First Tsukuba-CCS-RIKEN joint workshop on microscopic theories of nuclear structure and dynamics

Microscopic description of nuclear $\beta$-decay half-lives

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Outline

1. Introduction
2. Theoretical framework
3. Results and discussion
4. Summary and perspectives
The nuclear β-decay plays an important role not only in the nuclear physics, but also in other branches of science, such as astrophysics.

Since majority of neutron-rich nuclei relevant to the r-process are still out of the reach of experimental capabilities, theoretical predictions have to be used.

Theoretical models:

- **Phenomenological formula;** Zhang2006PRC, Zhang2007JPG
- **Gross theory;** Takahashi1973ADNDT, Takahashi1990PTP, Nakata1997NPA
  
  the reliability of the extrapolation is questionable
- **The shell model;** Pinedo1999PRL, Caurier2002PRC, Langanke2003RMP
  
  large configuration spaces ➔ mainly for nuclei with $A<60$ or near magic nums
- **The proton-neutron quasiparticle random phase approximation (PN-QRPA);** Möller1997ADNDT, Sarriguren2011PRC
Self-consistent QRPA (the residual interactions in the QRPA calculations are self-consistently derived with the effective interactions used in the g.s. calculations):

★ Traditional (non-relativistic) density functional:

- DF (Fayans)+CQRPA: Borzov1996ZPA, Borzov2003,2005PRC, Borzov2008NPA
- ETFSI (Skyrme)+CQRPA: Borzov1997NPA, Borzov2000PRC
- SHF BCS+QRPA: Minato2009PRC, Sarriguren2010PRC
- SHFB+QRPA: Engel1999PRC

★ Covariant (relativistic) density functional:


In present RHB+QRPA model, $\pi$ meson field is absent in the g.s. description and the strength parameter of counter-term of $\pi$ meson field ($g'$) in QRPA calculation is treated as an adjustable parameter.
Introduction

The fully consistent RHF+RPA model has achieved great success in the description of both nuclear ground state and charge-exchange excitations. Liang2008PRL

- $\pi$ is included in both the g.s. description and the p-h residual interaction;
- the exact zero-range counter-term with $g' = 1/3$ is maintained.

The RHF model has been extended to the RHFB, which gives a unified and self-consistent description of both mean field and pairing correlations. Long2010PRC, Long2010PRC(R)

Based on the RHFB approach, we have developed the fully consistent QRPA (RHFB+QRPA) model recently. Niu2013PLB

Present talk:

- employ the RHFB+QRPA model to calculate the nuclear $\beta$-decay rates and investigate the influence of Fock terms and pairing correlations.
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1. Introduction
2. Theoretical framework
   - Relativistic Hartree-Fock-Bogoliubov theory
   - Quasiparticle random phase approximation
   - Nuclear β-decay half-lives
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Relativistic Hartree-Fock-Bogoliubov theory

- Effective Lagrangian density:

\[
L = \overline{\psi} \left[ i\gamma^\mu \partial_\mu - M - g_\sigma \sigma - \gamma^\mu \left( g_\omega \omega_\mu + g_\rho \bar{\rho} \cdot \rho_\mu + e \frac{1 - \tau_3}{2} A_\mu \right) - \frac{f_\pi}{m_\pi} \gamma_5 \gamma^\mu \partial_\mu \bar{\pi} \cdot \bar{\pi} \right] \psi
\]

\[
+ \frac{1}{2} \partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{4} \Omega^{\mu\nu} \Omega_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu - \frac{1}{4} \bar{R}^{\mu\nu} \cdot \bar{R}_{\mu\nu} + \frac{1}{2} m_\rho^2 \bar{\rho}^\mu \cdot \bar{\rho}_\mu
\]

\[
+ \frac{1}{2} \partial^\mu \bar{\pi} \cdot \partial_\mu \bar{\pi} - \frac{1}{2} m_\pi^2 \bar{\pi} \cdot \bar{\pi} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}.
\]

- RHFB equation: Kucharek1991ZPA, Long2010PRC

\[
\int dr' \begin{pmatrix} h(r, r') - \lambda & \Delta(r, r') \\ \Delta(r, r') & -h(r, r') + \lambda \end{pmatrix} \begin{pmatrix} \psi_U(r') \\ \psi_V(r') \end{pmatrix} = E \begin{pmatrix} \psi_U(r) \\ \psi_V(r) \end{pmatrix},
\]

where \( h(r, r') \) and \( \Delta_\alpha(r, r') \) are the mean field and paring potential

\[
h(r, r') = h^{\text{kin}}(r, r') + h^D(r, r') + h^E(r, r'), \quad \Delta_\alpha(r, r') = -\frac{1}{2} \sum_\beta V^{pp}_{\alpha\beta}(r, r') \kappa_\beta(r, r').
\]

- \( h^{\text{kin}}, h^D, \) and \( h^E \): PKO1. Long2006PLB
- \( V^{pp} \): Gogny pairing force with D1S parameter set. Berger1991CPC
Quasiparticle random phase approximation

QRPA equations: Ring1995Springer

\[
\begin{pmatrix}
A & B \\
-B & -A
\end{pmatrix}
\begin{pmatrix}
X' \\
Y'
\end{pmatrix}
= \omega_v
\begin{pmatrix}
X' \\
Y'
\end{pmatrix}
\]

where \(\omega_v\) is the excitation energy, \(X_v\) and \(Y_v\) denote the 2qp amplitudes. The QRPA matrices \(A\) and \(B\) read:

\[
A_{kk'\ell\ell'} = (E_k + E_{k'})\delta_{kl}\delta_{k'l'} + \frac{\delta^2 E}{\delta R_{kk'}^* \delta R_{\ell\ell'}^*}, \quad B_{kk'\ell\ell'} = \frac{\delta^2 E}{\delta R_{kk'}^* \delta R_{\ell\ell'}^*}.
\]

In the canonical basis, the matrices \(A\) and \(B\) for the charge-exchange channel read:

\[
A_{pnp'n'} = H_{pp'}^{11} \delta_{nn'} + H_{np'}^{11} \delta_{pp'}
+ V_{pp'}^{ph}(u_p v_n u_{p'} v_{n'}) + V_{pp'}^{pp}(u_p v_n u_{p'} v_{n'}) + V_{pp'}^{ph}(u_p v_n u_{p'} u_{n'}) + V_{pp'}^{pp}(u_p v_n u_{p'} u_{n'})
\]

\[
B_{pnp'n'} = V_{pp'}^{ph}(u_p v_n u_{p'} v_{n'}) - V_{pp'}^{ph}(u_p v_n u_{p'} u_{n'})
\]

where \(H_{kl}^{11} = (u_k u_l - v_k v_l)h_{kl} - (u_k v_l - v_k u_l)\Delta_{kl}\).

Paar2003, 2004PRC
Nuclear β-decay half-lives

The nuclear β-decay half-life in the allowed Gamow-Teller approximation reads as follows:

\[ T_{1/2} = \frac{\ln 2}{\lambda_\beta} = \frac{D}{g_A^2 \sum_m \left| \sum_{\rho n} \langle 1^+_m | \sigma \tau | 0^+ \rangle \right|^2 f(Z,A,E_m)}, \]

where \( D = \frac{\hbar^2 2\pi^3 \ln 2}{g^2 m_e^5 c^4} = 6163.4 \) s, \( g_A = 1 \). The transition probability \( \langle 1^+_m | \sigma \tau | 0^+ \rangle \) can be directly taken from the QRPA calculations.

- The integrated \( (e, \bar{\nu}_e) \) phase volume \( f(Z,A,E_m) \):

\[ f(Z,A,E_m) = \frac{1}{m_e^5} \int_{m_e}^{E_m} p_e E_e (E_m - E_e)^2 F(Z,A,E_m) dE_e, \]

- The maximum value of β-decay energy \( E_m \):

\[ E_m = E_i - E_f = (m_n - m_p) - E_{QRPA} = \Delta_{np} - E_{QRPA}. \]

Due to \( E_m > m_e \), the sum on \( m \) runs over all final states with \( E_{QRPA} \) smaller than \( \Delta_{nH} = \Delta_{np} - m_e = 0.782 \) MeV.
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**β-decay rates in QRPA calculations**

Figure: β-decay half-lives of $^{134}$Sn. The results based on the (Q)RPA calculations without any residual interactions and the calculations gradually including the residual interactions of σ and ω fields, ρ field, π field, and $T = 0$ pairing are presented.

- **ph residual interactions:** increase the calculated β-decay half-lives.
- **RHF(B)+(Q)RPA:** σ- and ω-mesons play an important role via exchange terms.

- **pp interactions:**
  - $T=1$: are necessary to reproduce data.
  - $T=0$: can reduce the calculated β-decay half-lives significantly.
The influence of $T=0$ pairing

**Results and discussion**

By fitting to the experimental half-lives of Ca-Sn nuclei, an isospin-dependent function similar to the Woods-Saxon potential is proposed:

$$V_0 = V_1 + \frac{V_2}{1 + e^{a+b(N-Z)}},$$

$V_1=134$ MeV, $V_2=121$ MeV, $a=8.5$, $b=-0.4$.

**Figure:** β-decay half-lives for Fe and Cd isotopes calculated in RHFB+QRPA model with the PKO1 parameter set.

β-decay rates of Ca-Sn isotopes

Figure: The ratios of theoretical half-lives to the experimental values as a function of the experimental half-lives for Ca-Sn isotopes. The circles and diamonds represent the results calculated by the RHFB+QRPA and FRDM+QRPA approaches, respectively.

- RHFB+QRPA: well reproduces the experimental half-lives of these neutron-rich nuclei except for some magic nuclei, such as the Ni isotopes.
- FRDM+QRPA: generally overestimates the nuclear half-lives, which can be attributed partially to the neglect of the isoscalar pn pairing.
Influence of $Q_\beta$ values

Figure: $Q_\beta$ values and its influence on $\beta$-decay half-lives of Ni isotopes.

- The experimental $Q_\beta$ values are systematically underestimated by the RHFB theory.

- The new results are in excellent agreement with the experimental data, which reflects the importance of accurate nuclear mass predictions in half-life calculations.

- This modification of $Q_\beta$ is not self-consistent in the predictions of nuclear $\beta$-decay half-lives.
The effect of the PVC decreases the half-lives by large factors compared to RPA, substantially improving the agreement with experimental data.


Figure: The β-decay half-lives of 132Sn, 68Ni,34Si, and 78Ni, calculated by RPA and RPA+PVC approaches, respectively, in comparison with experimental values.
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Summary and perspectives

**Summary:**
- The nuclear β-decay half-lives are sensitive to the pp residual interactions and that in the T=0 channel can significantly reduce the β-decay half-lives.
- The self-consistent RHFB+QRPA calculations well reproduce the experimental half-lives of Ca-Sn isotopes with an isospin-dependent T=0 pairing except for some magic nuclei.
- The effect of the PVC decreases the half-lives by large factors compared to RPA, substantially improving the agreement with experimental data for magic nuclei.

**Perspectives:**
- QRPA → QRPA+QPVC
- Deformation degree
- Other applications: 2β decay and neutrino-nucleus scattering
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Thank you!