

Isoscalar pairing and spin-isospin response

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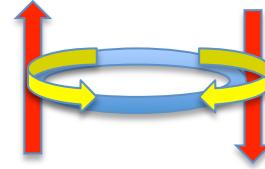
First Tsukuba-CCS-RIKEN joint workshop (Dec. 12-16, 2016)

- Introduction
- Pairing correlations $T=1, S=0$ and $T=0, S=1$ pairs
Energy spectra of $N=Z$ odd-odd nuclei.
- Gamow-Teller excitations in $N=Z$ pf - shell nuclei
interplay between $T=1$ and $T=0$ pairing
- IS and IV spin response and spin polarizability in sd -shell nuclei



Two particle systems

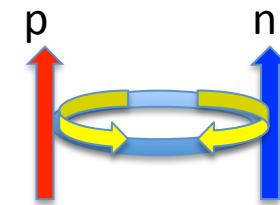
T=1, S=0 pair

$$|(L = S = 0)J = 0, T = 1\rangle \Rightarrow |(j = j')J = 0, T = 1\rangle$$


$p(n)$ $p(n)$

T=0, S=1 pair

$$|(L = 0, S = 1)J = 1, T = 0\rangle \Rightarrow$$



$$a|(l = l' j = j')J = 1, T = 0\rangle + b|((l = l')j, j' = j \pm 1)J = 1, T = 0\rangle$$

If there is strong spin-orbit splitting, it is difficult to make (T=0,S=1)pair.

T=0 J= 1⁺ state could be M1 or Gamow-Teller states in nuclei with N~Z
 ➔ strong M1 or GT states in N~Z nuclei

(J=0,T=1) and (J=1,T=0) are SU(4) supermultiplet in spin-isospin space

Well-known in light p-shell nuclei (LS coupling dominance)

The spin-singlet $T=1$ pairing $V^{(T=1)}(\mathbf{r}, \mathbf{r}') = -G^{(T=1)} \sum_{i,j} P_{i,i}^{(1,0)\dagger}(\mathbf{r}, \mathbf{r}') P_{j,j}^{(1,0)}(\mathbf{r}, \mathbf{r}')$

$$\begin{aligned} \langle (j_i j_i) T = 1, J = 0 | V^{(T=1)} | (j_j j_j) T = 1, J = 0 \rangle \\ = -\sqrt{(j_i + 1/2)(j_j + 1/2)} G^{(T=1)} I_{ij}^2 \end{aligned} \quad (5)$$

where I_{ij} is the overlap integral given by,

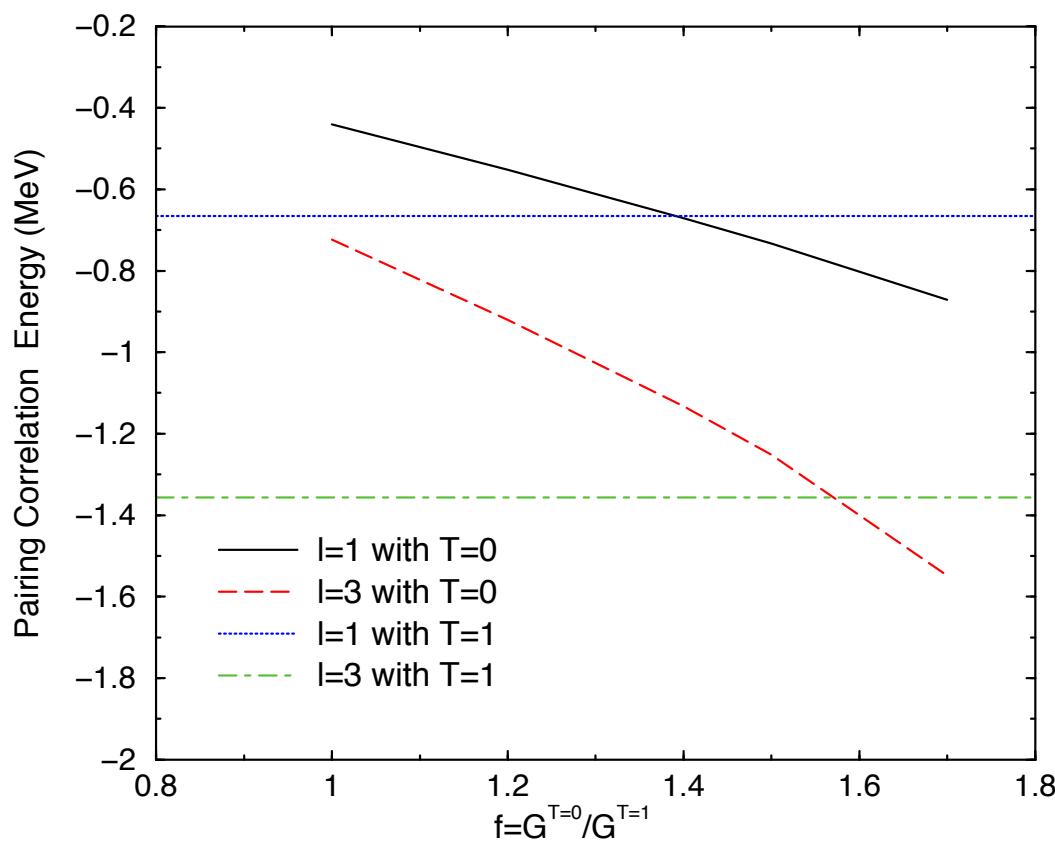
$$I_{ij} = \int \psi_i(\mathbf{r})^* \psi_j(\mathbf{r}) d\mathbf{r} \quad (6)$$

$(T=0, S=1)$ pairing $V^{(T=0)}(\mathbf{r}, \mathbf{r}') = -f G^{(T=1)} \sum_{i \geqq i', j \geqq j'} P_{i,i'}^{(0,1)\dagger}(\mathbf{r}, \mathbf{r}') P_{j,j'}^{(0,1)}(\mathbf{r}, \mathbf{r}')$

$$\begin{aligned} \langle (j_1 j_2) T = 0, J = 1 | V^{(T=0)} | (j'_1 j'_2) T = 0, J = 1 \rangle = \\ - \left\langle \left[\left(l_1 \frac{1}{2} \right)^{j_1} \left(l_2 \frac{1}{2} \right)^{j_2} \right]^{J=1} \left| \left[(l_1 l_2)^{L=0} \left(\frac{1}{2} \frac{1}{2} \right)^{S=1} \right]^{J=1} \right\rangle \right. \\ \times \left. \left\langle \left[\left(l'_1 \frac{1}{2} \right)^{j'_1} \left(l'_2 \frac{1}{2} \right)^{j'_2} \right]^{J=1} \left| \left[(l'_1 l'_2)^{L=0} \left(\frac{1}{2} \frac{1}{2} \right)^{S=1} \right]^{J=1} \right\rangle \right. \\ \times \left. \frac{\sqrt{2l_1 + 1} \sqrt{2l'_1 + 1}}{\sqrt{1 + \delta_{j_1, j_2}} \sqrt{1 + \delta_{j'_1, j'_2}}} f G^{T=1} (I_{j_1 j'_1} I_{j_2 j'_2} + I_{j_1 j'_2} I_{j_2 j'_1}), \right. \end{aligned}$$

TABLE I: The transformation coefficient R between the jj coupling and the LS coupling for the pair wave functions,
 $R = \langle [(l\frac{1}{2})^j(l\frac{1}{2})^{j'}]^{J=1} | [(ll)^{L=0}(\frac{1}{2}\frac{1}{2})^{S=1}]^{J=1} \rangle$. Ω is defined as
 $\Omega \equiv 3(2l+1)^2$.

j	j'	R	$l = 1$	$l = 3$
$l + 1/2$	$l + 1/2$	$\sqrt{\frac{(2l+2)(2l+3)}{2\Omega}}$	$\frac{1}{3}\sqrt{\frac{10}{3}}$	$\frac{2\sqrt{3}}{7}$
$l + 1/2$	$l - 1/2$	$-\sqrt{\frac{4l(l+1)}{\Omega}}$	$-\frac{2}{3}\sqrt{\frac{2}{3}}$	$-\frac{4}{7}$
$l - 1/2$	$l - 1/2$	$-\sqrt{\frac{2l(2l-1)}{2\Omega}}$	$-\frac{1}{3}\sqrt{\frac{1}{3}}$	$-\frac{\sqrt{5}}{7}$
$l - 1/2$	$l + 1/2$	$\sqrt{\frac{4l(l+1)}{\Omega}}$	$\frac{2}{3}\sqrt{\frac{2}{3}}$	$\frac{4}{7}$



HS, Y. Tanimura and K. Hagino, PRC87, 034310 (2013)

TABLE I. Strengths of triplet and singlet interactions from shell-model fits and their ratios. See text for details.

Source	v_s (MeV fm 3)	v_t (MeV fm 3)	Ratio
<i>sd</i> shell [8]	280	465	1.65
<i>fp</i> shell [9]	291	475	1.63

G.F. Bertsch and Y. Luo, PRC81, 064320 (2010)

N=Z odd-odd nuclei with 3-body model

Y. Tanimura, HS, K. Hagino, PTEP 053D02 (2014)

- n-p pairing interactions
 - ✓ T=0, 1 two channels
 - ✓ T=0, S=1 is attractive stronger than T=1, S=0 pair
cf. deuteron, matrix elements in shell models
 - ✓ In finite nuclei N>Z , the strong spin-orbit coupling may quench or even kill T=0 pairing

when λ is larger , the spin-orbit is larger and T=0 pair correlations decrease

$(J^\pi, T) =$

Measured 1^+_1 and 0^+_1 levels of odd-odd N=Z nuclei

$(0^+, 1)$

2.31 MeV

1.04 MeV

0.68 MeV

$(1^+, 0)$

g. S. $^{14}_{\text{N}_7}$

$^{18}_{\text{F}_9}$

$^{30}_{\text{P}_{15}}$

$^{34}_{\text{Cl}_{17}}$

$^{42}_{\text{Sc}_{21}}$

$^{58}_{\text{Cu}_{29}}$

-0.46 MeV

-0.61 MeV

0.20 MeV

Three-body Model

Total 3-body Hamiltonian

$$H = \frac{\mathbf{p}_p^2}{2m} + \frac{\mathbf{p}_n^2}{2m} + V_{pC}(\mathbf{r}_p) + V_{nC}(\mathbf{r}_n) + V_{pn}(\mathbf{r}_p, \mathbf{r}_n) + \frac{(\mathbf{p}_p + \mathbf{p}_n)^2}{2A_C m}$$

Core-N mean field

$$V_{(p/n)C}(r) = v_0 f(r) + v_{ls} \frac{1}{r} \frac{d}{dr} f(r) (\mathbf{l} \cdot \mathbf{s}) (+\text{Coulomb})$$

$$f(r) = \frac{1}{1+e^{(r-R)/a}}$$

p-n interaction

$$V_{pn} = \hat{P}_s v_s \delta(\mathbf{r}_p - \mathbf{r}_n) [1 + x_s (\frac{\rho(r)}{\rho_0})^\alpha] + \hat{P}_t v_t \delta(\mathbf{r}_p - \mathbf{r}_n) [1 + x_t (\frac{\rho(r)}{\rho_0})^\alpha]$$

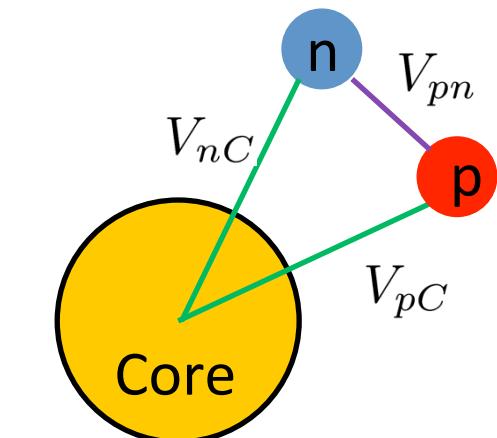
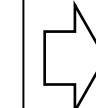
Determination of parameters

v_0, v_{ls} : neutron separation energy

v_s, v_t : pn scattering length with E_{cut} ($= 20$ MeV)

$v_s/v_t = 1.7$ (spin-triplet pairing is
much stronger than spin-singlet)

x_s, x_t, α : $1^+, 3^+, 0^+$ in ^{18}F energies are fitted



Diagonalization in a
large model space

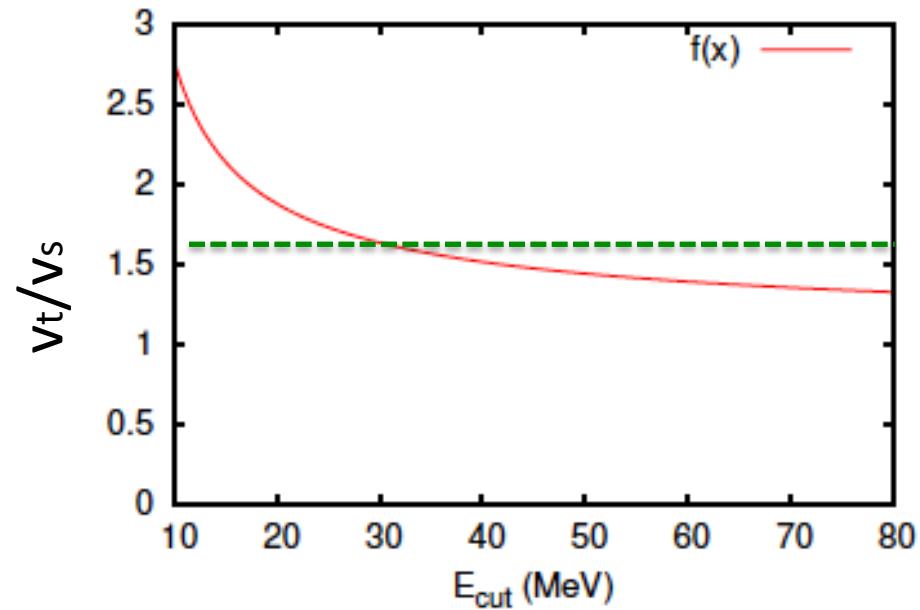
pn pairing interaction

$$V_{np}(\mathbf{r}_1, \mathbf{r}_2) = \hat{P}_s v_s \delta(\mathbf{r}_1 - \mathbf{r}_2) \left[1 + x_s \left(\frac{\rho(r)}{\rho_0} \right)^\alpha \right] \\ + \hat{P}_t v_t \delta(\mathbf{r}_1 - \mathbf{r}_2) \left[1 + x_t \left(\frac{\rho(r)}{\rho_0} \right)^\alpha \right]$$

$$\hat{P}_s = \frac{1}{4} - \frac{1}{4} \boldsymbol{\sigma}_p \cdot \boldsymbol{\sigma}_n, \quad \hat{P}_t = \frac{3}{4} + \frac{1}{4} \boldsymbol{\sigma}_p \cdot \boldsymbol{\sigma}_n$$

$$v_s = \frac{2\pi^2 \hbar^2}{m} \frac{2a_{pn}^{(s)}}{\pi - 2a_{pn}^{(s)} k_{\text{cut}}},$$

$$v_t = \frac{2\pi^2 \hbar^2}{m} \frac{2a_{pn}^{(t)}}{\pi - 2a_{pn}^{(t)} k_{\text{cut}}},$$



$$a_{pn}^{(s)} = -23.749 \text{ fm} \text{ and } a_{pn}^{(t)} = 5.424 \text{ fm}$$

$$E_{\text{cut}} = k_{\text{cut}}^2 / 2m$$

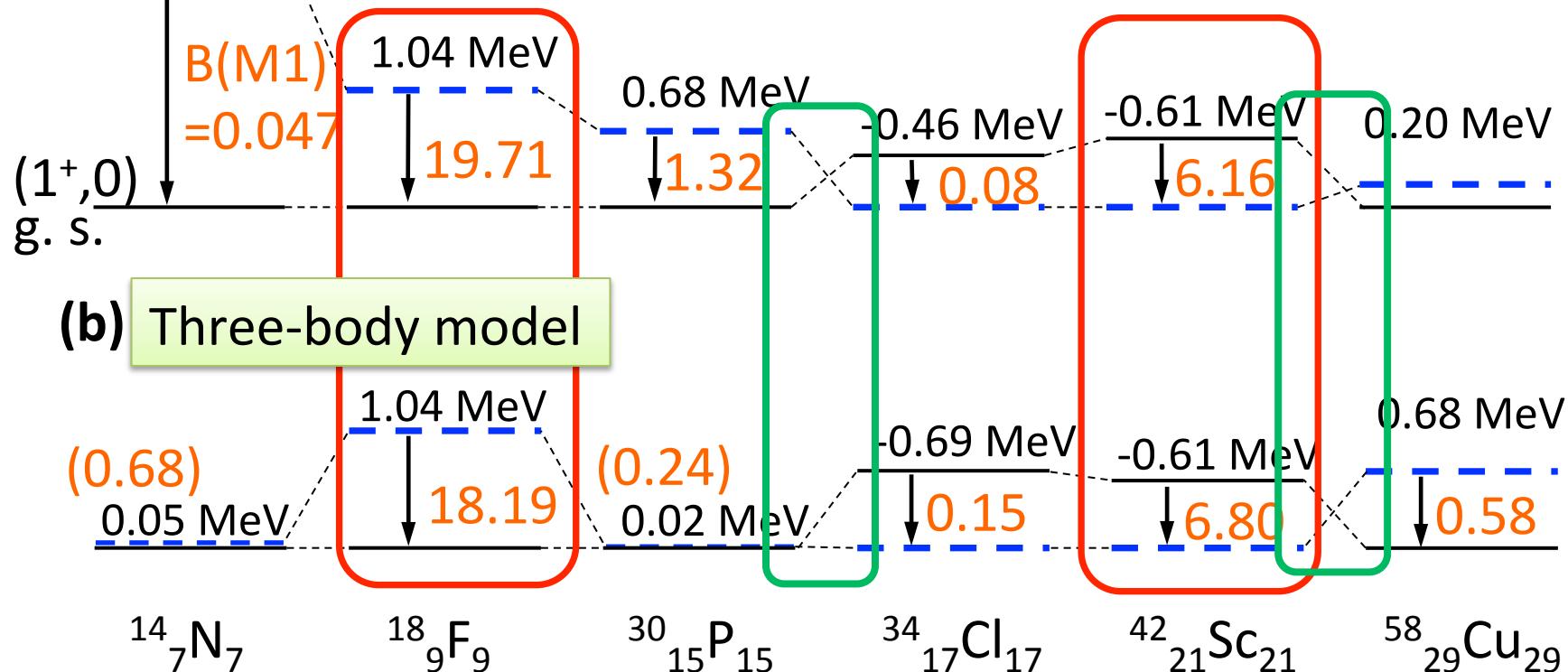
1. $E_{0+}-E_{1+}$ and $B(M1)$

(a) Experiment <http://www.nndc.bnl.gov/>)

$(J^\pi, T) =$

$(0^+, 1)$ 2.31 MeV

- ✓ Inversion of 1^+ and 0^+
- ✓ $^{18}\text{F}, ^{42}\text{Sc}$
- Large $B(M1)$
- Accurate $E_{0+}-E_{1+}$ (^{42}Sc)



The inversion of 1^+ and 0^+ shows a clear manifestation of the competition between spin-orbit and the spin-triplet pairing.

Results

Large B(M1) in ^{18}F and ^{42}Sc

^{18}F :

$$\left\{ \begin{array}{l} 1^+ \rightarrow P(S=1) = 90.1\%, (1d)^2 \\ 0^+ \rightarrow P(S=0) = 82.2\%, (1d)^2 \end{array} \right.$$

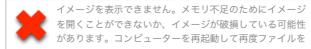
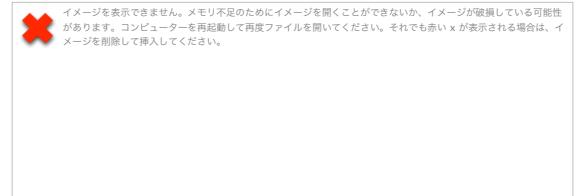
1⁺ and 0⁺ can be considered
as the states in the same SU(4) multiplets
(LST) = (0,1,0), (0,0,1)
The same as 42Sc in 1f-orbits

$(J^\pi, T) =$	\vdots	\vdots	\vdots
$(0^+, 1)$	$(0^+, 1)$	$(0^+, 1)$	
^{18}O	$(1^+, 0)$		^{18}Ne
		^{18}F	

SU(4) multiplet

$$\begin{aligned} O(\text{M1}) &\propto \sum_i [g_s(i)s(i) + g_\ell(i)\ell(i)] \\ &= \frac{g_s^{IV}}{\text{Large}} \boxed{\sum_i \tau_3(i)s(i)} + \frac{g_s^{IS}}{\text{(small)}} \sum_i s(i) + \sum_i g_\ell(i)\ell(i) \end{aligned}$$

SU(4)generator



results

^{18}F and ^{42}Sc : large B(M1)

Separate Contribution to $\langle f | | O(\text{M1}) | | i \rangle (\mu_N)$

	^{14}N	^{18}F	^{30}P	^{34}Cl	^{42}Sc	^{58}Cu
Valence orbital	p1/2	d5/2	s1/2	d3/2	f7/2	p3/2
 orbital	1.09	1.28	0.21	2.28	2.91	0.09
$g_s^{IV} \sum_i \tau_3(i) s(i)$	-2.78	7.44	-1.21	-3.65	6.34	1.47
$g_s^{IS} \sum_i s(i)$	5×10^{-5}	3×10^{-3}	3×10^{-5}	-1×10^{-4}	2×10^{-3}	-2×10^{-3}
B(M1) ↓ (μ_N^2) Exp.	0.047	19.71	1.32	0.08	6.16	---
Calc.	0.68	18.19	0.24	0.15	6.80	0.58

- ✓ $(j=l-1/2)^2$ spin and orbital are cancelled
(Lisetskiy et al., PRC60, 064310 ('99))

→ $^{14}\text{N}, ^{34}\text{Cl}$ B(M1) **small**

- ✓ $(j=l+1/2)^2$ spin and orbital coherent
(Lisetskiy et al., PRC60, 064310 ('99))
- ✓ good SU(4) symmetry

→ $^{18}\text{F}, ^{42}\text{Sc}$ B(M1) **large**

- ✓ even $j=l+1/2$ not good SU(4) symmetry

→ ^{58}Cu は(M1) **small**

Cooperation of T=0 and T=1 pairing in Gamow-Teller states in N=Z nuclei

C. L. Bai, H.S., M.Sasano, T. Uesaka, K. Hagino, H.Q. Zhang, X.Z. Zhang, F.R.Xu

Phys. Lett. B719, pp. 116-121 (2013)

HFB+QRPA with T=1 and T=0 pairing

T=1 pairing in HFB

T=0 pairing in QRPA

How large is the spin-triplet T=0 pairing?

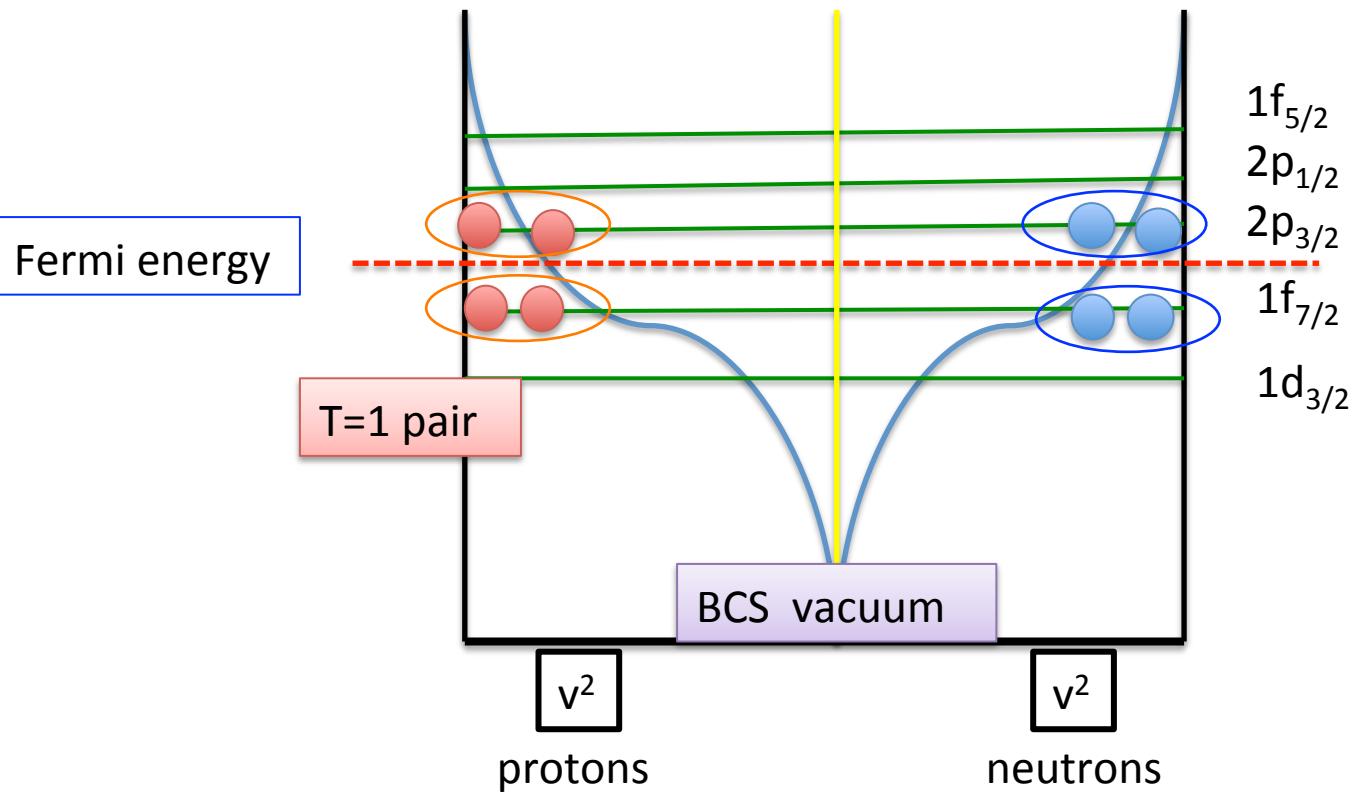
$$V_{T=1}(\mathbf{r}_1, \mathbf{r}_2) = V_0 \frac{1 - P_\sigma}{2} \left(1 - \frac{\rho(\mathbf{r})}{\rho_o}\right) \delta(\mathbf{r}_1 - \mathbf{r}_2), \quad (1)$$

$$V_{T=0}(\mathbf{r}_1, \mathbf{r}_2) = f V_0 \frac{1 + P_\sigma}{2} \left(1 - \frac{\rho(\mathbf{r})}{\rho_o}\right) \delta(\mathbf{r}_1 - \mathbf{r}_2), \quad (2)$$

As a possible manifestation of T=0 S=1 pairing correlations in nuclei N=Z.

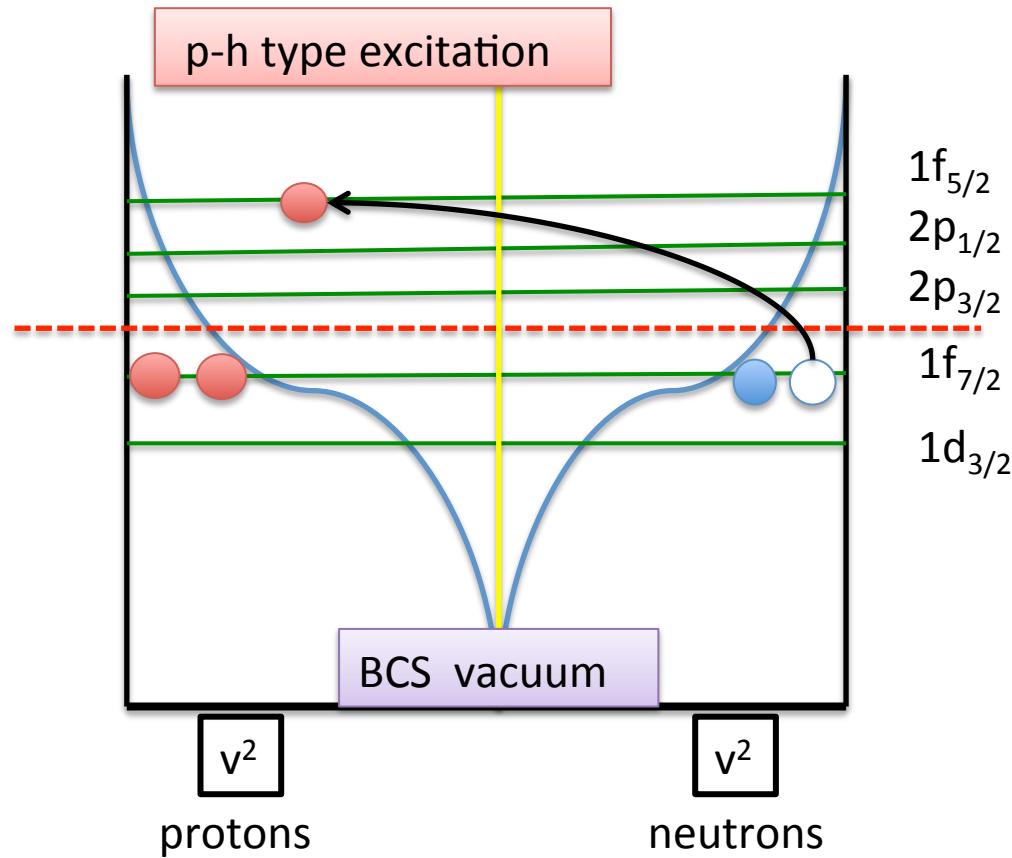
$$\hat{O}(GT) = \sigma \tau_\pm$$

Gamow-Teller transitions from BCS vacuum in N=Z nuclei



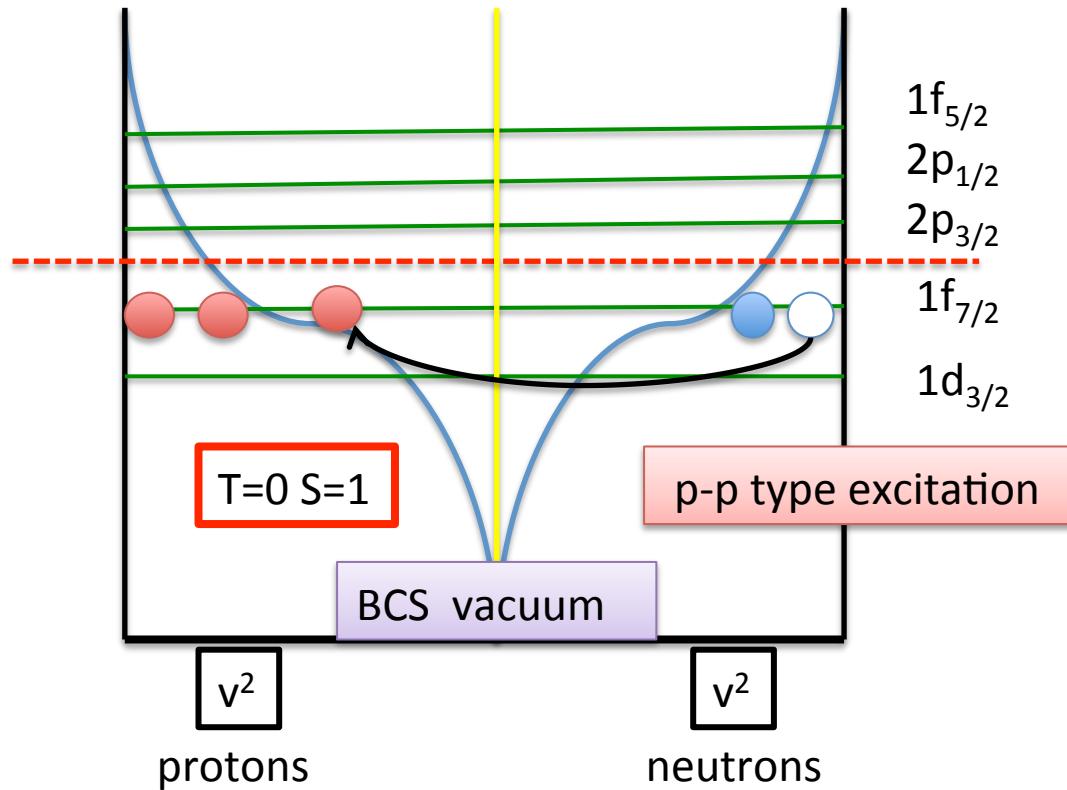
Gamow-Teller transitions in BCS vacuum

Fermi energy



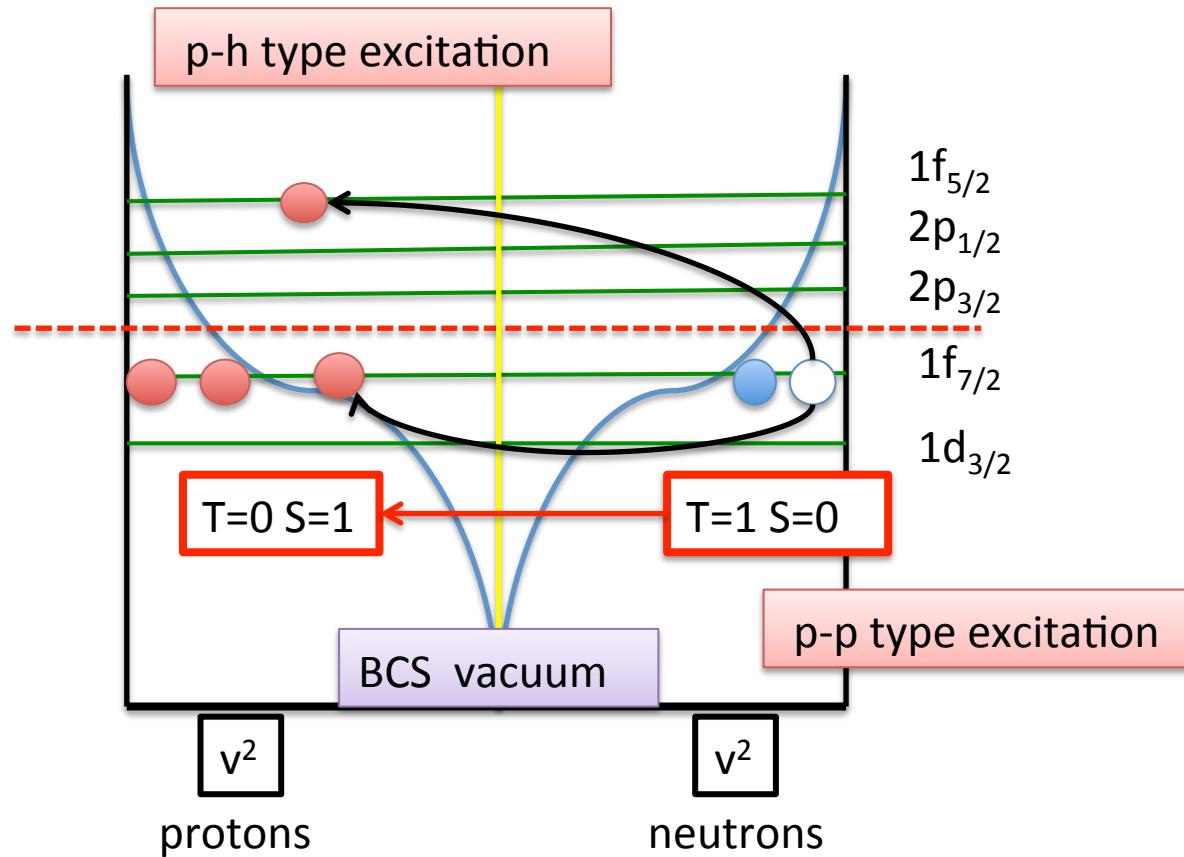
Gamow-Teller transitions in BCS vacuum

Fermi energy



Gamow-Teller transitions in BCS vacuum

Fermi energy



A pair of SU(4) supermultiplet

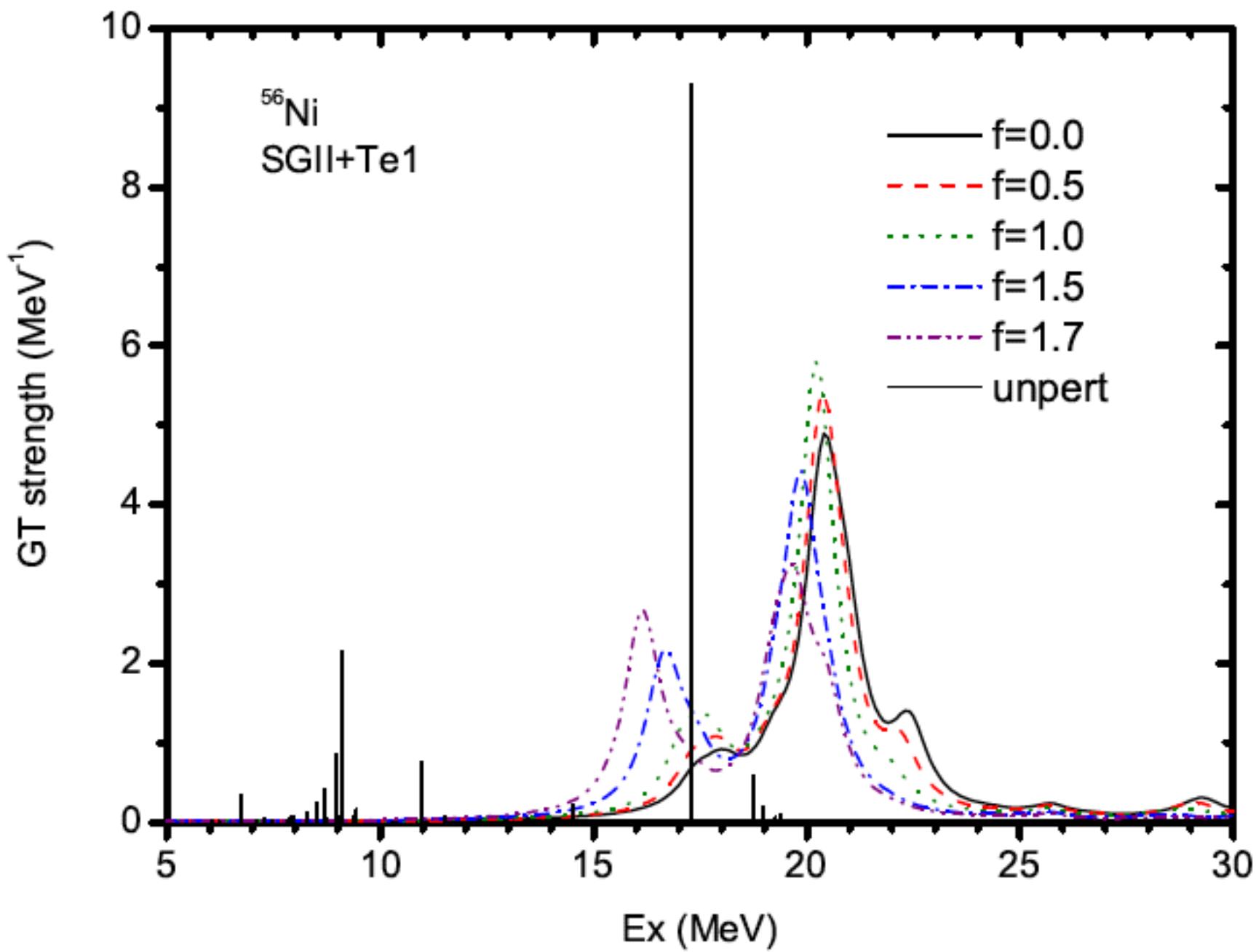


TABLE I: Amplitudes of main (np) particle-hole and particle-particle type configurations of GT states in ^{56}Ni . The QRPA calculations are performed without and with the T=0 pairing interaction in the cases of $f = 0$ and $f = 1.5$, respectively. The Skyrme interaction T21 is used for HF and p-h matrix calculations. The abbreviations B and C correspond to the GT reduced matrix element $B=(X u_\pi v_\nu - Y u_\nu v_\pi) \langle \pi || \hat{O}(\text{GT}) || \nu \rangle$ and the normalization factor $C=X^2 - Y^2$, respectively, where X,Y are QRPA amplitudes and $\hat{O}(\text{GT})$ is GT transition operator in Eq. (3).

^{56}Ni		$f = 0$			
E_x (MeV)	B(GT)	$\nu(v_\nu^2)$	$\pi(v_\pi^2)$	B	C
17.5	1.41	2p _{3/2} (0.20)	2p _{3/2} (0.21)	0.358	0.127
		1f _{7/2} (0.69)	1f _{5/2} (0.09)	-0.229	0.006
		1f _{7/2} (0.69)	1f _{7/2} (0.67)	1.341	0.802
18.3	1.49	2p _{1/2} (0.10)	2p _{3/2} (0.21)	0.260	0.153
		2p _{3/2} (0.20)	2p _{1/2} (0.11)	0.846	0.740
21.3	7.88	1f _{7/2} (0.69)	1f _{5/2} (0.09)	2.48	0.742
$S_{-}(\text{GT})=18.28$					
^{56}Ni		$f = 1.5$			
E_x (MeV)	B(GT)	$\nu(v_\nu^2)$	$\pi(v_\pi^2)$	B	C
16.6	4.82	2p _{3/2} (0.20)	2p _{1/2} (0.11)	-0.203	0.049
		2p _{3/2} (0.20)	2p _{3/2} (0.21)	-0.682	0.491
		1f _{7/2} (0.69)	1f _{5/2} (0.09)	-0.237	0.007
		1f _{7/2} (0.69)	1f _{7/2} (0.67)	-0.790	0.339
20.5	4.10	1f _{7/2} (0.69)	1f _{5/2} (0.09)	-1.900	0.437
$S_{-}(\text{GT})=14.75$					

$$B = (X u_\pi v_\nu - Y u_\nu v_\pi) \left\langle \pi \middle\| \hat{O}(\text{GT}) \middle\| \nu \right\rangle$$

$$C = X^2 - Y^2$$

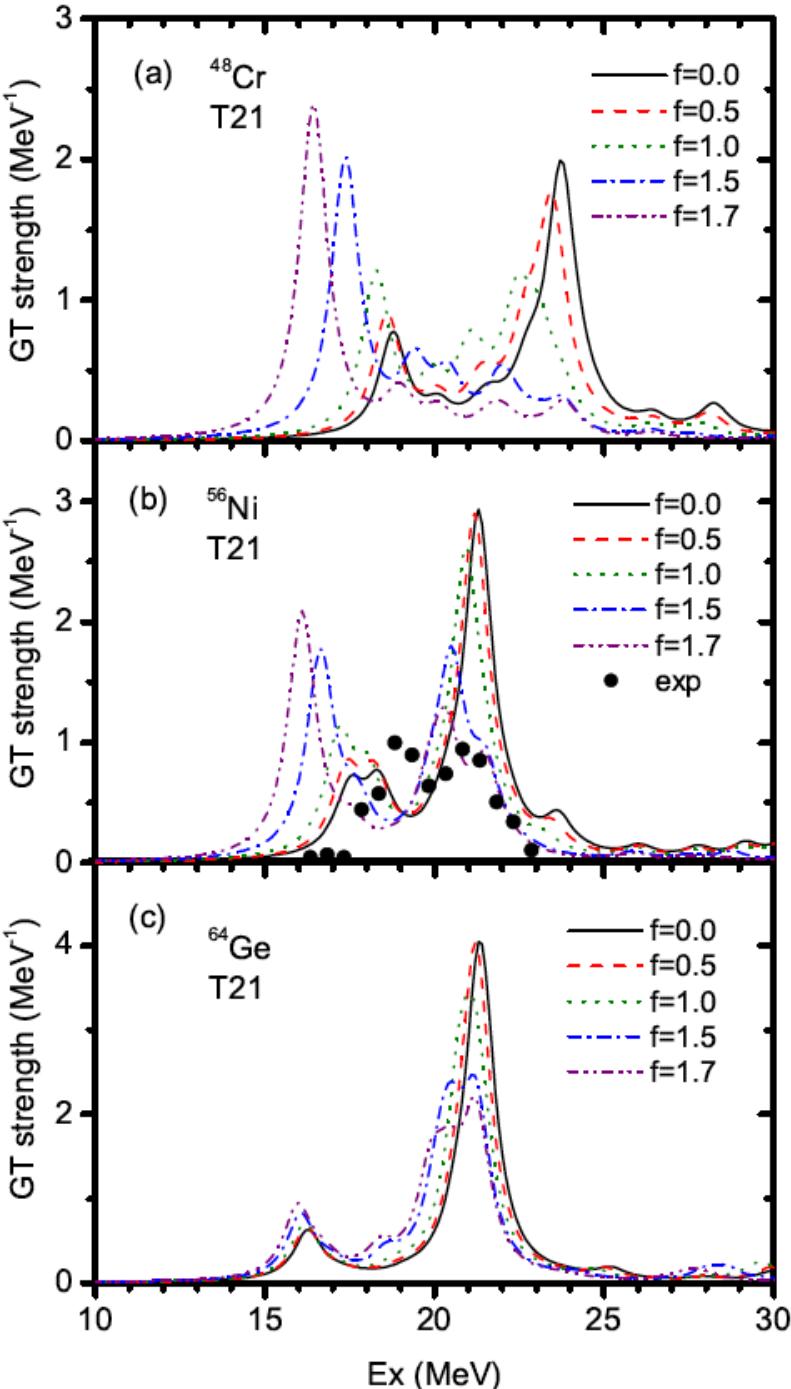
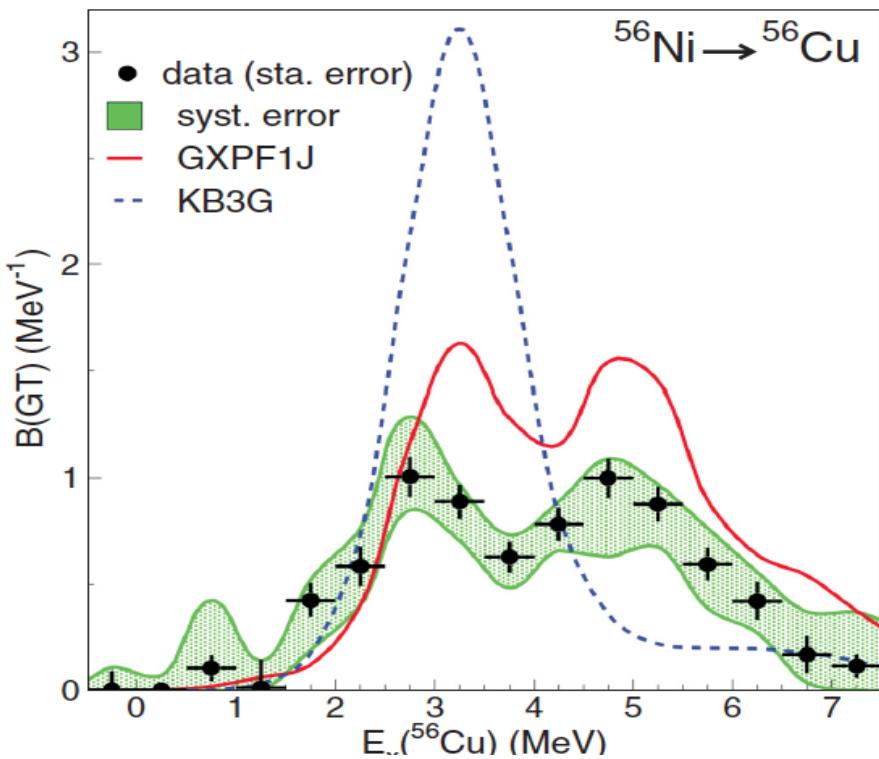


TABLE II: Same as Table I, but for ^{48}Cr and ^{64}Ge with the T=0 pairing interaction $f = 1.5$.

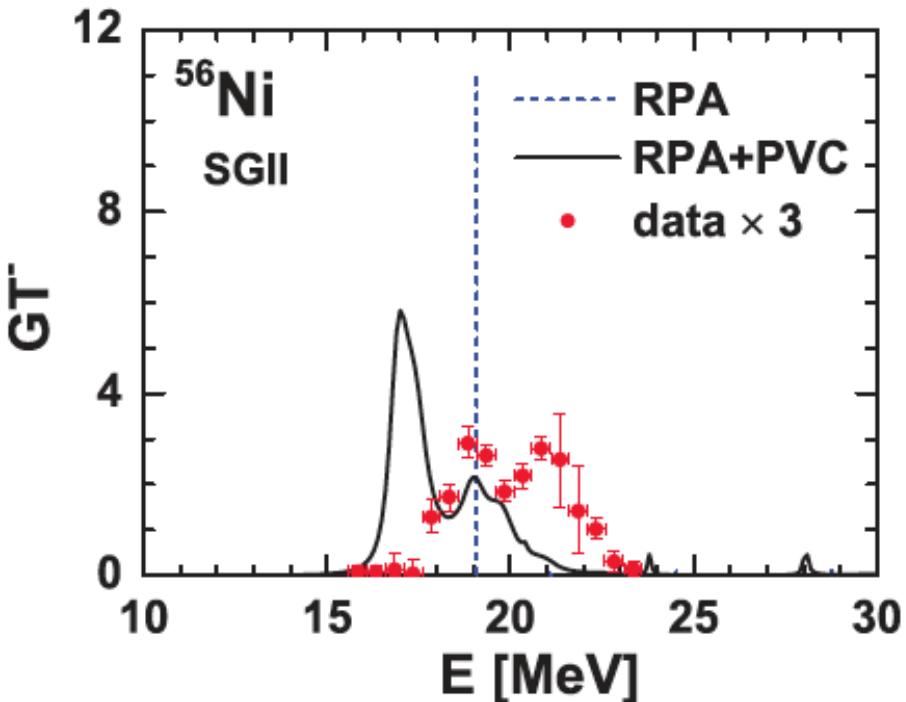
^{48}Cr		$f = 1.5$		
E_x (MeV)	B(GT)	$\nu(v_\nu^2)$	$\pi(v_\pi^2)$	
17.4	5.68	$2\text{p}_{3/2}(0.12)$	$2\text{p}_{3/2}(0.12)$	
		$1\text{d}_{3/2}(0.85)$	$1\text{d}_{3/2}(0.84)$	
		$1\text{f}_{7/2}(0.33)$	$1\text{f}_{5/2}(0.07)$	
		$1\text{f}_{7/2}(0.33)$	$1\text{f}_{7/2}(0.32)$	
19.4	1.19	$2\text{p}_{3/2}(0.12)$	$2\text{p}_{1/2}(0.07)$	
		$2\text{p}_{3/2}(0.12)$	$2\text{p}_{3/2}(0.12)$	
		$1\text{d}_{3/2}(0.85)$	$1\text{d}_{3/2}(0.84)$	
		$1\text{f}_{7/2}(0.33)$	$1\text{f}_{5/2}(0.07)$	
		$1\text{f}_{7/2}(0.33)$	$1\text{f}_{7/2}(0.32)$	
$S_-(\text{GT})=11.77$				
^{64}Ge		$f = 1.5$		
E_x (MeV)	B(GT)	$\nu(v_\nu^2)$	$\pi(v_\pi^2)$	
16.1	2.15	$2\text{p}_{3/2}(0.48)$	$2\text{p}_{1/2}(0.21)$	
21.2	5.21	$1\text{f}_{5/2}(0.14)$	$1\text{f}_{7/2}(0.90)$	
		$1\text{f}_{7/2}(0.92)$	$1\text{f}_{5/2}(0.15)$	
$S_-(\text{GT})=17.26$				

Fine adjustment of shell model int.



Sasano et al., PRC86, 034324(2012)

E_x (MeV)



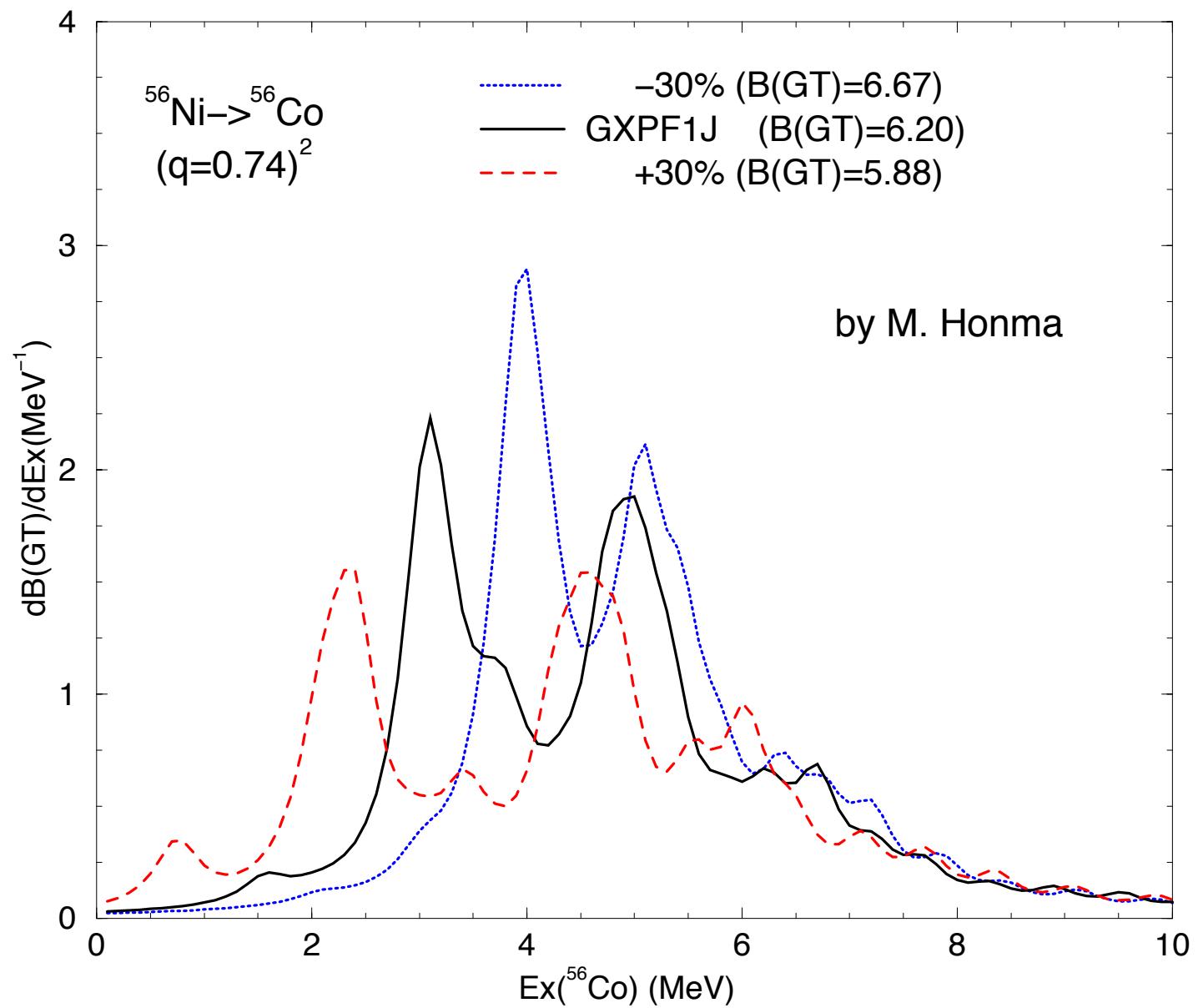
Beyond mean field effect:
Niu et al., PRC85, 034314(2012)

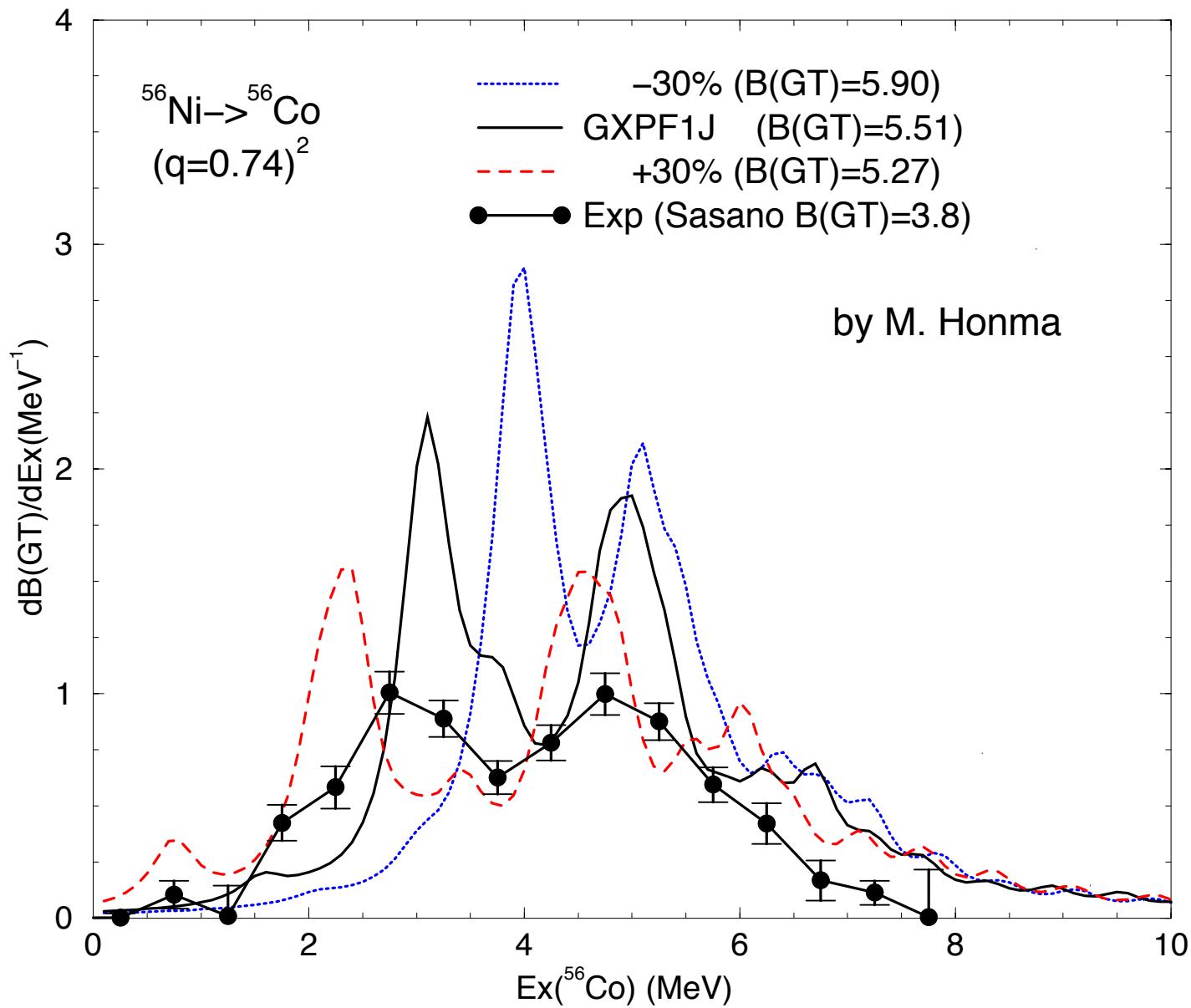
With $(0.74)^2$ quenching factor, still 30% missing strength

$^{56}\text{Ni} \rightarrow ^{56}\text{Cu}$	0–7	$3.8 \pm 0.2(\text{stat.}) \pm 0.8(\text{syst.})$	5.5 (5.2)
$^{55}\text{Co} \rightarrow ^{55}\text{Ni(g.s.)}$	0	0.267 (from Ref. [30])	0.209 (0.253)
$^{55}\text{Co} \rightarrow ^{55}\text{Ni}$	0–15	$5.3 \pm 0.5(\text{stat.})^{+2.5}_{-1.5}(\text{syst.})$	6.8 (6.2)

^aThe quoted error margins do not include the uncertainty in the value for the unit cross section (15%), which would change all strengths by a common scaling factor.

^bA quenching factor of $(0.74)^2$ [44] has been applied to the shell-model summed strengths.





IS and IV M1 response and T=0 spin-triplet pairing correlations

HS, T. Suzuki and M. Sasano (PRC94, 041303(R), 24. Oct. 2016)

USDB*=T=0 pairing matrix elements are 20% stronger than USDB
USDB**=USDB*+ Δ -isobar coupling($q=0.9$)

Exp. Data, Matsubara, et al., PRL115, 102501(2015)

High energy resolution proton inelastic scattering with $E_p=295\text{MeV}$

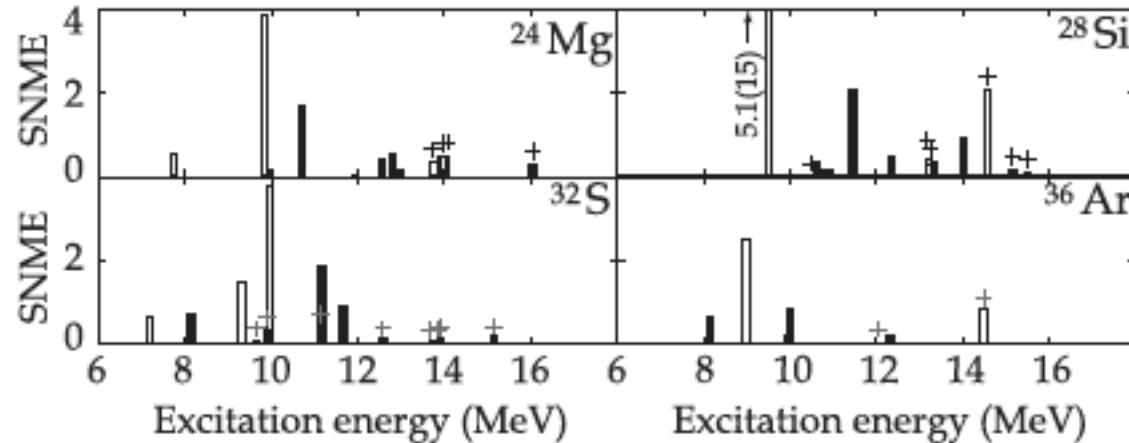
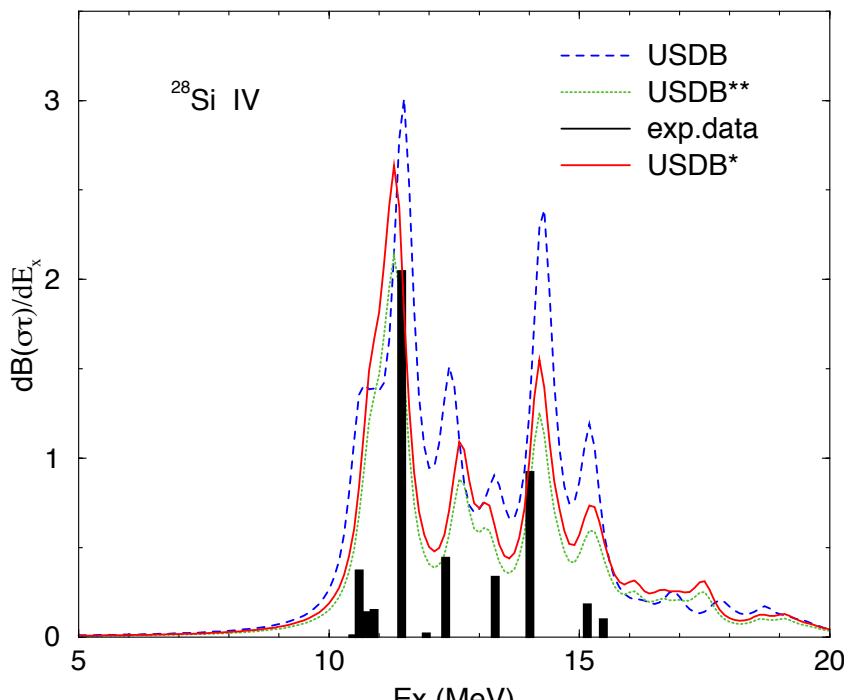
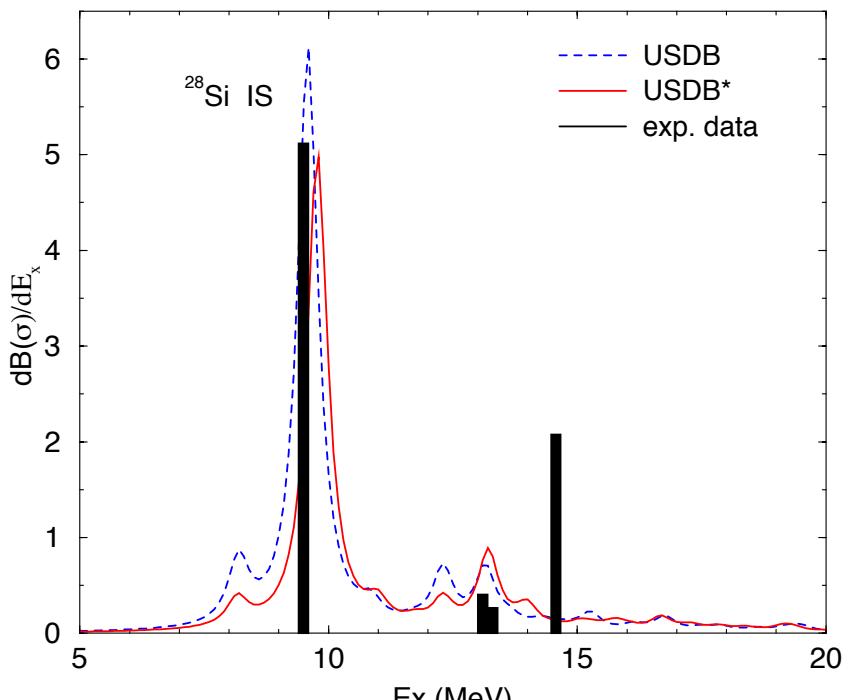
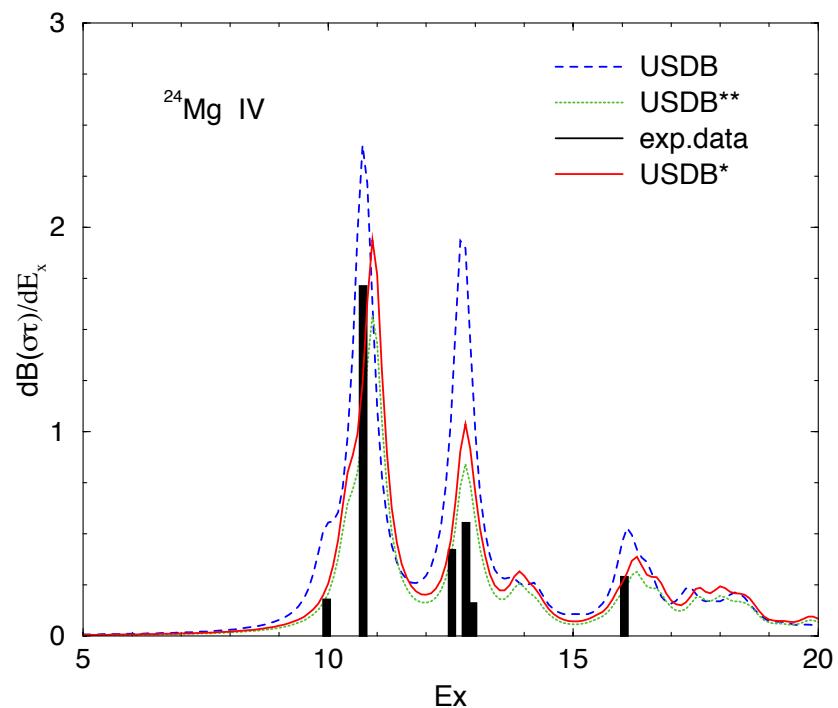
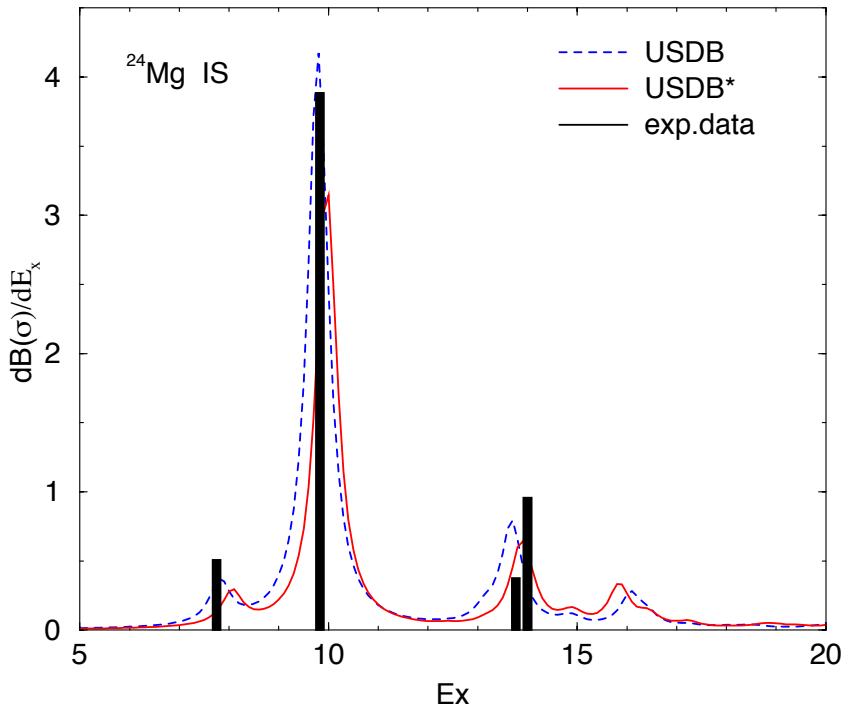
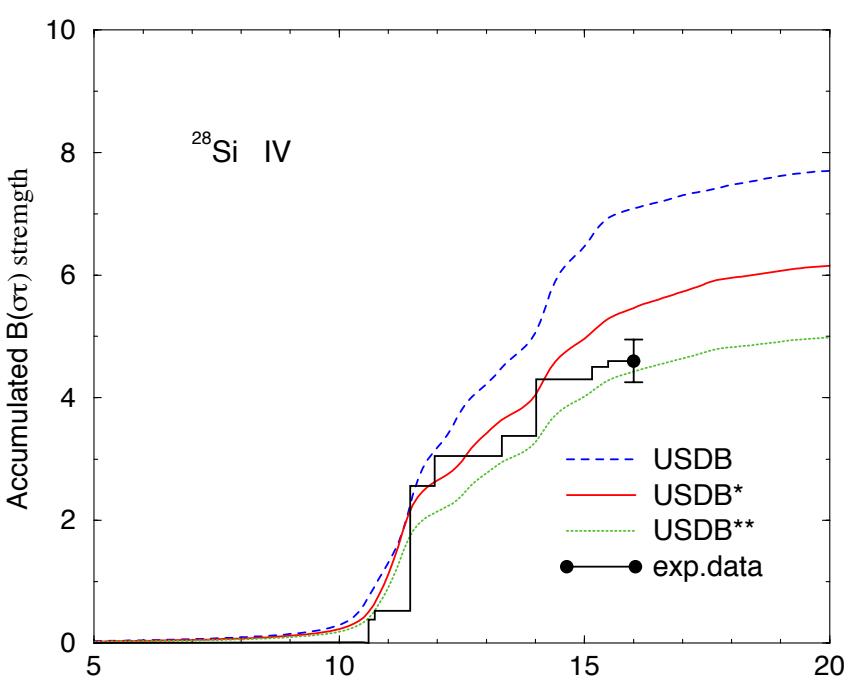
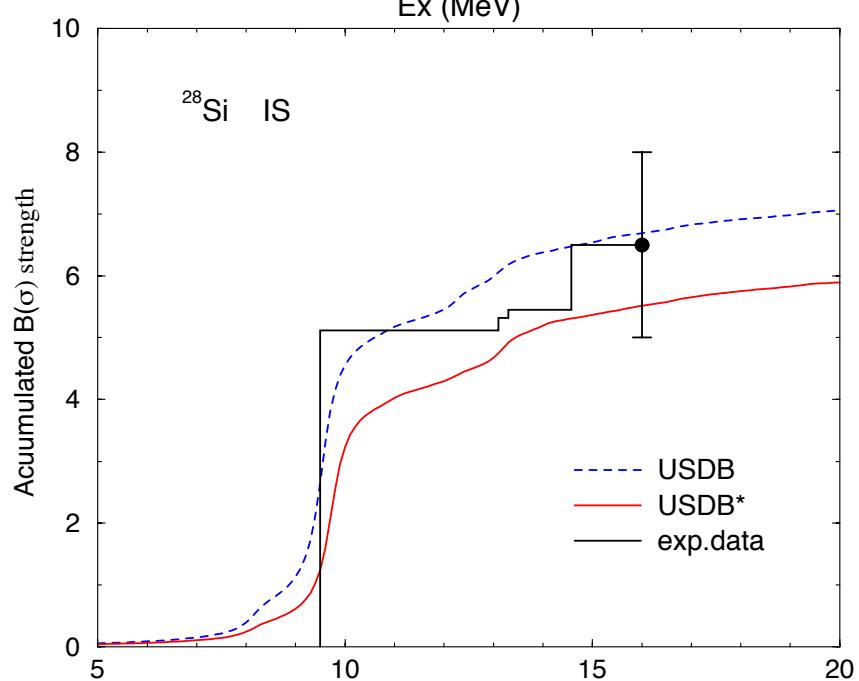
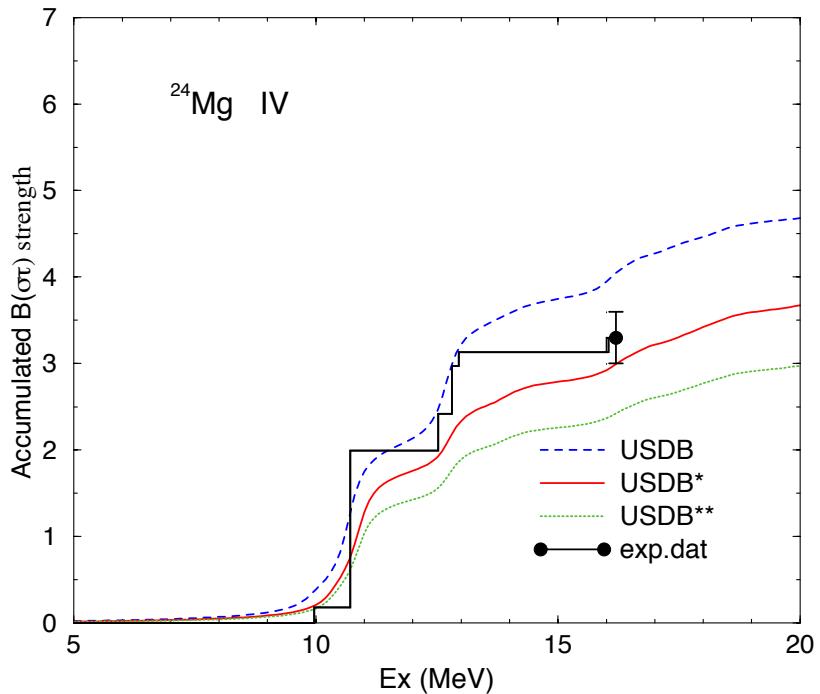
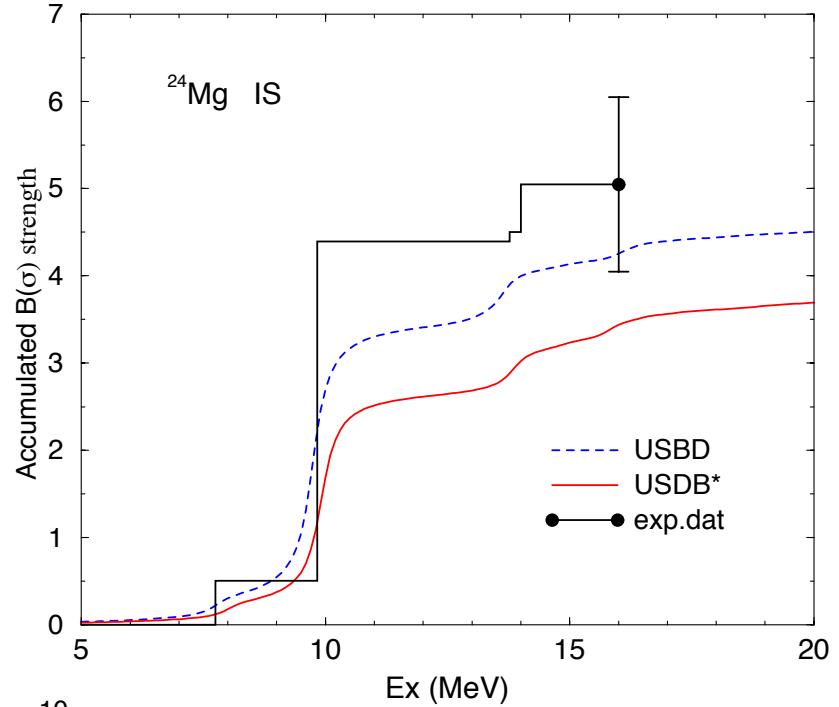
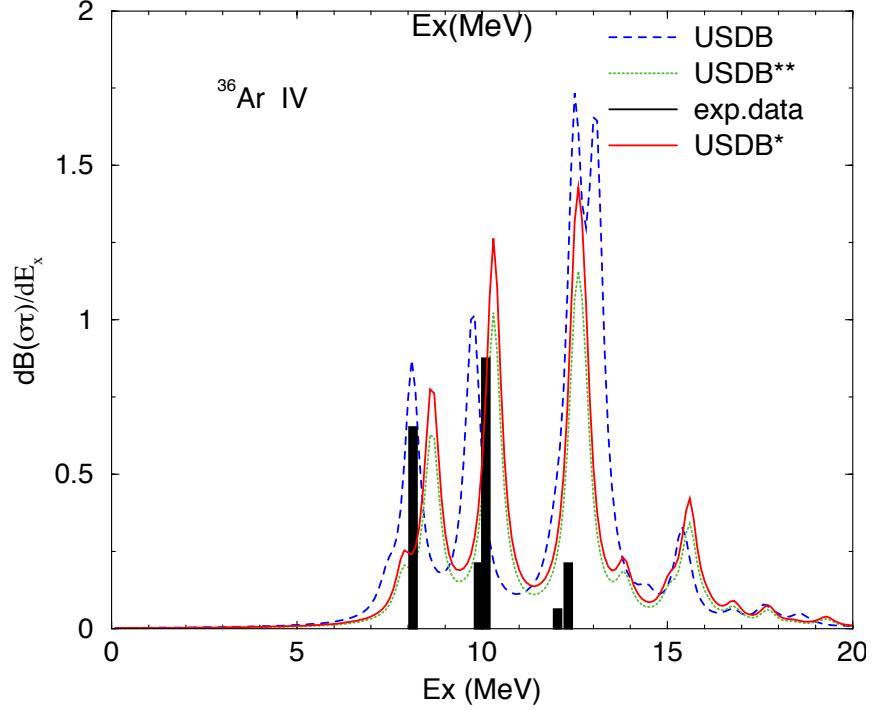
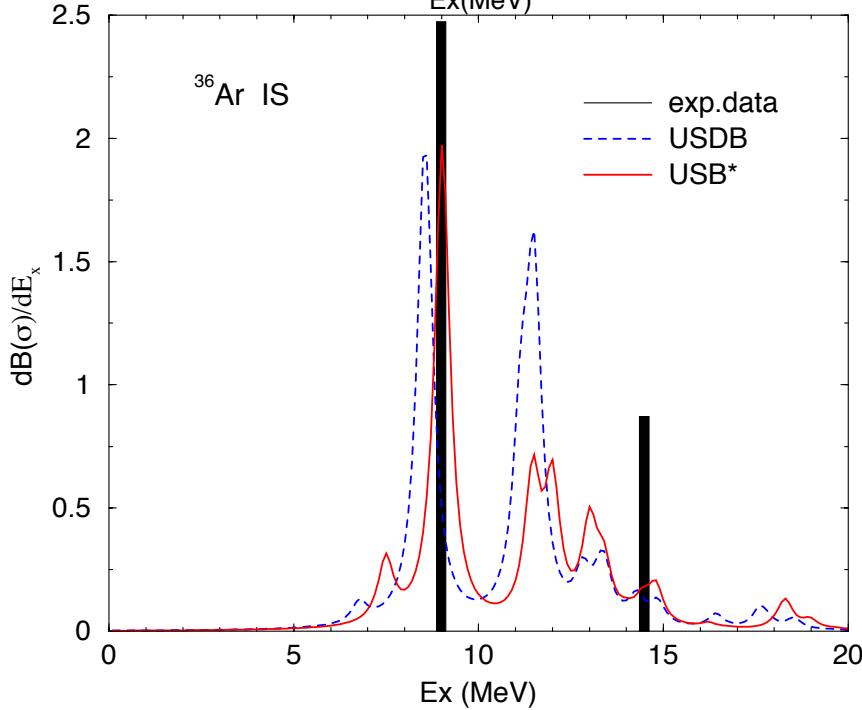
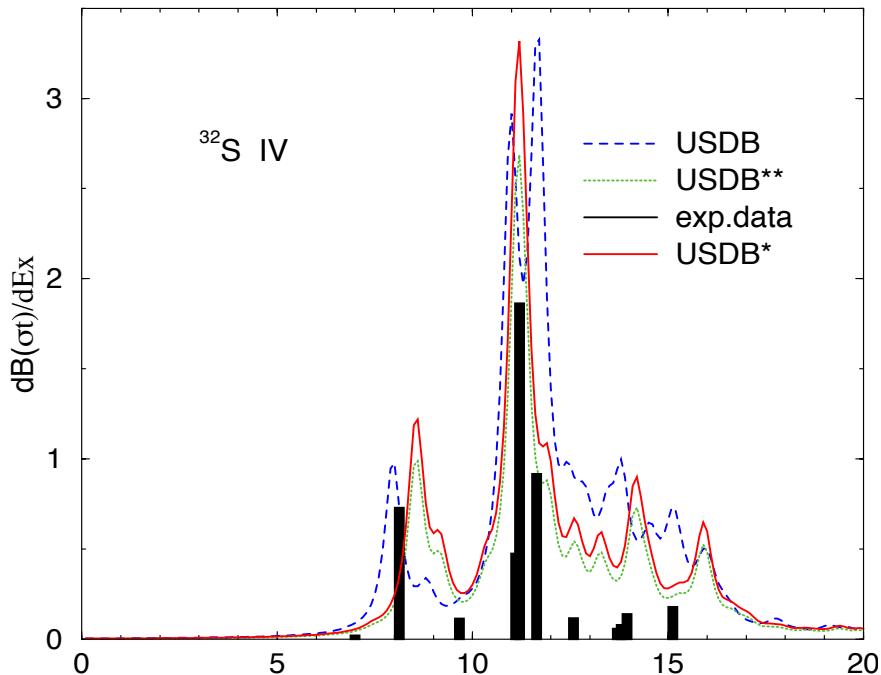
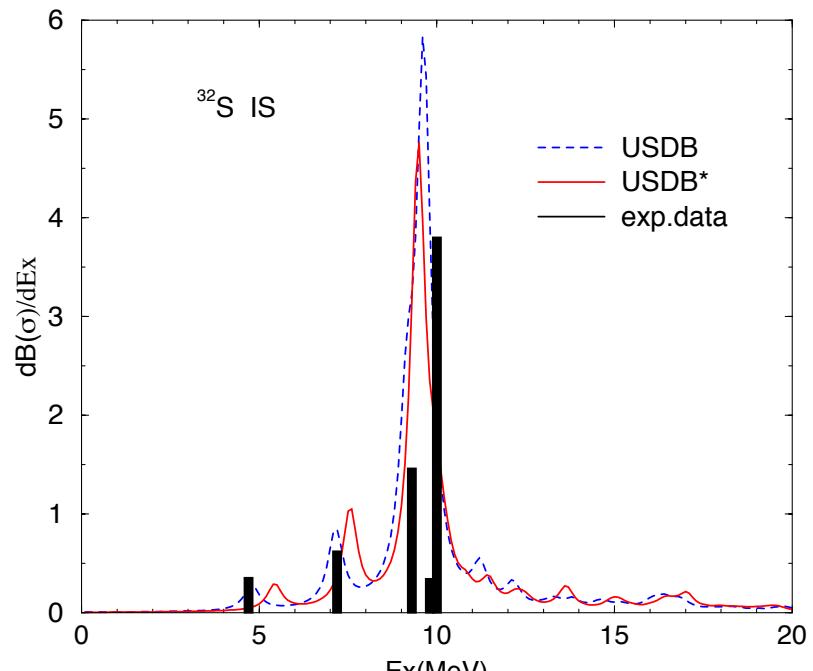
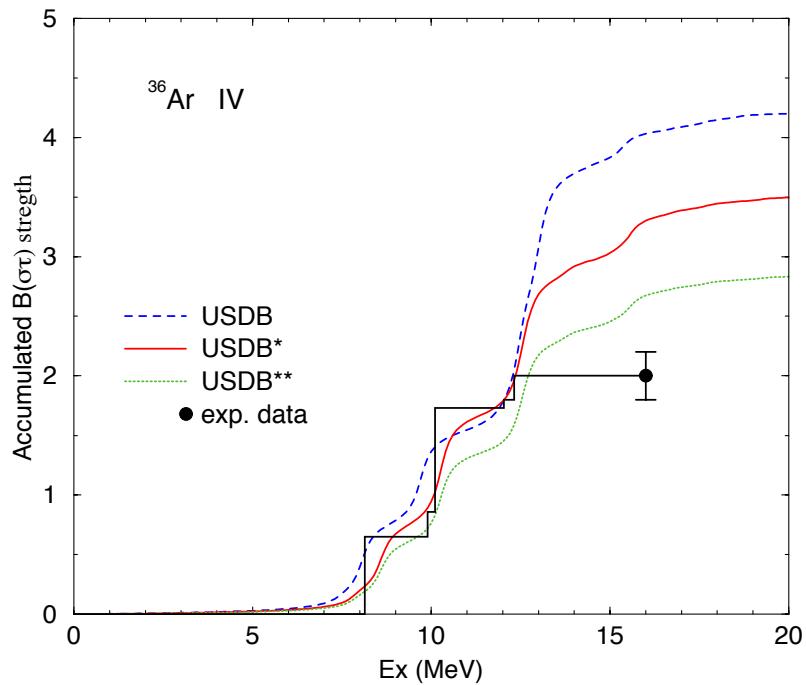
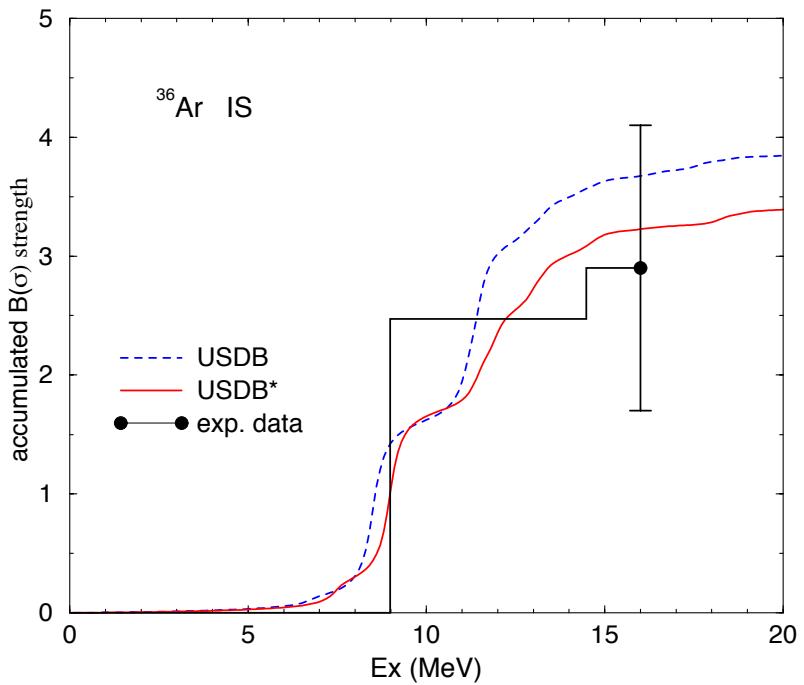
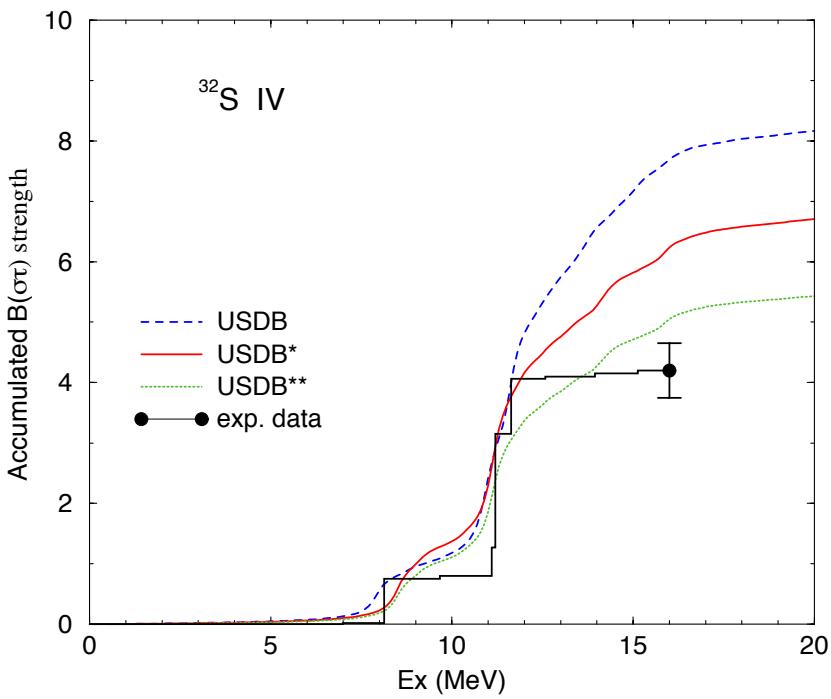
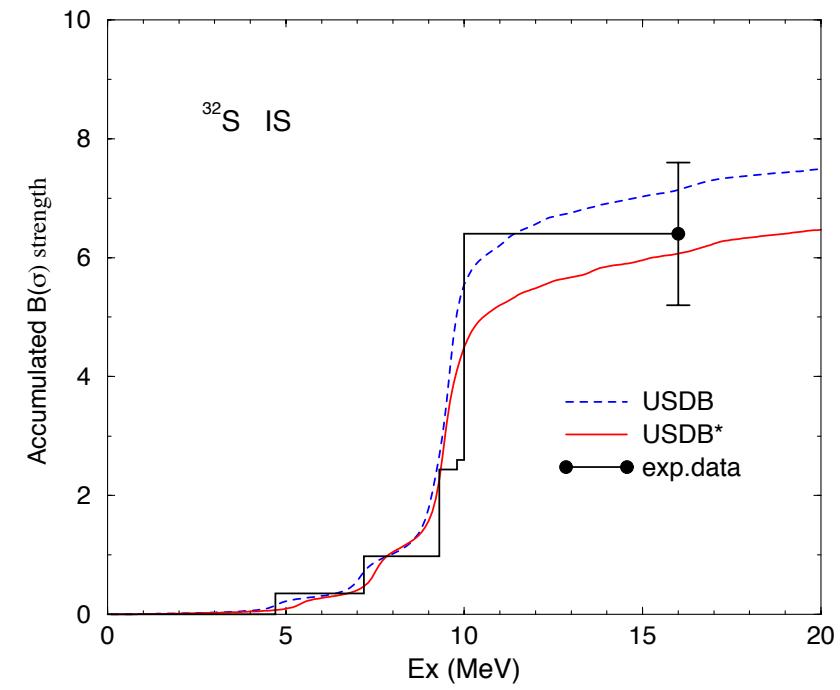


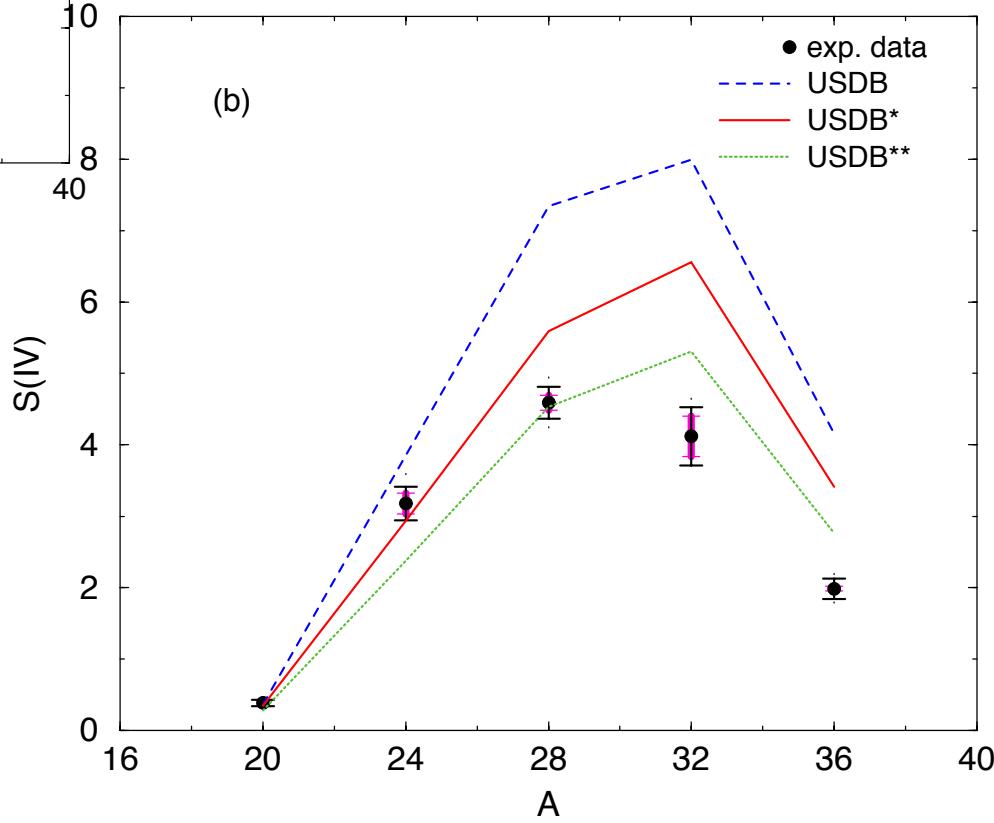
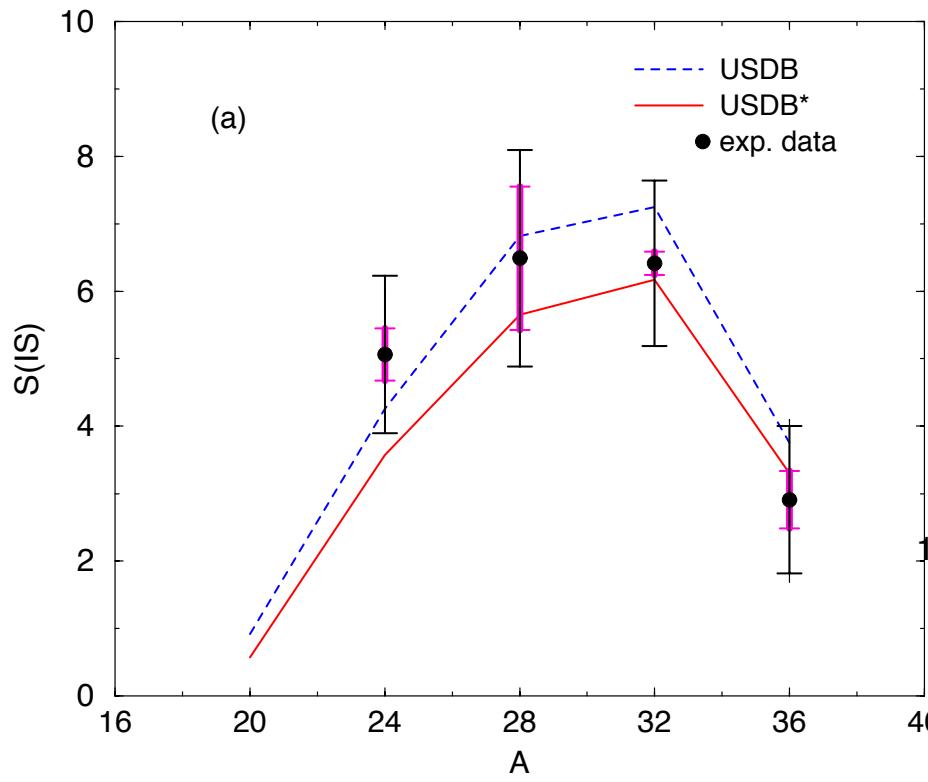
FIG. 2. Observed distributions of IS and IV-spin-*M1* SNME [open (filled) bars represent IS (IV) transitions]. The bars labeled + indicate states with a less confident spin assignment.





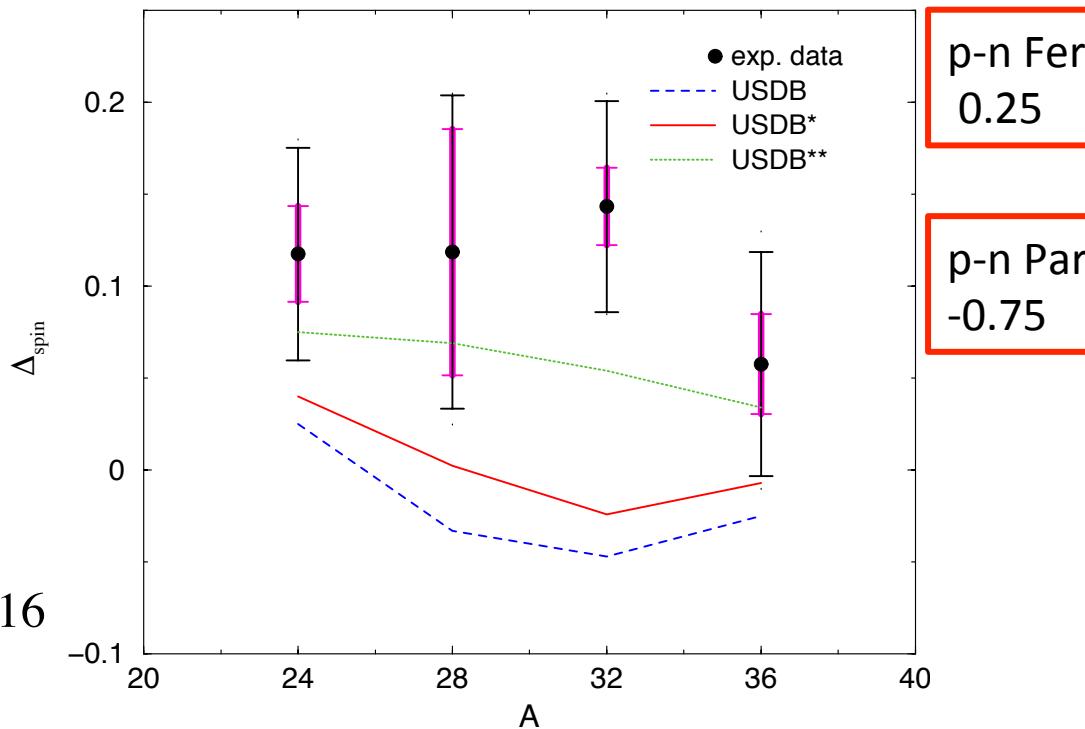






$$\Delta_{\text{spin}} = (S(\text{IS}) - S(\text{IV})) / 16$$

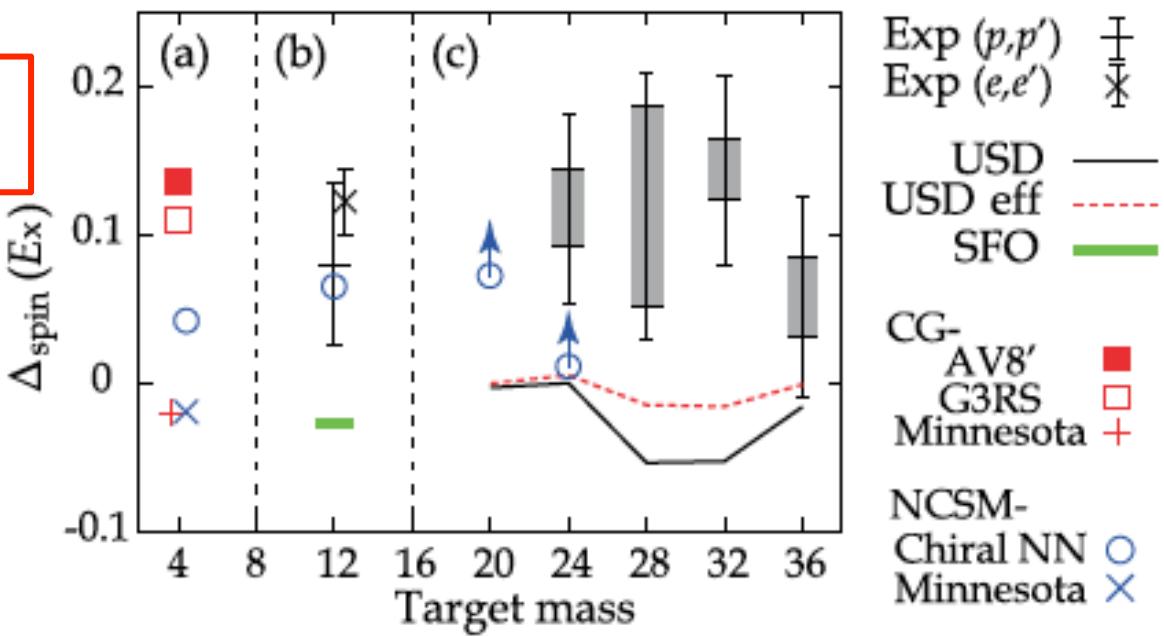
$$= \langle \mathbf{S}_p \cdot \mathbf{S}_n \rangle$$



p-n Ferromagnetic limit
0.25

p-n Paramagnetic limit
-0.75

Matsubara et al.,
PRL115, 102501(2015)



Summary: N=Z nucleus

1. Inversion of 1+ and 0+ states in the energy spectra and strong M1 transitions in odd-odd N=Z nuclei is induced by a strong T=0 pairing correlations competing with T=1 pairing and spin-orbit force.
2. Cooperative role of T=0 and T=1 pairings is studied in Gamow-Teller transitions of N=Z nuclei
3. It is pointed out that the low energy peak appear due to the strong T=0 pairing correlations in the final states.
Supermultiplets of T=1,S=0 and T=0 and S=1 pair
4. two GT peaks in ^{56}Ni → large quenching
 - effective interactions (IS pairing, smaller spin-orbit splitting)
 - beyond mean field effect
5. Future perspective (experiment):
New experiments in N=Z nuclei, ^{48}Cr and ^{64}Ge are approved by PAC Dec. 2016 in RIKEN .
6. Future perspectives (theory)
 - a. HFB+QRPA+PV (Particle-vibration coupling
Yifei Niu, Gianluca Colo)
 - b. Fine fittings of energy density functions for spin-isospin excitations
(which was done already for Shell model interactions: GPFX1J
BY Toshio Suzuki, Michio Honma)

Collaborators

- Pairing and Tensor correlations on spin-isospin excitations
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Gianluca Colo (University of Milano)
- IS Pairing correlations (Three-body model)
Y. Tanimura (Orsay -> GSI)
K. Hagino (Tohoku University)
- IS and IV M1 and spin-spin correlations
T. Suzuki (Nihon University)
M. Sasano (RIKEN)