Lattice QCD calculation of the ρ meson decay width

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We calculate ρ meson decay width
from scattering phase shift for I=1 ππ system.

Results have already been presented in arXiv:0708.3705.

1. Introduction

Previous works of ρ meson decay

1) S. Gottlieb, P.B. Mackenzie, H.B. Thacker, D. Weingarten PLB134(1984)346.

   Quench , $m_\pi/m_\rho = 0.84 - 0.91$

2) R.D. Loft, T.A. DeGrand PRD9(1989)2692.

   Quench , $m_\pi/m_\rho = 0.9$

3) C. McNeile, C. Michae + UKQCD PLB556(2003)177.

   $N_f = 2$, $m_\pi/m_\rho = 0.578^{+13}_{-19}$

problems

1) Quench

   $\rho \rightarrow \tilde{\pi}\tilde{\pi}$

   How can we extract physical decay width from "value in Quenched Theory" ?

2) $m_\pi/m_\rho > 1/2$

   ρ meson behaves as "stable particle" !!

This work

$$N_f = 2 \quad , \quad m_\pi / m_\rho = 0.42$$
$$L = 2.53 \text{ fm } (La = 12) \quad , \quad Ta = 24$$
$$1/a = 0.91 \text{ GeV}$$

generated by CP-PACS col. PRD70(2004)074503.

Calc. of SC. phase shift for I=1 ππ system
from energy eigenvalue by Rummikainen - Gottlieb formula.

$$W \Longrightarrow \tan\delta \quad \longrightarrow \Gamma_\rho$$

All calc. is carried out with VPP5000
at Information Processing Center of U. Tsukuba.

2. Method

Moving frame

- CM. frame

$$\rho(0) \;\rightarrow\; \pi(p)\pi(-p) \qquad p \neq 0$$

$$p_{\text{min.}} = 2\pi/L$$

$$\sqrt{s} = 2 \cdot \sqrt{m_\pi^2 + p_{\text{min.}}^2} \;>>\; m_\rho \qquad \text{in typical case}$$

- Moving frame with $\mathbf{P} = (0,0,1) \times 2\pi/L$

$$\rho(P) \;\rightarrow\; \pi(P)\pi(0) \quad \Longrightarrow \quad \rho(0) \;\rightarrow\; \pi(k)\pi(-k)$$

Lorents trans.

$$k_{\text{min.}} = 2\pi/L \times \underline{1/(2\gamma)}$$

$$\sqrt{s} = 2 \cdot \sqrt{m_\pi^2 + k_{\text{min.}}^2} \;\sim\; m_\rho$$

In our case $\left(m_\pi/m_\rho = 0.42 \;,\; L = 2.53 \text{ fm} \;,\; \gamma \sim 1.2 \right)$

CM. frame	$\sqrt{s} \;/\; m_\rho = 1.47$
Moving frame	$\sqrt{s} \;/\; m_\rho = 0.97$

Rummukainen - Gottlieb formula

Extension of Lüscher's formula to moving frame
NPB450(1995)395.

$$\begin{aligned} W \quad &: \quad \text{energy in moving frame} \\ \tan\delta \quad &: \quad \text{scattering phase shift for } I=1\ \pi\pi \text{ system} \end{aligned}$$

$$\tan\delta = \ Z(k \cdot L/(2\pi))$$

$$\frac{1}{Z(q)} = \frac{1}{2\pi^2 q\gamma} \sum_{\mathbf{r}\in\Gamma} \frac{1 + (3r_3^2 - r^2)/q^2}{r^2 - q^2}$$

$$\Gamma = \{\ \mathbf{r} \mid \mathbf{r} = \gamma^{-1}[\mathbf{n} + 1/2 \cdot \mathbf{P} \cdot L/(2\pi)]\ ,\ \mathbf{n} \in \mathbf{Z}^3\ \}$$

$$\sqrt{s} = \sqrt{W^2 - P^2} = 2\sqrt{m_\pi^2 + k^2} \quad , \quad \gamma = W/\sqrt{s}$$

Diag. method

We set $\mathbf{P} = (0,0,1) \times 2\pi/L$

$$\mathcal{O}_1(t) \equiv \rho_3(\mathbf{P},t) = \sum_{\mathbf{x}} \frac{1}{\sqrt{2}} \Big(\bar{u}(\mathbf{x},t)\,\gamma_3\,u(\mathbf{x},t) \;-\; \bar{d}(\mathbf{x},t)\,\gamma_3\,d(\mathbf{x},t) \Big) \cdot \mathrm{e}^{i\mathbf{P}\cdot\mathbf{x}}$$

$$\mathcal{O}_2(t) \equiv (\pi\pi)(\mathbf{P},t) = \frac{1}{\sqrt{2}} \Big(\pi^-(\mathbf{P},t)\,\pi^+(\mathbf{0},t) \;-\; \pi^+(\mathbf{P},t)\,\pi^-(\mathbf{0},t) \Big)$$

$$G_{ij}(t) = \langle\, \mathcal{O}_i^\dagger(t) \;\; \mathcal{O}_j(0) \,\rangle \qquad (\; : \; 2 \times 2 \;\; \text{matrix} \;)$$

$$= \sum_\alpha V_{i\alpha}\, \lambda_\alpha(t)\, V^\dagger_{\alpha j} \qquad \text{for large } t$$

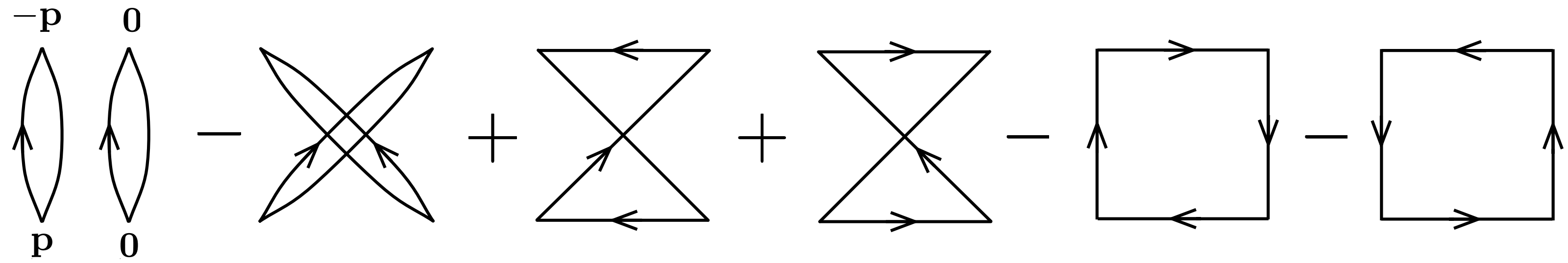
$$\Big(\; V_{i\alpha} = \langle 0|\mathcal{O}_i^\dagger|\alpha\rangle \;\; , \;\; \lambda_\alpha(t) = \exp(-W_\alpha \cdot t) \; \Big)$$

Assuming higher states $(\; \alpha \geq 3 \;)$ are negligible,

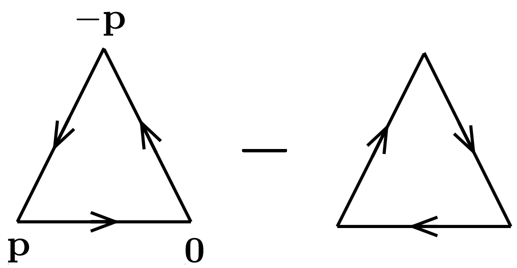
$$\mathrm{Ev}[\; G(t)\; G^{-1}(t_0) \;] \; = \; \lambda(t) \; \sim \; \mathrm{e}^{-Wt} \quad \text{for large } t$$

Calc. of $G(t)$

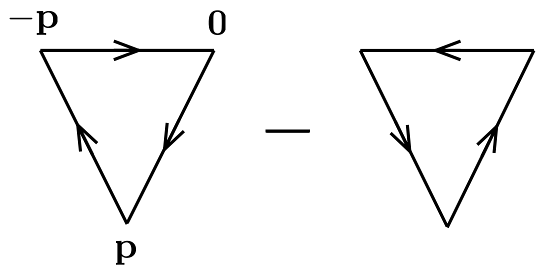
$G_{\pi\pi\to\pi\pi}(t) =$



$G_{\pi\pi\to\rho}(t) =$



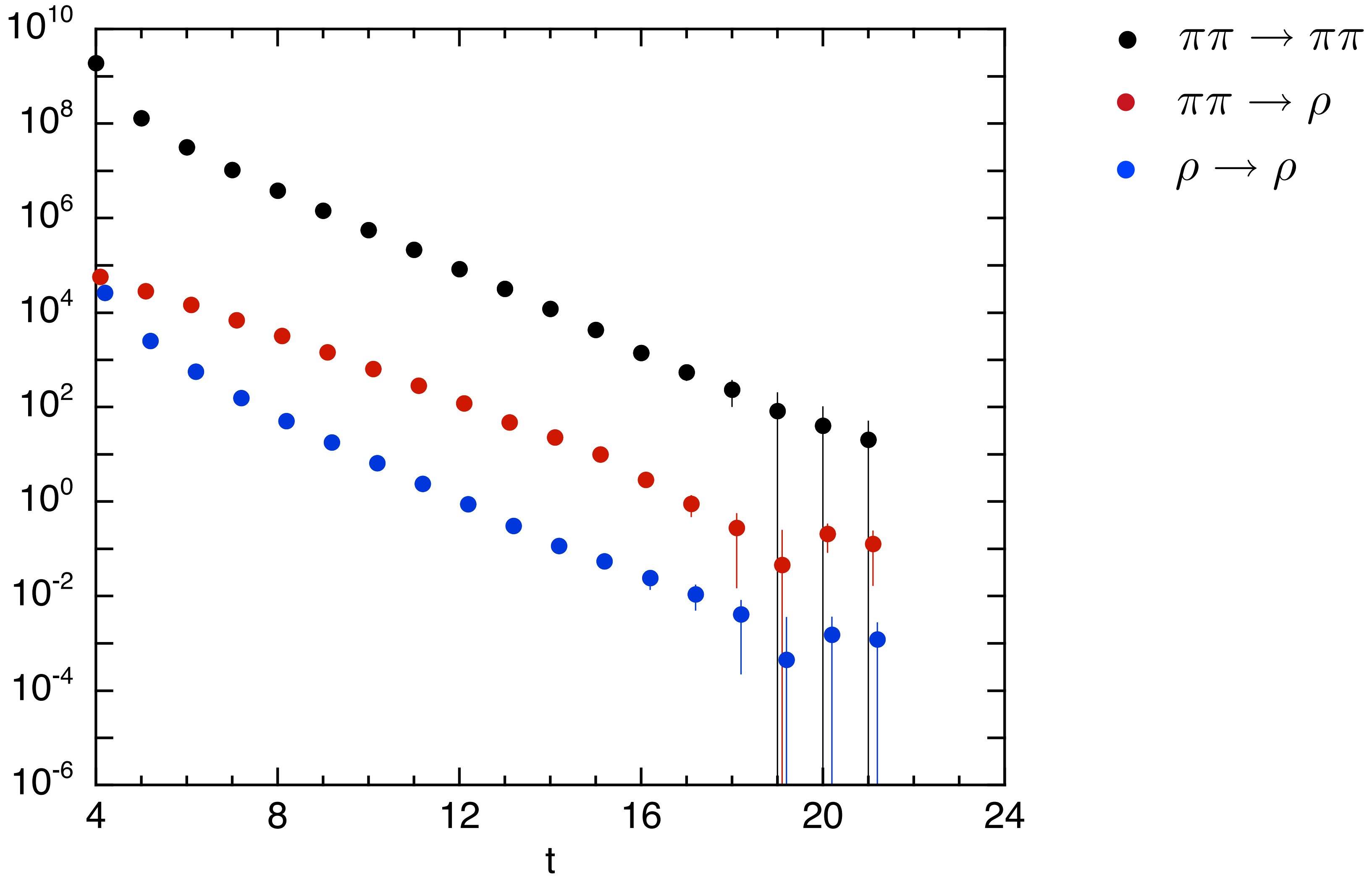
$G_{\rho\to\pi\pi}(t) =$



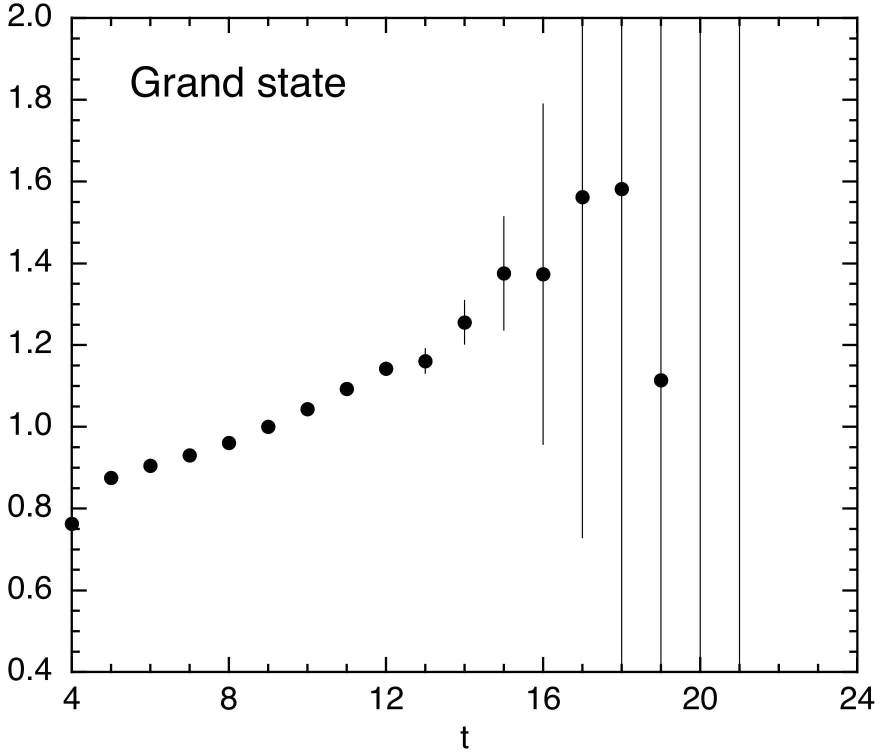
These are calculated by combination
of the stochastic source and the source method.

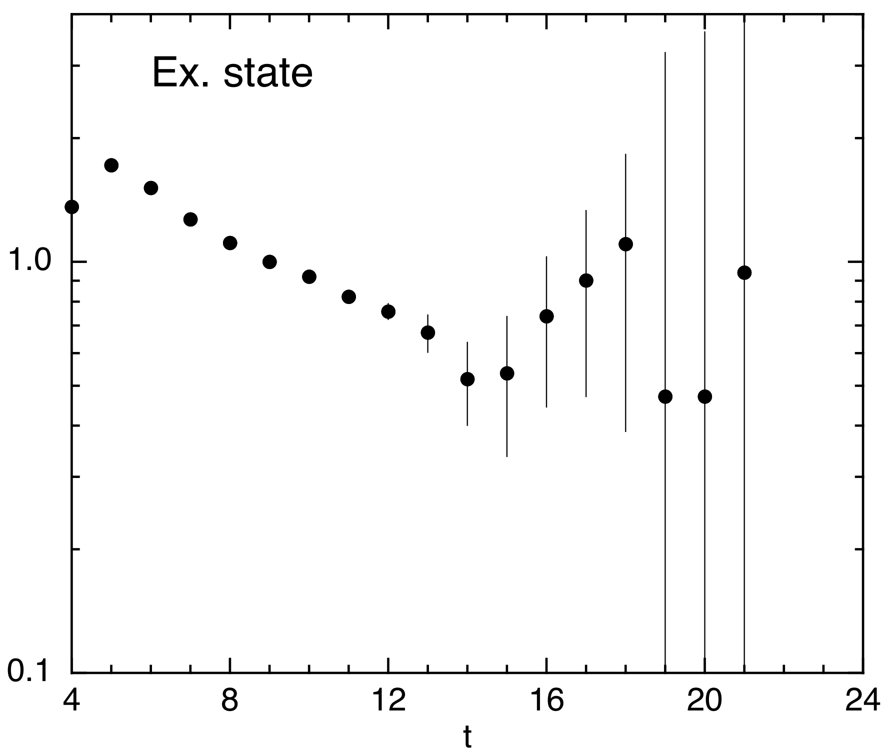
3. Results

Results. of $G(t)$



Eigenvalue $\lambda(t) = \mathrm{Ev}[\ G(t)\ G^{-1}(t_0)\ ] \ \sim\ \exp(-Wt)$





$\lambda_1(t)\ /\ \mathrm{e}^{-E_0 t}\ \sim\ \mathrm{e}^{-\Delta W_1 \cdot t}$

$\lambda_2(t)\ /\ \mathrm{e}^{-E_0 t}\ \sim\ \mathrm{e}^{-\Delta W_2 \cdot t}$

(E_0 : energy of free two pions)

$\Delta W_1 = -4.41(60) \times 10^{-2}$
$W_1 = E_0 + \Delta W_1 = 0.9364 \pm 0.0063$

$\Delta W_2 = 1.05(22) \times 10^{-1}$
$W_2 = E_0 + \Delta W_2 = 1.085 \pm 0.022$

Calc. of SC. phase shift

1) energy : W_1 , W_2

using two dispersion relations

- continuum (Cont) :

$$\sqrt{s} = \sqrt{W^2 - P^2}$$

- lattice (Lat) :

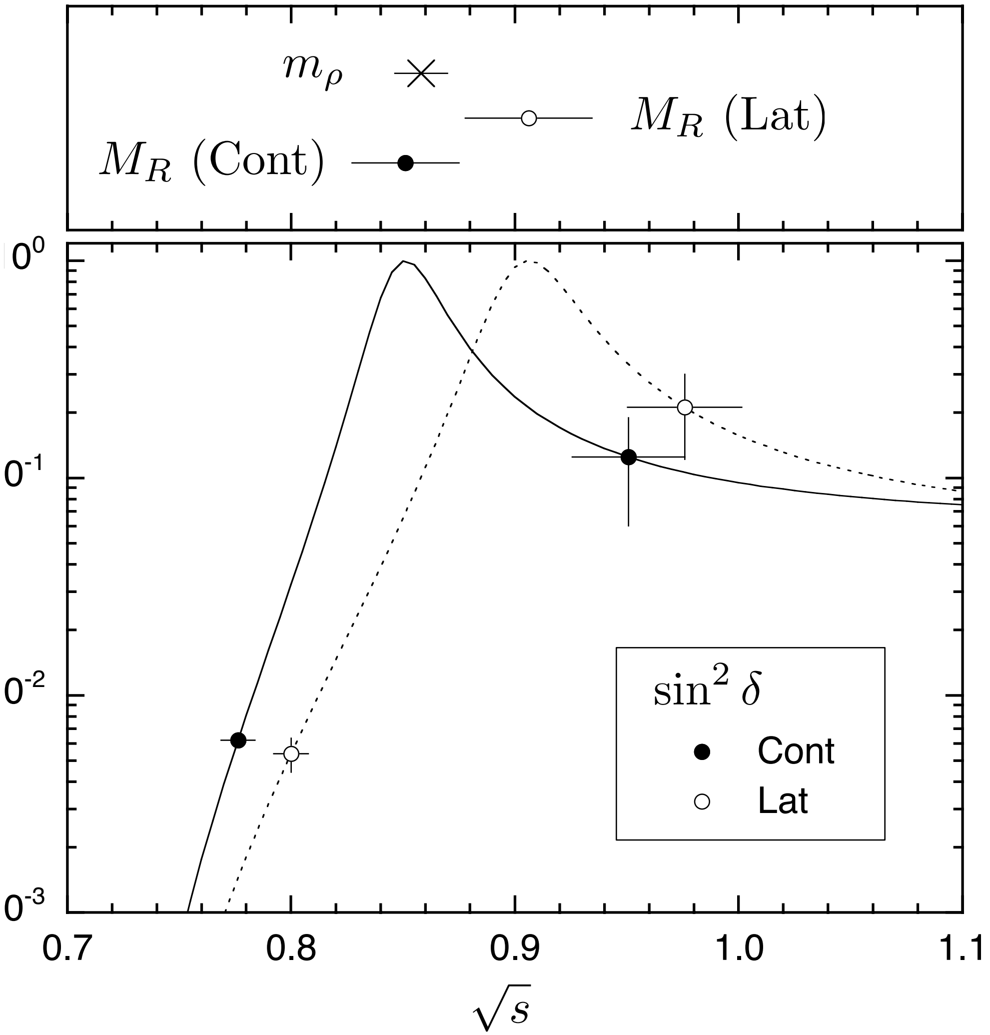
$$\cosh\sqrt{s} = \cosh W \ - \ 2 \cdot \sin^2 P/2$$

difference = O(a) error

2) $\sqrt{s}$ $\longrightarrow$ RG. formula $\longrightarrow$ $\tan\delta$

	Grand. $(\sqrt{s} < m_\rho)$		**Ex.** $(\sqrt{s} > m_\rho)$	
	Cont	Lat	Cont	Lat
$\sqrt{s}$	0.7764(75)	0.8000(77)	0.951(25)	0.976(26)
$\tan\delta$	0.07906(93)	0.0736(66)	−0.38(11)	−0.52(14)

$m_\rho = 0.858(12)$ (from time correlator)



$$\tan\delta = \frac{g_{\rho\pi\pi}^2}{6\pi}\frac{k^3}{\sqrt{s}\,(M_R^2 - s)}$$

$$\Gamma_\rho = \frac{g_{\rho\pi\pi}^2}{6\pi}\frac{\left(k^{\mathrm{Ph}}\right)^3}{\left(m_\rho^{\mathrm{Ph}}\right)^2} = \frac{g_{\rho\pi\pi}^2}{6\pi} \times 4.128\ \mathrm{MeV}$$

assumption : $g_{\rho\pi\pi}$: const.

Cont : $\Gamma_\rho = 162 \pm 35$ MeV $\quad M_R/m_\rho = 0.992 \pm 0.033$

Lat : $\Gamma_\rho = 140 \pm 27$ MeV $\quad M_R/m_\rho = 1.056 \pm 0.038$

cf. Expt. : $\Gamma_\rho = 150$ MeV

4. Summary

We calculate ρ meson decay width
from scattering phase shift for I=1 ππ system.

We find

(1) Calculation is possible.

(2) $\Gamma_\rho = 162 \pm 35 \text{ MeV}$

$\Gamma_\rho = 140 \pm 27 \text{ MeV}$ cf. Expt. : $\Gamma_\rho = 150 \text{ MeV}$

But

(1) Long extrapolation from $m_\pi / m_\rho = 0.42$ to phys. point.

Calc. near phys. point !!

(2) Large O(a) error in each steps of analysis.

Calc. near cont. limit !!

(3) Huge stat. error (about 25% for final results of decay width) .

Consider more efficient method !!