

Second proton-neutron random phase approximation
studied by the Lipkin-Meshkov-Glick (LMG) model
in $SU(4)$ group

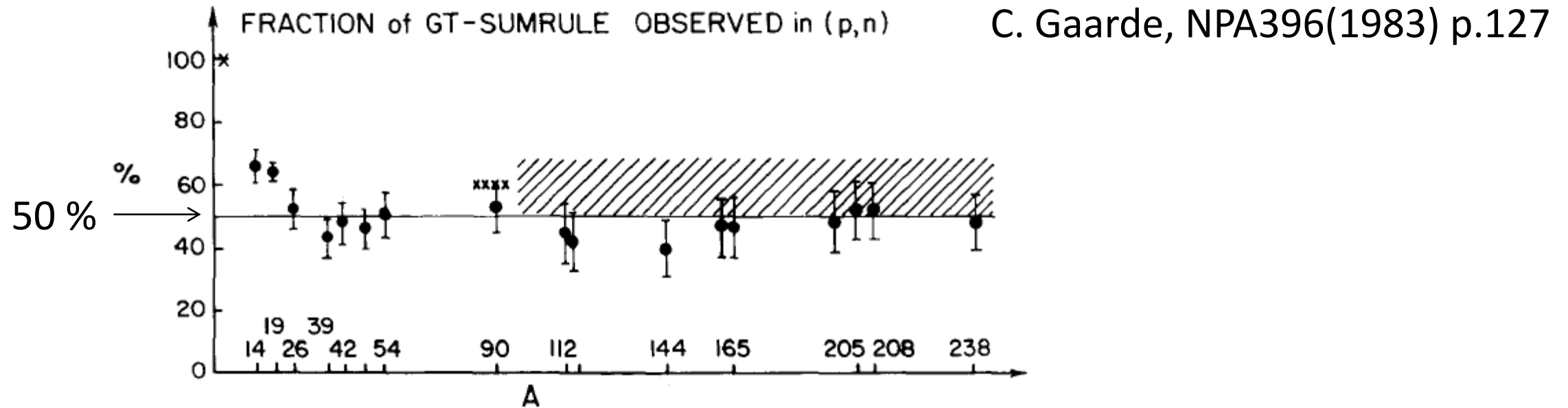
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Background ~ GT Quenching Problem ~

Gamow-Teller (GT) Strength up to $E^*=30$ MeV



Two causes which might produce the quenching

1. Δ -h configuration

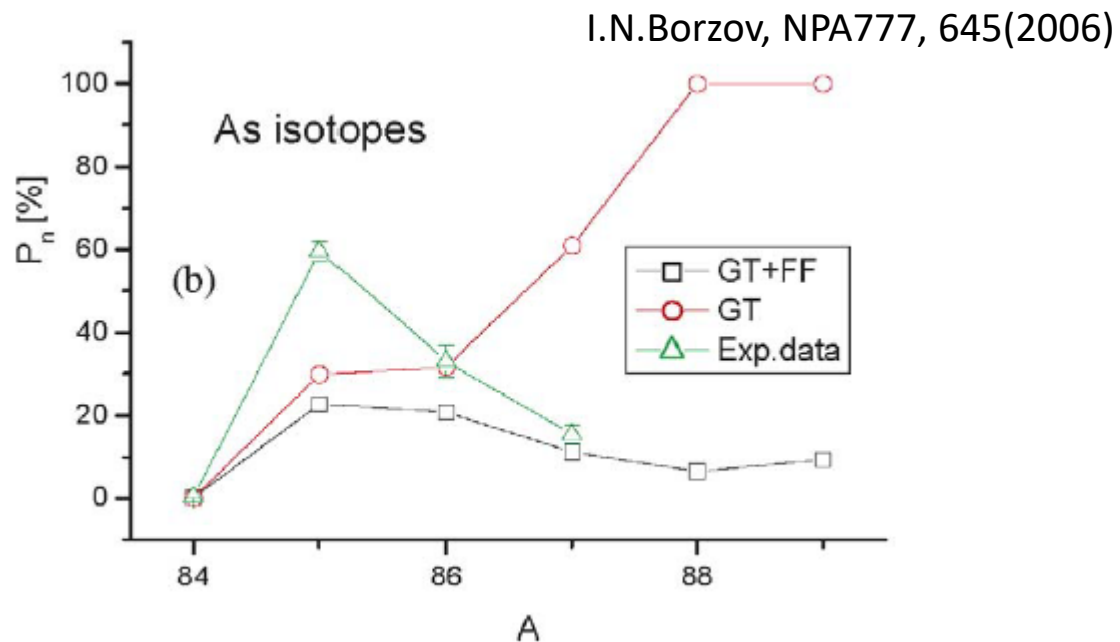
2. multi particle-multi hole configuration

Background \sim β -delayed Neutron \sim

Theory can reproduce β Half-lives ...

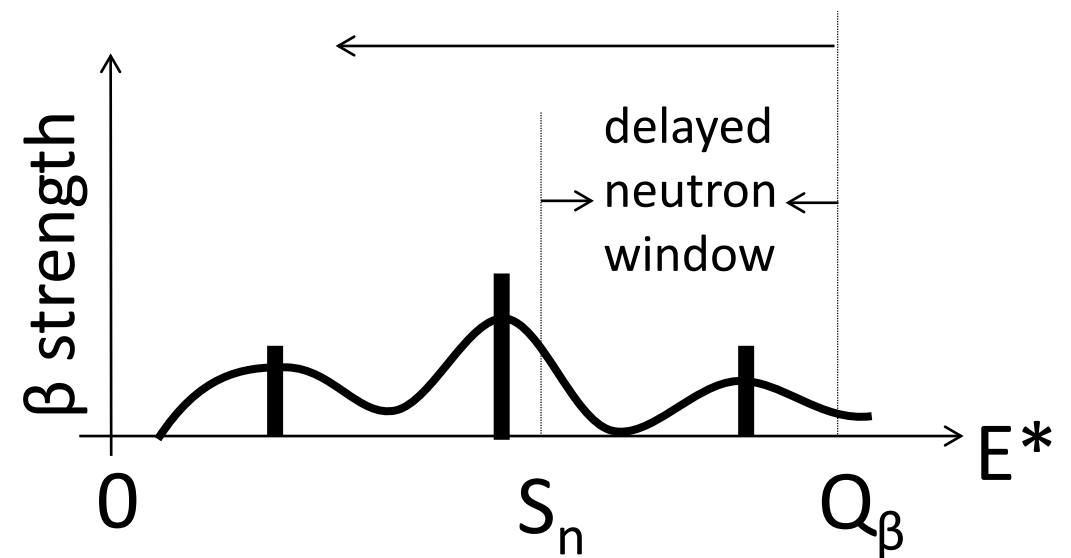
FRDM+QRPA, RHB+QRPA, HFB+QRPA

β -delayed Neutron Branching Ratio



Difference between Theory & Exp.

Main Problem causing the difference may be “Spreading Width.”



Background ~ Theoretical Works ~

- ◆ 1p1h RPA and TDA with Skyrme force
96-99% of sum-rule found in $E^* < 20$ MeV
- ◆ 2p2h TDA (STDA) with Skyrme FM, PRC 93, 044319 (2016)
80-88% of Sum-rule in $E^* < 20$ MeV for 48Ca
- ◆ Second RPA with G-matrix obtained by Bonn potential (HM3A)
70-75% of Sum-rule in $E^* < \sim 20$ MeV for 48Ca & 90Zr S. Drozdz, et al., PLB166, 18 (1986)
- ◆ Perturbative calculation G.F. Bertsch and I. Hamamoto, PRC26, 1323 (1982).
About 50 % of Sum-rule for 90Zr.

Does really proton-neutron STDA and SRPA work well?

To confirm reliability of proton-neutron STDA & SRPA

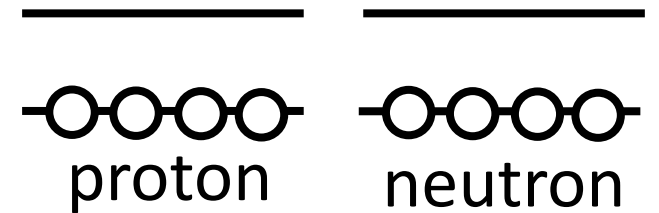
We adopt the Lipkin-Meshkov-Glick (LMG) Model

(enable to compare the model with exact solution)

Several Examples:

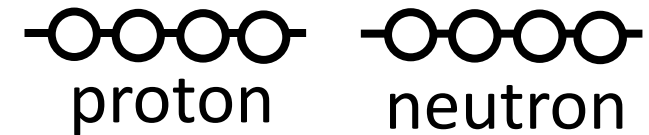
1) proton-neutron QRPA in $SO(5)$ group,

J. G. Hirsch et al., PRC56, 199 (1997)



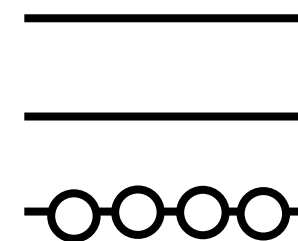
2) proton-neutron RPA in $SU(2) \times SU(2)$ group,

S. Stoica et al., PRC64, 017303 (2001)



3) SRPA in $SU(3)$ group,

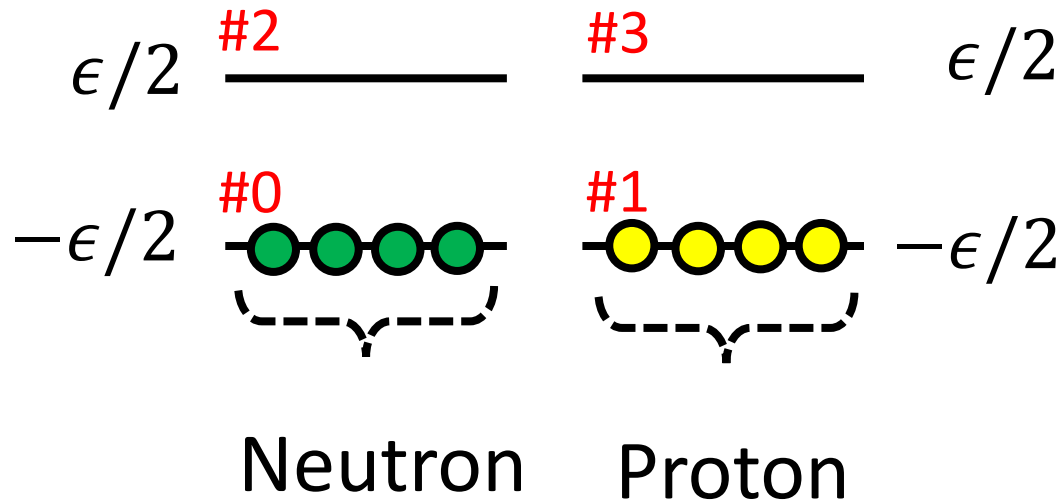
D. Gambacurta, PRC73, 024319 (2006)



Formalism

pnSRPA and pnSTDA are Investigated by LMG model in SU(4) group

Levels



Commutation Relation

$$[K_{ij}, K_{kl}] = \delta_{kj}K_{il} - \delta_{il}K_{kj},$$

where
$$K_{ij} = \sum_m a_{im}^\dagger a_{jm}$$

$$K_z \equiv \sum_i K_{ii}$$

Phonon Operator 1p1h $\Theta^+ = K_{30} + K_{21}$

2p2h $\theta^+ = (K_{30} + K_{21})(K_{20} + K_{31})$

- #0 & #1 states are fully occupied by neutron and proton.
- #0 & #2 states are assigned to neutron low and upper states.
- #1 & #3 states are assigned to proton low and upper states, respectively.

Model Hamiltonian

Only $\tau_+\tau_-$, $\tau_-\tau_+$ channels are considered in two-body interaction

$$H = \sum_i e_i K_{ii} + \underbrace{V_0(K_{21}K_{12} + K_{30}K_{03})}_{\text{p-h}} + \underbrace{V_1(2K_{21}K_{30} + 2K_{12}K_{30})}_{\text{p-p}} \\ + \underbrace{V_2(K_{30}K_{23} + K_{21}K_{32} + K_{30}K_{01} + K_{21}K_{10} + c.c.)}_{\text{3p-1h \& 1p-3h}} + \underbrace{V_3(K_{23}K_{32})}_{\text{4p}}$$

--- Model parameters ---

$$V_0 = 0.2\epsilon/N \quad V_1 = \kappa\epsilon/N \quad V_2 = -\kappa\epsilon/(5N) \quad V_3 = \kappa\epsilon/(25N) \quad N = N_n + N_p$$

We will see the excitation energy and transition strength as a function of κ

Exact Wave Function

$$|\Phi\rangle = \sum_{\alpha,\beta,\gamma,\delta} C_{\alpha\beta\gamma\delta} |N_n - \beta - \gamma, N_p - \alpha - \delta, \alpha + \gamma, \beta + \delta\rangle \\ = \sum_{\alpha,\beta,\gamma,\delta} C_{\alpha\beta\gamma\delta} (K_{21})^\alpha (K_{30})^\beta (K_{20})^\gamma (K_{31})^\delta |HF\rangle$$

Dimension to be solved $\propto N^4$

$C_{\alpha\beta\gamma\delta}$ are derived from diagonalization of H with $|\Phi\rangle$ 7

Formalism

D. Gambacurta, PRC73, 024319 (2006)

proton neutron RPA

$$Q^\dagger = \frac{X\Theta^+ - Y\Theta^-}{\sqrt{\langle HF | [\Theta^-, \Theta^+] | HF \rangle}}$$

RPA ground state

$$\begin{aligned} &\downarrow \\ Q |RPA\rangle &= 0 \\ |RPA\rangle &\Rightarrow |HF\rangle \end{aligned}$$

RPA Equation

$$\begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \hbar\omega \begin{pmatrix} G & 0 \\ 0 & -G^* \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix}$$

proton neutron SRPA

$$Q^\dagger = \frac{X\Theta^+ - Y\Theta^-}{\sqrt{\langle HF | [\Theta^-, \Theta^+] | HF \rangle}} + \frac{\chi\theta^+ - \gamma\theta^-}{\sqrt{\langle HF | [\theta^-, \theta^+] | HF \rangle}}$$

SRPA Equation

$$\begin{pmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{B}^* & \mathcal{A}^* \end{pmatrix} \begin{pmatrix} \mathcal{X} \\ \mathcal{Y} \end{pmatrix} = \hbar\omega \begin{pmatrix} \mathcal{G} & 0 \\ 0 & -\mathcal{G}^* \end{pmatrix} \begin{pmatrix} \mathcal{X} \\ \mathcal{Y} \end{pmatrix}$$

$$\mathcal{A} = \begin{pmatrix} A & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \quad \mathcal{B} = \begin{pmatrix} B & 0 \\ 0 & 0 \end{pmatrix}$$

Transition Operator

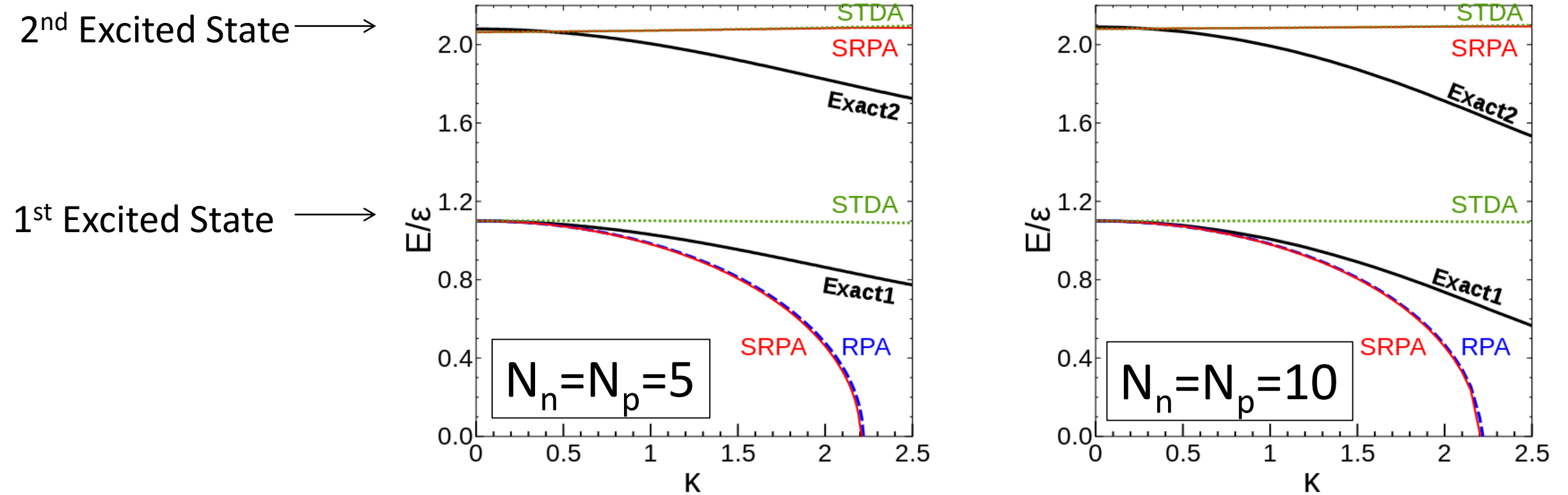
β^- transition

$$M^- = \chi \sum_{mm'} a_{3m}^\dagger a_{0m'}$$

β^+ transition

$$M^+ = \chi \sum_{mm'} a_{2m}^\dagger a_{1m'}$$

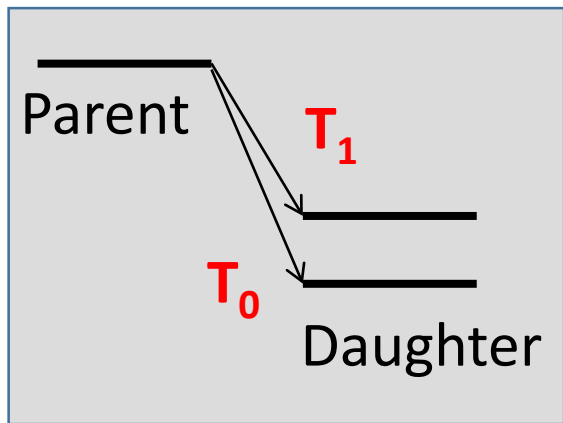
Result: Excitation Energy



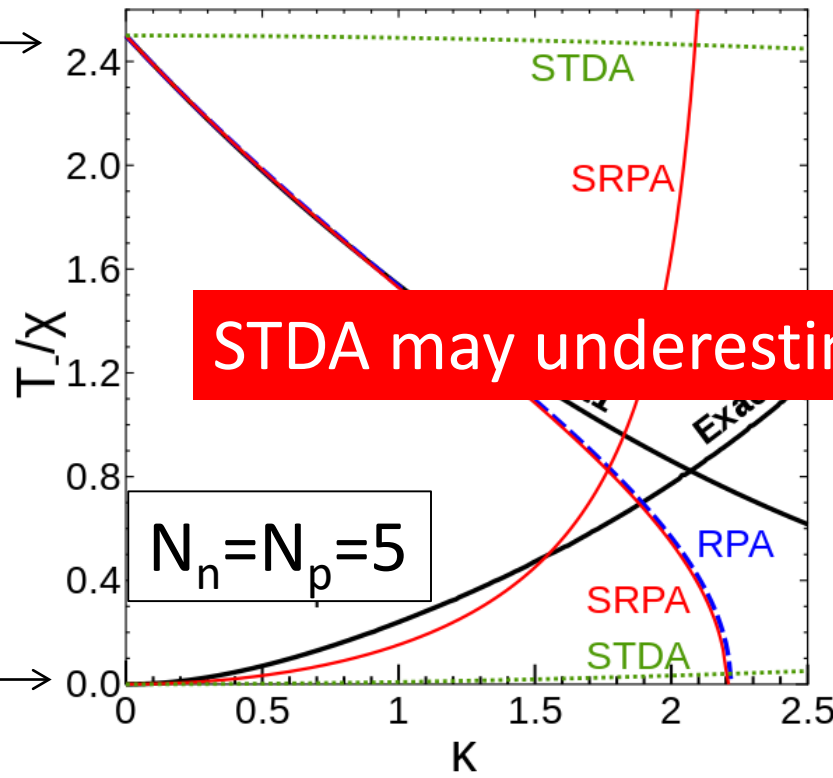
1. At small κ , RPA, SRPA, STDA are in a good agreement with the exact.
2. For 1st Excitation energy, RPA & SRPA are close.
3. For 2nd Excitation energy, SRPA & STDA are close.

Result: Transition Strength of β^-

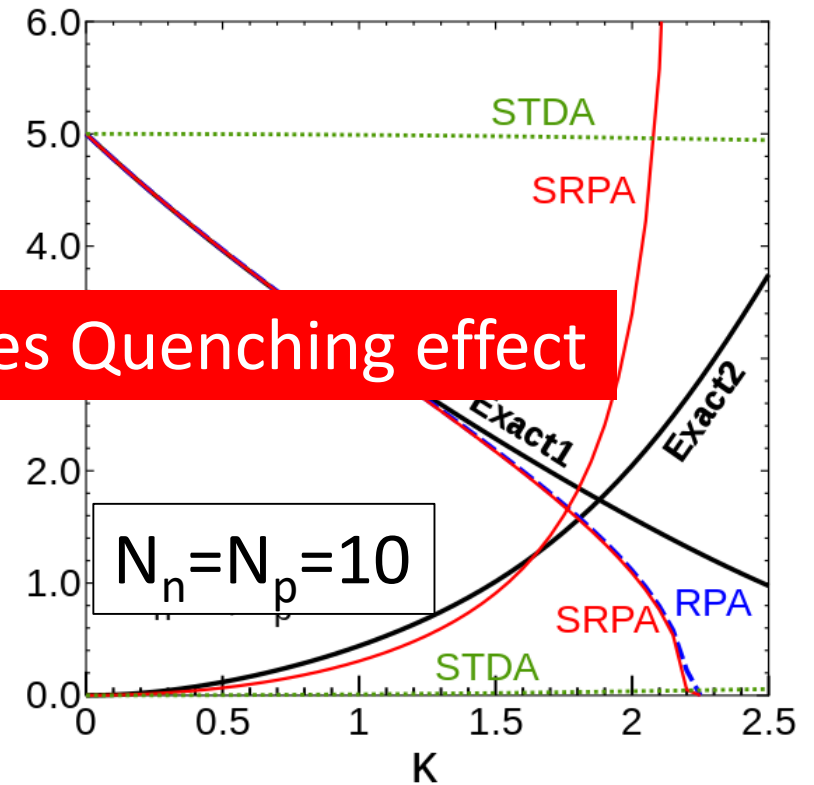
T_0 of 1st
Excited State



T_1 of 2nd
Excited State



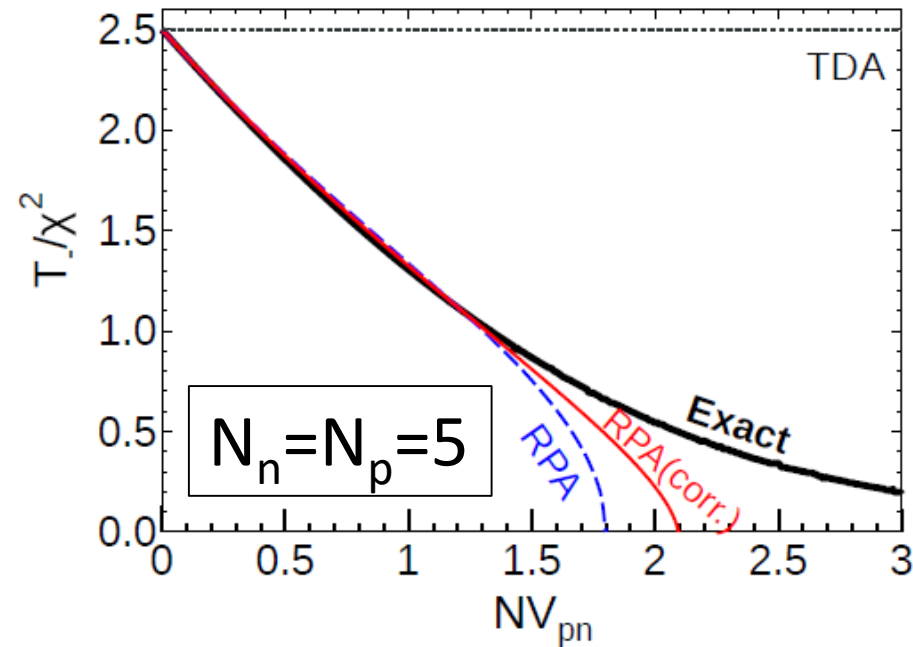
STDA may underestimate Quenching effect



1. For T_0 , RPA and SRPA work well up to $\kappa=1.5$.
2. For T_1 , SRPA works reasonably up to $\kappa=1.5$.
3. For both T_0 and T_1 , STDA shows a large deviation from the exact from a small κ .

1) SRPA takes into account ground state correlation to some extent, which STDA doesn't.

2) The same difference is obtained between RPA and TDA →

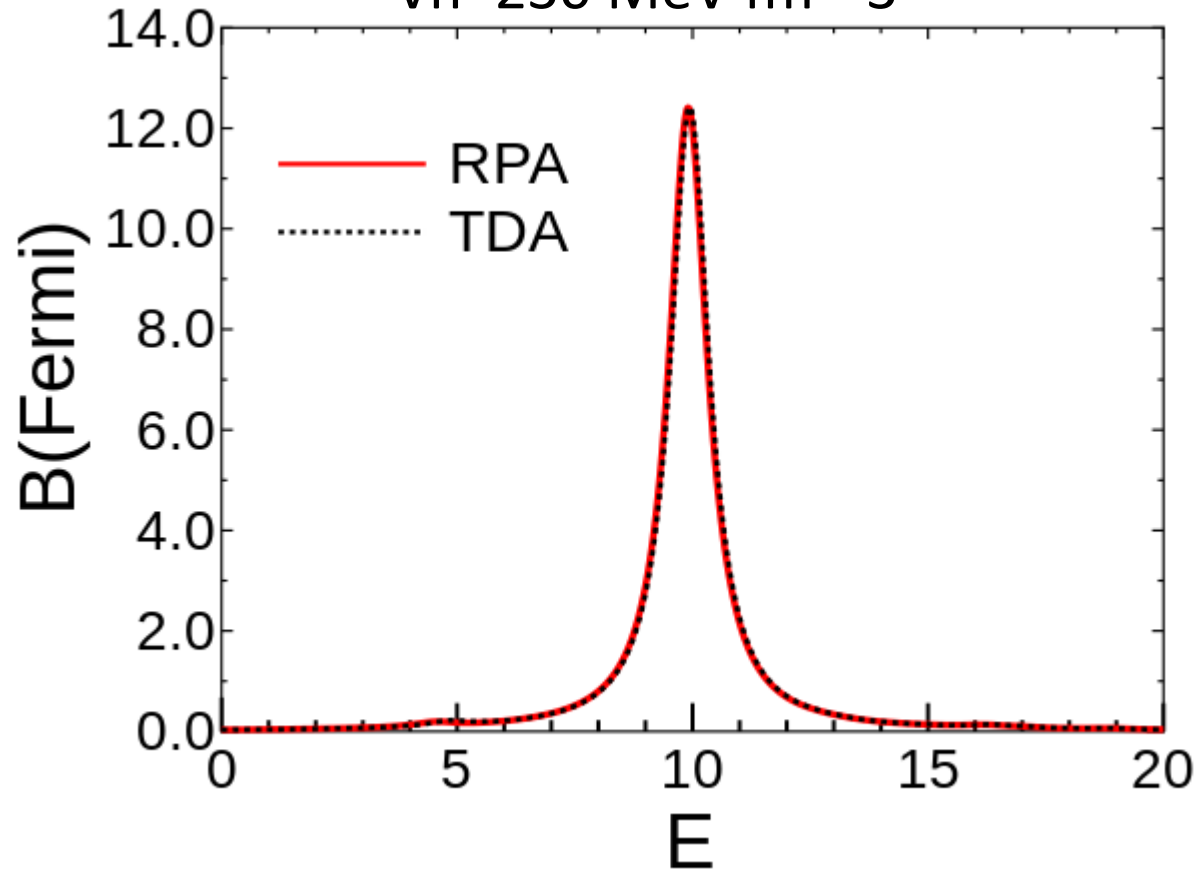


3) In the charge exchange channel, the ground state correlation generally doesn't play a significant role. Namely, we usually see results at small κ in a realistic nucleus (see Next Slide)

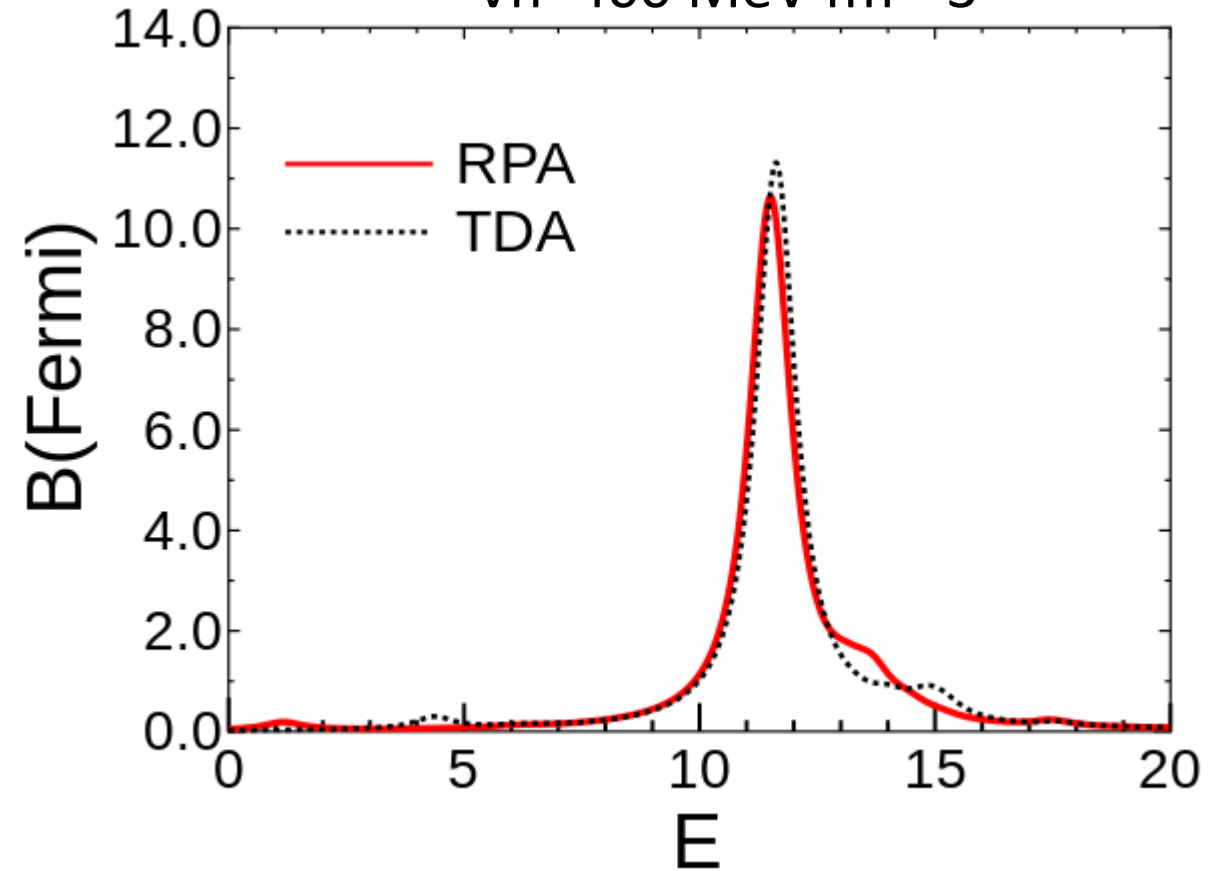
4) But we have used zero-range force in STDA. We might have used a large κ ?? ← Further analysis is in progress.

Result: ^{120}Sn Fermi Transition

Volume type pairing
 $V_n=230 \text{ MeV fm}^{-3}$



Volume type pairing
 $V_n=400 \text{ MeV fm}^{-3}$



If ground state significantly differs from HF state, TDA shows different distribution from RPA.

Summary

Investigate proton-neutron SRPA & STDA with LMG Model in SU(4) group

1) At small κ , SRPA & STDA work well for excited state
But, STDA is now accurate enough for transition strength

This might be one of the factors producing difference SRPA and STDA calculations on the GT quenching effect.

2) SRPA seems to work well as compared the exact solution.
Higher correlation is required ? or we cannot exclude a contribution of Δ -h coupling to explain the quenching problem.