Exotic states -- cluster model v.s. DFT

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First Tsukuba-CCS-RIKEN joint workshop on microscopic theories of nuclear structure and dynamics

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Purpose of the study

- Discuss exotic cluster states based on not only cluster model but also mean field model
- We extend cluster model to include jj coupling shell model states for the general understanding of nuclear structure

Previously we have been studying exotic cluster states using cluster models, but here we compare the results with those of DFT

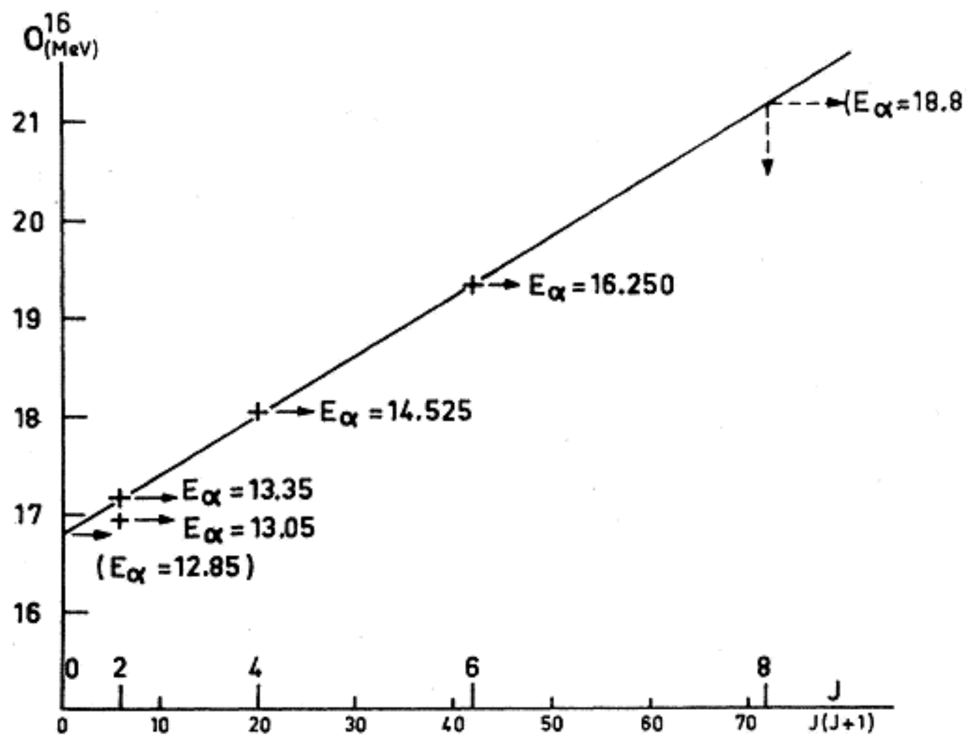
TABLE II. Properties of identified resonances. Even-parity resonances observed in $C^{12}(\alpha,\alpha)C^{12}$ in this region[a] are also shown for comparison.

		$C^{12}+\alpha \rightarrow Be^8+Be^8$				$C^{12}+\alpha \rightarrow C^{12}+\alpha$			
E_α in (MeV)	$E^*(O^{16})$ (MeV)	J^π	Γ (keV)	σ^{tot} (mb)	ΓBe^8 (keV)	E_α in (MeV)	E^* (MeV)	J^π	Γ (keV)
13.05	16.95	2^+	370	7	6	12.9	16.8	(4^+)	525
13.35	17.15	2^+	260	11	7	13.3	17.1	$(1^-,2^+,0^+)$	110
						13.9	17.55	(4^+)	225
(14.1	17.7	$0^+,2^+)$							
(14.2	17.8	4^+)							
14.52	18.05	4^+	20	16	0.5	14.49	18.01	(4^+)	45
(15.2	18.6	$0^+,2^+)$							
(15.6	18.9	4^+)							
16.25	19.35	6^+	70	10	0.8	15.96	19.10	$(2^+,4^+)$	55
						16.30	19.35	$(4^+,0^+)$	30

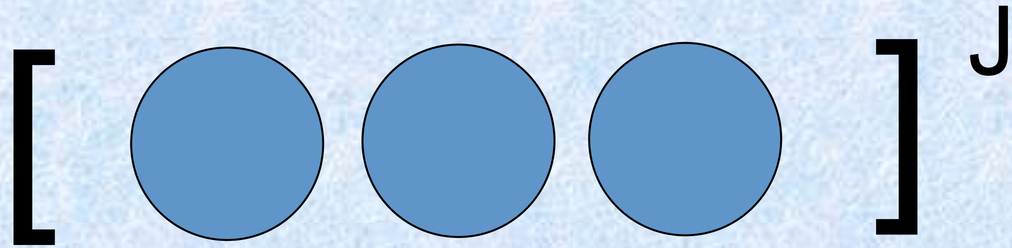
[a] Reference 1.

^{16}O four α states

P. Chevallier et al.
Phys. Rev **160**, 827 (1967)



The effect of Pauli principle



The second and third alpha-clusters are excited to higher-nodal configurations.

If linear-chain is stable, there must exist some very strong mechanism in the interaction side.

How can we stabilize geometric cluster shapes like linear chain configurations?

- Adding valence neutrons
- Rotating the system

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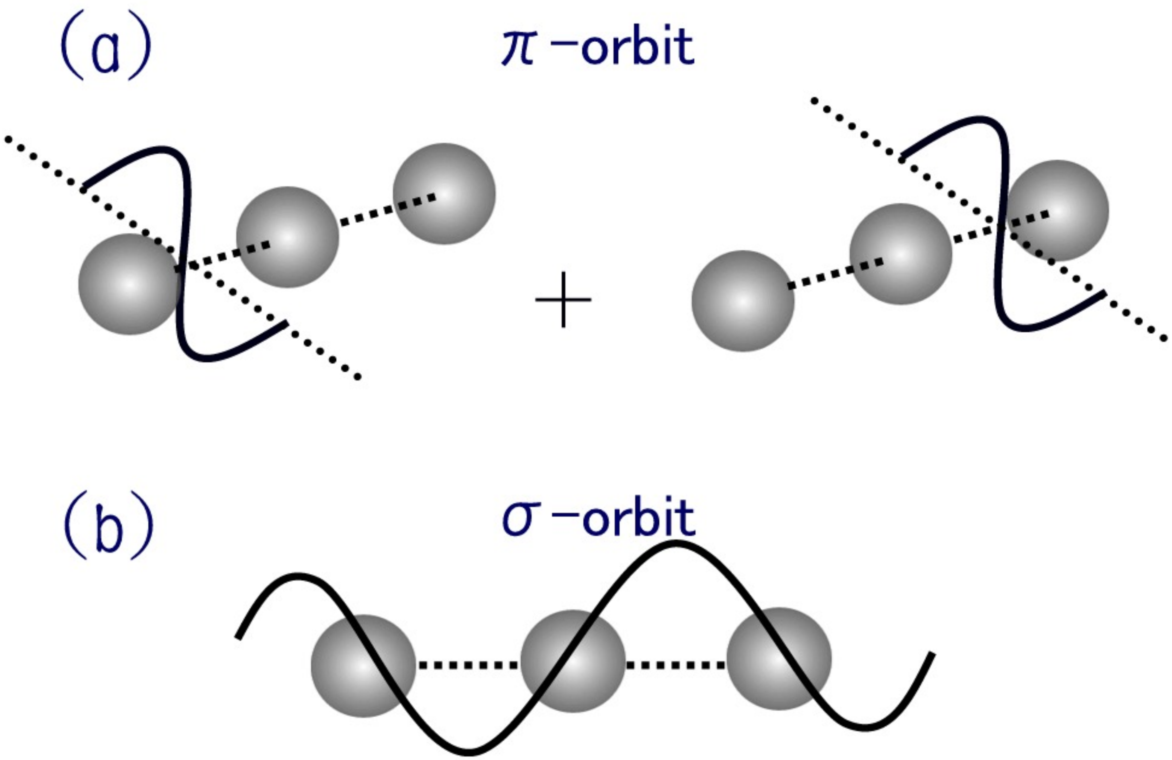
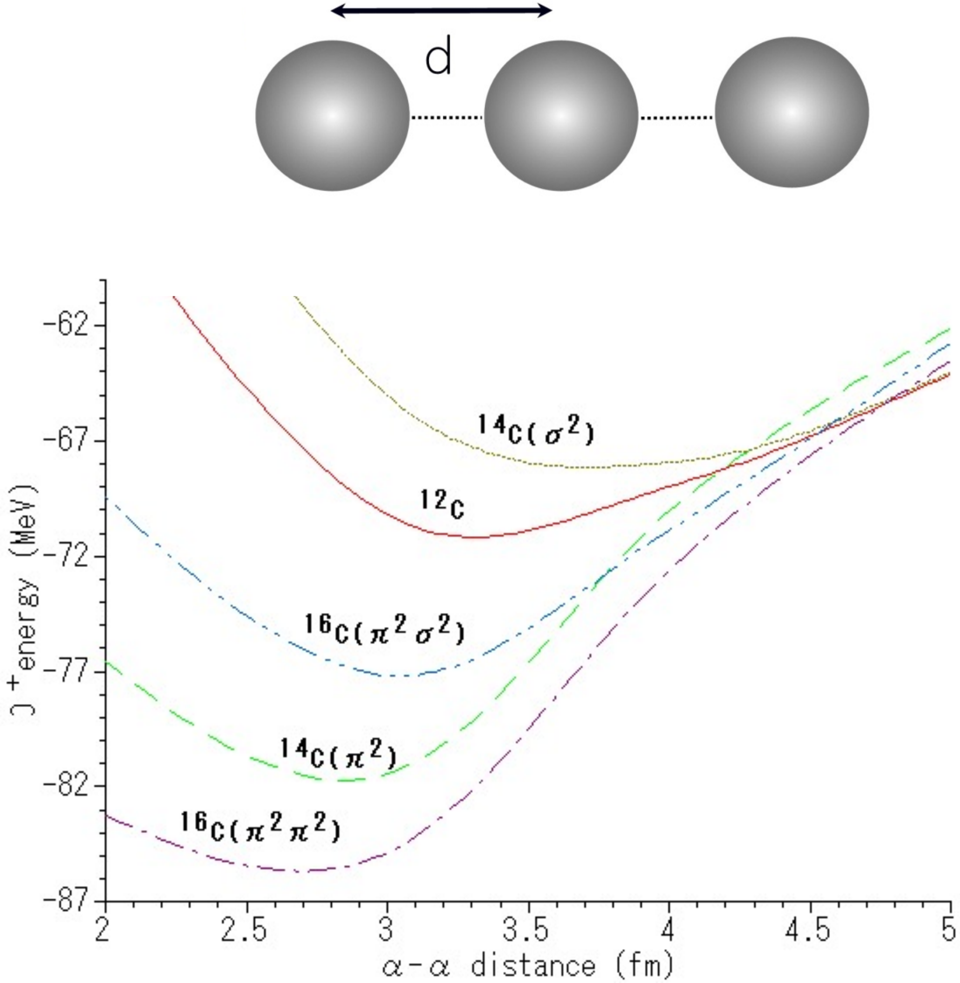
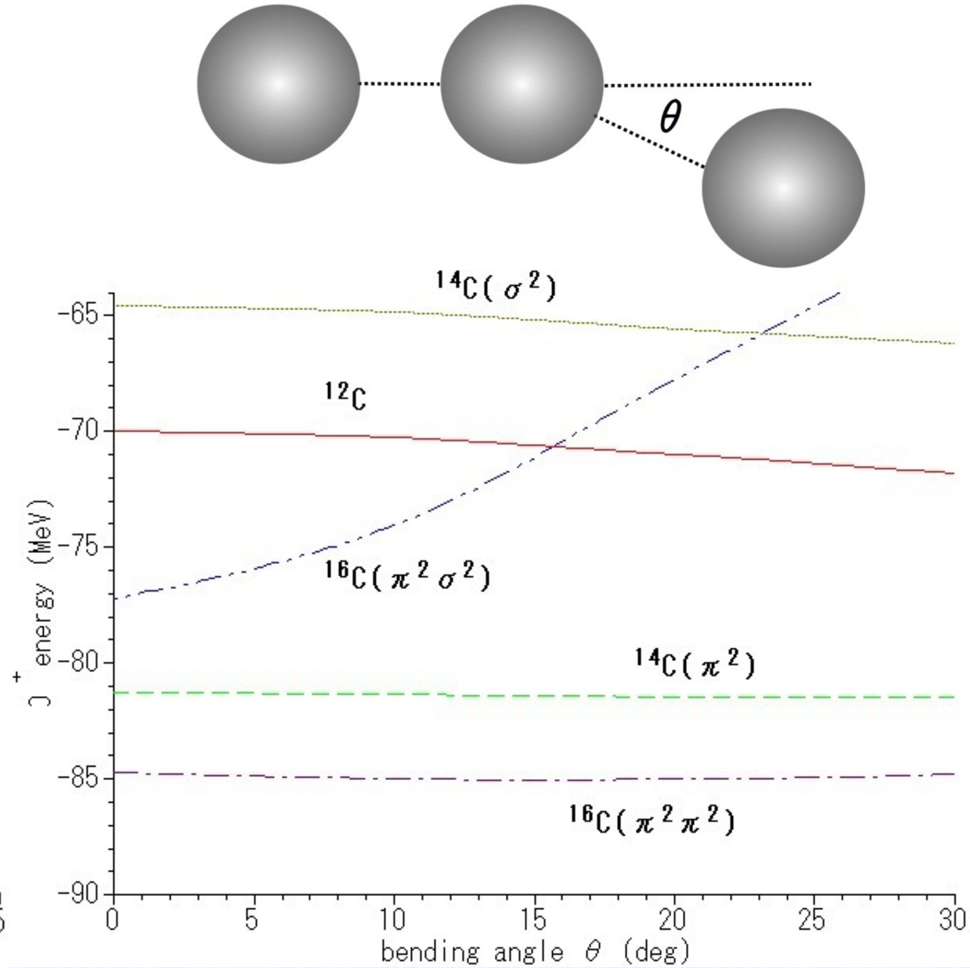
- Adding valence neutrons
- Rotating the system



N. Itagaki, S. Okabe, K. Ikeda, and I. Tanihata
Phys. Rev. C **64** 014301 (2001).





σ-orbit is important for the linear chain, but not the lowest configuration around 3 alpha linear chain

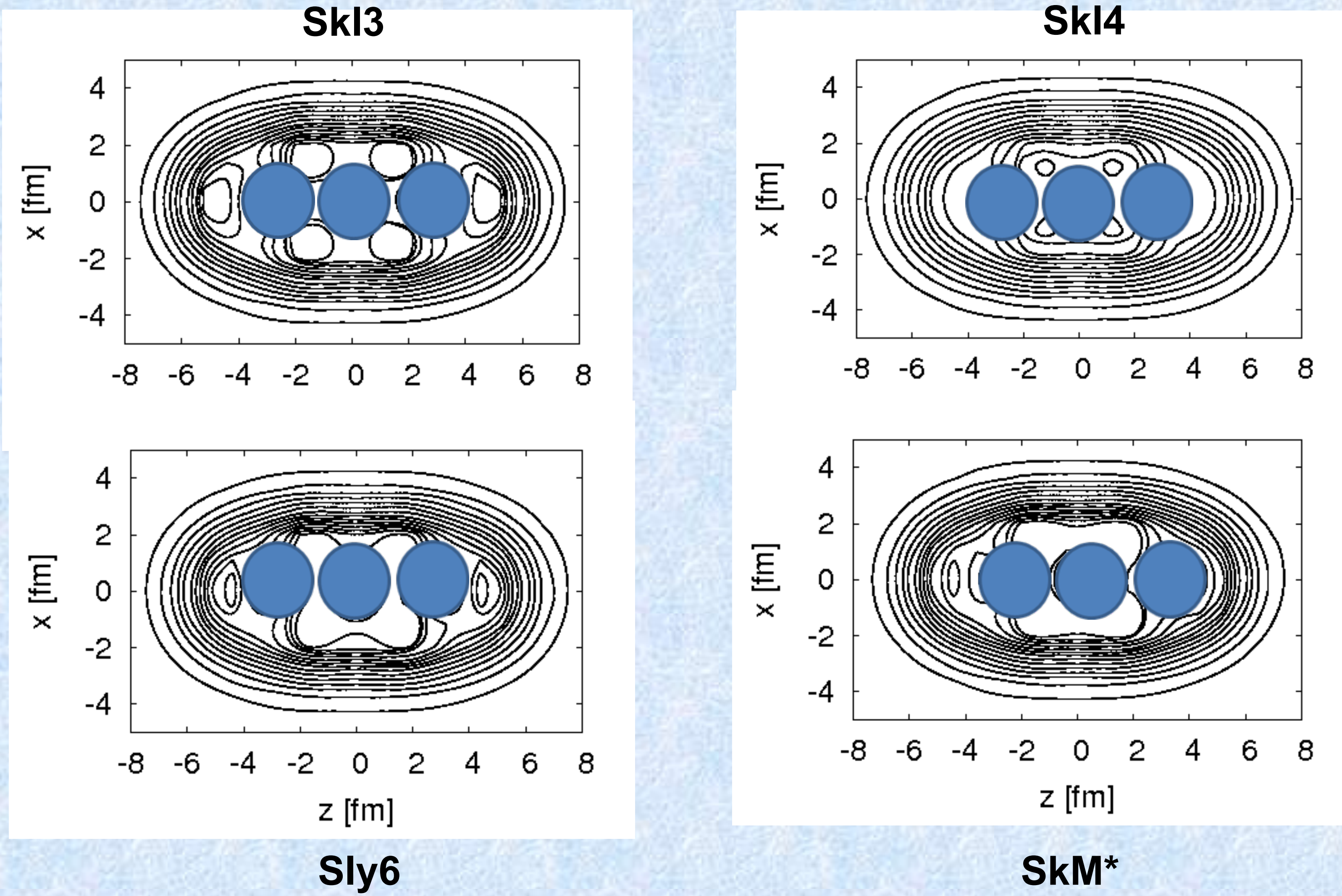
N. Itagaki, S. Okabe, K. Ikeda, and I. Tanihata
Phys. Rev. C **64** 014301 (2001).

Mean field models

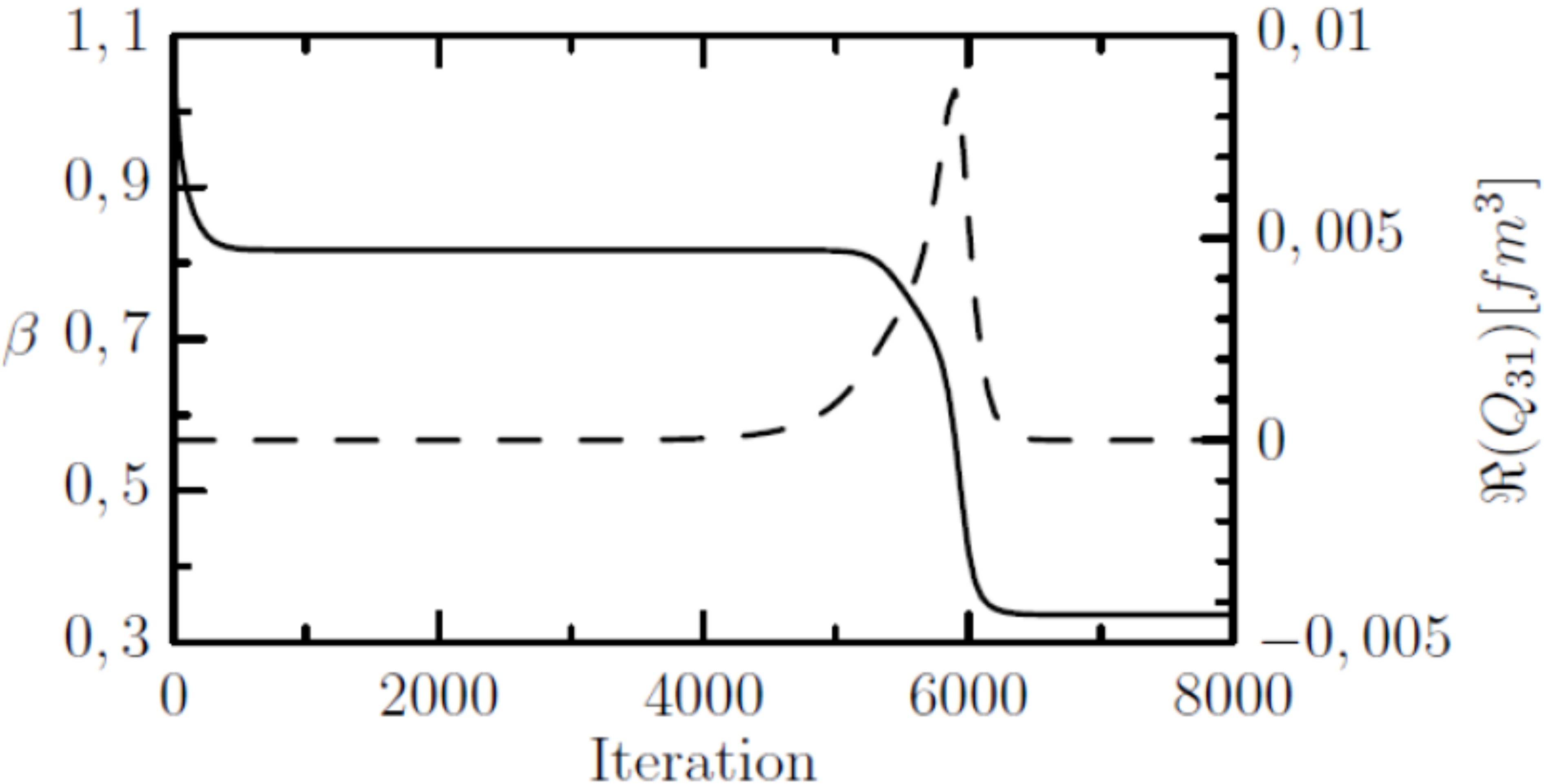
- Quite general models designed for nuclei of all the mass regions (exotic cluster structure is not assumed a priori).
- Appearance of cluster structure as results of studies using such general models give us more confidence for their existence.

Many people started analyzing cluster states with mean field models

^{20}C alpha chain states , Ex ~ 15 MeV region
Skyrme Hartree-Fock calculation



J.A. Maruhn, N. Loebl, N. Itagaki, and M. Kimura, Nucl. Phys. A **833** (2010).

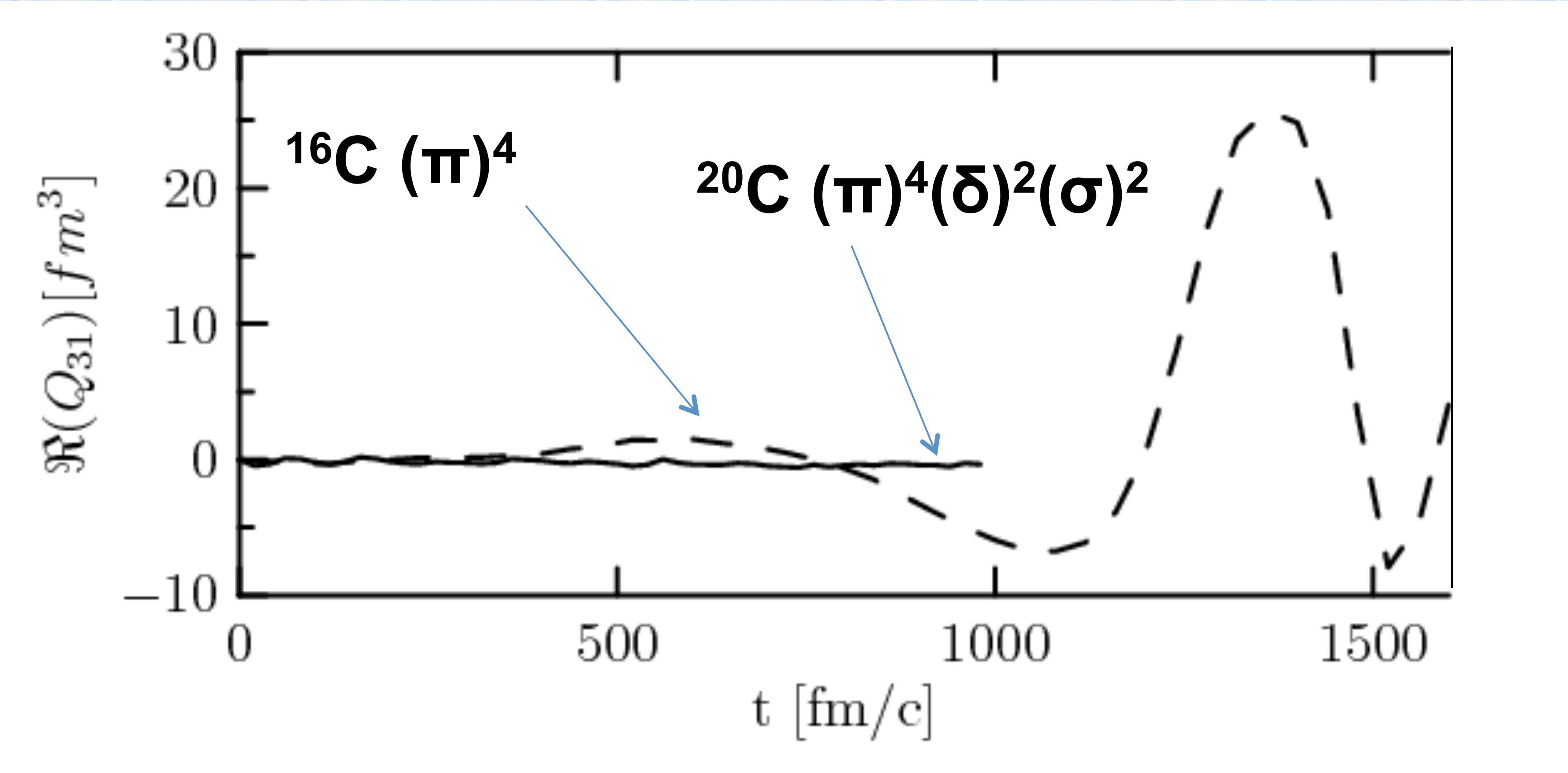


J.A. Maruhn, N. Loebl, N. Itagaki, and M. Kimura, Nucl. Phys. A **833** (2010).

Stability of 3 alpha linear chain with respect to the bending motion

Time Dependent Hartree-Fock calculation

Geometric shape is stabilized by adding neutrons in $(\sigma)^2$



How can we stabilize geometric cluster shapes like linear chain configurations?

- Adding valence neutrons
- Rotating the system

4 alpha linear chain in rotating frame

Progress of Theoretical Physics, Vol. 72, No. 5, November 1984

Configuration Space, Cranked Hartree-Fock Calculations for the Nuclei ^{16}O, ^{24}Mg and ^{32}S

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(Received March 10, 1984)

A method for performing basis free self-consistent calculations directly in coordinate space is applied to study rotating nuclei. The method is uniquely suited to accurately describe extremely deformed nuclei such as may be found near scission. Because of technical complications, the present study is limited to light nuclei cranked about a single fixed axis, and pairing and spin-orbit effects are not included. However these limitations are not significant at the high values of angular momentum in which we are interested, and we believe that our calculations may significantly be compared with high spin data. Calculations are presented for a number of bands in ^{16}O, ^{32}S and in ^{24}Mg which exhibit a number of different fission modes, most of which seem to have been observed. Correlation with experimental data is found where they exist for very deformed states.

Pioneering work, but no spin-orbit, no path to bending motion

PRL 107, 112501 (2011) PHYSICAL REVIEW LETTERS week ending 9 SEPTEMBER 2011

Linear Chain Structure of Four-α Clusters in ^{16}O

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[2]*Institut fuer Theoretische Physik, Universitaet Frankfurt, D-60438 Frankfurt, Germany*
[3]*Department of Applied Science and Environment, University of Kochi, Kochi 780-8515, Japan*
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(Received 17 June 2011; published 9 September 2011)

We investigate the linear chain configurations of four-α clusters in ^{16}O using a Skyrme cranked Hartree-Fock method and discuss the relationship between the stability of such states and angular momentum. We show the existence of a region of angular momentum (13–18$\hbar$) where the linear chain configuration is stabilized. For the first time we demonstrate that stable exotic states with a large moment of inertia ($\hbar^2/2\Theta \sim 0.06$–$0.08$ MeV) can exist.

DOI: 10.1103/PhysRevLett.107.112501 PACS numbers: 21.60.Jz, 21.30.Fe, 21.60.Cs

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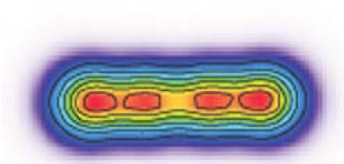
Phys. Rev. Lett. **107**, 112501
(issue of 9 September 2011)
Title and Authors

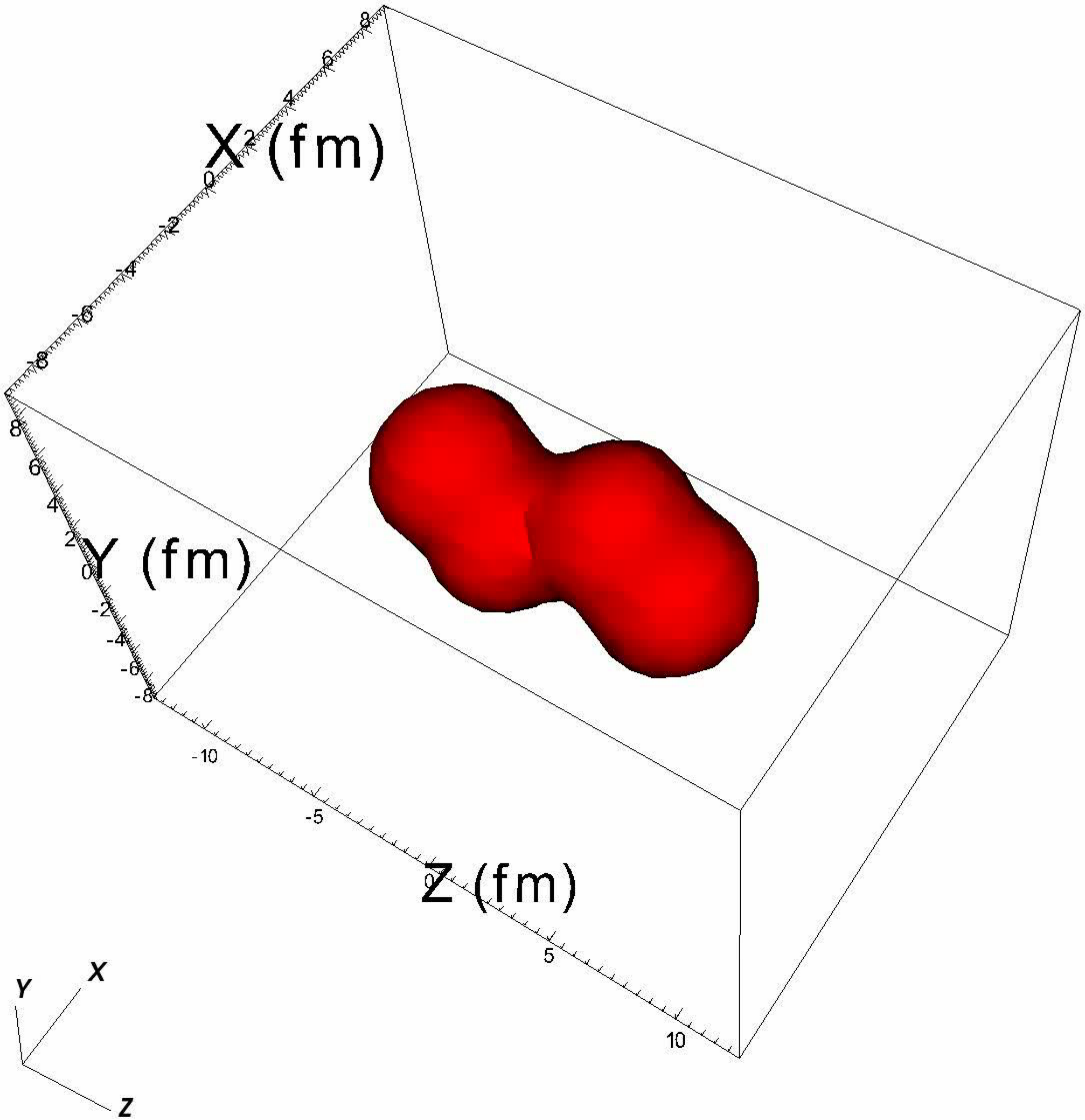
9 September 2011

Rod-Shaped Nucleus



We picture atomic nuclei as spherical globs of protons and neutrons, although they can also be egg-shaped. Now calculations published 9 September in *Physical Review Letters* show that an even more exotic shape is possible: a rapidly spinning nucleus can form into a linear chain of several small clusters of neutrons and protons. Such exotic nuclear states could play important intermediary roles in the formation of carbon-12 and oxygen-16--elements essential for life--in the interiors

Cranked Hartree-Fock calculation



X (fm)
Y (fm)
Z (fm)
8
6
4
2
0
-2
-4
-6
-8
-10
-5
0
5
10
Y
X
Z

Ichikawa et al.

MOI = 0.06 MeV
Ex(0) = 40 MeV

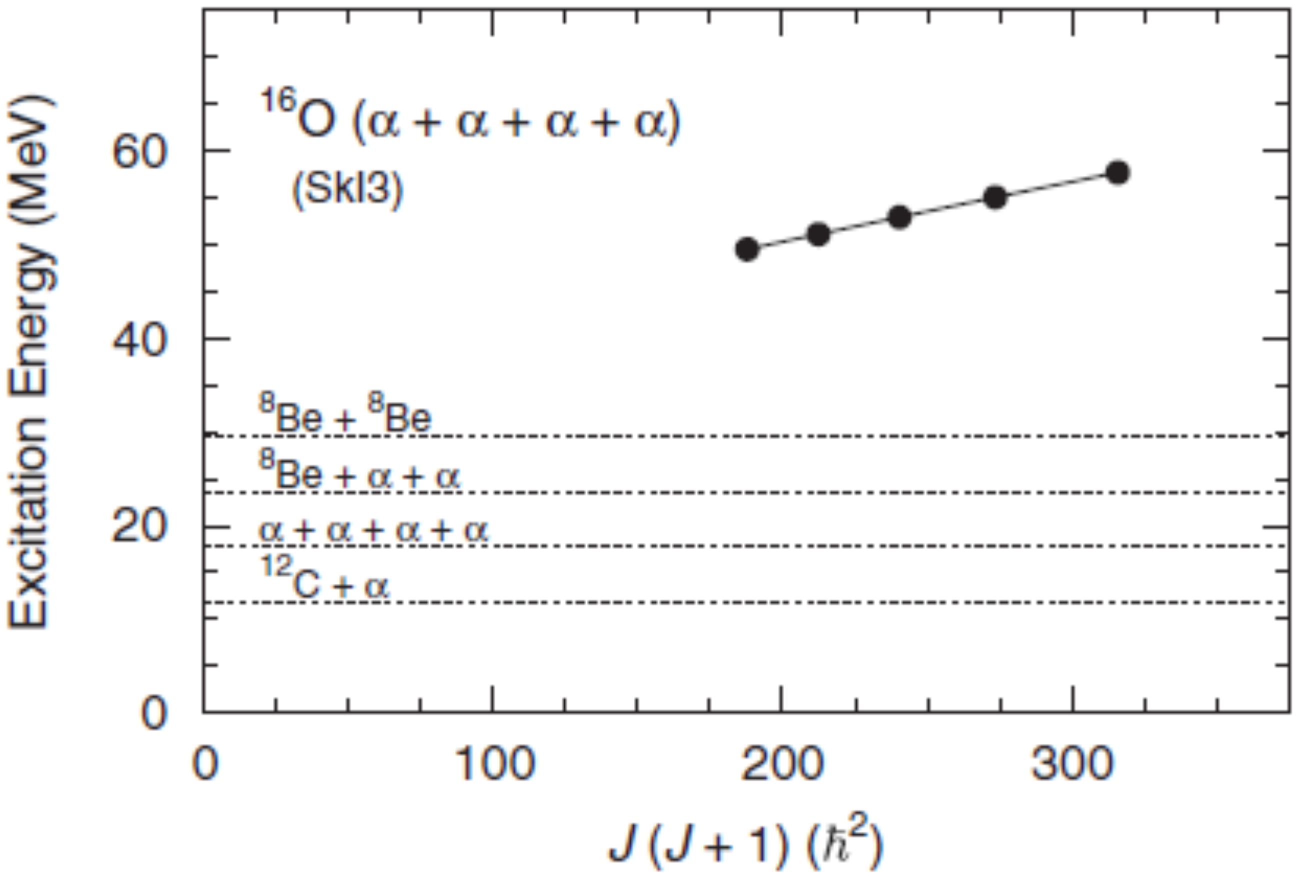


FIG. 6. Calculated excitation energies of the four-α linear chain states with the SkI3 force versus the angular momentum. The dotted lines denote the corresponding cluster-decomposition threshold energies.

PHYSICAL REVIEW C **90**, 054307 (2014)

Searching for a 4α linear-chain structure in excited states of ^{16}O with covariant density functional theory

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[2]*School of Physical Science and Technology, Southwest University, Chongqing 400715, China*
[3]*Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan*
[4]*State Key Laboratory of Nuclear Physics and Technology, School of Physics, Peking University, Beijing 100871, China*
[5]*School of Physics and Nuclear Energy Engineering, Beihang University, Beijing 100191, China*
[6]*Department of Physics, University of Stellenbosch, Stellenbosch, South Africa*
(Received 31 March 2014; revised manuscript received 19 September 2014; published 10 November 2014)

A study of the 4α linear-chain structure in high-lying collective excitation states of ^{16}O with covariant density functional theory is presented. The low-spin states are obtained by configuration mixing of particle-number and angular-momentum projected quadrupole deformed mean-field states with the generator coordinate method. The high-spin states are determined by cranking calculations. These two calculations are based on the same energy density functional PC-PK1. We have found a rotational band at low spin with the dominant intrinsic configuration considered to be the one whereby 4α clusters stay along a common axis. The strongly deformed rod shape also appears in the high-spin region with the angular momentum 13$\hbar$ to 18$\hbar$; however, whether the state is a pure 4α linear chain is less obvious than for the low-spin states.

DOI: 10.1103/PhysRevC.90.054307 PACS number(s): 21.60.Jz, 21.10.Re, 23.20.−g, 21.10.Gv

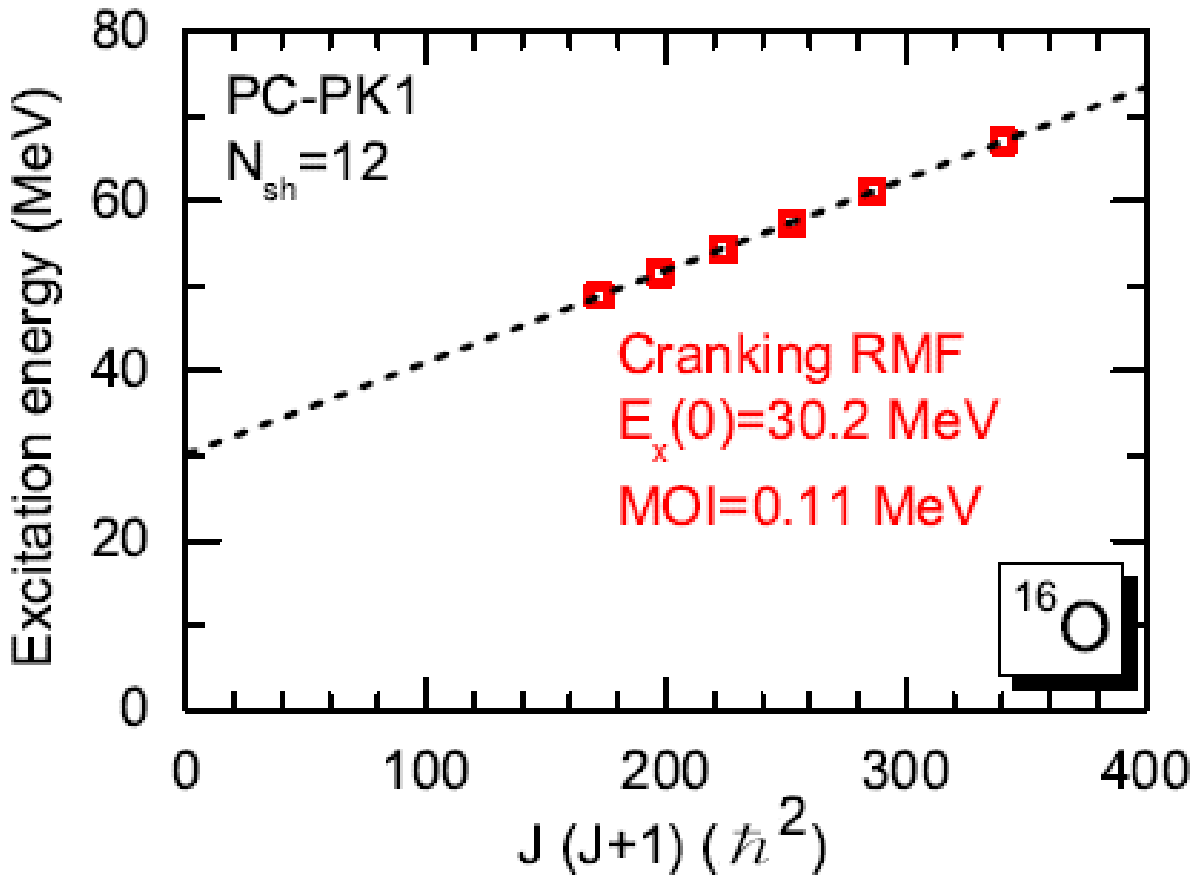


FIG. 11: (Color online) Excitation energy of the state in Tab.III as a function of angular momentum $J(J+1)$ predicted by the cranking RMF calculation. The dashed line is by the rotational formula $E(J_{\rm cra}) = J(J+1)/(2\mathscr{J})$ with the MOI $\hbar^2/(2\mathscr{J}) = 0.11$ MeV

Tohsaki interaction

$$\hat{V}_{central} = \frac{1}{2}\sum_{ij} V_{ij}^{(2)} + \frac{1}{6}\sum_{ijk} V_{ijk}^{(3)}, \qquad (3)$$

where $V_{ij}^{(2)}$ and $V_{ijk}^{(3)}$ consist of three terms,

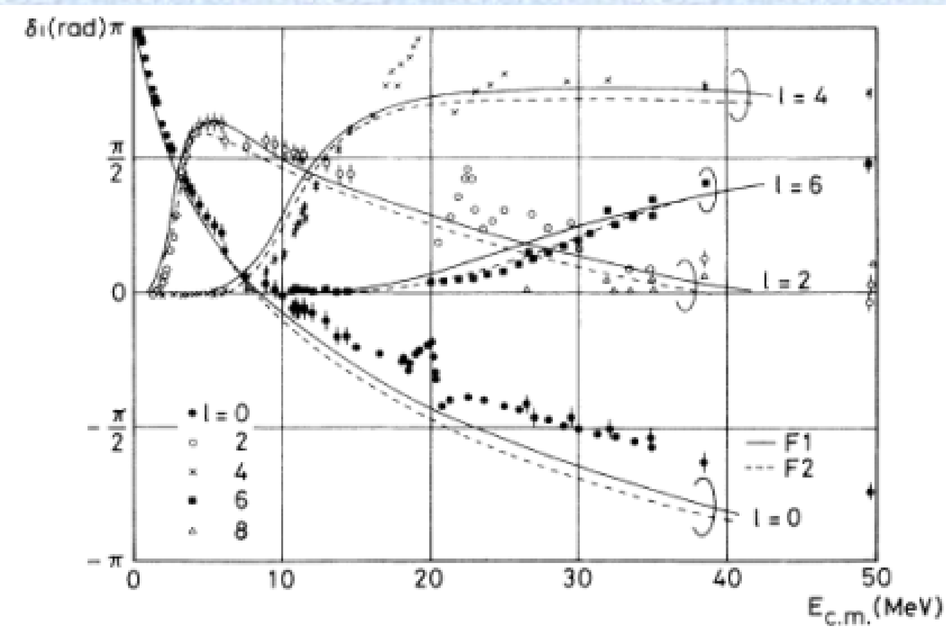
$$V_{ij}^{(2)} = \sum_{\alpha=1}^{3} V_{\alpha}^{(2)} \exp[-(\vec{r}_i - \vec{r}_j)^2/\mu_\alpha^2](W_\alpha^{(2)} + M_\alpha^{(2)} P^r)_{ij}, \qquad (4)$$

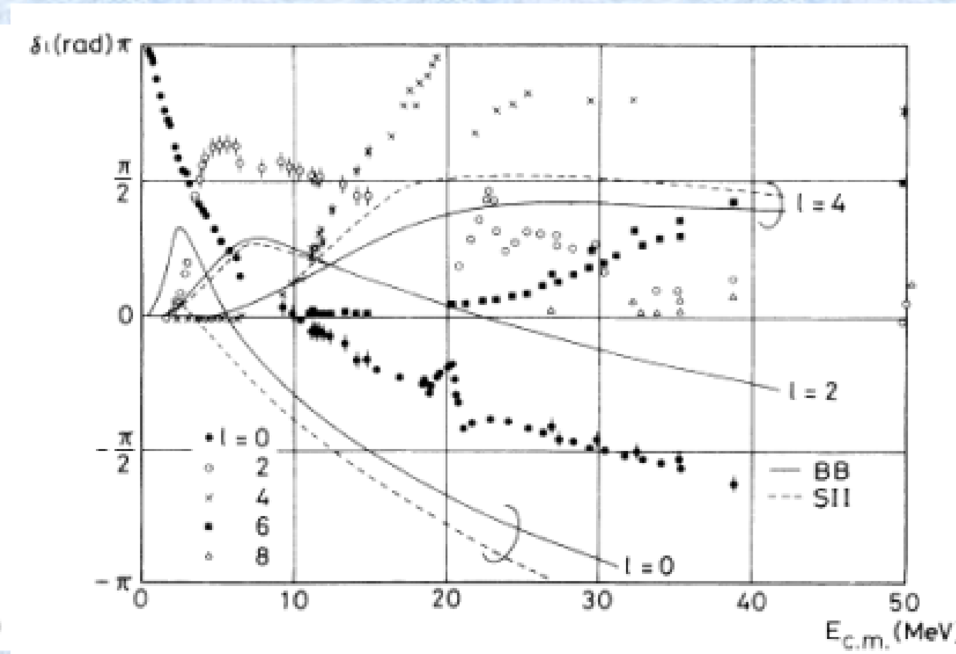
$$V_{ijk}^{(3)} = \sum_{\alpha=1}^{3} \quad V_{\alpha}^{(3)} \exp[-(\vec{r}_i - \vec{r}_j)^2/\mu_\alpha^2 - (\vec{r}_i - \vec{r}_k)^2/\mu_\alpha^2]$$
$$\times \quad (W_\alpha^{(3)} + M_\alpha^{(3)} P^r)_{ij}(W_\alpha^{(3)} + M_\alpha^{(3)} P^r)_{ik}. \qquad (5)$$

The advantages of Tohsaki interaction

- Saturation property is satisfied
- Size parameter dependence of ^{4}He is small and radius and binding energy of ^{4}He are reasonably reproduced
- ^{4}He- ^{4}He scattering phase shift is reproduced
- It is rather easy to perform angular momentum projection and/or superposition of different states, since the Hamiltonian is in the operator form (density dependence is expressed as finite-range three-body interaction)

α-α scattering phase shift





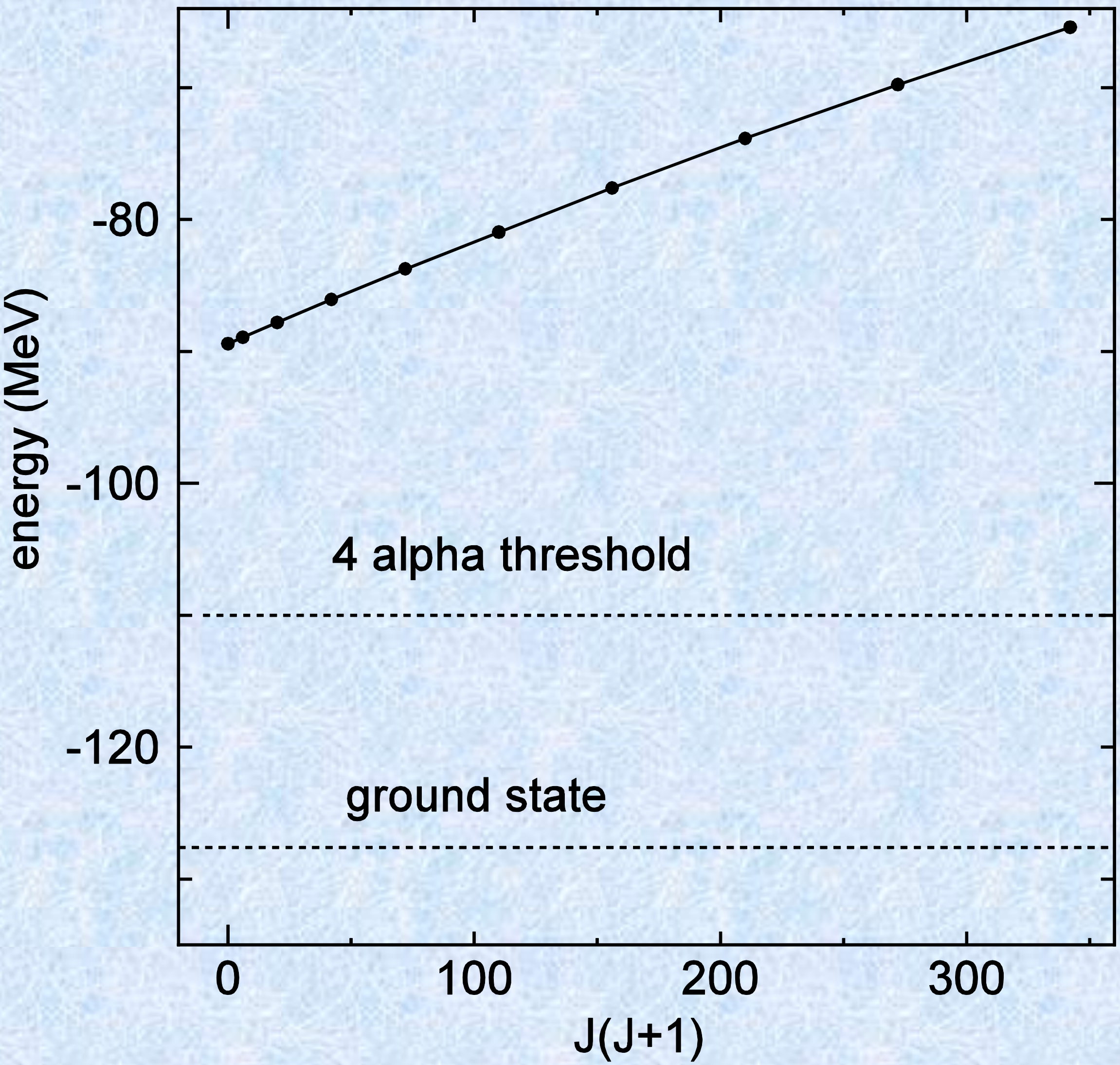
Tohsaki F1, F2

BB, SII

A. Tohsaki, Phys. Rev. C **49** 1814 (1994)

4 alpha chain using Tohsaki interaction

MOI = 0.07 MeV
Ex(0) = 38 MeV



Coherent effect of adding neutrons and rotating the system

P. W. Zhao, N. Itagaki, and J. Meng,
Phys. Rev. Lett. **115** 022501 (2015).

Exotic shape in extreme spin and isospin

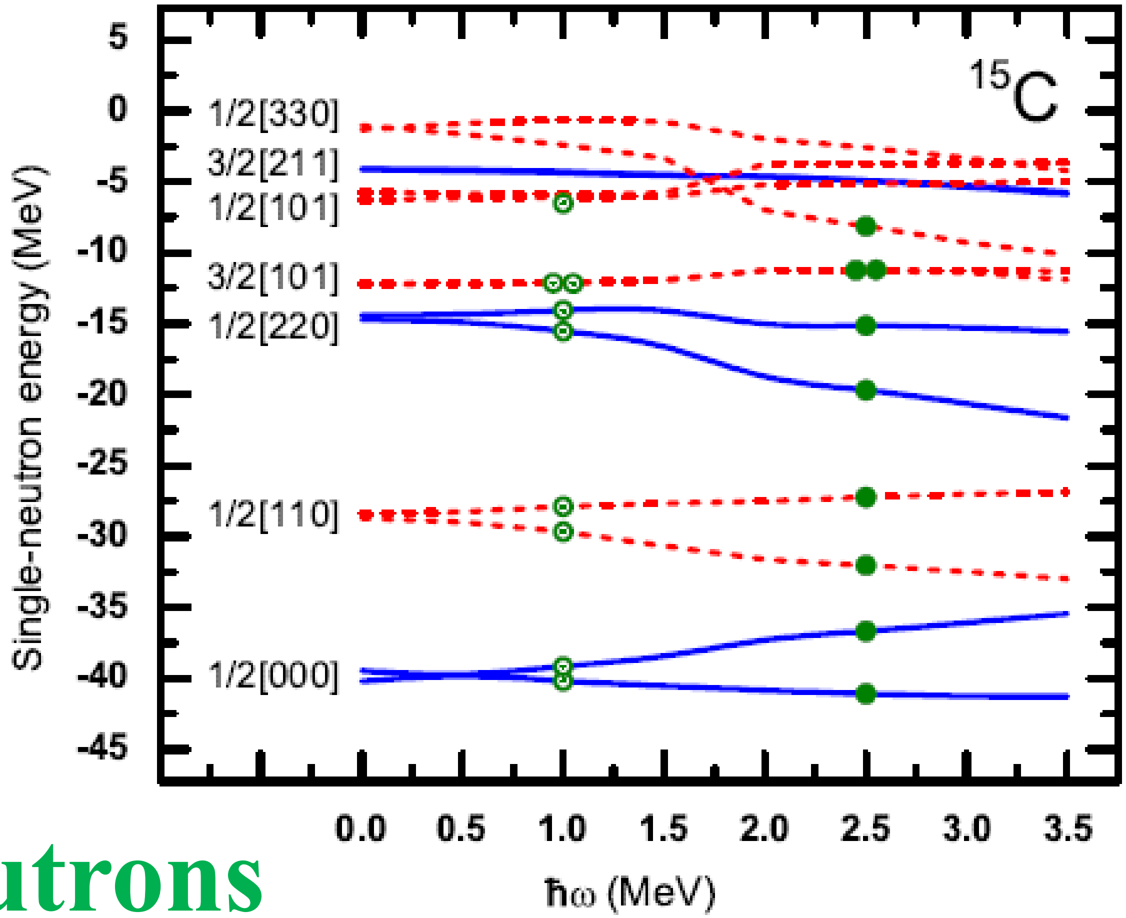
- Code – TAC

  3D Cartesian harmonic oscillator basis

  with N=12 major shells

- Density functional DD-ME2

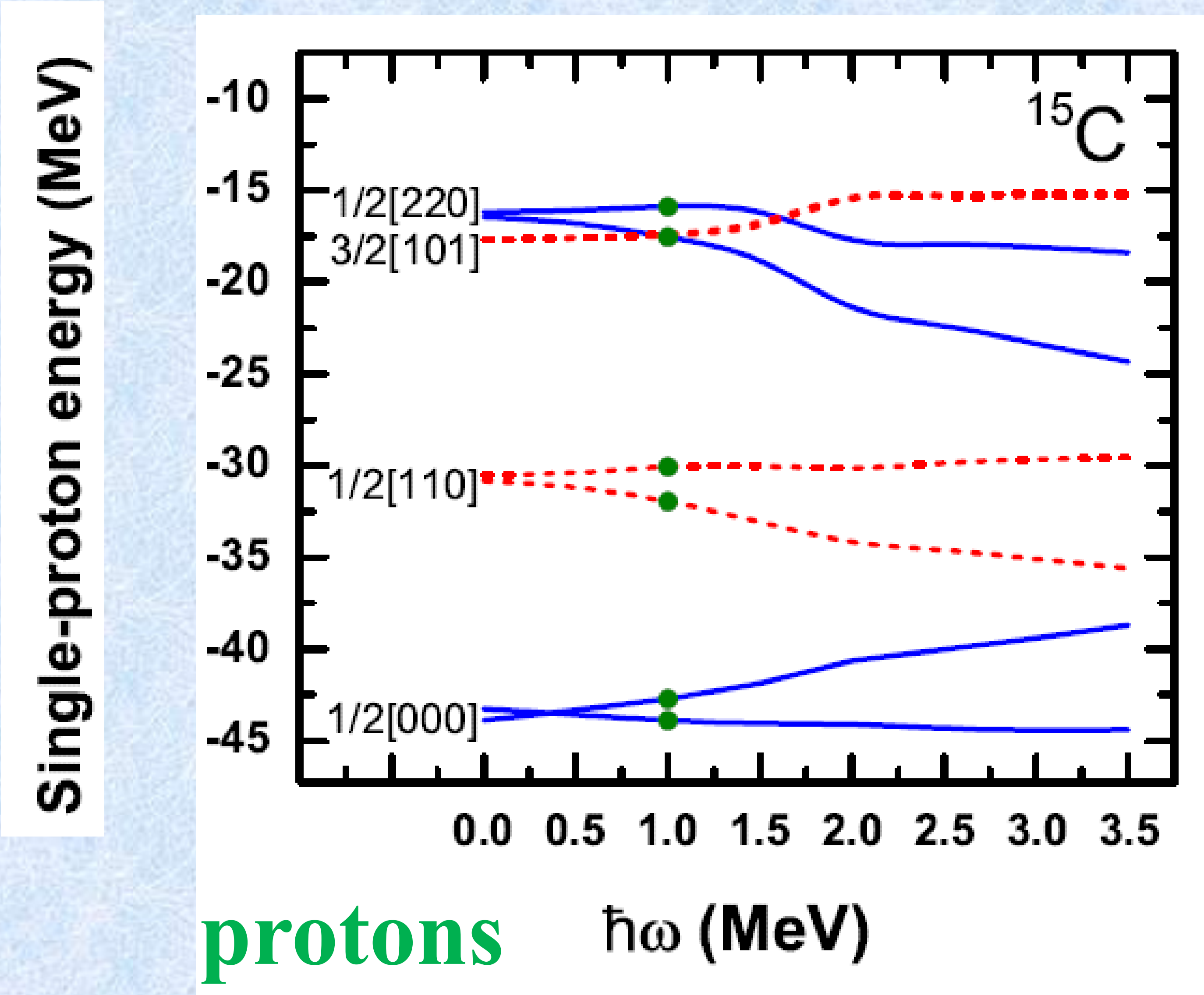
  G. A. Lalazissis, T. Nikšić, D. Vretenar, and P. Ring,
  Phys. Rev. C **71**, 024312 (2005).



neutrons

FIG. 5. (color online). Neutron single-particle energies as functions of the rotational frequency for ^{15}C. The open and solid circles denote respectively the occupied orbitals before and after the level crossing near $\hbar\omega = 1.75$ MeV.

P. W. Zhao, N. Itagaki, and J. Meng,
Phys. Rev. Lett. **115** 022501 (2015).



P. W. Zhao, N. Itagaki, and J. Meng, Phys. Rev. Lett. **115** 022501 (2015).

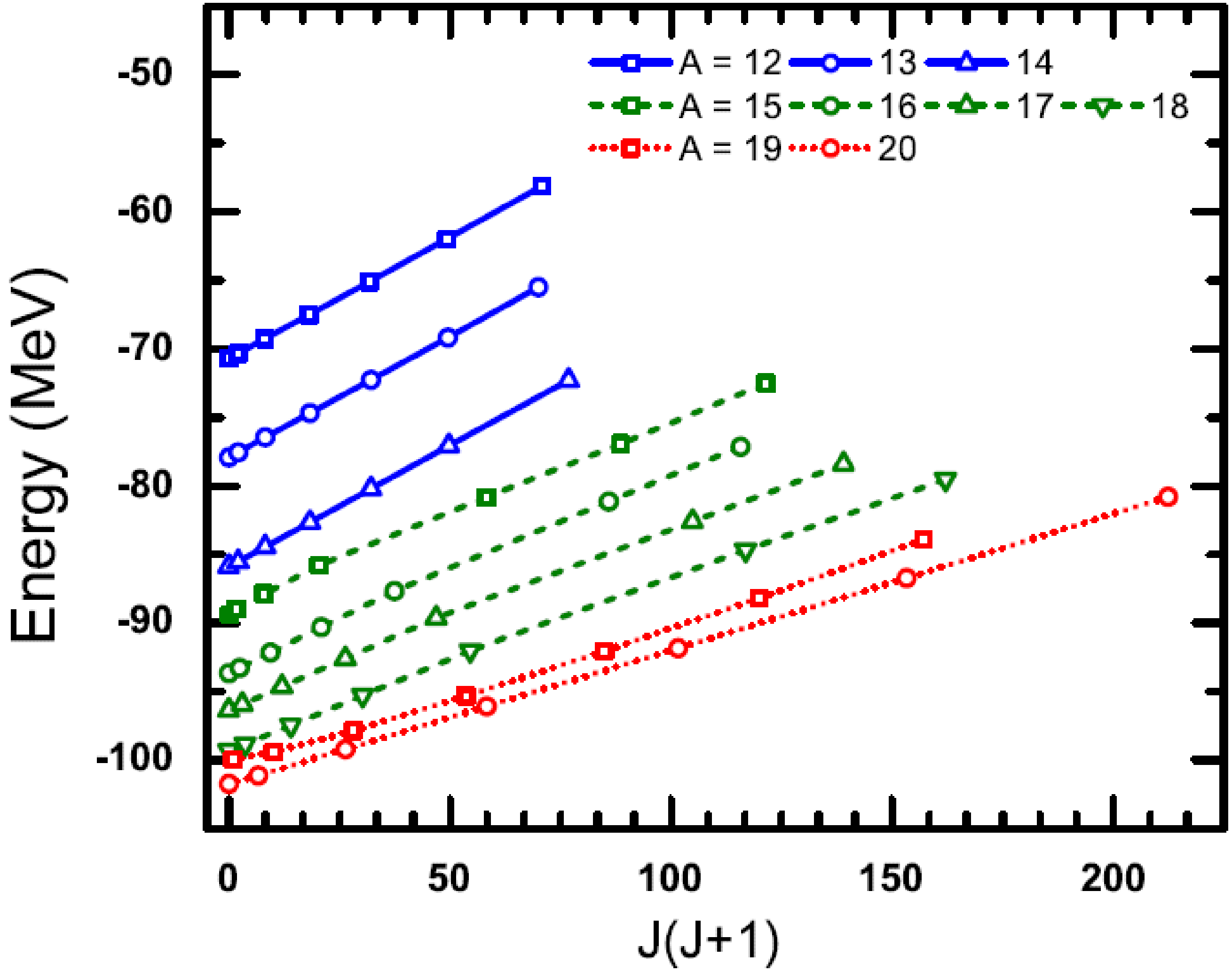


FIG. 1. Energy spectra for linear-alpha-chain bands in C isotopes.

P. W. Zhao, N. Itagaki, and J. Meng,
Phys. Rev. Lett. **115** 022501 (2015).

Cluster model and DFT give consistent results for the exotic cluster states

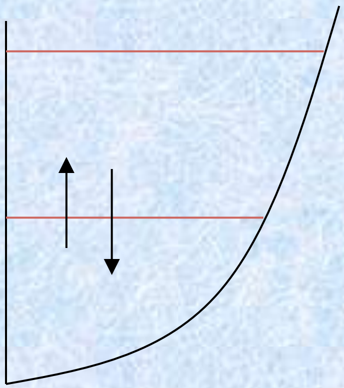
But low-lying states are shell model states

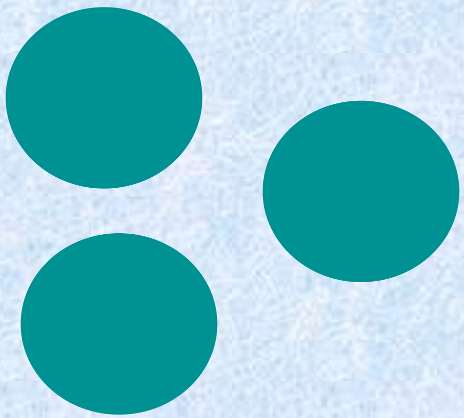
What is the relation between cluster and shell models?

Combining shell and cluster models may be important for the general understanding of the nuclear structure

Shell model side

Cluster side





Big computational challenge

Our strategy

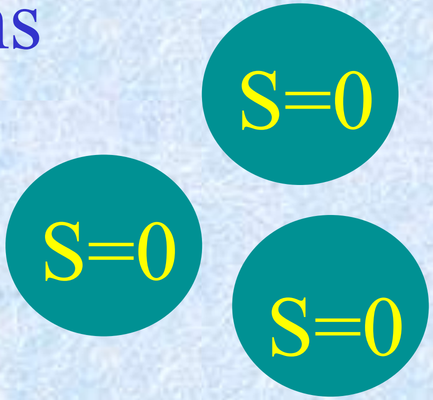
α-cluster model

Each ^{4}He:

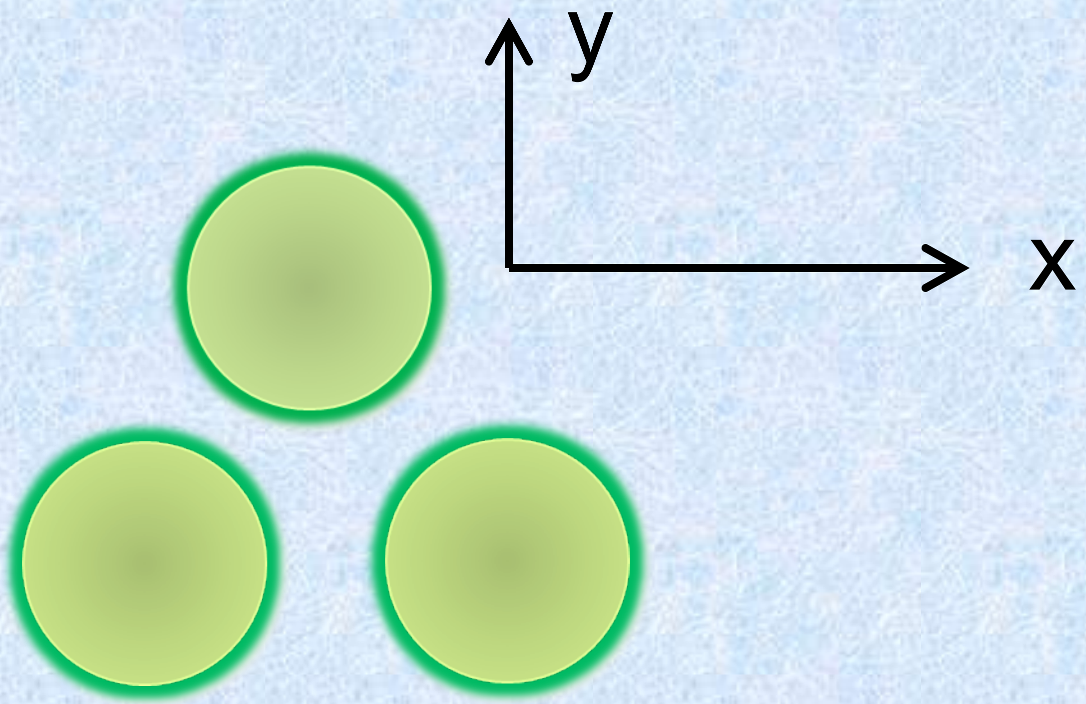
$(0s)^4$ configuration at some localized position

→ spin zero because of the antisymmetrization effect

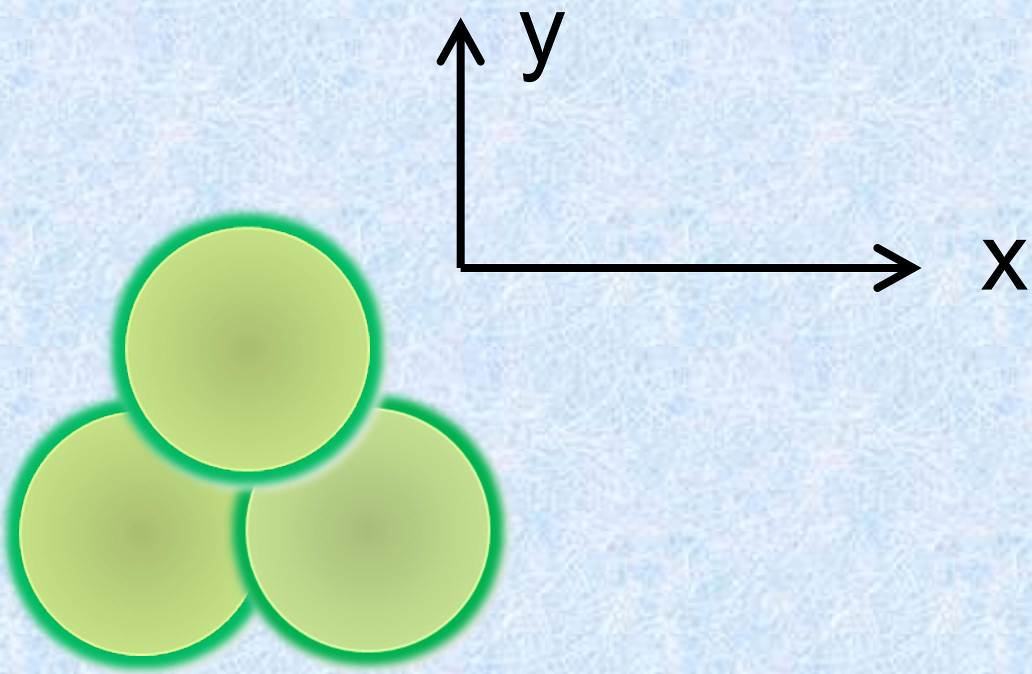
Non-central interactions between nucleons (spin-orbit, tensor) do not contribute



Cluster model partially covers the model space of the shell model

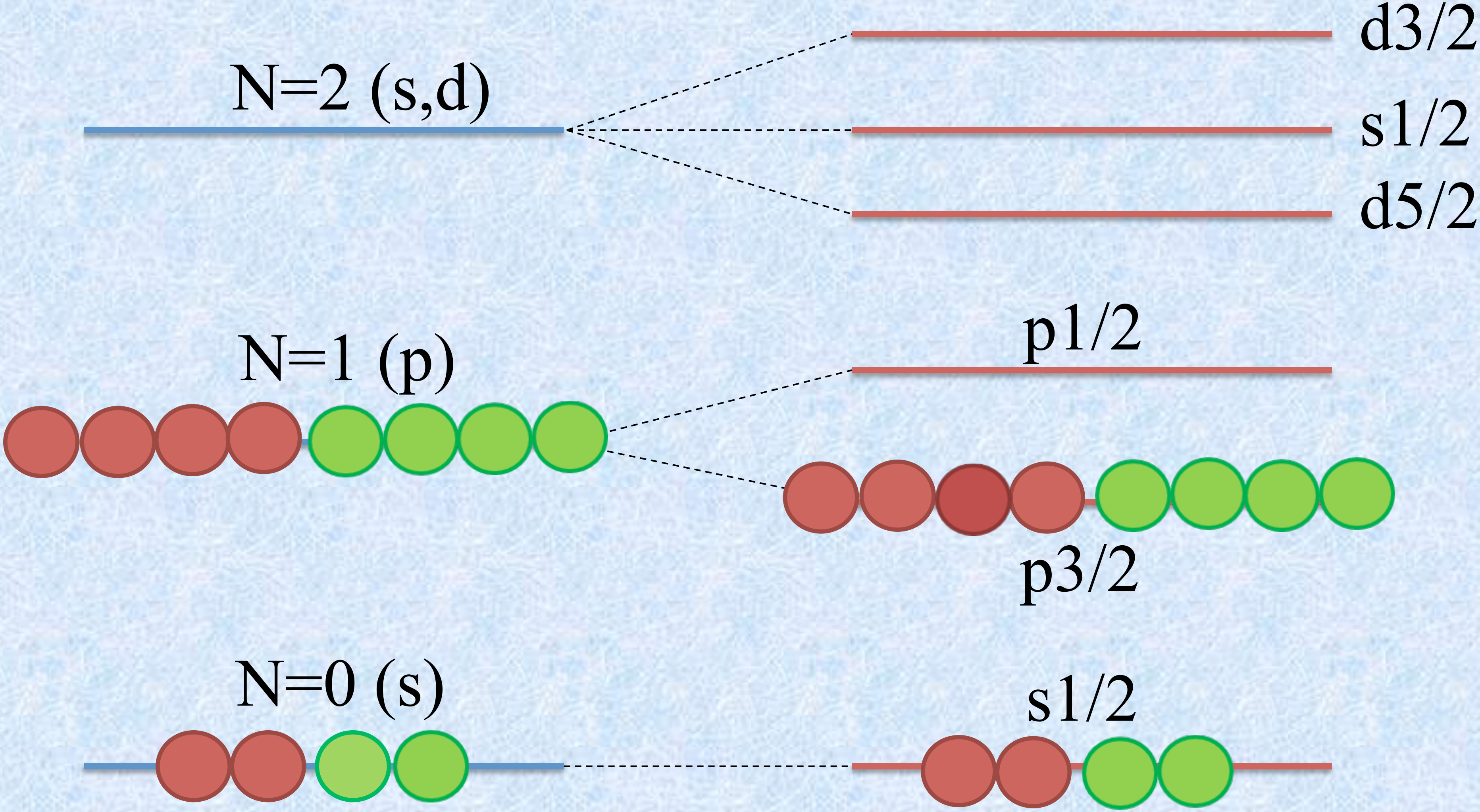


Cluster model partially covers the model space of the shell model



Elliott SU(3) limit

This is $(s)^4(p_x)^4(p_y)^4$, but not $(s_{1/2})^4(p_{3/2})^8$



Three dimensional Harmonic oscillator

jj coupling Shell Model

How we can include the spin-orbit contribution?

spatial part of the single particle wave function

$$\exp[-\nu(\mathbf{r} - \mathbf{Ri})^2]\ s_i t_i$$

In the Brink's model,
4 nucleons share the same Ri value in each α cluster

The spin-orbit interaction: (r x p) • s

r → Gaussian center parameter Ri

p → imaginary part of Ri

(r x p) • s = (s x r) • p

For the nucleons in the quasi cluster:

Ri → Ri + i Λ (e_spin x Ri)   quasi cluster model

N. Itagaki, H. Masui, M. Ito, and S. Aoyama, Phys. Rev. C **71** 064307 (2005).

^{12}C

https://arxiv.org/abs/1609.00466

Ex(2^+_1) : 4.44 MeV

Ex(0^+_1) : -7.27 MeV

Ex(0^+_2) : 0.38 MeV

Experimental values

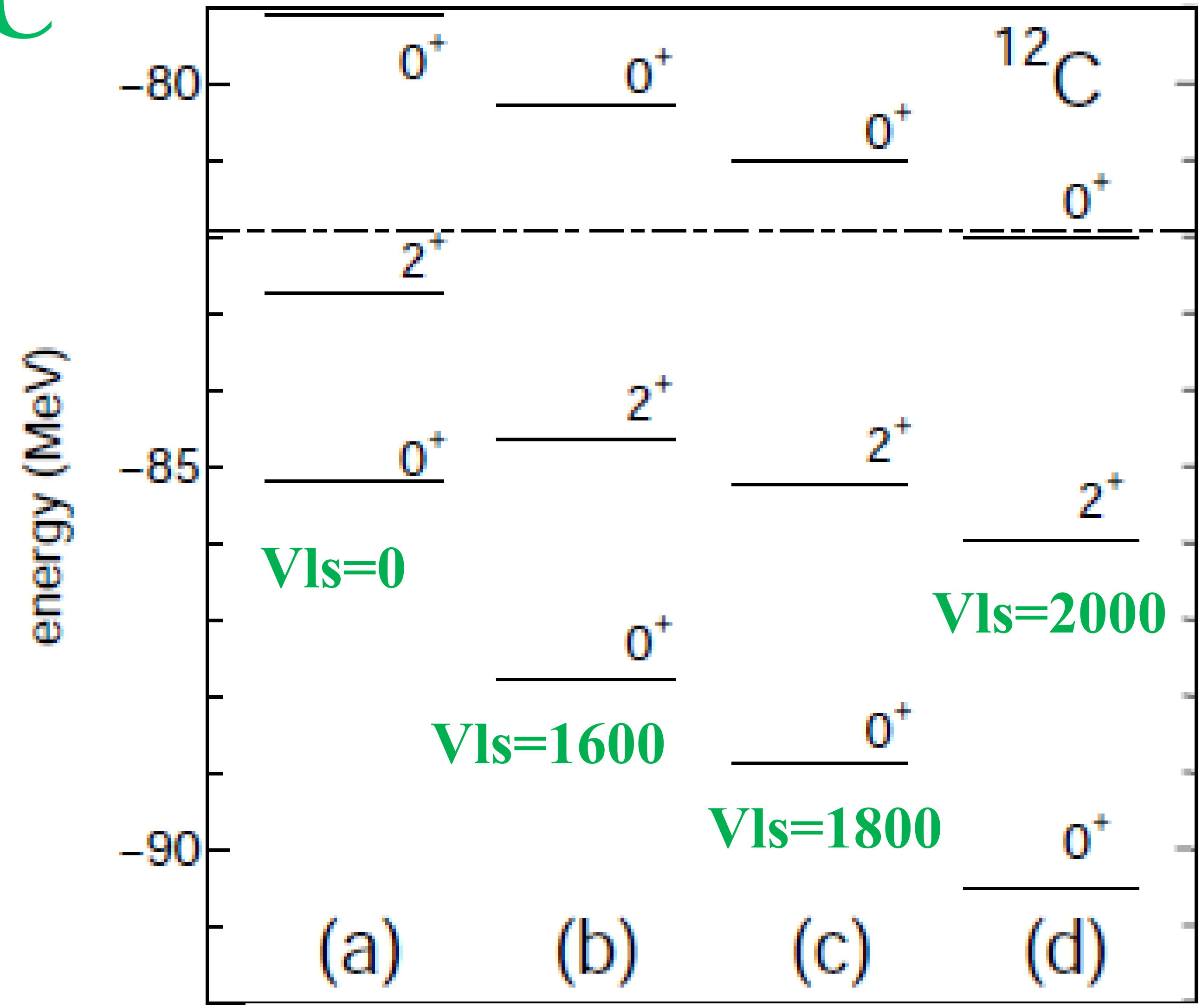
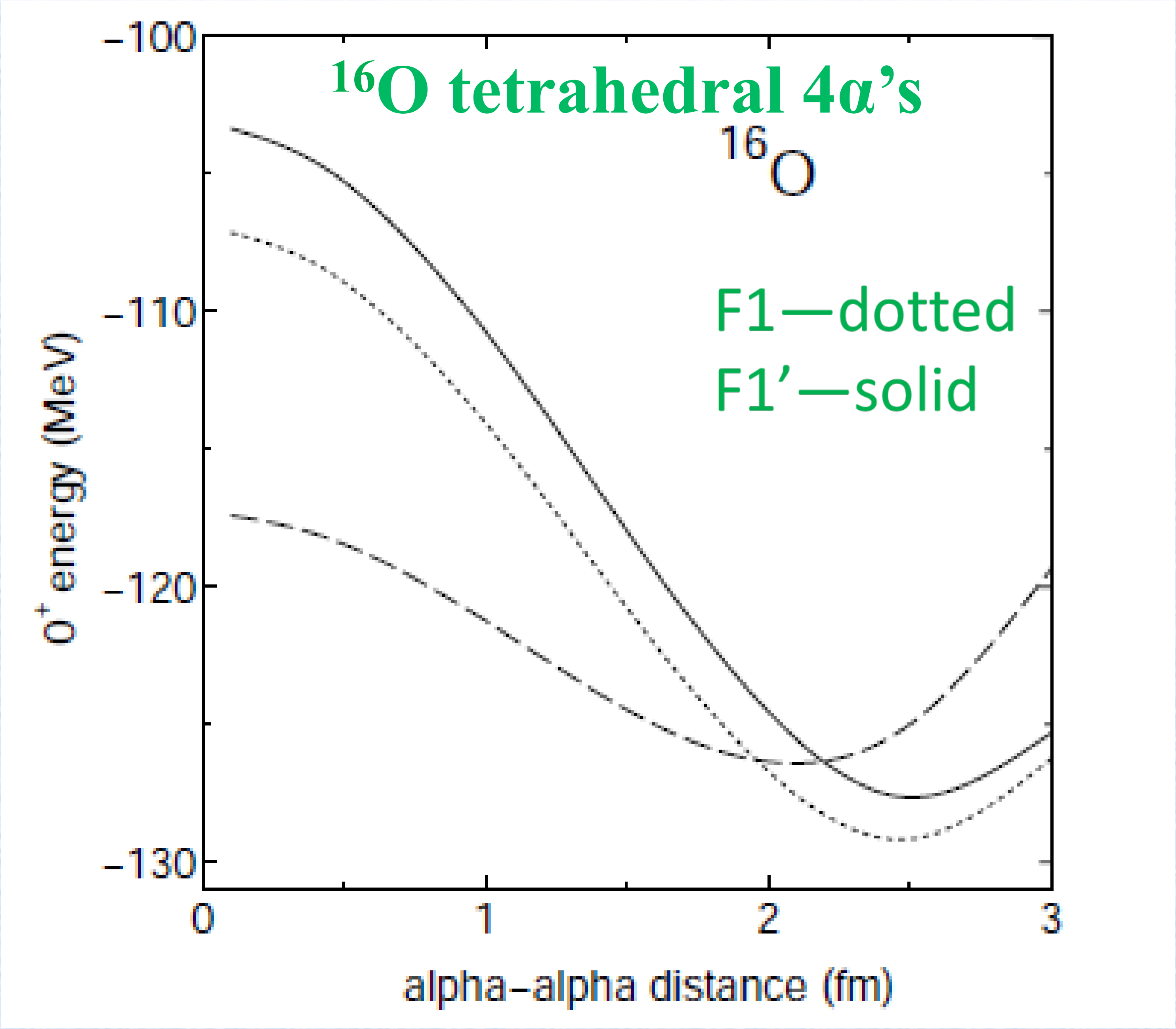


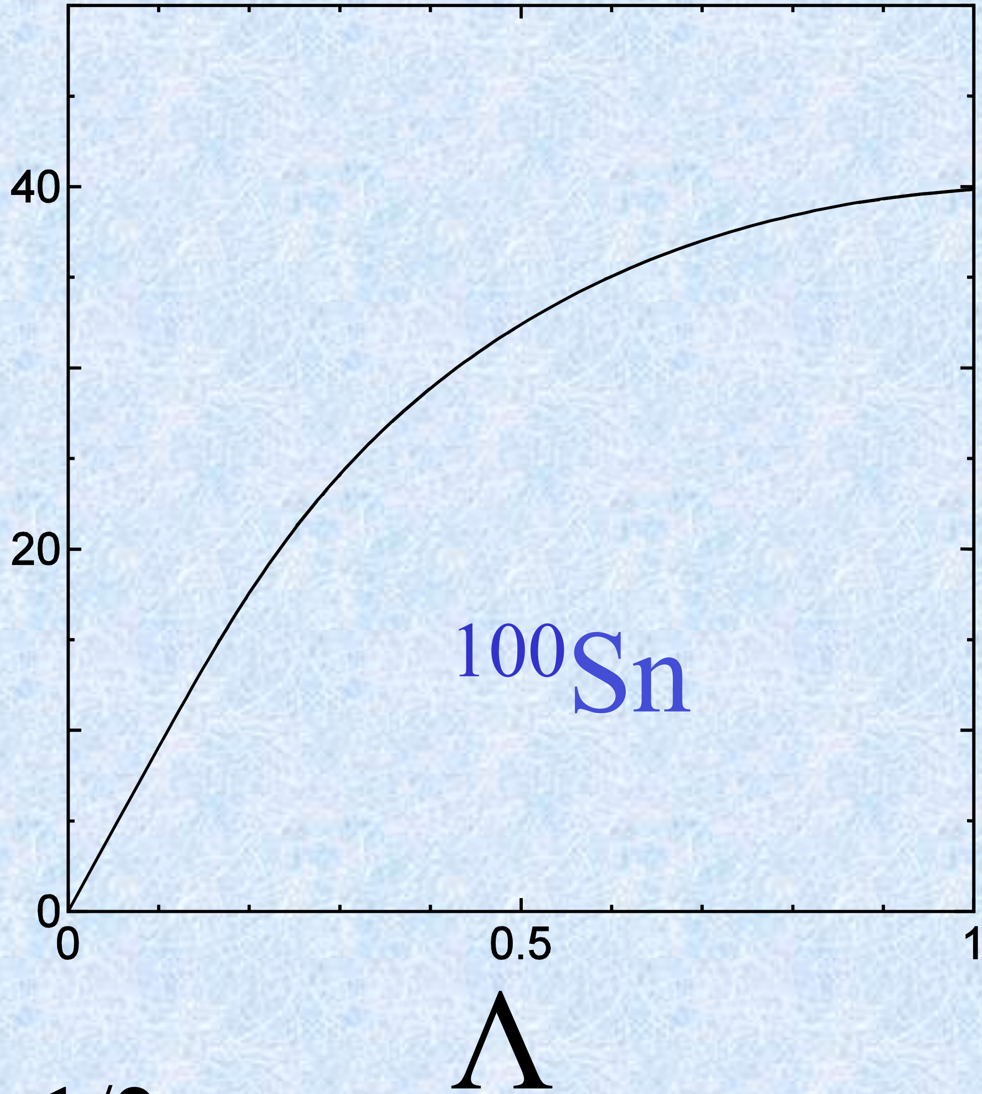
FIG. 6: The energy levels of ^{12}C. Here (a) is the result without the spin-orbit interaction, and (b), (c), and (d) show the results calculated with 1600 MeV, 1800 MeV, and 2000 MeV for the strength of the spin-orbit terms of the G3RS interaction (V_{ls} in Eq. 7), respectively. The dot-dashed line at −81.92 MeV shows three α threshold energy.

https://arxiv.org/abs/1609.00466



at R = 0.01 fm

$$\sum_i \mathbf{l_i} \cdot \mathbf{s_i}$$



for $g_{9/2}$

for j=9/2, l=4, s=1/2,

(j(j+1) - l(l+1) - s(s+1))/2

= (99/4 - 20 - 3/4) / 2 = 2

Summary

The stability of exotic cluster state can be studied with mean field models as well as cluster models, and they give consistent results

Two mechanisms,
rotation (high spin) and adding neutrons (high Isospin)
are important in stabilizing the rod shape,
and they coherently work in C isotopes

It is possible to extend the cluster model framework to include the jj coupling shell model states, this could be useful for the general understanding of nuclear structure