Non-perturbative renormalization of B_K and quark mass in SF scheme with quenched domain-wall QCD

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Plan of talk

- **Definiton of B_K and its renormalization factor.**
- **Strategy to derive RGI $\widehat{B}_K$**
- **SF scheme (definition of non-perturbative renormalization factor)**
  Alpha collaboration '05
- **Simulation results**
  - **RGI $\hat{B}_K$**
  - **Scaling behavior of SSF at $g^2(L) = 3.480$**
- **Renormalization of quark mass**
- **Summary**

Definition of B_K and its renormalization factor

- **The kaon B parameter: B_K**

$$B_K \equiv \frac{\langle \bar{K^0} | \bar{s}\gamma^\mu (1-\gamma_5) d \bar{s}\gamma^\mu (1-\gamma_5) d | K^0 \rangle}{\frac{8}{3}\langle \bar{K^0} | \bar{s}\gamma^\mu (1-\gamma_5) d | 0 \rangle \langle 0 | \bar{s}\gamma^\mu (1-\gamma_5) d | K^0 \rangle}$$

CP-PACS obtained B_K with quenched domain-wall fermion 2001

$\Rightarrow$ We need non-perturbative renormalization for B_K

- **Renormalization factor for B_K**

$$Z_{B_K}(a\mu) = \frac{Z_{VV+AA}(g_0, a\mu)}{Z_A^2(g_0)} = \frac{Z_{VA+AV}(g_0, a\mu)}{Z_V^2(g_0)}$$

  - **DWF has a good chiral symmetry even on lattice**
    - $Z_V(g_0) = Z_A(g_0)$ **has been shown by CP-PACS 2003**
    - $Z_{VV+AA}(g_0, a\mu) = Z_{VA+AV}(g_0, a\mu)$
      - **$\Rightarrow$this relation will be checked later**

Our strategy

- **Lattice bare $B_K^{(0)}$ with DWF**

  $\Rightarrow$ Renormalization group invariant (RGI) $\hat{B}_K$

- $$\mathcal{Z}_{B_K}(g_0) = \underbrace{Z^{\mathrm{PT}}_{VA+AV}(\infty,\mu_{\max})}_{\mathrm{III}}\,\underbrace{Z^{\mathrm{NP}}_{VA+AV}(\mu_{\max},\mu_{\min})}_{\mathrm{II}}\,\underbrace{Z^{\mathrm{NP}}_{B_K}(g_0,a\mu_{\min})}_{\mathrm{I}}$$

- **Three steps for RGI $\hat{B}_K$ via SF scheme.**

  I. $B_K^{(0)} \Rightarrow B_K^{(\mathrm{SF})}(a\mu_{\min})$ **at hadronic scale**

  suppress lattice artifact $\mathcal{O}(a\mu)$: $a\mu \ll 1$

  II. Non-pert. RG running : $B_K^{(\mathrm{SF})}(\mu_{\min}) \Rightarrow B_K^{(\mathrm{SF})}(\mu_{\max})$

  III. Perturbative running to RGI: $B_K^{(\mathrm{SF})}(\mu_{\max}) \Rightarrow \hat{B}_K$

- **II. III. are reguralization independent.**

  Possible to evaluate with cheap Wilson fermion.

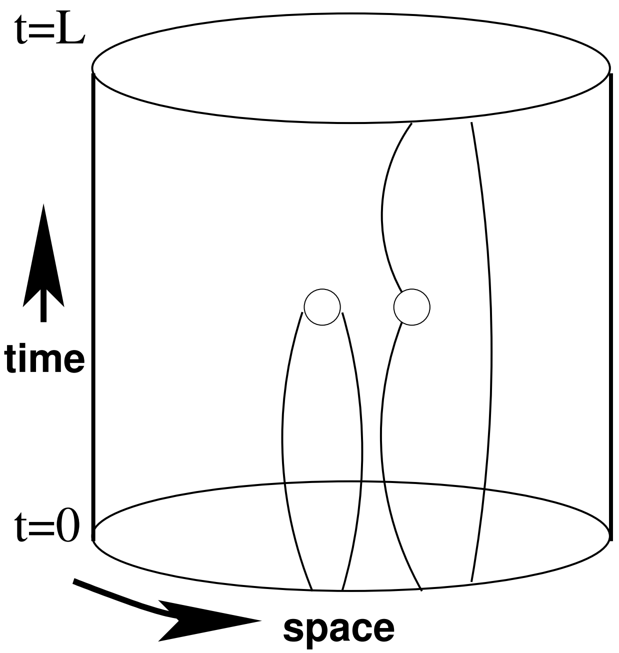
  Already calculated by Alpha collab. for O_{VA+AV} '05

  $\Rightarrow O_{VA+AV}$ has no mixing problem even for wilson fermion.

- **Our target: I. $Z_{VA+AV}(g_0, a\mu_{\min})$ with DWF.**

Set up for SF scheme Alpha '05

- **SF formalism**



$\mathcal{O}'_{ij} = a^6 \sum_{\vec{x}\vec{y}} \bar{\zeta}'_i(\vec{x})\Gamma\zeta'_j(\vec{y})$ **parity odd**

$\mathcal{O}'^{(R)}_{ij} = Z_\zeta \mathcal{O}'^{(0)}_{ij}$

$O_{VA+AV} = (\bar{\psi}_1\gamma_\mu\psi_2)(\bar{\psi}_3\gamma_\mu\gamma_5\psi_4) + (\bar{\psi}_1\gamma_\mu\gamma_5\psi_2)(\bar{\psi}_3\gamma_\mu\psi_4)$

$O^{(R)}_{VA+AV} = Z_{VA+AV} O^{(0)}_{VA+AV}$

$\mathcal{O}_{ij} = a^6 \sum_{\vec{x}\vec{y}} \bar{\zeta}_i(\vec{x})\Gamma\zeta_j(\vec{y})$ $\quad \mathcal{O}^{(R)}_{ij} = Z_\zeta \mathcal{O}^{(0)}_{ij}$

- **Correlation function (Alpha collaboration '05)**

$$\mathcal{F}^{\pm}_{[\Gamma_A,\Gamma_B,\Gamma_C]}(x_0) = \frac{1}{L^3}\langle \mathcal{O}_{21}[\Gamma_A]\mathcal{O}_{45}[\Gamma_B]O^{\pm}_{VA+AV}(x)\mathcal{O}'_{53}[\Gamma_C]\rangle$$

- **Orbifolding construction for SF with DWF Y.T '04,'06**

- **In order to cancel divergence due to boundary operator**
  - **boundary-boundary correlation function**

$$f_1 = -\frac{1}{2L^6}\langle O'_{12}[\gamma_5]O_{21}[\gamma_5]\rangle, \quad k_1 = -\frac{1}{2L^6}\sum_k \langle O'_{12}[\gamma_k]O_{21}[\gamma_k]\rangle$$

- **Three specific cases**

Scheme 1 | **Scheme 3** | **Scheme 7**

$$h_1^{\pm}(x_0) = \frac{\mathcal{F}^{\pm}_{[\gamma_5,\gamma_5,\gamma_5]}(x_0)}{f_1^{3/2}}, \qquad h_3^{\pm}(x_0) = \frac{\mathcal{F}^{\pm}_{[\gamma_5,\gamma_k,\gamma_k]}(x_0)}{f_1^{3/2}}, \qquad h_7^{\pm}(x_0) = \frac{\mathcal{F}^{\pm}_{[\gamma_5,\gamma_k,\gamma_k]}(x_0)}{f_1^{1/2}k_1}$$

- **Renormalization condition**

$$Z_{VA+AV;s}(g_0, a\mu)h_s^{\pm}(x_0; g_0) = h_s^{\text{cont}}(x_0, g_0)^{\pm}|_{g_0=0}, \qquad s = 1, 3, 7$$

- **Renormalization factor**

$$Z^{\pm}_{VA+AV;s} = \frac{h_s^{\pm}(x_0; g_0)|_{g_0=0}}{h_s^{\pm}(x_0; g_0)}$$

Simulation parameters

- **Same parameters as in previous CP-PACS calculation of B_K**

  Phys.Rev. D64(2001)114506

  - **Domain-wall fermion with $M = 1.8$ and $N_5 = 16$**
  - **Iwasaki gauge action at $\beta = 2.6$, $\beta = 2.9$, $\beta = 3.2$(new,not in the paper)**

    corresponding to $a^{-1} \sim 2$, $a^{-1} \sim 3$, $a^{-1} \sim 4$ GeV

- **Renormalization factor $Z_{VA+AV}(\beta, a\mu)$**

  - $1/\mu_{\min} = 2L_{\max}$ **Def. of $L_{\max}$: $\bar{g}^2_{\mathrm{SF}}(L_{\max}) = 3.480$**

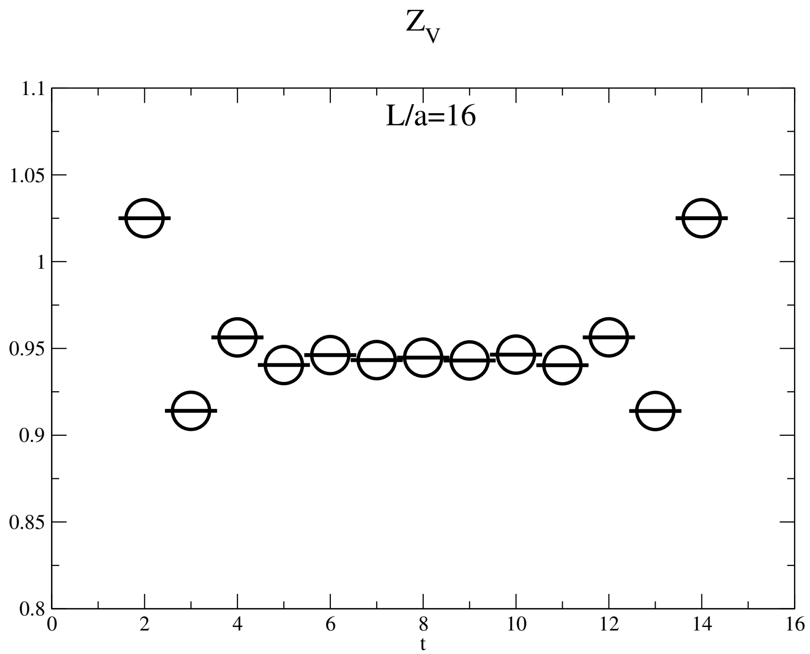
  Z_{VA+AV} **at $\beta = 2.6$, $\beta = 2.9$, $\beta = 3.2$**

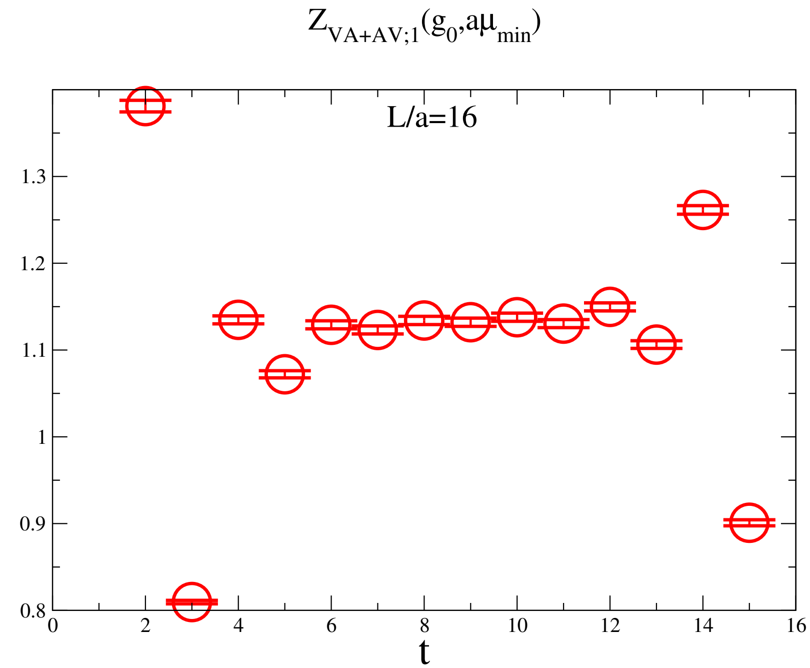
- **Fine tuning of β to give $aN_T = aN_L = 2L_{\max} = 1.498r_0$**

$L_{\max}/a$	6	8	10	12	14	16	18
β	2.446	2.6339	2.7873	2.9175	3.03133	3.0313	3.2254
# of conf	1000	1000	1000	1018	716	556	564

Simulation results

- **Numerical result for Z_V and $Z_{VA+AV;s}$**





We take the value at $x_0 = N_T/2$ (Alpha collaboration '05)

$$Z_{B_K}(a\mu_{\mathrm{min}}) = \frac{Z_{VA+AV;s}(x_0 = 16/2, a\mu_{\mathrm{min}})}{Z_V^2(x_0 = 16/2, a\mu_{\mathrm{min}})} = 1.2708(53)$$

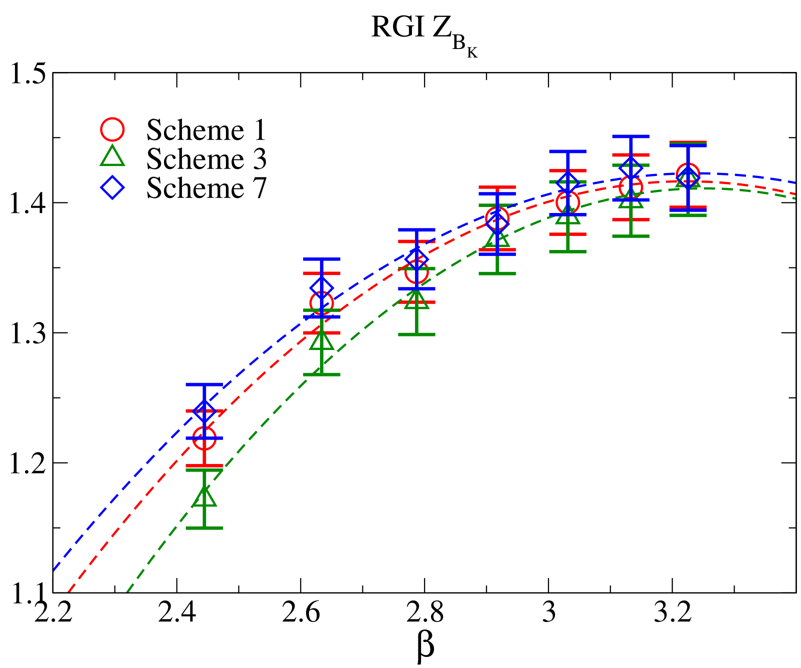
Strategy to derive RGI $\widehat{B}_K$

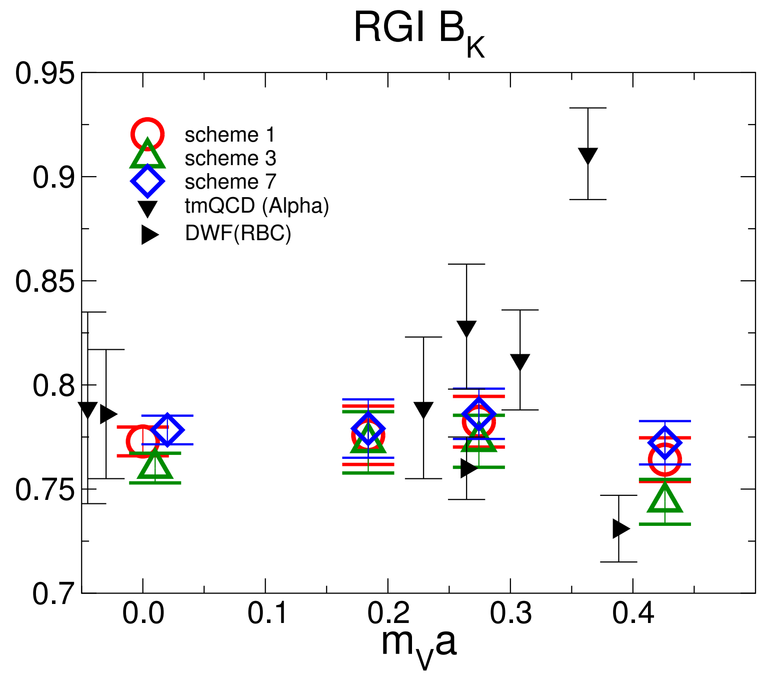
- **Lattice bare $B_K^{(0)}$ with DWF**

  $\Rightarrow$ Renormalization group invariant (RGI) $\hat{B}_K$

- $$\mathcal{Z}_{B_K}(g_0) = \underbrace{Z^{\mathrm{PT}}_{VA+AV}(\infty, \mu_{\mathrm{max}})}_{\mathrm{III}} \underbrace{Z^{\mathrm{NP}}_{VA+AV}(\mu_{\mathrm{max}}, \mu_{\mathrm{min}})}_{\mathrm{II}} \underbrace{Z^{\mathrm{NP}}_{B_K}(g_0, a\mu_{\mathrm{min}})}_{\mathrm{I}}$$

Result for RGI $\mathcal{Z}_{B_K}$ and $\hat{B}_K$ (quench)





- **Fitting form** $\hat{Z}_{B_K}(\beta) = a_1 + b_1(\beta - 3) + c_1(\beta - 3)^2$

  interpolated at $\beta = 2.6, \beta = 2.9, \beta = 3.2$

$\hat{B}_K = 0.773\ (7)$**(preliminary)** : **Scheme 1**

$\hat{B}_K = 0.760\ (7)$**(preliminary)** : **Scheme 3**

$\hat{B}_K = 0.778\ (7)$**(preliminary)** : **Scheme 7**

$\hat{B}_K = 0.786(31)$: **RBC**

$\hat{B}_K = 0.789(46)$: **Alpha**

- **Error is smaller**
  - **Scaling behaviour is better**
  - **Error of each data is smaller**
  - **The smallest lattice spacing is finer**

Conversion of RGI B_K to that in $\overline{\mathrm{MS}}$ scheme

- **Conversion from RGI to $\overline{\mathrm{MS}}$ NDR at $\mu = 2\mathrm{GeV}$ by NLO PT**

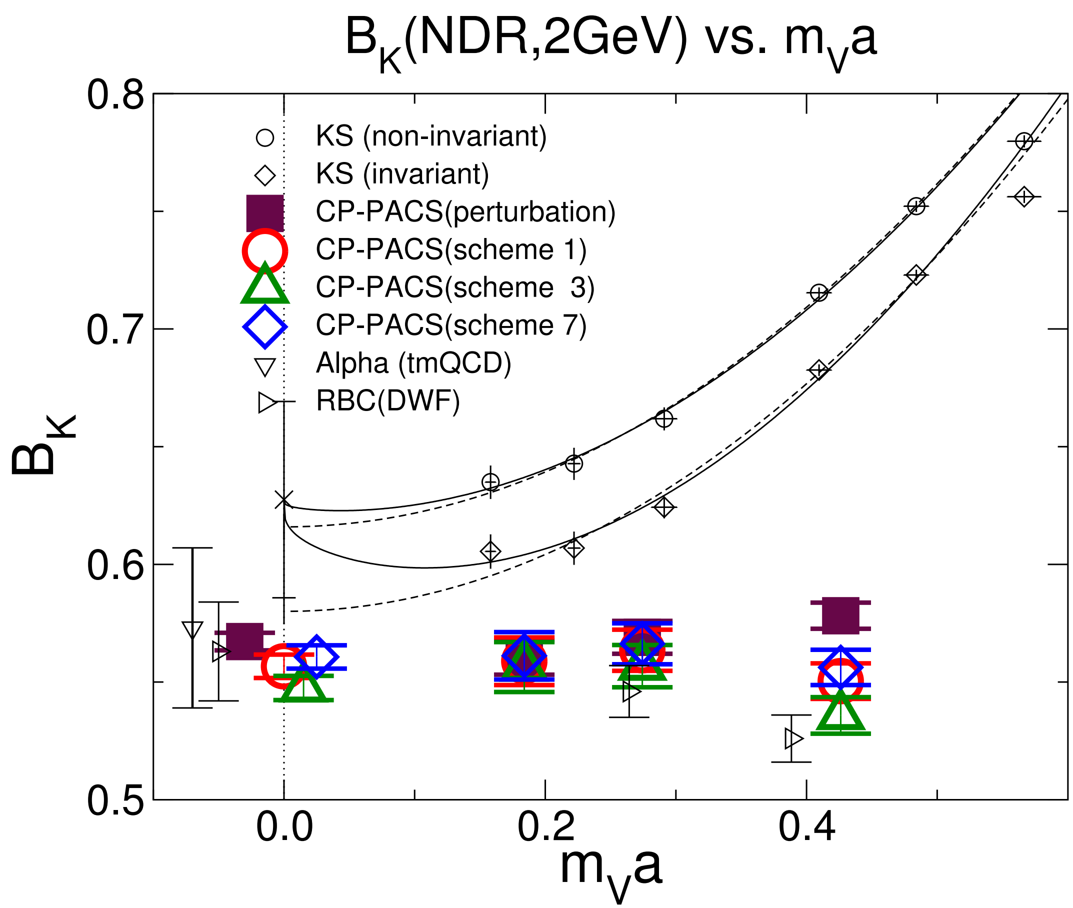
$$B_K^{\overline{\mathrm{MS}}}(\mathrm{NDR},\mu) = \left[\frac{g^2_{\overline{\mathrm{MS}}}(\mu)}{4\pi}\right]^{\frac{\gamma_O^+}{2b_0}} \exp\left[\int_0^{g_{\overline{\mathrm{MS}}}(\mu)} dg\left(\frac{\gamma^+}{\beta(g)} - \frac{\gamma_0^+}{b_0 g}\right)\right]\hat{B}_K$$

- **Gauge coupling**

$$\Lambda_{\overline{\mathrm{MS}}} = \mu(b_0 g^2_{\overline{\mathrm{MS}}})^{-\frac{b_1}{2b_0^2}} \exp\left[-\frac{1}{2b_0 g^2_{\overline{\mathrm{MS}}}}\right]\exp\left[-\int_0^{g_{\overline{\mathrm{MS}}}} dg\left(\frac{1}{\beta_{\overline{\mathrm{MS}}}(0)} + \frac{1}{b_0 g^3} - \frac{b_1}{b_0^2 g}\right)\right]$$

  - $\Lambda_{\overline{\mathrm{MS}}} = 0.586(48)/r_0$ Necco&Sommer '02

Comparison of our result for $B_K(\overline{\mathrm{MS}}, 2\mathrm{GeV})$ with previous ones



- $B_K^{\overline{\mathrm{MS}}}(\mathrm{NDR}, 2\mathrm{GeV}) = 0.557(5)(^{+4}_{-10})$ **(preliminary)**

  $B_K^{\overline{\mathrm{MS}}}(\mathrm{NDR}, 2\mathrm{GeV}) = 0.567(4)$ **: CP-PACS(perturbative ren.)**

  $B_K^{\overline{\mathrm{MS}}}(\mathrm{NDR}, 2\mathrm{GeV}) = 0.563(21)$ **: RBC**

  $B_K^{\overline{\mathrm{MS}}}(\mathrm{NDR}, 2\mathrm{GeV}) = 0.573(34)$ **: Alpha**

Further checks

- **Scaling behavior of Step Scaling Function**

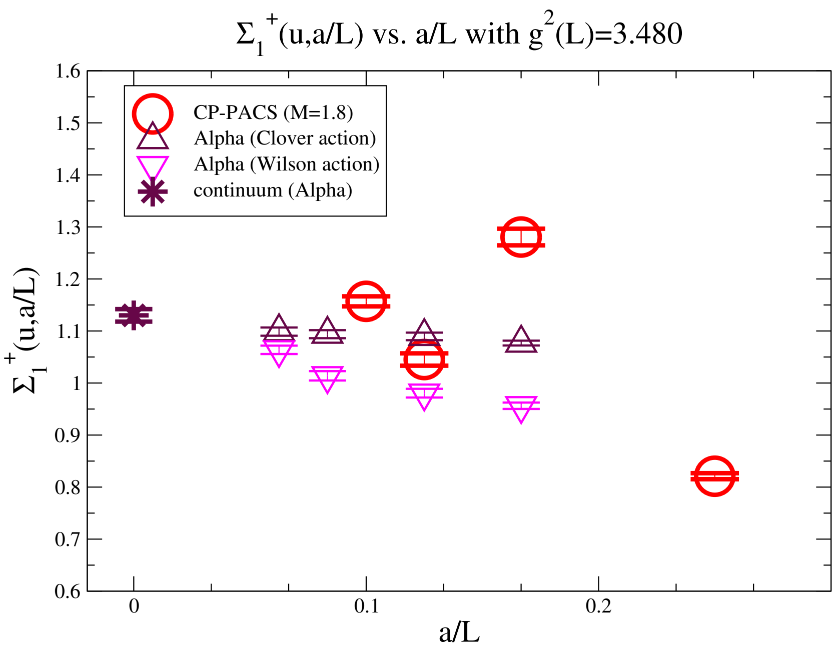
$$\Sigma_{VA+AV;s}(u, L_{\max}/a) = \frac{Z_{VA+AV;s}(g_0, a/2L_{\max})}{Z_{VA+AV;s}(g_0, a/L_{\max})}$$

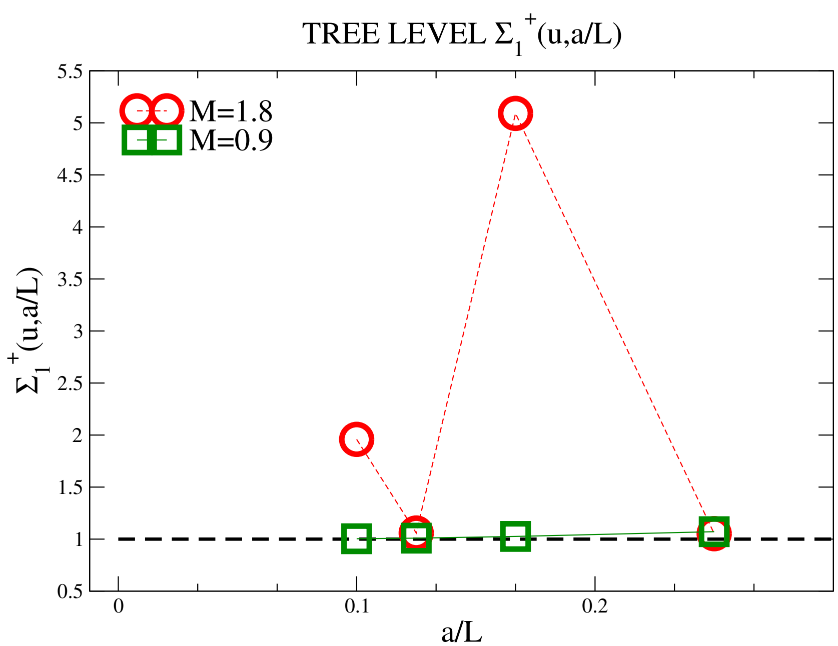
- **Chiral symmetry breaking effect in renormalization factor**

$$Z_{VA+AV} = Z_{VV+AA}$$

Scaling behavior of SSF at $g^2(L) = u = 3.480$

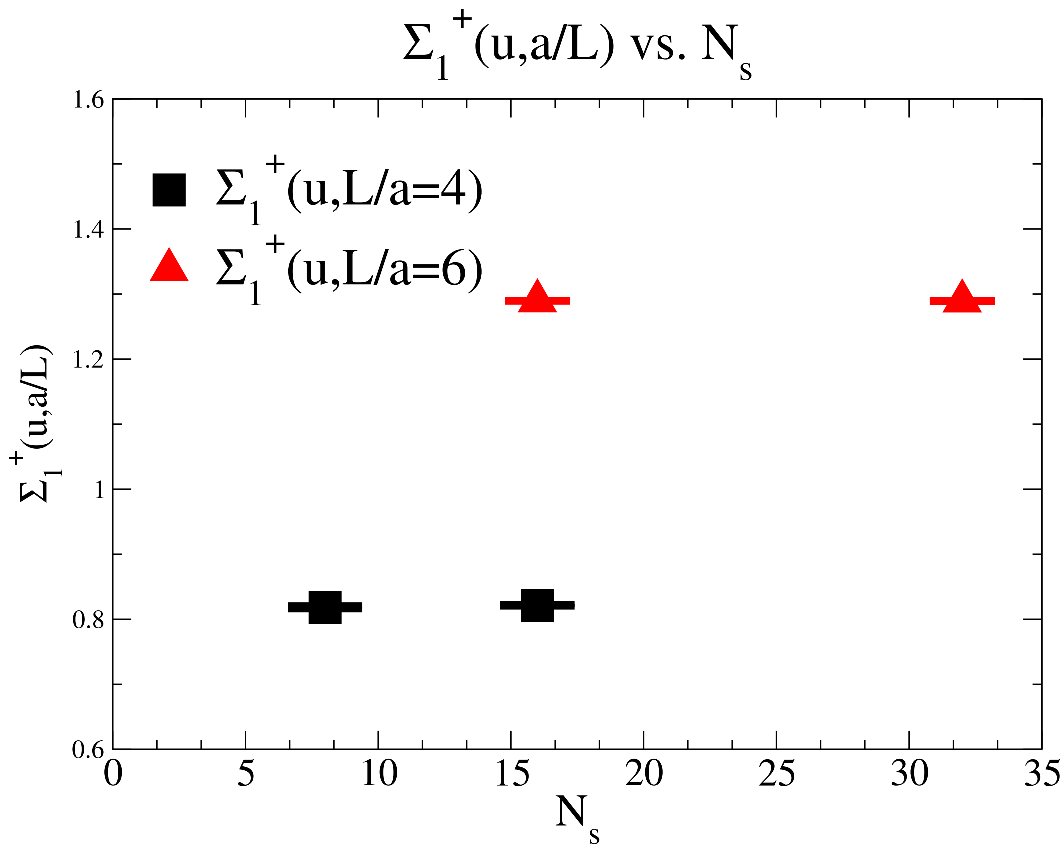
$$\sigma_{VA+AV;s}(u) = \lim_{a\to 0} \Sigma_{VA+AV;s}(u, a/L_{\max}) = \left.\frac{Z_{VA+AV;s}(g_0, a/2L_{\max})}{Z_{VA+AV;s}(g_0, a/L_{\max})}\right|_{\bar{g}^2=u}$$





- **Boundary $O(a)$ effect is large at $M = 1.8$**

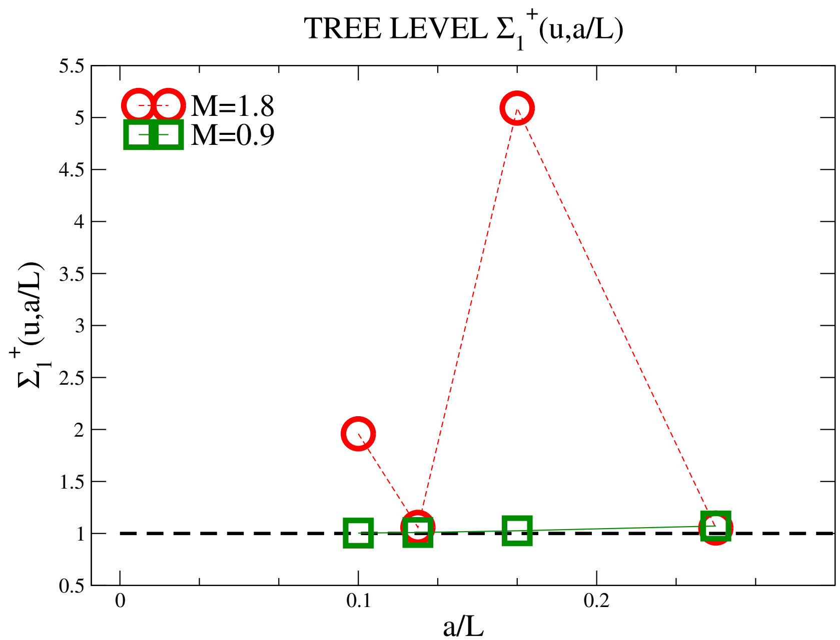
Results at $N_5 = 8$ and $N_5 = 32$

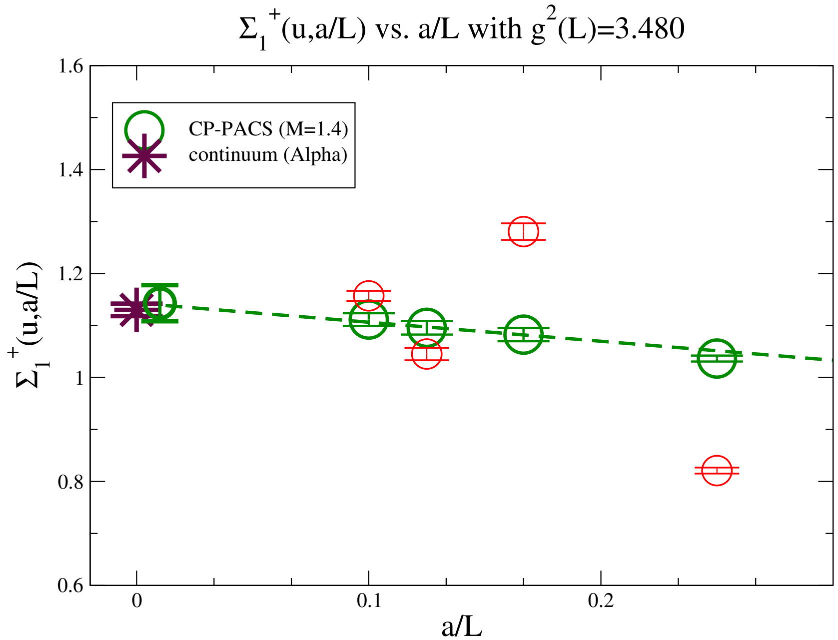


- **No N_5 dependence**

Our solution

- **SOLUTION 1 : results at** $M = 1.4$



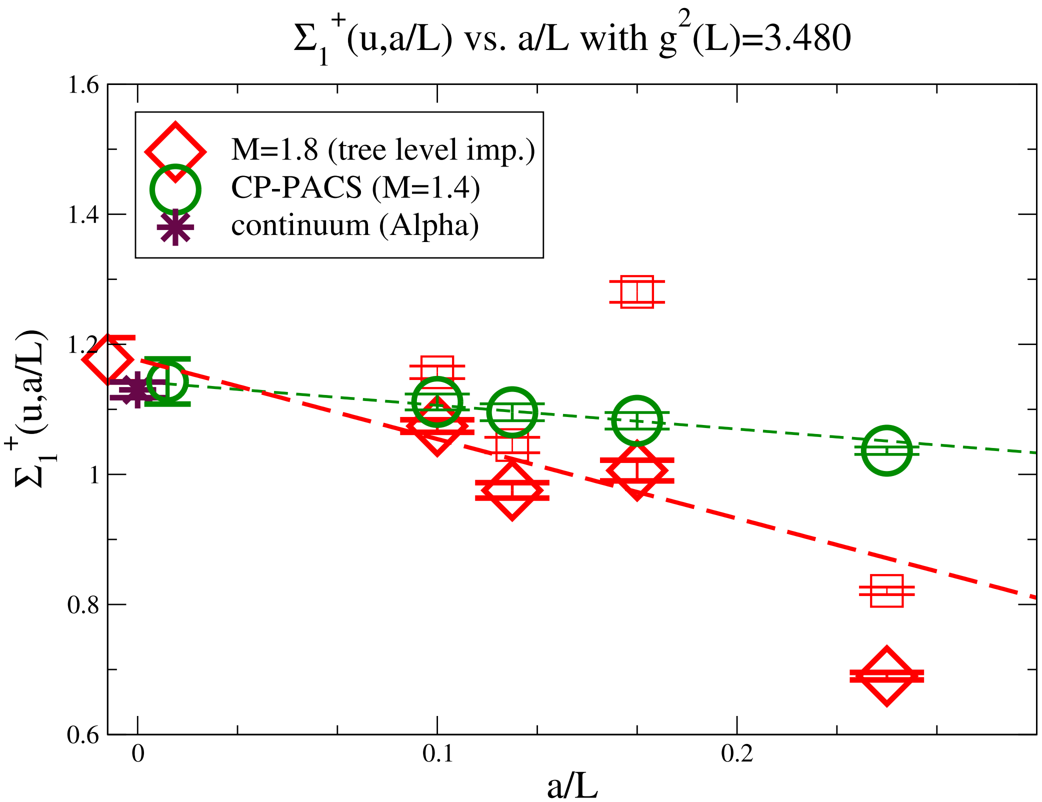


- $M_{\text{tadpole}} \sim 1$

- **Scaling violation is small**
- **Continuum limit is consistent with Alpha's**

SOLUTION 2 : Tree-level improvement

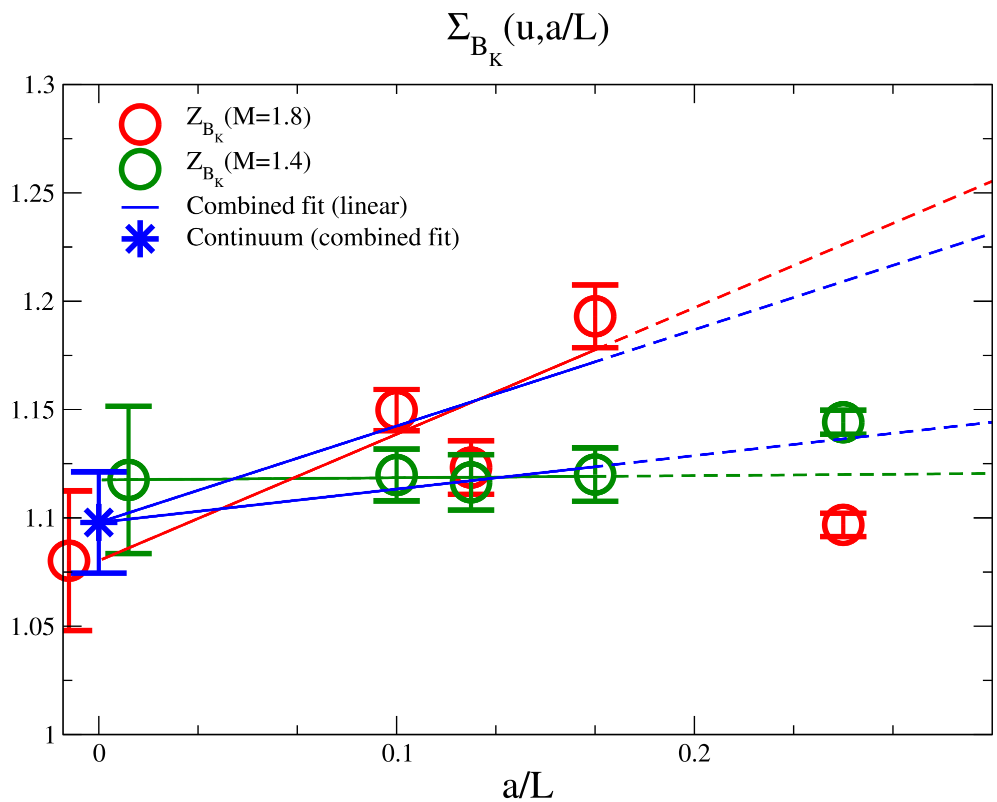
$$Z_{VA+AV;s}(g_0, a\mu) h_s^{\pm}(x_0; g_0) = h_s(x_0, g_0)^{\pm}|_{g_0=0}{}^{\mathrm{CONT}} \Rightarrow h_s(x_0, g_0)^{\pm}|_{\mathrm{tree}}{}^{\mathrm{Lattice}}$$



- **Scaling behavior is improved**
- **Continuum limit is consistent with Alpha's**

Scaling behavior of SSF for Z_{B_K}

$$\Sigma_{B_K}(u, L_{\max}/a) = \frac{Z_{B_K;s}(g_0, a/2L_{\max})}{Z_{B_K;s}(g_0, a/L_{\max})}$$



- **Results at $M = 1.4$ and $M = 1.8$ are consistent**
- **$\mathcal{O}(a)$ error is patrly cancelled in Z_{B_K}**

- **Chiral symmetry breaking effect**
  - **WT Identity**
    - $\delta q_1 = -i\gamma_5 \tilde{q}_1, \quad \delta\zeta_1 = i\gamma_5\tilde{\zeta}_1, \quad \delta\zeta_1' = i\gamma_5\tilde{\zeta}_1{}'$

$$\langle O_{VA+AV} O[\zeta]\rangle_S = \langle O_{VV+AA} O'[\zeta]\rangle_S$$
$$\Rightarrow Z_{VV+AA} = Z_{VA+AV}$$

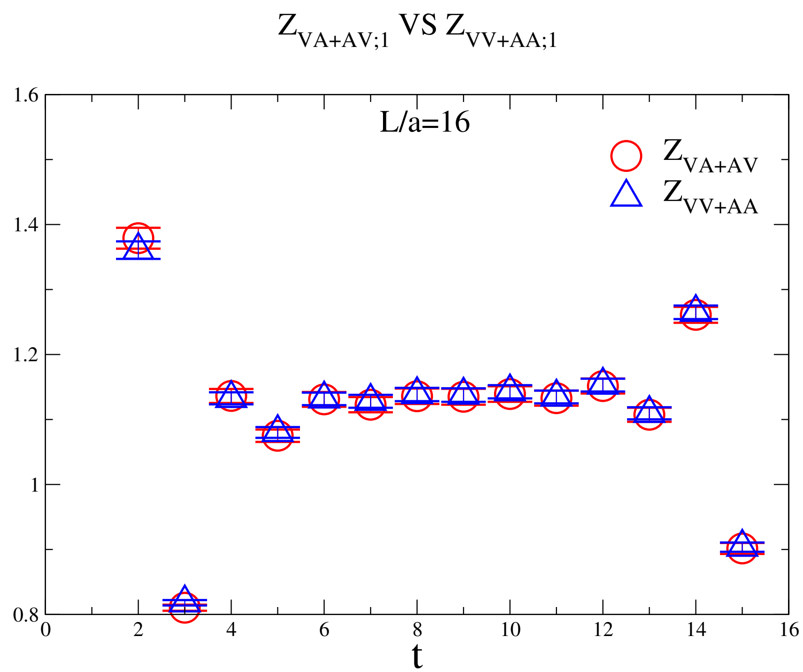
  - **For the domain-wall case**
    - $S \to S + Y$ **:** Y **is the chiral symmetry breaking term**

$$\begin{aligned}\langle O_{VA+AV} O[\zeta]\rangle_S &= \langle O_{VV+AA} O'[\zeta]\rangle_{S+Y} \\ &\neq \langle O_{VV+AA} O'[\zeta]\rangle_S\end{aligned}$$

$$\langle O_{VA+AV} O[\zeta]\rangle_S \Leftrightarrow \langle O_{VV+AA} O'[\zeta]\rangle_S$$

We investigate if $Z_{VV+AA} = Z_{VA+AV}$ holds or not.

Result for $Z_{VA+AV;1}$ and $Z_{VV+AA;1}$ ($N_5 = 16$)



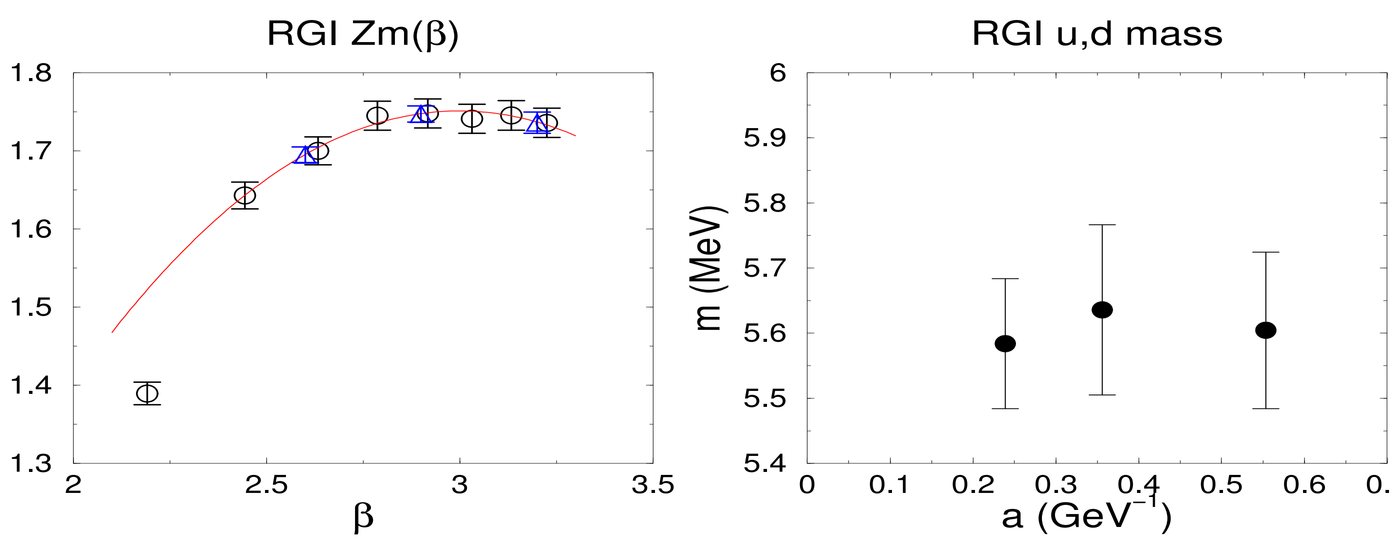
- **Good agreement between $Z_{VA+AV;1}$ and $Z_{VV+AA;1}$**
- $Z_{B_K} = \frac{Z_{VA+AV}}{Z_V^2}$ **: justified**

Quark mass renormalization

- **RGI quark mass**

$$M = \overline{m}(\mu)\left(2b_0\overline{g}^2(\mu)\right)^{-\frac{d_0}{2b_0}} \exp\left(-\int_0^{\overline{g}(\mu)} dg \left(\frac{\tau(g)}{\beta(g)} - \frac{d_0}{b_0 g}\right)\right)$$

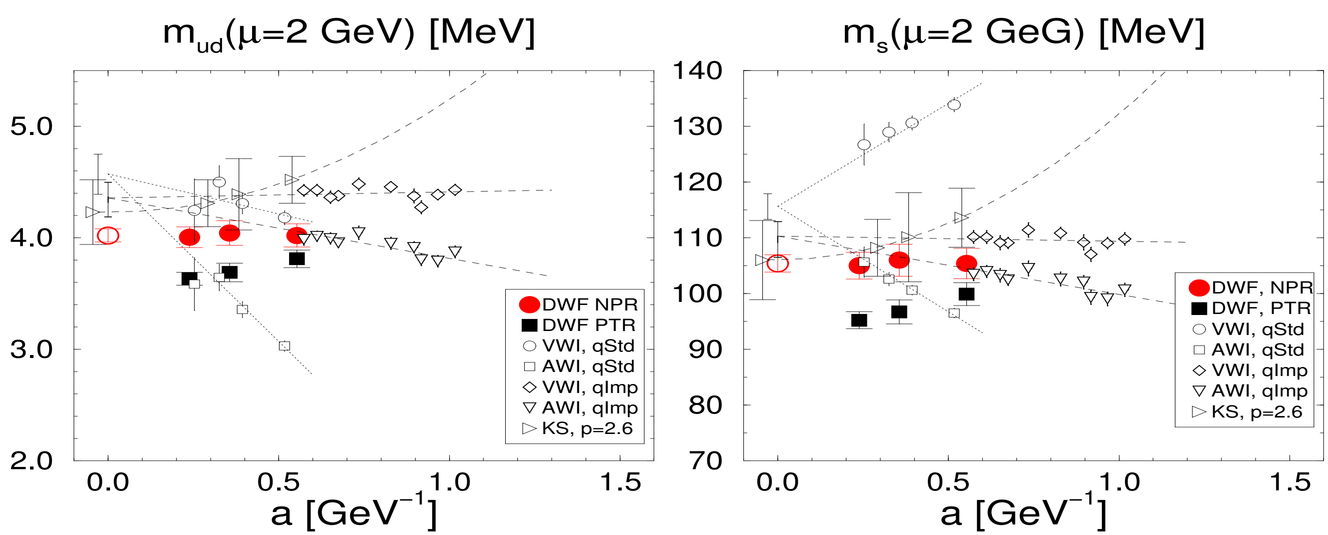
- $\mathcal{Z}_m(g_0) = \underbrace{Z_m^{\rm PT}(\infty,\mu_{\rm max})}_{\rm III}\underbrace{Z_m^{\rm NP}(\mu_{\rm max},\mu_{\rm min})}_{\rm II}\underbrace{Z_m^{\rm NP}(g_0,a\mu_{\rm min})}_{\rm I}$
  - **Lattice bare mass with DWF $\Rightarrow$ RGI mass**
  - **II, III: given by Alpha collab. for PCAC mass**
  - $Z_m(g_0,a\mu_{\rm min}) = 1/Z_P(g_0,a\mu_{\rm min})$ **for DWF**



$\overline{\text{MS}}$ scheme at $\mu = 2$ GeV

$$m^{\overline{\text{MS}}}(\mu) = M\left(2b_0\left(g^{\overline{\text{MS}}}(\mu)\right)^2\right)^{\frac{d_0}{2b_0}} \exp\left(\int_0^{g^{\overline{\text{MS}}}(\mu)} dg\left(\frac{\tau^{\overline{\text{MS}}}(g)}{\beta^{\overline{\text{MS}}}(g)} - \frac{d_0}{b_0 g}\right)\right)$$

- $\Lambda_{\overline{\text{MS}}} = 0.586(48)/r_0$ **as an input.**



$$m_{u,d}(\mu = 2\text{GeV}) = 4.020(58)\ \text{MeV}, \quad m_s(\mu = 2\text{GeV}) = 105.4(15)\ \text{MeV}$$

Summary

- **Evaluation of NP renormalization factor for** B_K
  - **Bare** $B_K^{(0)}$ **on lattice with DWF** $\Rightarrow$ **RGI** $\hat{B}_K$
  - **Investigation of chiral breaking effect**
    - $Z_{VA+AV} = Z_{VV+AA}$ **in DWQCD**
- **RGI** $\hat{B}_K = 0.773(7)$ **(preliminary)**
- $\overline{\text{MS}}$ $B_K(\text{NDR}, 2\text{GeV}) = 0.557(5)(\,^{+4}_{-10}\,)$ **(preliminary)**
  - **Consistent with previous results**
- **Check of scaling behavior of SSF**
- **Evaluation of NP renormalization factor for quark mass**

$$m_{u,d}(\mu = 2\text{GeV}) = 4.020(58)\ \text{MeV}, \quad m_s(\mu = 2\text{GeV}) = 105.4(15)\ \text{MeV}$$