New pairing observable: binding energy differences of even-even nuclei

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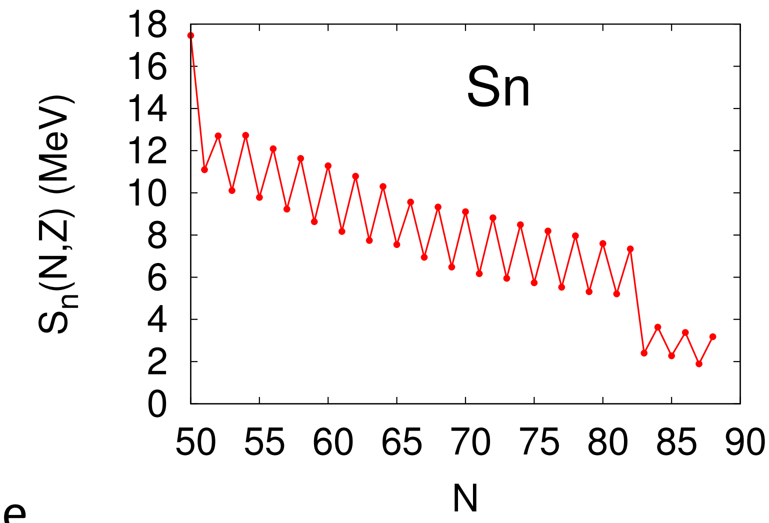
Pairing observables

odd-even staggering (OES)

$$\Delta^{(3)}(N) = \frac{(-1)^N}{2}[B(N-1) - 2B(N) + B(N+1)]$$

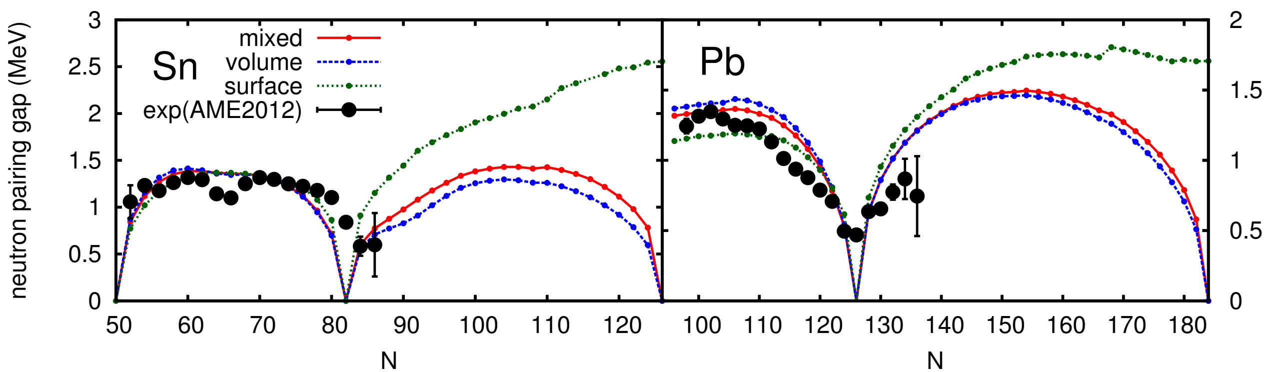
$$\Delta^{\exp}(N,Z) = \frac{1}{2}[\Delta^{(3)}(N-1) + \Delta^{(3)}(N+1)]$$



Problem

odd-mass nuclei in DFT is not very accurate

experimental OES ⟷ theoretical pairing gap



Problems

pairing gap: not an experimental observable

time-reversal symmetry: broken in OES, pairing gap is from even-system

multiple definition

Pairing observables

pairing (superconductivity) : U(1) gauge-symmetry breaking

spontaneous symmetry breaking

-- zero-energy collective mode (Anderson-Nambu-Goldstone,1958-61)

-- NG mode for gauge-symmetry breaking: pairing rotation

<u>finite nuclei:</u>

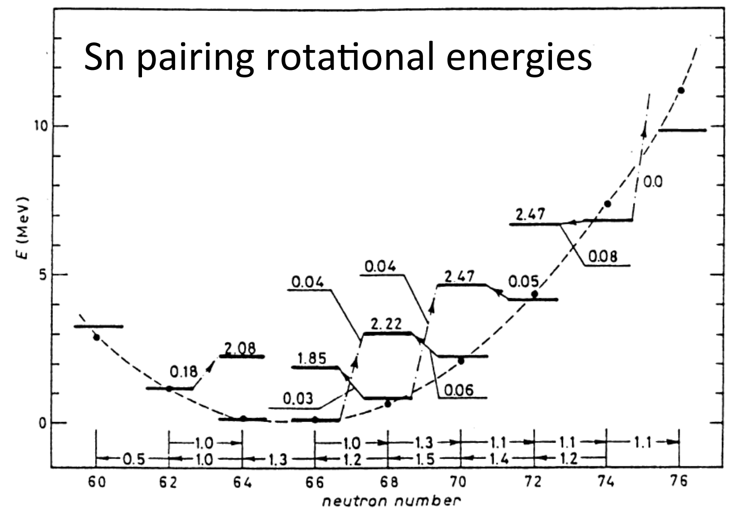
symmetry breaking is not exact: NG mode has finite excitation energy

<u>pairing rotational energy</u> $$E(N) = \frac{1}{2\mathcal{J}_\mathrm{N}}(N - N_0)^2$$

new pairing observable:

pairing rotational moment of inertia

* quantity from symmetry breaking
* double binding-energy differences
* involves even-even systems only

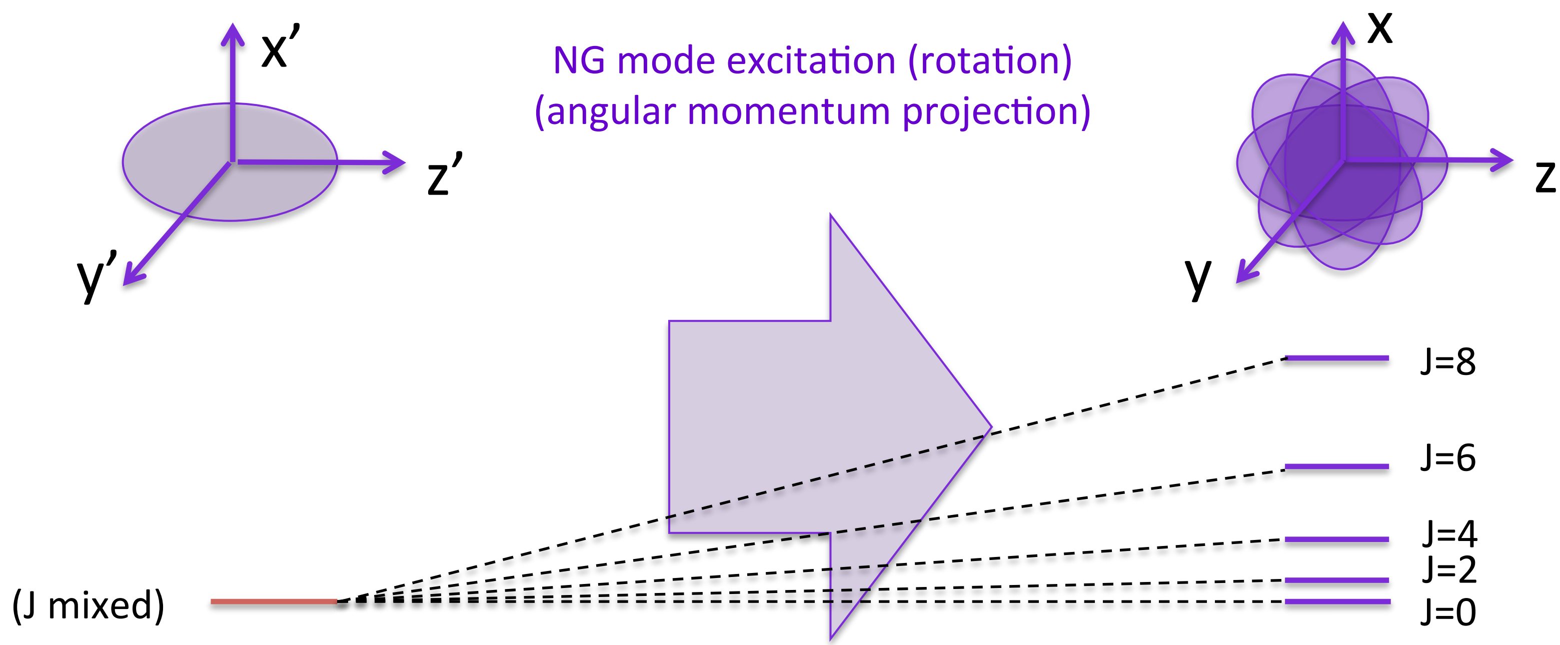


Brink and Broglia "Nuclear Superconductivity"
review: Broglia et al., Phys. Rep. **335**, 1(2000)

Rotational symmetry breaking (for comparison)

symmetry-broken deformed state
(body-fixed intrinsic frame)

symmetry-restored state
(Laboratory frame)



intrinsic, axially deformed state
(even-even nucleus)

eigenstates of
angular momentum operator

rotational band

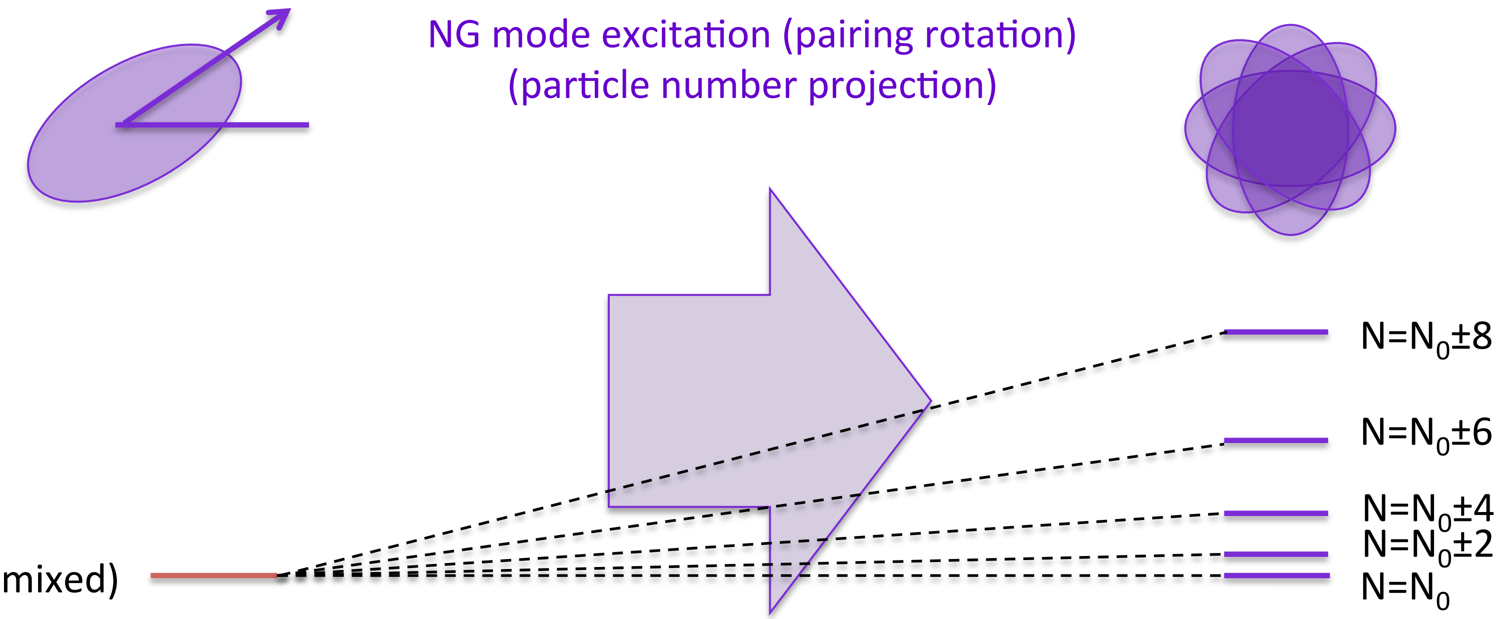
rotational energy $$E(J) = \frac{\hbar^2}{2\mathcal{J}_{\rm rot}} J(J+1)$$

moment of inertia ($E(2_1^+)$): magnitude of quadrupole collectivity

Gauge symmetry breaking

symmetry-broken superconducting state
(body-fixed intrinsic frame)

symmetry-restored state
(Laboratory frame)



intrinsic, superconducting state
with a coherent complex phase

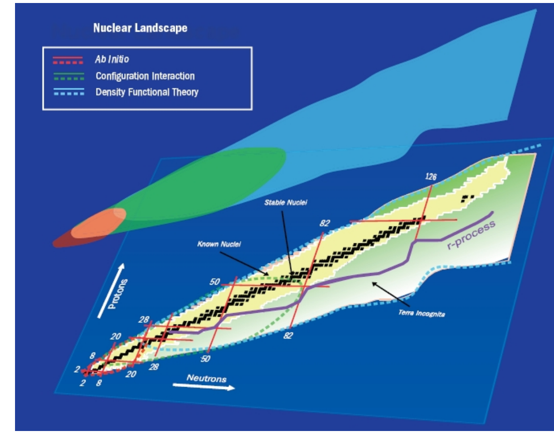
eigenstates of
particle-number operator

pairing rotational band

pairing rotational energy $E(N) = \frac{1}{2\mathcal{J}_{\mathrm{N}}}(N - N_0)^2$

pairing rotational moment of inertia: magnitude of pairing collectivity

Theoretical description of NG mode



Symmetry-broken state

Density Functional Theory (DFT)

Skyrme HFB (HFBTHO, UNEDF1-HFB)

strength fitted to reproduce pairing gaps of ^{120}Sn (neutron) and ^{92}Mo(proton)

NG mode excitation

Quasiparticle Random-Phase Approximation (time-dependent DFT)

$$[\hat{H}_{\rm QRPA}, \hat{\mathcal{P}}_{\rm NG}] = i\hbar^2 \omega_{\rm NG}^2 \mathcal{J}_{\rm TV} \hat{Q}_{\rm NG} = 0$$

$$[\hat{H}_{\rm QRPA}, \hat{Q}_{\rm NG}] = -\frac{i}{\mathcal{J}_{\rm TV}} \hat{\mathcal{P}}_{\rm NG}$$

P_{NG}: particle number operator

moment of inertia from QRPA: Thouless-Valatin inertia

Thouless and Valatin, Nucl. Phys. **31** (1962)211

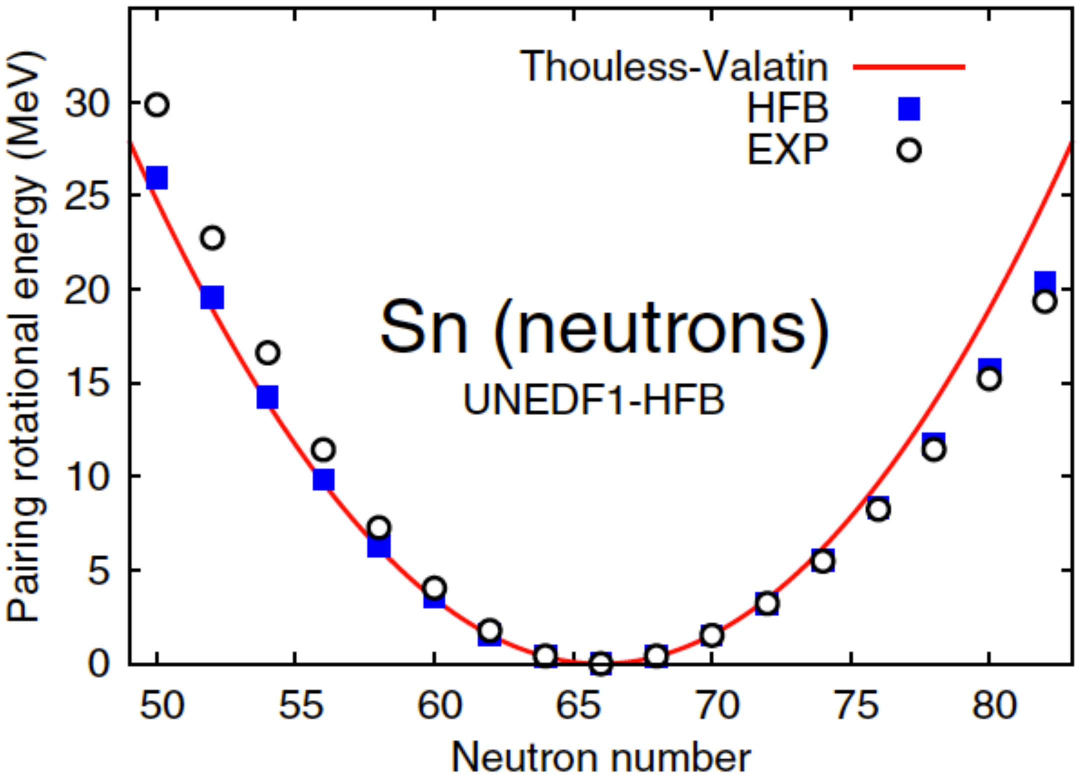
efficient solution based on linear response theory: Finite-amplitude method (FAM)

Nakatsukasa et al., PRC**76**, 024318 (2007)

NH, PRC**92**, 034321 (2015)

Pairing rotational energy

neutron pairing rotational energy



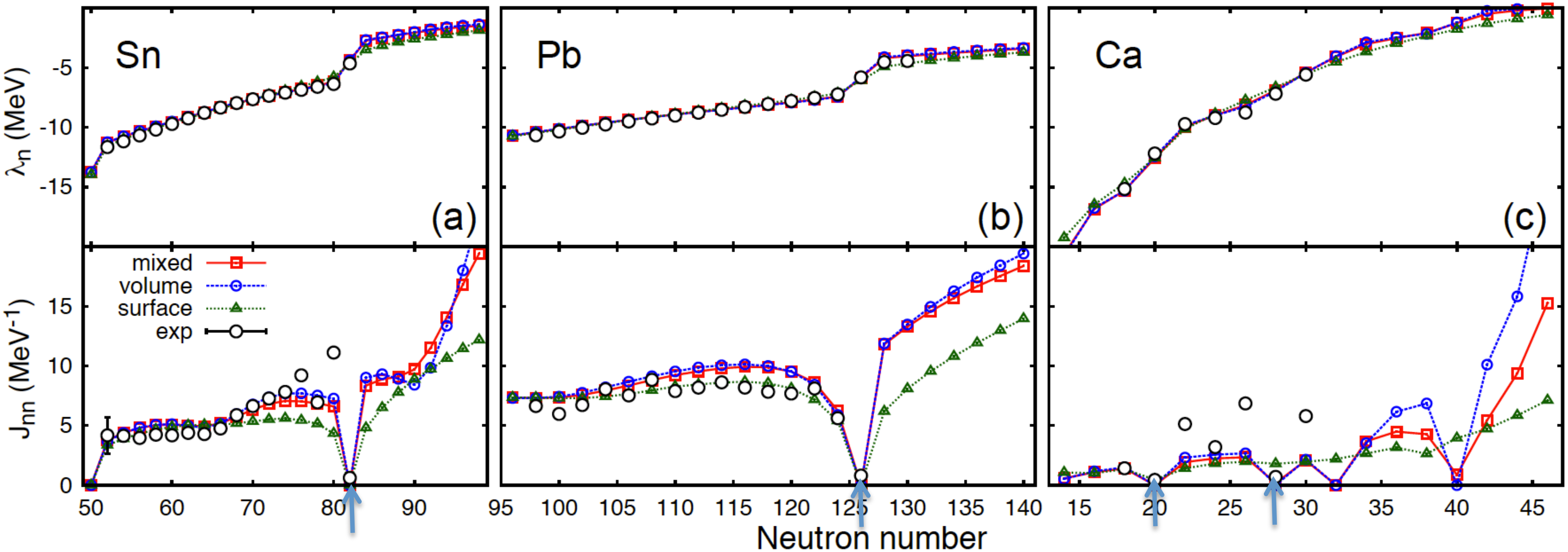
- reference state: ^{116}Sn (N=66)
- deviation from parabola: change of intrinsic structure with N

$$E_{\mathrm{pair}}^{\mathrm{exp}}(N) = E^{\mathrm{exp}}(N) - E^{\mathrm{exp}}(66) - \lambda_{\mathrm{n}}^{\mathrm{exp}}(66)(N-66)$$

$$\lambda_{\mathrm{n}}^{\mathrm{exp}}(66) = \frac{1}{4}[E^{\mathrm{exp}}(64) - E^{\mathrm{exp}}(68)]$$

Neutron pairing rotational MOI in single-closed shell nuclei

broken symmetry: neutron number U(1), Δn≠0



experimental Fermi energy and pairing rotational moment of inertia

$$\mathcal{J}_{\rm nn,exp}(N) = \frac{4}{E_{\rm exp}(N+2) - 2E_{\rm exp}(N) + E_{\rm exp}(N-2)}$$

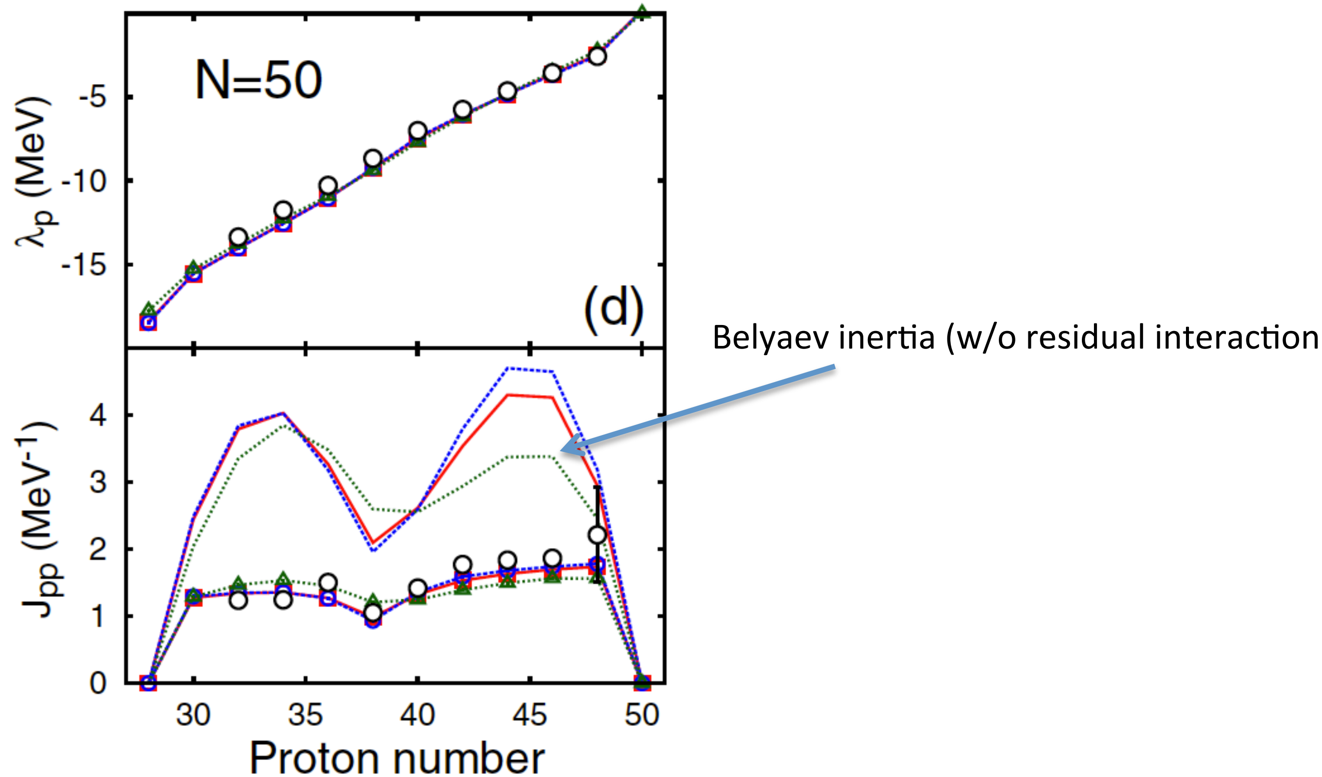
$$\lambda_{\rm n,exp}(N) = \frac{1}{4}[E_{\rm exp}(N-2) - E_{\rm exp}(N+2)]$$

$$E(N+\Delta N) = E(N) + \lambda_{\rm n}(N)\Delta N + \frac{(\Delta N)^2}{2\mathcal{J}_{\rm nn}(N)} + O((\Delta N)^3)$$

- ☐ experimental MOI: sensitive to the symmetry breaking
- ☐ pairing rotational assumption is not valid at next to magic numbers
- ☐ three kinds of pairing functional: sensitivity to pairing form in n-rich Ca

Proton pairing rotational MOI in single-closed shell nuclei

broken symmetry: proton number U(1), Δp≠0

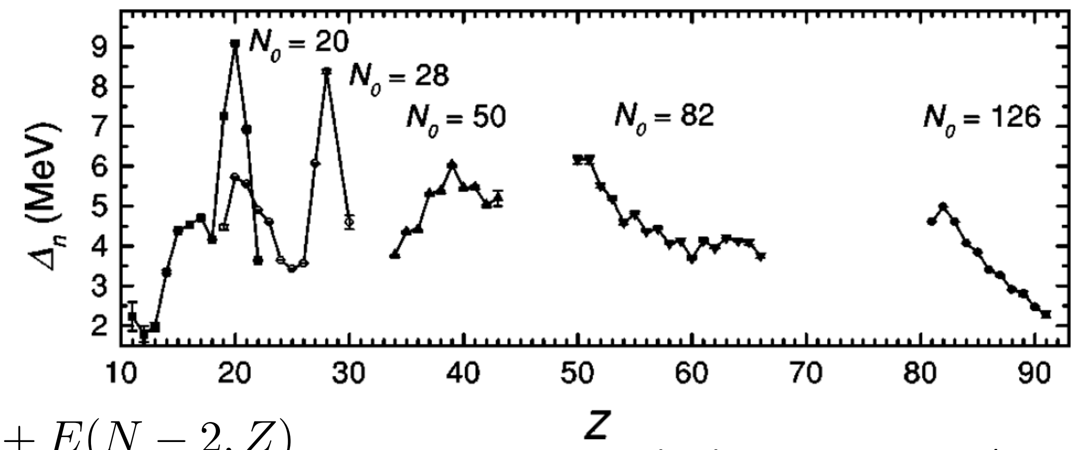


- smaller pairing rotational MOI: (neutron ~ 5 MeV^{-1})
- contribution from residual Coulomb suppresses the MOI (ph Coulomb force ~ Z^2)
- Coulomb suppresses the proton pairing [Anguiano NPA683,227(2001)]

Shell gap and pairing rotational moment of inertia

Experimental pairing rotational MOI

$$\mathcal{J}_{\rm nn}^{-1}(N) = \frac{1}{4}\delta_{\rm 2n}(N)$$



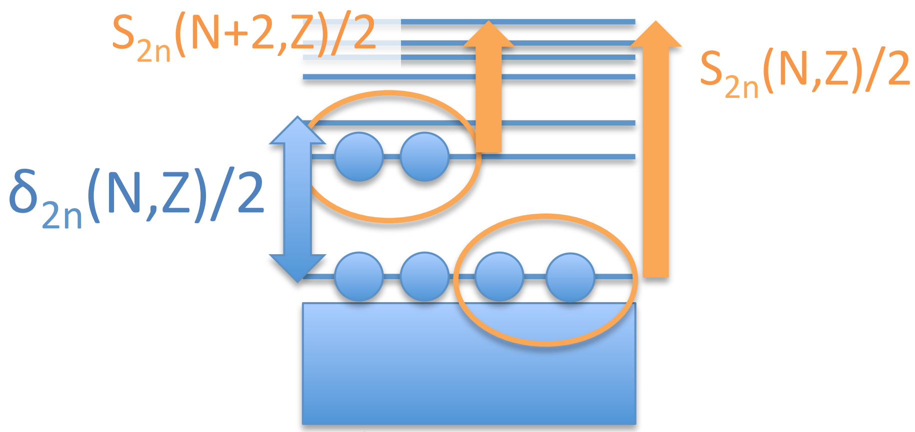
Lunney et al., Phys. Rep. **75**, 1021(2003)

empirical shell gap

$$\begin{aligned}\delta_{\rm 2n}(N,Z) =& E(N+2,Z) - 2E(N,Z) + E(N-2,Z)\\ =& S_{\rm 2n}(N,Z) - S_{\rm 2n}(N+2,Z)\end{aligned}$$

weak pairing collectivity

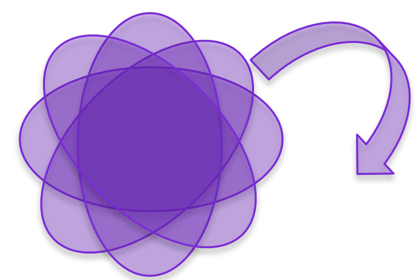
shell model picture
(before symmetry breaking)



size of magic shell gap

strong pairing collectivity

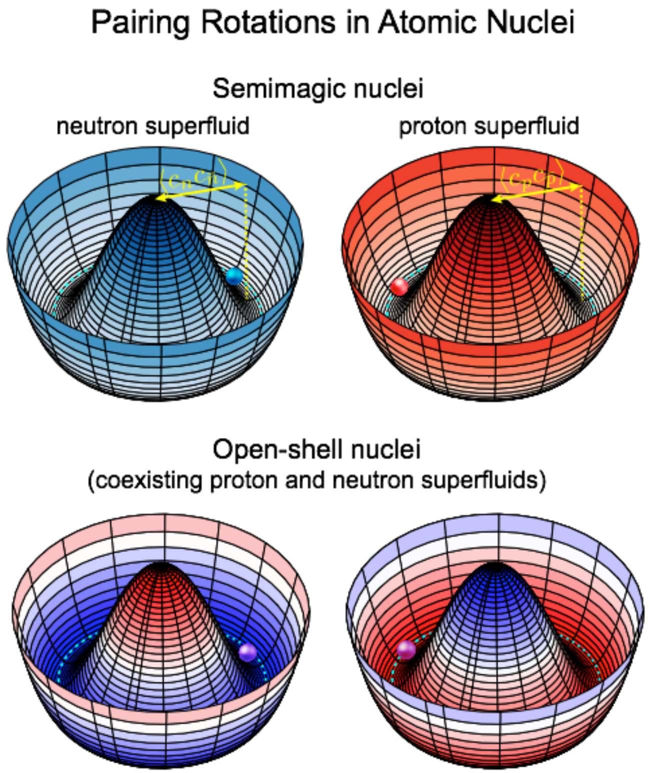
collective pairing picture
(after symmetry breaking)



$$E_{\rm pairrot}(N) = \frac{1}{2\mathcal{J}_{\rm nn}(N)}(\Delta N)^2$$

measure of gauge symmetry breaking

Pairing NG modes in doubly open-shell nuclei



neutron and proton gauge symmetry broken
(Δn≠0 and Δp≠0, $U(1)_n \mathrm{x} U(1)_p$)

$$[\hat{H}_{\mathrm{HFB}}, \hat{N}_{\mathrm{n}}] \neq 0 \qquad [\hat{H}_{\mathrm{HFB}}, \hat{N}_{\mathrm{p}}] \neq 0$$

neutron pairing rotational energy
proton pairing rotational energy

NG modes as QRPA eigenmodes are neutron-proton mixed

formal theory: Marshalek, Nucl. Phys. A **275**,416 (1977)
first calculation: NH, Phys. Rev. C **92**,034321 (2015)

$$\hat{N}_1 = \hat{N}_{\mathrm{n}} \cos\theta + \alpha \hat{N}_{\mathrm{p}} \sin\theta$$
$$\hat{N}_2 = -\hat{N}_{\mathrm{n}} \sin\theta + \alpha \hat{N}_{\mathrm{p}} \cos\theta$$

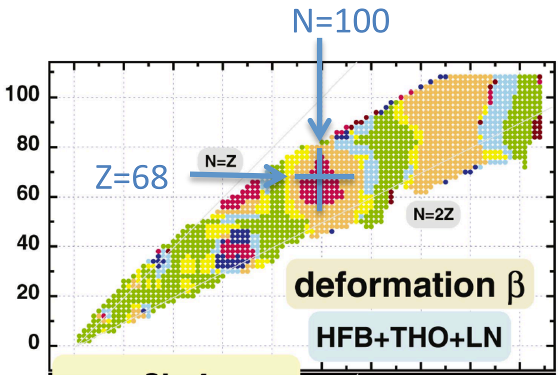
$$[\hat{H}_{\mathrm{HFB}}, \hat{N}_1] \neq 0 \qquad [\hat{H}_{\mathrm{HFB}}, \hat{N}_2] \neq 0$$

$$E(N,Z) = E(N_0,Z_0) + \lambda_1(N_0,Z_0)\Delta N_1 + \lambda_2(N_0,Z_0)\Delta N_2 + \frac{(\Delta N_1)^2}{2\mathcal{J}_1(N_0,Z_0)} + \frac{(\Delta N_2)^2}{2\mathcal{J}_2(N_0,Z_0)}$$

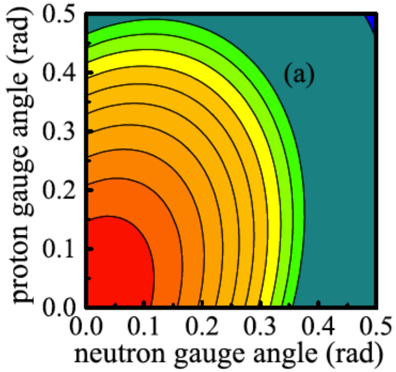
$$E_{\mathrm{pairrot}}(N,Z) = \frac{(\Delta N)^2}{2\mathcal{J}_{\mathrm{nn}}(N_0,Z_0)} + \frac{2(\Delta N)(\Delta Z)}{2\mathcal{J}_{\mathrm{np}}(N_0,Z_0)} + \frac{(\Delta Z)^2}{2\mathcal{J}_{\mathrm{pp}}(N_0,Z_0)}$$

note: no neutron-proton pairing

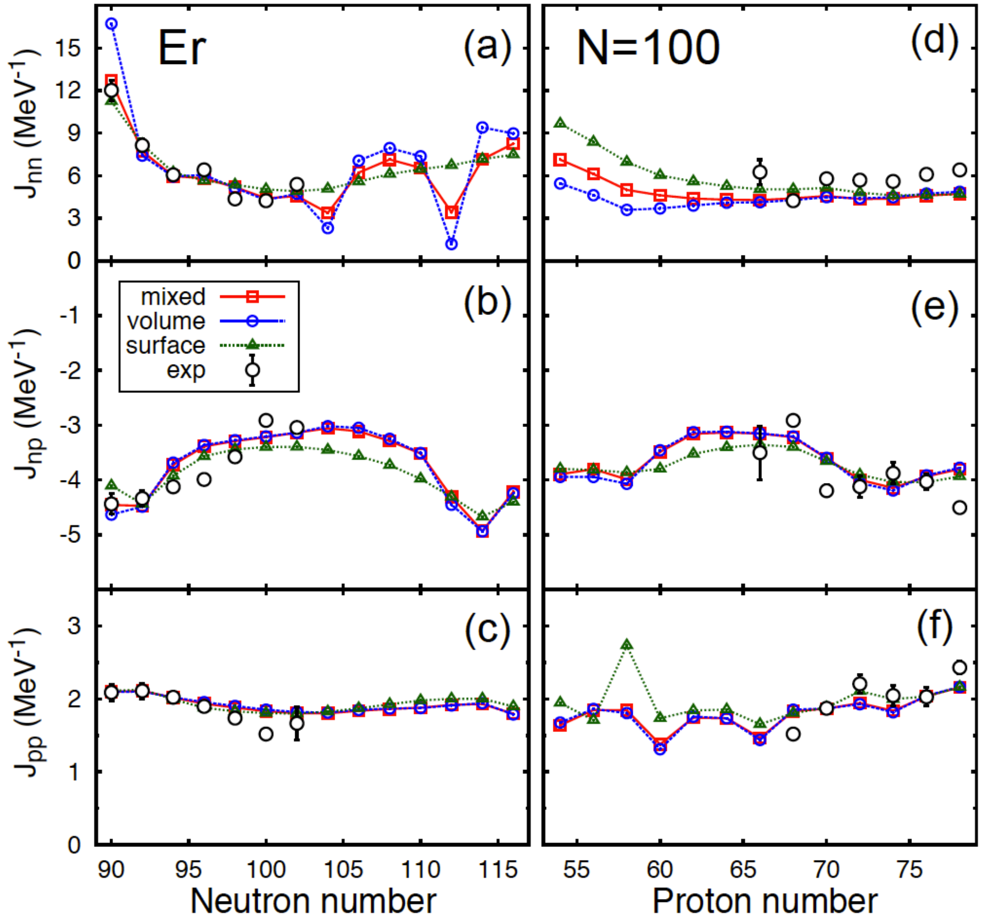
Pairing rotational MOI in doubly open-shell nuclei



Stoitsov et al (2003)



Wang et al., PRC**90**,014312(2014)



- simultaneous agreement of three pairing rotational moments of inertia in prolately deformed open-shell nuclei

  evidence for mixing of neutron and proton NG modes
- some sensitivity to density dependence in neutron-rich (proton-deficient) nuclei?

δV_{pn} and pairing rotational MOI

$$\mathcal{J}_{\rm np}^{-1}(N,Z) = -\delta V_{\rm pn}(N+2,Z+2)$$

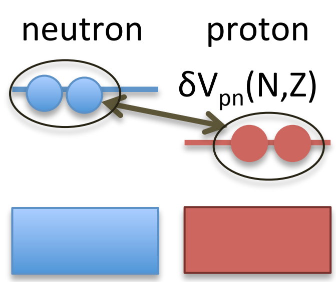
proton-neutron interaction energy

Zhang et al., Phys. Lett. B **227**, 1 (1989)

$$\begin{aligned}\delta V_{\rm pn}(N,Z) &= -\frac{1}{4}[E(N,Z)-E(N-2,Z)-E(N,Z-2)+E(N-2,Z-2)] \\ &= \frac{1}{4}[S_{\rm 2n}(N,Z)-S_{\rm 2n}(N,Z-2)] = \frac{1}{4}[S_{\rm 2p}(N,Z)-S_{\rm 2p}(N-2,Z)]\end{aligned}$$

weak pairing collectivity

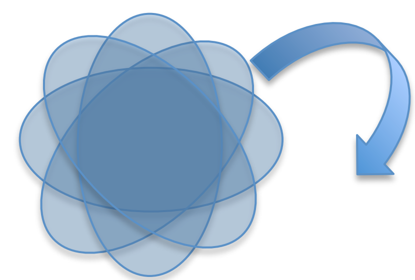
shell model picture
(before symmetry breaking)



interaction energy of
valence 2n and 2p

strong pairing collectivity

collective pairing picture
(after simultaneous symmetry breaking)



$$E_{\rm pairrot}(N,Z) = \frac{(\Delta N)^2}{2\mathcal{J}_{\rm nn}(N_0,Z_0)} + \frac{2(\Delta N)(\Delta Z)}{2\mathcal{J}_{\rm np}(N_0,Z_0)} + \frac{(\Delta Z)^2}{2\mathcal{J}_{\rm pp}(N_0,Z_0)}$$

measure of gauge symmetry breakings of n and p

Pairing picture of binding energy differences

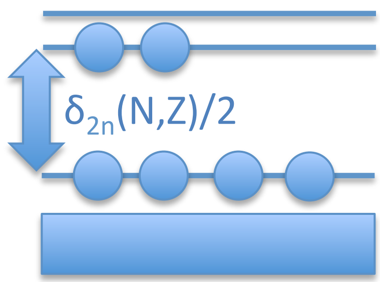
pairing collectivity

quadrupole collectivity

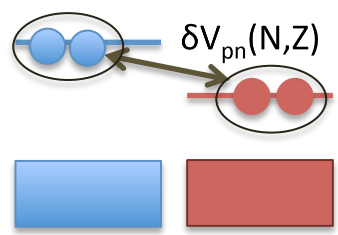
empirical shell gap   δVpn

magic nuclei

signature of magic number



valence proton-neutron interaction energy



gauge symmetry breaking

superconducting nuclei

excitation of two NG modes

$$E_{\text{pairrot}}(N,Z) = \frac{(\Delta N)^2}{2\mathcal{J}_{\text{nn}}(N_0,Z_0)} + \frac{2(\Delta N)(\Delta Z)}{2\mathcal{J}_{\text{np}}(N_0,Z_0)} + \frac{(\Delta Z)^2}{2\mathcal{J}_{\text{pp}}(N_0,Z_0)}$$

$$\mathcal{J}_{\text{nn}}^{-1}(N) = \frac{1}{4}\delta_{2\text{n}}(N) \qquad \mathcal{J}_{\text{np}}^{-1}(N,Z) = -\delta V_{\text{pn}}(N+2,Z+2)$$

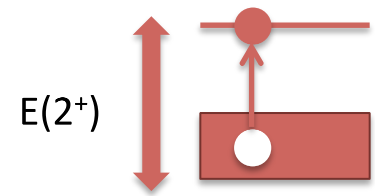
$$\mathcal{J}_{\text{pp}}^{-1}(N,Z) = \frac{1}{4}\delta_{2\text{p}}(N,Z)$$

magnitude of pairing collectivity
(gauge symmetry breaking)

$E(2_1^+)$

doubly magic nuclei

^{208}Pb: $E(2^+)$=4.085 MeV
proton h9/2-h11/2
ph excitation



rotational symmetry breaking

deformed nuclei

excitation of NG mode

$$E(2_1^+) \sim \frac{3}{\mathcal{J}_{\text{rot}}}$$

magnitude of quadrupole collectivity
(deformation)

Summary

first systematic calculation of pairing rotational MOI in doubly open-shell nuclei
Evidence of neutron and proton mixed pairing Nambu-Goldstone modes

1) Pairing rotational MOIs as good indicators of nuclear pairing:
we should use pairing rotational MOIs instead of pairing gaps for comparing with data, adjusting pairing strengths.

2) New interpretation of double binding energy differences:
shell gap δ_{2n} and proton-neutron interaction energy δV_{pn} in terms of gauge symmetry breaking

3) New motivation for mass measurement:
binding energy difference may determine unknown pairing property

Collaborator: Witek Nazarewicz (FRIB/MSU)

References: NH and Nazarewicz, Phys. Rev. Lett. **116**, 152502 (2016)
NH, Phys. Rev. C **92**, 034321 (2015)