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Effects of pairing correlation on low-energy s-wave scattering in neutron-rich nuclei

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Background

- Pairing correlation influences strongly low-lying excitation mode in weakly bound nuclei.
- Quasi-particle resonance (Belyaev et.al. 1987) is an characteristic phenomenon caused by the pairing correlation.

Purpose of present study

- We intend to disclose novel features of the q.p. resonance.
- ・Superfluid nucleus plus an unbound nucleon (is treated as an unbound quasi-particle state built on a pair-correlated nucleus).
- **Numerical example: s-wave resonance in (^{20}C + n)*=^{21}C***

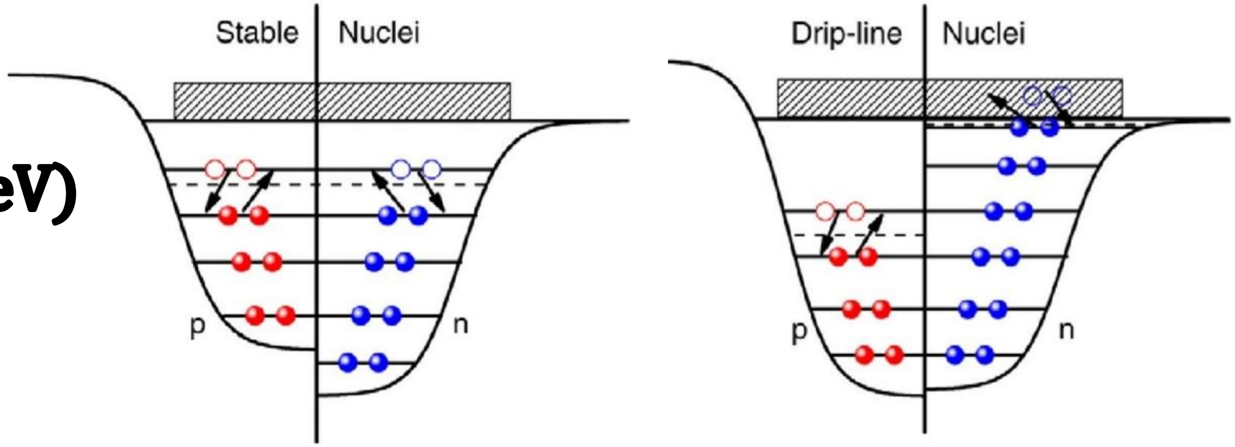
Pairing correlation influences the continuum

- Many nuclei with open-shell configuration have **superfluidity** generated by **the pairing correlation**.

Pair gap in nuclei

$$\Delta \sim 12.0/\sqrt{A} \sim O(1\ \mathrm{MeV})$$

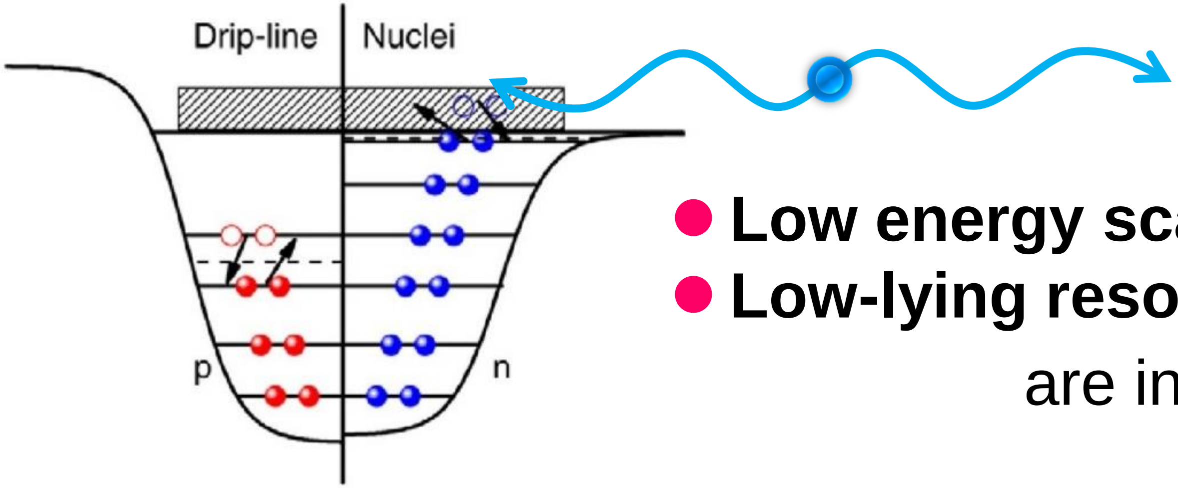
A. Bohr and B. R. Mottelson
Nuclear Structure



Figures are taken from J. Meng, *et al*., Prog. Part. Nucl. Phys. 57, 470 (2006)

- The pairing correlation causes configuration mixing
  - among bound orbits in well bound nuclei. $\lambda \sim 8$ MeV
  - involving both **bound and unbound (continuum) orbits** in weakly bound nuclei. $\lambda \sim 0-1$ MeV

Nucleon in continuum is influenced by pairing



- **Low energy scattering particle**
- **Low-lying resonance**

are influenced by the pairing.

Figures is taken from J. Meng, *et al*., Prog. Part. Nucl. Phys. 57, 470 (2006)

- **We focus on low-energy s-wave scattering in neutron-rich nuclei with the pairing correlation.**

- Low angular momentum wave (s and p) can approach nuclei easily due to no or small centrifugal barriers.
  →main contributors in halo formation and capture phenomena

Bogoliubov equation and quasi-particle resonance

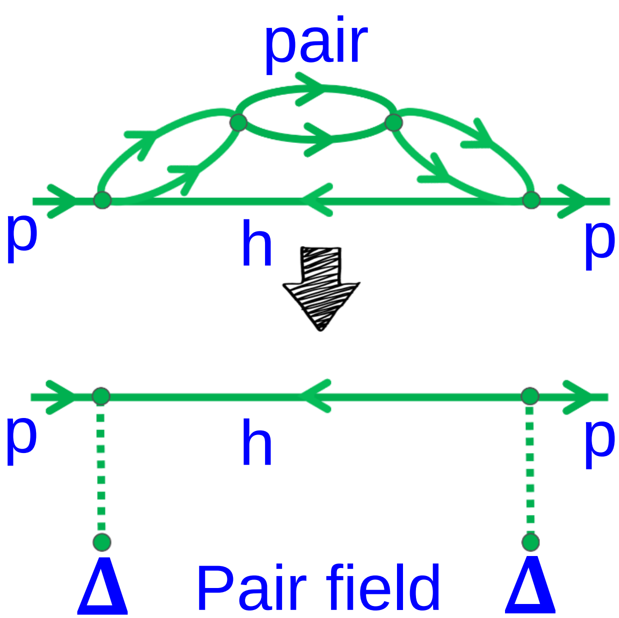
- Hartree-Fock-Bogoliubov equation for the coupled single-particle motion (hole & particle components)

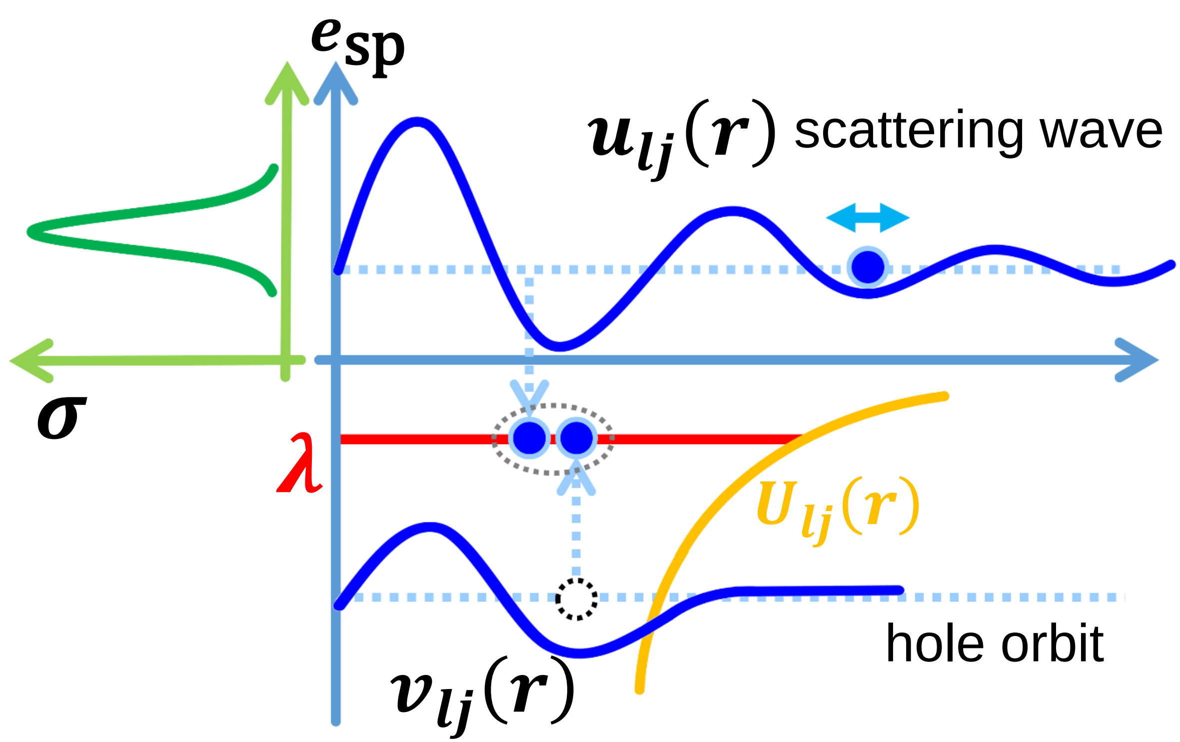
$$\begin{pmatrix} \frac{-\hbar^2}{2m}\frac{d^2}{dr^2} + U_{lj}(r) - \lambda & \Delta(r) \\ \Delta(r) & \frac{\hbar^2}{2m}\frac{d^2}{dr^2} - U_{lj}(r) + \lambda \end{pmatrix} \begin{pmatrix} u_{lj}(r) \\ v_{lj}(r) \end{pmatrix} = E \begin{pmatrix} u_{lj}(r) \\ v_{lj}(r) \end{pmatrix}$$

←unbound (scattering) w.f.
←bound w.f.

S. T. Belyaev et al., Sov. J. Nucl. Phys, 45 783 (1987)
A. Bulgac, Preprint(1980); nucl-th/9907088
J. Dobaczewski et al., Nucl. Phys. A 422 103 (1984)

$U_{lj}(r)$: HF potential with $\vec{l}\cdot\vec{s}$ interaction
$\Delta(r)$: Pair potential





Boundary condition and numerical model

- **Scattering boundary condition for the quasi-particle**

$$\frac{1}{r}\begin{pmatrix} u_{lj}(r) \\ v_{lj}(r) \end{pmatrix} = C\begin{pmatrix} \cos\delta_{lj} j_l(k_1 r) - \sin\delta_{lj} n_l(k_1 r) \\ D h_l^{(1)}(i\kappa_2 r) \end{pmatrix} \xrightarrow[r\to\infty]{} C\begin{pmatrix} \dfrac{\sin\left(k_1 r - \frac{l\pi}{2} + \delta_{lj}\right)}{k_1 r} \\ 0 \end{pmatrix}$$

$$k_1 = \sqrt{\frac{2m(\lambda+E)}{\hbar^2}}, \quad \kappa_2 = \sqrt{-\frac{2m(\lambda-E)}{\hbar^2}} \quad C = \sqrt{\frac{2mk_1}{\hbar^2\pi}}$$

S. T. Belyaev et al., Sov. J. Nucl. Phys, 45 783 (1987)
M. Grasso et al., Phys. Rev. C 64 064321 (2001)
I. Hamamoto et al., Phys. Rev. C 68 034312 (2003)

- Phase shift, elastic cross section and S-matrix.

$$\sigma_{lj} = \frac{4\pi}{k_1^2}\left(j+\frac{1}{2}\right)\sin^2\delta_{lj}$$

- **Numerical model**
  - ✓ Mean-field potential $U_{lj}(r)$: Woods-Saxon potential

    A. Bohr and B. R. Mottelson, *Nuclear Structure*
  - ✓ Pair potential $\Delta(r)$: Woods-Saxon form

$$U_{lj}(r) = \left[V_0 + (\vec{l}\cdot\vec{s})V_{SO}\frac{r_0^2}{r}\frac{d}{dr}\right]f_{WS}(r) \qquad \Delta(r) = \Delta_0 f_{WS}(r) \qquad f_{WS}(r) = \left[1+\exp\left(\frac{r-R}{a}\right)\right]^{-1}$$

Numerical example

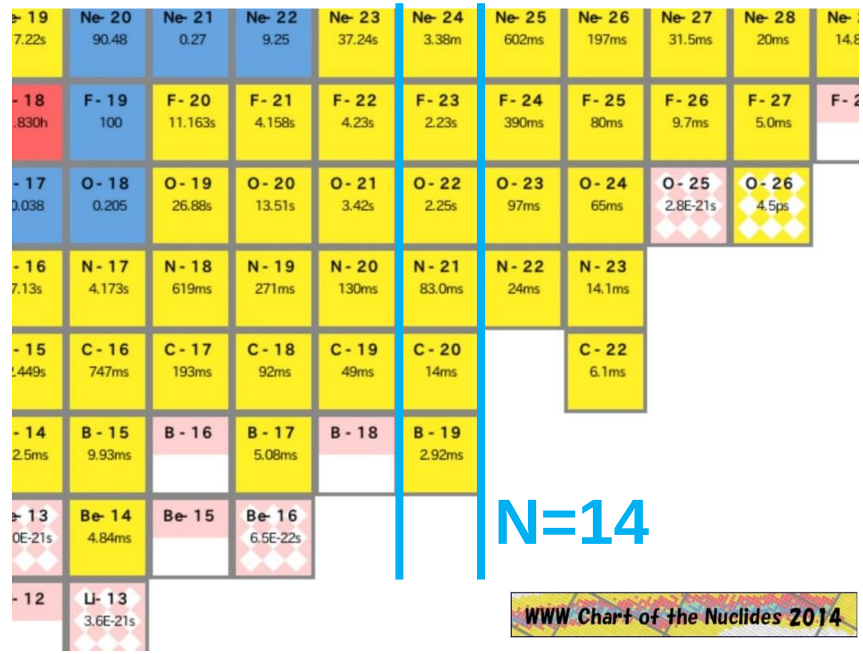
: s-wave resonance in (^{20}C + n)*=^{21}C*

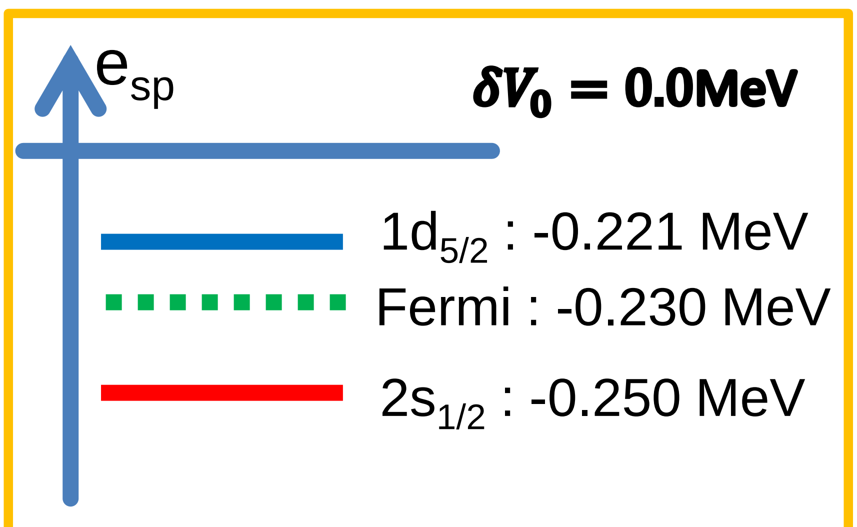
Single-neutron elastic scattering on ^{20}C: (^{20}C+n)*

- Low energy s-wave scattering on ^{20}C: (^{20}C+n)*=^{21}C*

$2s_{1/2}$ orbit is located around the continuum threshold in ^{20}C

- 21C* is subsystem of 22C.
- Weakly bound $2s_{1/2}$ orbit in the Woods-Saxon potential (↓).





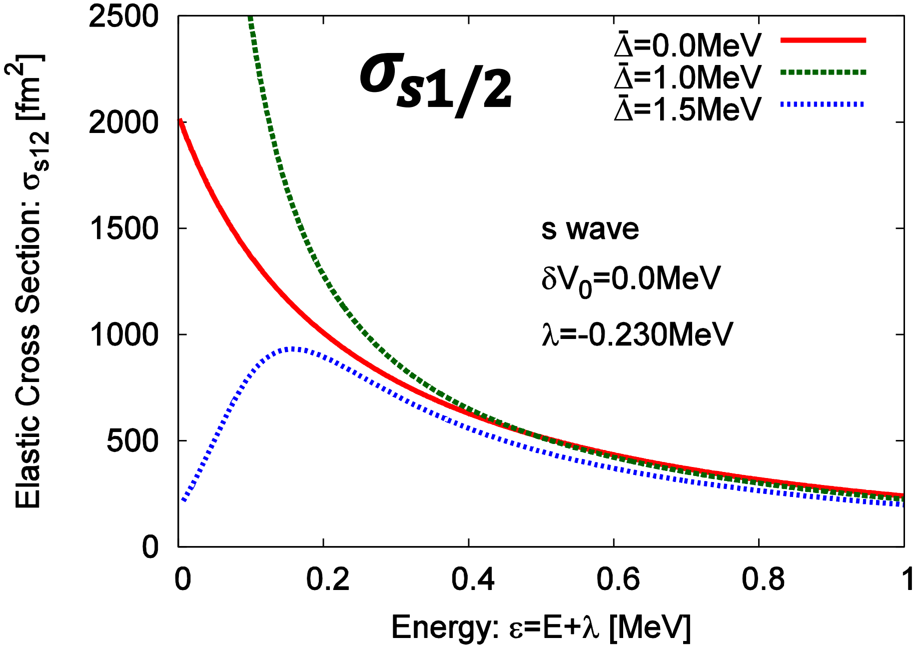
- ^{22}C is an 2 neutron s-wave halo nucleus

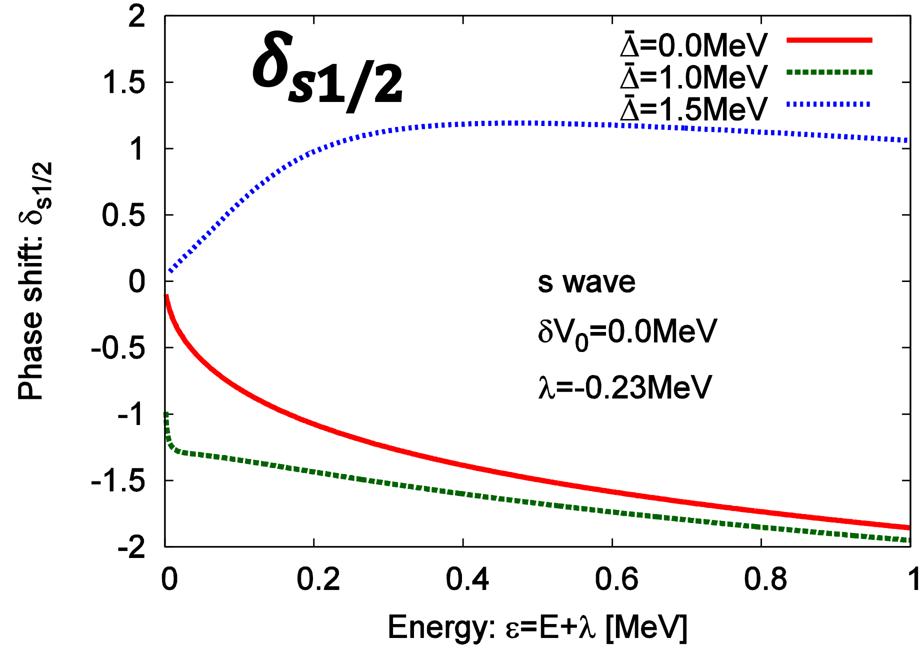
  W. Horiuchi and Y. Suzuki, Phys. Rev. C 89, 034607 (2006).
  K. Tanaka et al., Phys. Rev. Lett. 104, 062701 (2010).
  Y. Togano et al., Phys. Lett. B 761, 412 (2016) etc…

- ^{20}C has $2s_{1/2}$ component

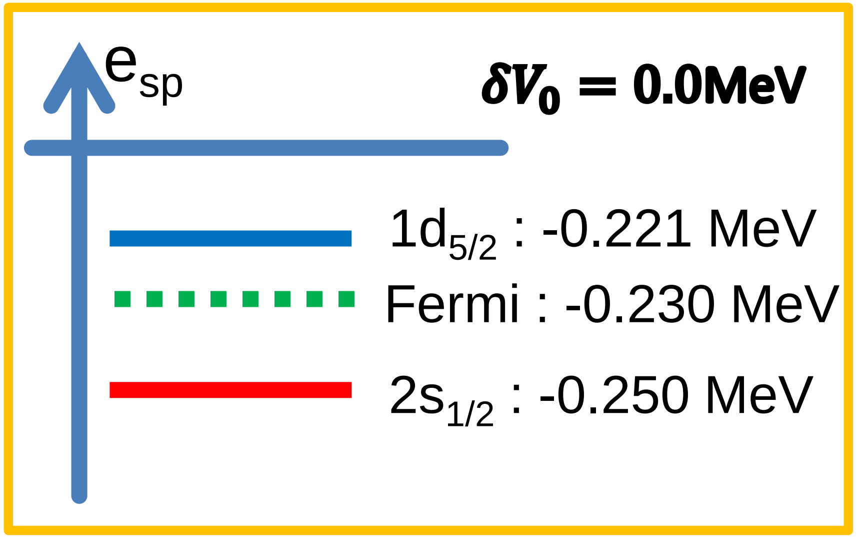
  Y. Togano et al., Phys. Lett. B 761, 412 (2016)

$\sigma_{s1/2}$ and $\delta_{s1/2}$ are depend on the pairing correlation





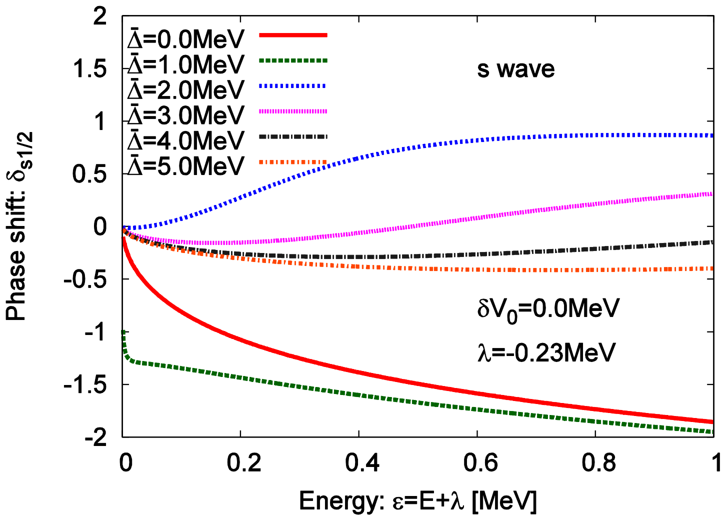
- σ and δ are influenced by the pairing strength $\bar{\Delta}$ drastically.
- For $\bar{\Delta}= \mathbf{1.0}$ MeV:
  Virtual state-like.
  For $\bar{\Delta}= \mathbf{1.5}$ MeV:
  Resonance-like.



Low-energy effective range formula does not work

- We extract the scattering length ($\boldsymbol{a}$) and the effective range ($\boldsymbol{r}_{\mathrm{eff}}$) from calculated phase shift ($\boldsymbol{\delta}$).

$$k\cot\delta \cong -\frac{1}{a}+\frac{1}{2}k^2 r_{\mathrm{eff}} \qquad k = k_1 = \sqrt{\frac{2m(\lambda+E)}{\hbar^2}}$$



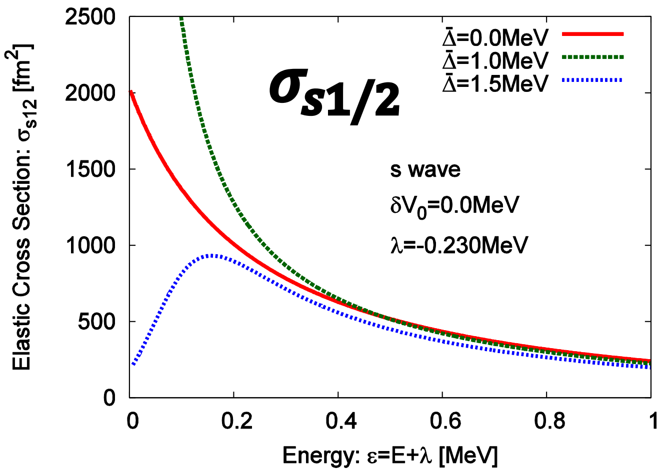
$\bar{\Delta}$ [MeV]	$1/a$ [fm^{-1}]	r_{eff} [fm]
0.0	0.0790	5.373
1.0	0.00825	-1.478
2.0	-0.9279	-109.617
3.0	0.3160	-69.521
4.0	0.3018	-14.192
5.0	0.2862	-5.711

- The sign of scattering length and effective range is strange.
- The effective range becomes negative.

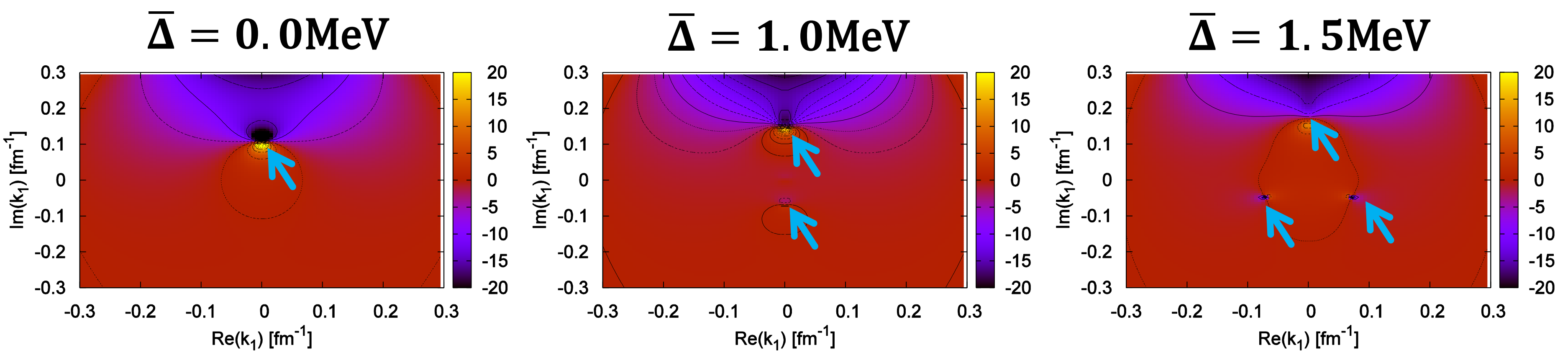
→ **beyond the low-energy effective formula**

“Additional” S-matrix poles are emerged by pairing

- S-matrix poles are calculated in order to understand the behavior of $\boldsymbol{\sigma}_{s1/2}(\bar{\Delta})$.
- $\bar{\Delta}$ causes continuum coupling. (“additional” poles emerge in $\mathrm{Im}(k_1) < 0$.)



$\mathrm{Re}(S_{s1/2})$



Bound state | Virtual state-like | Resonance-like

- The pairing dependence of elastic cross sections $\boldsymbol{\sigma}_{s1/2}(\bar{\Delta})$ can be described by the “additional” poles qualitatively.

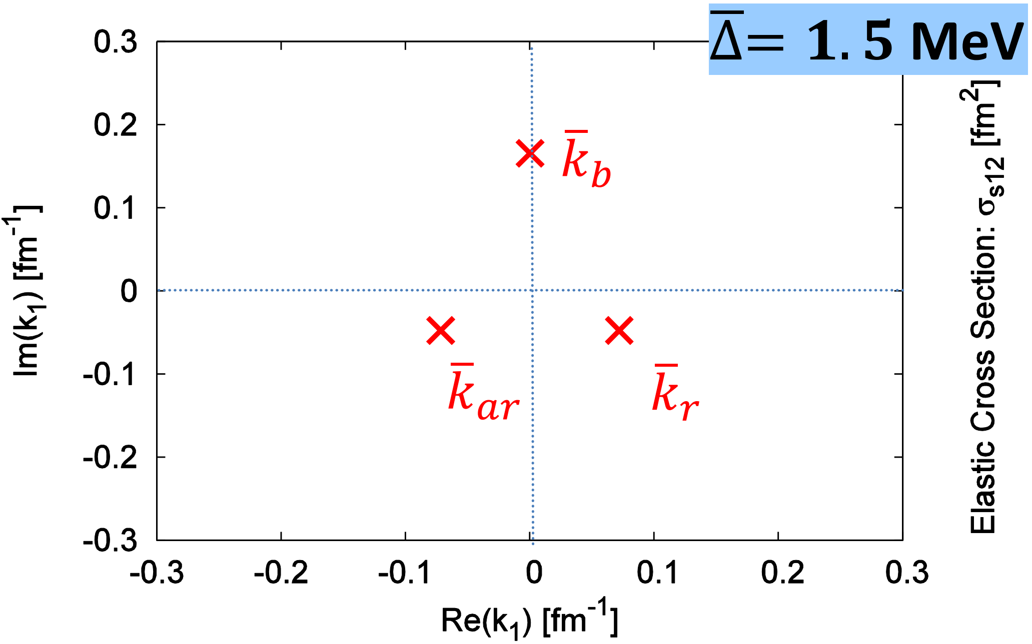
$\sigma_{s1/2}$ is described pole contributions qualitatively

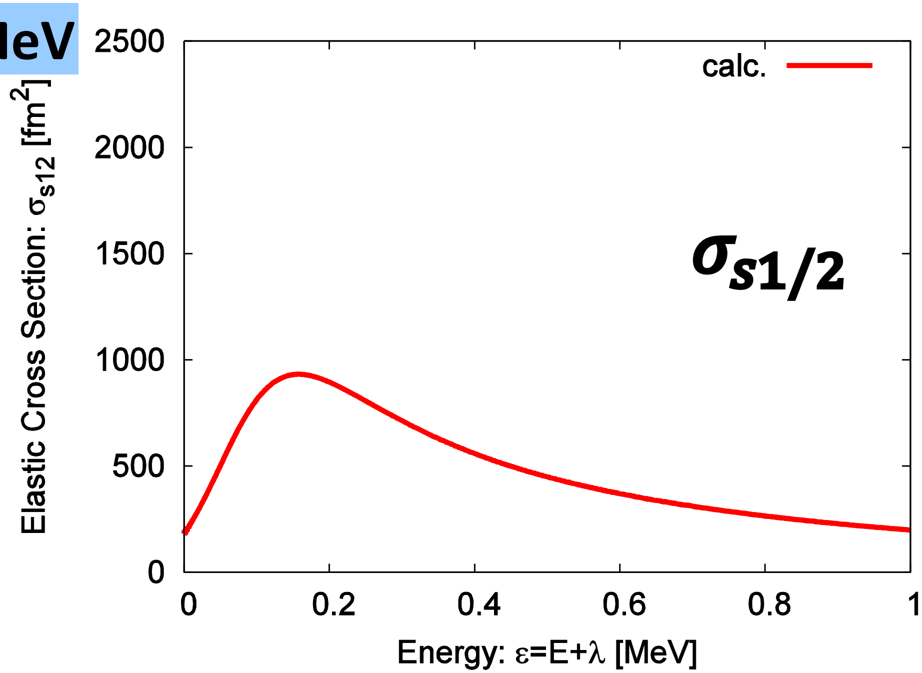
- We try to extract pole contribution with following eqs.

$$S(k) \sim \frac{k-\bar{k}_b^*}{k-\bar{k}_b} \cdot \frac{k-\bar{k}_r^*}{k-\bar{k}_r} \cdot \frac{k-\bar{k}_{ar}^*}{k-\bar{k}_{ar}} \longrightarrow \sigma(k) = \frac{\pi}{k^2}|S(k)-1|^2$$

$$\bar{k}_b = 0.166i \qquad \bar{k}_r = 0.0719 - 0.0473i \qquad \delta_{bg} = 0$$

$$\bar{k}_{ar} = -0.0719 - 0.0473i \qquad S^*(k^*)S(k) = 1$$





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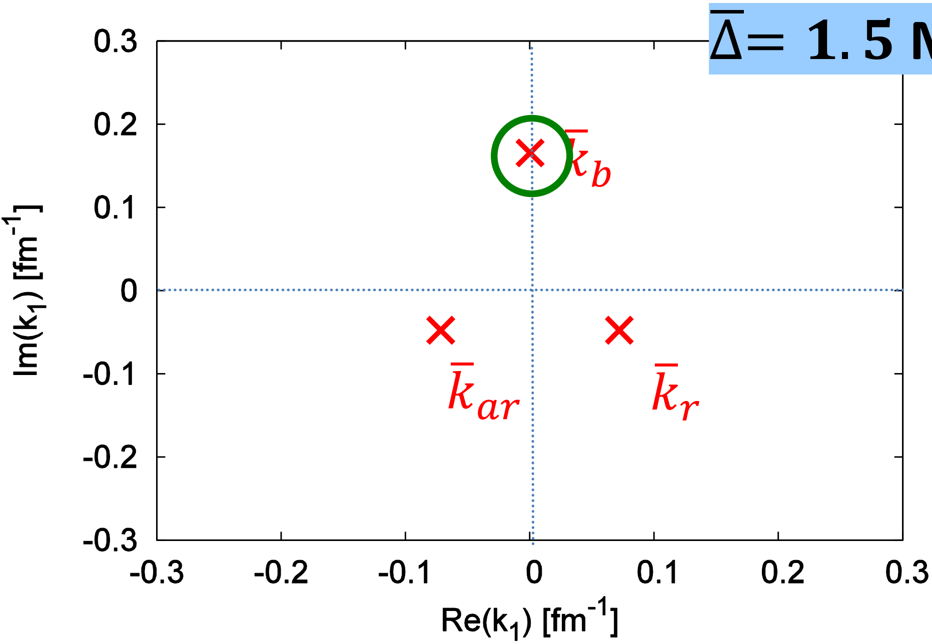
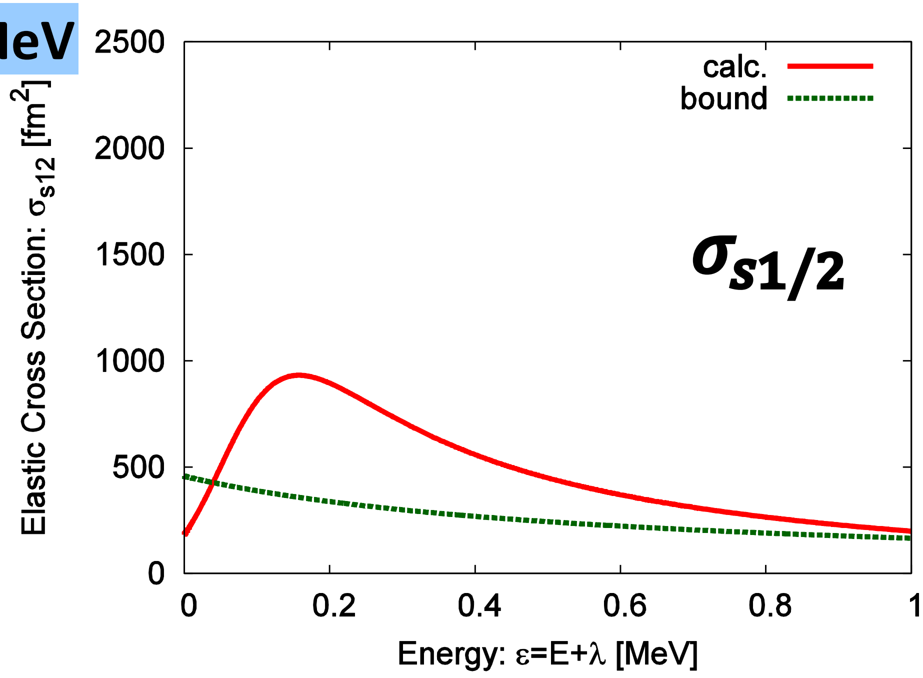
$\bar{\Delta} = 1.5$ MeV





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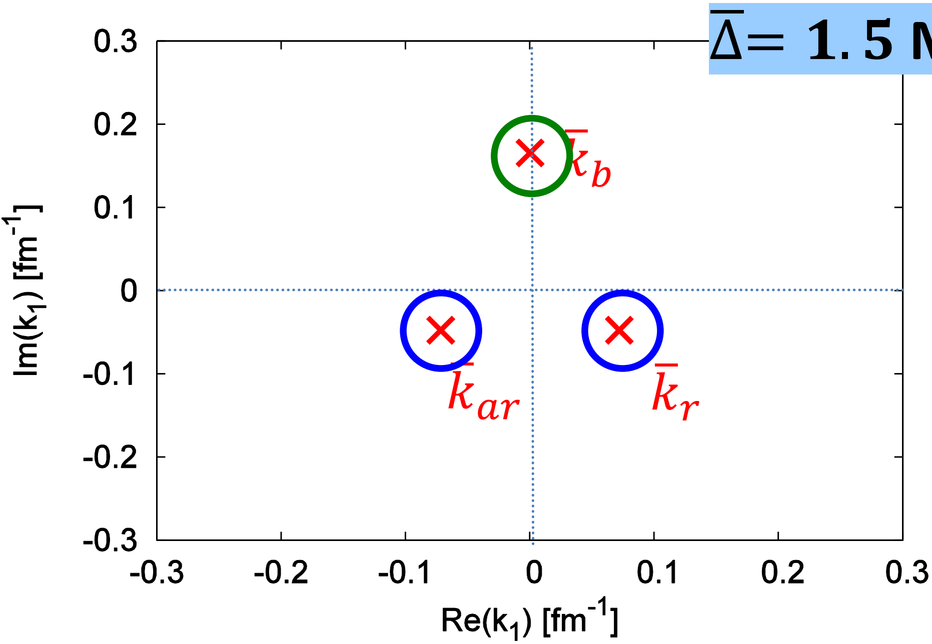

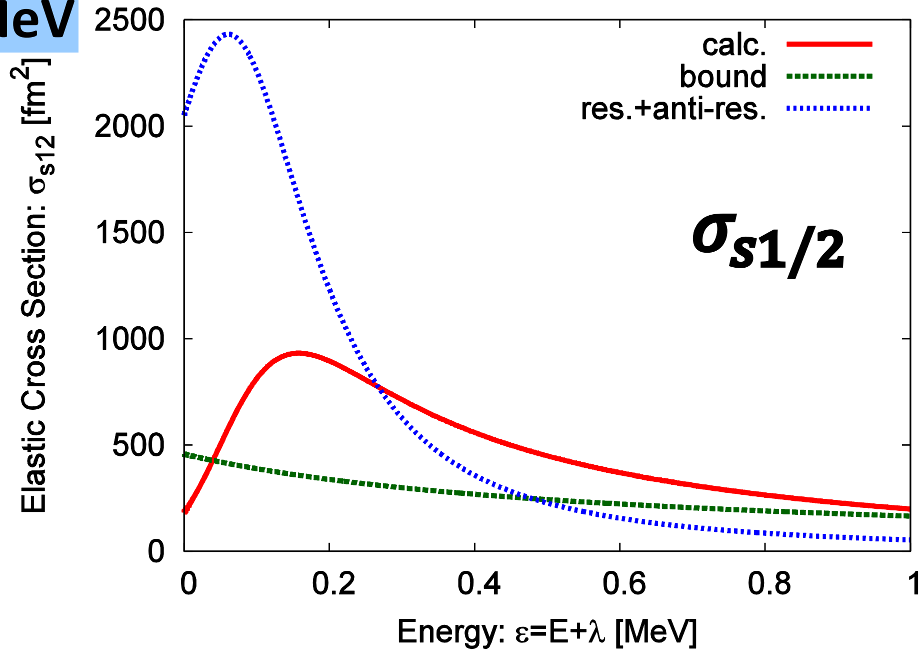

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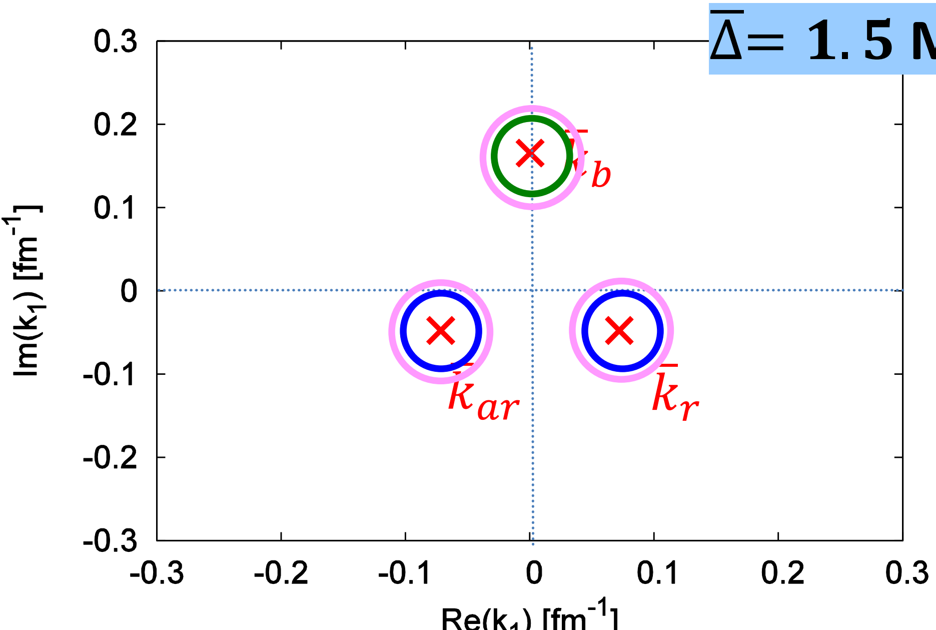
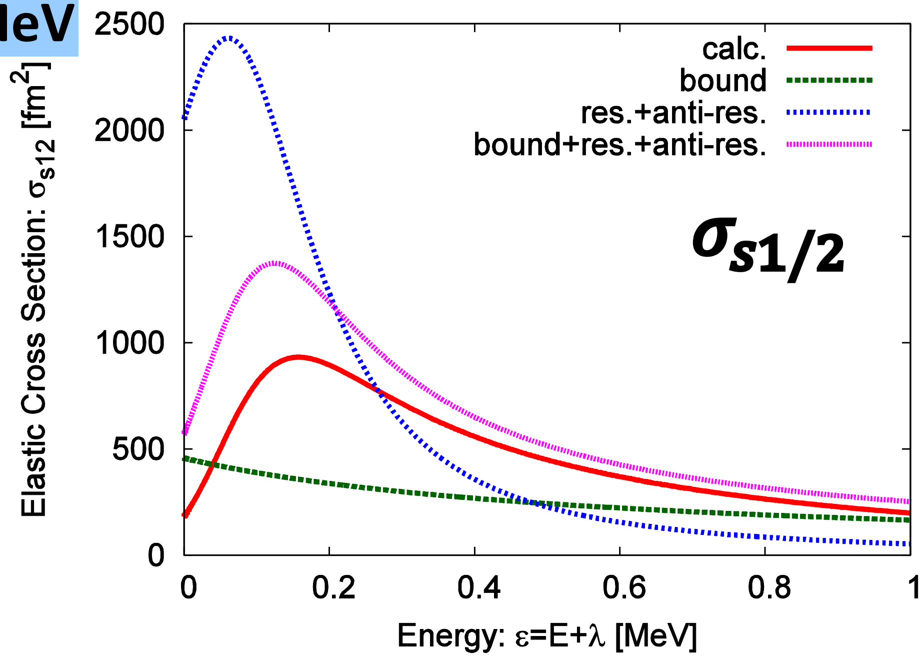
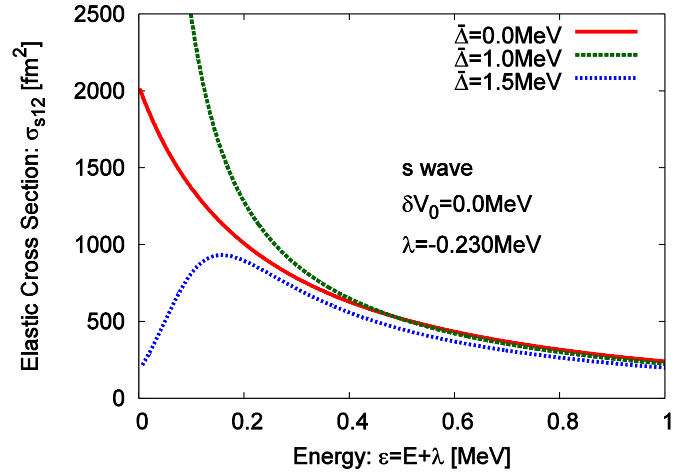




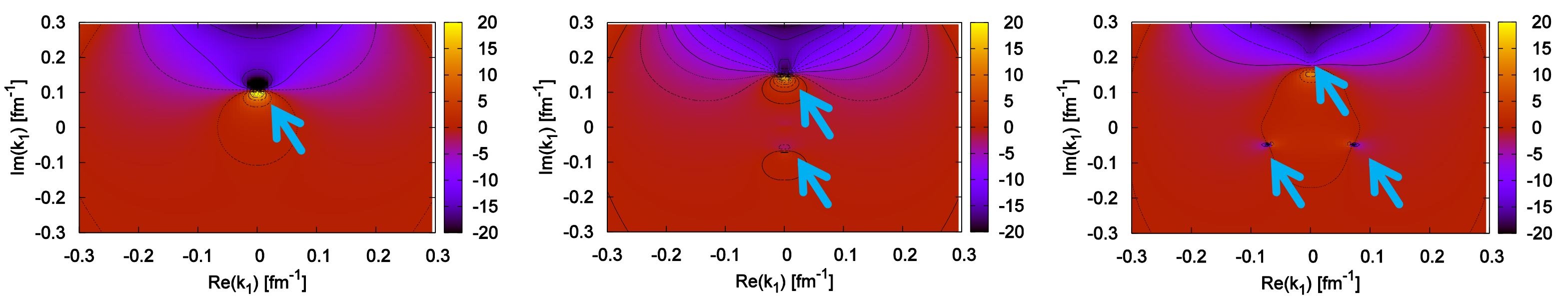
Conclusion

- The pairing correlation effects on the scattering length (a) and the effective range (r_{eff}) cannot be described by the low-energy effective range formula.
- Pairing dependence of elastic cross section is understood by character of "additional" S-matrix poles.
- Scattering on superfluid nucleus which has weakly bound s orbit can be Virtual state-like and Resonance-like.



$\bar{\Delta} = 0.0$ MeV $\bar{\Delta} = 1.0$ MeV $\bar{\Delta} = 1.5$ MeV

$\mathrm{Re}(S_{s1/2})$



Bound state   Virtual state-like   Resonance-like