η’ meson mass from lattice QCD

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CP-PACS/JLQCD Collaborations

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Introduction

- U(1) problem : one of outstanding issues in light hadron spectroscopy
  - η’ (960MeV) is much heavier than π (140MeV)
  - naive expectation (1975,S.Weinberg)

$$m_{\eta'} = 960MeV < \sqrt{3}m_{\pi} = 234MeV$$

$$m_{\eta} + m_{\eta'} = 1510MeV \neq 2m_{K} = 990MeV$$

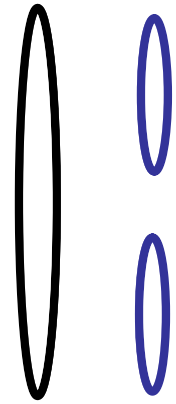
  - QCD: topological vacuum structure, axial U(1) anomaly

- Deriving the η’ mass from first principles of QCD
  - give solution of the problem
  - demonstrate departure of QCD from models

- Lattice QCD calculations
  - in quenched QCD ~< 2000
    - importance of disconnected (2-loop) loop diagram
    - relation with topological structure
    - mass calculation, theoretically not consistent
  - in two-flavor full QCD ~< 2005
    - large mass of η' is demonstrated in e.g.

      CP-PACS Collab. “Flavor singlet meson mass in the continuum limit in two-flavor lattice QCD", Phys. Rev. D67 (2003) 074503
  - N_f=2+1 simulation is essential for quantitative studies
    - $m_s >> m_u = m_d$ mixing of singlet and octet PS

      leads to physical η and η'



- first study in 2+1 flavor full QCD, taking account of mixing

Configurations

- a set of 2+1 flavor full QCD configurations from a joint project of CP-PACS/JLQCD collaborations
- Actions
  - Iwasaki RG gauge action
  - Non-perturbatively O(a) improved Wilson quark action
- Parameters
  - β =1.83 a=0.122fm (coarsest of three lattice spacings we have)
  - 16^3x32 L=2 fm
  - 10 combinations of (K_{ud}, K_s)

    K_{ud} = 0.13655, 0.13710, 0.13760, 0.13800, 0.13825

    m_{PS}/m_V = 0.78 – 0.61 (for Light-Light)

    K_s = 0.13710 and 0.13760

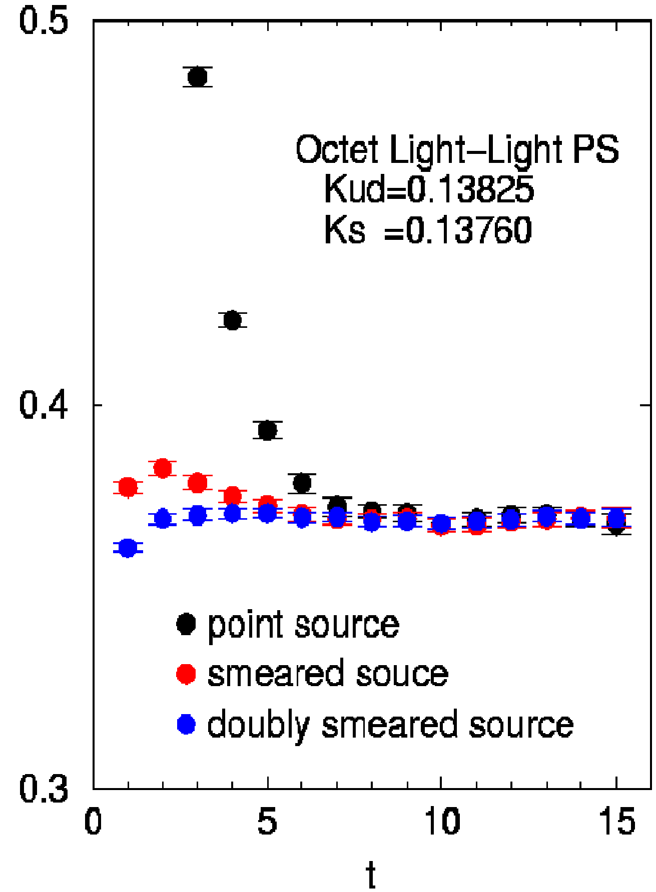
    m_{PS}/m_V = 0.75, 0.70

    physical strange quark mass m_{PS}/m_V = 0.73
  - 7000 – 8000 traj for each (K_{ud}, K_s), measurement at each 10 traj

Method

- numerical challenge
  - disconnected loop diagram
  - extract mass from *G(t)* at small *t*
  - η' as an excited state
- a combination of known techniques
  - Stochastic Noise Estimator technique
  - Smearing method (Coulomb gauge)
  - Propagator matrix diagonalization (variational method)

typical effective mass plot



- smearing kernel

$$K(\vec{n}-\vec{m}) = A\exp(-B\,|\,\vec{n}-\vec{m}\,|) \qquad \vec{n}\neq\vec{m}$$

$$K(\vec{0})=1,\ \ A=0.12,\ \ B=0.10$$

  - point sink
  - with doubly smeared source, a good estimate of octet PS meson masses at relatively small time slices

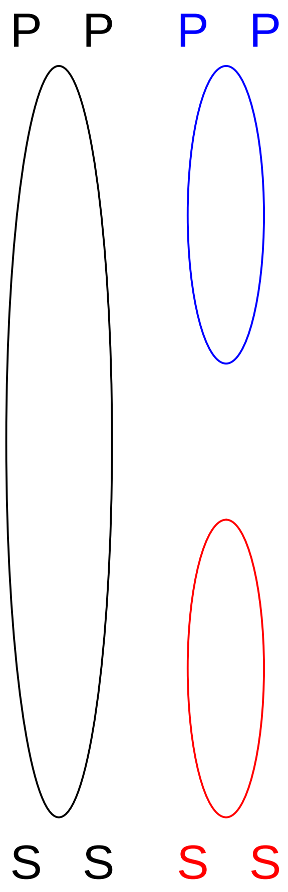
- stochastic noise for 2-loop diagram
  - prepare U(1) random numbers for all sites $\exp(i\theta(\vec{n},t))$ and solve quark propagator $q(\vec{n},t)$

$$\xi(t_{src}) = \sum_{\vec{l},\vec{m},\vec{n}} \exp(-i\theta(\vec{l},t_{src}))K(\vec{l}-\vec{m})K(\vec{m}-\vec{n})q(\vec{n},t_{src})$$

$$\zeta(t_{snk}) = \sum_{\vec{n}} \exp(-i\theta(\vec{n},t_{snk}))q(\vec{n},t_{snk})$$

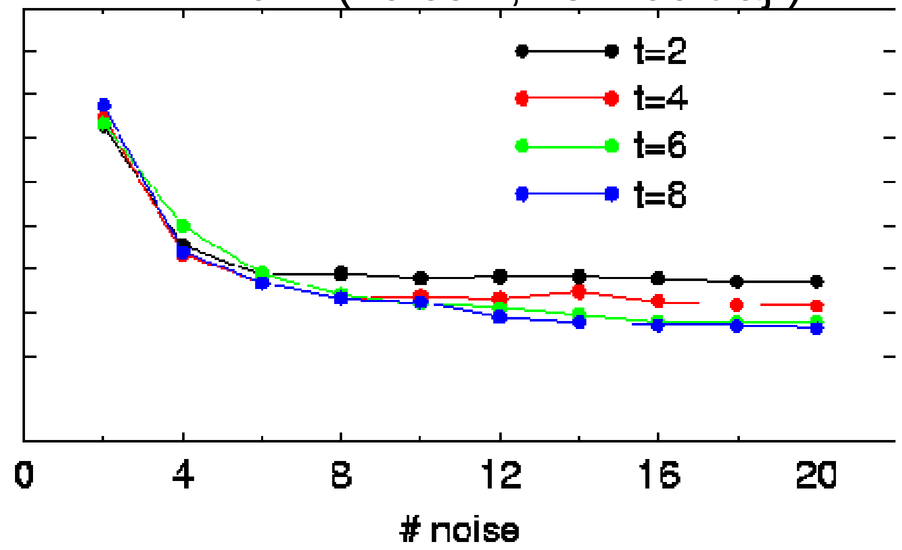
$$G_{2-loop}(t) = \frac{1}{L^3T} \sum_{t=t_{snk}-t_{src}} \langle Tr(\gamma_5\zeta(t_{snk}))Tr(\gamma_5\xi(t_{src}))\rangle_{noise}$$

  - 20 sets of stochastic noise for each color and spin 240 prop. on a config.
  - all information of configuration
    - projection to zero momentum made at both source and sink
    - average over time slice, time difference of two loops fixed



- statistical error of 2-loop diagram
  - statistical error consists of
    1) stochastic noise
    2) configuration fluctuations

Error of 2-loop diagram for all configurations (800), bin=(10 conf, i.e. 100 traj.)



  - as increase of #noise, error reaches plateaus
  - error are (almost) dominated by configuration fluctuations

- propagator matrix and mass extraction
  - prepare flavor SU(2)-singlet and strange PS operators

$$\eta_n = (\bar{u}\gamma_5 u + \bar{d}\gamma_5 d)/\sqrt{2} \qquad \eta_s = (\bar{s}\gamma_5 s)$$

  - calculate 2x2 correlators

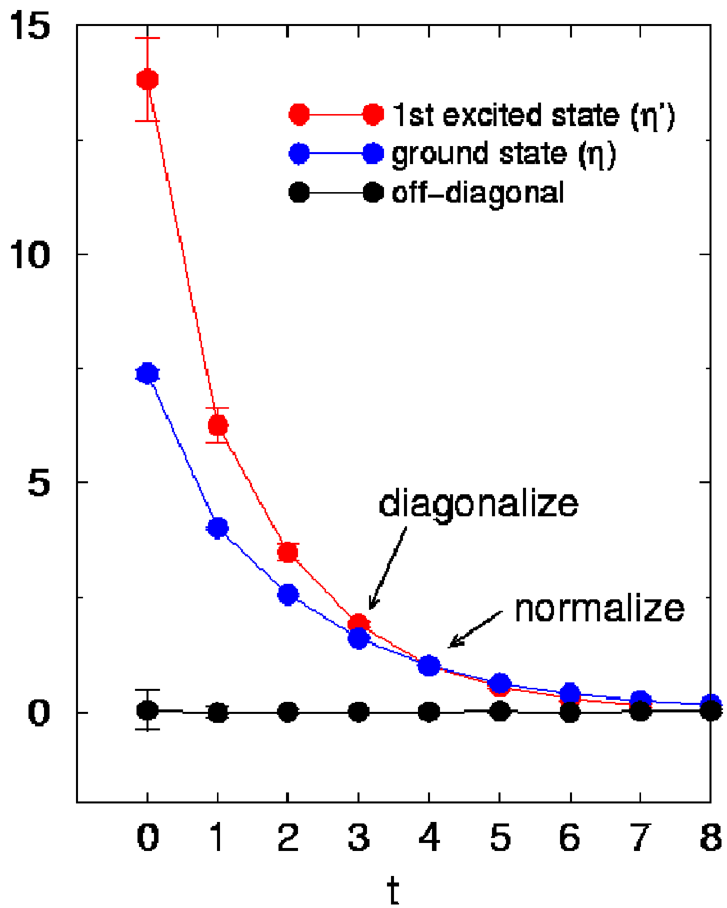
$$G(t) = \begin{pmatrix} \eta_n(t)\eta_n(0) & \eta_n(t)\eta_s(0) \\ \eta_s(t)\eta_n(0) & \eta_s(t)\eta_s(0) \end{pmatrix}$$

  - normalize the correlator matrix at some reference time slice t_0 =4, and diagonalize it to extact masses of the ground state η and the fisrt excited state η ':

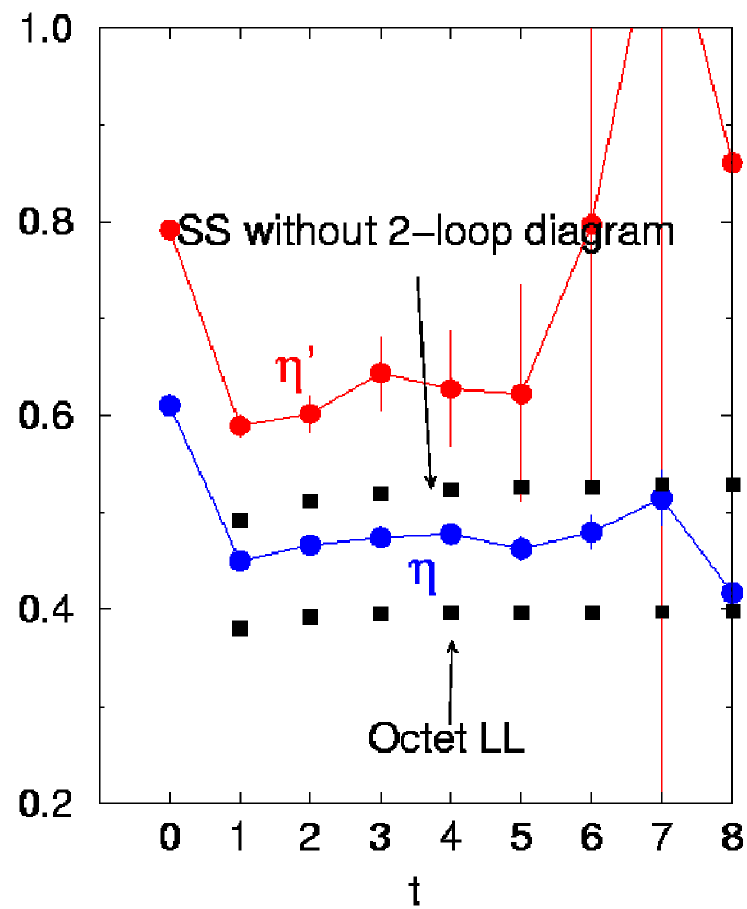
$$C\,G(t)G^{-1}(t_0)\,C^{-1} \approx \delta_{ij}\exp(-m_i(t-t_0)) \qquad C\begin{pmatrix} \eta_n \\ \eta_s \end{pmatrix} = \begin{pmatrix} \eta \\ \eta' \end{pmatrix}$$

  - assuming that the ground and 1st excited states dominate at t_d =3, C is estimated from diagonalization at t_d

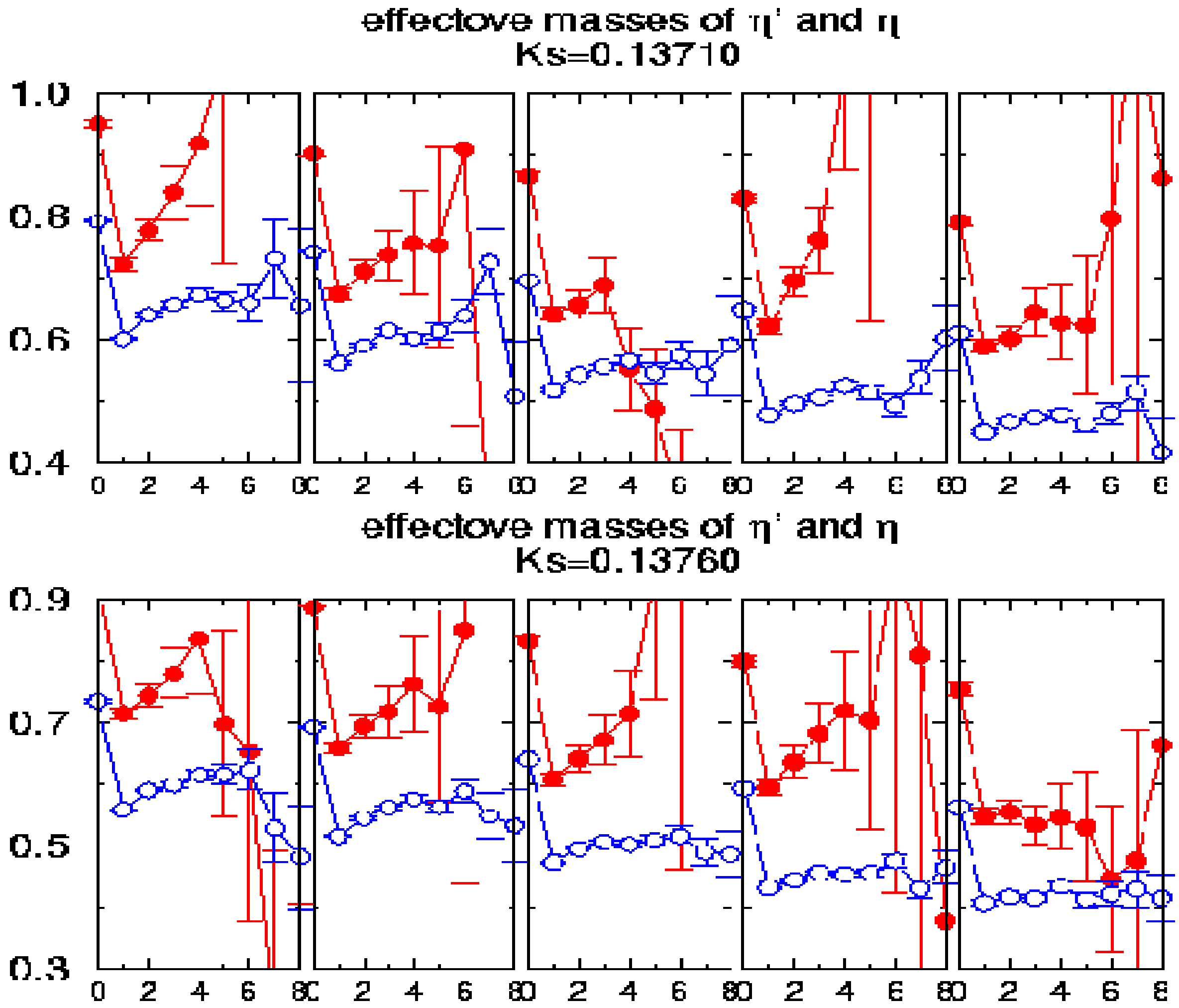
- diagonalize at $t=t_d=3$
- diagonal parts decay exponentially
- off-diagonal parts are negligible for all *t*
- η η ' contribution dominates our propagators already at *t*~2



diagonalized propagator



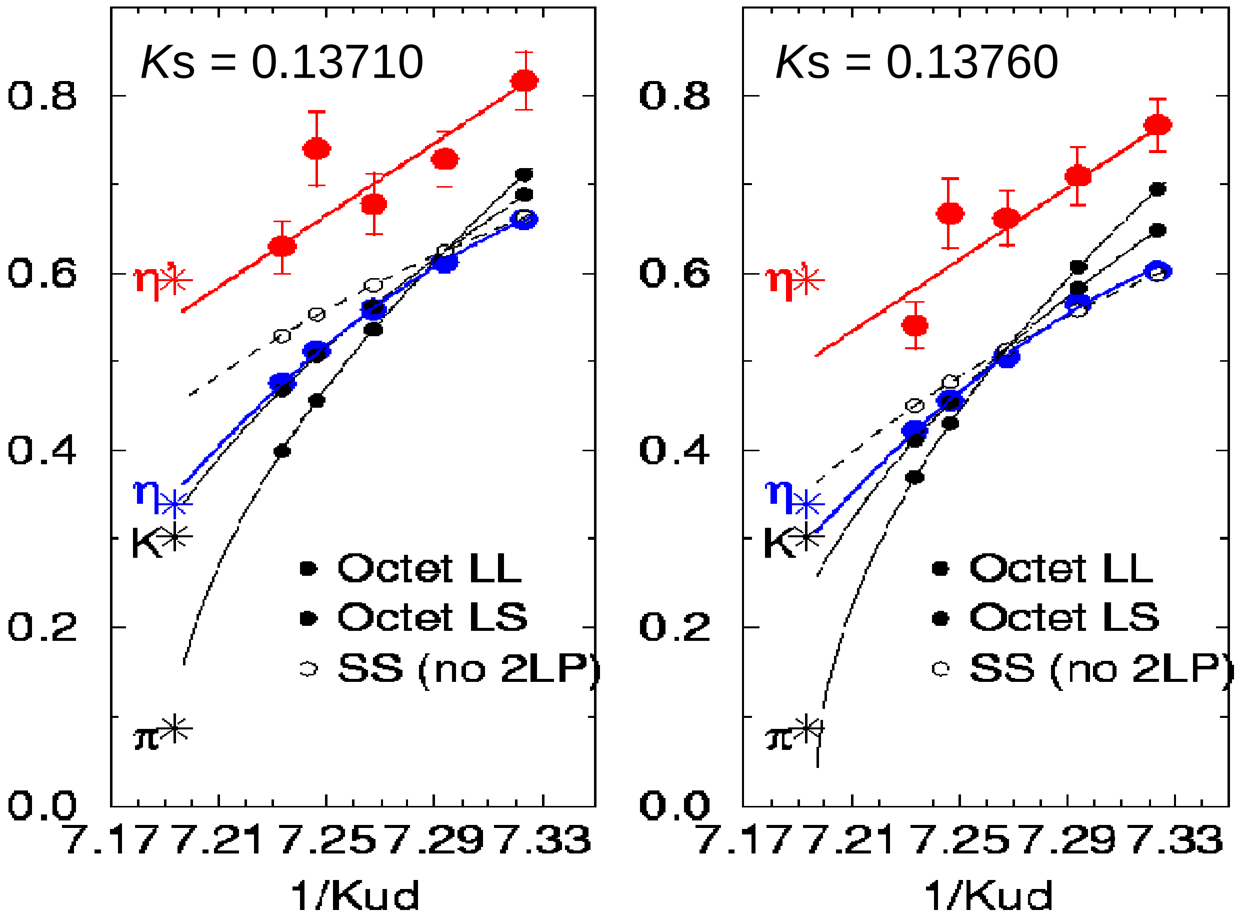
effective mass plot



*K*ud = 0.13655, 0.13710, 0.13760, 0.13800, 0.13825

– determine η mass from fit with t=3-5
– estimate η' mass from fit with t=2-4

Results



$$m_{\eta} = A + B_{ud}\, m_{ud} + B_s\, m_s + D_{ud}\, m_{ud}^2 \qquad m_{\eta'} = A' + B_{ud}'\, m_{ud} + B_s' m_s$$

$$m_{ud} = (1/K_{ud} - 1/K_c)/2 \qquad m_s = (1/K_s - 1/K_c)/2$$

- masses at the physical point

$$m_{\eta'} = 0.871(46)\ \text{GeV} \qquad \text{exp. } 0.960\ \text{GeV}$$

$$m_{\eta} = 0.545(16)\ \text{GeV} \qquad \text{exp. } 0.550\ \text{GeV}$$

(input π ρ K)

  - η mass: consistent with experiment
  - η' mass: much heavier than octet masses

    smaller than experiment only by 100MeV (2 sigma)

- comparison with another work
  - similar work by UKQCD Collab. with KS action
  - η' ~ 0.99 GeV, η ~ 0.60 GeV (an estimate from variational propagator at a simulation point)

Summary and Future Work

- η and η' masses are estimated in N_f=2+1 lattice QCD, albeit at one lattice spacing
- Mass of η' larger than octet PS masses, only 100 MeV below experiment
- Importance of U(1) contribution demonstrate

- lighter quarks, finer lattices toward a quantitative solution
- reduce computational cost
  - implement an efficient SET algorithm with e.g. a variance reduction, truncated eigen-mode ex., domain-decomposition ….
- remove systematic errors from excited states
  - fine-tune smearing function
  - use more operators