Probing nuclear clustering from nuclear responses

M. Kimura (Hokkaido University)

Part1. Introduction

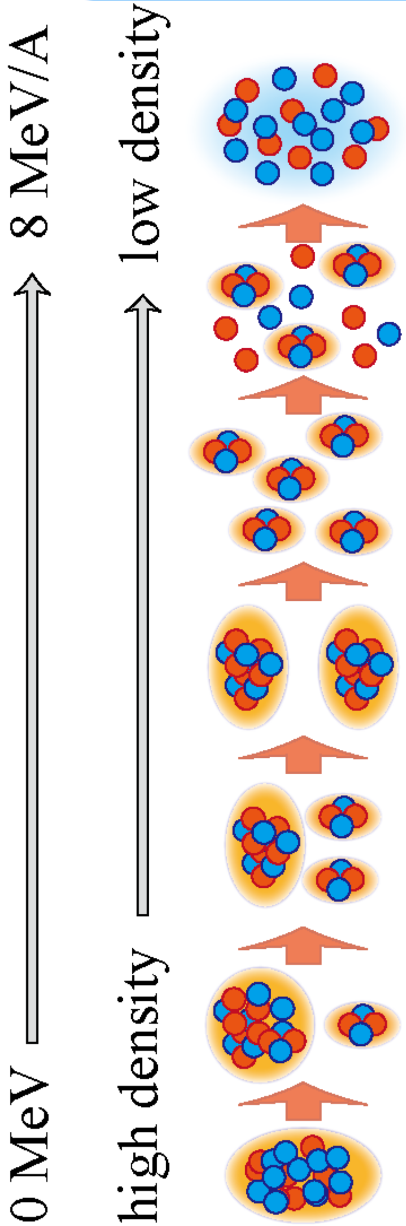
Part2. Why IS monopole/dipole transitions strongly feed cluster states?

Part3. Numerical calculation by AMD

Part4. Comment on PDR

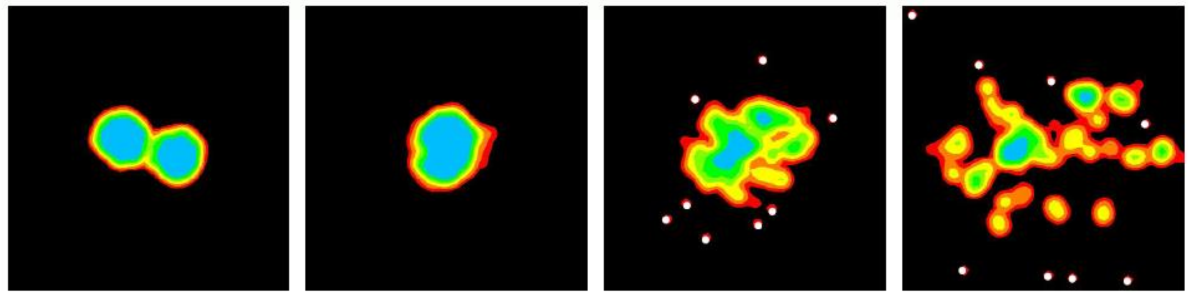
Part5. Summary

Part 1. Nuclear clustering



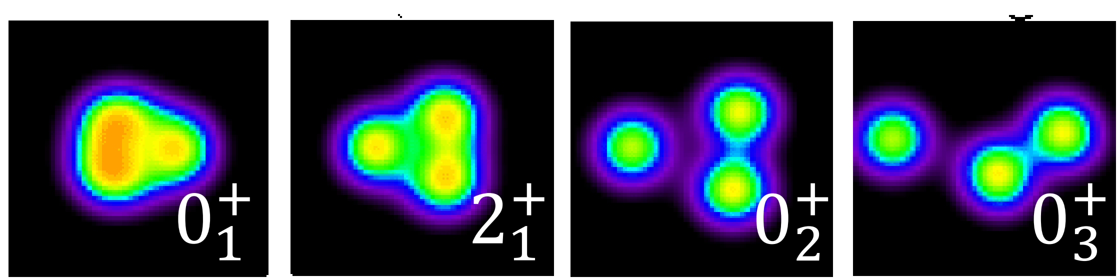
Multi fragmentation in Heavy Ion Collision

^{129}Xe+Sn E/A 50 MeV, A. Ono, PRC66 (2002).



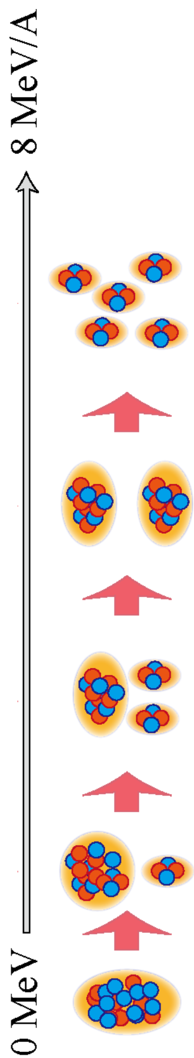
Clustering in the excited states of nuclei

^{12}C excited states, Y. Kanada-En'yo, PRL81 (1998)



Part 1. Nuclear clustering

When clustering happens?



- If the clusters mutually interact too strong, they are strongly distorted ($V_{rel} \simeq 0$)
- If the relative kinetic energy is too large, they cannot form a bound (resonant) state ($T_{rel} \simeq 0$).

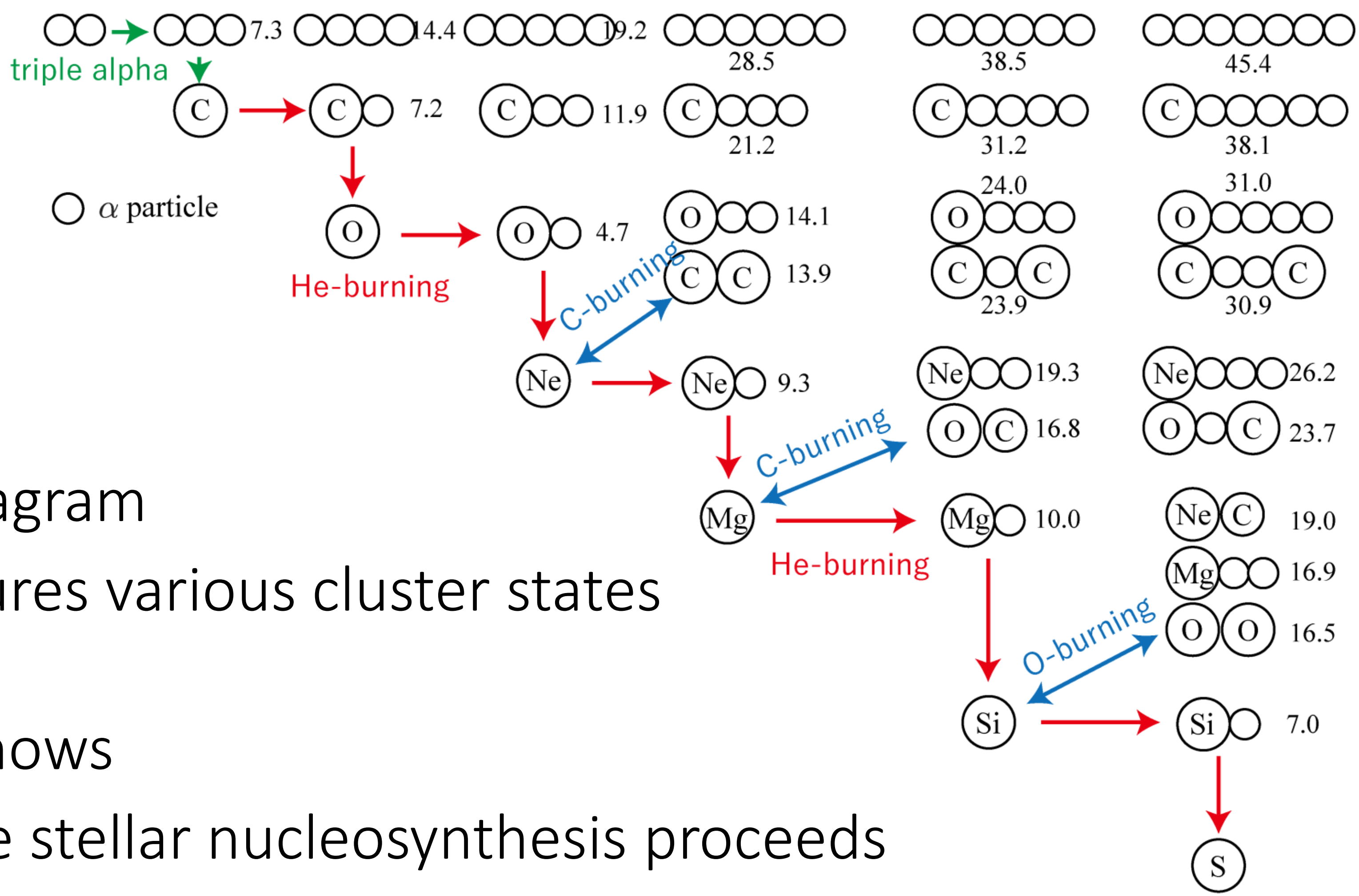
$$E_{cluster} = E_{C1} + E_{C2} + T_{rel} + V_{rel}$$
$$\simeq E_{C1} + E_{C2} \quad (T_{rel} \simeq V_{rel} \simeq 0)$$

Clustering should occur at the threshold energies which decompose the system into clusters

Part 1. Nuclear clustering

Ikeda diagram

K. Ikeda, PTPS Ex. 464(1968)



Ikeda diagram

conjectures various cluster states

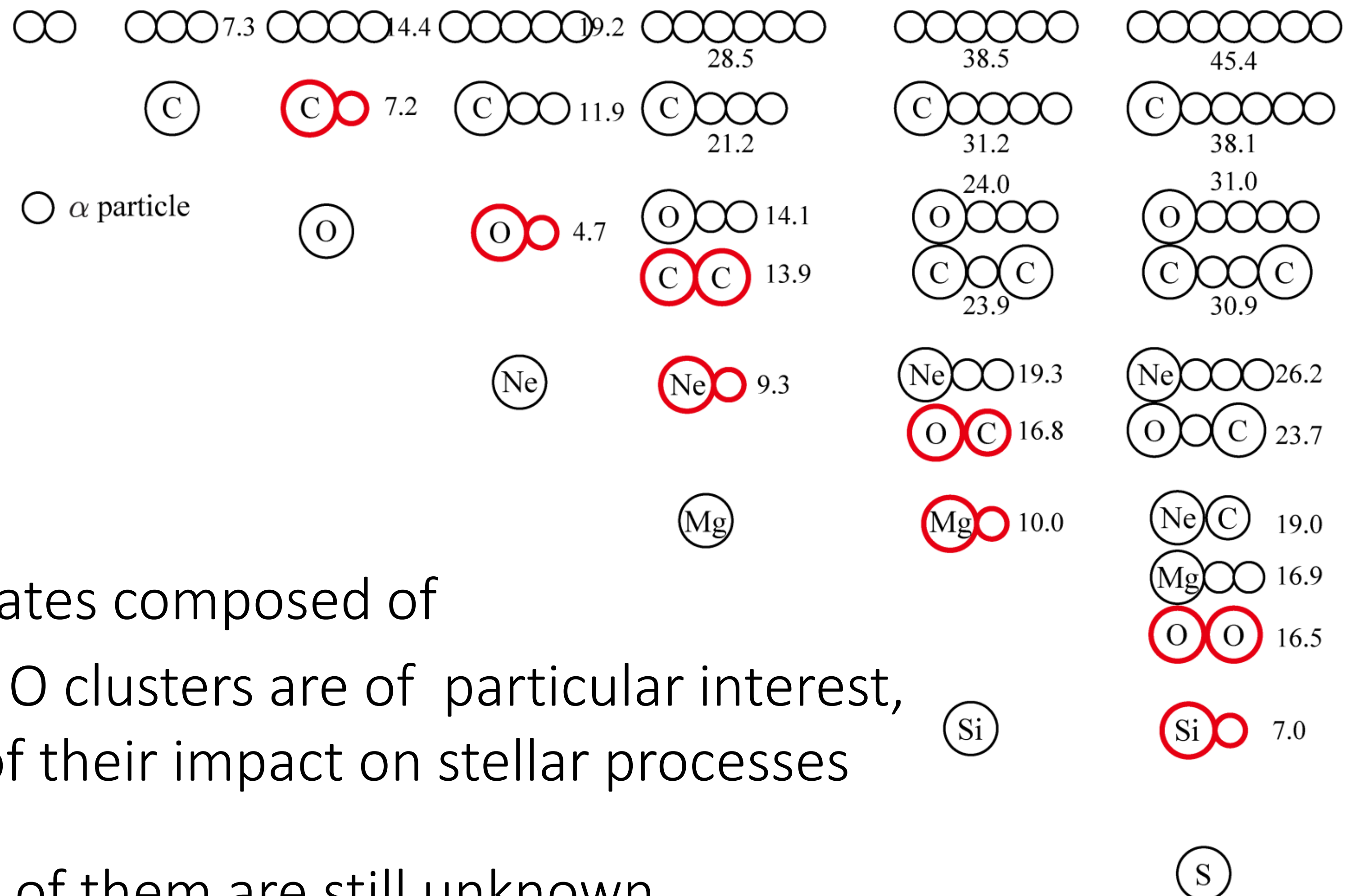
It also shows

how the stellar nucleosynthesis proceeds

Part 1. Nuclear clustering

Ikeda diagram

K. Ikeda, PTPS Ex. 464(1968)



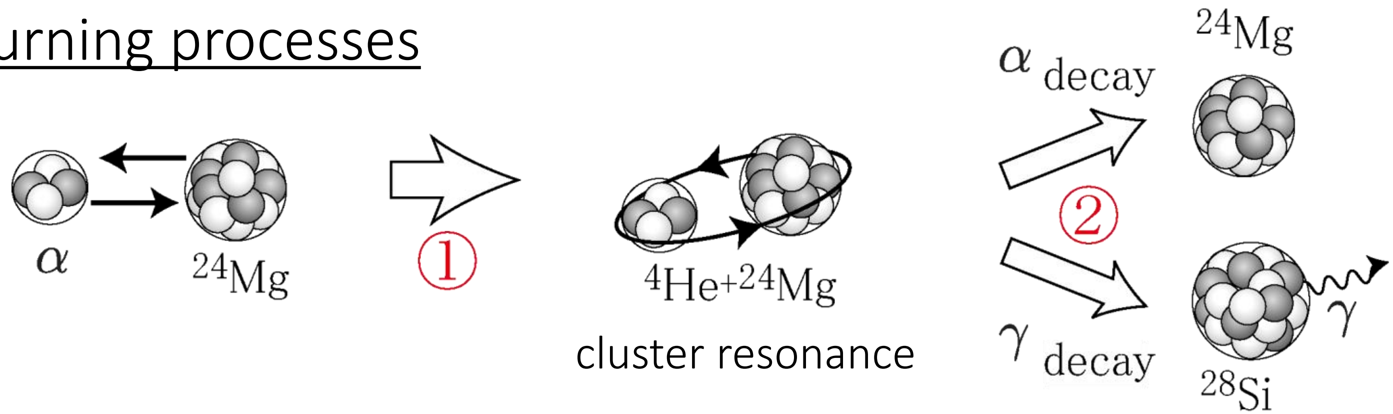
Cluster states composed of

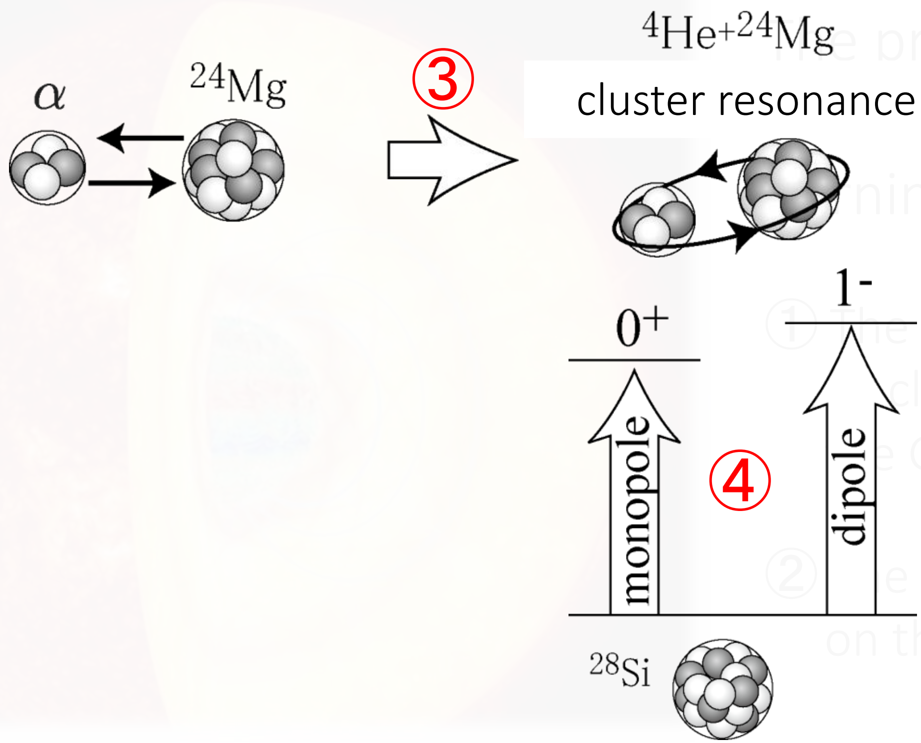
He, C and O clusters are of particular interest, because of their impact on stellar processes

But many of them are still unknown

He- C- O-burning processes and cluster resonances

Light element burning processes





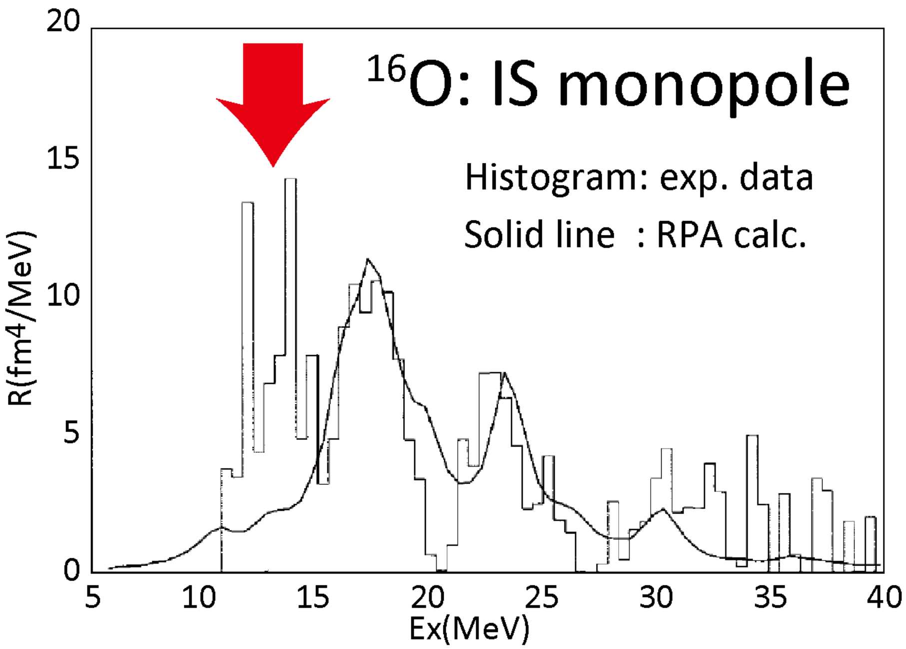
③ The serious problem is that the reaction rates are too small for direct observation

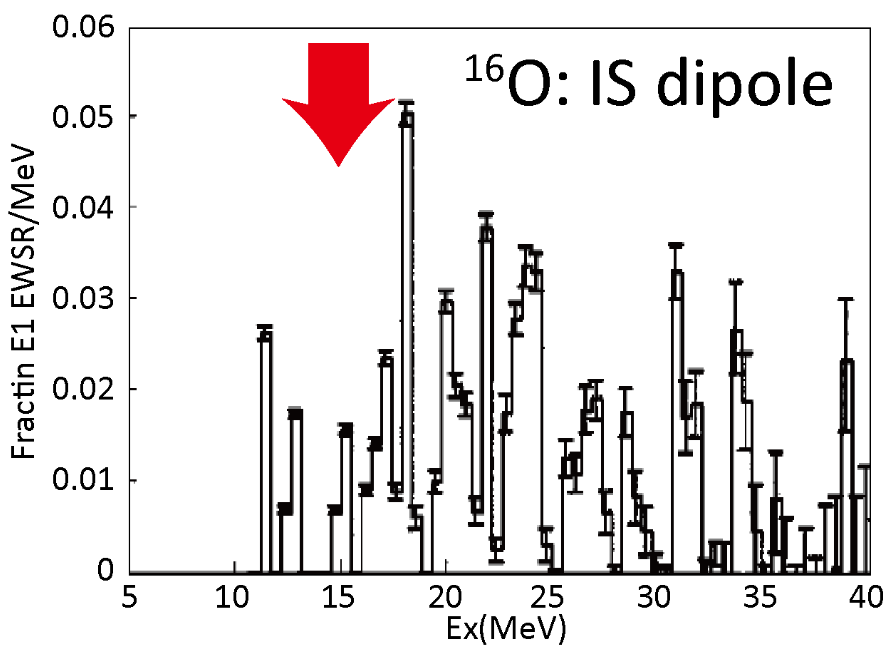
④ The IS monopole/dipole transitions strongly populate cluster resonances

Part 1. IS monopole/dipole responses

The data for IS monopole/dipole responses of light nuclei show many narrow resonances well below Giant Resonance.

Y. –W. Lui et al., PRC64, 064308 (2001).





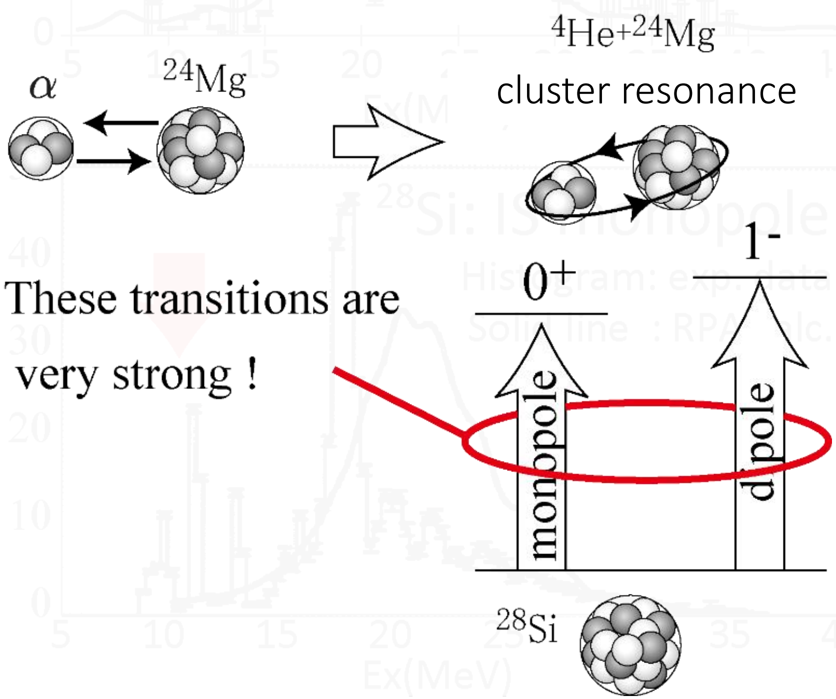
RPA calculation does not explain them.

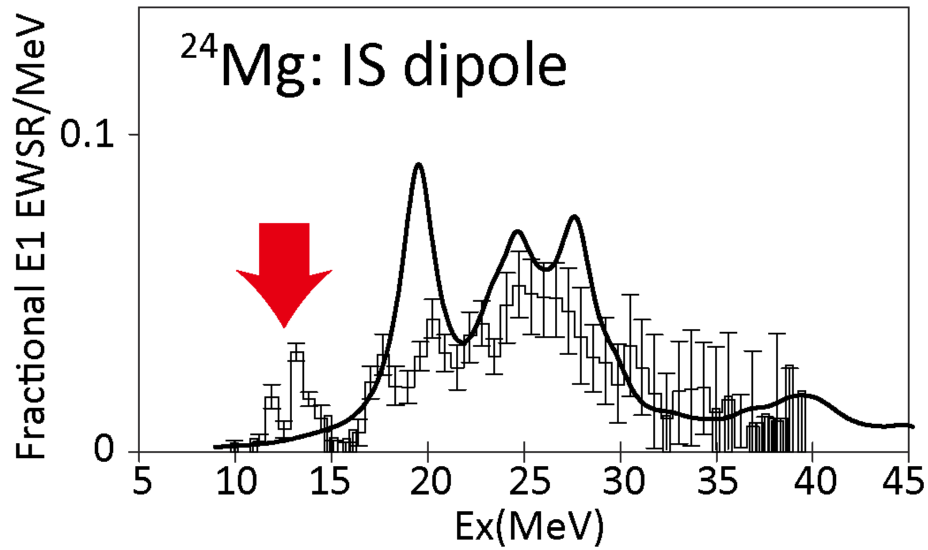
Part 1. IS monopole/dipole responses

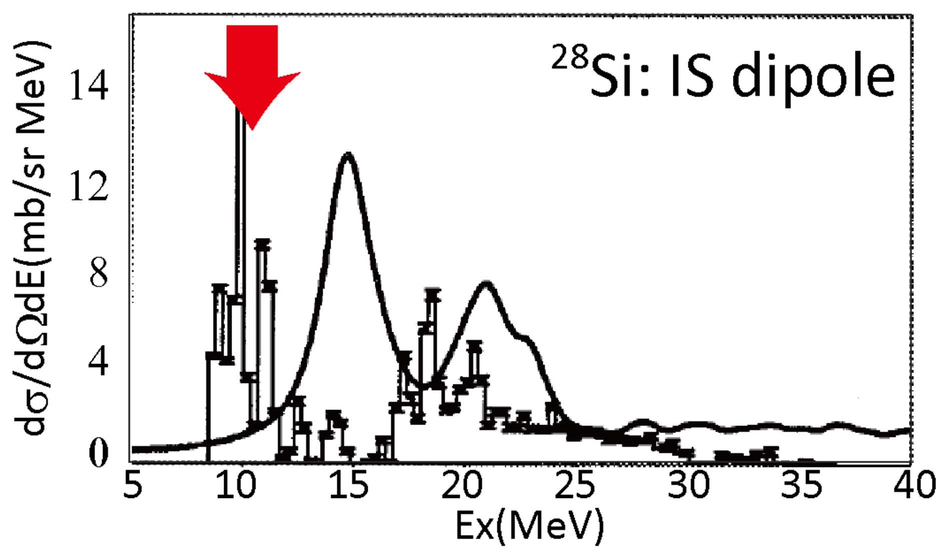
X. Chen et al.,PRC80, 014312 (2009). D. H. Young-Blood et al.,PRC65, 034302 (2002).

They are what we are looking for!

They are the 0^+ and 1^- resonances having cluster structure such as α+^{20}Ne, α+^{24}Mg,...

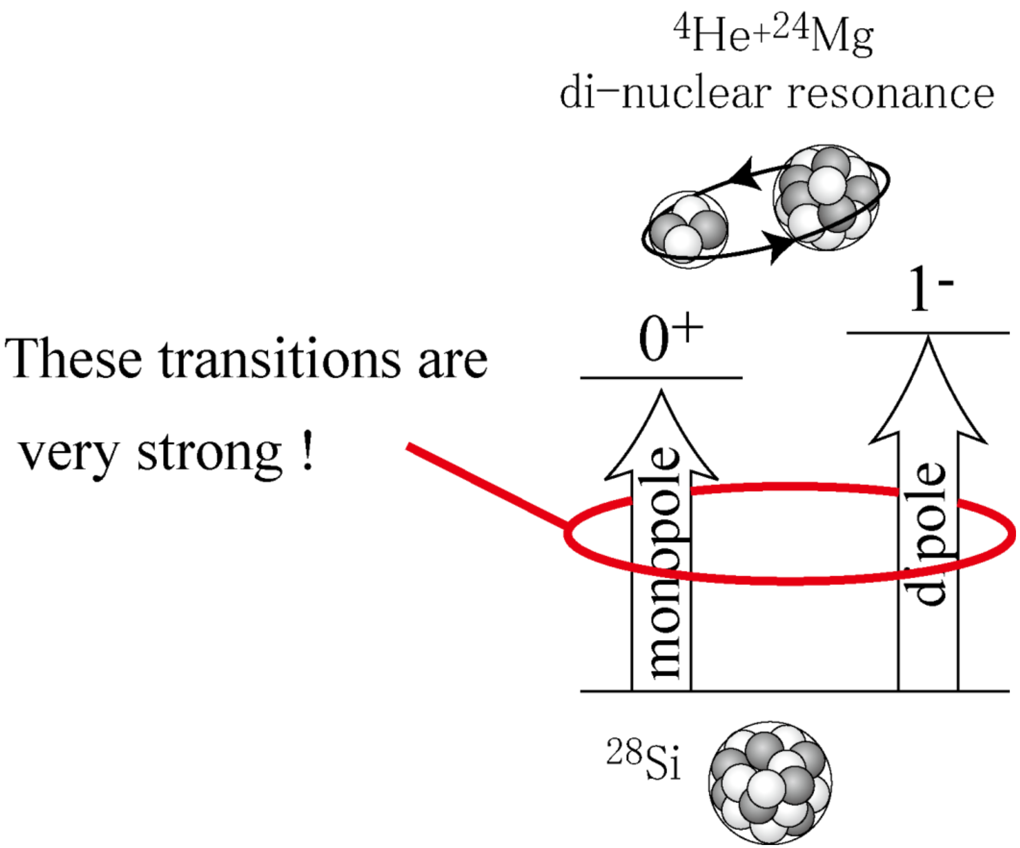






Part 2.

Why IS monopole/dipole transitions strongly populate cluster resonances?



Part 2. Duality of “shell” and “cluster”

The ground state wave function has duality of “shell” and “cluster”

Two well known facts

1. The ground states of light nuclei are described well by SU(3) shell model
   J. P. Elliott, Proc. R. Soc. London A 245, 562 (1958).
2. A single SU(3) shell model wave function is
   mathematically equivalent to a cluster model wave function
   “Bayman-Bohr theorem” , NP9, 596 (1958/1959).

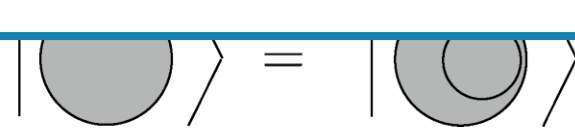
Example

This duality of the ground state wave function means that

“Even in an ideal shell model ground state,
the degrees-of-freedom of cluster excitation is embedded”

shell model wave function

= completely overlapping α and ^{12}C clusters



Part 2. Duality of "shell" and "cluster"

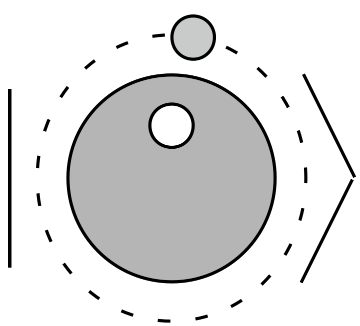
In the shell model representation,

$$\Phi_{g.s.}(^{20}\mathrm{Ne}) = \mathcal{A}\{\underline{(0s)^4(0p)^{12}(0d1s)^4}\}$$

degrees-of-freedom of single particle excitation

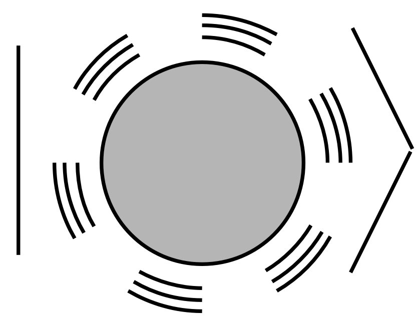
Single particle excitation

$$\mathcal{A}\{(0s)^4(0p)^{12}(0d1s)^3(0f)^1\}$$



Collective excitation

$$\mathcal{A}\{(0s)^4(0p)^{12}(0d1s)^3(0f)^1\}$$
$$+\mathcal{A}\{(0s)^4(0p)^{12}(0d1s)^3(1p)^1\}$$
$$+\ \dots$$



Part 2. Nodal and angular excitations

In the cluster model representation

$$\Phi_{g.s.}(^{20}\mathrm{Ne}) = n\mathcal{A}\{\,\underline{R_{80}(r)Y_{00}(\hat{r})}\phi_\alpha\phi_{^{16}\mathrm{O}}\,\}$$

degrees-of-freedom of cluster excitation

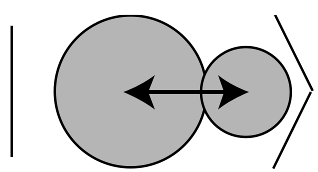
Nodal excited state

$$\sum_{\underline{N=N_0+2}}^{\infty} f_N n_N \mathcal{A}\{\underline{R_{\boldsymbol{N}0}(r)}Y_{00}(\hat{r})\phi_\alpha\phi_{^{16}\mathrm{O}}\}$$

increase of nodal quantum number

superposition of 2,4,6,.. $\hbar\omega$ excitations

Excited 0^+ state



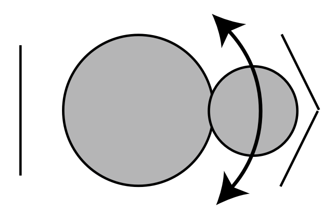
Angular excited state

$$\sum_{\underline{N=N_0+1}}^{\infty} f_N n_N \mathcal{A}\{R_{N0}(r)\underline{Y_{\ell=1}(\hat{r})}\phi_\alpha\phi_{^{16}\mathrm{O}}\}$$

increase of angular momentum

superposition of 1,3,5,.. $\hbar\omega$ excitations

Excited 1^- state



Part 2. Nodal and angular excitations

In the cluster model representation

$$\Phi_{g.s.}(^{20}\mathrm{Ne}) = n\mathcal{A}\{\,\underline{R_{80}(r)Y_{00}(\hat{r})}\phi_\alpha\phi_{^{16}\mathrm{O}}\,\}$$

degrees-of-freedom of cluster excitation

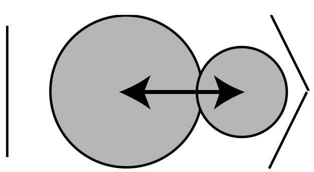
Nodal excited state

$$\sum_{\underline{N=N_0+2}}^{\infty} f_N n_N \mathcal{A}\{\underline{R_{\boldsymbol{N}0}(r)}Y_{00}(\hat{r})\phi_\alpha\phi_{^{16}\mathrm{O}}\}$$

increase of nodal quantum number

superposition of 2,4,6,.. $\hbar\omega$ excitations

Excited 0^+ state



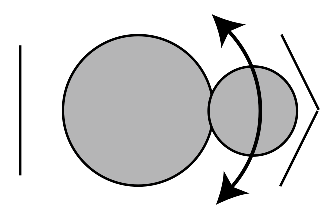
Angular excited state

$$\sum_{\underline{N=N_0+1}}^{\infty} f_N n_N \mathcal{A}\{R_{N0}(r)\underline{Y_{\ell=1}(\hat{r})}\phi_\alpha\phi_{^{16}\mathrm{O}}\}$$

increase of angular momentum

superposition of 1,3,5,.. $\hbar\omega$ excitations

Excited 1^- state

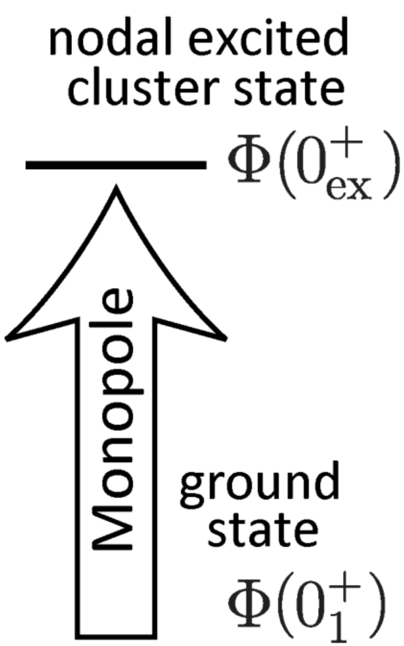


Part2. IS monopole/dipole transitions

Monopole transition induces "nodal excitation"

Y. Suzuki et al., PRC39, 658 (1989). T. Yamada et al., PTP120, 1139 (2008)
Y. Kanada-Enyo., PRC75, 024302 (2007).

$$\mathcal{M}^{IS0}_{\mu} = \sum_{i=1}^{A} (\boldsymbol{r}_i - \boldsymbol{r}_{\mathrm{cm}})^2 = \sum_{i\in C_1} \xi_i^2 + \sum_{i\in C_2} \xi_i^2 + \frac{C_1 C_2}{C_1 + C_2} r^2$$



Cluster estimate for ^{20}Ne (analytical)

$$M^{IS0} = \langle \Phi(0^+_{\mathrm{ex}}) | \mathcal{M}^{IS0} | \Phi(0^+_1) \rangle$$

$$= f_{N_0+2} \sqrt{\frac{\mu_{N_0}}{\mu_{N_0+2}}} \langle R_{N_0 0} | r^2 | R_{N_0+2 0} \rangle$$

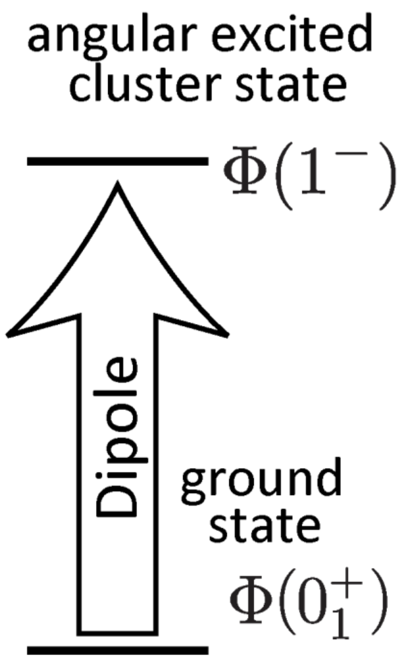
$$\simeq 7.67 \, f_{N_0+2} = 5.48 \ \mathrm{fm}^2$$

Single particle estimate

$$M^{IS0}_{\mathrm{WU}} = \frac{3}{5} (1.2 A^{1/3})^2 \simeq 6.37 \ \mathrm{fm}^2$$

Part2. IS monopole/dipole transitions

Even if the ground state is an ideal shell model state, the monopole/dipole transitions from the g.s. to the cluster states are as large as single particle estimates



Cluster estimate (analytical)

$$M^{IS1} \simeq 3.08\, f_{N_0+1} - 7.36\, f_{N_0+3} = 5.82\ \mathrm{fm}^3$$

Single particle estimate

$$M^{IS1}_{\mathrm{WU}} = \sqrt{\frac{3}{4\pi}}\frac{3}{6}(1.2A^{1/3})^3 \simeq 8.44\ \mathrm{fm}^3$$

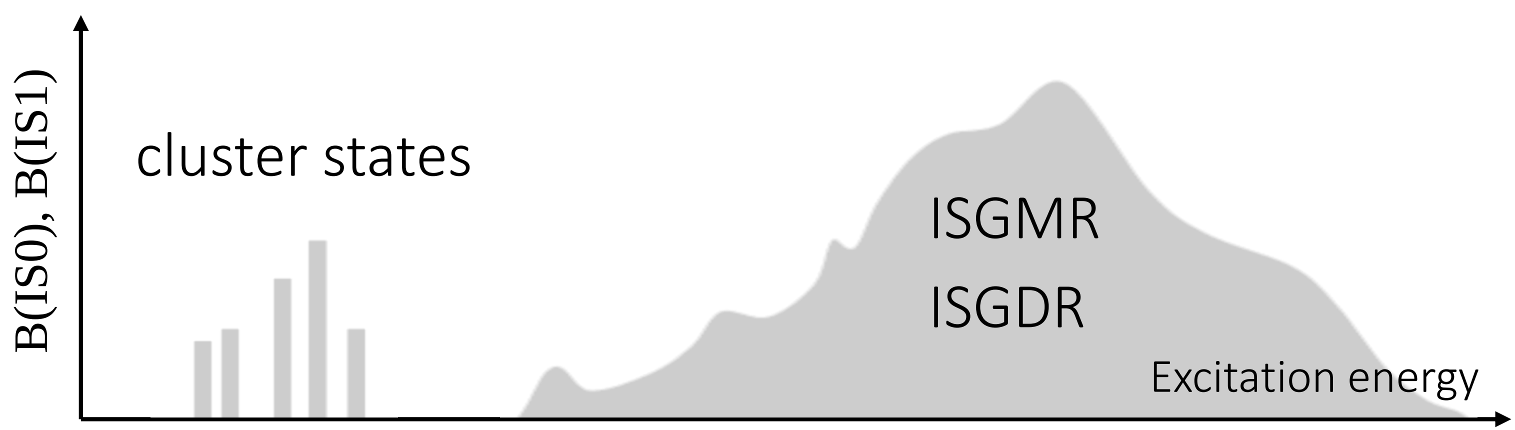
Part2. IS monopole/dipole transitions

Now we can understand
why narrow resonances exist well below the Giant Resonances

Two different types of nuclear excitations

◎ Collective excitation: Stronger than s.p. estimate, $E_x > 15\ \mathrm{MeV}$

◎ Cluster excitation: Comparable with s.p. estimate, $E_x < 15\ \mathrm{MeV}$



Part 3.
Realistic calculations by Antisymmetrized Molecular Dynamics

3.1 Formulation of AMD.

3.2 Results for ^{20}Ne, ^{28}Si and ^{24}Mg

Part 3. Formulation of AMD

Microscopic Hamiltonian (A-nucleons)

Gogny D1S interaction, No spurious center-of-mass energy

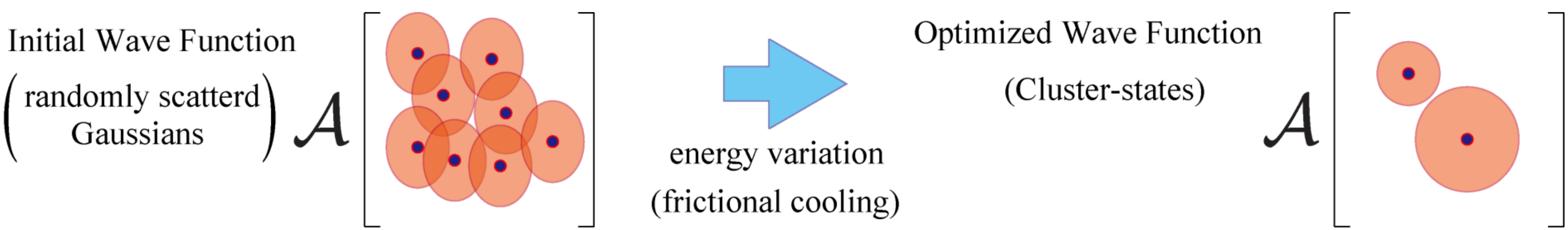
$$\hat{H} = \sum_{i}^{A} \hat{t}_i - \hat{t}_{c.m.} + \sum_{i<j}^{A} \hat{v}_{\mathrm{GognyD1S}}(r_{ij}) + \sum_{i<j}^{Z} \hat{v}_{\mathrm{Coulomb}}(r_{ij})$$

Variational wave function: Antisymmetrized product of Gaussian wave packets

No a-priori assumption on cluster structure

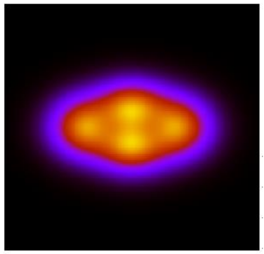
$$\Psi^{\pi} = \frac{1+\pi\hat{P}_r}{2}\Psi_{int} = \frac{1+\pi\hat{P}_r}{2}\mathcal{A}\{\varphi_1, \varphi_2, ..., \varphi_A\}$$

$$\varphi_i(\boldsymbol{r}) \propto \exp\left\{-\boldsymbol{\nu_x}\left(x-\frac{\boldsymbol{Z_{ix}}}{\sqrt{\boldsymbol{\nu_x}}}\right)^2 - \boldsymbol{\nu_y}\left(y-\frac{\boldsymbol{Z_{iy}}}{\sqrt{\boldsymbol{\nu_y}}}\right)^2 - \boldsymbol{\nu_z}\left(z-\frac{\boldsymbol{Z_{iz}}}{\sqrt{\boldsymbol{\nu_z}}}\right)^2\right\} \otimes \{\boldsymbol{a_i}|\uparrow\rangle + \boldsymbol{b_i}|\uparrow\rangle\} \otimes (|n\rangle \text{ or } |p\rangle)$$

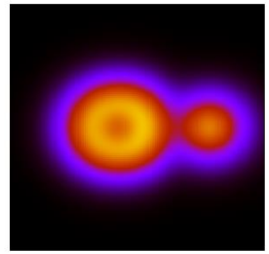


Part 3. Result for ^{20}Ne (α+^{16}O resonances)

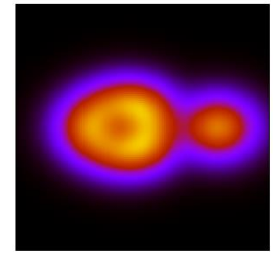
MK, PRC 69, 044319 (2004). Y. Taniguchi, MK, and H. Horiuchi, PTP112, 475 (2004).
Y. Chiba, M.K., and Y. Taniguchi, PRC93, 034319 (2016).



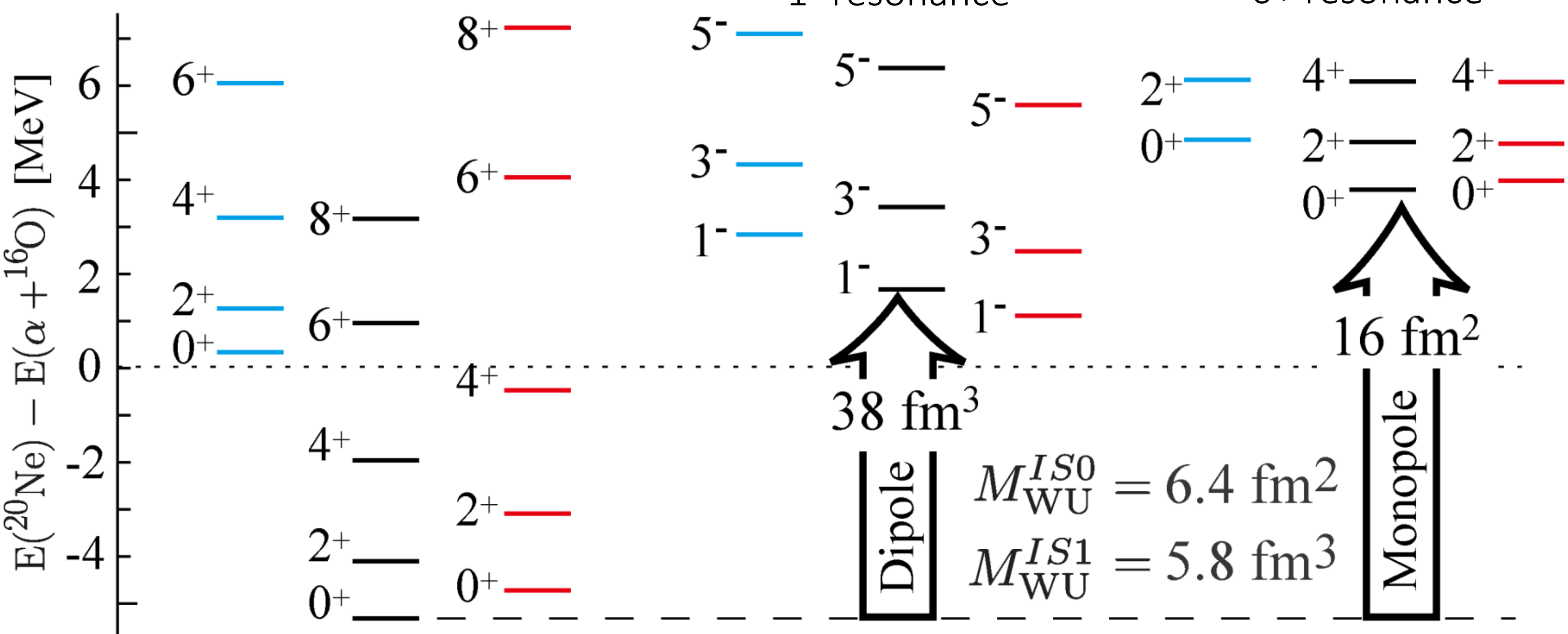
Ground band



α+^{16}O
1- resonance



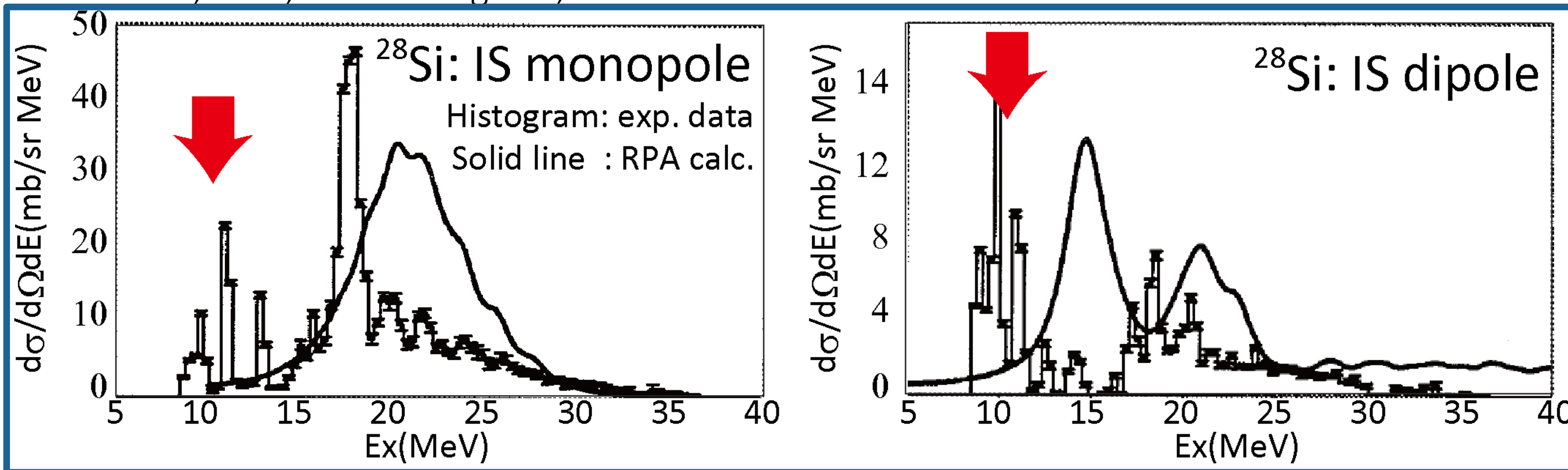
α+^{16}O
0+ resonance

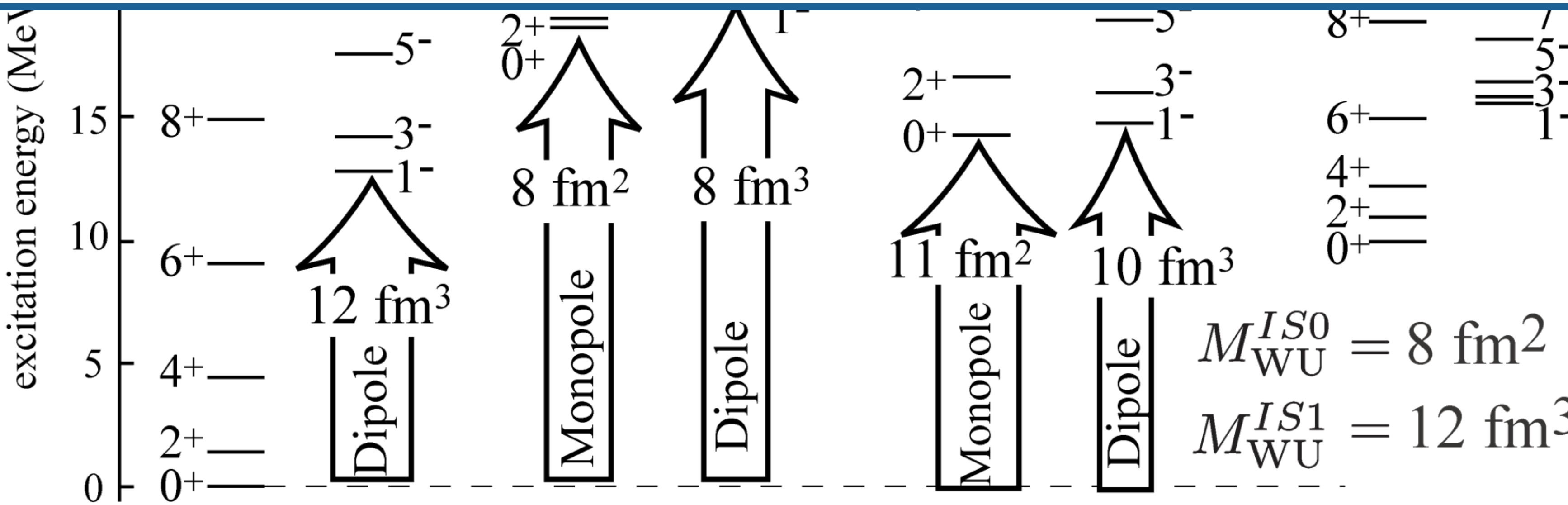


Part 3. Result for ^{28}Si (α+^{24}Mg and ^{8}Be+^{20}Ne resonances)

Y. Taniguchi, Y. Kanada-En'yo and M.K. PRC80, 044316 (2009).

Y. Chiba, M.K., and Y. Taniguchi, arXiv:1610.04000





$M_{\mathrm{WU}}^{IS0} = 8\ \mathrm{fm}^2$

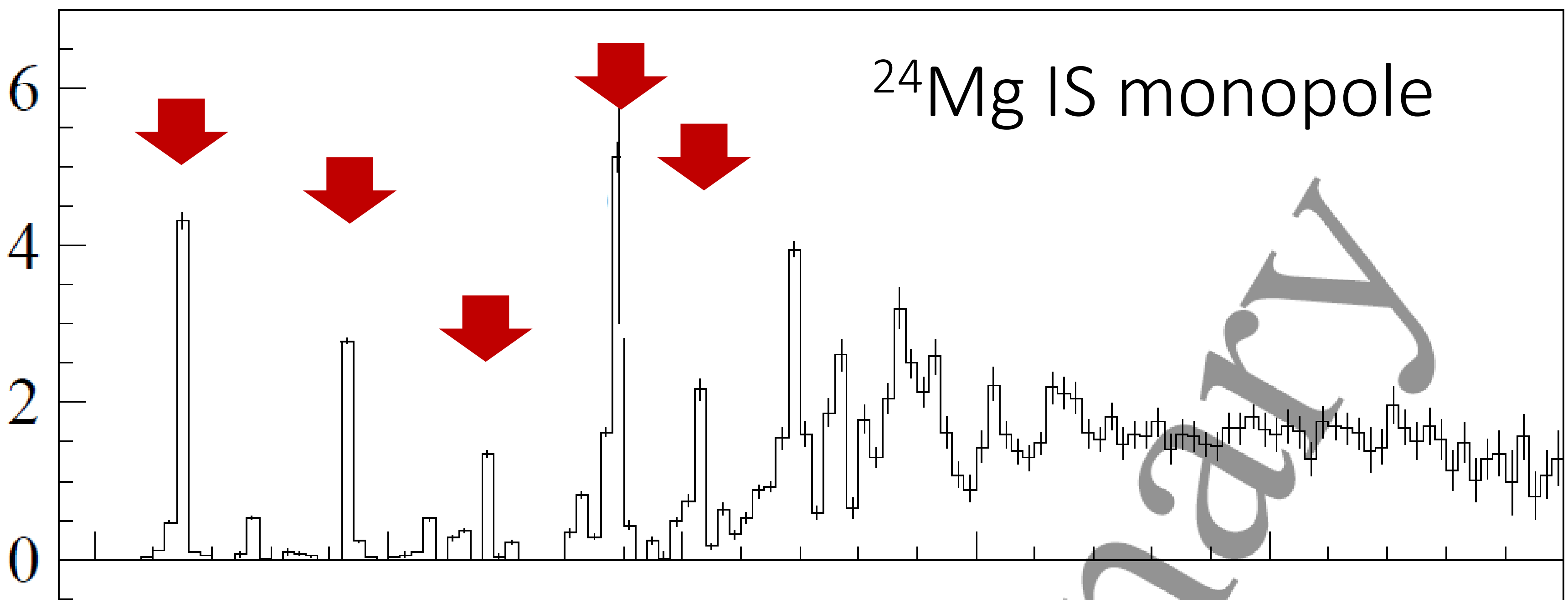
$M_{\mathrm{WU}}^{IS1} = 12\ \mathrm{fm}^3$

Part 3. Result for ^{24}Mg (^{12}C+^{12}C and α+^{20}Ne resonances)

High resolution data from RCNP(Osaka)

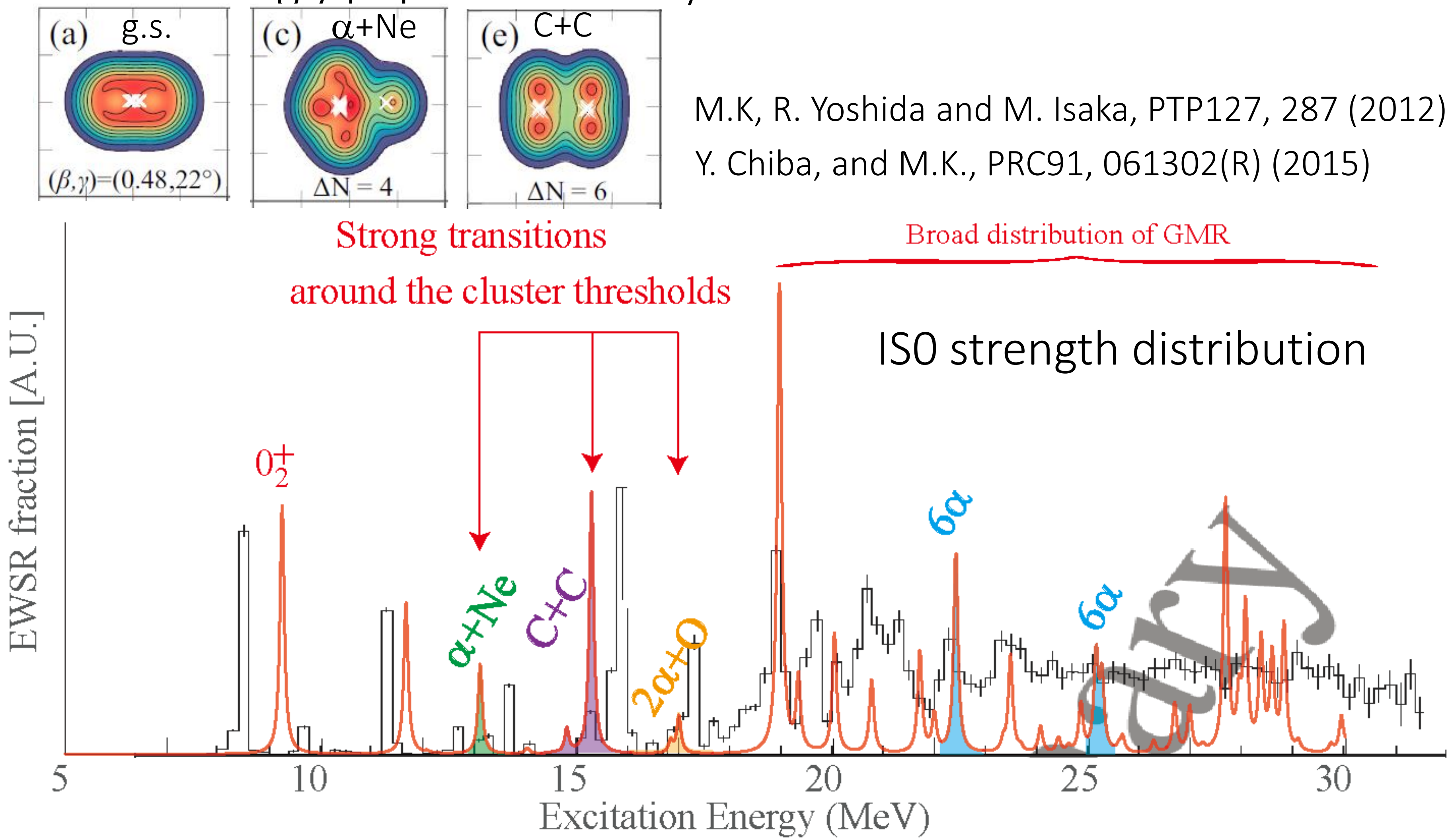
Strong peaks appear well blow the Giant resonance

T. Kawabata, Reported at the last Cluster conf. in 2012



Part 3. Result for ^{24}Mg (^{12}C+^{12}C and α+^{20}Ne resonances)

IS monopole/dipole transitions of ^{24}Mg strongly populate α+^{20}Ne/^{12}C+^{12}C resonances



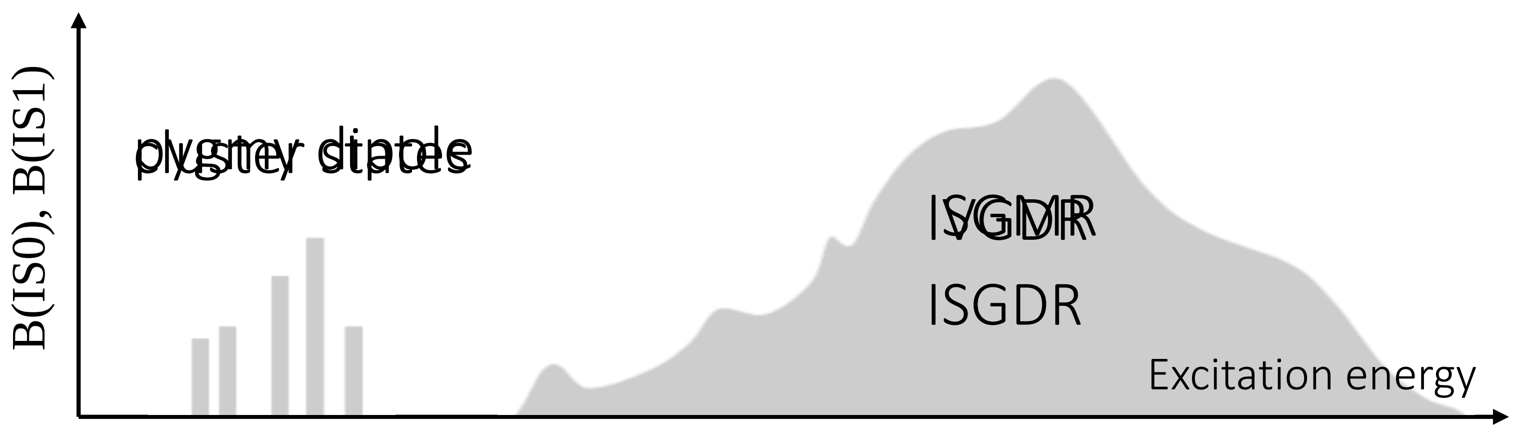
M.K, R. Yoshida and M. Isaka, PTP127, 287 (2012)

Y. Chiba, and M.K., PRC91, 061302(R) (2015)

Part. 4 Comment on PDR

This pattern of the strength distribution reminds us PDR, and we wonder if the argument also applies to PDR.

Yes, it’s true.



Applying the same argument, we can explain the decay pattern of pygmy dipole resonances

What is the "^{26}Ne PDR Puzzle" ?

◎ PDR of ^{26}Ne have been studied in detail

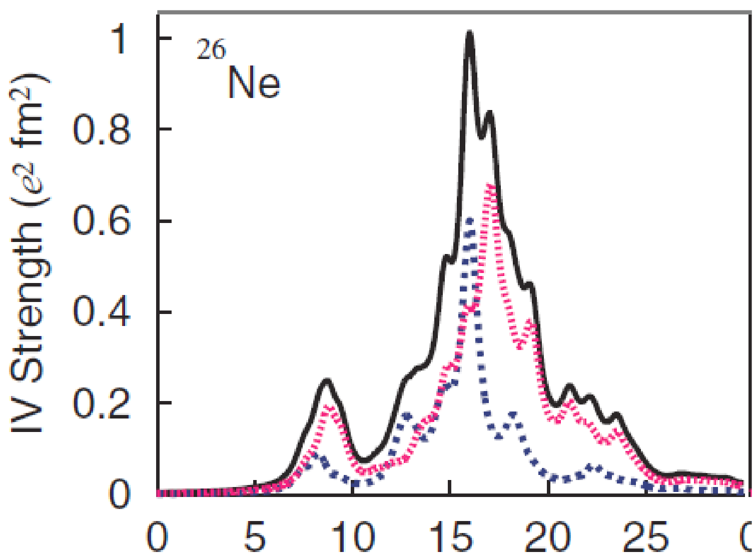
◎ Reasonable agreement between theory and experiment for the energy and B(E1) of PDR.

Theory: QRPA

Energy: E_x = 6 ~ 10 MeV

Strength: 5~10 % of TRK sum

K. Yoshida et al., PRC78, 014305 (2008).

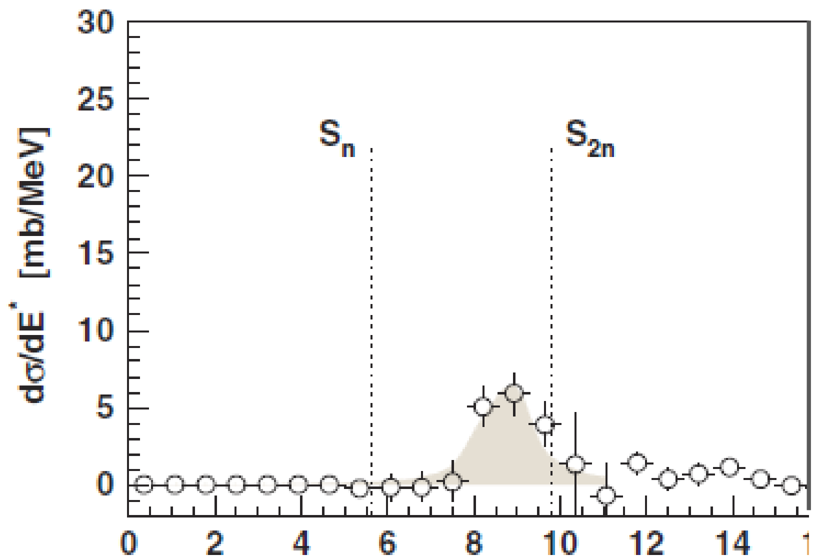


Experiment@RIKEN

Energy: E_x = 9 MeV

Strength: 5 % of TRK sum

J. Gibelin et al., PRL101, 212503 (2008)



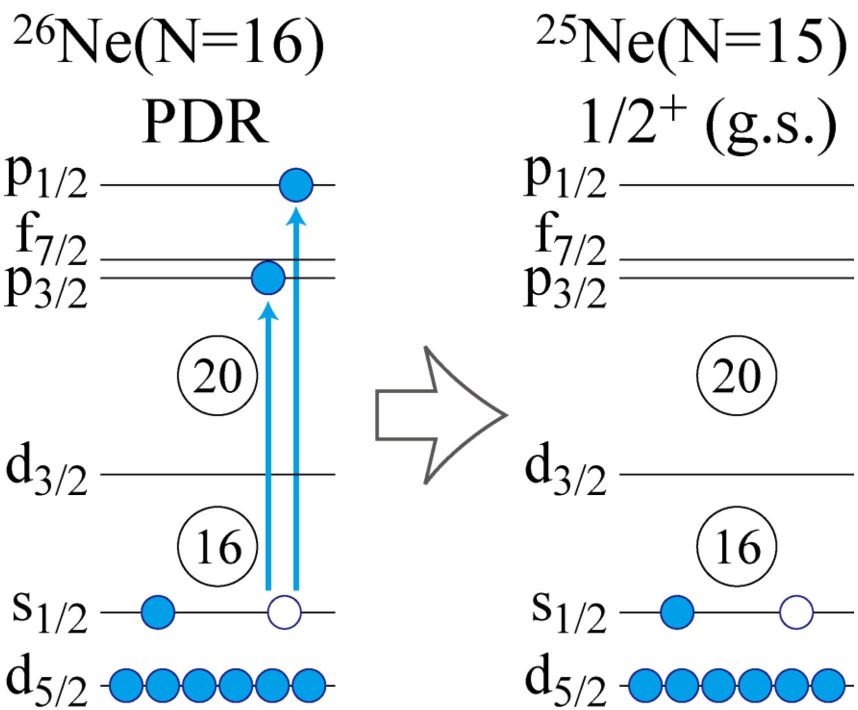
What is the "^{26}Ne PDR Puzzle" ?

◎ Theory cannot explain the observed decay pattern

Theory: QRPA

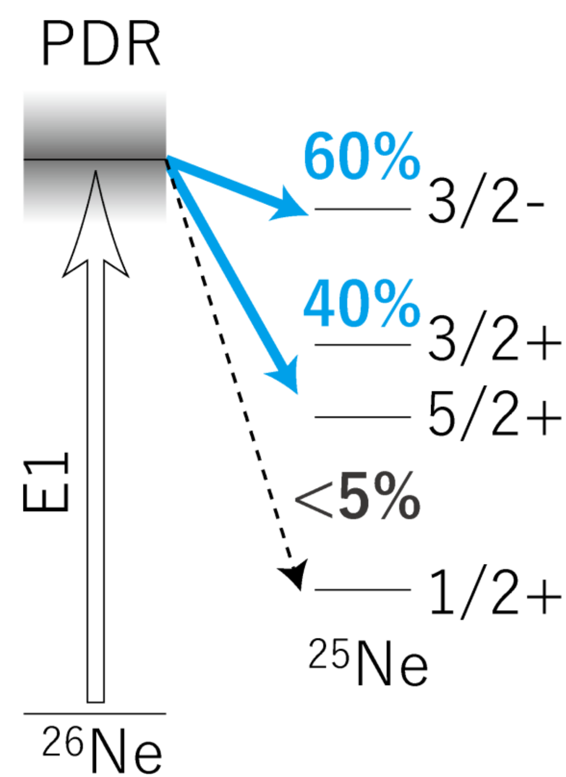
Leading configurations of PDR

$\nu(s_{1/2})^{-1}(p_{3/2})^1$ and $\nu(s_{1/2})^{-1}(p_{1/2})^1$



Experiment@RIKEN

PDR decays to ^{25}Ne*
not to ^{25}Ne(g.s.)



Part. 4 Comment on PDR

PDR is dominated by the neutron excitation.

It's not the eigenmode of isospin, but a sum of IS, IV components

$$|PDR\rangle = \mathcal{M}^{E1}|g.s.\rangle + \mathcal{M}^{IS1}|g.s.\rangle$$

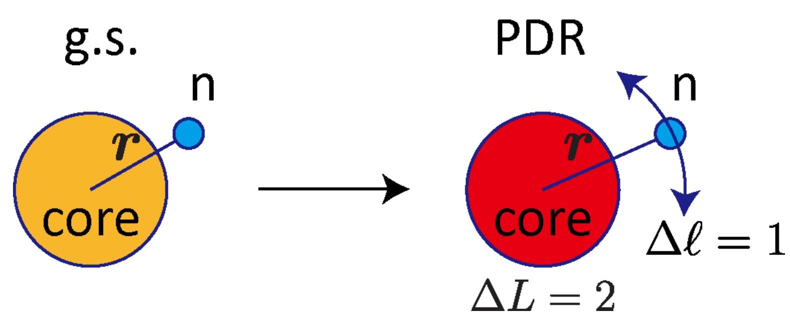
If we regard PDR as (A-1)+n cluster system, IS dipole operator reads

$$\mathcal{M}^{IS1} = \sum_{i\in\mathrm{Core}} \xi_i^2 \mathcal{Y}_{1\mu}(\boldsymbol{\xi}_i) - \frac{(A-1)(A-2)}{A^2} r^2 \mathcal{Y}_{1\mu}(\boldsymbol{r}) + \frac{5}{3A}\sum_{i\in\mathrm{Core}} \xi_i^2 \mathcal{Y}_{1\mu}(\boldsymbol{r})$$

$$- \frac{4\sqrt{2}\pi}{3A}\Big[\sum_{i\in\mathrm{Core}} \underline{\mathcal{Y}_2(\boldsymbol{\xi}_i)} \otimes \underline{\mathcal{Y}_1(\boldsymbol{r})}\Big]_{1\mu}$$

quadrupole excitation of the core

$\Delta\ell = 1$ excitation of intercluster motion



Part. 4 Comment on PDR

① PDR is a linear combination of IV and IS components

② IS component involves quadurupole core excitation

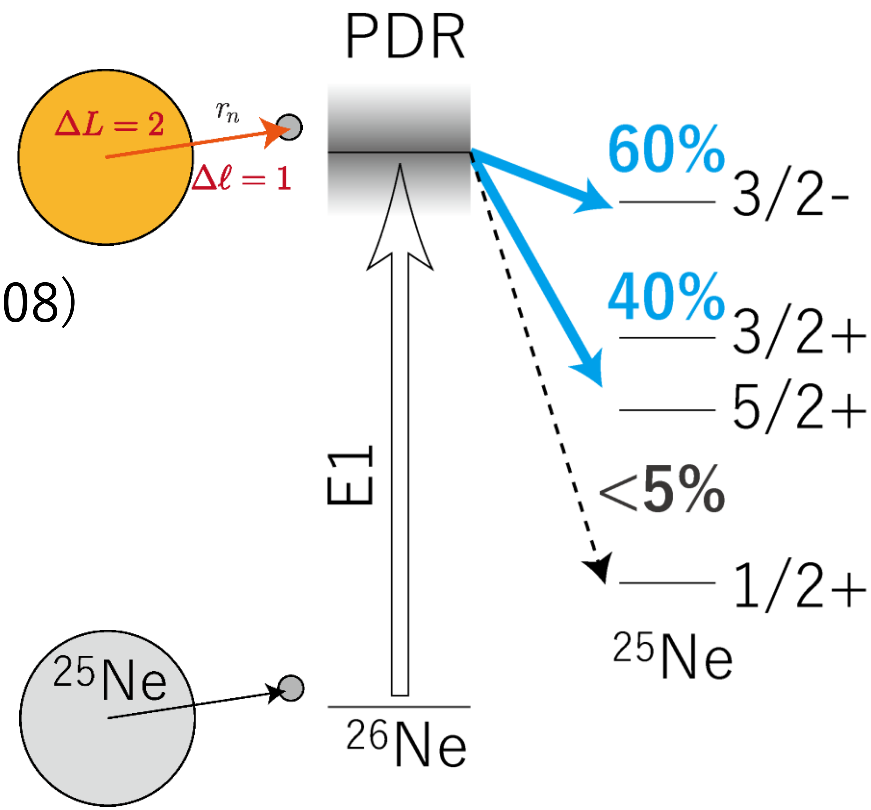
$$\mathcal{M}(IS1) \simeq -\frac{4\sqrt{2\pi}}{3A}\left[\left(\sum_{i\in \mathrm{Core}} \xi_i^2 Y_2(\hat{\xi}_i)\right)\otimes r_n Y_1(\hat{r}_n)\right]_{1\mu}$$

◎ This explains the observed decay pattern of ^{26}Ne PDR

J. Gibelin et al., PRL101, 212503 (2008)

◎ AMD calculation also supports this idea,

M. Kimura, arXiv:1612.02086



Part 5. Summary

Observed data show many narrow resonances well below the giant resonance, which are attributed to the cluster states

Analytical formula were derived to show that IS monopole/dipole transitions strongly feed cluster states

More realistic calculation by AMD showed many candidates of cluster states in the IS monople/dipole response functions

The same story also applies to PDR which explains the obeved core excitation in PDF

References

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