Probing Resonances of the Dirac Equation with Complex Momentum Representation

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12 December 2016

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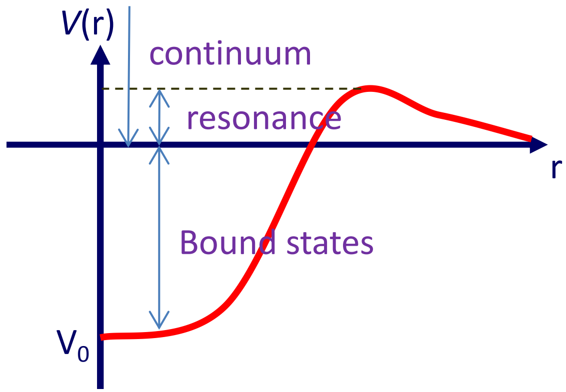
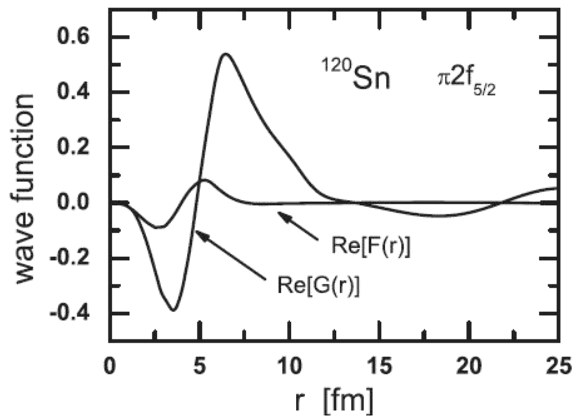
Introduction of Resonance

➢ When the particle energy meets the condition $0<E<V_{max}$, the particle lies in resonant state. And the wave function of resonant state oscillates even at large radius r.





➢ Resonant states play an important role not only in nuclear physics, but also in many branches of science, such as atomic, molecular, and nanophysics.

G. Gamow 1928 ZPA, T. Sommerfeld 1998 PRL, N. Moiseyev 1979 PRA,M. Bylicki 2005 PRB

➢ The resonances has been thought to be the cause of some exotic nuclear phenomena, such as halo, giant halo, and deformed halo.

I. Tanihata 1985 PRL, J. Meng 1996 PRL, W. Pöschl 1997 PRL, N. Sandulescu 2000 PRC; J. Meng 1998 PRL, Y. Zhang 2012 PRC, I. Hamamoto 2010 PRC(R), S.G. Zhou 2010 PRC(R)

➢ The resonances in the continuum play an important role in the description of the nuclear dynamical processes, such as the collective giant resonances.

P. Curutchet 1989 PRC, L.G. Cao 2002 PRC

Methods for exploring resonant states

- Several methods based on scattering theory have been employed to study resonant states, such as

  R-matrix theory E. P. Wigner 1947 Phys. Rev., G. M. Hale 1987 PRL

  K-matrix theory J. Humblet 1991 PRC

  S-matrix method J. R. Taylor 1972

- Some bound-state-like methods have been developed, including

  The real stabilization method (RSM) A.U. Hazi 1970 PRA

  The analytic continuation in the coupling constant (ACCC) method V.I. Kukuli 1989 Kluwer Academic

  The complex scaling method (CSM) Y.K. Ho 1983 Phys. Rep.

  The complex momentum representation (CMR) T.Berggre 1968 NPA

- The development of these methods in the relativistic framework

  RMF-ACCC S.C. Yang 2001 CPL, S.S. Zhang 2004 PRC
  J.Y. Guo 2005 PRC, J.Y. Guo 2006 PRC, S. S. Zhang 2012 PRC

  RMF-RSM L. Zhang 2008 PRC, Z. Z. Zhang 2010 MPLA

  RMF-CSM J.Y. Guo 2010 PRC, Q. Liu 2013 PRA

  Jost function approach B.N. Lu 2012 PRL, B.N. Lu 2013 PRC

  RMF-CMR N.Li 2016 PRL

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The relativistic mean-field model

The basic ansatz of RMF model is a Lagrangian density where nucleons are described as Dirac particles which interact via the exchange of mesons (σ, ω and ρ) and photon.

$$L = \overline{\psi}_i \left(i\gamma^\mu \partial_\mu - M - g_\sigma \sigma - g_\omega \gamma^\mu \omega_\mu - g_\rho \gamma^\mu \vec{\tau} \cdot \vec{\rho}_\mu - e\gamma^\mu \frac{1-\tau_3}{2} A_\mu \right) \psi_i$$

$$+\frac{1}{2}\partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4$$

$$-\frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu$$

$$-\frac{1}{4}\vec{R}^{\mu\nu} \cdot \vec{R}_{\mu\nu} + \frac{1}{2} m_\rho^2 \vec{\rho}^\mu \cdot \vec{\rho}_\mu$$

$$-\frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

$$\begin{cases} \Omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu \\ \vec{R}_{\mu\nu} = \partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu - g_\rho \left(\rho_\mu \times \rho_\nu \right) \\ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \end{cases}$$

where M is the nucleon mass and m_σ (g_σ), m_ω (g_ω), m_ρ (g_ρ) are the masses (coupling constants) of the respective mesons.

From the Lagrangian density, the Dirac equation for nucleon and the Klein-Gordon equation for mesons and photon can be obtained by the classical variation principle.

The relativistic mean-field model

The Dirac equation for nucleon:

$$[\alpha \bullet p + V(\boldsymbol{r}) + \beta(M + S(\boldsymbol{r}))]\psi_i(\boldsymbol{r}) = \varepsilon_i \psi_i(\boldsymbol{r})$$

The corresponding density

$$\begin{cases} \rho_s(\boldsymbol{r}) = \sum_{i=1}^{A} \overline{\psi}_i(\boldsymbol{r})\psi_i(\boldsymbol{r}) \\ \rho_v(\boldsymbol{r}) = \sum_{i=1}^{A} \psi_i^+(\boldsymbol{r})\psi_i(\boldsymbol{r}) \\ \rho_3(\boldsymbol{r}) = \sum_{i=1}^{A} \psi_i^+(\boldsymbol{r})\tau_3\psi_i(\boldsymbol{r}) \\ \rho_c(\boldsymbol{r}) = \sum_{i=1}^{A} \psi_i^+(\boldsymbol{r})\frac{1-\tau_3}{2}\psi_i(\boldsymbol{r}) \end{cases}$$

The Klein-Gordon equation for mesons and photon:

$$\begin{cases} \left(-\Delta + \partial_\sigma U(\sigma)\right)\sigma(\boldsymbol{r}) = -g_\sigma \rho_s(\boldsymbol{r}) \\ \left(-\Delta + m_\omega^2\right)\omega^0(\boldsymbol{r}) = g_\omega \rho_v(\boldsymbol{r}) \\ \left(-\Delta + m_\rho^2\right)\rho^0(\boldsymbol{r}) = g_\rho \rho_3(\boldsymbol{r}) \\ -\Delta A^0(\boldsymbol{r}) = e\rho_c(\boldsymbol{r}) \end{cases}$$

the vector and scalar potentials are as following

$$\begin{cases} V(\boldsymbol{r}) = g_\omega \omega^0(\boldsymbol{r}) + g_\rho \tau_3 \rho^0(\boldsymbol{r}) + e\frac{1-\tau_3}{2}A^0(\boldsymbol{r}) \\ S(\boldsymbol{r}) = g_\sigma \sigma(\boldsymbol{r}) \end{cases}$$

By solving these coupled equations iteratively with the no-sea and the mean-field approximations, we get self-consistent nuclear potential V(r) and S(r).

Complex momentum representation

The wave function of a free particle with momentum $\vec{p}$ or wave vector $\vec{k} = \vec{p} / \hbar$ is denoted as $\left|\vec{k}\right\rangle$. In the momentum representaion, the Dirac equation can be expressed as

$$\int d\vec{k}' \left\langle \vec{k} \left| H \right| \vec{k}' \right\rangle \psi\left(\vec{k}'\right) = \varepsilon \psi\left(\vec{k}\right) \qquad (1)$$

where $H = \vec{\alpha} \bullet \vec{p} + V(\boldsymbol{r}) + \beta(M + S(\boldsymbol{r}))$

$\psi(\vec{k})$ can be split into the radial and angular parts as $\psi\left(\vec{k}\right) = \begin{pmatrix} f(k)\phi_{ljm_j}(\Omega_k) \\ g(k)\phi_{\tilde{l}jm_j}(\Omega_k) \end{pmatrix}$

Putting the wave function into Eq.(1), the Dirac equation is reduced to the following form:

$$\begin{aligned} Mf(k) - kg(k) + \int k'^2 dk' V^+(k,k') f(k') &= \varepsilon f(k) \\ -kf(k) - Mg(k) + \int k'^2 dk' V^-(k,k') g(k') &= \varepsilon g(k) \end{aligned} \qquad (2)$$

with

$$V^+(k,k') = \frac{2}{\pi} \int r^2 dr \left[V(r) + S(r) \right] j_l(k'r) j_l(kr)$$

$$V^-(k,k') = \frac{2}{\pi} \int r^2 dr \left[V(r) - S(r) \right] j_{\tilde{l}}(k'r) j_{\tilde{l}}(kr)$$

Complex momentum representation

By turning the integral in Eq.(2) into a sum over a finite set of points k_i and dk with a set of weights w_j , it is then transformed into a matrix equation

$$\sum_{j=1}^{N}\begin{pmatrix} A_{ij}^{+} & B_{ij} \\ B_{ij} & A_{ij}^{-} \end{pmatrix}\begin{pmatrix} f(k_j) \\ g(k_j) \end{pmatrix} = \varepsilon \begin{pmatrix} f(k_i) \\ g(k_i) \end{pmatrix} \qquad (3)$$

For simplicity in computation, we symmetrize it by the transformation

$$\sum_{j=1}^{N}\begin{pmatrix} \mathrm{A}^{+} & \mathrm{B} \\ \mathrm{B} & \mathrm{A}^{-} \end{pmatrix}\begin{pmatrix} F(k_j) \\ G(k_j) \end{pmatrix} = \varepsilon \begin{pmatrix} F(k_i) \\ G(k_i) \end{pmatrix} \qquad (4)$$

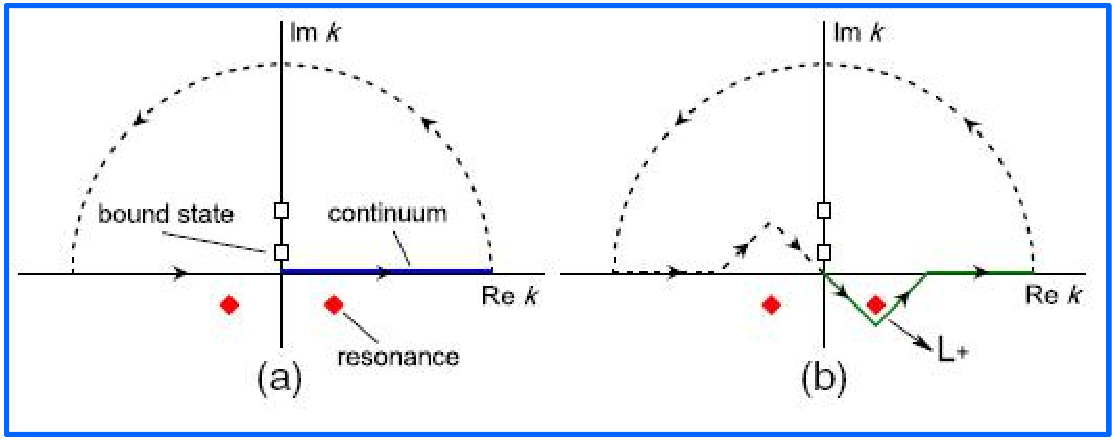
where

$$F(k_j) = \sqrt{w_i}\, k_i f(k_i) \qquad \mathrm{A}_{ij}^{\pm} = \mp M \delta_{ij} + \sqrt{w_i}\sqrt{w_j}\, k_i k_j V^{\pm}(k_i, k_j)$$
$$G(k_j) = \sqrt{w_i}\, k_i g(k_i) \qquad B_{ij} = -k_i \delta_{ij} \qquad (5)$$

RMF-CMR Method

To calculate the symmetric matrix, several key points need to be clarified. As the integration in Eq. (2) is from zero to infinity, it is necessary to truncate the integration to a large enough momentum k_{max}. When k_{max} is fixed, the integration can be calculated by a sum shown in Eq. (3).

As a sum with evenly spaced *dk* and a constant weight w_j converges slowly, so we replace the sum by the Gauss-Legendre quadrature with a finite grid number N, which gives us a $2N \times 2N$ Dirac Hamiltonian matrix (4).



The bound states are located on the imaginary axis, the continuum states follow the contour, and the resonances are located on the fourth quadrant.

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Results and Discussion

To display how the resonance expose in the complex energy plane.

Li, Shi, Guo, Niu, Liang
Phys. Rev. Lett. 117, 062502 (2016)

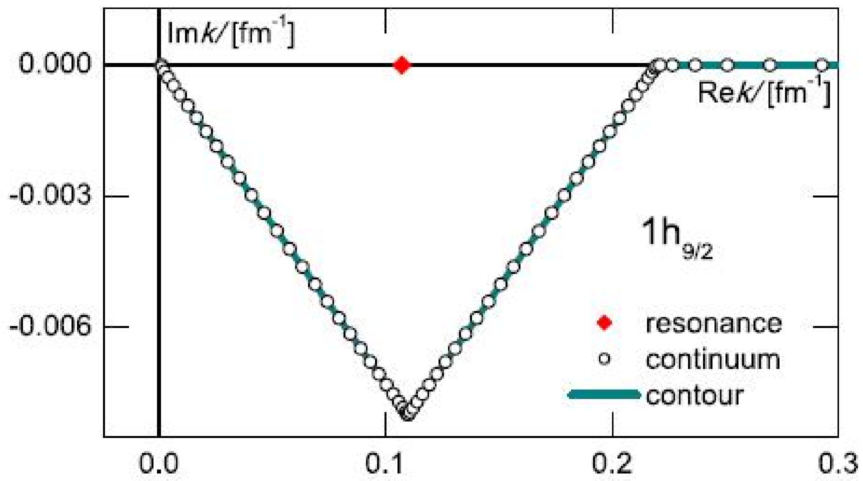

Fig.1 Single-particle spectra for the state $h_{9/2}$ in the complex k plane.

- From the scattering theory, it is known that the bound states populate on the imaginary axis, while the resonances locate at the fourth quadrant in the momentum plane.
- Most solutions follow the contour, corresponding to the nonresonant continuum states(black circle).
- The resonant state (red diamond) separated completely from the continuum and exposed clearly in the complex momentum plane.

Results and Discussion

To check if the present calculations depend on the choice of contour.

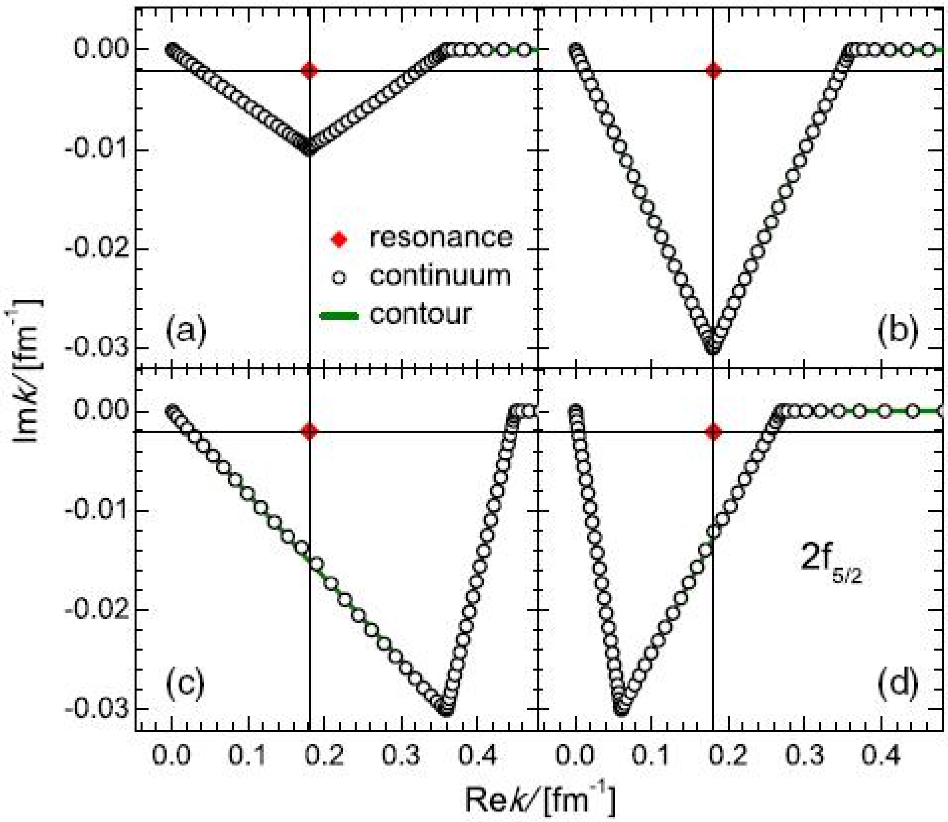

Fig.2 The resonant state $2f_{5/2}$ in four different contours in the momentum integration.

- The resonant state exposed clearly in differnt contours.
- When the contour is deeper, the conrresponding continuous spectra drop down with the contour, while the resonant state $2f_{5/2}$ doesn't change.
- When the contour moves from left to right or from right to left, the coninuum follows the contour, while the resonant state keeps at original position.

Results and Discussion

To obtain all of the resoance which we want to concern.

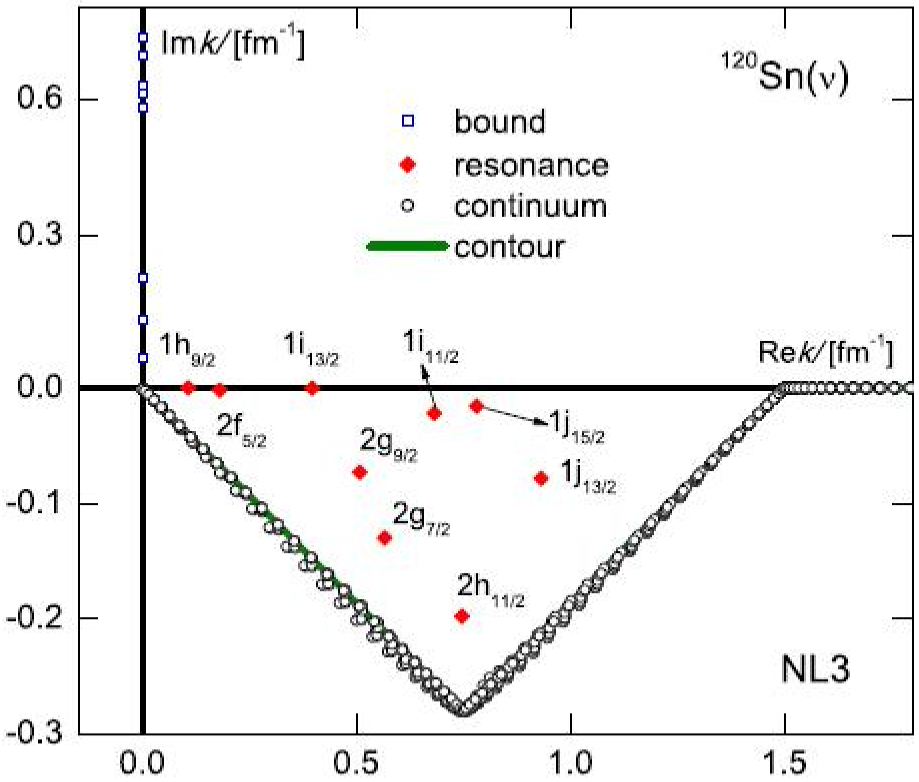

Fig.3 The sigle-neutron spectra in ^{120}Sn in the RMF-CMR calculations with the interaction NL3.

- Choose a large enough contour to expose all the concerned resonances.
- We can see clearly that the bound states populate on the imaginary axis in the momentum plane, while the resonances locate at the fourth quadrant, and the continuum statefollows the integration contour.
- We can also find it is easily to get the broad resonances.

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Summary:

- Combining with the RMF theory, complex momentum representation (CMR) is applied to probe the resonances in ^{120}Sn with the energies, widths, and wavefunctions being obtained.

- Resonant states are exposed clearly in complex momentum plane and the resonance parameters can be determined precisely without imposing unphysical parameters.

Prespective:

- Deal with the pairing correlation by BCS approximation.

Thank you for your attention！