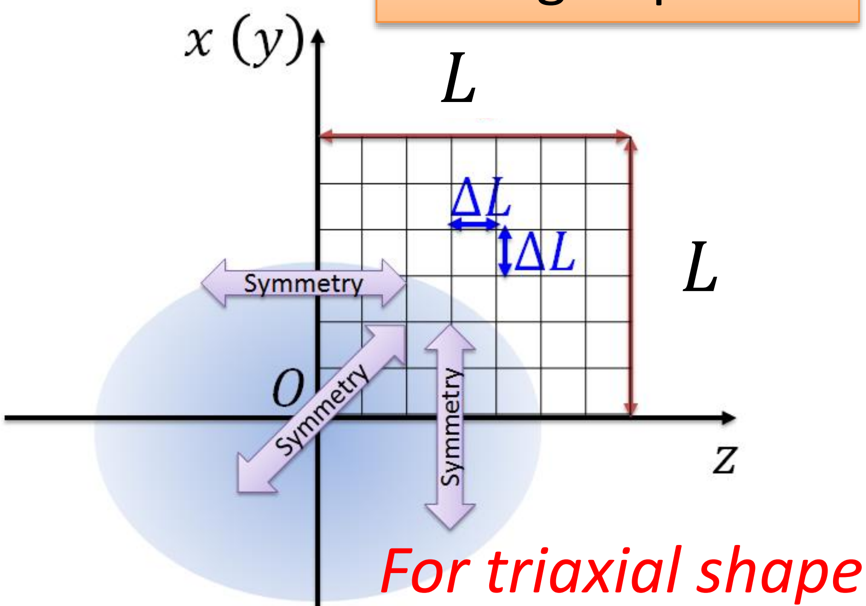
Model space for convergence of RPA calculation

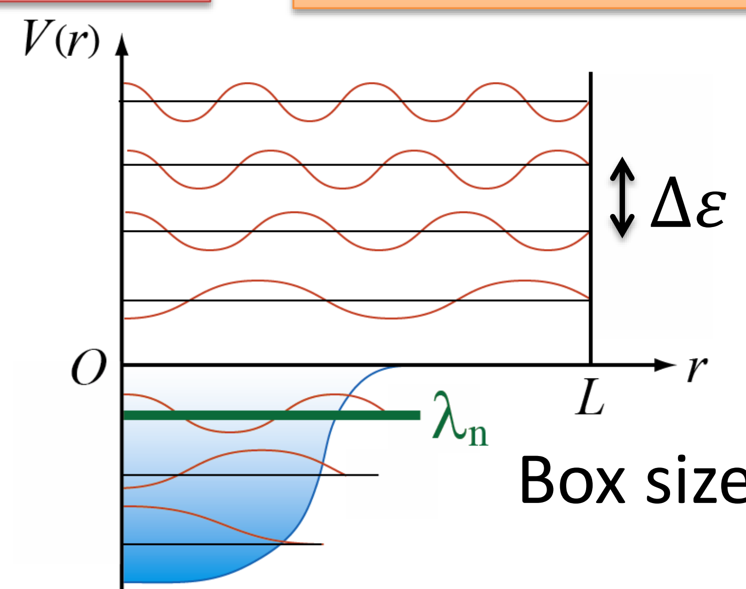
(Octupole excitations of SD in ^{40}Ca (Triaxial deformation, $\omega_{rot} = 0$)

	Grid points N	Box size L	$\Delta k (= \pi/L)$	$(\Delta k)^2 \propto \Delta\varepsilon$
k-space rep.	560	-	0.40 fm^{-1}	0.160 fm^{-2}
r-space rep.*	5625	14.7 fm	0.21 fm^{-1}	0.044 fm^{-2}

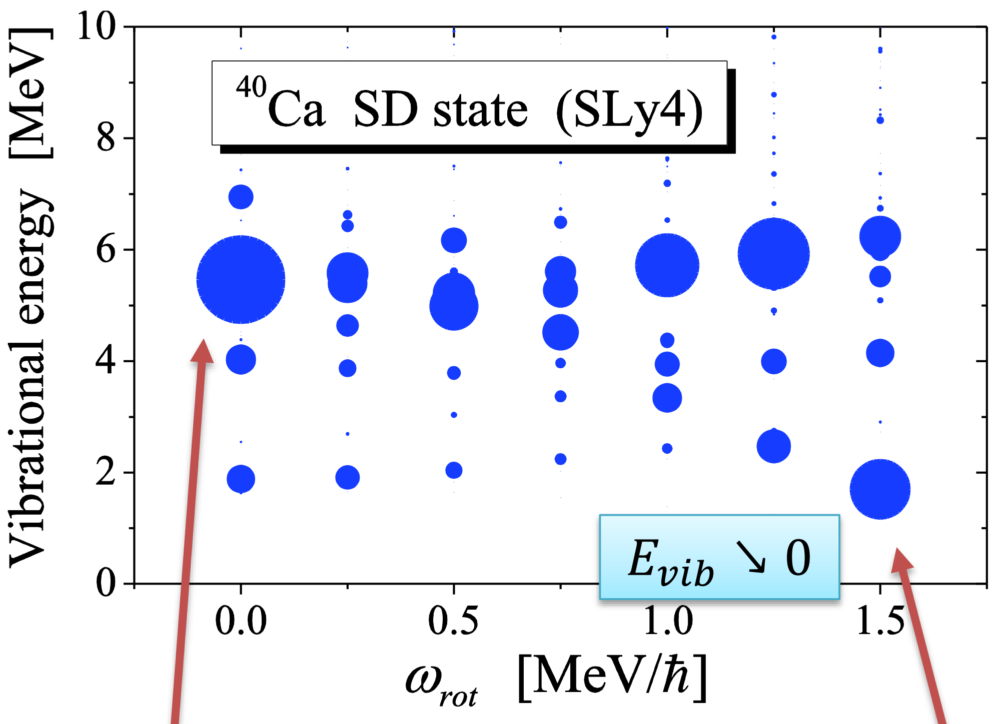
* T.Inakura *et al.*, NPA768 (2006)

10 times smaller # of grid points → **40 times smaller** ← **4 times smaller # of p-h configurations**

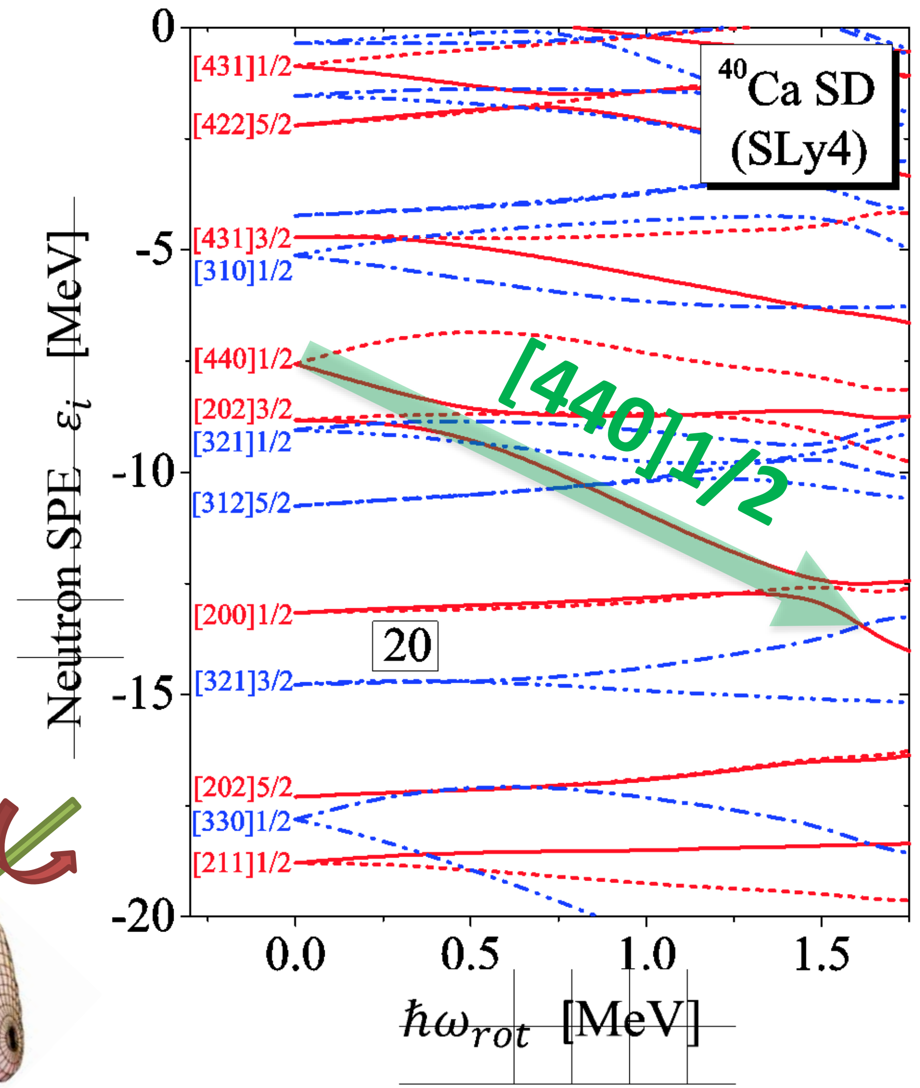


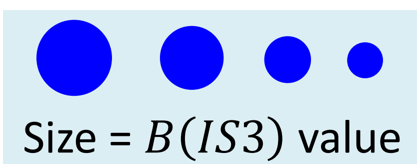


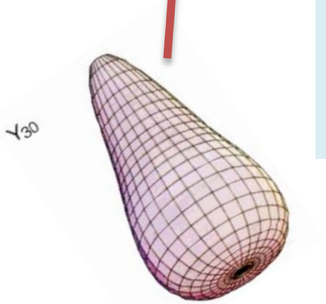
Soft banana mode at $\hbar\omega_{rot} \approx 1.5$ MeV

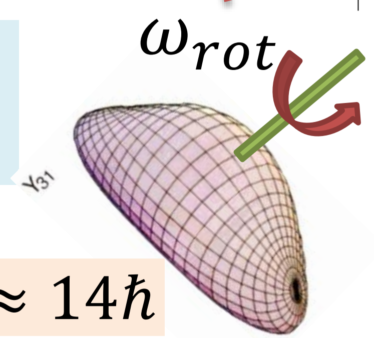


$E_{vib} \searrow 0$









$J_x \approx 14\hbar$

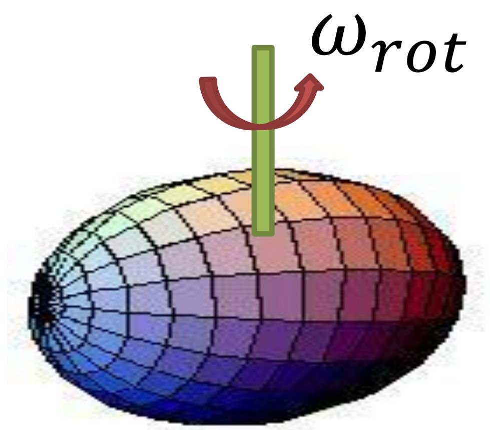
Pear mode
(Y_{30}: 98%)

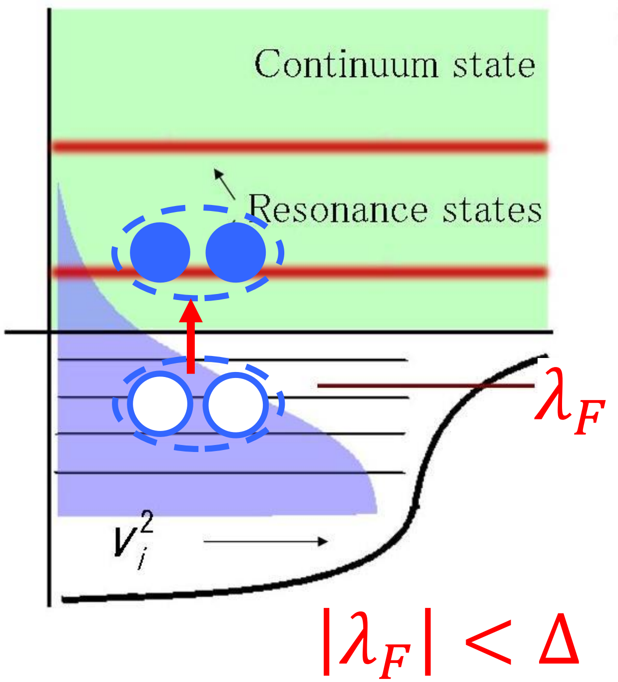
Soft Banana mode
(Y_{31}: 78%)

Application 2:
Quasiparticle RPA calculation for rotating unstable nuclei

Key points for describing pairing correlations

1. Breaking of time-reversal symmetry
2. Coupling to continuum states



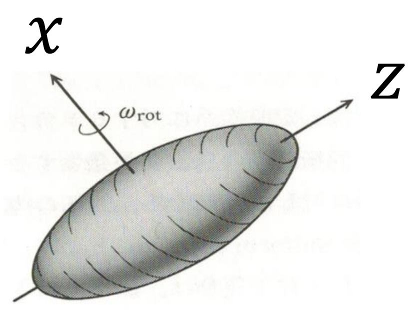


Hartree-Fock-Bogoliubov theory for *rotating* *triaxial* nuclei

Basic idea ⇒ A. Goodman, Nucl. Phys. A230, 466 (1974)

1) Signature symmetry: quasiparticle pair $\left(\alpha_k^\dagger, \alpha_{\bar{k}}^\dagger\right)$ with $x-$sig. $(\pm i, \mp i)$

2) HFB equation $\left[\text{HF base}: C_k^{(HF)\dagger}, C_{\bar{k}}^{(HF)\dagger}\right]$



$$\begin{pmatrix} h-\omega_{rot}j_x & \Delta \\ -\Delta^* & -h^*+\omega_{rot}j_x^* \end{pmatrix}\begin{pmatrix} U_k \\ V_k \end{pmatrix} = E_k \begin{pmatrix} U_k \\ V_k \end{pmatrix}$$

3) Bloch-Messiah theorem

Canonical base

$$k: \ C_k^{(Can)\dagger} = \sum_l D_{kl} C_l^{(HF)\dagger} \Rightarrow \rho_{kk'} = \left\langle \Phi \middle| C_{k'}^{(Can)\dagger} C_k^{(Can)} \middle| \Phi \right\rangle = (v_k)^2 \delta_{kk'}$$

$$\bar{k}: \ C_{\bar{k}}^{(Can)\dagger} = \sum_{\bar{l}} E_{\bar{k}\bar{l}} C_{\bar{l}}^{(HF)\dagger} \Rightarrow \rho_{\bar{k}\bar{k}'} = \left\langle \Phi \middle| C_{\bar{k}'}^{(Can)\dagger} C_{\bar{k}}^{(Can)} \middle| \Phi \right\rangle = (v_k)^2 \delta_{kk'}$$

BCS-form wave function

$$|\Phi\rangle = \prod_k \left(u_k + v_k C_k^{(Can)\dagger} C_{\bar{k}}^{(Can)\dagger}\right)|0\rangle$$

⇒ **Canonical-basis QRPA**

QRPA in canonical-basis (BCS-form quasiparticle WF)

QRPA equation

$$\sum_{ll'}\begin{pmatrix} A_{kk'll'} & B_{kk'll'} \\ -B^*_{kk'll'} & -A^*_{kk'll'} \end{pmatrix}\begin{pmatrix} f^{(\lambda)}_{ll'} \\ g^{(\lambda)}_{ll'} \end{pmatrix} = E_\lambda \begin{pmatrix} f^{(\lambda)}_{kk'} \\ g^{(\lambda)}_{kk'} \end{pmatrix}$$

Symmetry of QP WFs

1) Parity symmetry

2) Signature symmetry $\left(\alpha^\dagger_k, \alpha^\dagger_{\bar{k}}\right)$

$$\begin{aligned} A_{kk'll'} &= (E_k + E_{k'})\delta_{kl}\delta_{k'l'} \\ &+ u_k u_{k'} u_l u_{l'} \langle kk' | V_{pair} | ll' \rangle + v_k v_{k'} v_l v_{l'} \langle \bar{l}\,\bar{l}' | V_{pair} | \bar{k}\bar{k}' \rangle \\ &+ u_k v_{k'} u_l v_{l'} \langle k\,\bar{l}' | V_{ph} | \bar{k}' l \rangle + v_k u_{k'} v_l u_{l'} \langle k' \bar{l} | V_{ph} | \bar{k} l' \rangle \\ &- u_k v_{k'} v_l u_{l'} \langle k\bar{l} | V_{ph} | \bar{k}' l' \rangle - v_k u_{k'} u_l v_{l'} \langle k'\,\bar{l}' | V_{ph} | \bar{k} l \rangle \end{aligned}$$

$$\begin{aligned} B_{kk'll'} &= -u_k u_{k'} v_l v_{l'} \langle kk' | V_{pair} | \bar{l}\,\bar{l}' \rangle - v_k v_{k'} u_l u_{l'} \langle ll' | V_{pair} | \bar{k}\bar{k}' \rangle \\ &+ u_k v_{k'} u_l v_{l'} \langle kl | V_{ph} | \bar{k}'\,\bar{l}' \rangle + v_k u_{k'} v_l u_{l'} \langle k'l' | V_{ph} | \bar{k}\bar{l} \rangle \\ &- u_k v_{k'} v_l u_{l'} \langle kl' | V_{ph} | \bar{k}'\bar{l} \rangle - v_k u_{k'} u_l v_{l'} \langle k'l | V_{ph} | \bar{k}\,\bar{l}' \rangle \end{aligned}$$

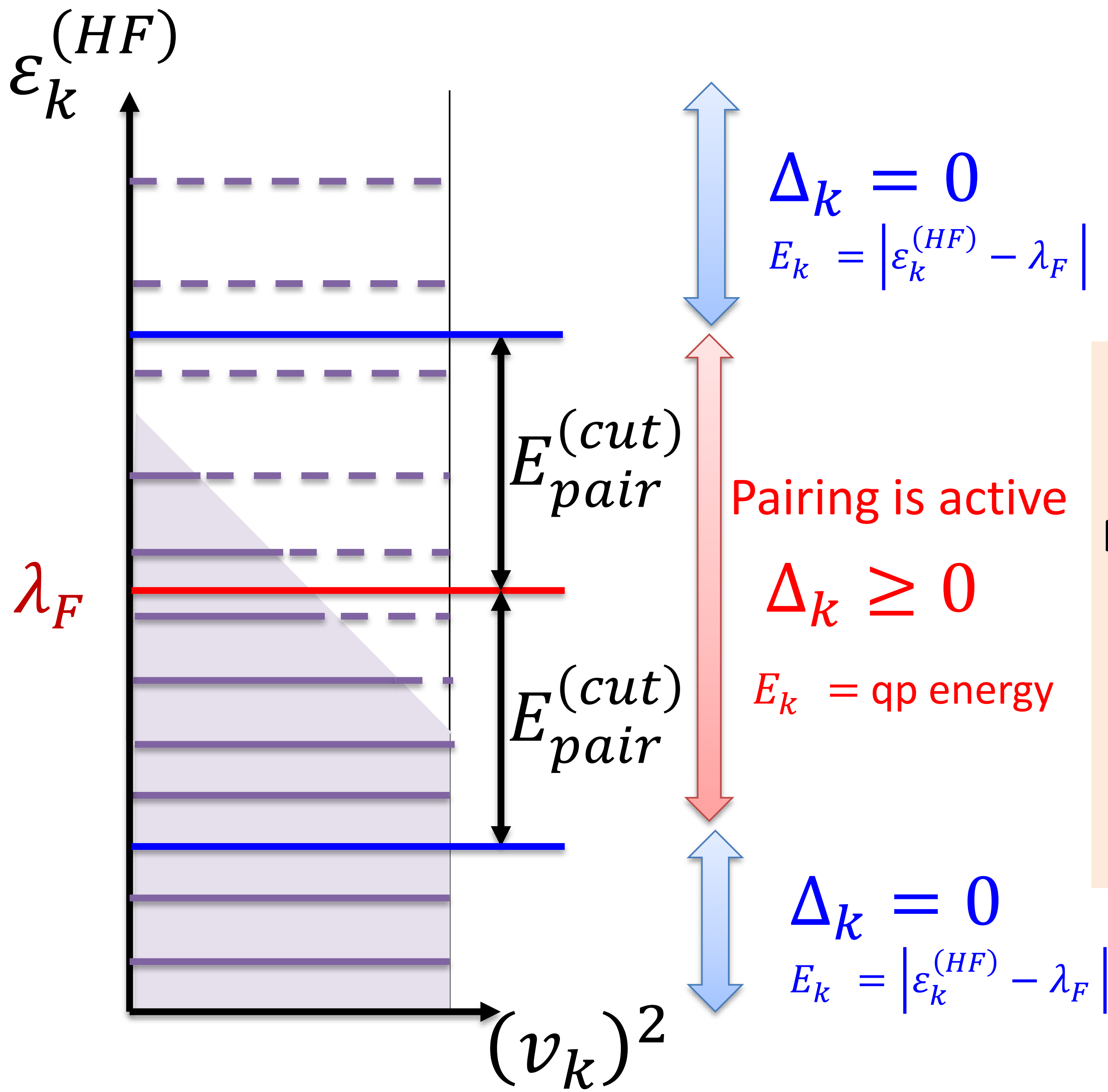
Cranked HFB calculation

- ■ Skyrme SkM* & Density-dep. δ pairing force (Mixed-type)
- ■ Triaxial deformation & No time-reversal symmetry

Residual interaction

- ■ Particle-hole ch. V_{ph} : Landau-Migdal approximation of Skyrme force
- ■ Pairing ch. V_{pair} : Density-dep. δ pairing force (Mixed-type)

Model space for HFB+QRPA calc.



Two-quasiparticle states

$E_k + E_{k'}$<35 MeV

Pairing active space

$E_{pair}^{(cut)}$=10 MeV

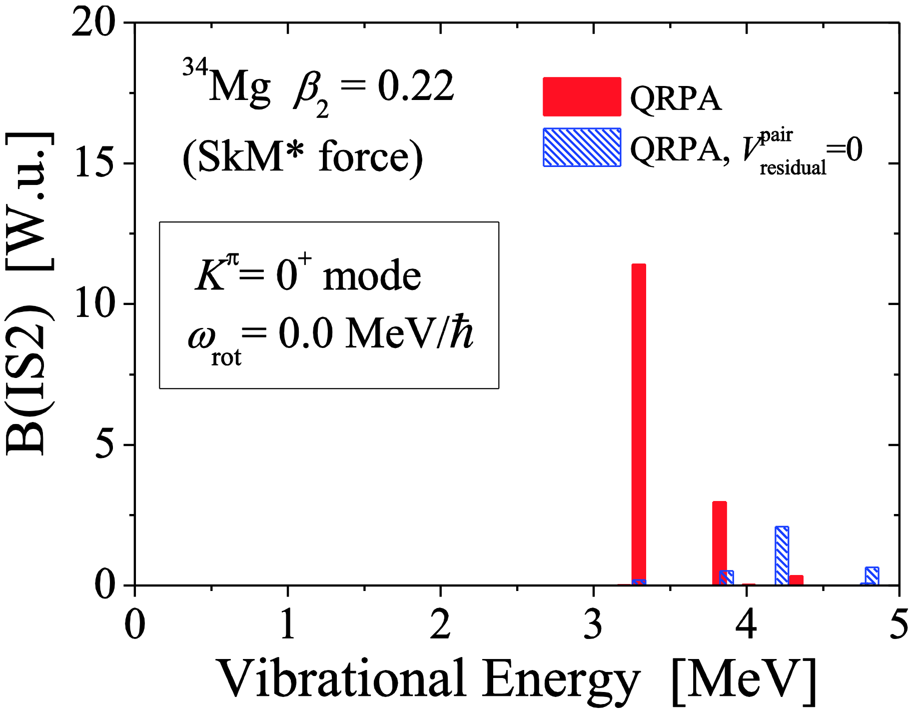
with strength

V_{pair}=−700 MeV fm^{-3}

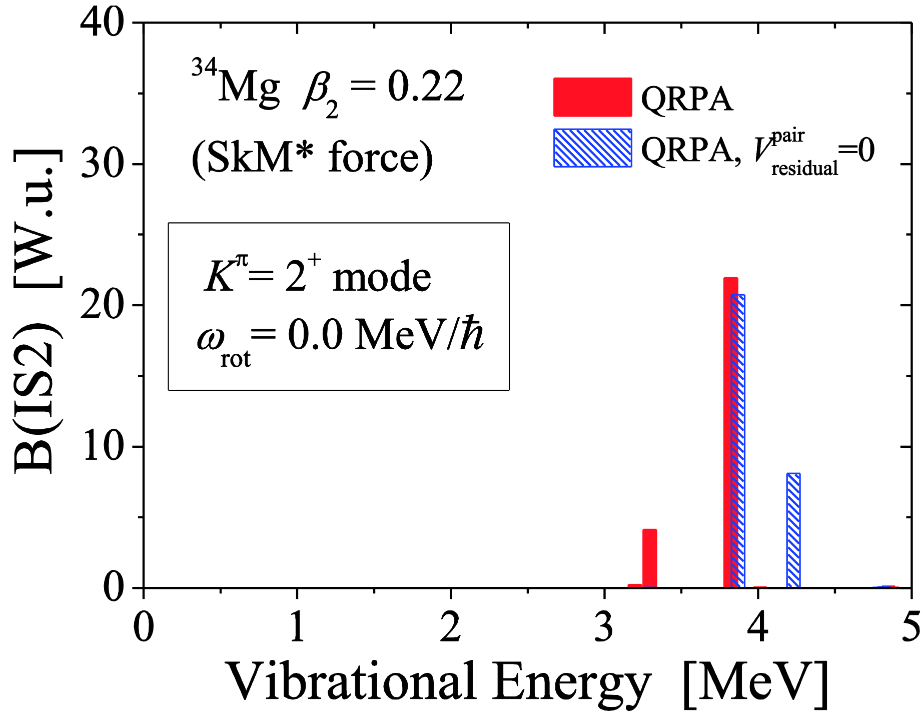
$(\Delta_n \approx 12/\sqrt{A}$ MeV$)$

Quadrupole excitations at $\omega_{rot} = 0$ in ^{34}Mg

$K^{\pi} = 0^{+}$ mode (β-vib.)



$K^{\pi} = 2^{+}$ mode (γ-vib.)



Pairing fluctuation is essential!

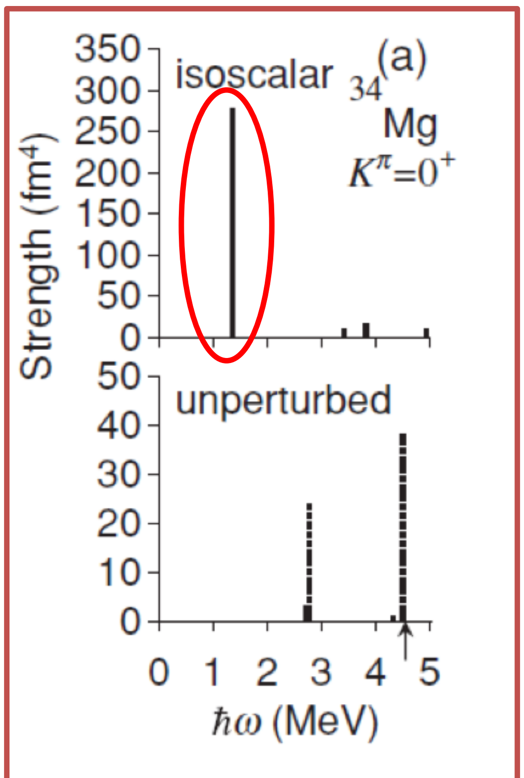
p-h fluctuation

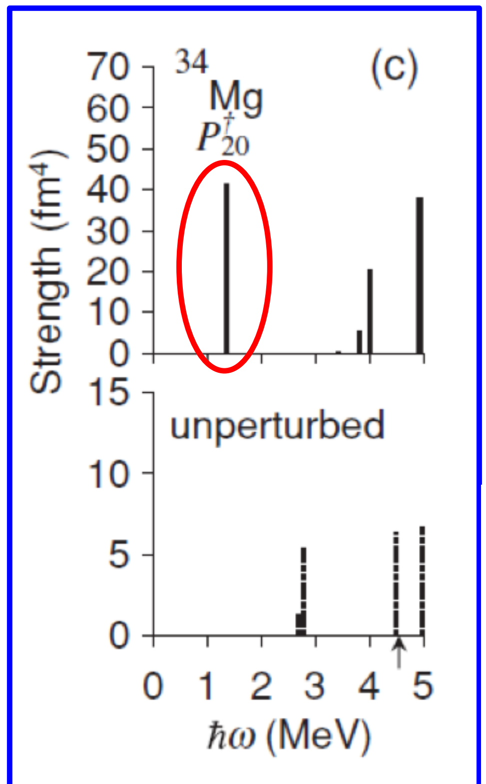
$K^{\pi} = 0^{+}$ mode in *deformed nuclei*

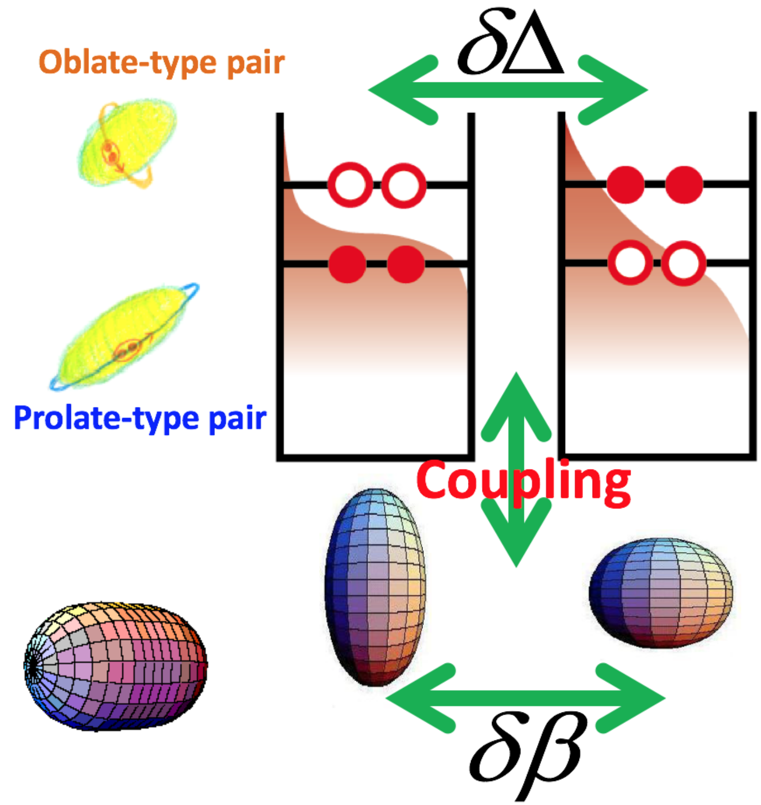
QRPA with axially-symmetric WS pot. (K.Yoshida, M.Y., PRC77, 044312 (2008))

$K^{\pi} = 0^{+}$ mode ⟺ Quadrupole pairing vibration

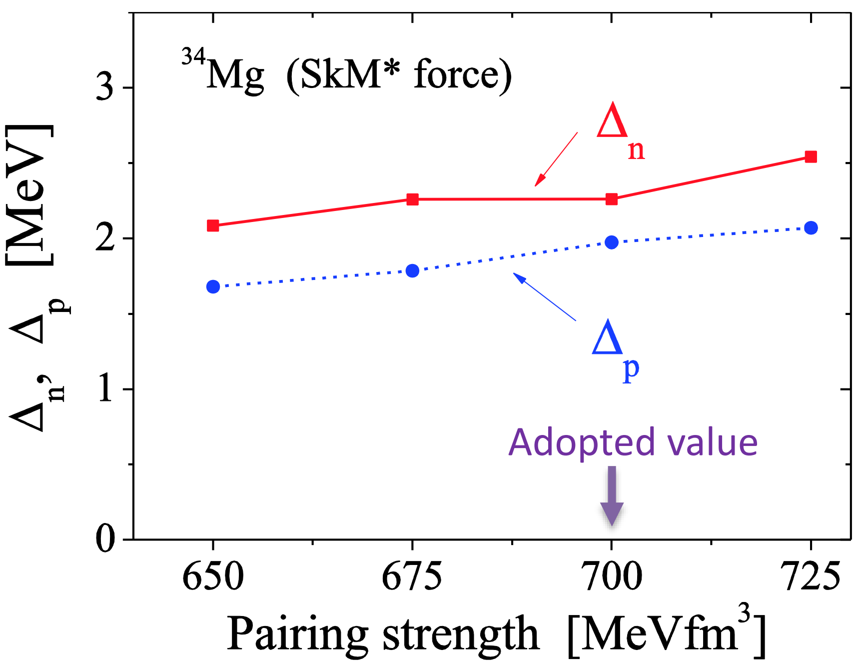
$r^2 Y_{20}$ **Coupling !** $P_{20}^{\dagger} = \int d\vec{r}\, r^2 Y_{20}(\hat{r}) \psi^{\dagger}(\vec{r},\uparrow) \psi^{\dagger}(\vec{r},\downarrow)$

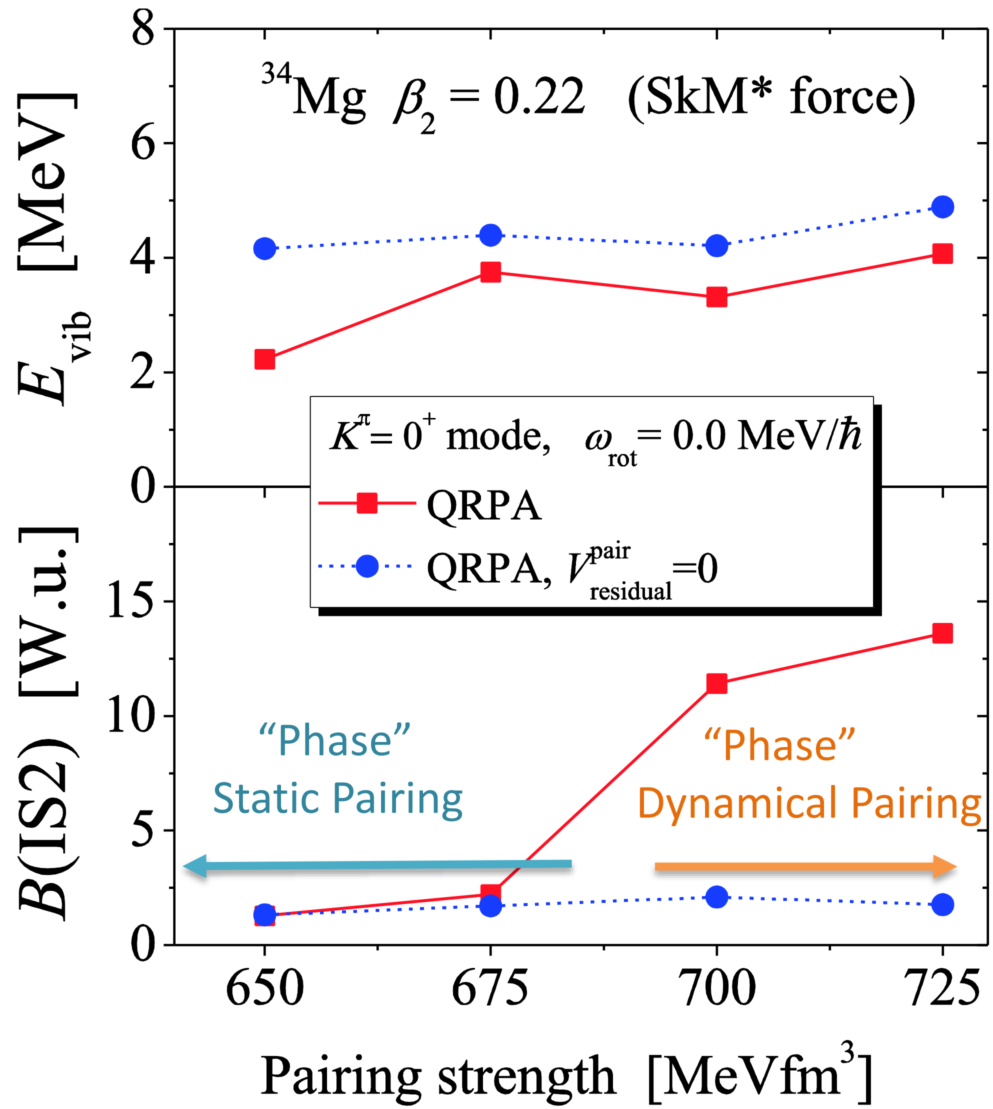




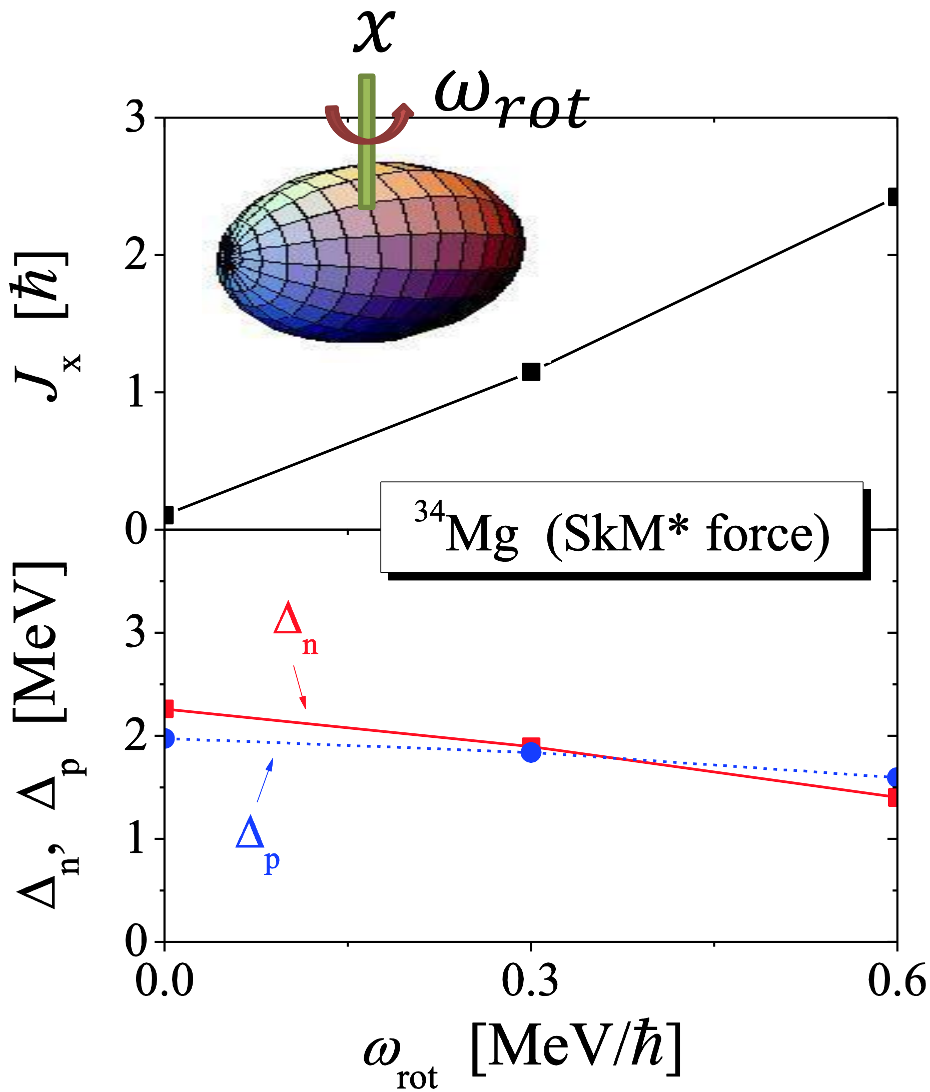


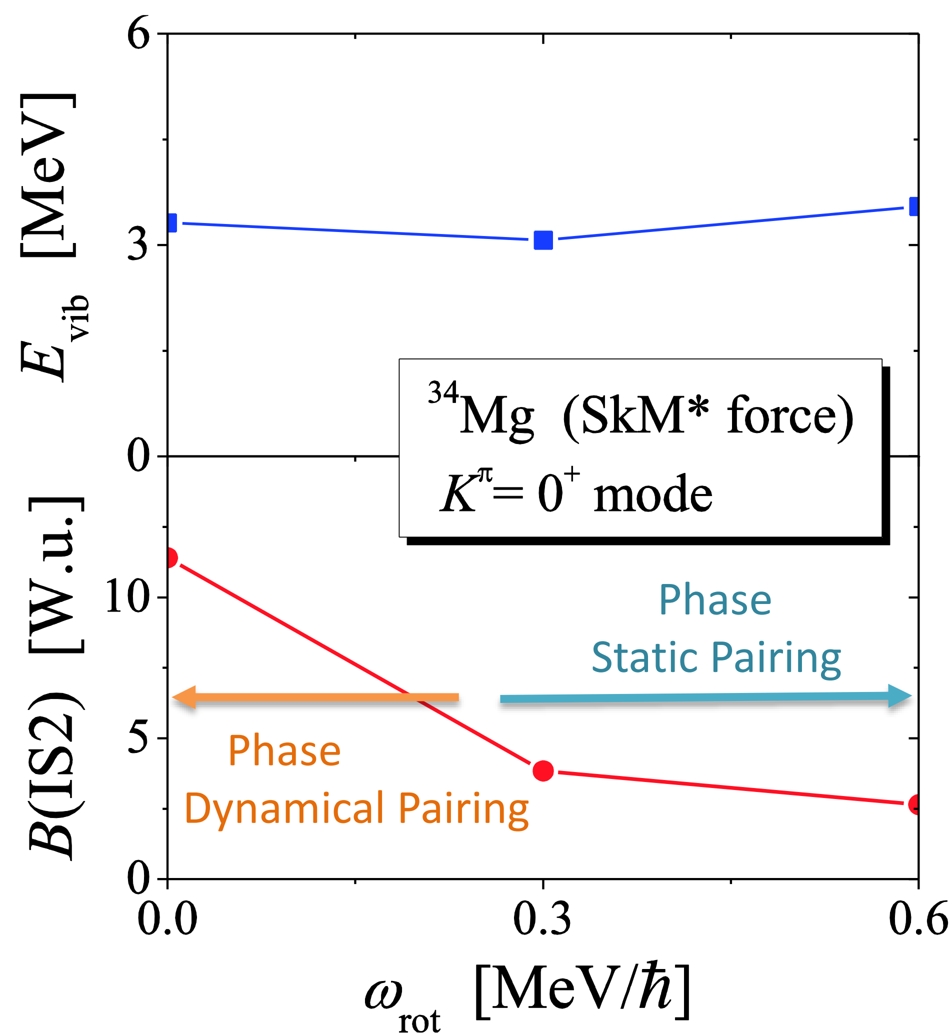
“Phase transition”: Sensitivity to pairing strength





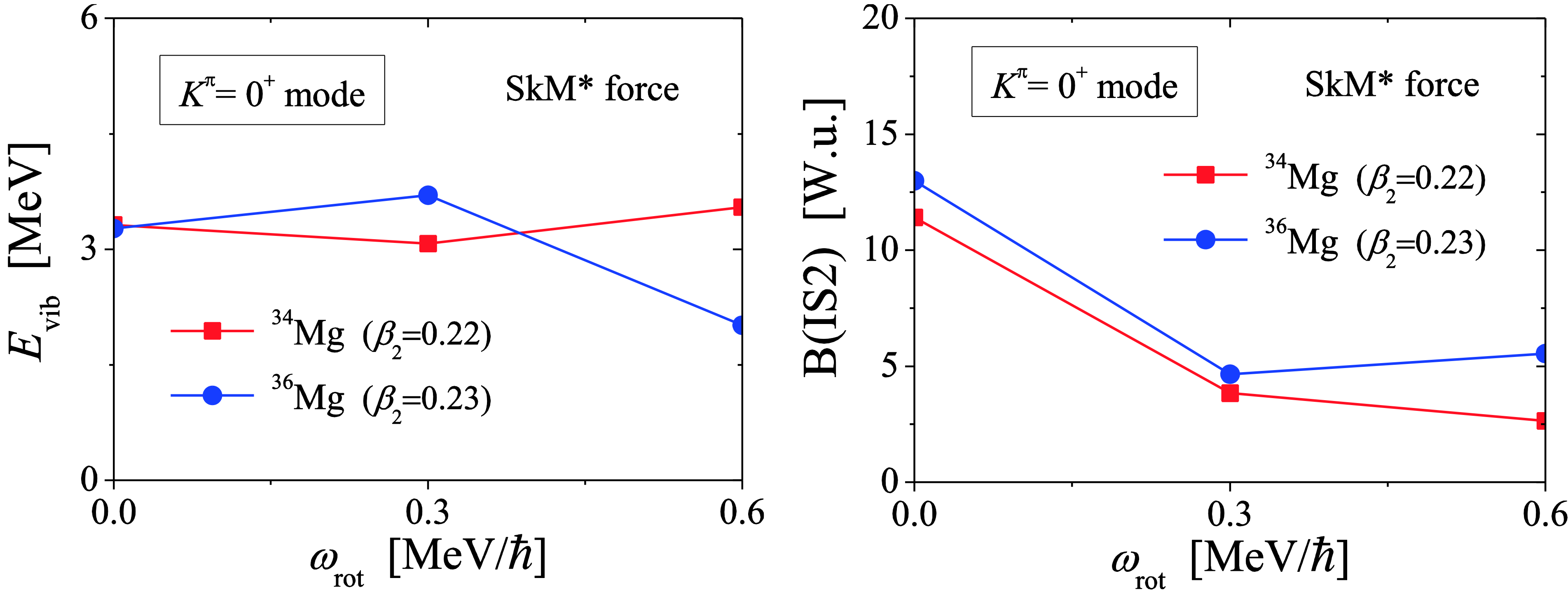
Rotational effect on $K^{\pi} = 0^{+}$ mode in ^{34}Mg



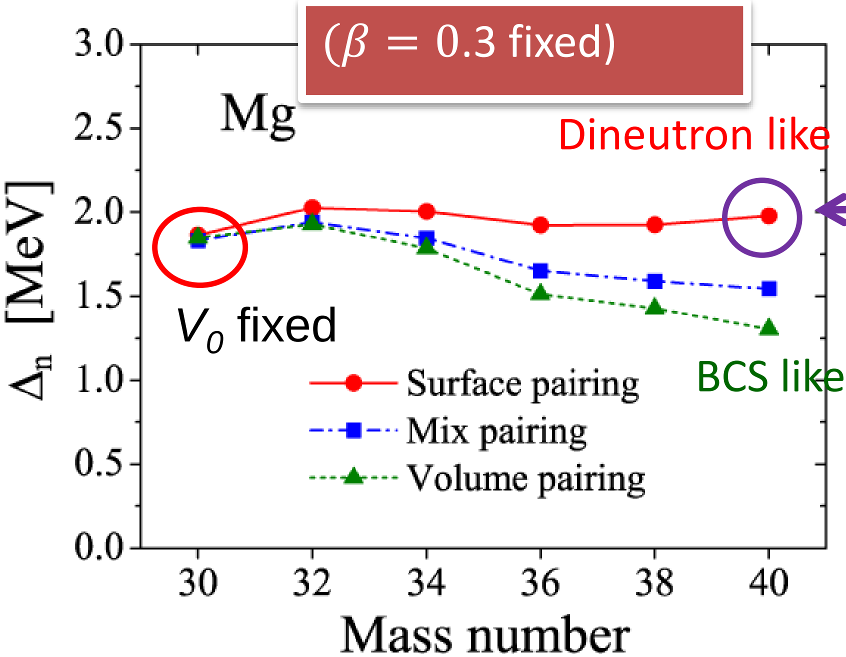


Slow rotation drastically changes the properties of quantum system.

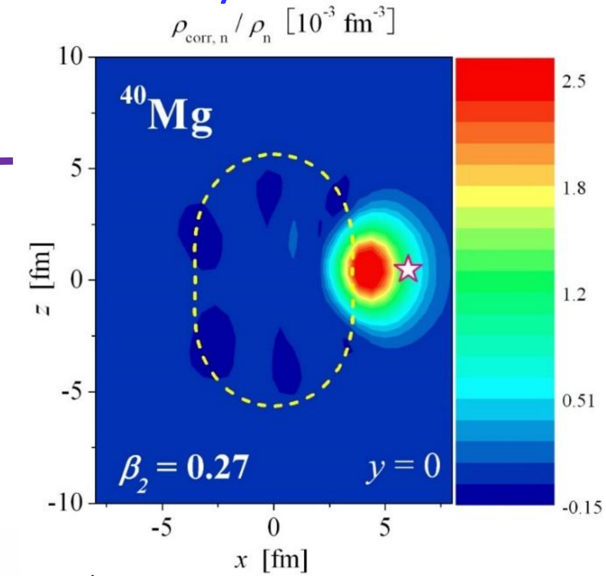
Comparison: ^{34}Mg and ^{36}Mg

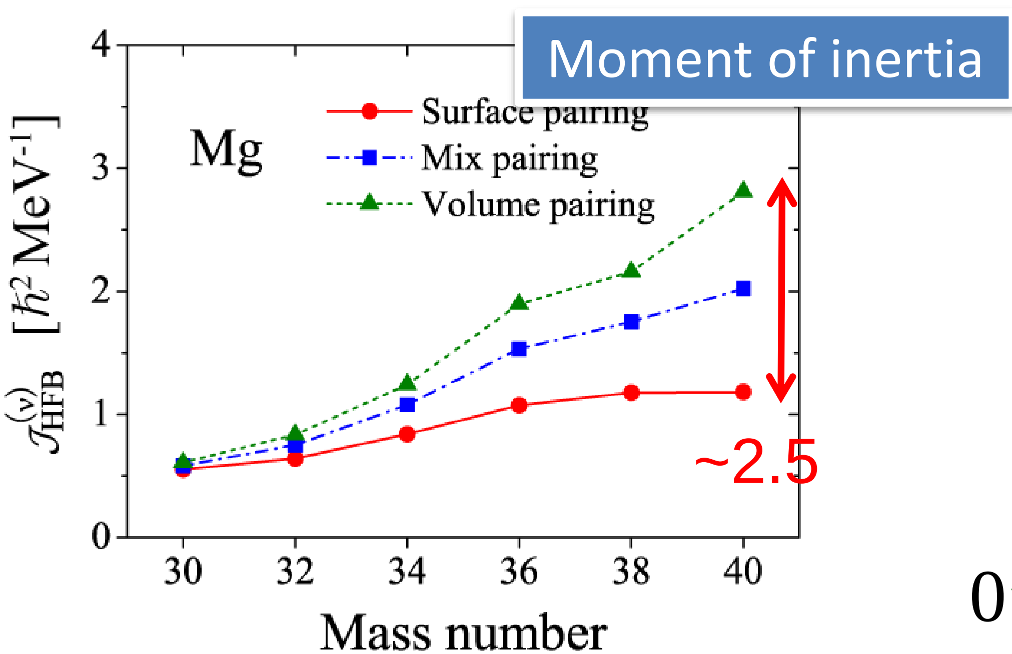


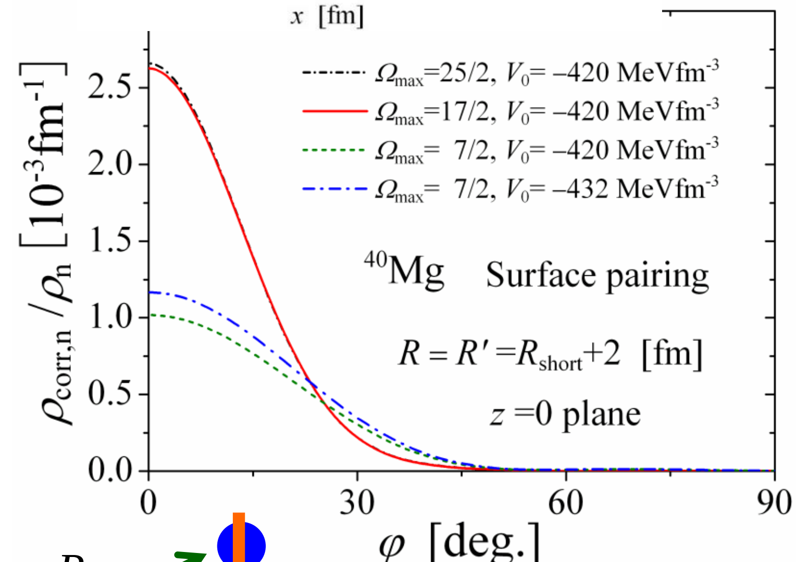
Dineutron correlations in deformed nuclei

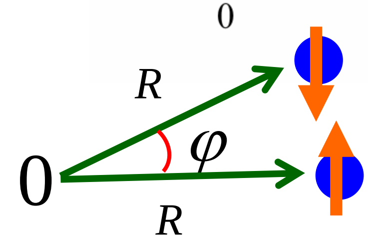


Two-body correlation density









M. Y., and Y. R. Shimizu,
Phys. Rev. C 77 (2008) 064319

Summary

New method: "Fourier-series expansion method" ($\boldsymbol{k}$-space rep.)

- Converge with smaller model space than $\vec{r}$-space rep..

Application 1: Cranked RPA with Skyrme-EDF

- Soft banana mode in SD of ^{40}Ca (Precursor of *Static* banana shape)
- Rotational effect is essential.

Application 2: Cranked Quasiparticle RPA with Skyrme-EDF

- Rotational effect on quadrupole-pairing vibrations in n-rich Mg
- K=0 mode is strongly sensitive to pairing fluctuation.

Future development

*Parallelization of code, *Selfconsistent QRPA, …,*Large-amp. motions