

## Multi-nucleon transfer reaction by TDHF

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In collaboration with

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Center for Computational Sciences, Univ. Tsukuba

# *OUTLINE*

## What I show today

- To what extent the TDHF theory describes the MNT reaction, quantitatively.

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1. Mechanisms of the MNT reaction
2. Particle number projection method
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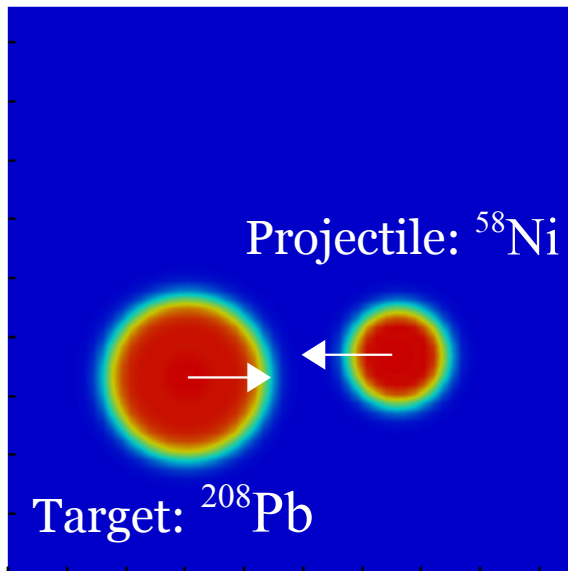
# Results of the TDHF calculation: ${}_{28}^{58}\text{Ni}_{30} + {}_{82}^{208}\text{Pb}_{126}$ at $E_{\text{lab}} = 328.4$ MeV

K. Sekizawa and K. Yabana, Phys. Rev. C **88**, 014614 (2013)

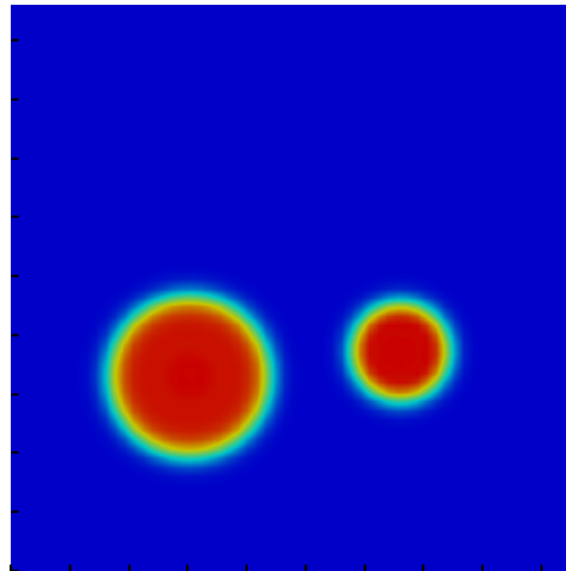
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Skyrme force: SLy5,  $\Delta t$ : 0.2 fm/c, Initial separation distance: 16 fm  
Calculated impact parameter:  $0 \leq b \leq 10$  fm  
Fusion reactions ( $b \leq 1.38$  fm), Binary reactions ( $1.39 \text{ fm} \leq b$ )

## Time evolution of density obtained from the TDHF calculation

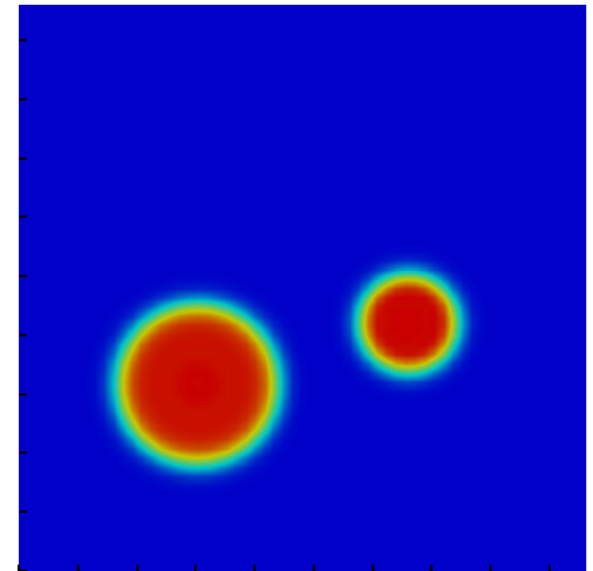
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$b = 1.60$  fm



$b = 4.00$  fm



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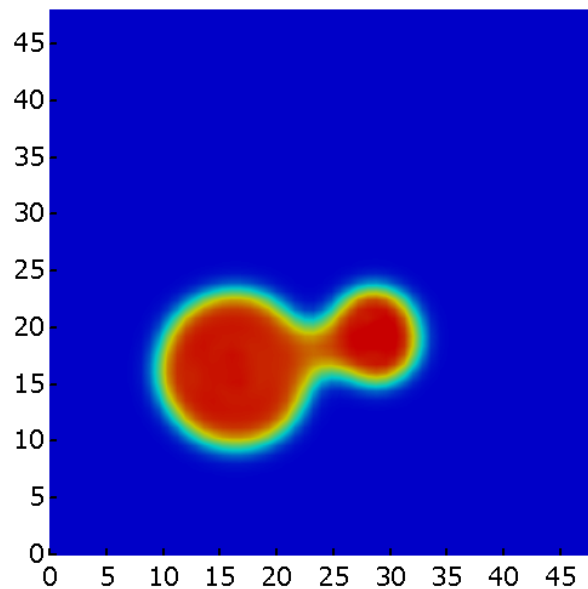
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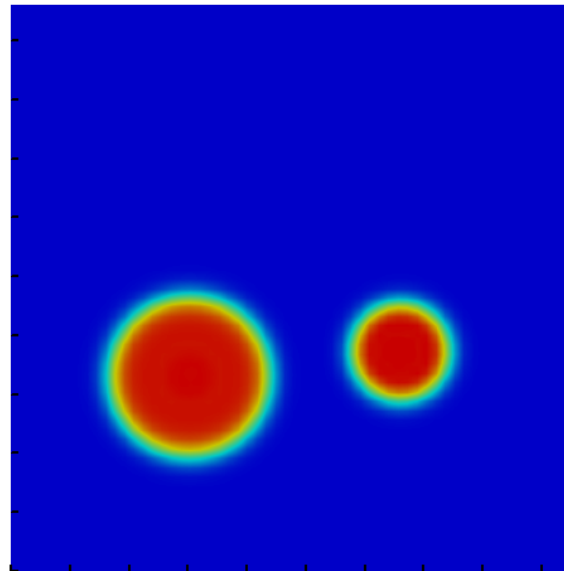
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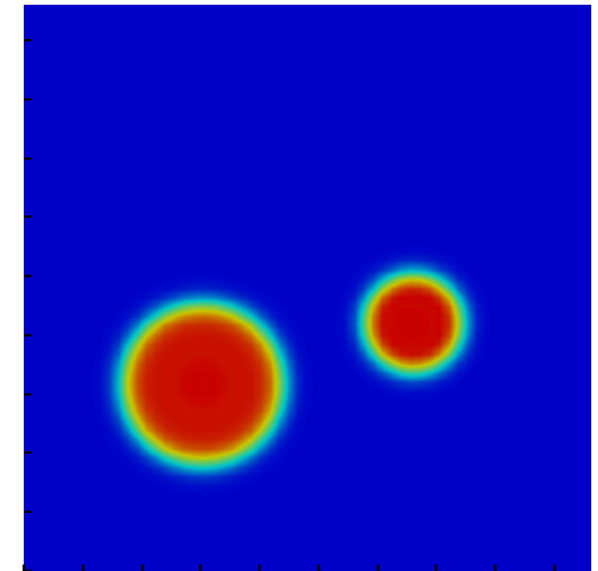
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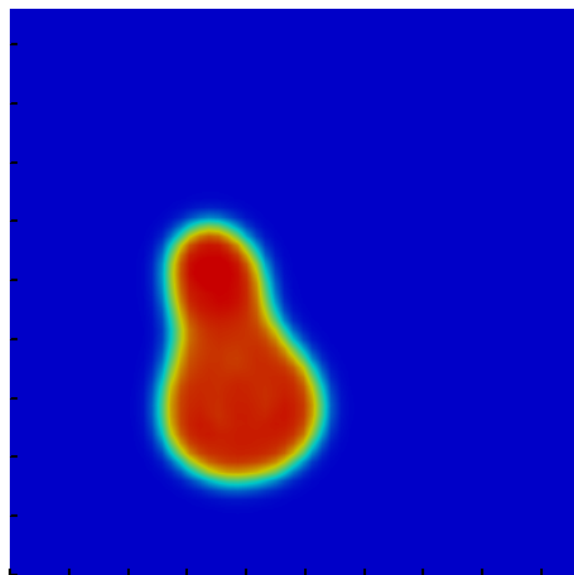
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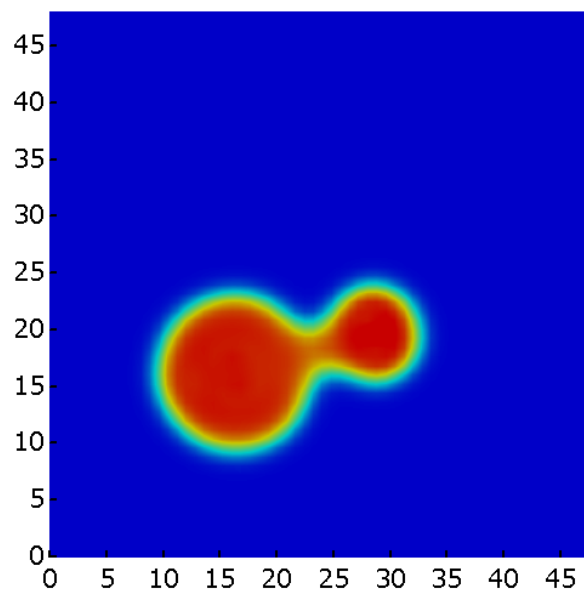
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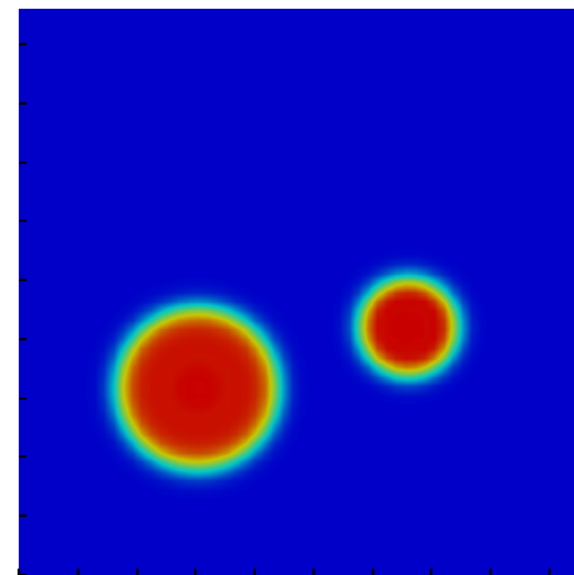
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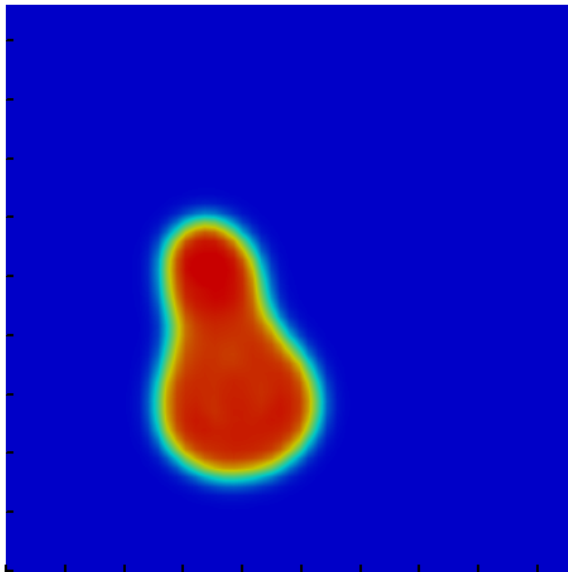
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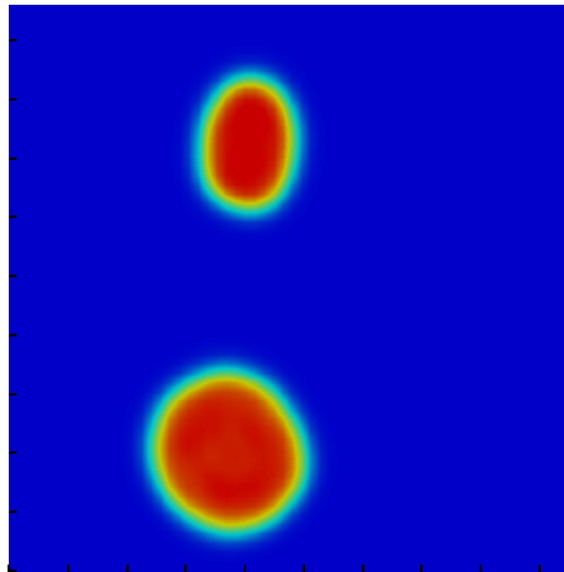
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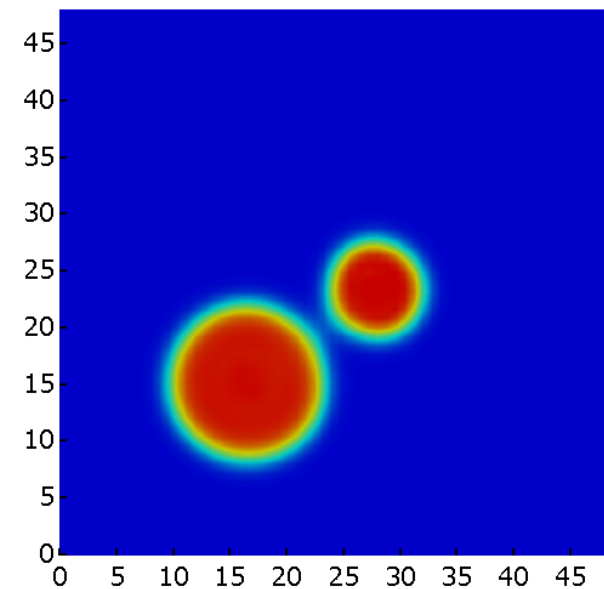
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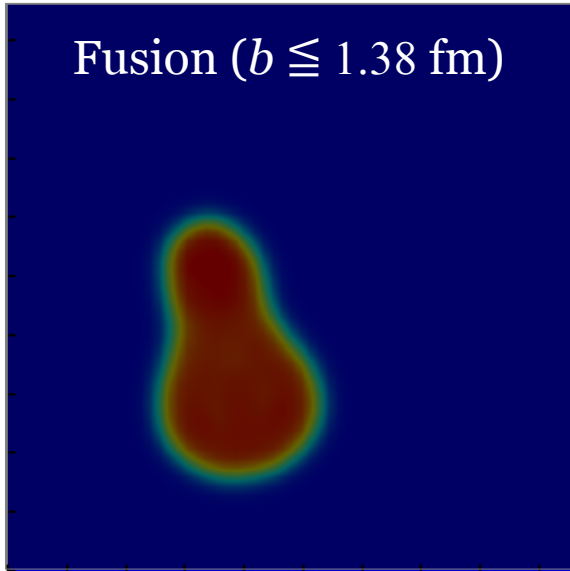
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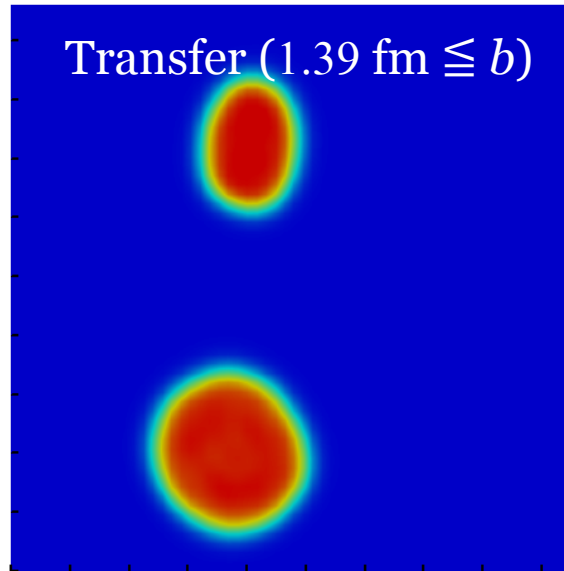
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Fusion ( $b \leq 1.38$  fm)

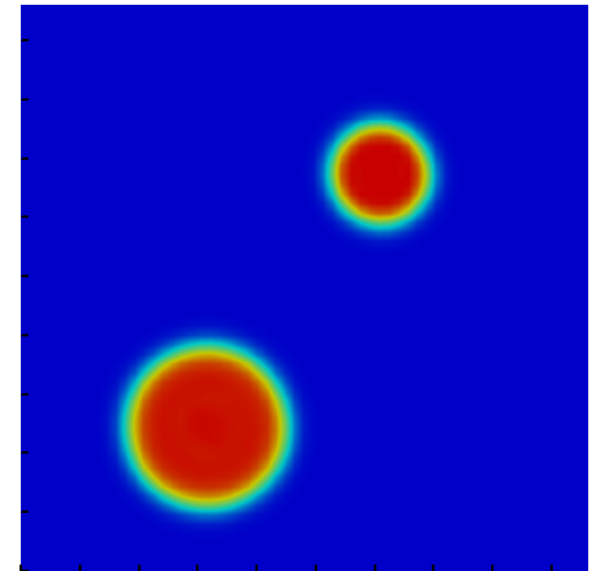


$b = 1.60$  fm

Transfer ( $1.39 \text{ fm} \leq b$ )



$b = 4.00$  fm



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# How to calculate the transfer probability

## Particle number projection method

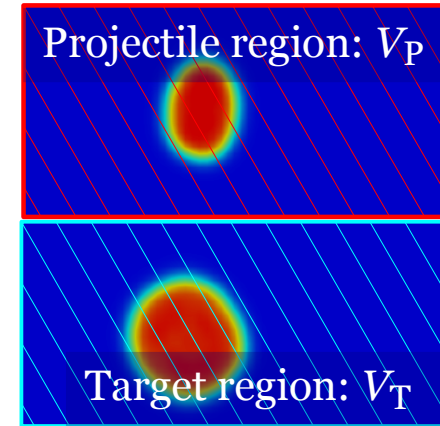
C. Simenel, Phys. Rev. Lett. **105**, 192701 (2010)

### ✓ Particle number projection operator

$$\hat{P}_n = \frac{1}{2\pi} \int_0^{2\pi} d\theta e^{i(n - \hat{N}_P)\theta}$$

$\hat{N}_P$ : Number operator of the spatial region  $V_P$

$$\hat{N}_P = \int_{V_P} d^3r \sum_{i=1}^{N_P + N_T} \delta(\mathbf{r} - \mathbf{r}_i)$$



$N = N_P + N_T$ : Total number of nucleons

### ➤ Probability $P_n$ : $n$ nucleons are in the $V_P$ and $N-n$ nucleons are in the $V_T$

$$\begin{aligned} P_n &= \langle \Phi | \hat{P}_n | \Phi \rangle \\ &= \frac{1}{2\pi} \int_0^{2\pi} d\theta e^{in\theta} \det \{ \langle \phi_i | \phi_j \rangle_{V_T} + e^{-i\theta} \langle \phi_i | \phi_j \rangle_{V_P} \} \end{aligned}$$

Slater determinant

$$\Phi(\mathbf{x}_1, \dots, \mathbf{x}_N) = \frac{1}{\sqrt{N!}} \det \{ \phi_i(\mathbf{x}_j) \}$$

Single-particle w.f.

$$\begin{aligned} \phi_i(\mathbf{x}) &\equiv \phi_i(\mathbf{r}, \sigma) \\ i &= 1, \dots, N_P + N_T \end{aligned}$$

Overlap integral in respective regions

$$\langle \phi_i | \phi_j \rangle_{\tau} = \int_{\tau} d^3x \phi_i^*(\mathbf{x}) \phi_j(\mathbf{x})$$

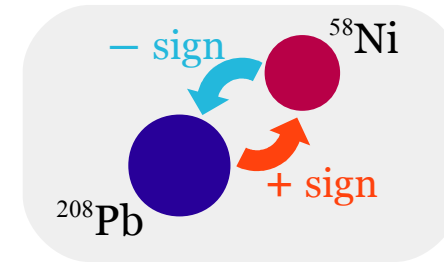
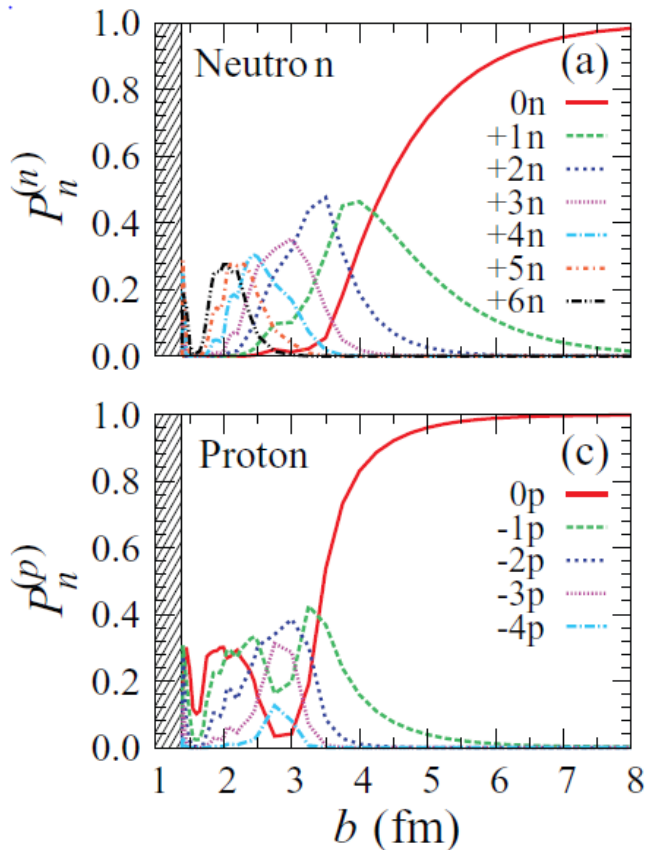
$\tau = V_P \text{ or } V_T$

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## Transfer probabilities

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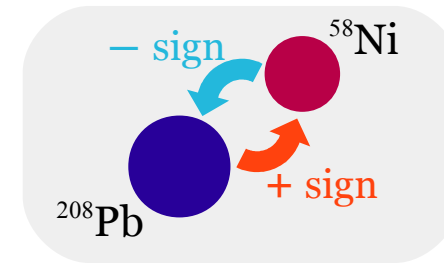
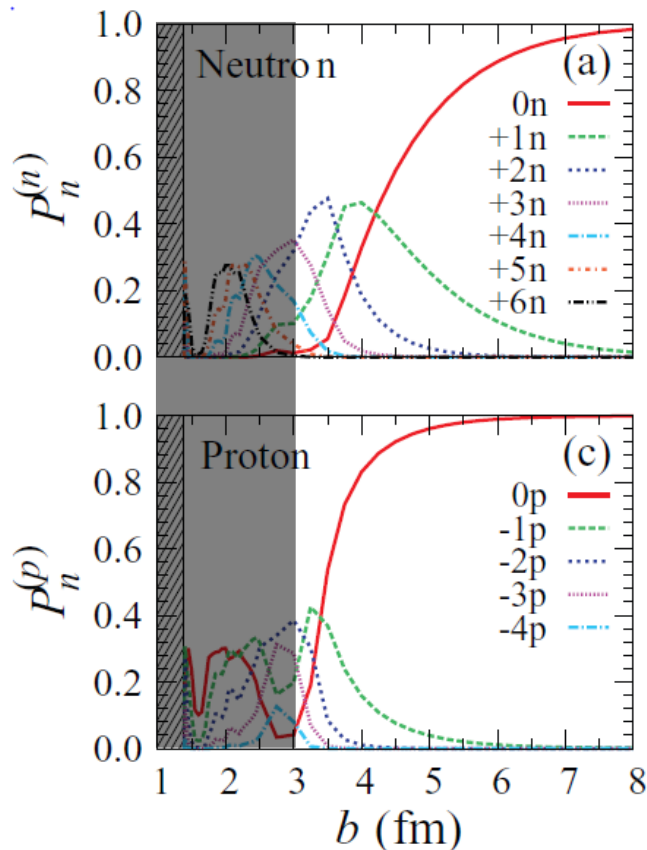


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- At large impact parameter ( $3 \text{ fm} < b$ )

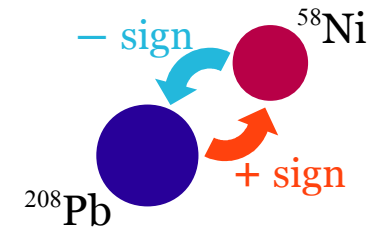
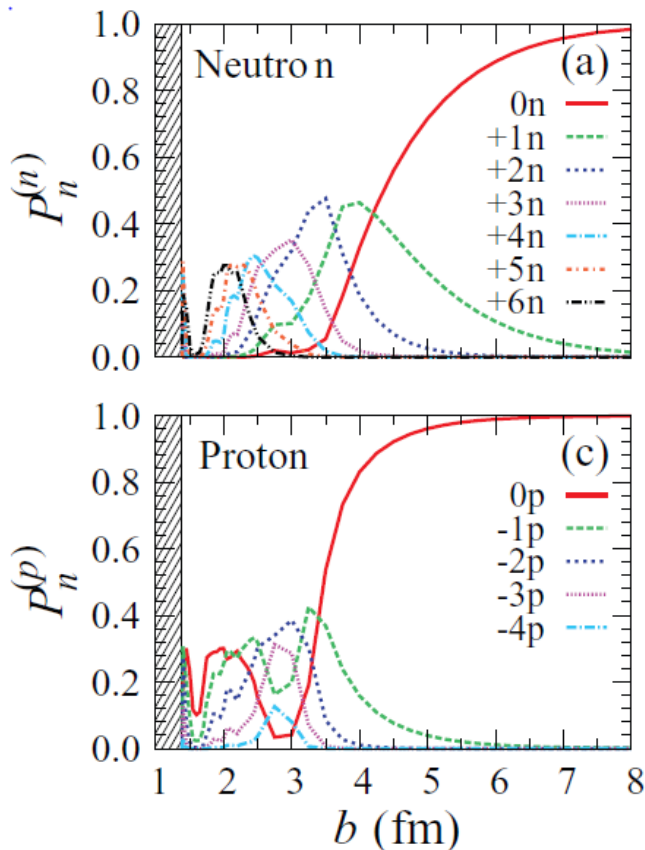
*Charge equilibration*

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- At large impact parameter ( $3 \text{ fm} < b$ )  
*Charge equilibration*
- At small impact parameter ( $b < 3 \text{ fm}$ )  
*Neck breaking*







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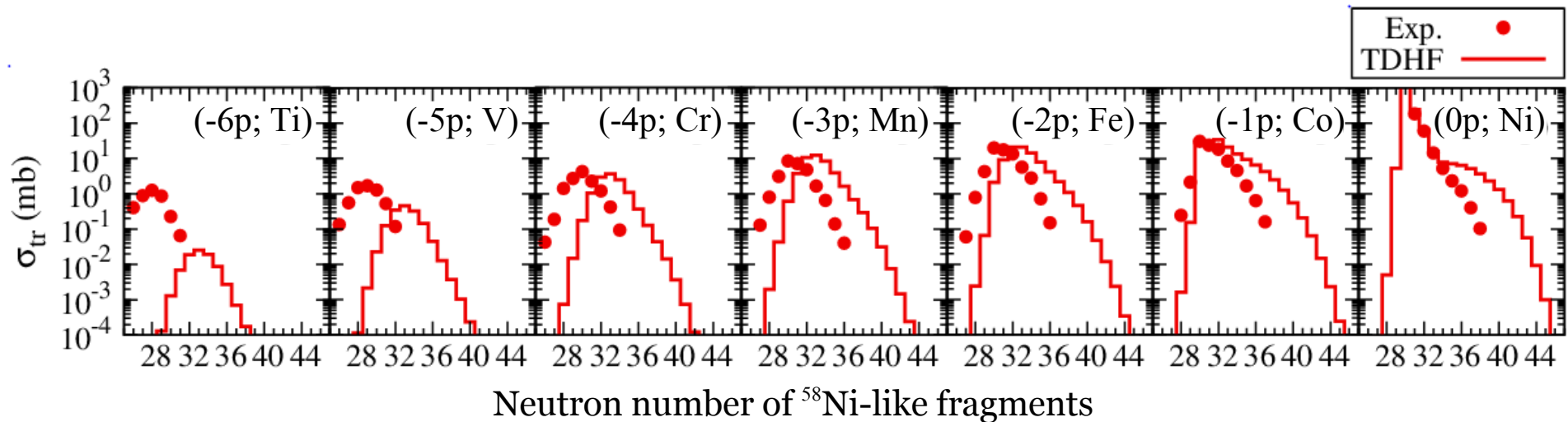
K. Sekizawa and K. Yabana, Phys. Rev. C **88**, 014614 (2013)

## Production cross sections for ${}^{58}\text{Ni}$ -like fragments

Exp.: L. Corradi *et al.*, Phys. Rev. C **66**, 024606 (2002)

$$\sigma_{\text{tr}}(Z, N) = 2\pi \int_{b_{\text{min}}}^{\infty} b P_Z^{(p)}(b) P_N^{(n)}(b) db \quad : \text{Production cross section}$$

- Horizontal axis: Number of neutrons in lighter ( ${}^{58}\text{Ni}$ -like) fragments
- Labels “(-  $x$  p)”,  $x=0, \dots, 6$ : Number of removed protons from  ${}^{58}\text{Ni}$



➤ TDHF reproduces measurement reasonably, when number of transferred nucleons is small

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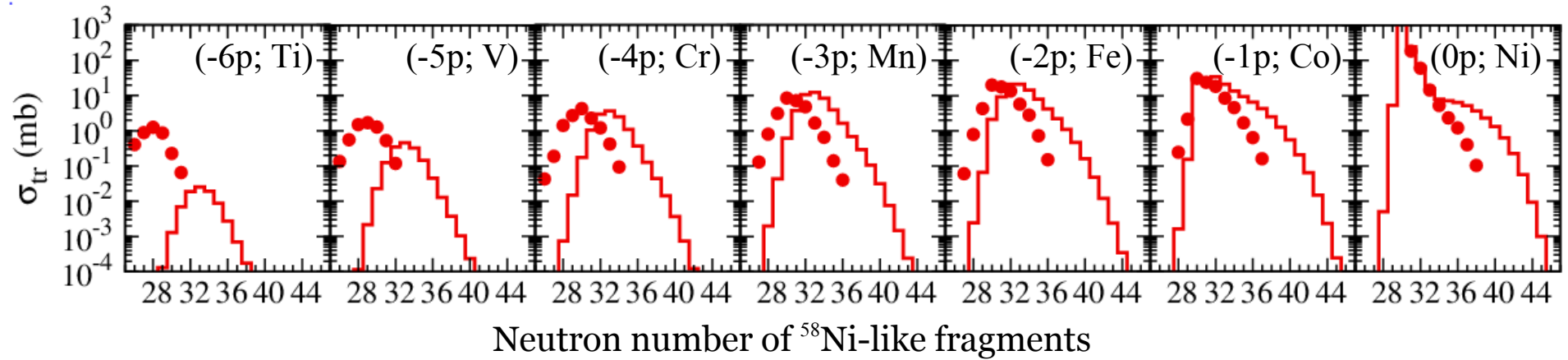
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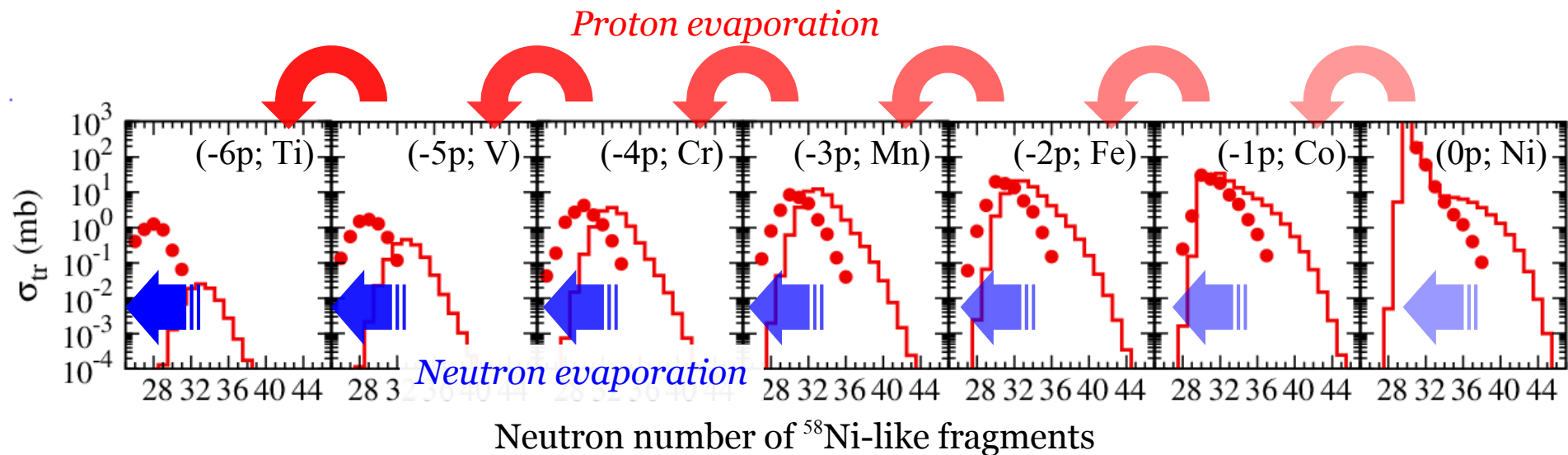
# Effects of particle evaporation

- ✓ Deexcitation processes by particle emission



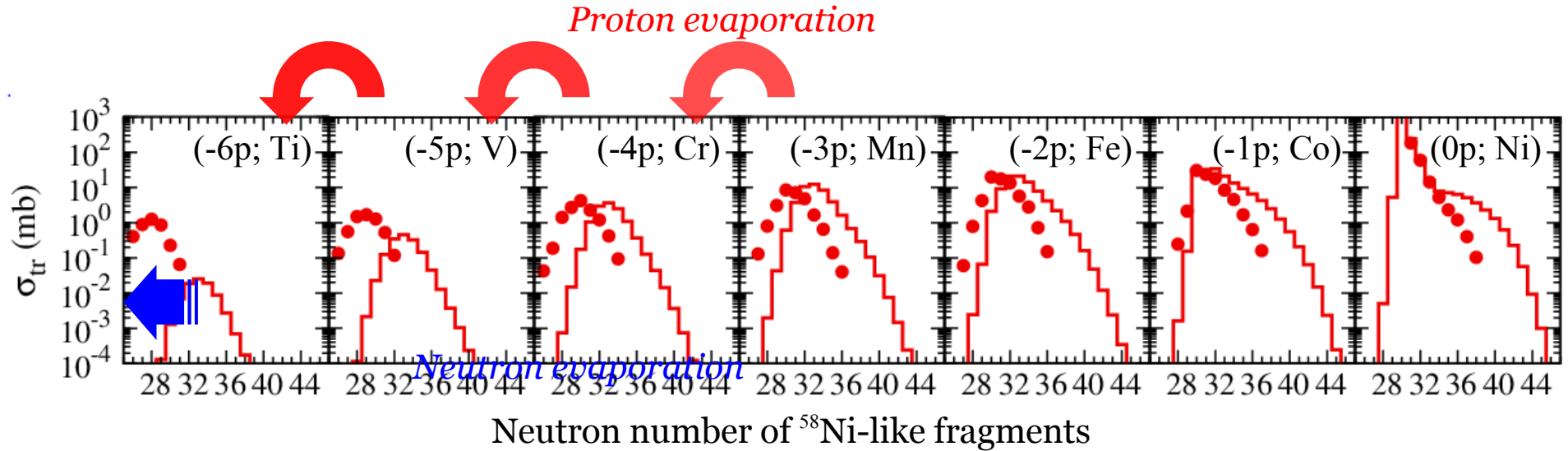
# Effects of particle evaporation

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# Effects of particle evaporation

## ✓ Deexcitation processes by particle emission



## ➤ Our approach to include the effects of particle evaporation

(1) Evaluate internal energy

$$E_{N,Z}$$

Projection method

(2) Subtract the g.s. energy

$$E_{N,Z}^* = E_{N,Z} - E_{\text{g.s.}}$$

Excitation energy

(3) Put  $E_{N,Z}^*$  into a statistical model

$$P_{n,z}^{\text{evap.}}$$

Evaporation probabilities





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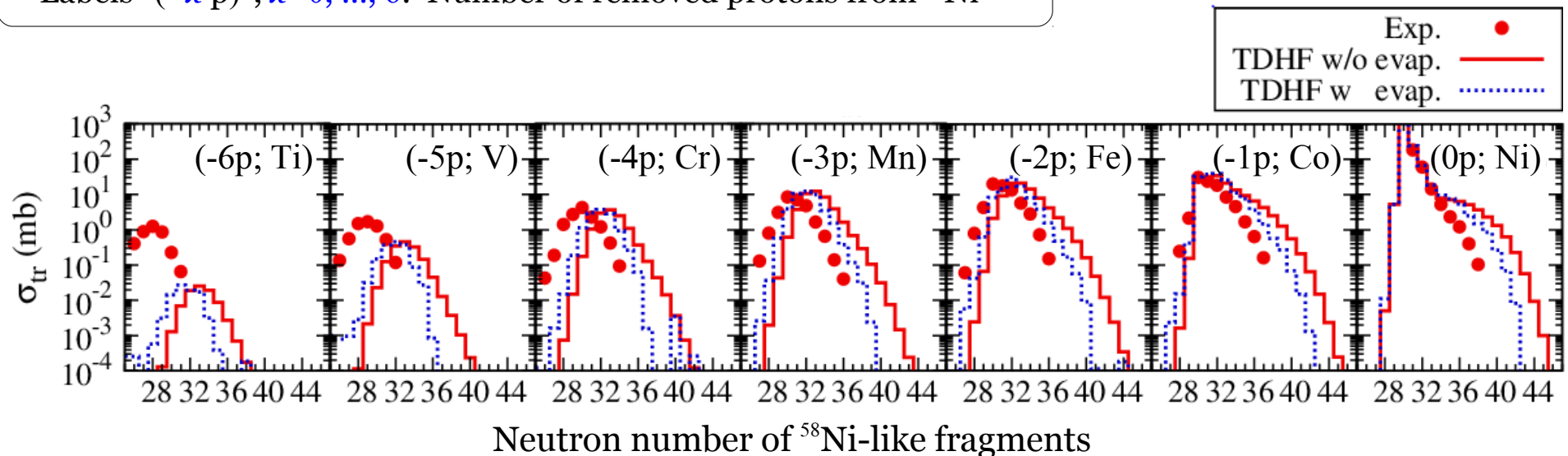
K. Sekizawa and K. Yabana, In preparation.

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(Preliminary)



➤ The discrepancy is somewhat remedied, but not enough.



**It may indicate the importance of correlation effects beyond the mean-field level.**

## Summary

- ✓ I showed how to calculate nucleon transfer probabilities from the TDHF wave function.  
(Projection method: C. Simenel, PRL**105**(2010)192701)
- ✓ I presented results of the TDHF calculation for  $^{58}\text{Ni}+^{208}\text{Pb}$  reaction.  
(K. Sekizawa and K. Yabana, PRC**88**(2013)014614)
- ✓ I discussed how to include effects of particle evaporation.

## Perspective

- We try to find a preferable condition to produce  $N$ -rich unstable nuclei.