

# Constraining the nuclear symmetry energy from collective excitations

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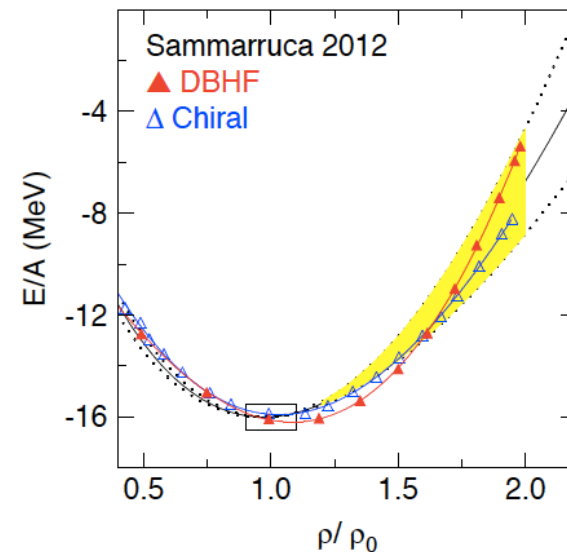
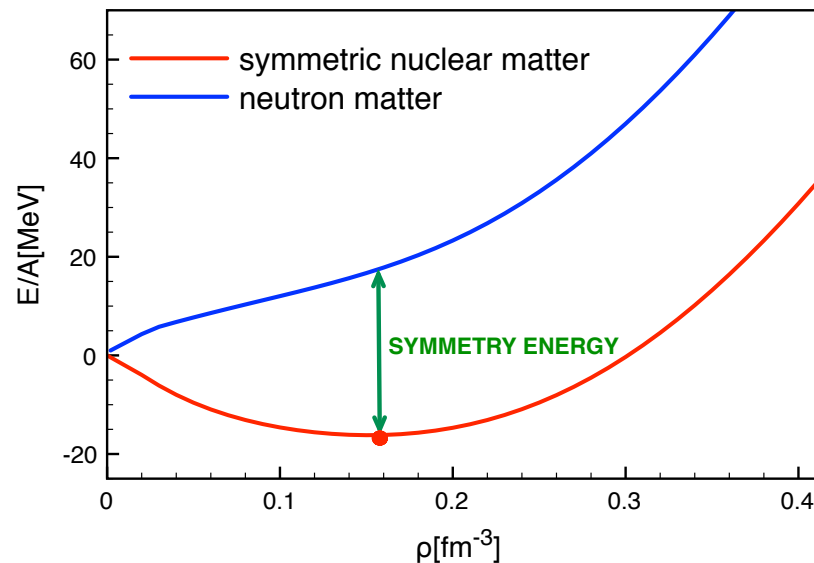
**QuantiXLie**

CENTER OF EXCELLENCE FOR THE THEORY OF QUANTUM AND COMPLEX SYSTEMS AND LIE ALGEBRA REPRESENTATION



# INTRODUCTION

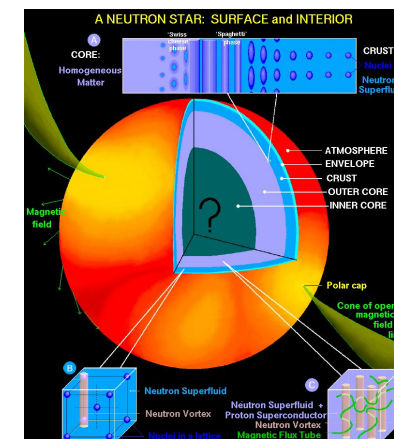
- Nuclear matter equation of state (EOS) plays important role in nuclear physics and astrophysics
- EOS of neutron matter is essential to understand the physics of neutron stars and binary mergers



A. Le Fevre et al., NPA 945, 112 (2016)

- EOS beyond the saturation point?

- Symmetry energy  $S(\rho)$  describes the increase in the energy of the  $N \neq Z$  system as protons are turned into neutrons;
- It is important for understanding the properties of neutron-rich matter and neutron rich nuclei
- $S(\rho)$  is constrained by data on finite nuclei near the saturation density

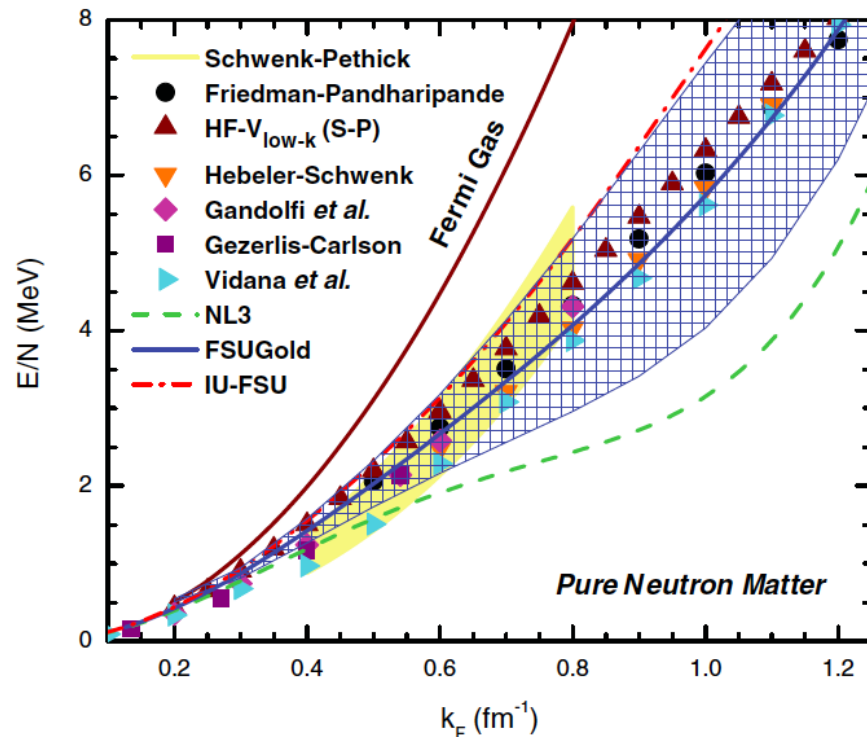


- Neutron star properties ?

# NEUTRON MATTER EOS

- Energy per particle for pure neutron matter with uncertainty estimates
  - from phenomenological toward ab-initio calculations

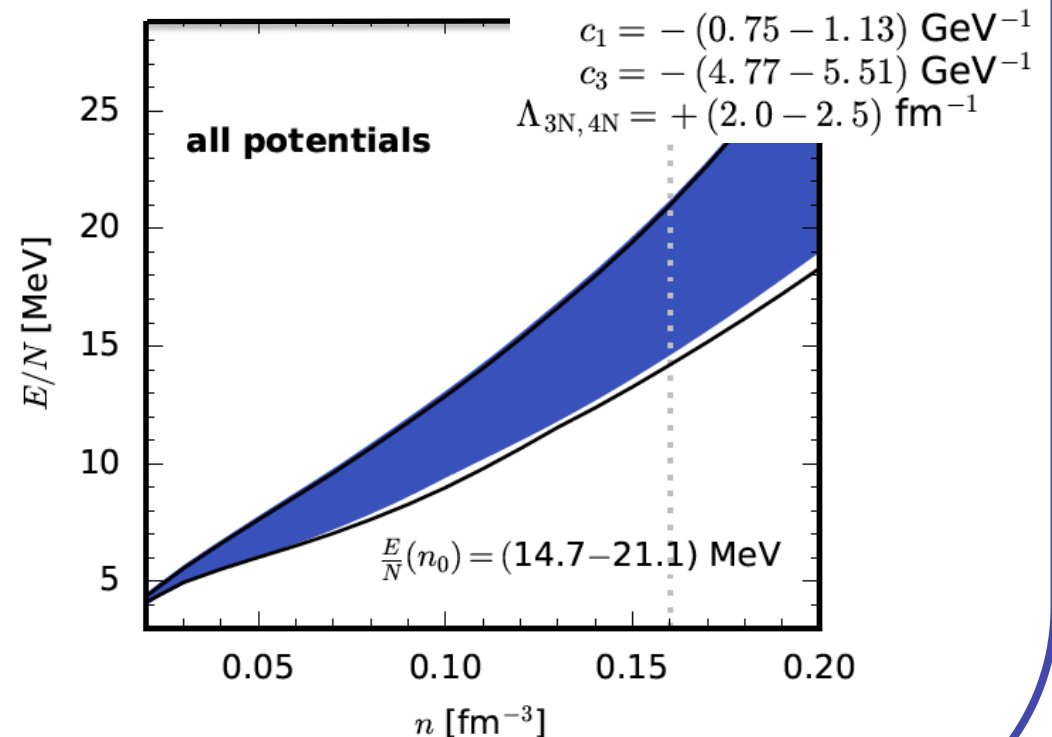
→ Theoretical uncertainties associated with the FSUGold model.



F.J. Fattoyev and J. Piekarewicz, PRC 86, 015802 (2012).

→ Theoretical uncertainties due to parameter variation in 3N forces

**full N<sup>3</sup>LO calculation**



Drischler, A. Carbone, K. Hebeler, A. Schwenk, arXiv:1608.05615 (2016).

# SYMMETRY ENERGY

- Nuclear matter equation of state:

$$E(\rho, \delta) = E_{SNM}(\rho) + E_{sym}(\rho)\delta^2 + \dots$$

$$\rho = \rho_n + \rho_p \quad \delta = \frac{\rho_n - \rho_p}{\rho}$$

- Symmetry energy term:

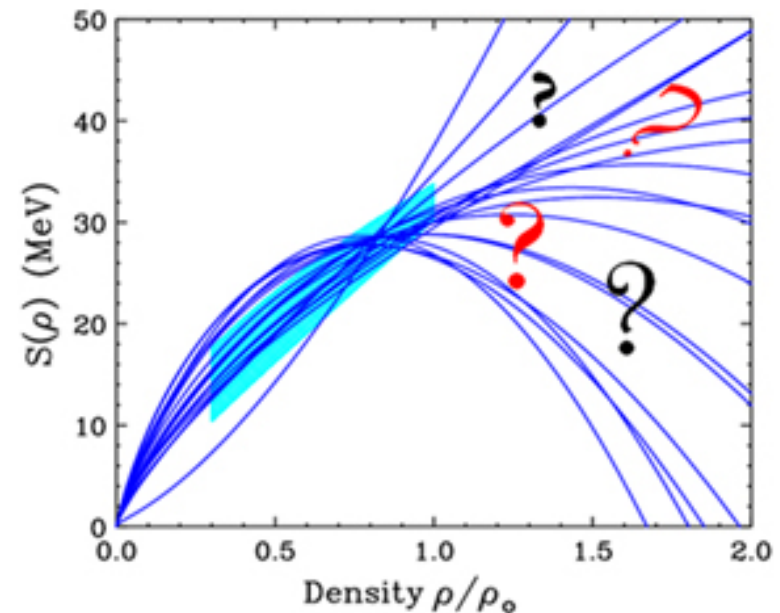
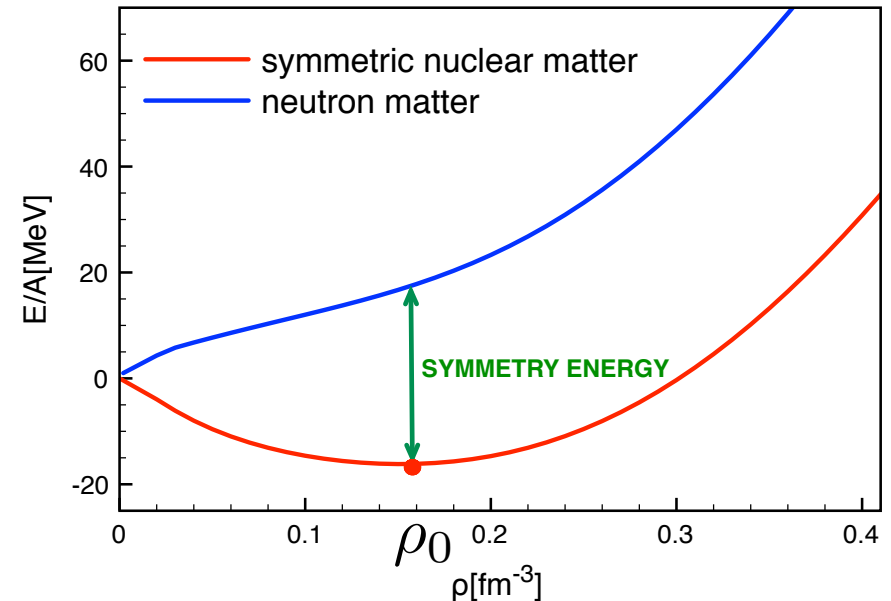
$$E_{sym}(\rho) \equiv S_2(\rho) = J - L\epsilon + \dots$$

$$\epsilon = (\rho_0 - \rho)/(3\rho_0)$$

$$L = 3\rho_0 \left. \frac{dS_2(\rho)}{d\rho} \right|_{\rho_0}$$

**J** – symmetry energy at saturation density

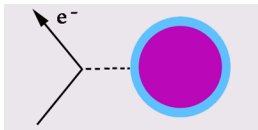
**L** – slope of the symmetry energy  
(related to the pressure of neutron matter)



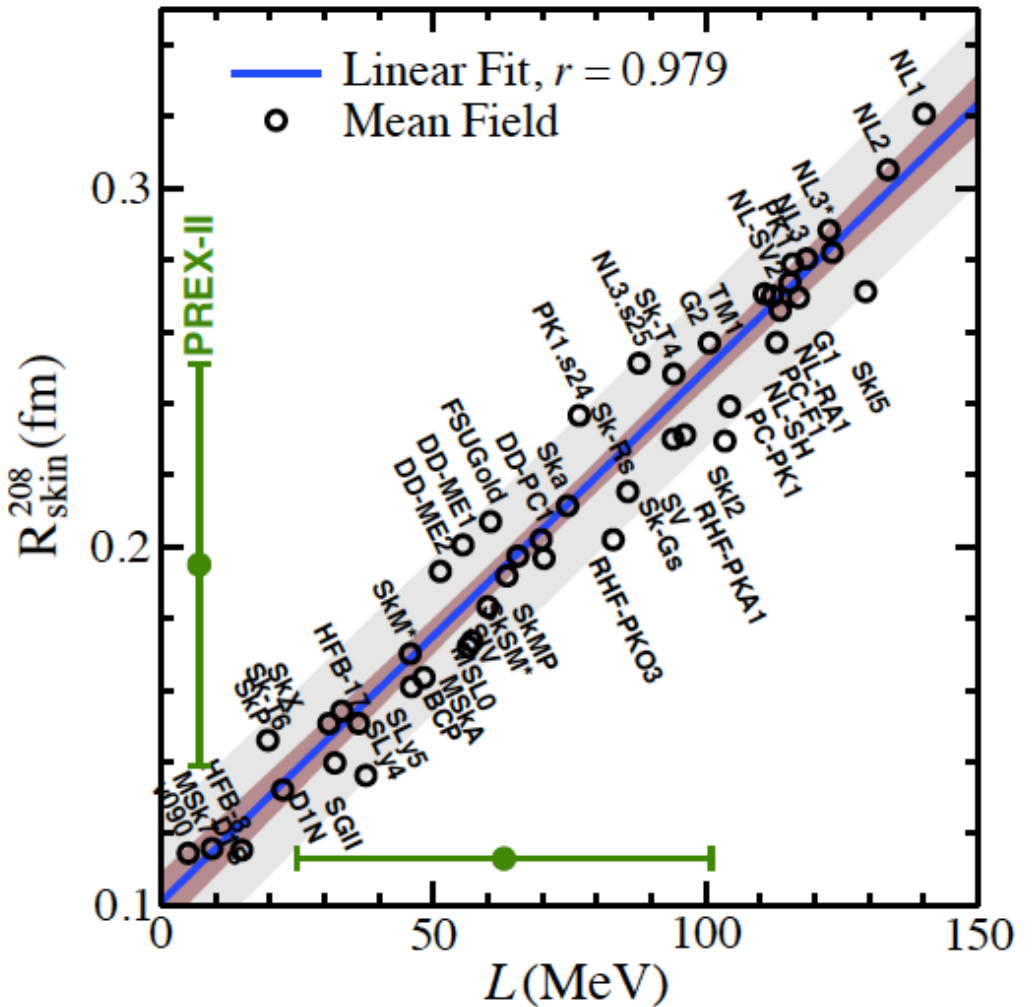
## NEUTRON SKINS AND THE SYMMETRY ENERGY

- In nuclei, thickness of the neutron skin  $r_{np} = r_n - r_p$  depends on the pressure of neutron matter  $P_{PNM} \sim L$
- the size of  $r_{np}$  increases with pressure as neutrons are pushed out against surface tension
- The pressure of neutron matter  $P_{PNM} \sim L$  is poorly constrained
- Parity violating electron scattering - Lead Radius Experiment (PREx) @ JLab:

$$R_n - R_p = 0.33^{+0.16}_{-0.18}$$



Abrahamyan et al. PRL 108, 112502 (2012)



X. Roca-Maza et al., PRL106, 252501 (2011)  
J. Piekarewicz, arXiv:1502.01559 (2015)

## COLLECTIVE EXCITATIONS AND THE SYMMETRY ENERGY

- There are various (isovector) modes of collective excitations that also provide constraints on the neutron skin thickness, with recent experimental data available

- **Isovector giant dipole resonances**

- **Dipole polarizability:** A. Tamii et al., PRL 107, 062502 (2011)

$$\alpha_D \sim m_{-1}$$

D.M. Rossi et al., PRL 111, 242503 (2013)

T. Hashimoto et al., Phys. Rev. C 92, 031305(R) (2015)

- **Pygmy dipole resonances:** A. Carbone et al., PRC 81, 041301(R) (2010)  
A. Klimkiewicz et al., PRC 76, 051603(R) (2007)

- **Anti-analog GDR:** A. Krasznahorkay et al., PLB 720, 428 (2013)

- **Isovector giant quadrupole resonances:** S.S. Henshaw, M.W. Ahmed, et al, PRL 107, 222501 (2011)

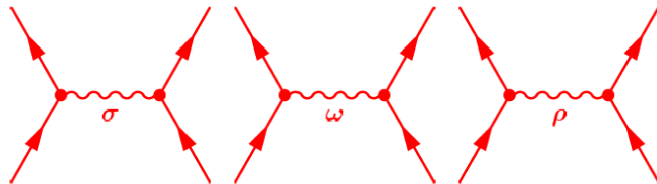
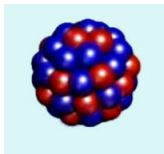
- ...

- The goal: use collective excitations to constrain the symmetry energy

# THEORY FRAMEWORK

## RELATIVISTIC NUCLEAR ENERGY DENSITY FUNCTIONAL

i) Nucleons are Dirac particles coupled by the exchange mesons and the photon field



ii) Four-fermion contact interaction (Point-coupling model)



- The model parameters are constrained directly by many-body observables (masses, charge radii, pseudo-data, ...)
- Complicated many body dynamics encoded in the functional and its empirical constants
- DIRHB -- a relativistic self-consistent mean-field framework for atomic nuclei  
Relativistic Hartree Bogoliubov model [T. Niksic et al., Comp. Phys. Comm. 185, 1808 \(2014\)](#).
- In the small amplitude limit, self-consistent quasiparticle random phase approximation (QRPA) is used to compute nuclear excitations, etc.



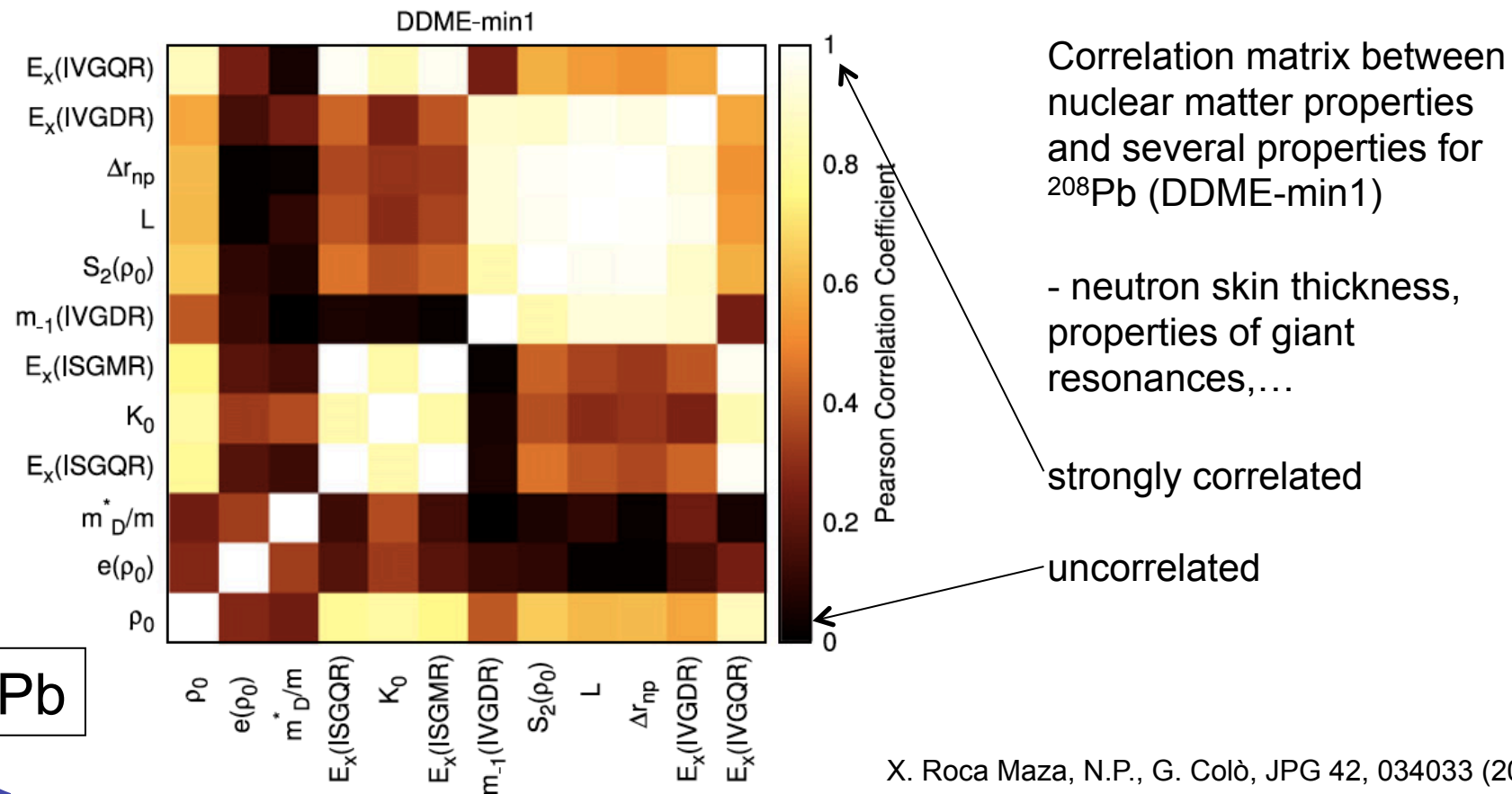
# CORRELATIONS: NUCLEAR MATTER vs. PROPERTIES OF NUCLEI

- Covariance analysis in the EDF framework - information on relevant correlations and statistical uncertainties

- Pearson product-moment correlation coefficient**  
provides a measure of the correlation (linear dependence)  
between two variables A and B.

Curvature matrix:

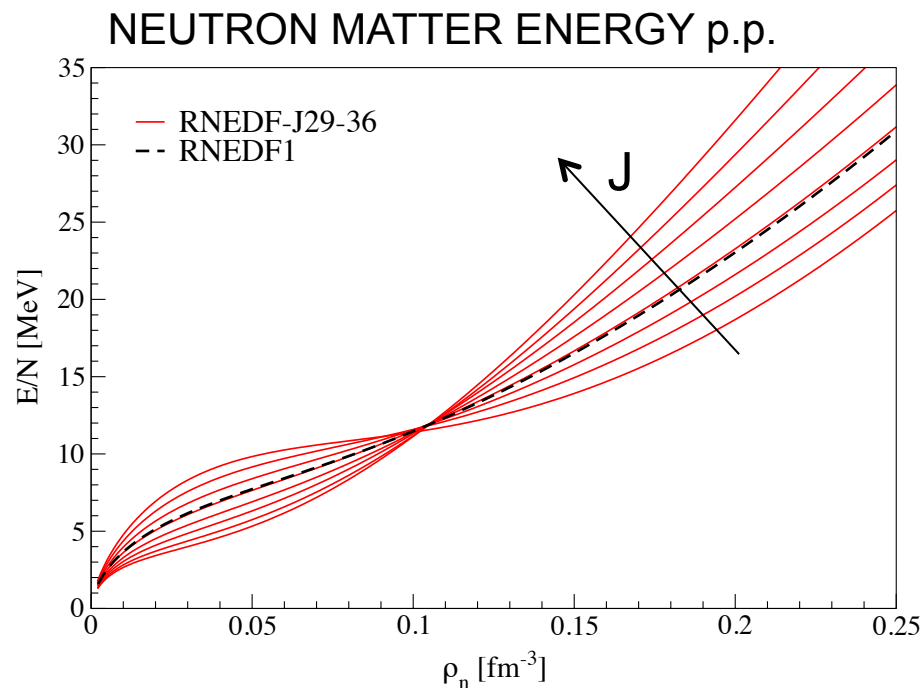
$$\mathcal{M}_{ij} = \frac{1}{2} \partial_{p_i} \partial_{p_j} \chi^2|_{\mathbf{p}_0}$$





## VARIATION OF THE SYMMETRY ENERGY IN CONSTRAINING THE EDF

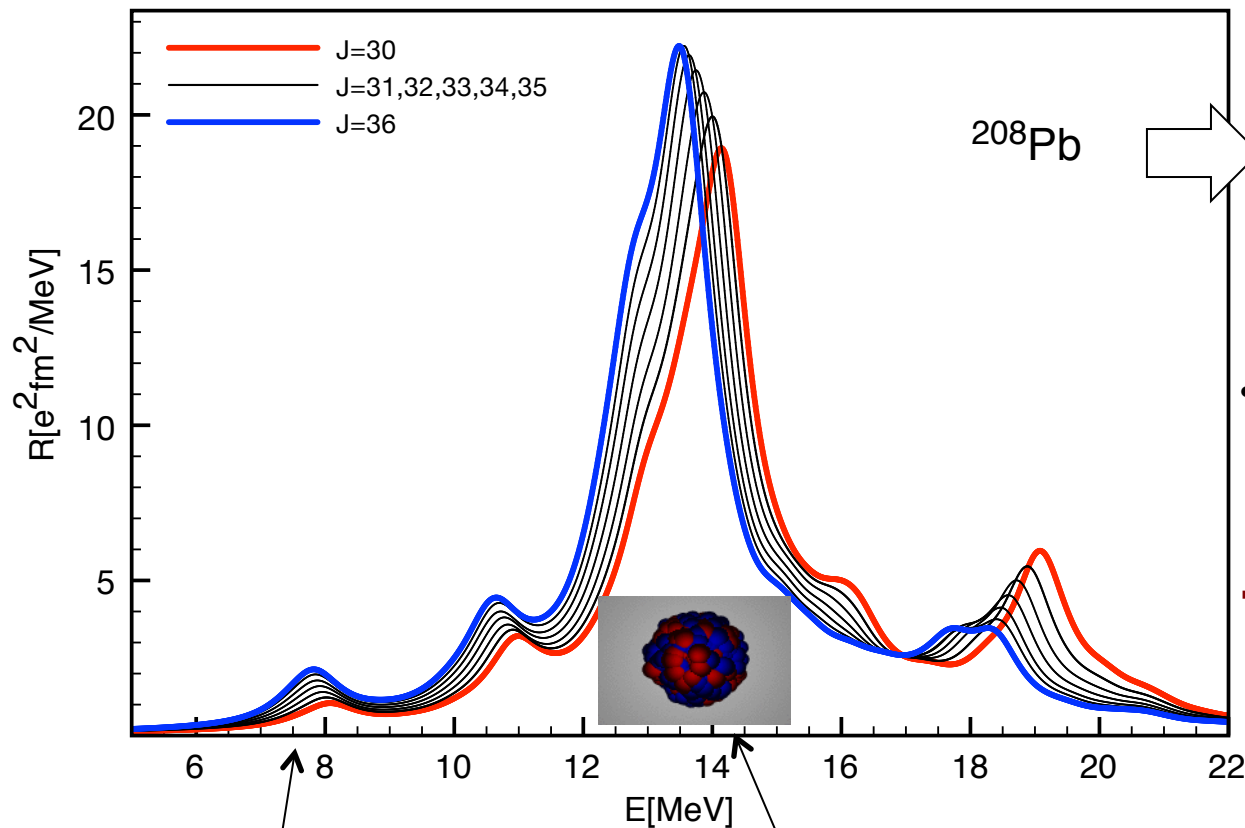
- Adjust the properties of 72 spherical nuclei to exp. data (binding energies, charge radii, diffraction radii, surface thickness, pairing gaps)
- Establish a set of 8 relativistic point coupling interactions that span the range of values of the symmetry energy at saturation density:  $J=29,30,\dots,36$  MeV
- Each interaction is determined independently using the same dataset supplemented with an additional constraint on  $J$



$J$ [MeV]	$L$ [MeV]
29	31.9
30	37.0
31	44.1
32	52.5
33	62.2
34	72.3
35	83.4
36	94.3

# CONSTRAINING THE SYMMETRY ENERGY

- Isovector dipole transition strength for  $^{208}\text{Pb}$  using a set of relativistic point coupling interactions which vary the symmetry energy properties ( $J=30,31,\dots,36$  MeV)



Pygmy strength

Isovector giant dipole resonance

- Isovector giant dipole resonance
- Pygmy dipole strengths
- Dipole polarizability ( $\alpha_D \sim m_{-1}$ )

- The transition strength is sensitive on the properties of symmetry energy - ( $J, L$ )

→ Dipole response can be used to constrain effective nuclear interactions (isovector channel)

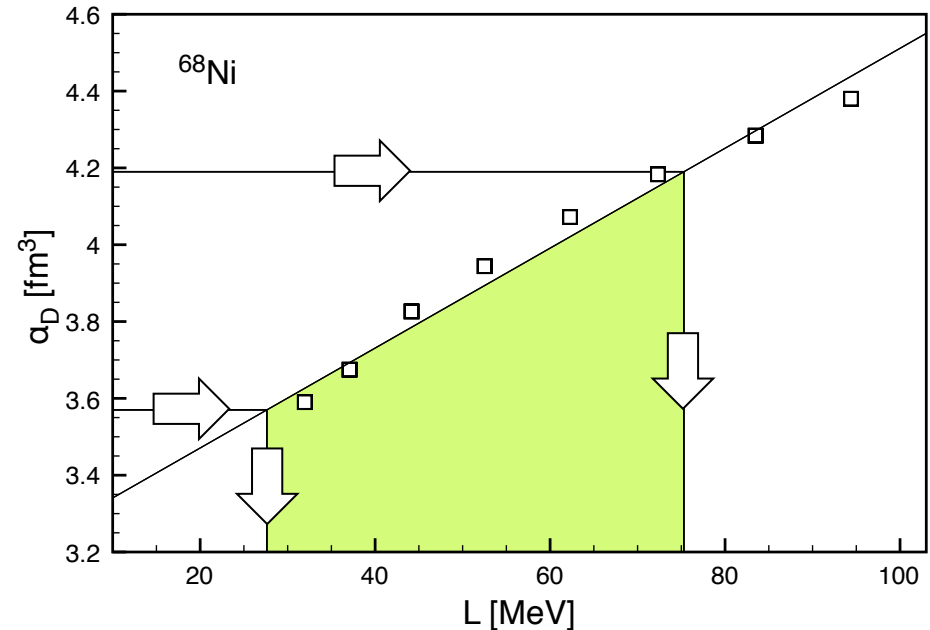
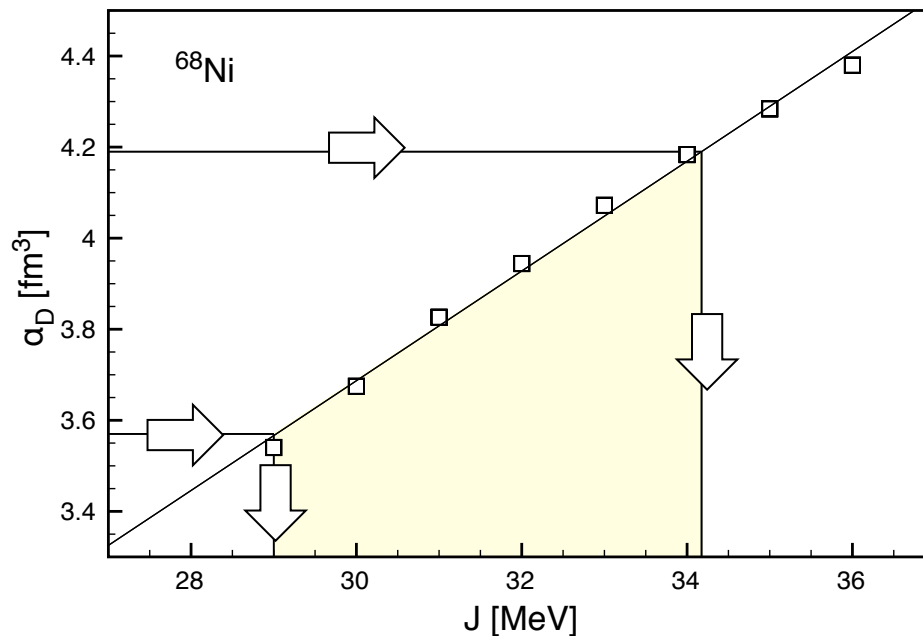
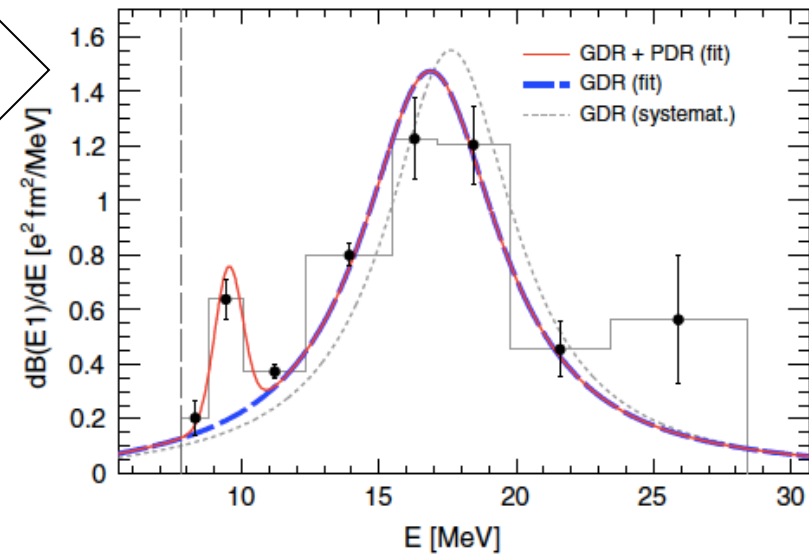
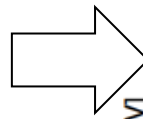
- There are exp. data available on the dipole response in nuclei ( $\alpha_D$ , IVGDR, pygmy strength)

# DIPOLE POLARIZABILITY IN $^{68}\text{Ni}$ AND SYMMETRY ENERGY

- Measurement of dipole polarizability of unstable neutron rich  $^{68}\text{Ni}$

D. Rossi et al, PRL 111, 242503 (2013)

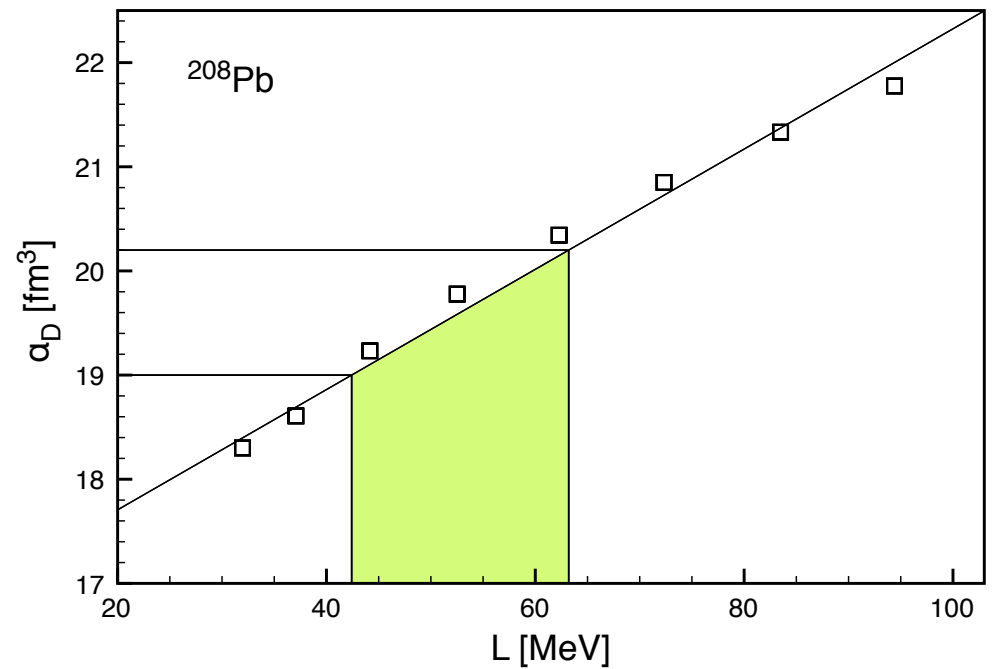
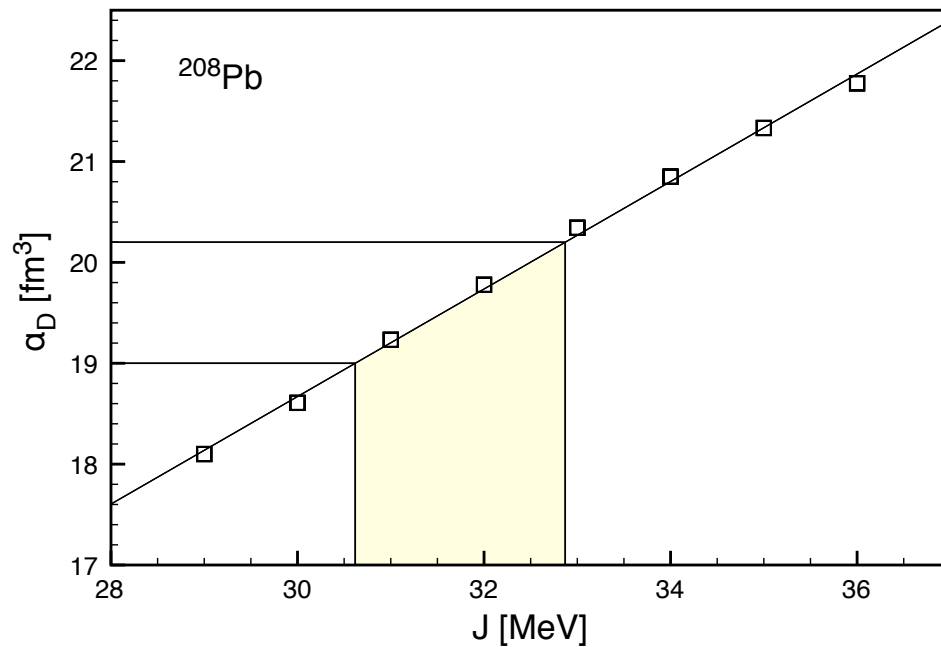
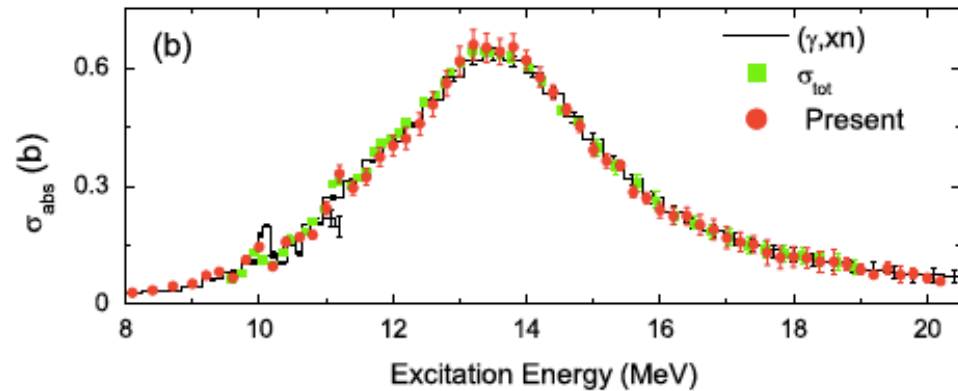
- Implementation to constrain the symmetry energy using relativistic point coupling interactions



# DIPOLE POLARIZABILITY IN $^{208}\text{Pb}$ AND SYMMETRY ENERGY

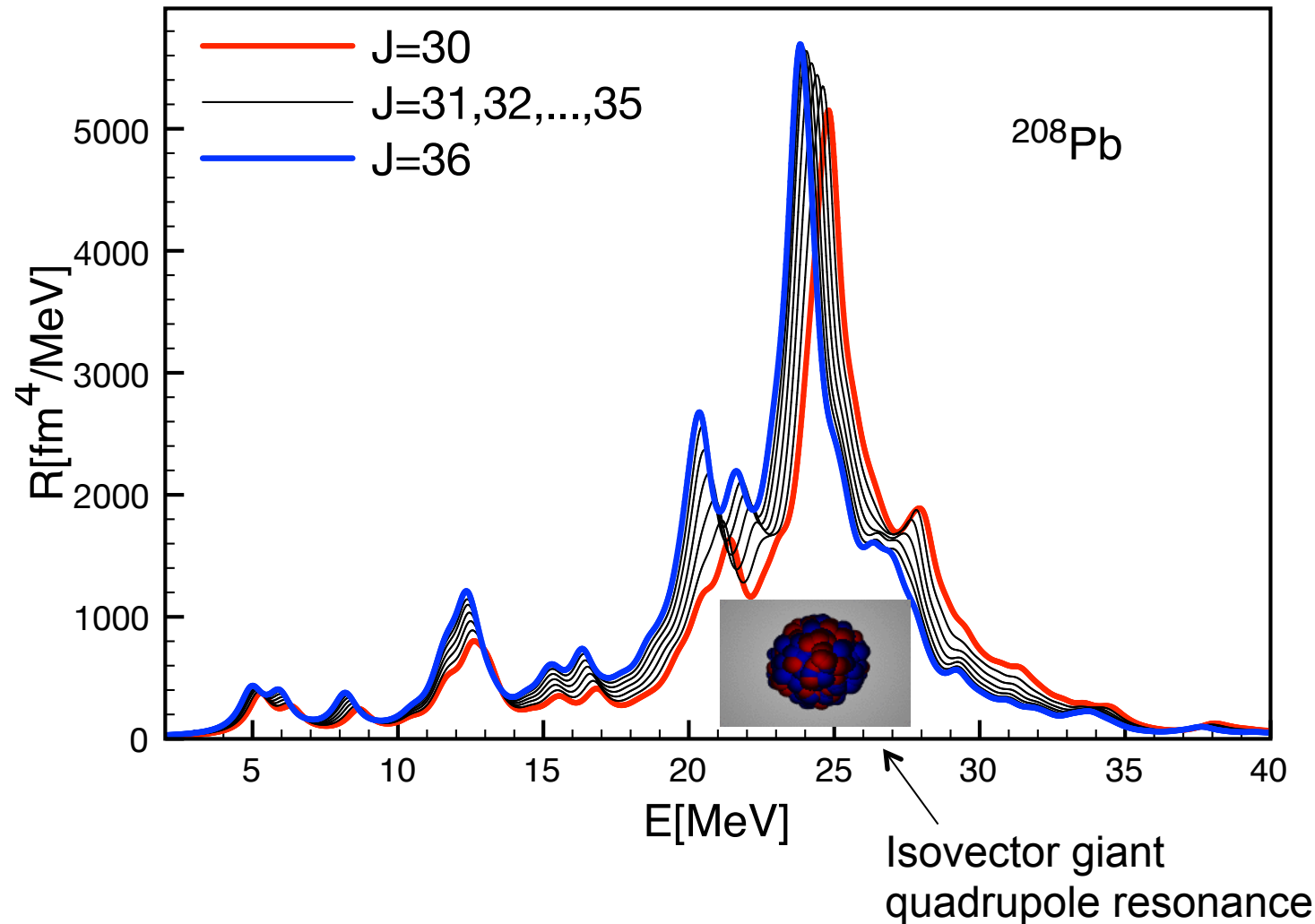
- Using relativistic point coupling interactions and measurement of dipole polarizability in  $^{208}\text{Pb}$

• A. Tamii et al., PRL 107, 062502 (2011)



## GIANT QUADRUPOLE RESONANCES IN $^{208}\text{Pb}$ AND SYMMETRY ENERGY

- Isovector quadrupole transition strength for  $^{208}\text{Pb}$  using a set of relativistic point coupling interactions which vary the symmetry energy properties ( $J=30,31,\dots,36$  MeV)

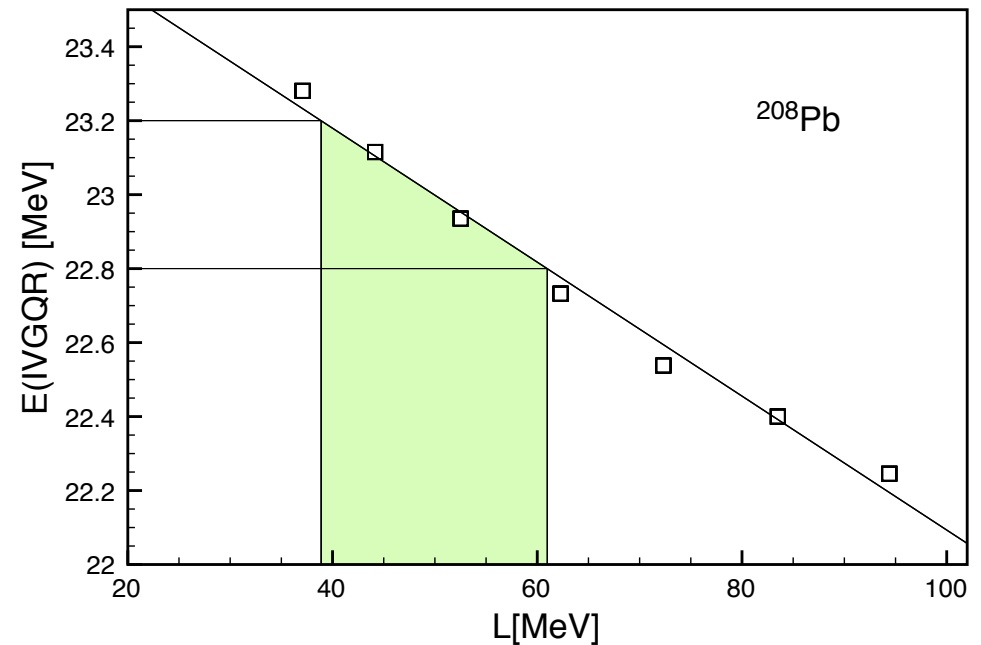
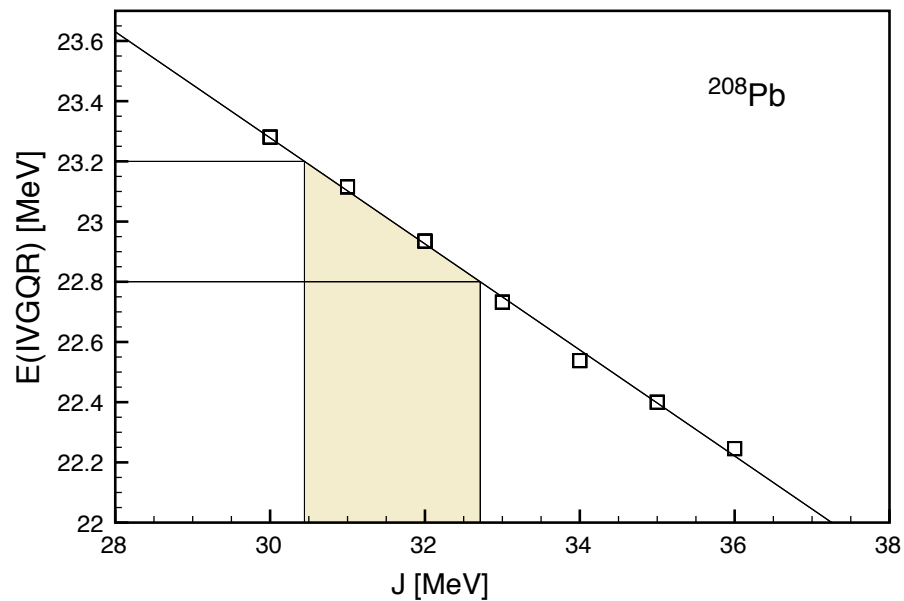


# GIANT QUADRUPOLE RESONANCES IN $^{208}\text{Pb}$ AND SYMMETRY ENERGY

- Precise determination of isovector giant quadrupole resonances

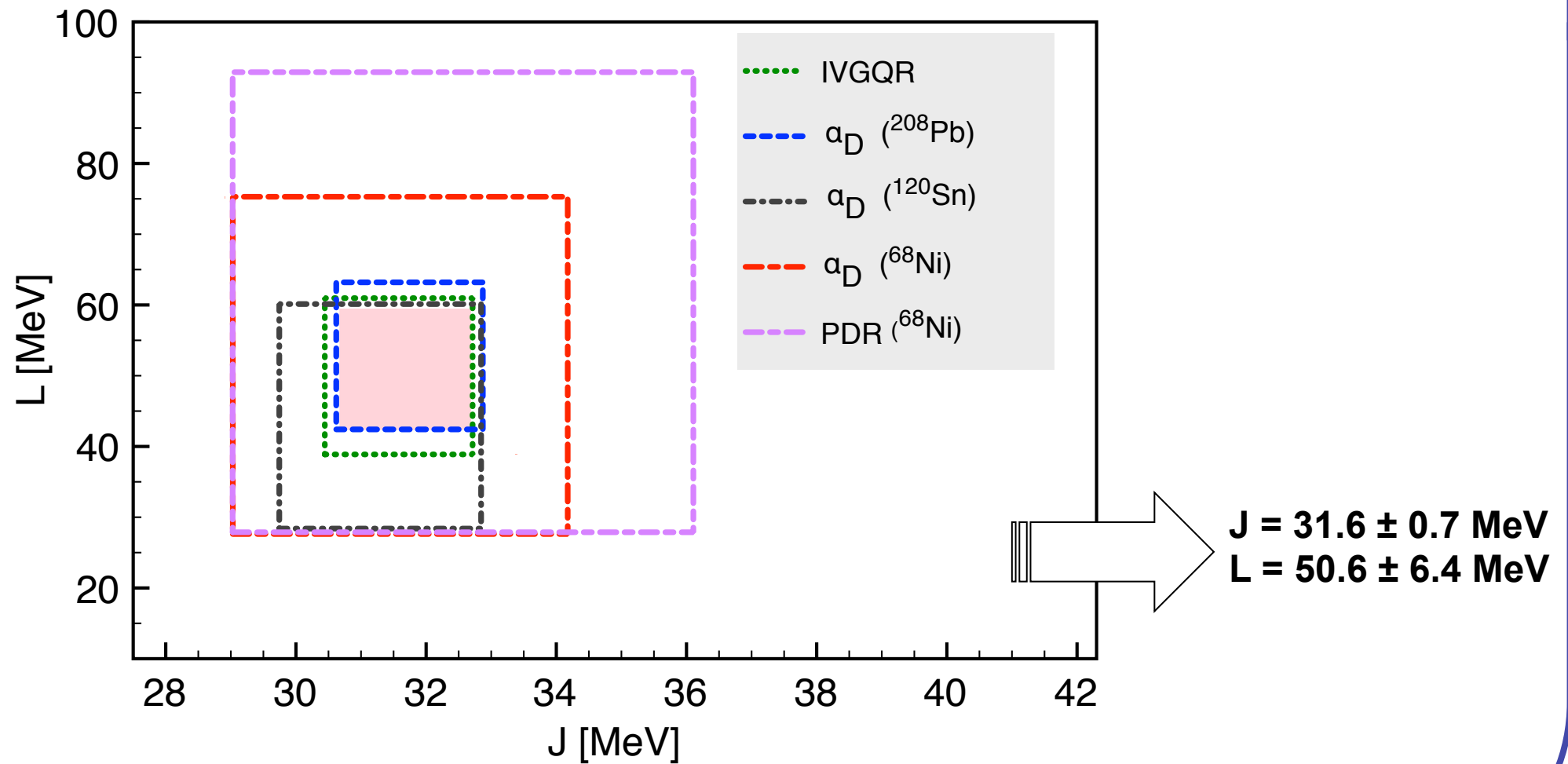
S.S. Henshaw, M.W. Ahmed, G. Feldman et al, PRL 107, 222501 (2011)

- IVGQR energy is strongly correlated with the symmetry energy parameters



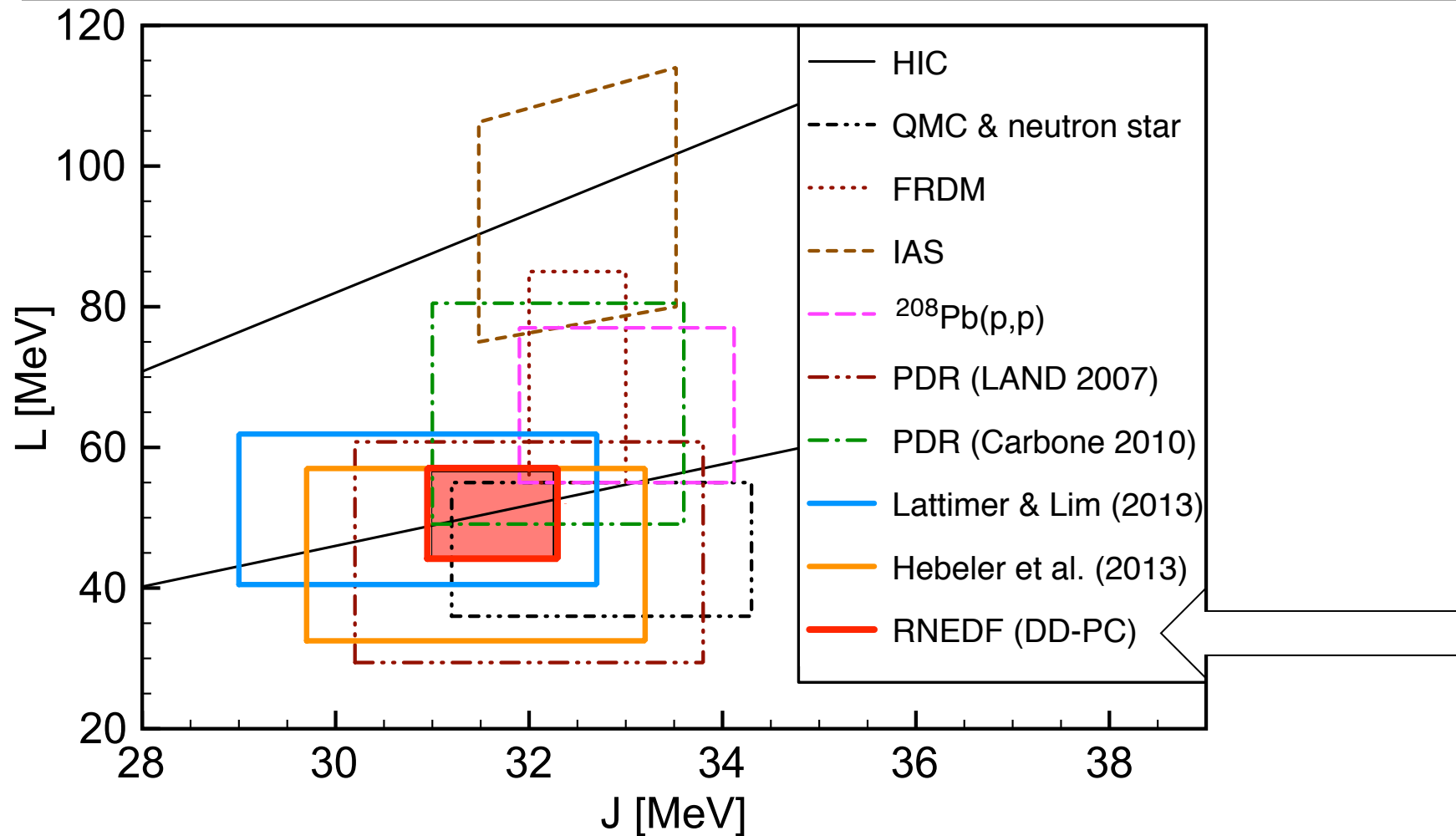
## CONSTRAINING THE SYMMETRY ENERGY (J-L)

- Based on relativistic density-dependent point coupling interactions





## CONSTRAINING THE SYMMETRY ENERGY (J-L)



- Lattimer & Lim, ApJ. 771, 51 (2013) – compilation from various approaches
- K. Hebeler et al., AJ 773, 11 (2013) – based on nuclear interactions derived from chiral EFT
- P. Danielewicz and J. Lee, Nucl. Phys. A 818, 36 (2009) – IAS
- A. Carbone, G. Colo, A. Bracco, et al., Phys. Rev. C 81, 041301 (2010) – PDR
- A. W. Steiner and S. Gandolfi, Phys. Rev. Lett. 108, 081102 (2012) – QMC (Av8') + neutron stars
- etc.

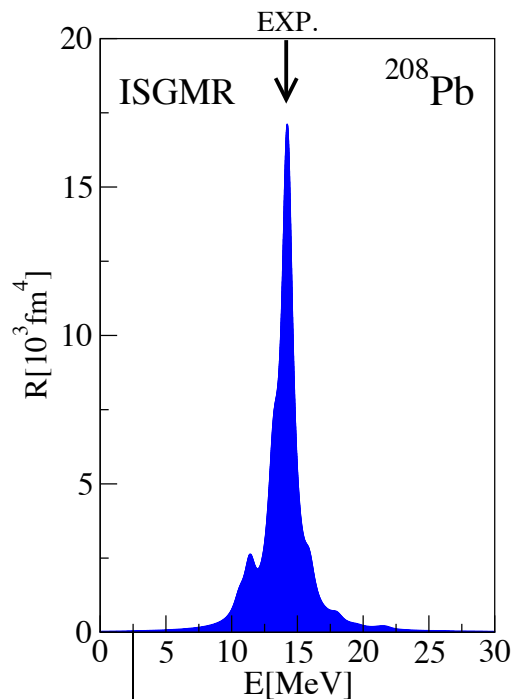
## CONSTRAINING THE SYMMETRY ENERGY: 2<sup>nd</sup> approach

- use the experimental data on collective excitations to constrain the symmetry energy **within the fitting protocol** to determine the parameters of the functional (relativistic point coupling interaction)
- Adjust the properties of 72 spherical nuclei to exp. data (binding energies, charge radii, diffraction radii, surface thickness, pairing gaps)
- constrain the symmetry energy  $S_2(\rho_0)=J$  (2%) from exp. data on dipole polarizability ( $^{208}\text{Pb}$ ) within an iterative procedure [A. Tamii et al., PRL 107, 062502 \(2011\) + update \(2015\)](#).
- constrain the nuclear matter incompressibility  $K_{\text{nm}}$  (2%) from exp. data on ISGMR modes ( $^{208}\text{Pb}$ ); [D. Patel et al., PLB 726, 178 \(2013\)](#).
- constrain the maximal neutron star mass by solving the Tolman-Oppenheimer-Volkov (TOV) equations + observational data from [J. Antoniadis, et al. Science 340, 448 \(2013\)](#)

 **“Correct symmetry energy” is the one obtained for the interaction that accurately reproduces the exp. data on dipole polarizability**

# CONSTRAINING THE SYMMETRY ENERGY: 2<sup>nd</sup> approach

ISOSCALAR GIANT  
MONOPOLE RESONANCE

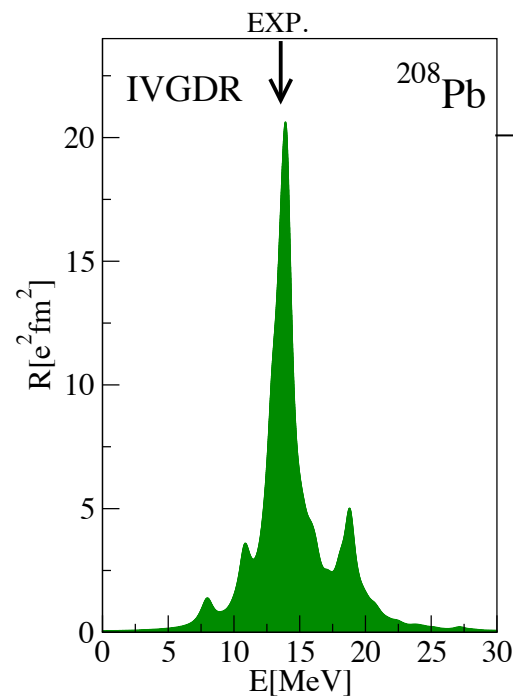


- ISGMR energy determines the nuclear matter incompressibility:  
 **$K_{nm}=232.4 \text{ MeV}$**

$E \text{ (Exp.)} = (13.91 \pm 0.11) \text{ MeV (TAMU)}$

$E \text{ (Exp.)} = (13.7 \pm 0.1) \text{ MeV (RCNP)}$

ISOVECTOR GIANT  
DIPOLE RESONANCE



Dipole polarizability:  
 $\alpha_D = (19.68 \pm 0.21) \text{ fm}^3$

Exp.  
 $\alpha_D = (19.6 \pm 0.6) \text{ fm}^3$

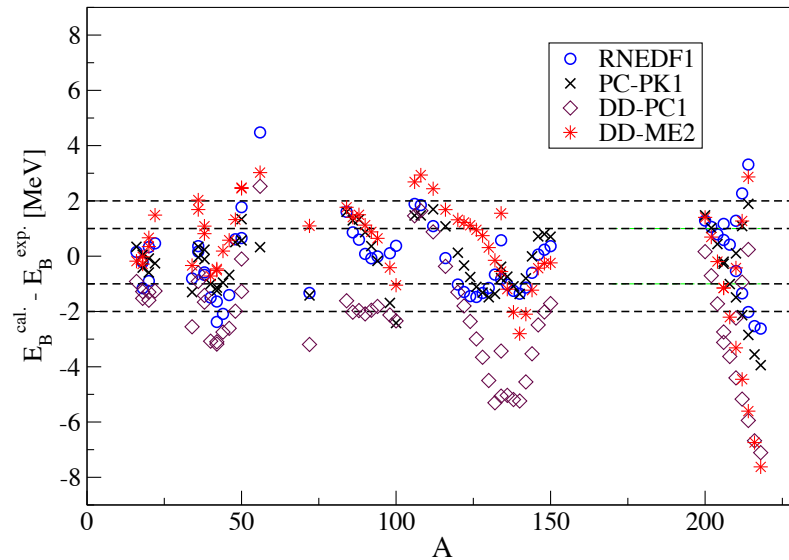
A.Tamii et al., PRL 107, 062502 (2011). + update (2015).

- IVGDR –  $\alpha_D$  determine the symmetry energy for the interaction

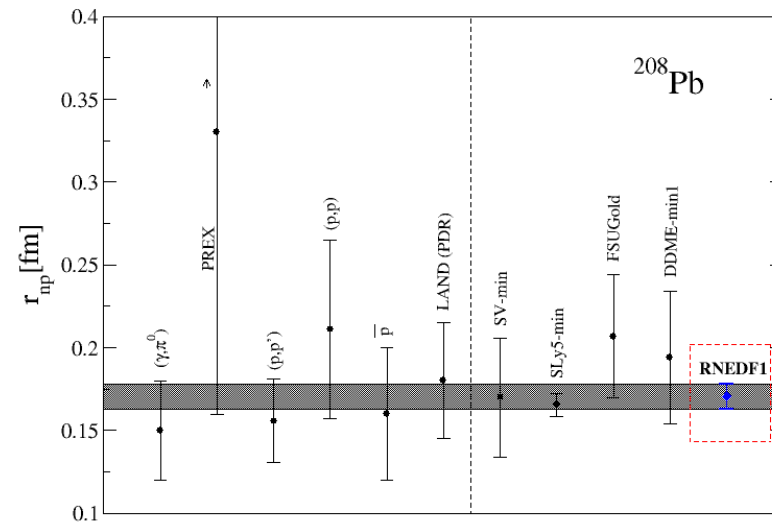
**$J = 31.89 \text{ MeV}$**   
 **$L = 51.48 \text{ MeV}$**

# SOME OTHER PROPERTIES...

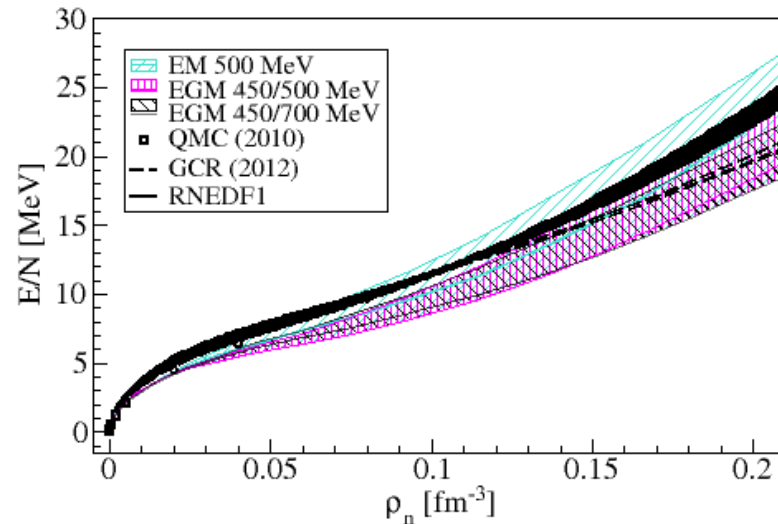
## Nuclear binding energies (calc. – exp.)



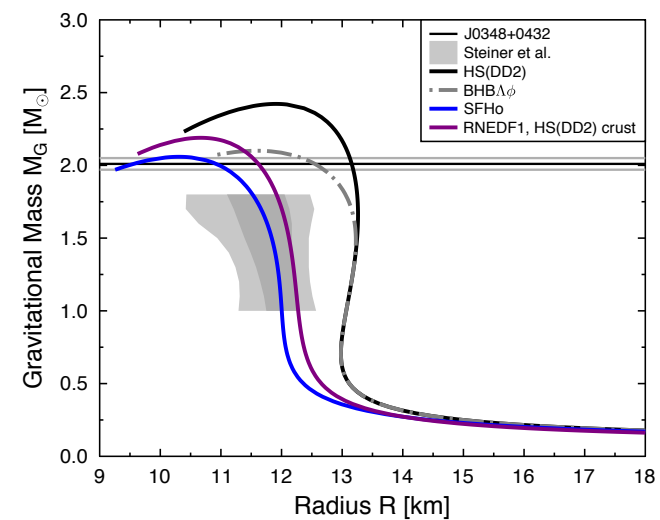
## Neutron skin thickness



## Pure neutron matter



## Neutron star mass-radius relationship

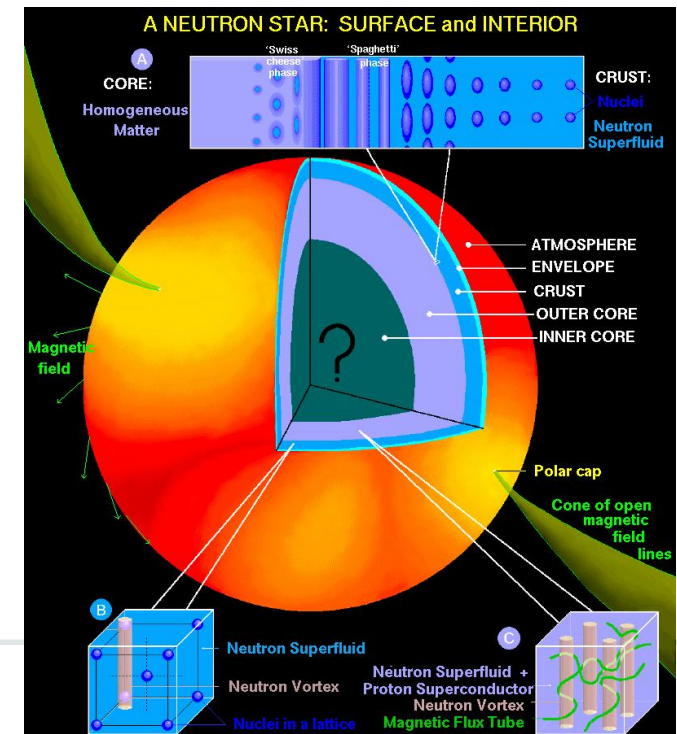


# Neutron star structure and excitations in finite nuclei

- exploit excitations in finite nuclei to constrain the neutron star core-to crust transition density ( $n_t$ ) and pressure ( $P_t$ ) - they are sensitive to the density dependence of the nuclear matter symmetry energy
- Find the density at which the uniform nuclear matter becomes unstable against small-amplitude density fluctuations

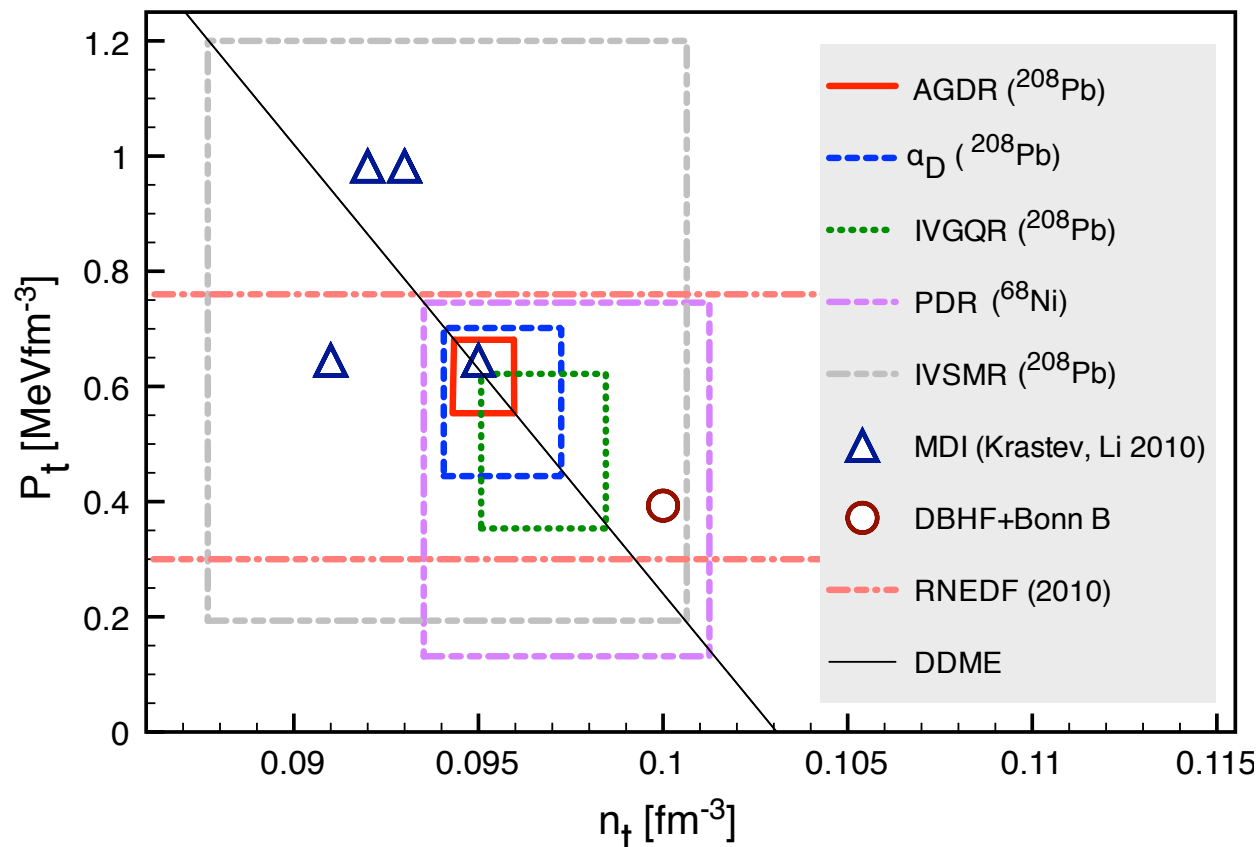
## Theory framework:

- Thermodynamic method to determine liquid-to-solid transition density and pressure, based on relativistic energy density functional. Ch. C. Moustakidis et al., PRC 81, 065803 (2010)
- Consistent approach: the same energy density functional for excitations in finite nuclei, nuclear matter equation of state and symmetry energy, and neutron star properties
- Constraints from the exp. data on excitations are used



# Neutron star structure and excitations in finite nuclei

- Constraints on the neutron star core-to-crust transition density  $n_t$  and pressure  $P_t$
- Calculations are based on the RNEDF, and experimental data for AGDR, IVGQR and IVSMR excitation energies ( $^{208}\text{Pb}$ ), dipole polarizability ( $^{208}\text{Pb}$ ) and PDR energy weighted strength ( $^{68}\text{Ni}$ ).



$n_t$ ([fm <sup>-3</sup> ])	
<b>RNEDF (EXC.)</b>	<b>0.0955</b>
A18+ $\delta v$ +UIX* (A. Akmal, 1998)	0.087
EOS Friedman Pandharipande (C.P. Lorenz, 1993)	0.096
Chiral EFT (NN+3N) (K. Hebeler, 2013)	0.076-0.088
HIC (B.A.Li, 2005)	0.040-0.065

## CONCLUDING REMARKS

- Dipole excitations in nuclei (PDR,  $\alpha_D$ , IVGDR, AGDR) and other modes (IVGQR,...) provide valuable constraints for the nuclear matter symmetry energy
- small uncertainty in the calculated symmetry energy (J,L) considerably limits the choice of currently available equations of state used in modeling neutron stars and supernova matter.
- Accurate measurements have important implications to reduce uncertainties in the symmetry energy
- Prospects to include the properties of collective excitations as new observables in the fitting protocols to determine the parameters of the EDF (especially to constrain better the isovector channel of the interaction)
- Neutron star core-to-crust transition density and pressure can be assessed using information on collective nuclear excitations / symmetry energy