Self-consistent Collective Coordinate in Richardson model

First Tsukuba-CCS-RIKEN joint workshop

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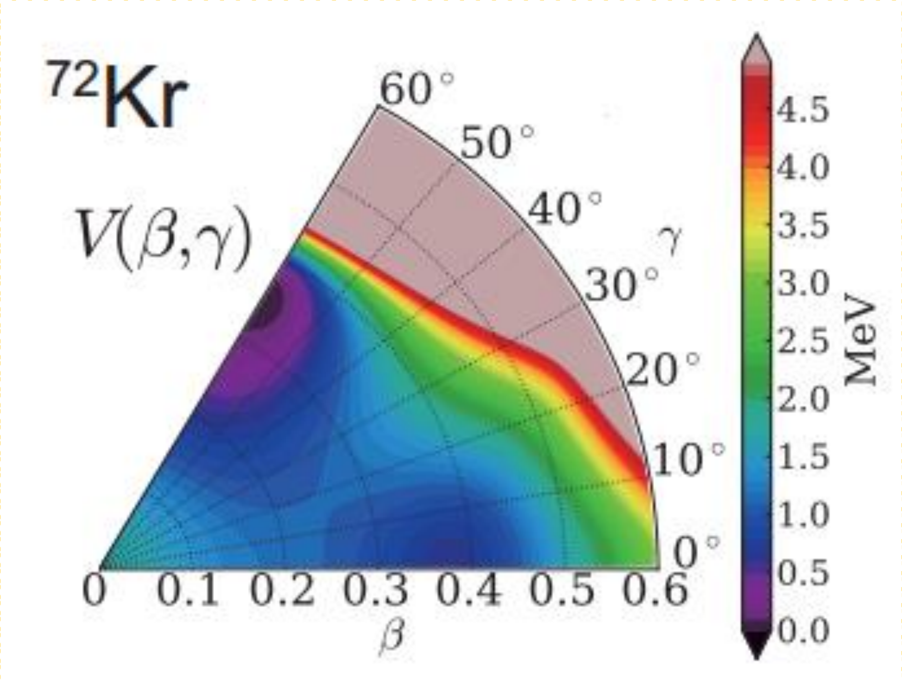
Outline

- **Introduction**
  - **Collective coordinate**
  - **Pairing, Richardson model**
- **How to obtain collective coordinate self-consistently?**
- **Result**
- **Summary**

Introduction

Collective coordinate (collective degree of freedom) “by hand”

ex) Quadrupole deformation parameter β, γ



K. Matsuyanagi, et al., JPG (2016)

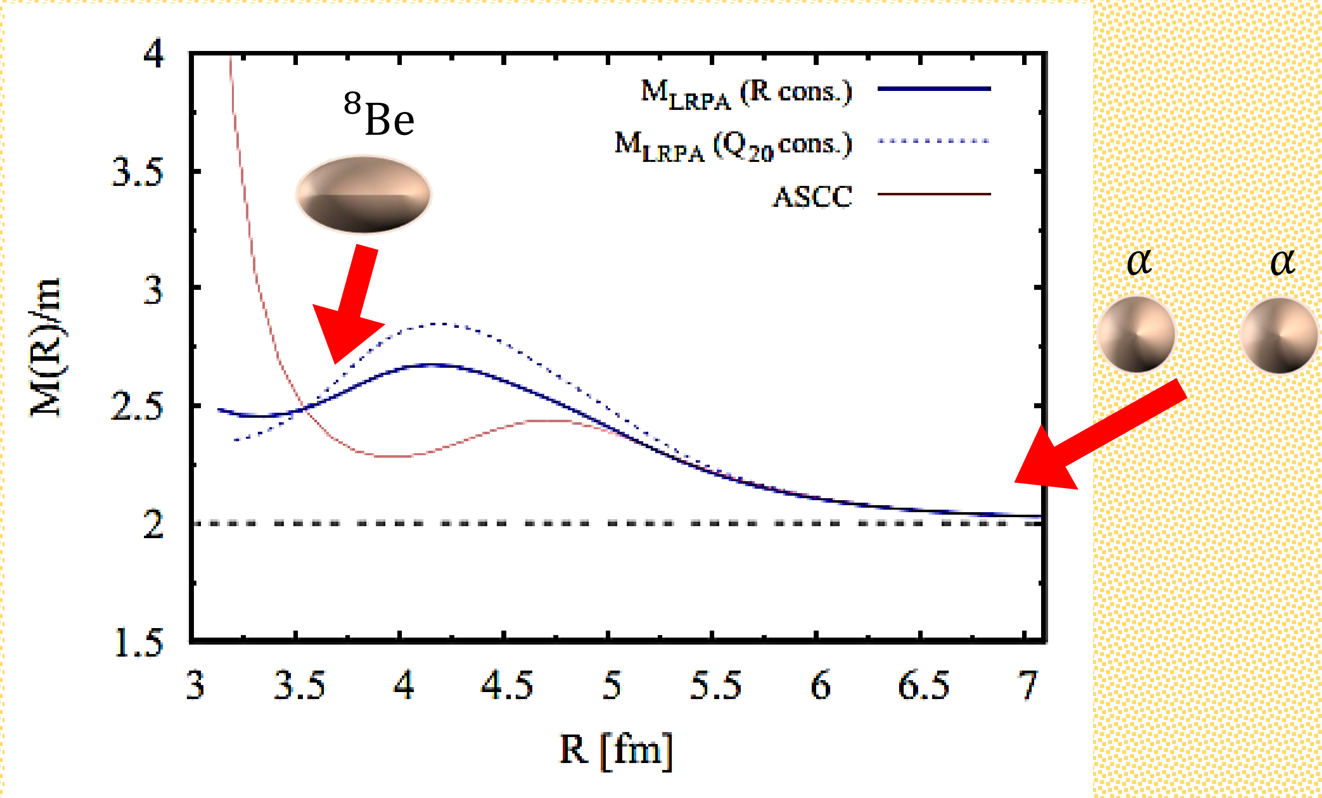
Theoretical approach:
Constrained Hartree-Fock(-Bogoliubov) method

Self-consistent collective coordinate (SCC)

T. Marumori, et al., PTP (1980)
M. Matsuo, et al., PTP (2000)

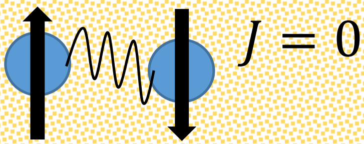
Q: Who decide the collective coordinate? **A: SYSTEM ITSELF!**

ex) Inertial mass in Reaction path in $\alpha + \alpha \leftrightarrow {}^{8}\mathrm{Be}$



K. Wen, T. Nakatsukasa, PRC (2016)

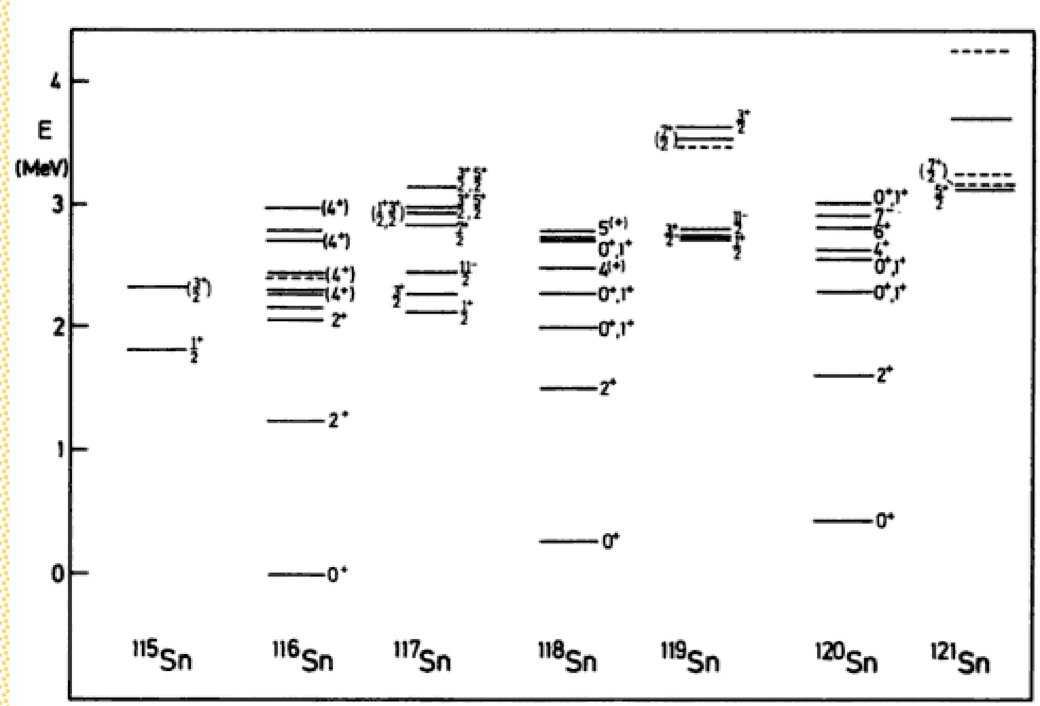
Pairing correlation



Quite different structure between odd-even and even-even nuclei

Ground states are well understood by BCS or HFB

Excitation spectra in Sn isotopes



P. Ring, P. Schuck, The Nuclear Many-Body Problem

Pairing collective motion

- How can we describe pairing collective motion?
- Are there suitable collective coordinates? Pairing gap Δ?

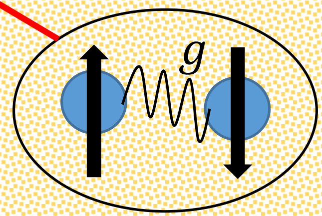
➔ Consider a simple pairing model: Richardson model

Richardson model

(pairing model, multi-seniority model)

$$H = \sum_l \epsilon_l n_l - \frac{g}{4} \sum_{ll'} P_l^\dagger P_{l'}$$

$$\begin{cases} n_l = \sum_m a_{lm}^\dagger a_{lm} \\ P_l^\dagger = \sum_m a_{lm}^\dagger a_{l\overline{m}}^\dagger \end{cases}$$



$J = 0$

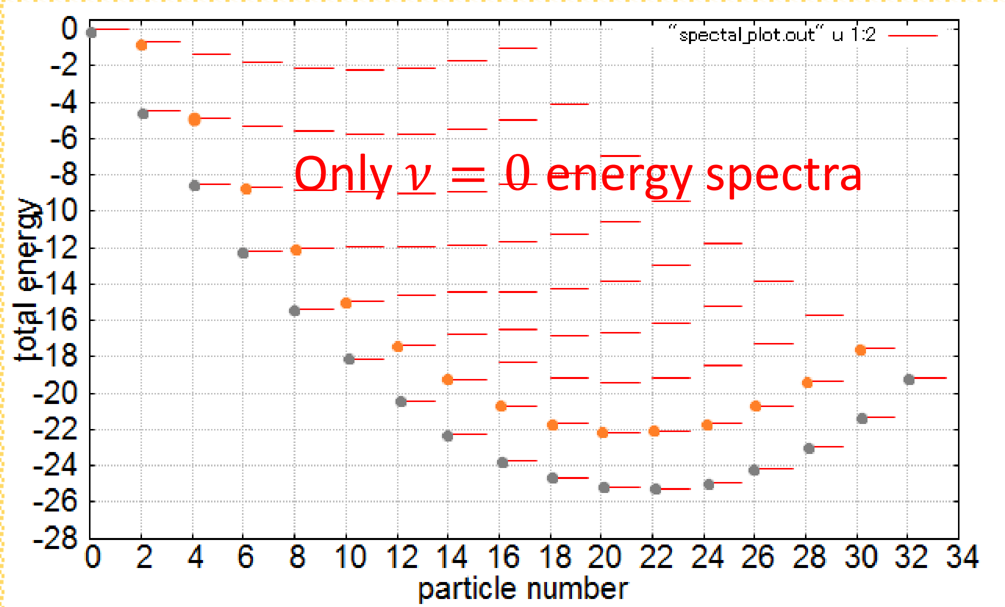
Initial parameters	
g	Coupling constant
ϵ_l	Single-particle energy
$\Omega_l = j_l + 1/2$	Degeneracy

l: Label of energy level

ν_l: Seniority (unpaired particles)

- Exact solvable model
- $\nu = 0$ excited states in multi-level system

Q: Can we describe these states by collective coordinate?



How to obtain collective coordinate?

Step 1. Create coherent state from Thouless theorem

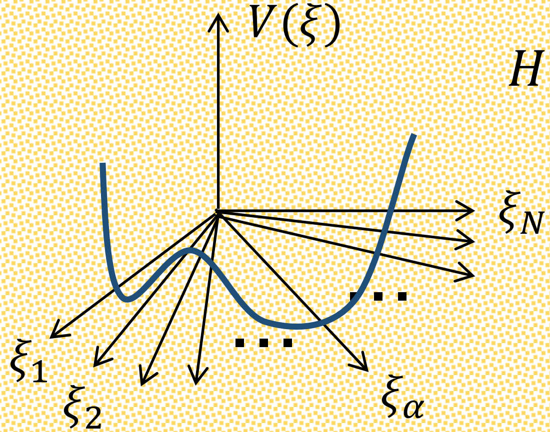
$$|\phi(t)\rangle \propto \prod_l e^{\chi_l S_l^+} |0\rangle \quad (\chi_l \in \boldsymbol{C})$$

Only contains
$\nu = 0$ element

Step 2. Obtain classical Hamiltonian

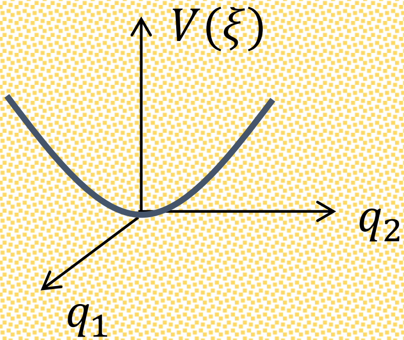
$$\mathcal{H}(\{\xi_l\},\{\pi_l\}) = \langle\phi(t)|H|\phi(t)\rangle \qquad \{\xi_l, \pi_{l'}\}_{PB} = \delta_{ll'}$$

Step 3. Extract collective coordinate



Many variables…

$$H(\xi,\pi) \Rightarrow H_{\text{coll}}(q,p)$$



A few variables!

Step 4. Requantization $\quad [q,p] = i$

Obtain 1D collective coordinate self-consistently

Step 1. Find energy minimum point
by solving HFB or BCS eq.

Step 2. Diagonalize moving QRPA eq.
choose the lowest mode basically

iteration

Step 3. Decide the neighborhood point
of collective path from eigenvector.

Basic equation

(1) Moving HFB eq.

$$V_{,\alpha} = \lambda f_{,\alpha}$$

(2) Moving covariant QRPA eq.

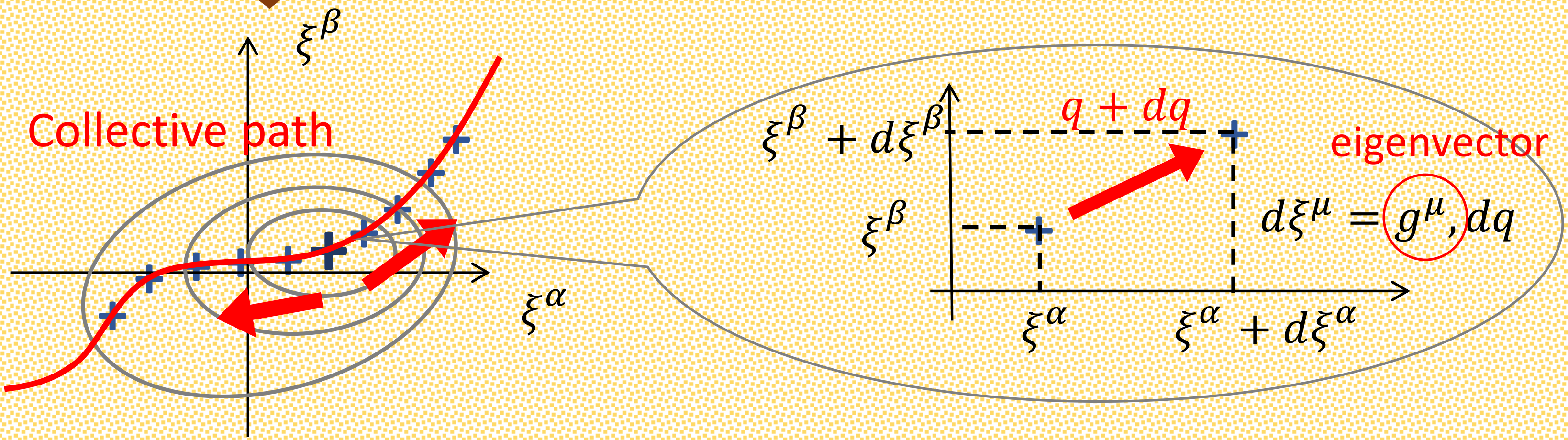
$$\begin{cases} B^{\beta\gamma} V_{;\alpha\gamma} f_{,\beta} = \omega^2 f_{,\alpha} \\ B^{\beta\gamma} V_{;\alpha\gamma} g^{\alpha}, = \omega^2 g^{\beta}, \end{cases}$$

$q^{\mu} = f^{\mu}(\xi), \xi^{\alpha} = g^{\alpha}(q)$

$F_{,\alpha} \equiv \frac{\partial F}{\partial \xi^{\alpha}}$ derivative

$V_{;\alpha\beta} \equiv V_{,\alpha\beta} - \Gamma^{\gamma}_{\alpha\beta} V_{,\gamma}$ covariant derivative

T. Nakatsukasa, PTEP 01A207 (2012)



Application

Classical Hamiltonian

expand ϕ up to 2nd order, π: coordinates, ϕ: momenta

$$\mathscr{H}(\{\phi_l\},\{\pi_l\}) = V(\pi) + \frac{1}{2}B^{\alpha\beta}(\pi)\phi_\alpha\phi_\beta$$

$$\begin{cases} V(\pi) = \mathscr{H}(\phi = 0) \\ B^{\alpha\beta}(\pi) = \left.\dfrac{\partial^2 \mathscr{H}}{\partial\phi_\alpha\partial\phi_\beta}\right|_{\phi=0} \end{cases}$$

Definition:

- q: collective coordinate
- $q = 0$ for energy minimum point
- Unify the scale of q by $\bar{B}(q) = f_{,\alpha} B^{\alpha\beta} f_{,\beta} = 1$

$\bar{B}(q)$: collective mass parameter

Focus on neutron's pairing in Sn isotope system

Set up

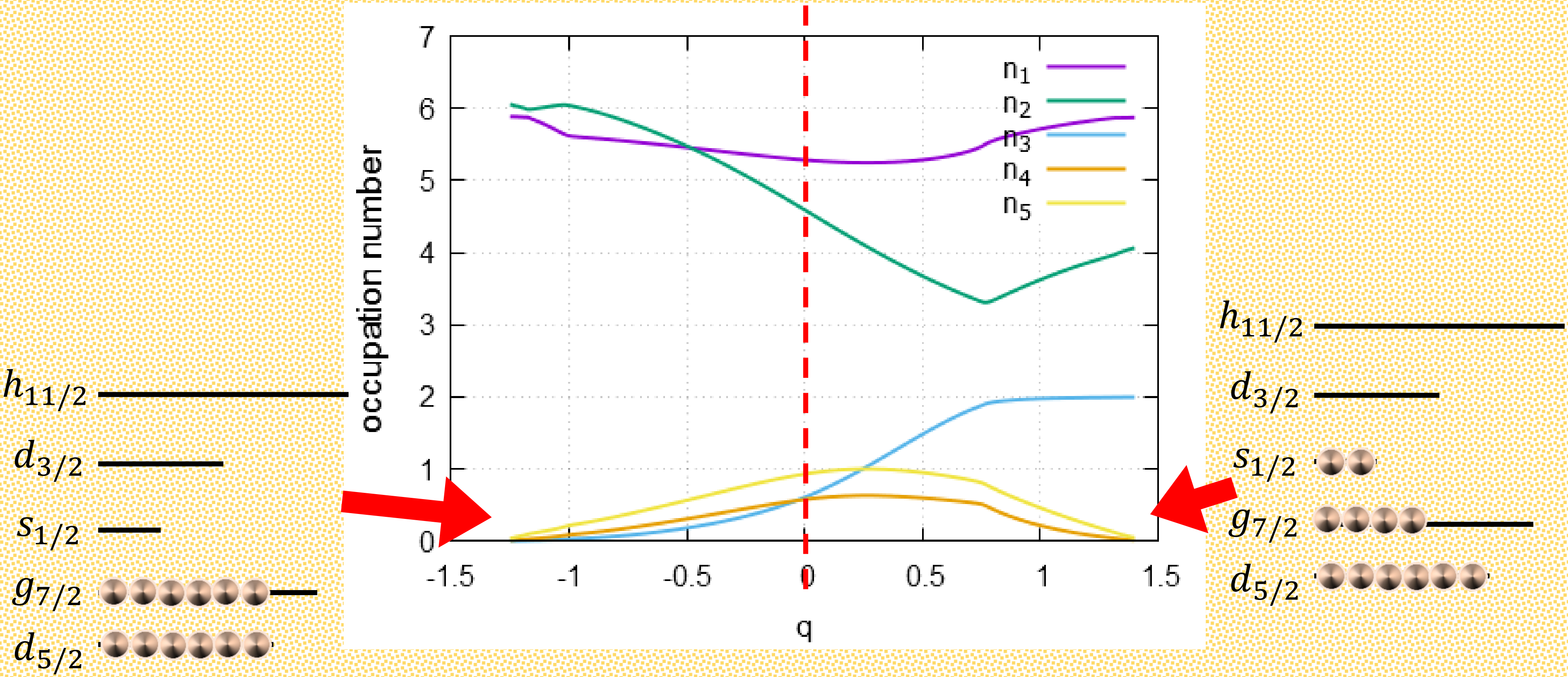
- sp-energy obtained from spherical WS potential + Spin-Orbit interaction in ^{100}Sn
- $g = 0.241$ (MeV) correspond to experimental value of pairing gap in ^{112}Sn

Single-particle level	Energy (MeV)
h11/2	-7.38
d3/2	-8.15
s1/2	-8.92
g7/2	-9.65
d5/2	-10.98

Result (in ^{112}Sn)

Occupation number of each level in collective path

BCS ground state



SYSTEM chose the collective path!!

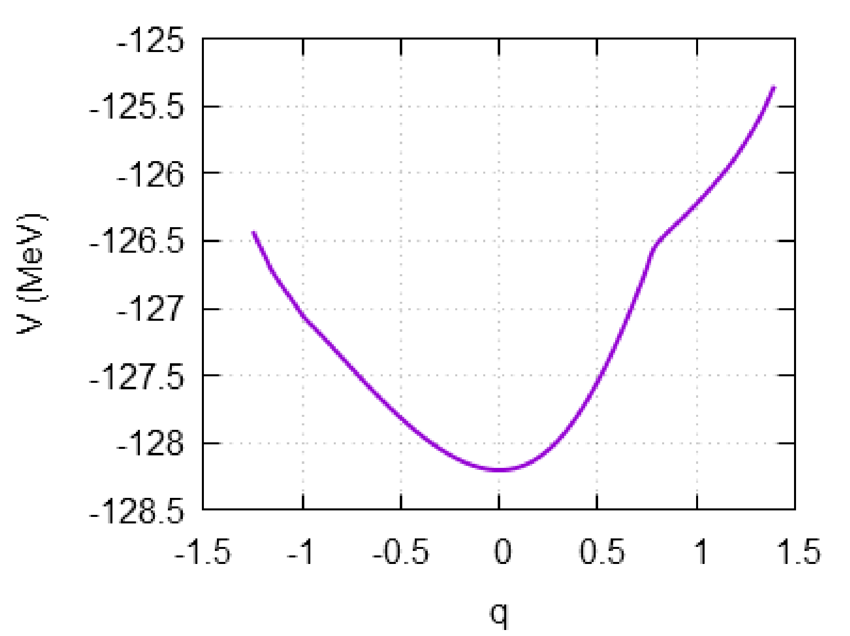
Requantization $[q, p] = i$

$$H_{coll} = -\frac{1}{2}\frac{d^2}{dq^2} + \bar{V}(q)$$

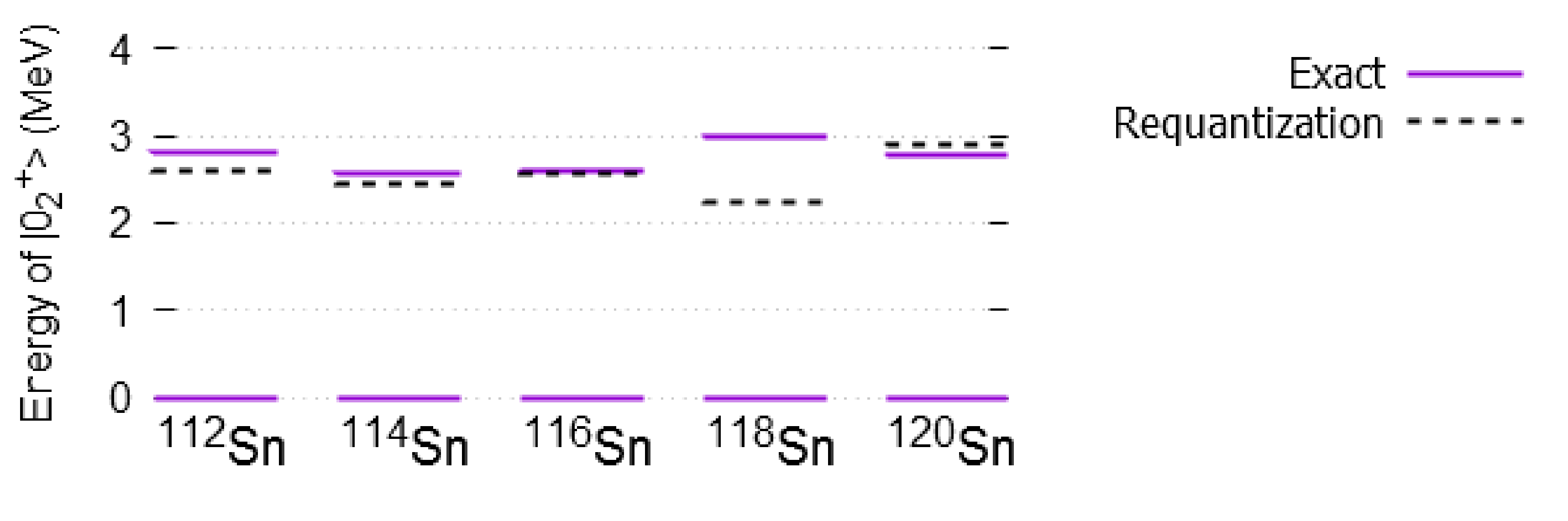
Dirichlet boundary condition:

$$\psi(q_{left}) = \psi(q_{right}) = 0$$

Collective potential of ^{112}Sn



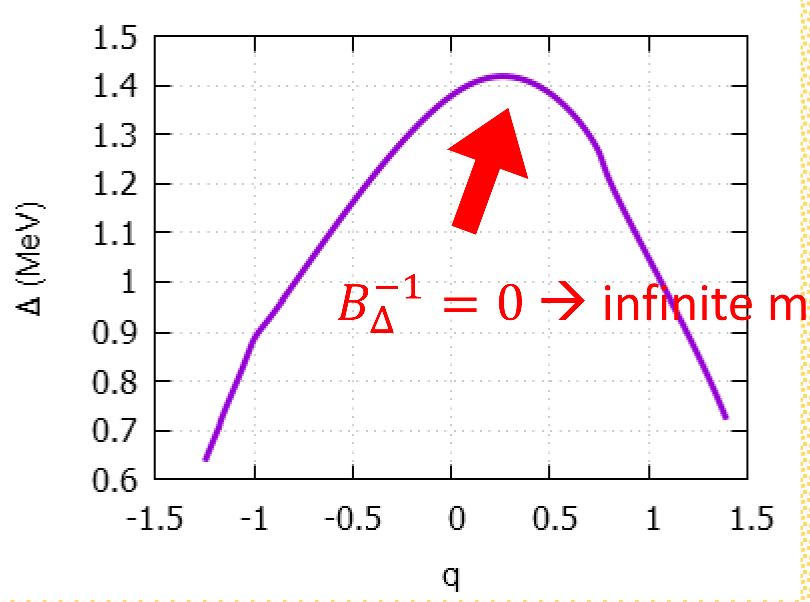
Other Sn isotopes…



Is pairing gap available as a collective coordinate?

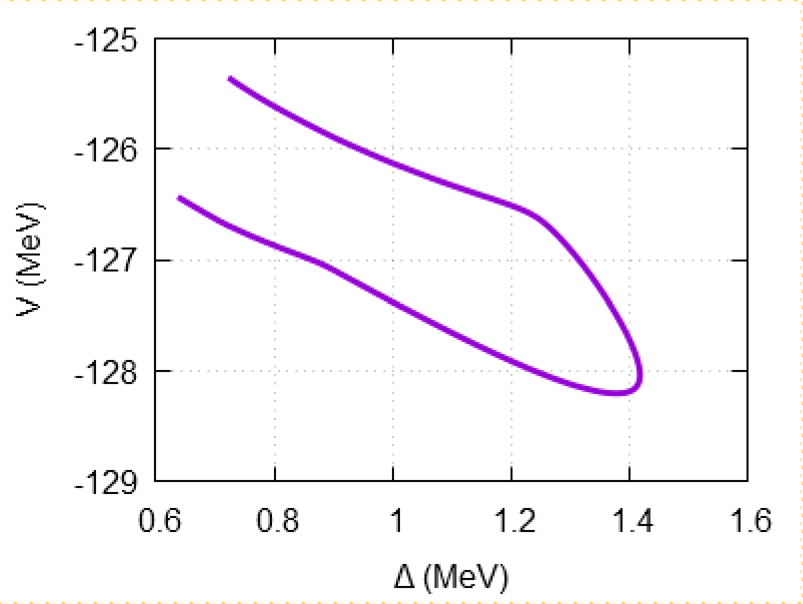
- Mapping q to Δ

Pairing gap $\Delta = g\sum_{k>0} u_k(q) v_k(q)$



$B_\Delta^{-1} = 0$ → infinite mass!

Potential $V_\Delta(\Delta)$



Collective mass parameter of Δ

$$\frac{1}{B_\Delta} = \left(\frac{\partial \Delta}{\partial q}\right)^2 \frac{1}{\bar{B}(q)}$$

No 1:1 correspondence!!

Summery

- We obtained collective path self-consistently in Richardson model
- The obtained collective path terminates at special states (Each orbit has integer(even) number of particles)
- Energy of $|0_2^+\rangle$ is well reproduced
- Pairing gap Δ is not suitable as a collective coordinate

Next step:

- Construct two-particle transfer operators in requantized form
- Search an available mapping parameter of collective coordinate
- Add quadrupole interaction

Thank you!

Back up

Explicit form of potential and mass parameter

$$V(\pi) = \sum_l \epsilon(-2S_l q_l + \Omega_l) - g\sum_l \frac{S_l}{2}\left(2S_l(1-q_l^2) + (1-q_l)^2\right)$$

$$- 2g \sum_{l_1<l_2} S_{l_1} S_{l_2} \sqrt{(1-q_{l_1}^2)(1-q_{l_2}^2)}$$

$$B^{\alpha\beta}(\pi) = \begin{cases} 2g\sum_{l\neq\alpha} S_\alpha S_l \sqrt{(1-q_\alpha^2)(1-q_l^2)} & (\text{for} \quad \alpha = \beta) \\ -2g S_\alpha S_\beta \sqrt{(1-q_\alpha^2)(1-q_\beta^2)} & (\text{for} \quad \alpha \neq \beta) \end{cases}$$

$$q_\alpha = \frac{1}{S_\alpha}\left(S_\alpha - \pi_\alpha - \frac{1}{L}\Pi\right) \quad (\text{for} \quad \alpha : 2 \sim \text{L})$$

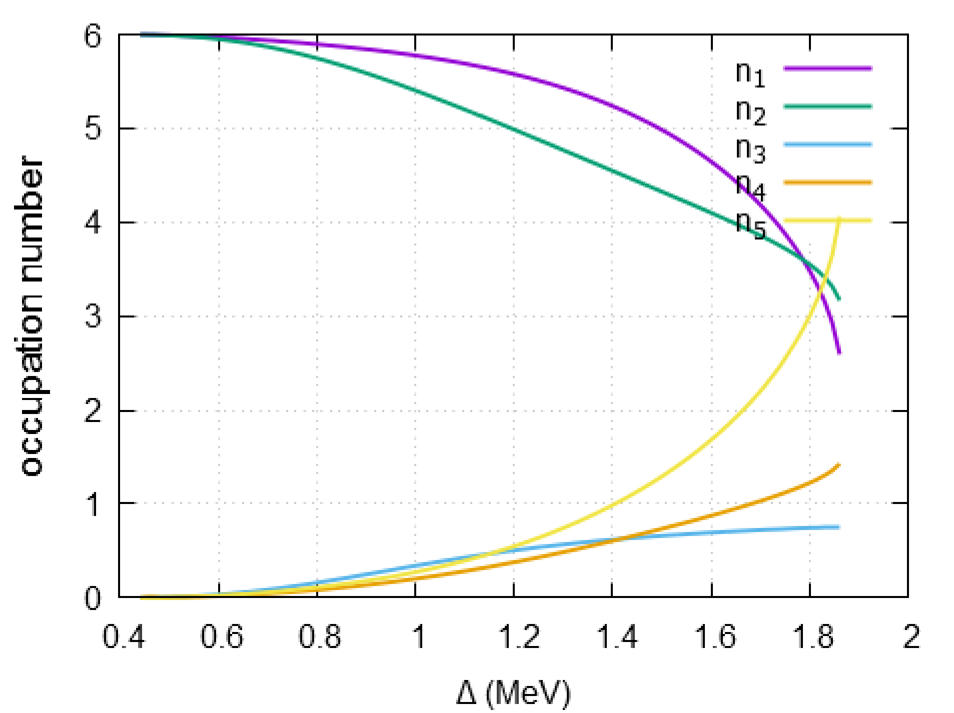
$$q_1 = \frac{1}{S_1}\left(S_1 + \sum_{\alpha=2}^{L} \pi_\alpha - \frac{1}{L}\Pi\right)$$

$$\Pi = (N - \nu)/2$$

Compare with Constraint BCS

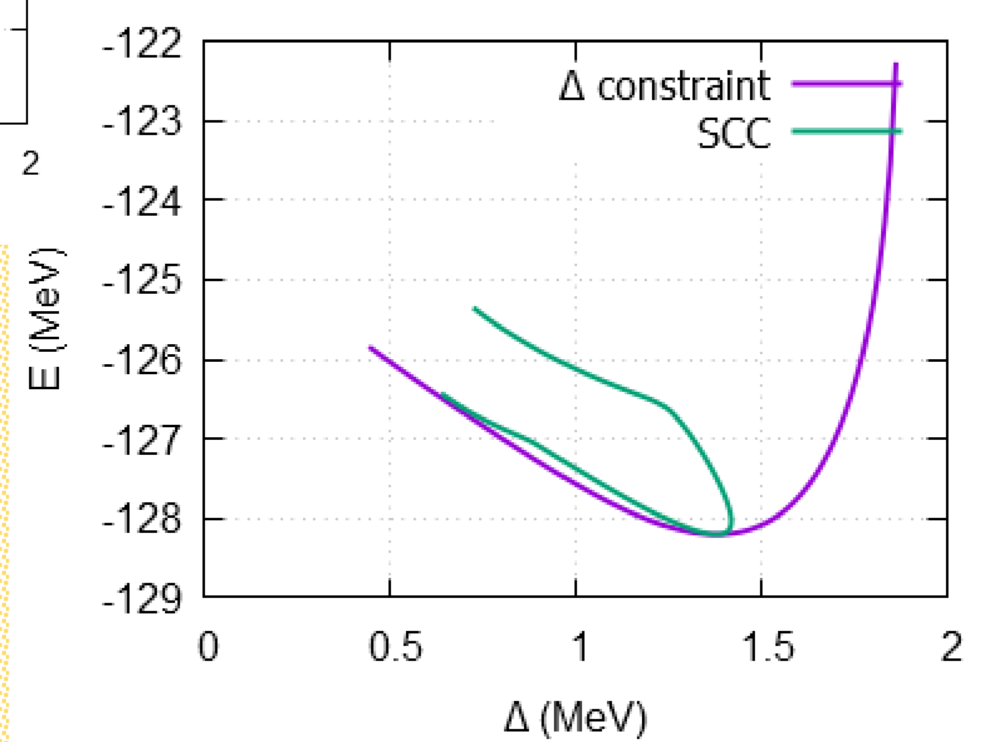
$$\hat{H}' = \hat{H} - \mu\hat{N} - \lambda\hat{P}$$

$$\hat{P} = \sum_{k>0} \hat{a}_k \hat{a}_{\bar{k}}$$



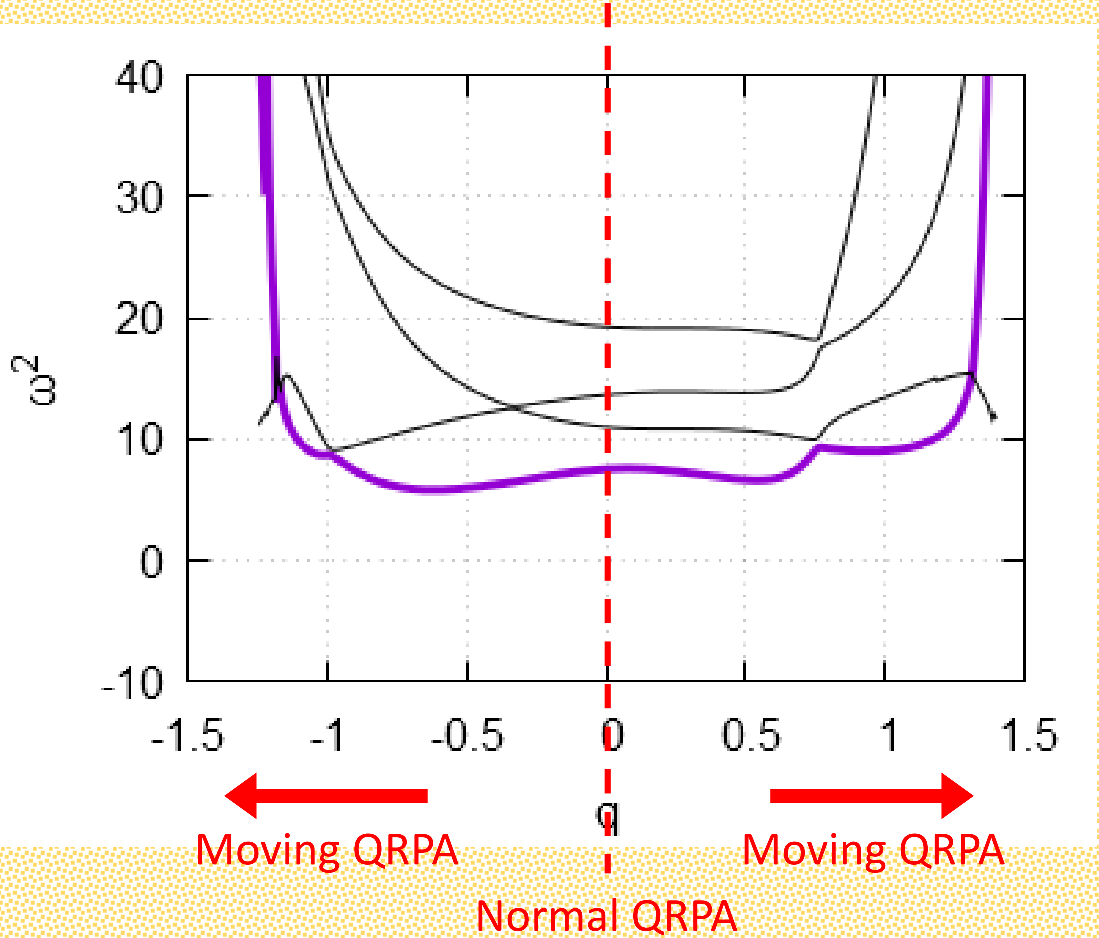
Occupation number

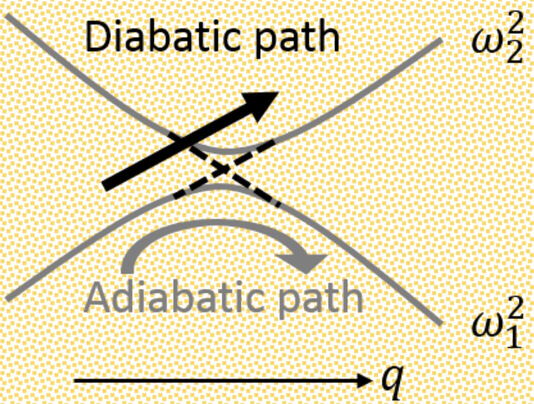
Potential



Result (in ^{112}Sn)

1. Moving QRPA solution in collective path

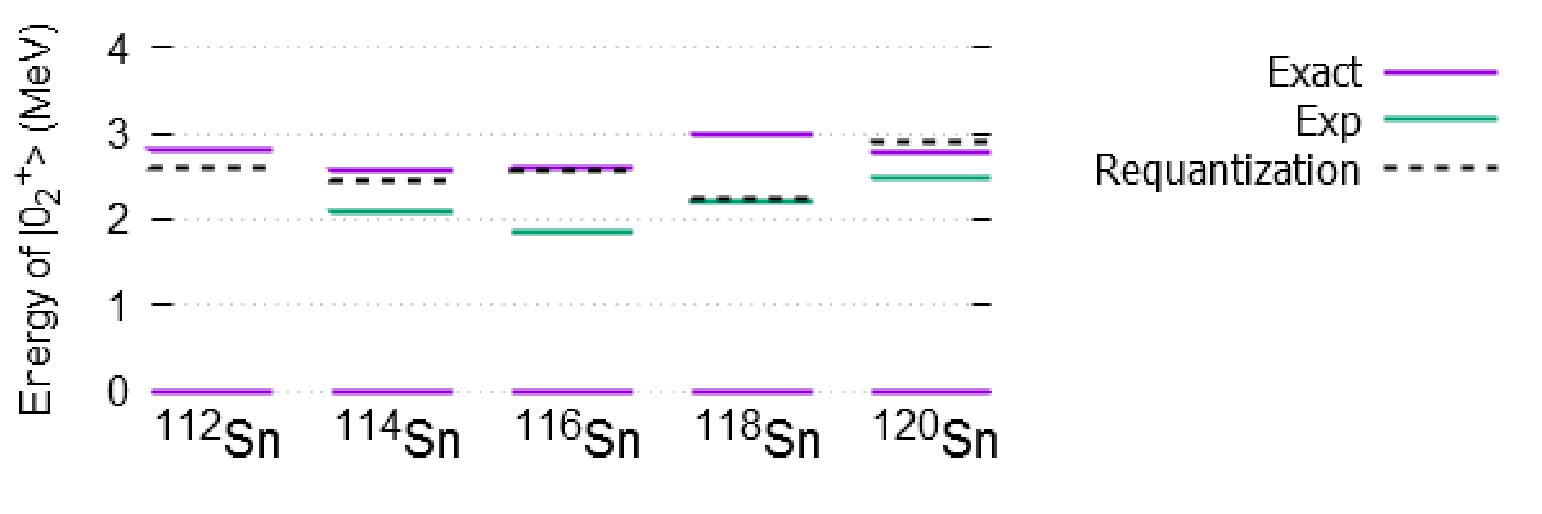




We choose the smoothest path!

$$D = |\sum_{\alpha}(f^{n}_{,\alpha})_{\mu}(f^{n-1}_{,\alpha})_{\nu}|$$

Choose max of D



$$E_{exp} = \frac{\sum_i \sigma_i(0_i^+) E(0_i^+)}{\sum_i \sigma_i(0_i^+)}$$

i: Excited $|0^+\rangle$ below 3MeV
σ_i: Relative cross section of (t,p) or (p,t)
with respect to g.s.

D. M. Brink, R. A. Broglia, Nuclear Superfluidity

energy of $\|0_2^+\rangle$	Exact	QRPA	Requantization
^{112}Sn	2.80	2.75	2.61
^{114}Sn	2.56	2.70	2.46
^{116}Sn	2.61	2.73	2.58
^{118}Sn	2.98	2.68	2.24
^{120}Sn	2.77	2.17	2.89