Baryon-Baryon Potential in Lattice QCD

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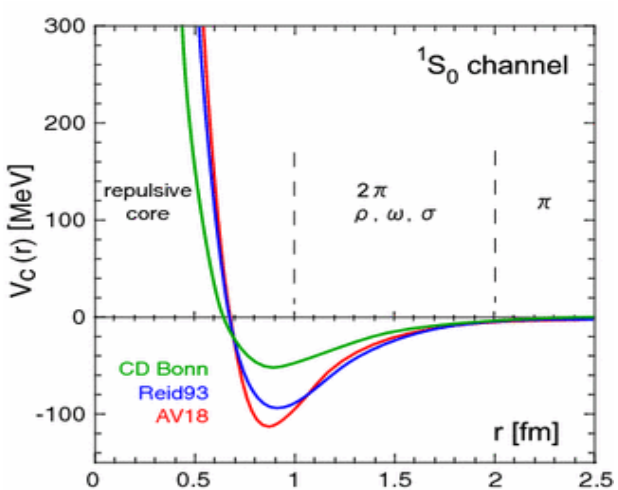
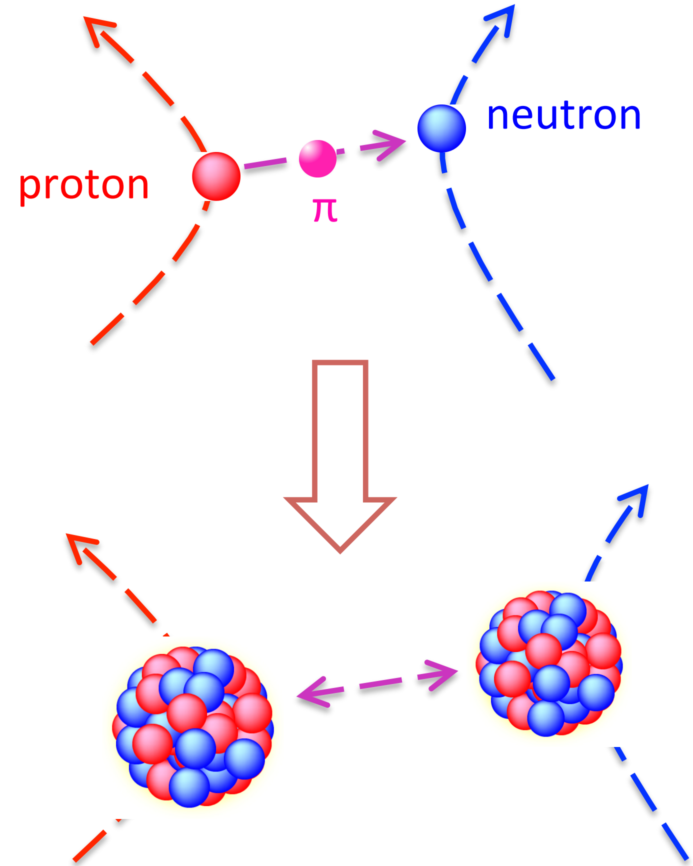
for HAL QCD collaboration

Background

Background

◆ The nuclear force is important for nuclear / astro phys.





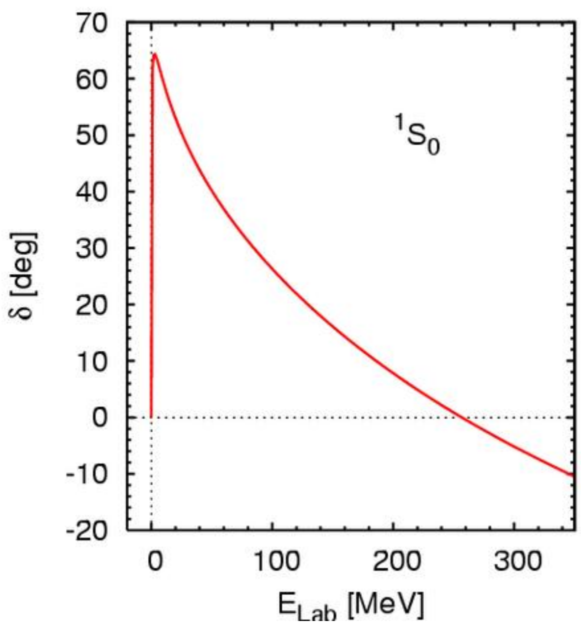
◆ Structures and reactions of atomic nuclei

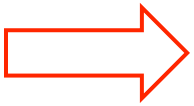
◆ Supernova explosions and neutron stars

Background

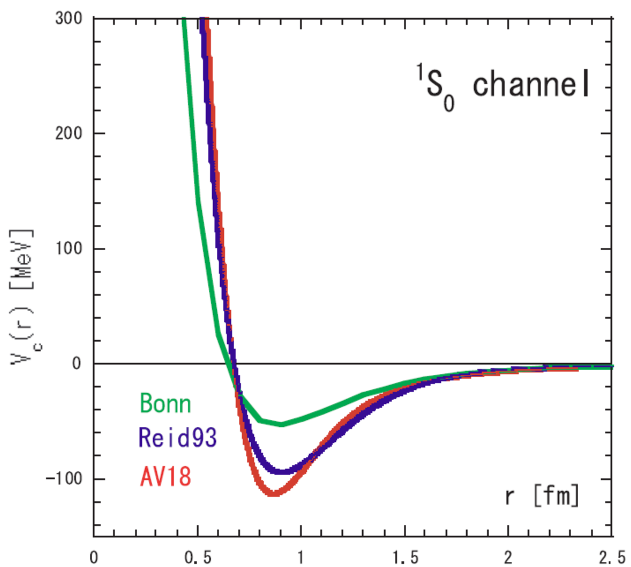
◆ Experimental determination of the nuclear force.

NN scattering data
(～4000 data)





Nuclear Force
(18 fit parameter ➔ χ^2/dof ～1[AV18])

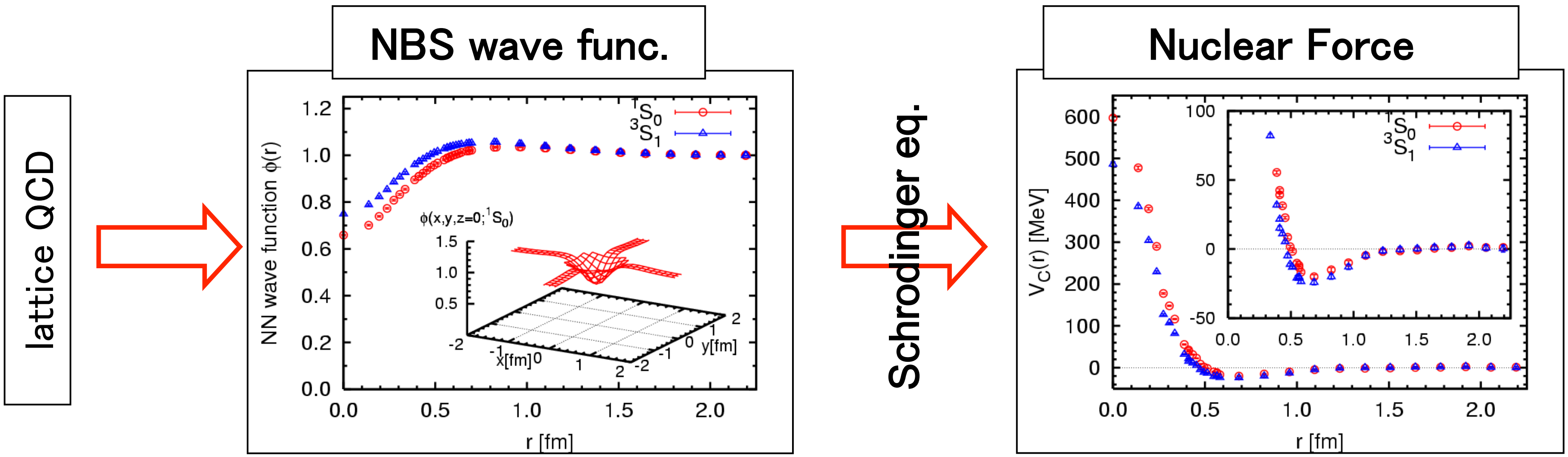


◆ The same method does not work for
- ◆ Hyperon-Hyperon interactions
- ◆ Three nucleon interactions

Background

◆ Lattice QCD method to determine the nuclear force. (HAL QCD method)

[Ishii,Aoki,Hatsuda,PRL99,022001(2007)]



◆ Advantadges

- ◆ It gives the potentials which are faithful to the scattering data.

- ◆ Experimental scattering information is not needed.
  - ➔ It can be applied to
    - ◆ Hyperon-Hyperon interaction
    - ◆ Three nucleon interaction

◆ We have been applied this method to many targets.

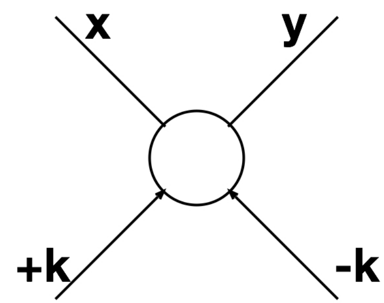
HAL QCD method

HALQCD method

[Aoki,Hatsuda,Ishii,PTP123(2010)89]

◆ Nambu-Bethe-Salpeter (NBS) wave function

$$\left\langle 0 \left| T\left[N(x)N(y) \right] \right| N(+k)N(-k),in \right\rangle$$



◆ Relation to S-matrix by reduction formula

Bosonic notation is to avoid lengthy notations.

$$\begin{aligned}&\left\langle N(p_1)N(p_2),out \middle| N(+k)N(-k),in \right\rangle \\ &= \mathrm{disc} + \left(iZ_N^{-1/2}\right)^2 \int d^4x_1 d^4x_2\, e^{ip_1x_1}\left(\Box_1 + m_N^2\right) e^{ip_2x_2}\left(\Box_2 + m_N^2\right) \left\langle 0 \left| T\left[N(x_1)N(x_2) \right] \right| N(+k)N(-k),in \right\rangle\end{aligned}$$

◆ **Equal-time** restriction of **NBS wave function** behaves at long distance

[C.-J.D.Lin et al., NPB619,467(2001).]

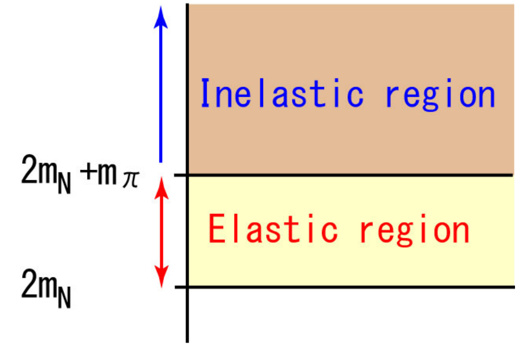
$$\begin{aligned}\psi_k(\vec{x}-\vec{y}) &\equiv \lim_{x_0\to+0} Z_N^{-1} \left\langle 0 \left| T\left[N(\vec{x},x_0)N(\vec{y},0) \right] \right| N(+k)N(-k),in \right\rangle \\ &= Z_N^{-1} \left\langle 0 \left| N(\vec{x},0)N(\vec{y},0) \right| N(+k)N(-k),in \right\rangle \\ &\simeq e^{i\delta(k)} \frac{\sin\left(kr + \delta(k)\right)}{kr} + \cdots \quad \text{as } r \equiv \left|\vec{x}-\vec{y}\right| \to \text{large}\end{aligned}$$

(for S-wave)

Exactly the same functional form
as that of scattering wave functions in quantum mechanics
(equal-time NBS wave function is a good candidate of "NN wave function")

HALQCD method

◆ **Def.** of potential from **equal-time NBS wave functions**:



$$\left(k^2/m_N - H_0\right)\psi_k(\vec{r}) = \int d^3r' U(\vec{r},\vec{r}')\psi_k(\vec{r}')$$

$$\text{for } 2\sqrt{m_N^2 + k^2} < E_{\text{th}} \equiv 2m_N + m_\pi$$

$$H_0 \equiv -\frac{\nabla^2}{m_N}$$

◆ U(r,r') is E-indep.
(Proof of existence of such U(r,r') is given in next slide)

◆ U(r,r') reproduces the scattering phase,
because
(1) U(r,r') reproduces equal-time NBS wave functions.
(2) The equal-time NBS wave functions behave at long distance as

$$\psi_k(\vec{x}-\vec{y}) \simeq e^{i\delta(k)} \frac{\sin\left(kr + \delta(k)\right)}{kr} + \cdots \quad \text{as } r \equiv \left|\vec{x}-\vec{y}\right| \to \text{large}$$

HALQCD method

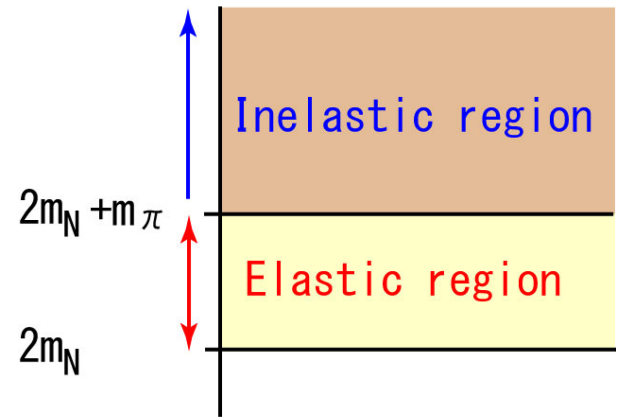
◆ Existence of E-indep. U(r,r')

◆ Assumption:

Linear independence of equal-time NBS wave func. for E < Eth.

➔ There exists dual basis:

$$\int d^3r\, \tilde{\psi}_{\vec{k}'}(\vec{r})\psi_{\vec{k}}(\vec{r}) = (2\pi)^3\delta^3(\vec{k}'-\vec{k})$$



◆ Proof:

$$K_{\vec{k}}(\vec{r}) \equiv \left(k^2/m_N - H_0\right)\psi_{\vec{k}}(\vec{r})$$

$$H_0 \equiv -\frac{\nabla^2}{m_N}$$

$$K_{\vec{k}}(\vec{r}) = \int \frac{d^3k'}{(2\pi)^3} K_{\vec{k}'}(\vec{r}) \int d^3r'\, \tilde{\psi}_{\vec{k}'}(\vec{r})\psi_{\vec{k}}(\vec{r})$$

$$= \int d^3r' \left\{ \int \frac{d^3k}{(2\pi)^3} K_{\vec{k}'}(\vec{r})\tilde{\psi}_{\vec{k}'}(\vec{r}') \right\} \psi_{\vec{k}}(\vec{r}')$$

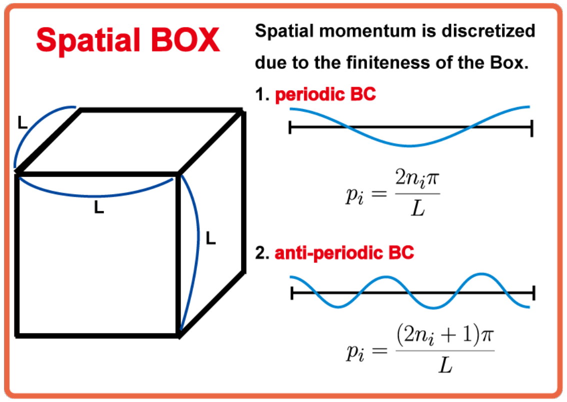
$$\left(k^2/m_N - H_0\right)\psi_{\vec{k}}(\vec{r}) = \int d^3r'\, U(\vec{r},\vec{r}')\psi_{\vec{k}}(\vec{r}')$$

$$U(\vec{r},\vec{r}') \equiv \int \frac{d^3k'}{(2\pi)^3} K_{\vec{k}'}(\vec{r})\tilde{\psi}_{\vec{k}'}(\vec{r})$$

U(r,r') does not depend on E because of the intergration of k'.

HALQCD method

- NBS wave function is obtained from nucleon 4-point correlator
  (Example: NBS wave func. for ground state)



$$C_{NN}(\vec{x}-\vec{y},t) \equiv \left\langle 0 \middle| N(\vec{x},t)N(\vec{y},t)\cdot \overline{NN}(t=0) \middle| 0 \right\rangle$$

$$= \sum_n \left\langle 0 \middle| N(\vec{x})N(\vec{y}) \middle| n \right\rangle \cdot e^{-E_n t} A_n$$

$$\to \psi_{\mathrm{G.S.}}(\vec{x}-\vec{y}) e^{-E_{\mathrm{G.S}}t} A_{\mathrm{G.S.}} \quad \text{for} \quad t \to \text{large}$$

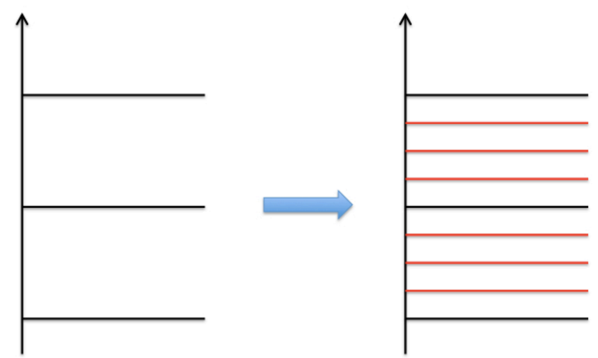
$$A_n \equiv \left\langle n \middle| \overline{NN}(t=0) \middle| 0 \right\rangle$$

- Ground state saturation
  becomes problematic at large volume.

  - For L → large, energy gap shrinks as

$$\Delta E = E_{n+1} - E_n \sim \frac{1}{m_N}\left(\frac{2\pi}{L}\right)^2$$

  - We have to be very careful against
    this problem when considering atomic nuclei.



If L becomes twice as large,
ΔE becomes 4 times as small.

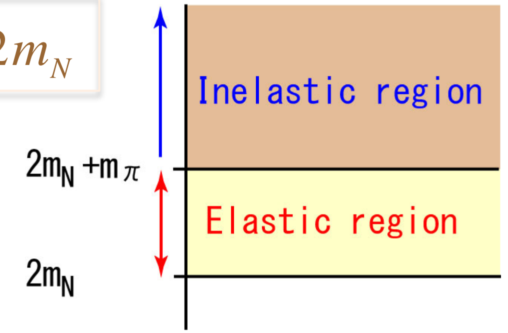
	L=3 fm	L=6 fm	L=9 fm	L=12 fm
Δ E	181.5 MeV	45.3 MeV	20.2 MeV	11.3 MeV

HALQCD method

[Ishii et al.,PLB712(2012)437]

Extraction of potential: Ground state saturation is not needed.

$$\Delta W(k) \equiv 2\sqrt{m_N^2 + k^2} - 2m_N$$



◆ Normalized NN correlator

$$R(t,\vec{x}-\vec{y}) \equiv e^{2m_N \cdot t}\left\langle 0 \left| T\left[N(\vec{x},t)N(\vec{y},t)\cdot \overline{NN}(t=0)\right]\right| 0 \right\rangle$$

$$= \sum_k a_k \exp\left(-t\Delta W(k)\right)\cdot \psi_k(\vec{x}-\vec{y})$$

Assumption:

"t" is large enough so that **elastic contributions** can **dominate** intermediate states.

◆ **"Time-dependent" Schrodinger-like equation**

to extract our potential.

$$\left(\frac{1}{4m_N}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t}\right) R(t,\vec{r}) = \sum_k a_k \frac{k^2}{m_N}\exp\left(-t\Delta W(k)\right)\cdot \psi_k(\vec{r})$$

$$(H_0 + U)\psi_k(\vec{r}) = \frac{k^2}{m_N}\psi_k(\vec{r})$$

$$\left(\frac{1}{4m_N}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0\right) R(t,\vec{r}) = \int d^3r' U(\vec{r},\vec{r}\,')\cdot R(t,\vec{r}\,')$$

An identity

$$\frac{\Delta W(k)^2}{4m_N} + \Delta W(k) = \frac{k^2}{m_N}$$

Only **Elastic saturation** is required to derive this equation.

(**Elastic saturation** is much easier than **single state saturation**.)

HALQCD method

(Example) **Resultant potential does not depend on excited state contamination.**

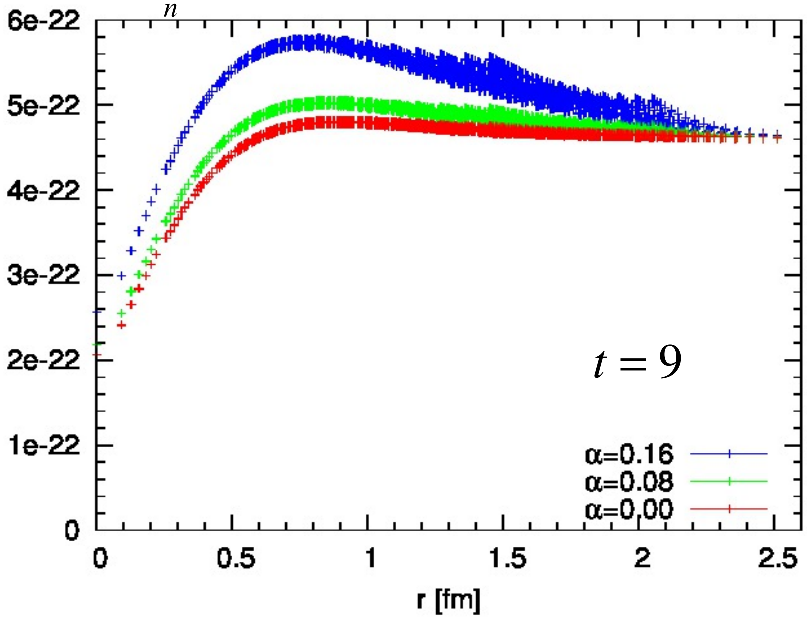
◆ Source function (with a single real parameter **alpha**)

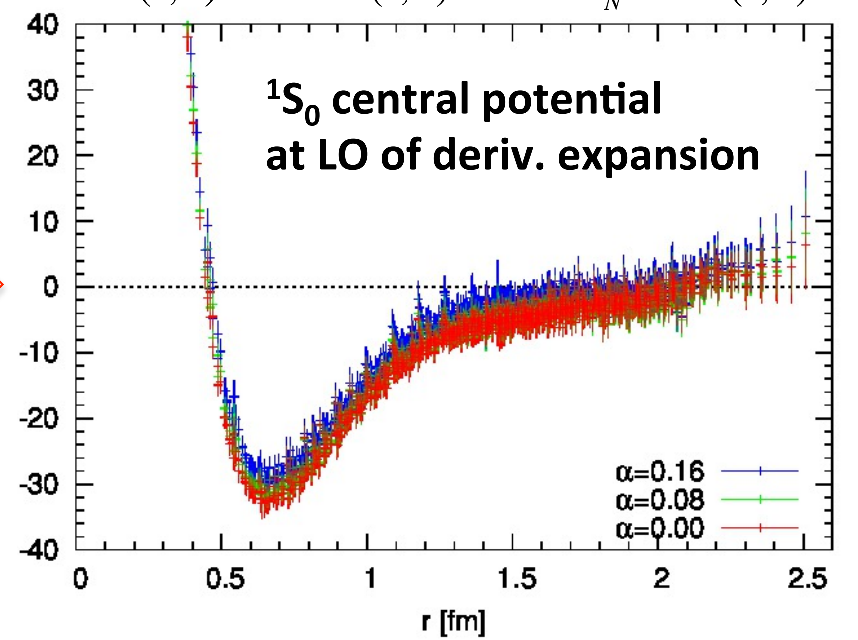
$$f(x,y,z)=1+\alpha\left(\cos(2\pi x/L)+\cos(2\pi y/L)+\cos(2\pi z/L)\right)$$

alpha is used to arrange possible mixture of excited states

$$\langle 0|T[N(\vec{x},t)N(\vec{y},t)\cdot\overline{NN}(t=0;\alpha)]|0\rangle = \sum_n \psi_n(\vec{x}-\vec{y})\cdot a_n(\alpha)\cdot\exp(-E_n t)$$

$$V_C(\vec{x}) = -\frac{H_0 R(t,\vec{x})}{R(t,\vec{x})} - \frac{(\partial/\partial t)R(t,\vec{x})}{R(t,\vec{x})} + \frac{1}{4m_N}\frac{(\partial/\partial t)^2 R(t,\vec{x})}{R(t,\vec{x})}$$





HAL QCD method

[Murano,Ishii,Aoki,Hatsuda, PTP125(2011)1225]

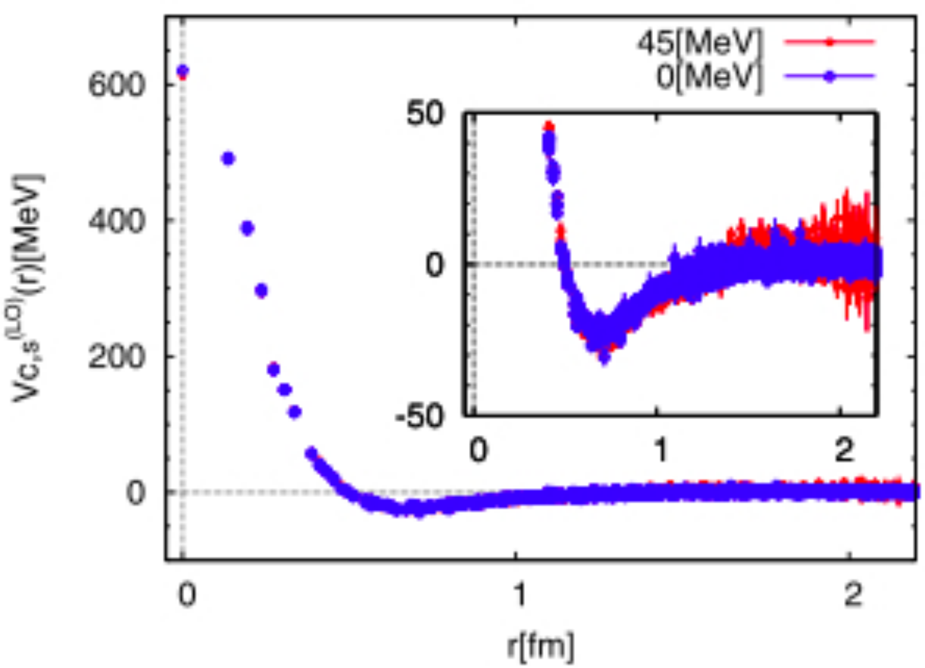
- General nonlocal potential is intractable.
  - ➔ We employ **derivative expansion**:

$$U(\vec{r},\vec{r}') \equiv V(\vec{r},\vec{\nabla})\delta(\vec{r}-\vec{r}')$$

$$V(\vec{r},\vec{\nabla}) \equiv V_C(r) + \underbrace{V_{ll}(r)\vec{L}^2 + \{V_{pp}(r),\nabla^2\}}_{O(\nabla^2)\text{ term}} + O(\nabla^4)$$

Convergence of Derivative expansion has to be checked.

(Example) $V(\vec{r},\vec{\nabla}) \equiv V_C(r) + O(\nabla^2)$ case:



We define $V_C(\vec{r};E) \equiv E - \dfrac{H_0\psi_E(\vec{r})}{\psi_E(\vec{r})}$

⟷ $\left(k^2/m_N - H_0\right)\psi_E(\vec{r}) = V_C(r;E)\psi_E(\vec{r})$

If $V_C(r;E)$ is E-indep. for $E_0 < E < E_1$, then

$$V(\vec{r},\vec{\nabla}) \equiv V_C(r) + \cancel{O(\nabla^2)}$$

with $V_C(\vec{r}) \equiv V_C(\vec{r};E)$

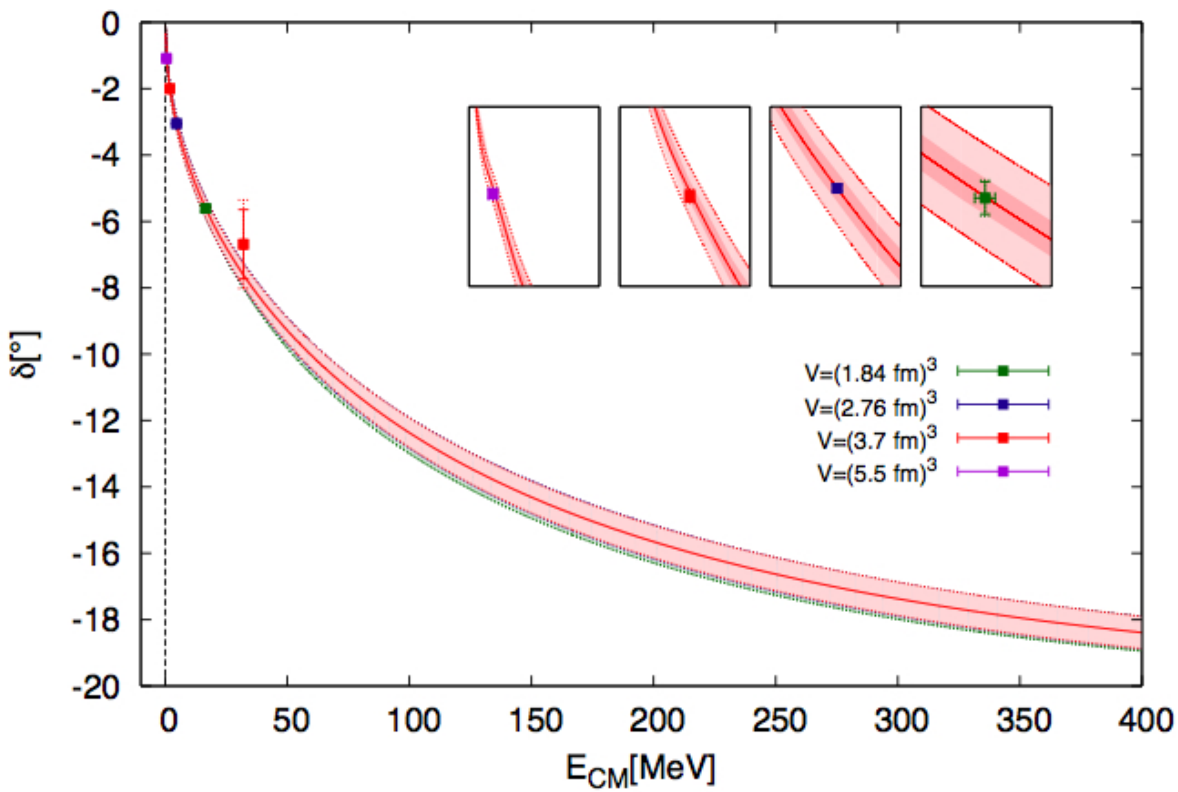
➔ O(nabla^2) terms are negligible.

Comment: The current result is obtained based on an older method.
The result should be replaced by the new method. "time-dependent" Schrodinger-like eq.

HAL QCD method

◆ Comparison of **the potential method** and **Lueschcer's finite volume method.**

ππ scattering in I = 2 channel



Ns=16,24,32,48, Nt=128, a=0.115

m_{pi} = 940 MeV by Quenched QCD

Good agreement !

[Kurth et al., arXiv:1305.4462[hep-lat]]

Nuclear Forces

Nuclear Forces

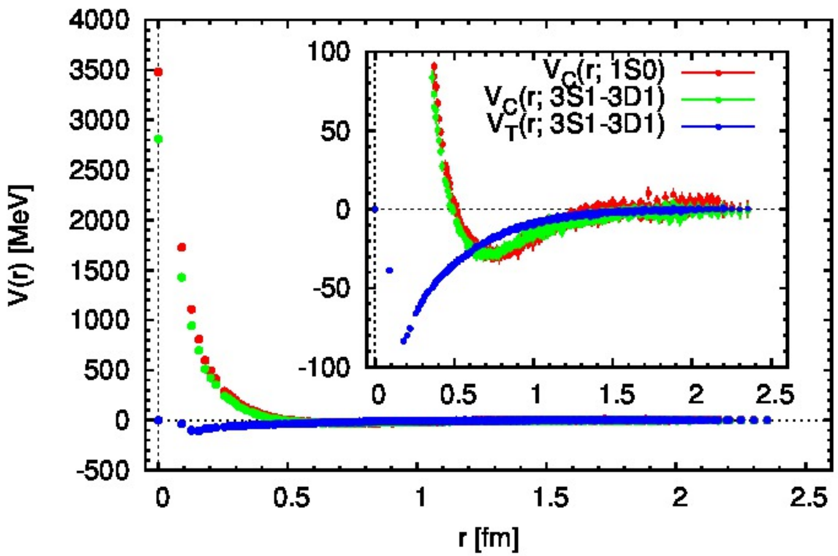
Nuclear Force at LO:

$$S_{12} \equiv 3(\hat{r}\cdot\vec{\sigma}_1)(\hat{r}\cdot\vec{\sigma}_2) - \vec{\sigma}_1\cdot\vec{\sigma}_2$$

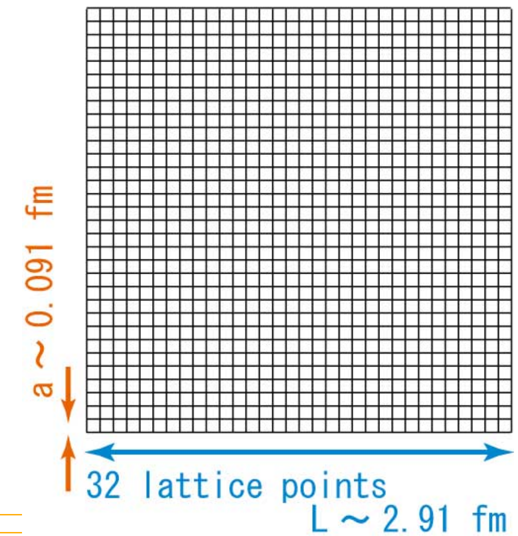
$$V_{NN} = V_{C;S=0}(r)\mathbb{P}^{(S=0)} + V_{C;S=1}(r)\mathbb{P}^{(S=1)} + V_T(r)S_{12} + O(\nabla)$$

ILDG Japan Lattice Data Grid

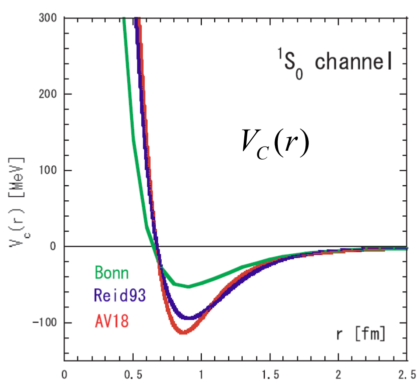
2+1 flavor QCD result of nuclear forces at LO for m(pion)=570 MeV.



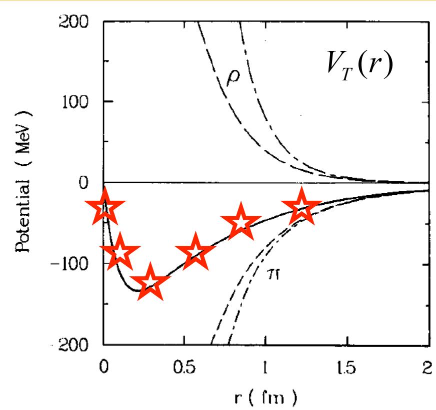
2+1 flavor config by PACS-CS Coll.
m(pion) = 570 MeV, m(N)=1412MeV



central force



tensor force



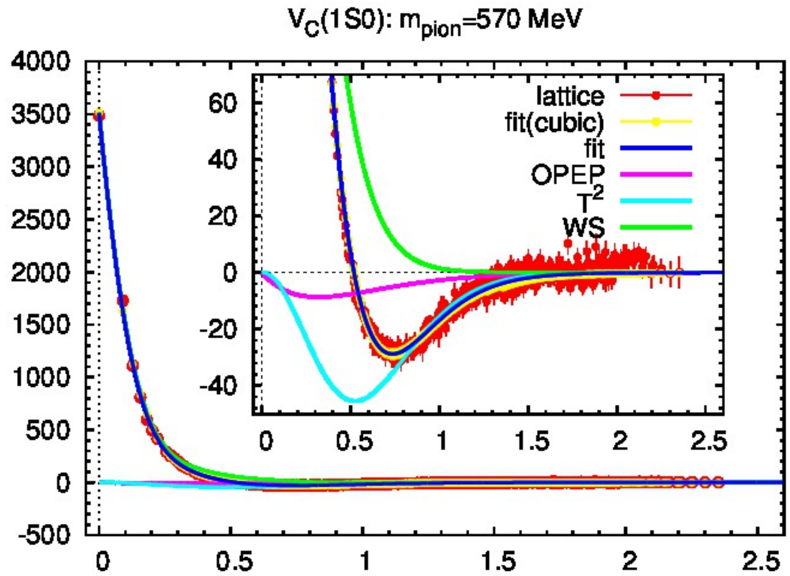
from R.Machleidt, Adv.Nucl.Phys.19

Fig. 3.7. The contributions from π and ρ (dashed) to the $T = 0$ tensor potential. The solid line is the full potential. The dash-dot lines are obtained when the cutoff is omitted.

Nuclear Forces

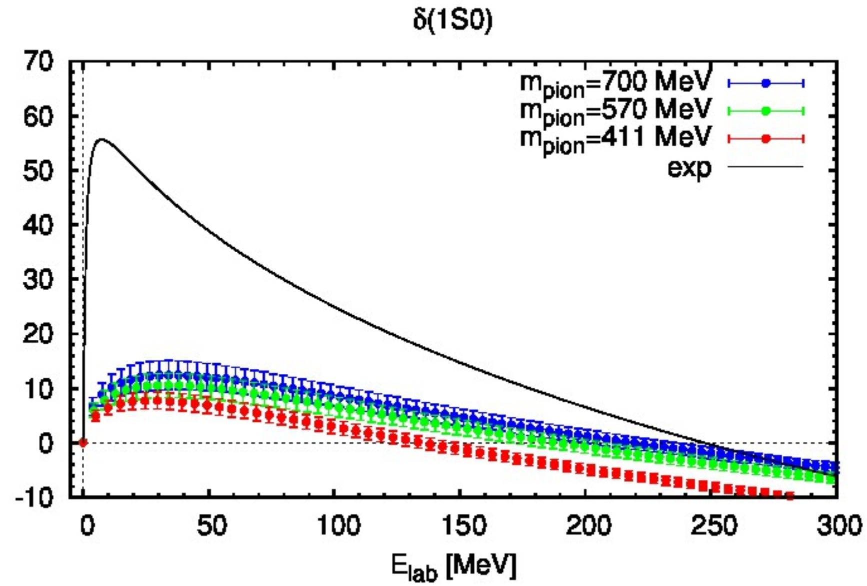
[Ishii,PoS(CD12)(2013)025]

Fit

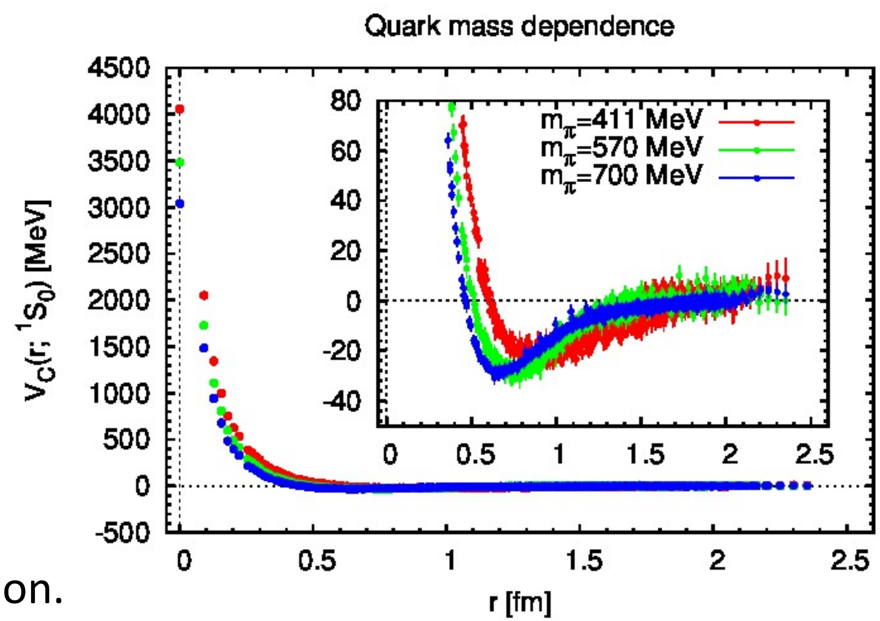


Schroedinger eq.

$^1S_0$ phase shift from Schrodinger eq.



- ❖ Qualitatively reasonable behavior. But the strength is significantly weak. (Attractive. No bound state.)
- ❖ Attraction shrinks as m_{pion} decreases.
  Reason:
  The **repulsive core** grows more rapidly than the **attractive pocket** in the region m_{pion} = 411-700 MeV.
- ❖ It is important to go to smaller quark mass region.

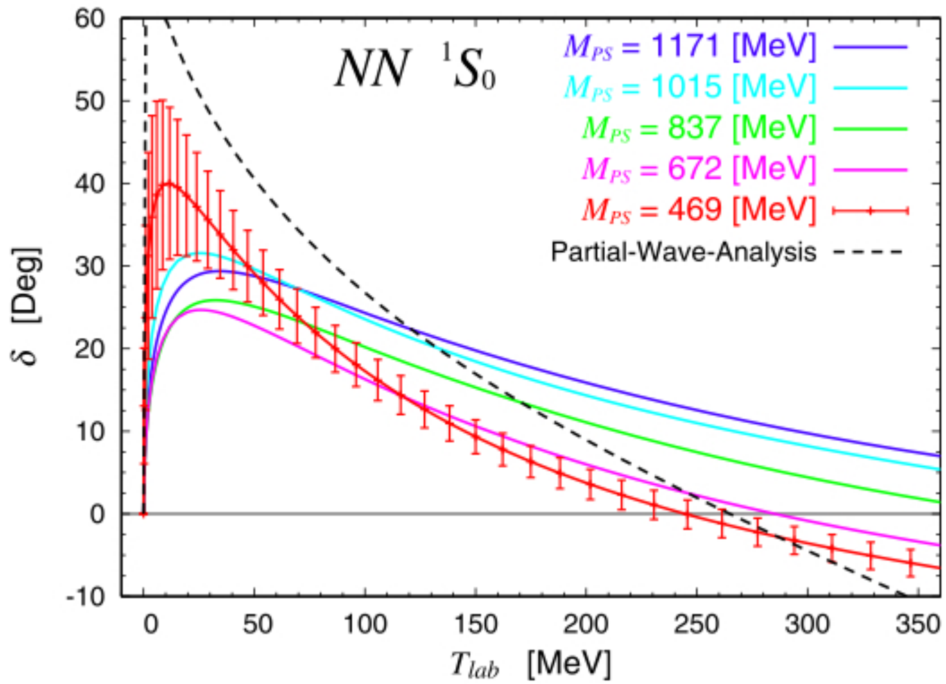


Nuclear Forces

[Inoue et al.,NPA881(2012)28]

Similar behavior is seen in NF=3 calculation (flavor SU(3) limit)

- m_{PS}=672-1171 MeV: attraction shrinks as decreasing quark mass.
- m_{PS}=**469**-672 MeV: **turning point**: attraction starts to increase.
- m_{PS}=0 -**469** MeV: attraction increase (← Our expectation !)



- For the similar thing to happen for NF=2+1,pion mass has to be smaller. Nuclear force for NF=3 is generally more attractive than NF=2+1.

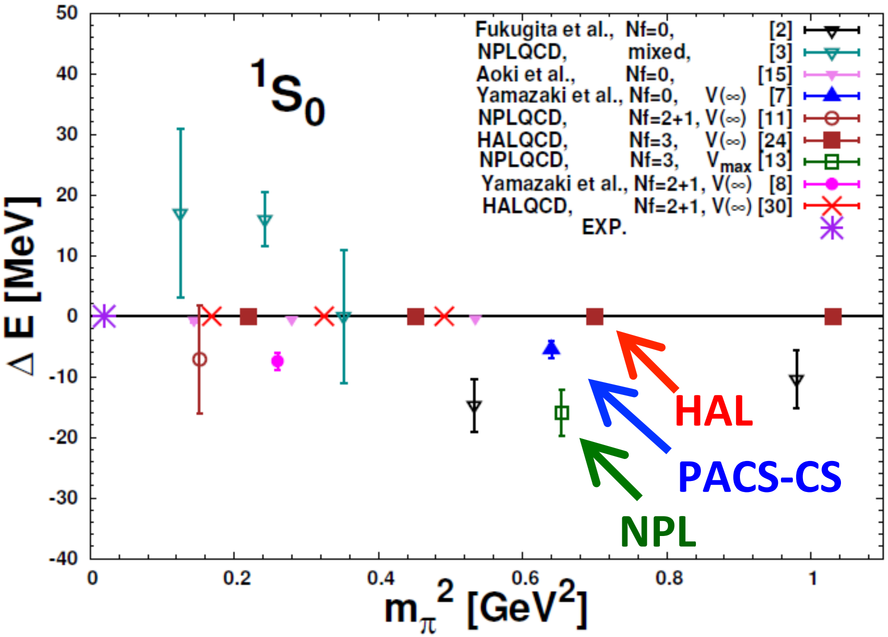
$$\#(\text{Goldstone mode}) = \begin{cases} 3 & (N_F = 2+1) \\ 8 & (N_F = 3) \end{cases}$$

Nuclear Forces

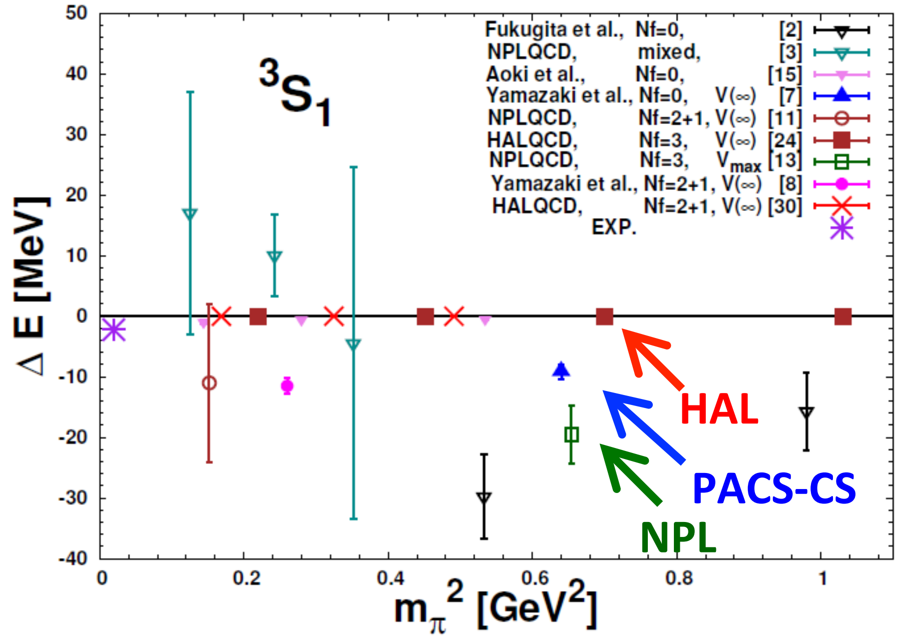
[T.Doi, PoS LAT2012,009]

Comparison with other collaborations (two-nucleon ΔE)

“di-neutron”



“deuteron”



Comments:

YN/YY are also inconsistent between HAL and NPL

HAL: B.E.(H) = 37.8(3.1)(4.2) MeV
NPL: B.E.(H) = 74.6(3.3)(3.3)(0.8) MeV

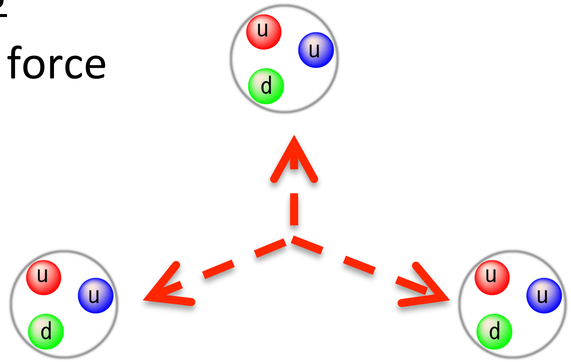
On-going study

Employ the same PACS-CS confs

Analyze both HAL & Luscher

Nuclear Forces

◆ Three-nucleon force



◆ Few body calculations shows its relevance

--- To understand qualitative trend,

two-nucleon force is enough.

Tor quantitative argument,

three-nucleon force is needed.

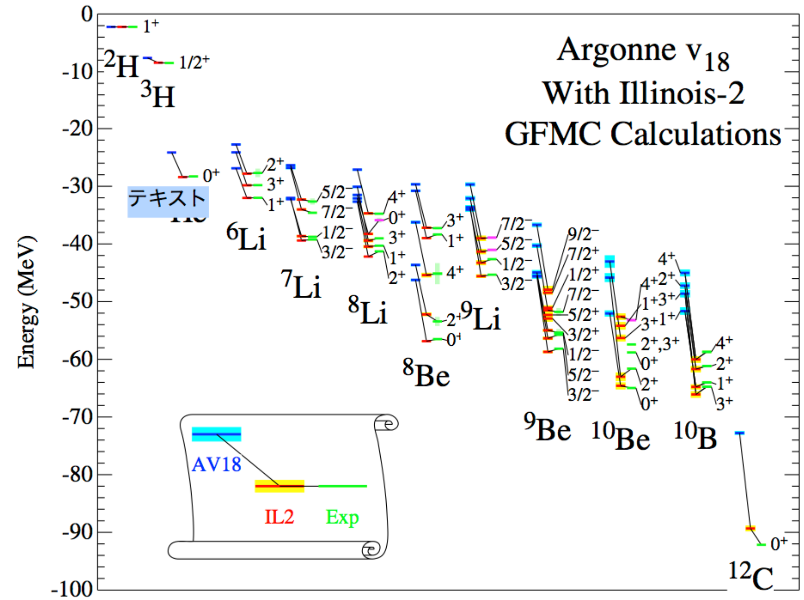


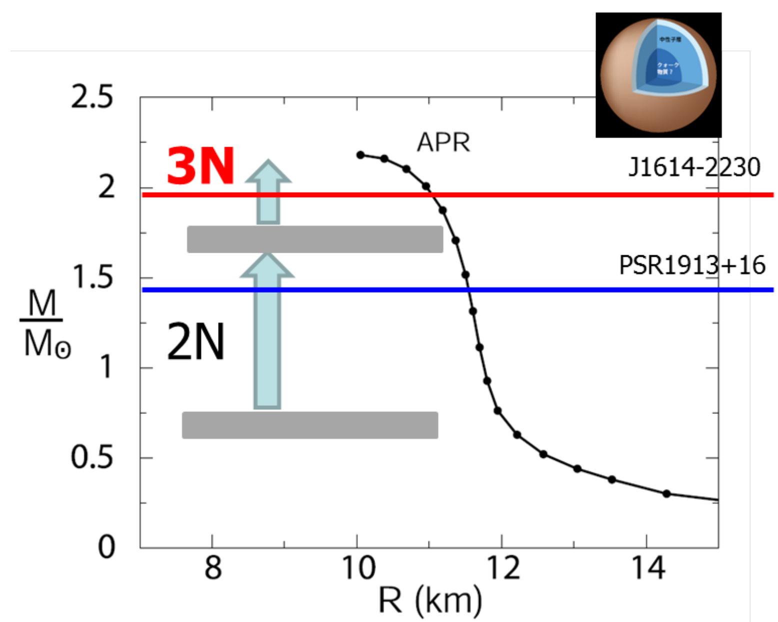
Fig. 3. – GFMC computations of energies for the AV18 and AV18+IL2 Hamiltonians compared with experiment.

◆ Important influence on neutron-rich nuclei.

---the magic number and the drip line.

◆ Important at higher density.

➔ supernova explosion and neutron star.

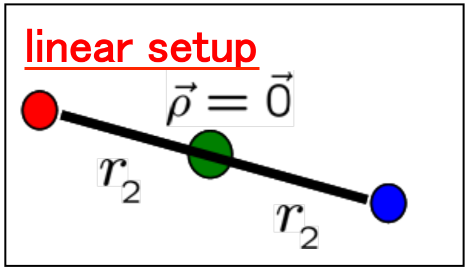


◆ Experimental information is limited

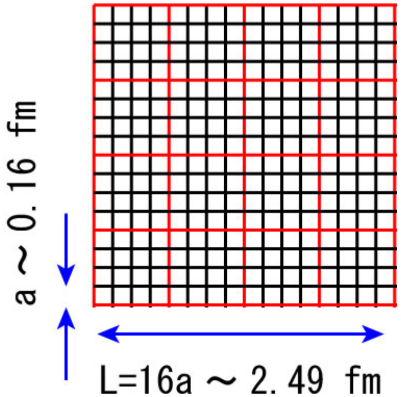
Nuclear Forces

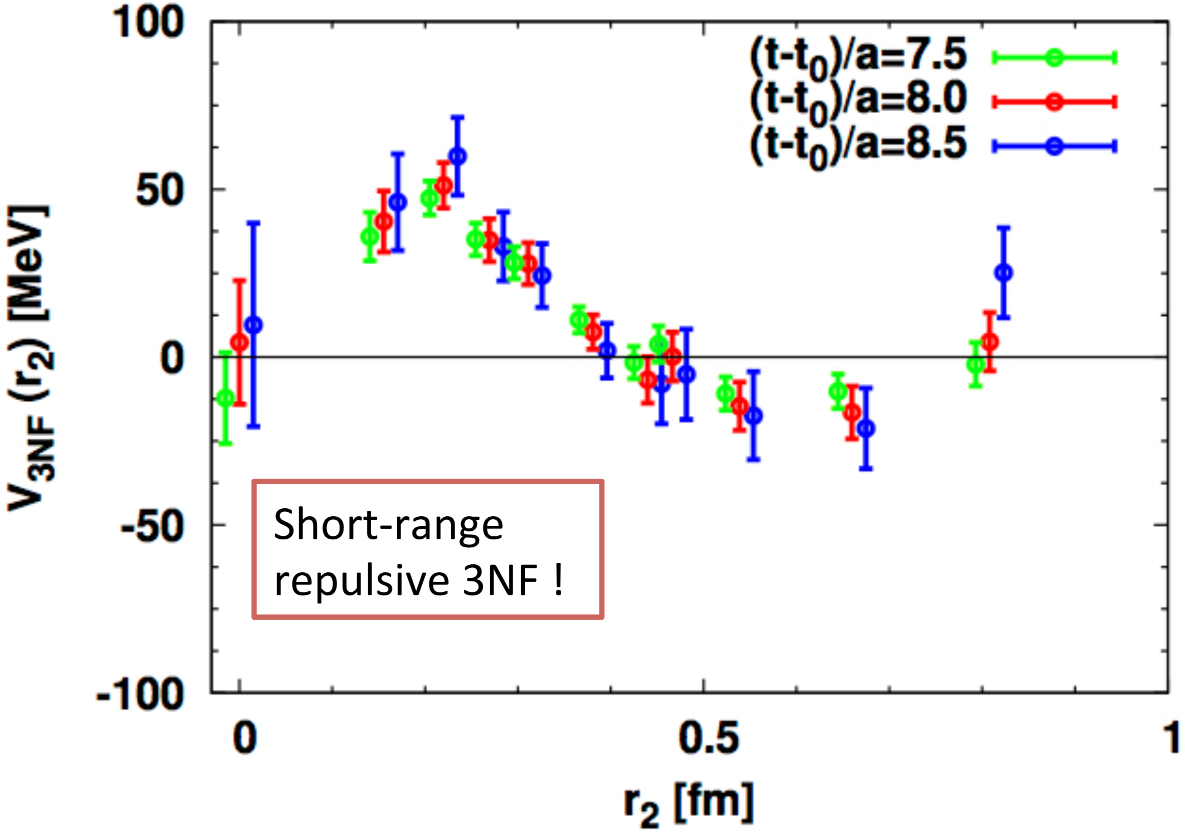
[T.Doi et al, PTP127,723(2012)]

Three-nucleon potential (on the linear setup)



2 flavor gauge config by CP-PACS Coll.
m(pion) = 1136 MeV, m(N) = 2165 MeV





Nuclear Forces

- Nuclear Force up to NLO

$$V^{(\pm)}(\vec{r},\vec{\nabla}) = \underbrace{V_{\mathrm{C;S=0}}^{(\pm)}(r)\mathbb{P}^{(S=0)} + V_{\mathrm{C;S=1}}^{(\pm)}(r)\mathbb{P}^{(S=1)} + V_{\mathrm{T}}^{(\pm)}(r)S_{12}(\hat{r})}_{\mathrm{LO}:O(\nabla^0)} + \underbrace{V_{\mathrm{LS}}^{(\pm)}(r)\vec{L}\cdot(\vec{s}_1+\vec{s}_2)}_{\mathrm{NLO}:O(\nabla^1)} + O(\nabla^2)$$

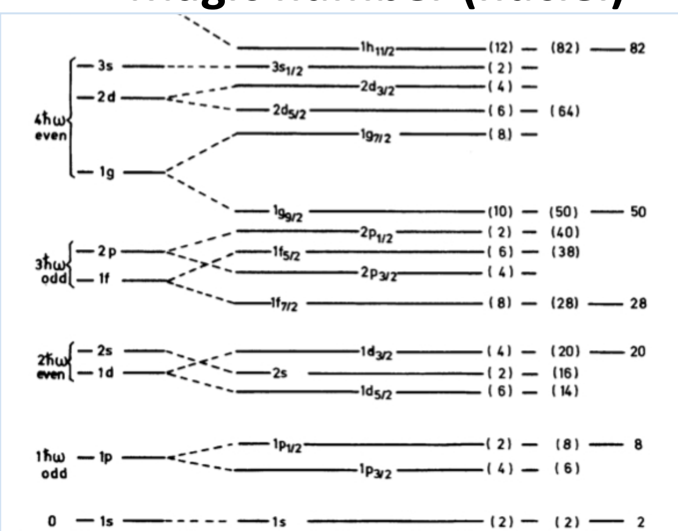
With **wall source**, we can access

	$O(\nabla^0)$	$O(\nabla^1)$	$O(\nabla^2)$	⋯
Parity-even	○	×	×	×
Parity-odd	×	×	×	×

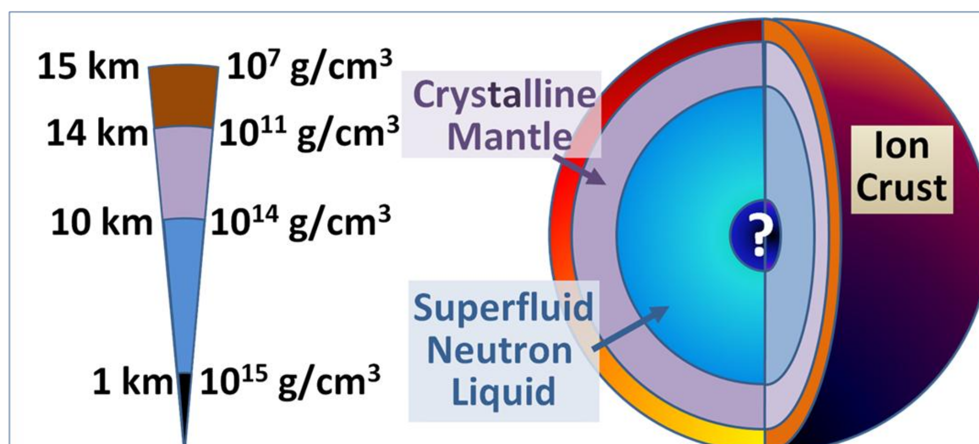
Momentum wall sources allows us to access to higher orders

- **Spin orbit (LS) force** is important in phenomenology.

Magic number (nuclei)



$^3P_2$ neutron superfuluid (neutron star cooling)



Momentum wall source

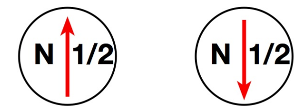
[K.Murano et al, arXiv.1305.2293]

◆ Wall source:

$$\overline{\mathcal{J}}_{\alpha\beta} \equiv \sum_{\vec{x}_1,\cdots,\vec{x}_6} \bar{N}_\alpha(\vec{x}_1,\vec{x}_2,\vec{x}_3)\bar{N}_\beta(\vec{x}_4,\vec{x}_5,\vec{x}_6)$$

$$N_\alpha(x_1,x_2,x_3) \equiv \begin{cases} q_{abc}\big(u_a(x_1)C\gamma_5 d_b(x_2)\big)u_{c;\alpha}(x_3) & \text{(proton)} \\ q_{abc}\big(u_a(x_1)C\gamma_5 d_b(x_2)\big)d_{c;\alpha}(x_3) & \text{(neutron)} \end{cases}$$

accessible only to $J^P = A_1^+(\sim 0^+)$ and $T_1^+(\sim 1^+)$.

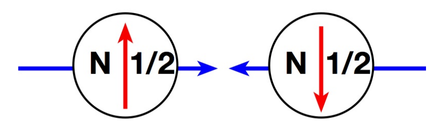


➔ Only LO potentials are calculable.

◆ Momentum wall source:

$$\overline{\mathcal{J}}_{\alpha\beta}(\vec{p}) \equiv \sum_{\vec{x}_1,\cdots,\vec{x}_6} \bar{N}_\alpha(\vec{x}_1,\vec{x}_2,\vec{x}_3)\bar{N}_\beta(\vec{x}_4,\vec{x}_5,\vec{x}_6)\cdot\exp\big(i\,\vec{p}\cdot(\vec{x}_3-\vec{x}_6)\big)$$

$$\overline{\mathcal{J}}^{\Gamma}_{\alpha\beta}(|\vec{p}|) \equiv \frac{1}{48}\sum_{g\in O_h}\chi^{(\Gamma)}(g^{-1})\cdot\overline{\mathcal{J}}_{\alpha'\beta'}(g\cdot\vec{p})S_{\alpha'\alpha}(g^{-1})S_{\beta'\beta}(g^{-1})$$



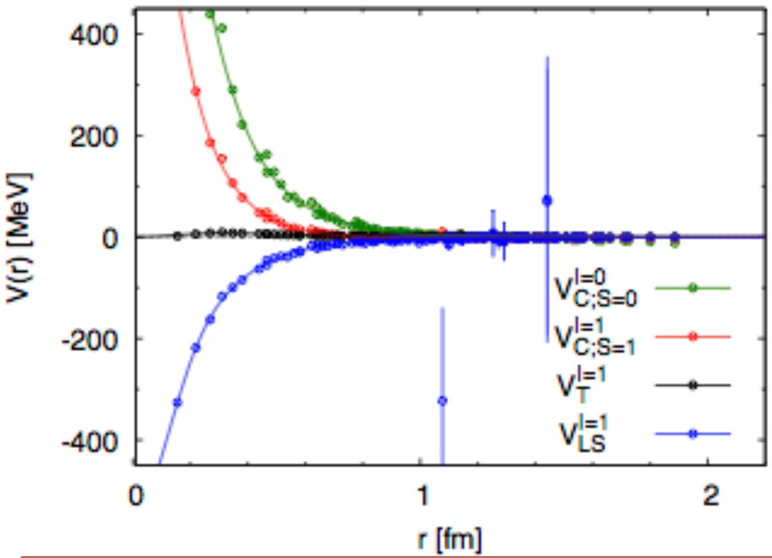
allows us to access varieties cubic group irreps. $J^P=\Gamma$.

➔ Potentials beyond NLO can be calculable.

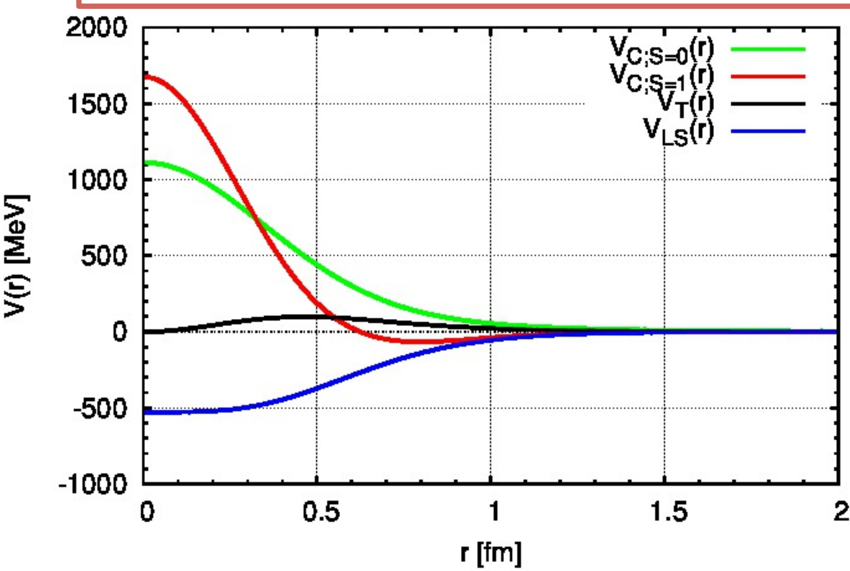
Nuclear Forces

[K.Murano et al., arXiv:1305.2293]

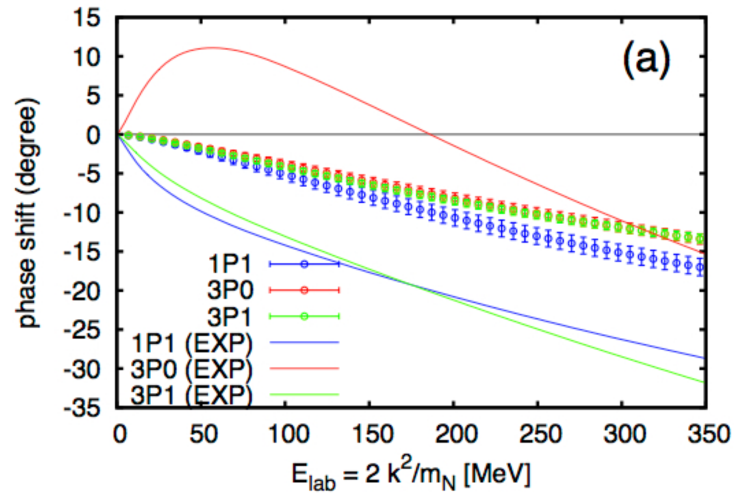
◆ Nuclear forces and LS force in parity-odd sector

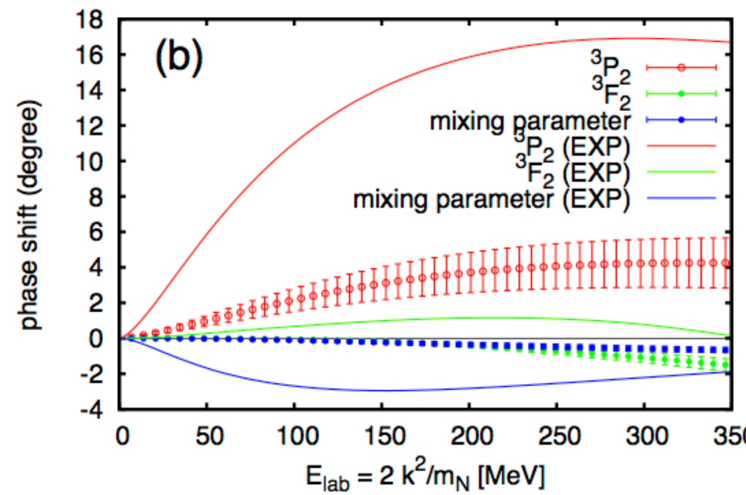


Phemenological one (AV18) for comparison



Scattering phase shifts





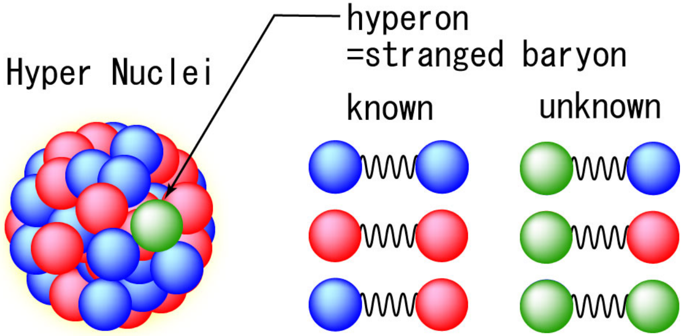
Hyperon Forces

Hyperon Forces

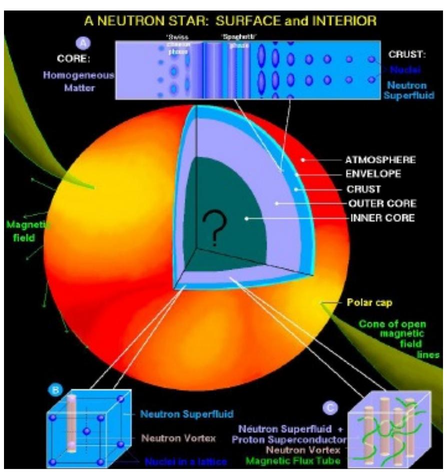
Our best target is hyperon force.

◆ Experimental information is limited due to the short life time of hyperons.

◆ Structure of hypernuclei

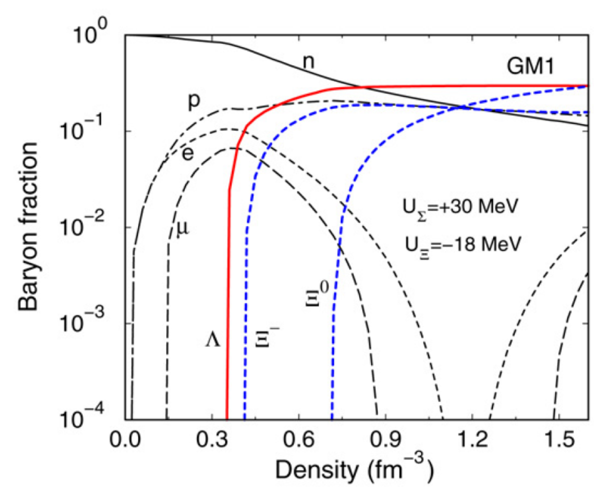


◆ Eq. of state of hyperon matter



J-PARC

Exploration of multi-strangeness world

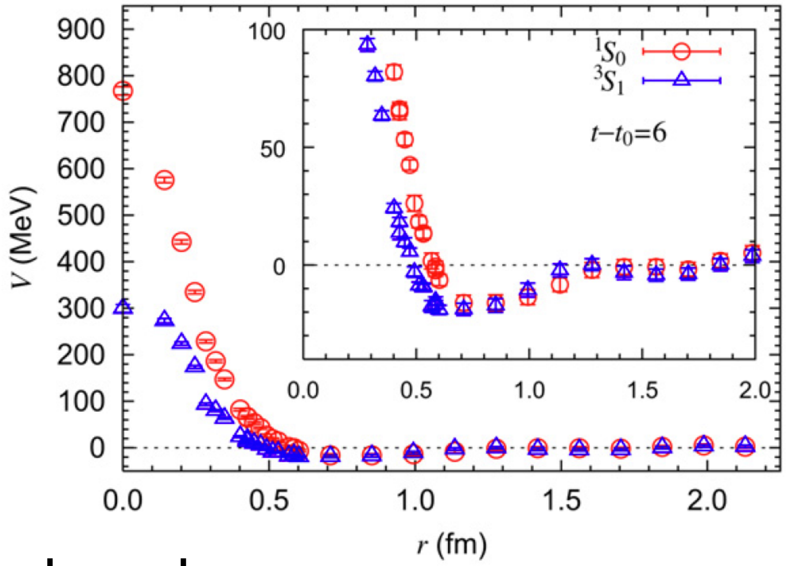


J.Schaffner-Bielich, NPA804('08)309.

Hyperon Forces

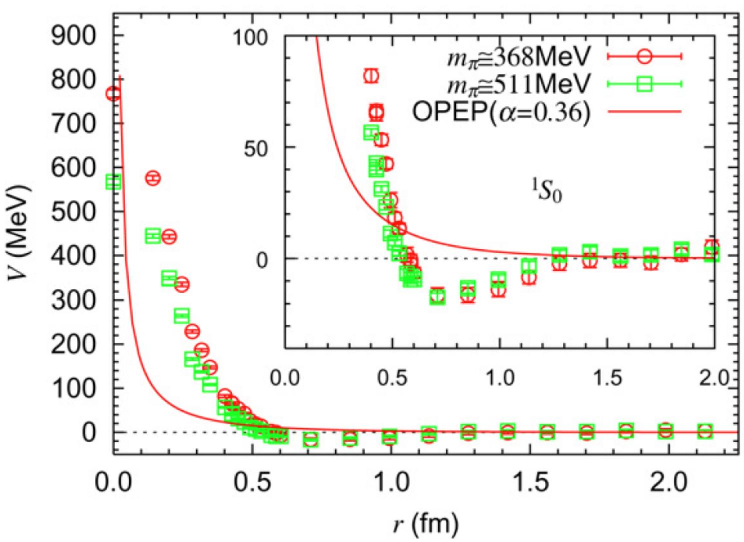
Nemura, Ishii, Aoki, Hatsuda, PLB673(2009)136.

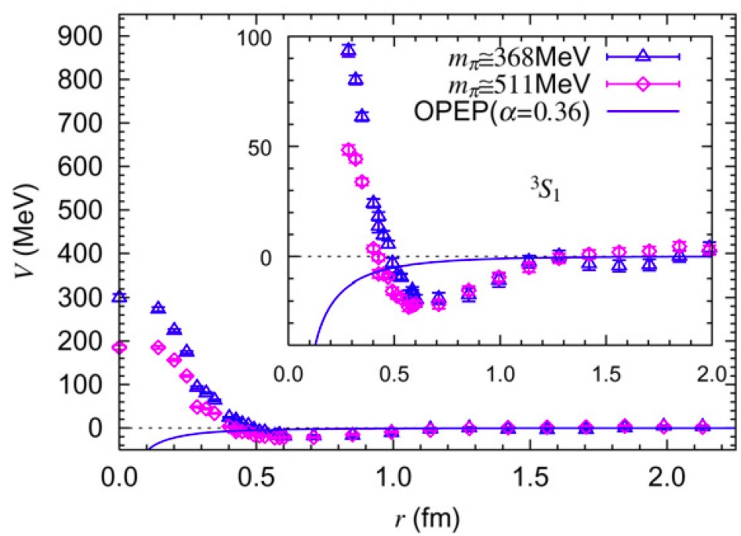
$\Xi N(I=1)$



- Repulsive core is surrounded by attraction like NN case.
- Strong spin dependence of repulsive core.

quark mass dependence





Repulsive core grows with decreasing quark mass.
No significant change in the attraction.

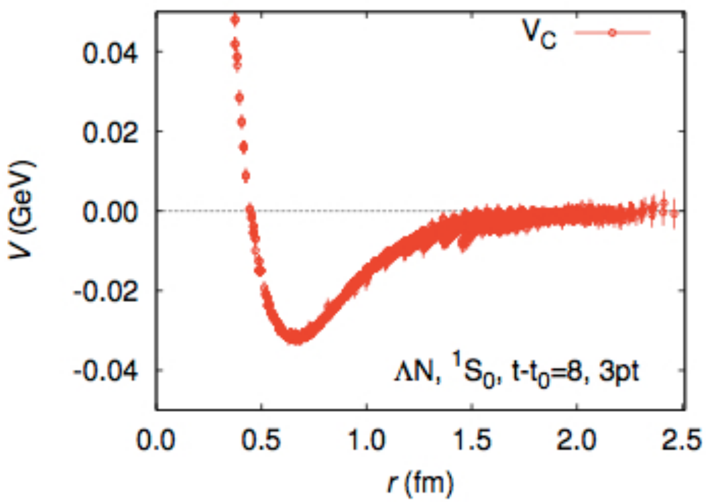
Hyperon Forces

[Nemura,PoS(LAT2011)]

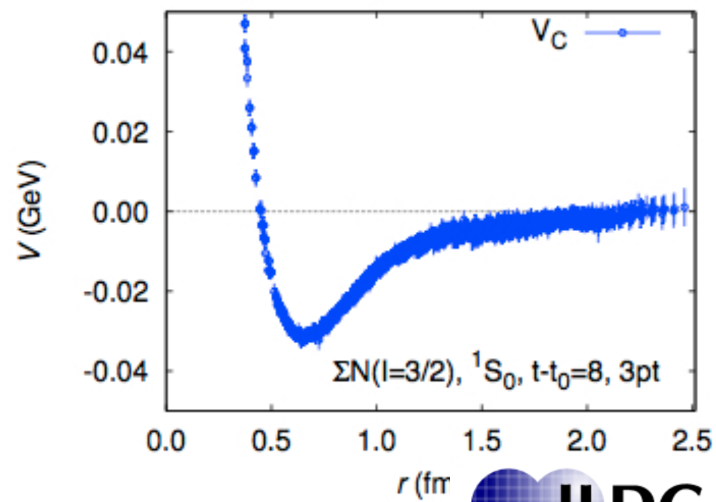
Spin-singlet sector

$^{1}S_{0}$

ΛN



$\Sigma N(I = 3/2)$



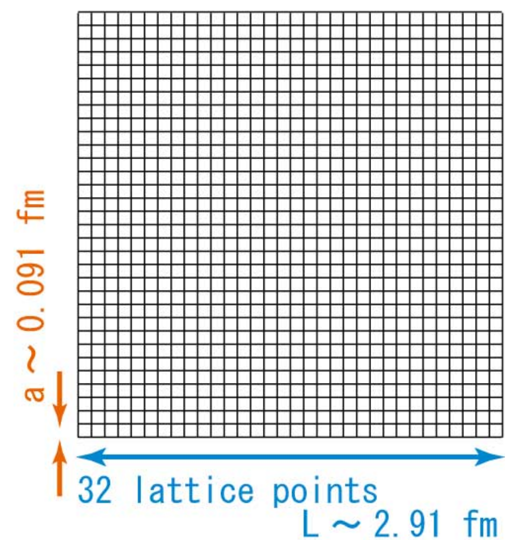
ILDG
Japan Lattice Data Grid

2+1 flavor config by PACS-CS Coll.
m(pion) = 570 MeV, m(N)=1412MeV

- Repulsive core is surrounded by attraction like NN case.
- These two potentials looks similar,
  which may be due to small flavor SU(3) breaking.

  They are not necessarily equal.

  - N-Lambda belongs to **27+8$_s$** rep. in flavor SU(3) limit.
  - N-Sigma belongs to **27** rep. in flavor SU(3) limit.



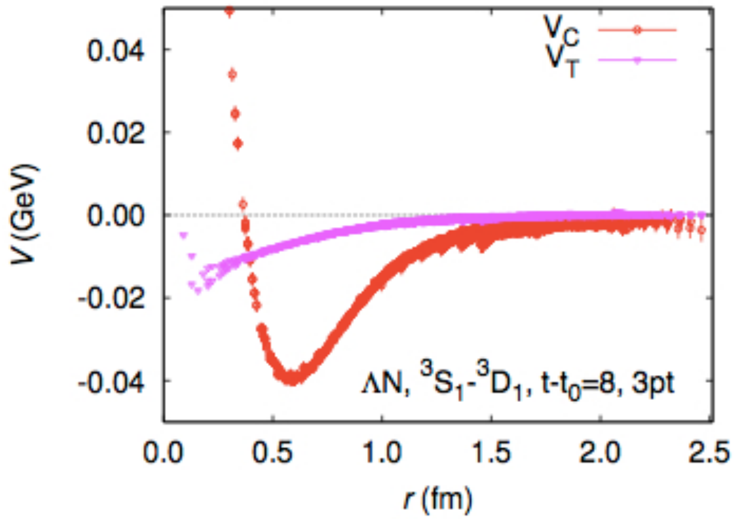
Hyperon Forces

[Nemura,PoS(LAT2011)]

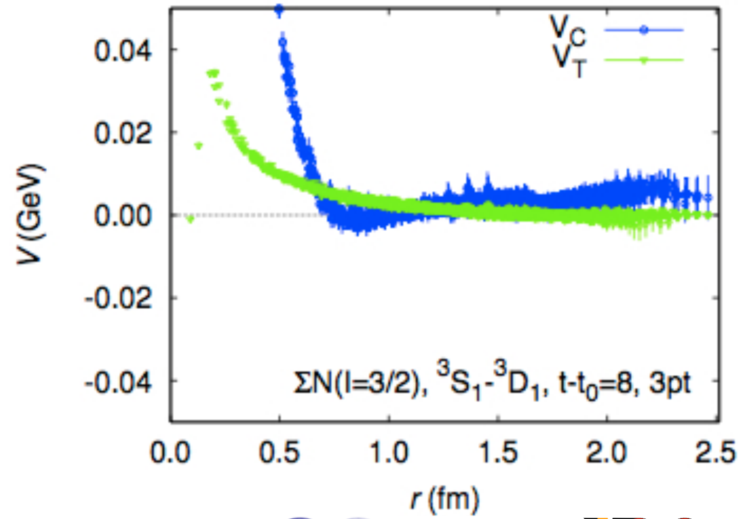
Spin-triplet sector

$^3S_1 - ^3D_1$

ΛN



$\Sigma N(I = 3/2)$

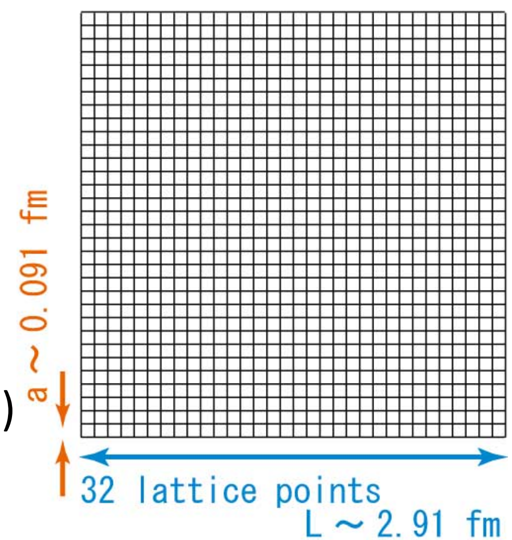


Japan Lattice Data Grid

2+1 flavor config by PACS-CS Coll.
m(pion) = 570 MeV, m(N)=1412MeV

- N-Lambda
  - Repulsive core is surrounded by attraction
  - The attraction is deeper than 1S0 case
  - Weak tensor force (no one-pion exchange is allowed)

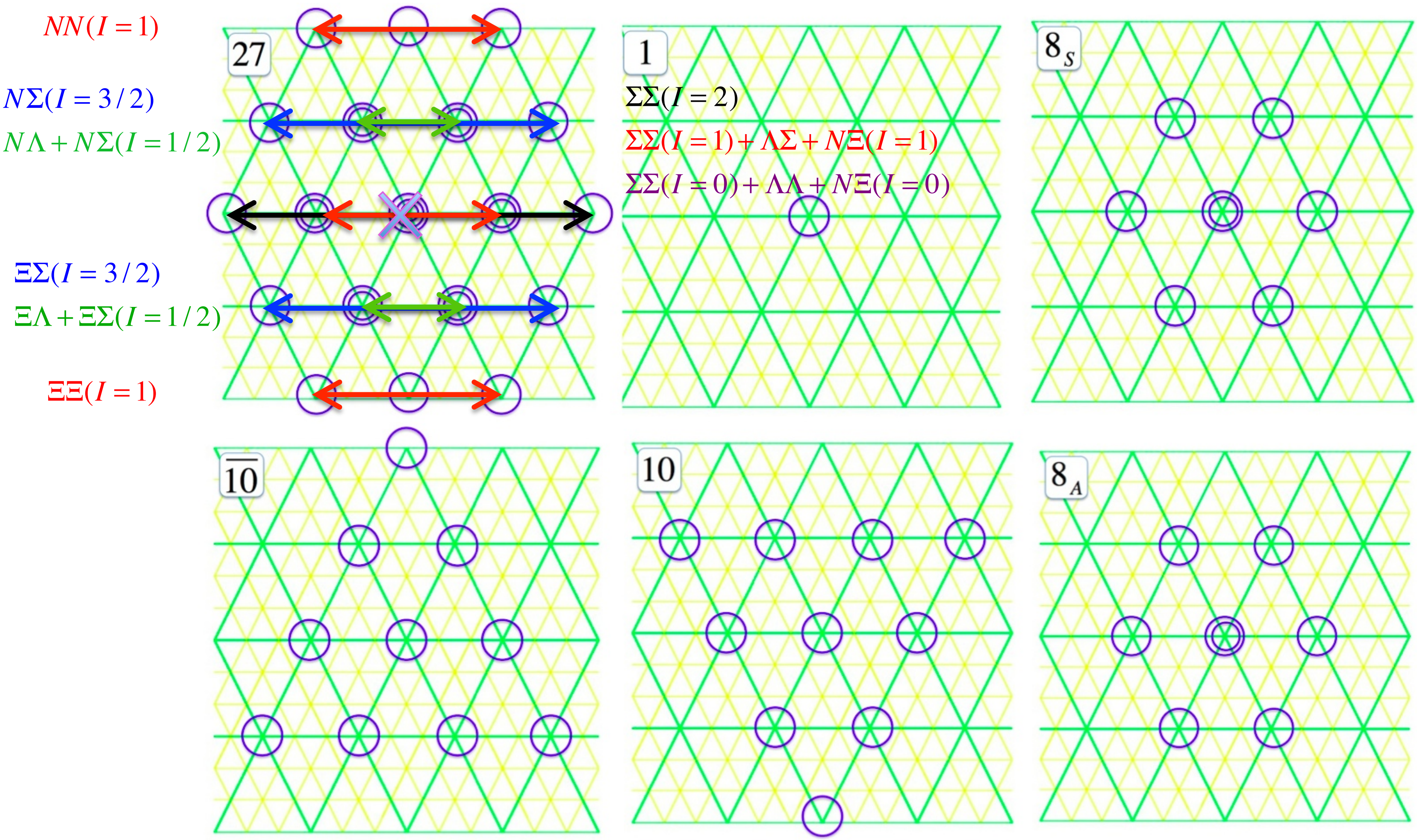
- N-Sigma
  - Repulsive core at short distance
  - No clear attractive well
    (Repulsive nature is consistent with the naïve quark model)
  - Strength of tensor force: N-N > N-Sigma > N-Lambda



Hyperon Forces

◆ Flavor SU(3) limit   to understand a general trend.

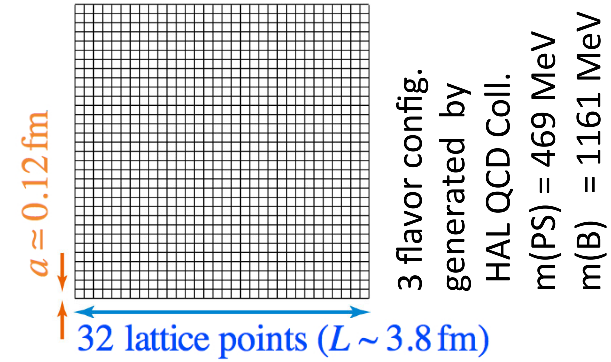
$$8 \otimes 8 = \underbrace{27 \oplus 8_S \oplus 1}_{\text{symmetric}} \oplus \underbrace{\overline{10} \oplus 10 \oplus 8_A}_{\text{anti-symmetric}}$$

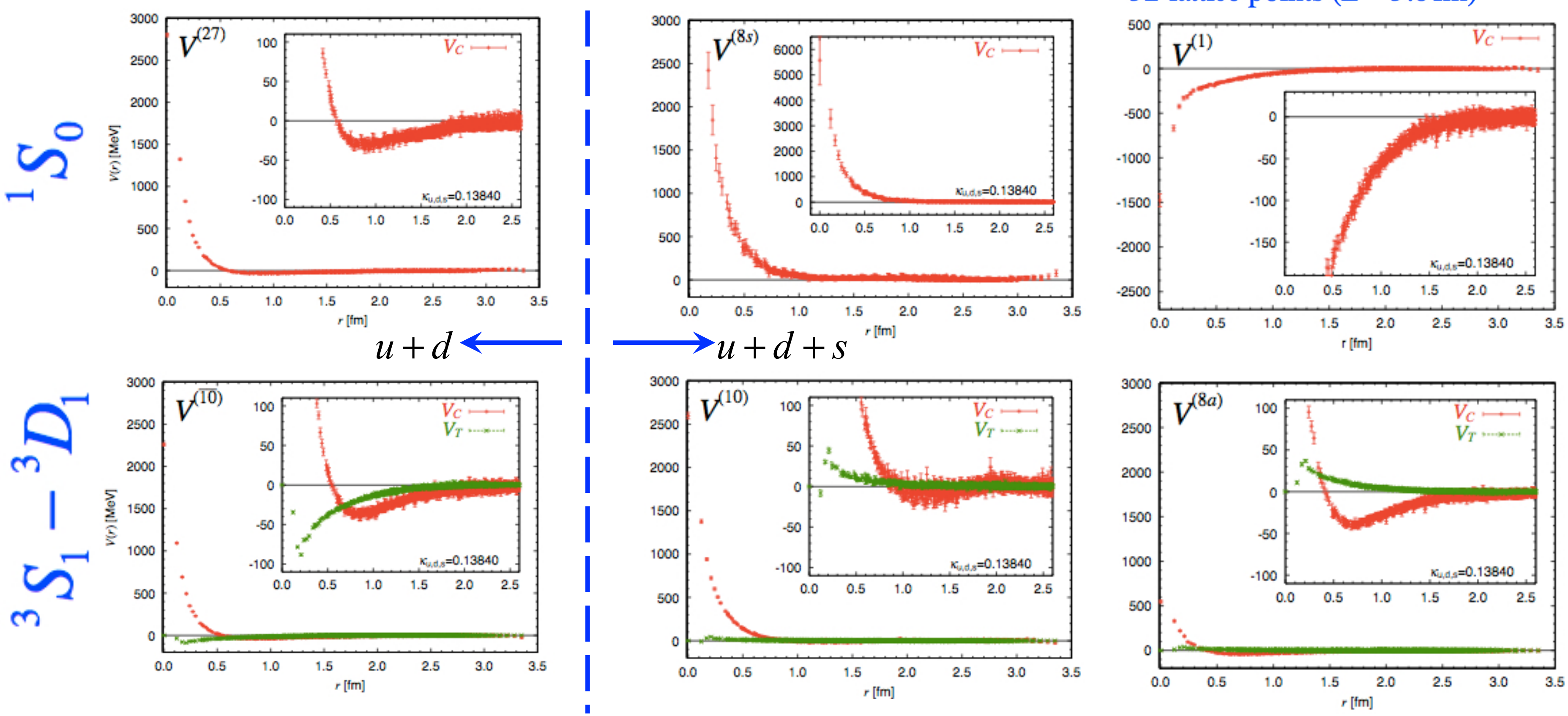


Hyperon Forces

[T.Inoue et al, PTP124,591(2010)]

Hyperon Potentials   in flavor SU(3) limit



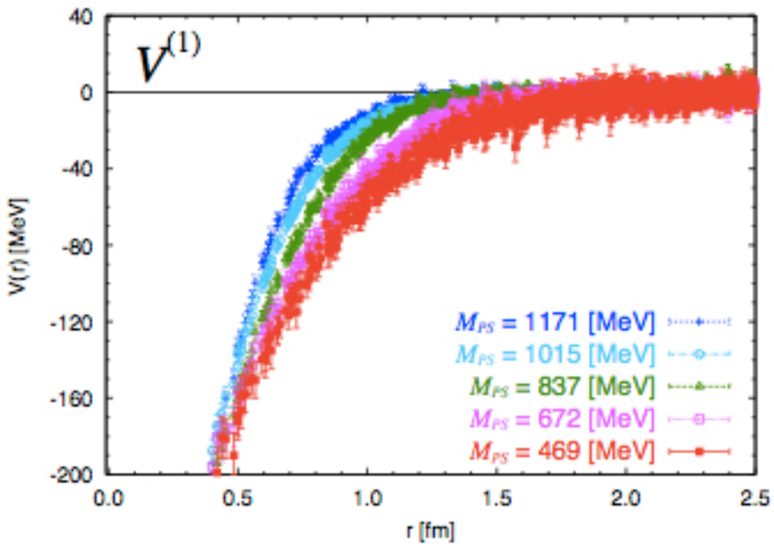


- Strong flavor dependence
- These short distance behaviors are consistent with quark Pauli blocking picture.

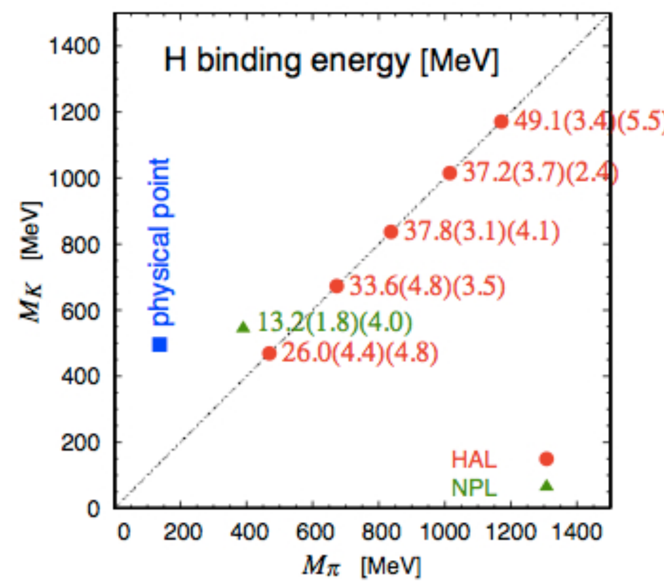
Hyperon Forces

[T.Inoue et al., PRL106(2011)162002.]

- Bound H-dibaryon in flavor SU(3) limit



Entirely attractive potential in flavor 1 channel leads to a bound H-dibaryon



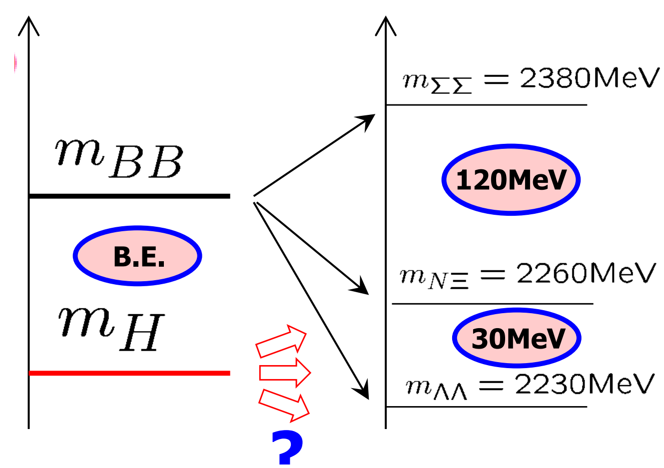
- Flavor SU(3) breaking for real world.

  → BB threshold in flavor SU(3) limit splits into three, i.e., ΛΛ, NΞ, ΣΣ thresholds

  → Coupled channel system of

$$\Lambda\Lambda - N\Xi - \Sigma\Sigma$$

SU(3) lat → **Physical point**



Hyperon Forces

Coupled channel extension

$$\Psi_n(\vec{x}-\vec{y}) \equiv \begin{bmatrix} \langle 0|\Lambda(\vec{x})\Lambda(\vec{y})|n,in\rangle \\ \langle 0|N(\vec{x})\Xi(\vec{y})|n,in\rangle \\ \langle 0|\Sigma(\vec{x})\Sigma(\vec{y})|n,in\rangle \end{bmatrix}$$

$$\begin{aligned} E &\equiv 2\sqrt{m_\Lambda^2 + \vec{p}_{\Lambda\Lambda}^{\,2}} \\ &= \sqrt{m_N^2 + \vec{p}_{N\Xi}^{\,2}} + \sqrt{m_\Xi^2 + \vec{p}_{N\Xi}^{\,2}} \\ &= 2\sqrt{m_\Sigma^2 + \vec{p}_{\Sigma\Sigma}^{\,2}} \end{aligned}$$

A parallel argument leads a “**coupled-channel Schrodinger eq.”.**

[S.Aoki et al., Proc.Japan Acad.B87(2011)509.]

$$\begin{bmatrix} \left(\frac{\vec{p}_{\Lambda\Lambda}^{\,2}}{2\mu_{\Lambda\Lambda}} + \frac{\Delta}{2\mu_{\Lambda\Lambda}}\right)\psi_{\Lambda\Lambda}(\vec{r};n) \\ \left(\frac{\vec{p}_{N\Xi}^{\,2}}{2\mu_{N\Xi}} + \frac{\Delta}{2\mu_{N\Xi}}\right)\psi_{N\Xi}(\vec{r};n) \\ \left(\frac{\vec{p}_{\Sigma\Sigma}^{\,2}}{2\mu_{\Sigma\Sigma}} + \frac{\Delta}{2\mu_{\Sigma\Sigma}}\right)\psi_{\Sigma\Sigma}(\vec{r};n) \end{bmatrix} = \int d^3r' \begin{bmatrix} U_{\Lambda\Lambda;\Lambda\Lambda}(\vec{r},\vec{r}') & U_{\Lambda\Lambda;N\Xi}(\vec{r},\vec{r}') & U_{\Lambda\Lambda;\Sigma\Sigma}(\vec{r},\vec{r}') \\ U_{N\Xi;\Lambda\Lambda}(\vec{r},\vec{r}') & U_{N\Xi;N\Xi}(\vec{r},\vec{r}') & U_{N\Xi;\Sigma\Sigma}(\vec{r},\vec{r}') \\ U_{\Sigma\Sigma;\Lambda\Lambda}(\vec{r},\vec{r}') & U_{\Sigma\Sigma;N\Xi}(\vec{r},\vec{r}') & U_{\Sigma\Sigma;\Sigma\Sigma}(\vec{r},\vec{r}') \end{bmatrix} \cdot \begin{bmatrix} \psi_{\Lambda\Lambda}(\vec{r}';n) \\ \psi_{N\Xi}(\vec{r}';n) \\ \psi_{\Sigma\Sigma}(\vec{r}';n) \end{bmatrix}$$

- This U(r,r’) is state-independent, i.e.,
  It works for any linear combinations |n,in> = |ΛΛ,in>α+ |NΞ,in>β+ |ΣΣ,in>γ.

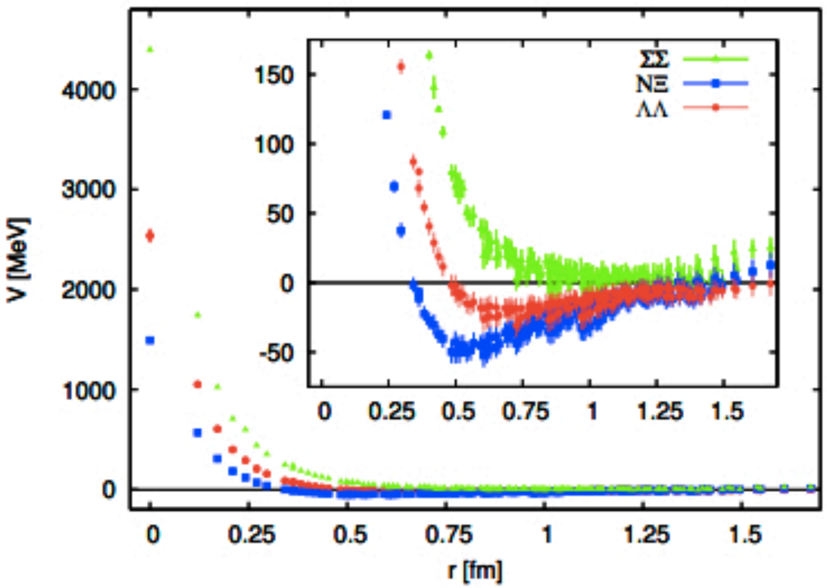
- Extract U(r,r’) in the **finite** volume.
  Use U(r,r’) in the **inifinite** volume
  to obtain the NBS wave functions of these states separately. ➔ S-matrix.

Hyperon Forces

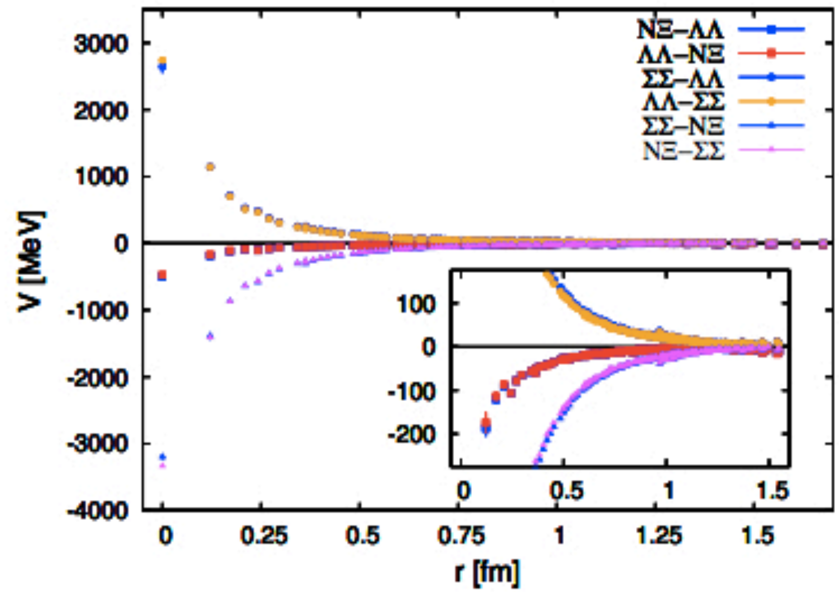
The numerical calculation is tough.
But it is doable. (work in progress)

[K.Sasaki@Lattice2012]

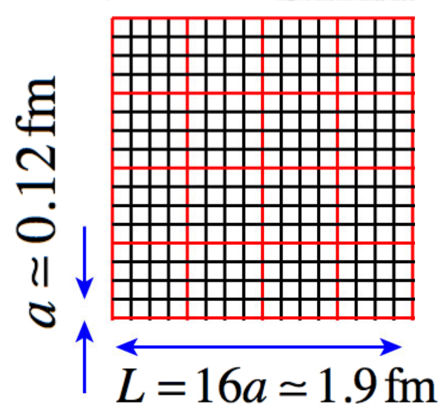
diagonal part



off-diagonal part



ILDG
Japan Lattice Data Grid



2+1 flavor gauge config
by CP-PACS/JLQCD Coll.
m(pion) = 875 MeV
m(K) = 916 MeV
m(N) = 1806 MeV
m(Lambda) = 1835 MeV
m(Sigma) = 1841 MeV
m(Xi) = 1867 MeV

Hyperon Forces

◆ Hyperon forces up to NLO

$$V_{BB} = V_{\mathrm{C;S=0}}(r)\mathbb{P}^{(S=0)} + V_{\mathrm{C;S=1}}(r)\mathbb{P}^{(S=1)} + V_{\mathrm{T}}(r)\left(3(\hat{r}\cdot\vec{\sigma}_1)(\hat{r}\cdot\vec{\sigma}_2) - \vec{\sigma}_1\cdot\vec{\sigma}_2\right)$$

$$+V_{\mathrm{LS}}(r)\vec{L}\cdot(\vec{s}_1+\vec{s}_2) + \underbrace{V_{\mathrm{ALS}}(r)\vec{L}\cdot(\vec{s}_1-\vec{s}_2)}_{\text{NEW TERM: Anti-symmetric LS}} + O(\nabla^2)$$

Momentum wall source allows us to access these terms for both parity sectors.

◆ Spin-orbit puzzle in ΛN sector

Convensional Nuclei

Very Large Spin-Orbit Splitting

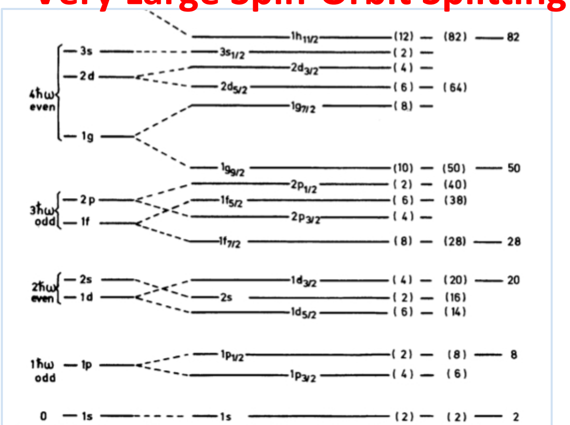
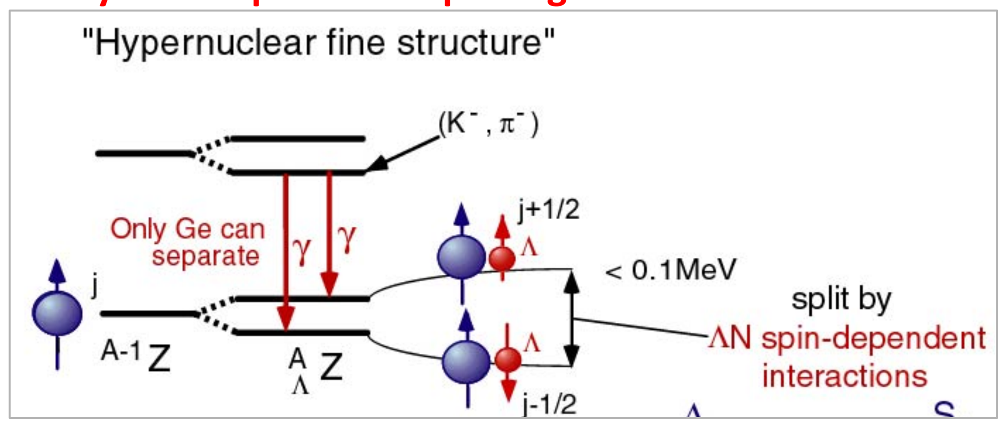


Figure 2.5. Schematic nuclear levels of the shell model with spin orbit term.

Λ Hyper Nuclei

Very Small Spin-Orbit Splitting for Λ



◆ One possible solution

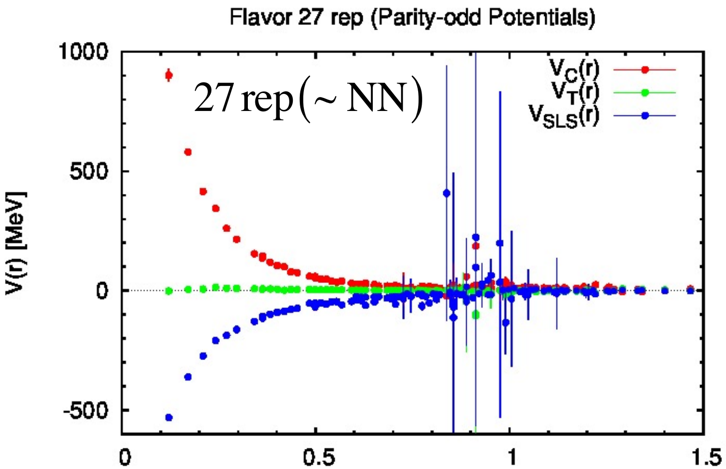
LS & Anti-LS cancellation of ΛN force

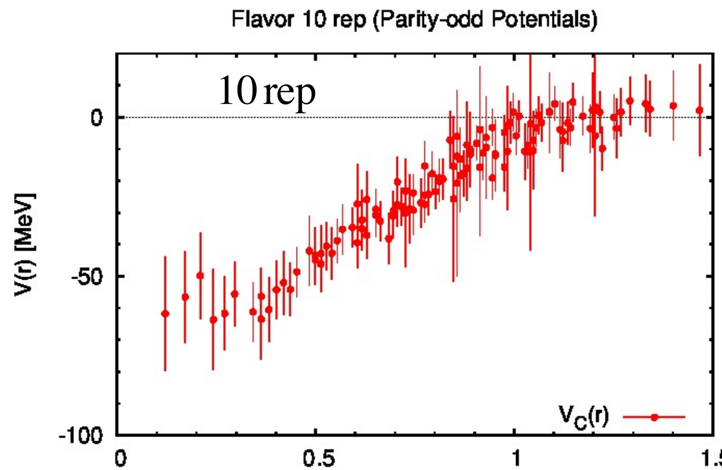
$$V_{\mathrm{LS}}^{(\Lambda)}(r) \equiv V_{\mathrm{LS}}(r) + V_{\mathrm{ALS}}(r) \sim 0$$

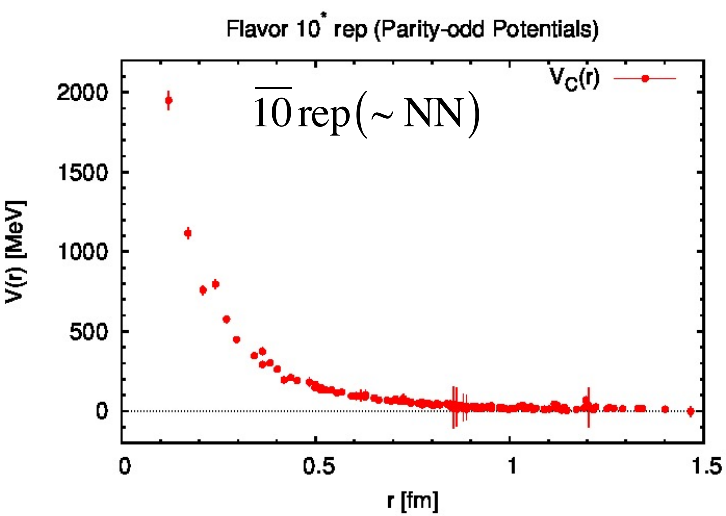
Experimental determination of anti-symmetric LS is difficult.

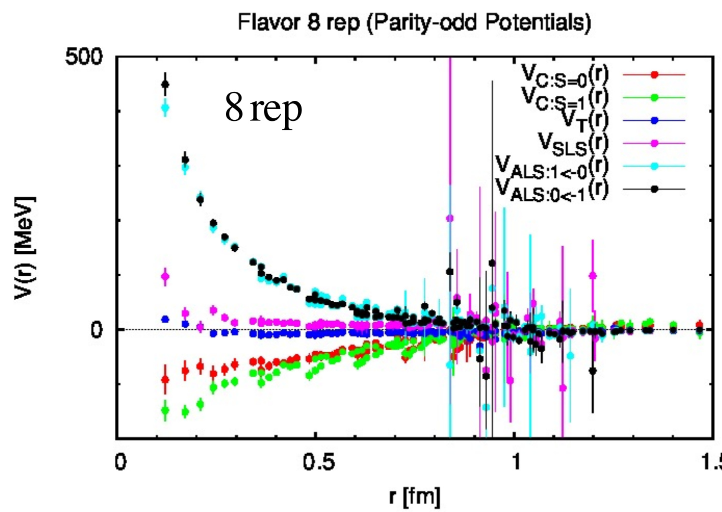
Hyperon Forces

Parity-odd hyperon potentials in the **flavor SU(3) limit**. [N.Ishii@Lattice 2013]









- Repulsive core for irreps. 27 and 10^*. No repulsive core for irreps. 10 and 8. (consistent with quark model)
- Strong LS for irrep. 27 (∼NN). Weak LS for irrep. 8.
- Strong anti-symmetric LS (irrep. 8).

Hyperon Forces

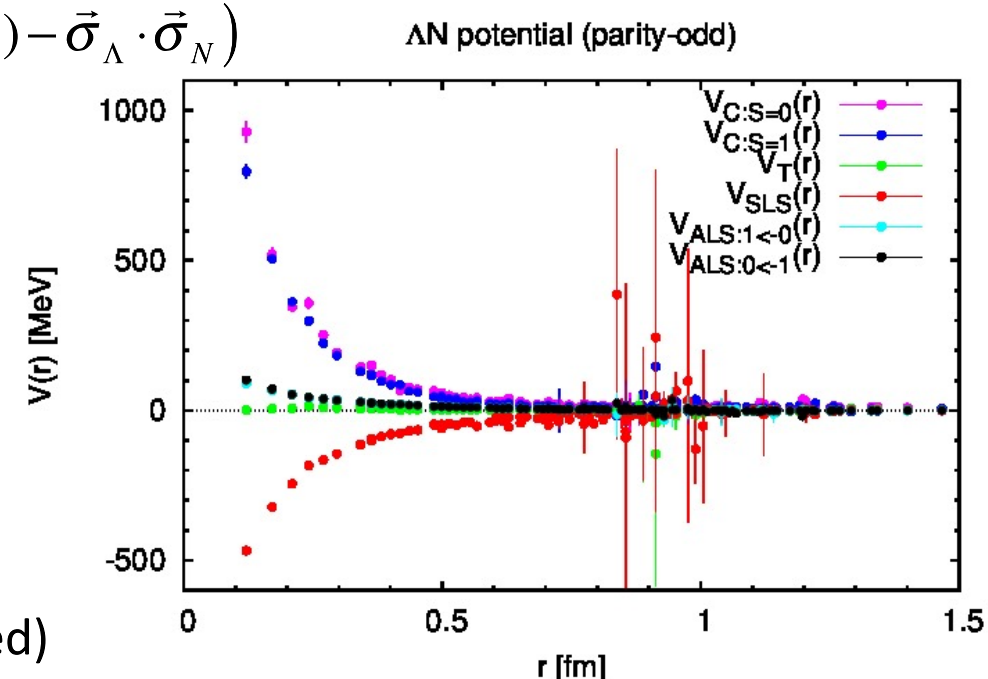
ΛN force (parity-odd sector) is obtained as linear combination of 8, 10^* and 27:

$$V_{\Lambda N} = \left(\frac{1}{2} V_{\rm C}^{(\overline{10})} + \frac{1}{2} V_{\rm C;S=0}^{(8)} \right) \mathbb{P}^{(S=0)} + \left(\frac{1}{10} V_{\rm C:S=1}^{(8)} + \frac{9}{10} V_{\rm C}^{(27)} \right) \mathbb{P}^{(S=1)}$$

$$+ \left(\frac{1}{10} V_{\rm T}^{(8)} + \frac{9}{10} V_{\rm T}^{(27)} \right) \left(3(\hat{r}\cdot\vec{\sigma}_\Lambda)(\hat{r}\cdot\vec{\sigma}_N) - \vec{\sigma}_\Lambda\cdot\vec{\sigma}_N \right)$$

$$+ \left(\frac{1}{10} V_{\rm LS}^{(8)} + \frac{9}{10} V_{\rm LS}^{(27)} \right) \vec{L}\cdot(\vec{s}_\Lambda + \vec{s}_N)$$

$$+ \frac{1}{2\sqrt{5}} V_{\rm ALS}^{(8)} \cdot \vec{L}\cdot(\vec{s}_\Lambda - \vec{s}_N)$$



- Weak cancellation (← to be continued)
  - Symmetric LS is strong. It comes from irrep. 27 (90%), i.e., NN LS

$$V_{\rm LS}^{(\Lambda N)} = \frac{1}{10} V_{\rm LS}^{(8)} + \frac{9}{10} V_{\rm LS}^{(27)}$$

  - Anti-symmetric LS is weak. It is weakened by a numerical factor 1/(2*sqrt(5))

$$V_{\rm ALS}^{(\Lambda N)} = \frac{1}{2\sqrt{5}} V_{\rm ALS}^{(8)}$$

- Quark mass dep. should be studied by breaking the flavor SU(3) symmetry.

Summary

Summary

- We have developed a method to determine inter-baryon potentials from Lattice QCD
  - Definition of the potentials which are faithful to scattering observables
  - Extraction of the potentials which do not rely on the ground state saturation
  - Many extensions, i.e., coupled channel, many-particle system, etc.
- LQCD numerical calculations for $m_{pi} > 400$ MeV.
  - Nuclear force
    - Parity-even sector:
      - Central and tensor forces (LO potentials).
        ➔ Attractive phase shifts. Strength is weak. (No bound states for $^1S_0$ and $^3S_1$).
      - Three nucleon force (linear alignment)
    - Parity-odd sector:
      - Central and tensor forces(LO potentials), and LS force(NLO).
  - Hyperon force
    - Parity-even sector:
      - Central and tensor forces (LO potentials).
        - Flavor SU(3) limit ➔ Bound H-dibaryon
        - Flavor SU(3) breaking ➔ coupled channel interactions
    - Parity-odd sector:
      - Flavor SU(3) limit:
        Central and tensor forces (LO potentials)
        LS and anti-symmetric LS forces (NLO potentials)
- Physical point simulation on a large spatial volume will start soon.

Summary

- Status for nuclear/hyperon forces for m_{pi} > 400 MeV

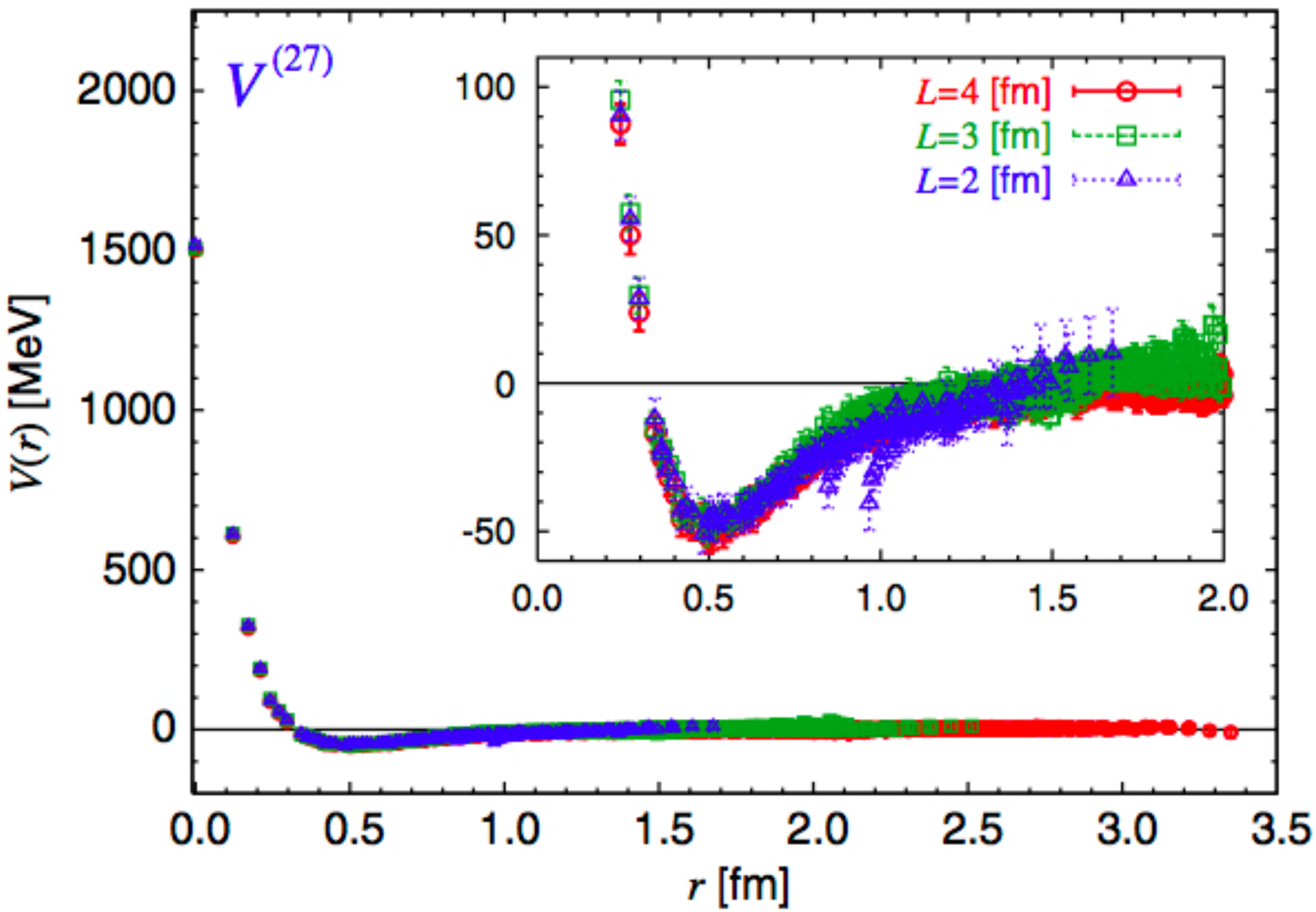
	Nucleon sector	Hyperon sector
1st stage: with flat wall source	DONE: $V^{(+)}_{C;S=0}, V^{(+)}_{C;S=1}, V^{(+)}_{T}$	DONE: $V^{(+)}_{C;S=0}, V^{(+)}_{C;S=1}, V^{(+)}_{T}$ (single ch.) $V^{(+)}_{C;S=0}$ (coupled ch.) **To be done:** $V^{(+)}_{C;S=1}, V^{(+)}_{T}$ (coupled ch.)
2nd stage: with momentum wall source	DONE: $V^{(-)}_{C;S=0}, V^{(-)}_{C;S=1}, V^{(-)}_{T}, V^{(-)}_{LS}$ **To be done:** $V^{(+)}_{LS}$	Work in progress: $V^{(-)}_{C;S=0}, V^{(-)}_{C;S=1}, V^{(-)}_{T}, V^{(-)}_{SLS}, V^{(-)}_{ALS}$ (single ch.) **To be done:** $V^{(-)}_{C;S=0}, V^{(-)}_{C;S=1}, V^{(-)}_{T}, V^{(-)}_{SLS}, V^{(-)}_{ALS}$ (coupled ch.) $V^{(+)}_{SLS}, V^{(+)}_{ALS}$ (single & coupled chs.)
Three body force	DONE: linear alignment	**To be done**

- Physical point calculation on a large spatial volume will start soon.

Backup Slides

Nuclear Force from Lattice QCD

Volume dependence of the potential.



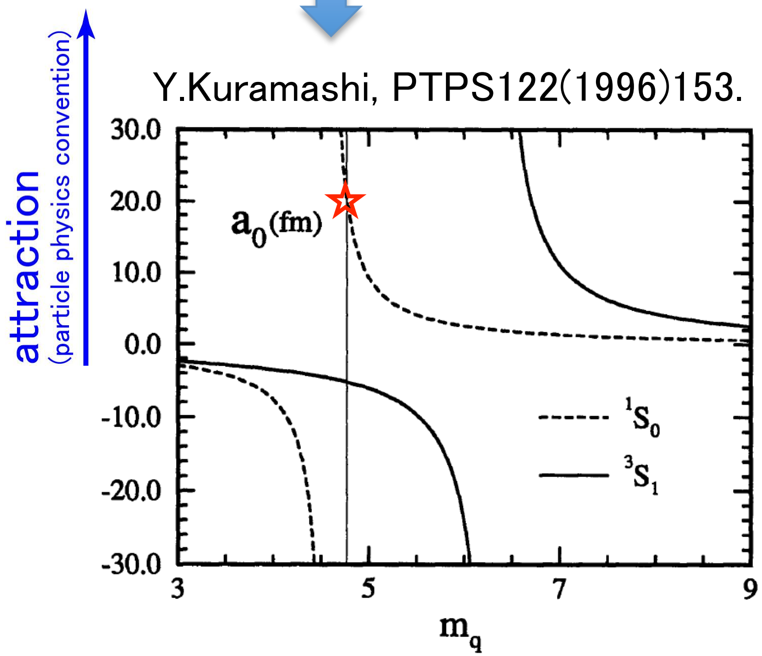
Nuclear Forces

❖ **HAL QCD**
No bound states for nn and deuteron

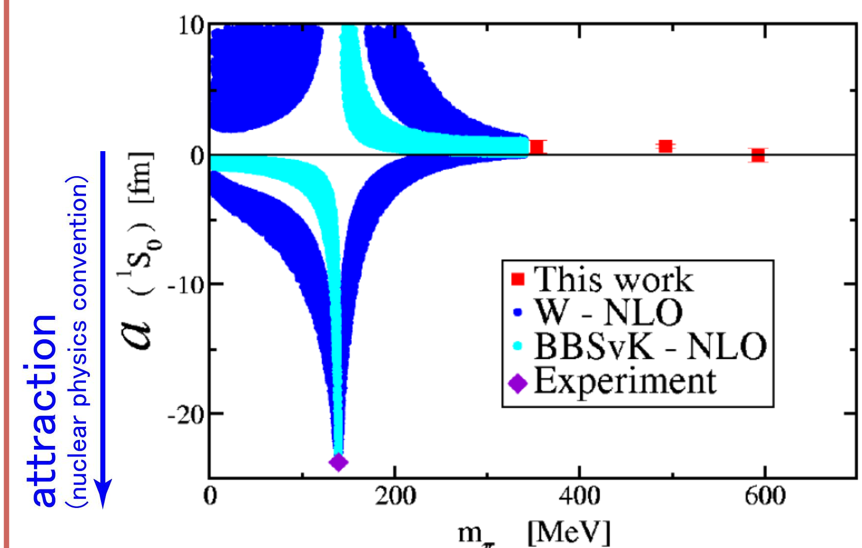
❖ **PACSCS & NPLQCD**
Bound nn and deuteron

quark mass behavior of scattering length

Y.Kuramashi, PTPS122(1996)153.



S.Beane et al., PRL97,012001(2006).



❖ less attractive for heavier quark mass
❖ no bound state for large quark mass

❖ more attractive for heavier quark mass
❖ bound state for large quark mass

m_{pi} > 390 MeV is too heavy.
LQCD calculation near physical point is important.