TDHFB calculations with Gogny interaction

Yukio Hashimoto

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4. Summary

1. Introduction

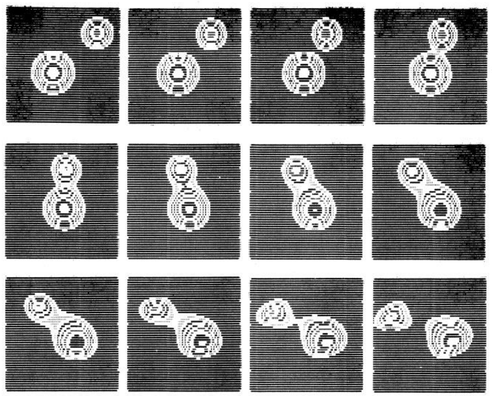
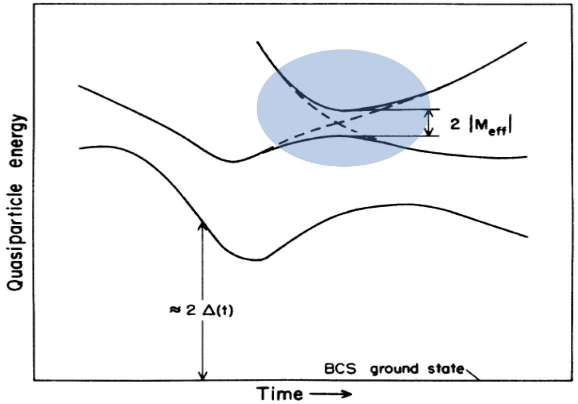


FIG. 40. Contour plots for the same reaction as in Fig. 38 with an initial angular momentum of $l = 80\hbar$.

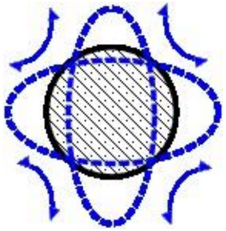
M.S.Weiss, Fizika 9, Suppl. 3(1977), 315.



S.E.Koonin and J.R.Nix, PRC 13(1976), 209.

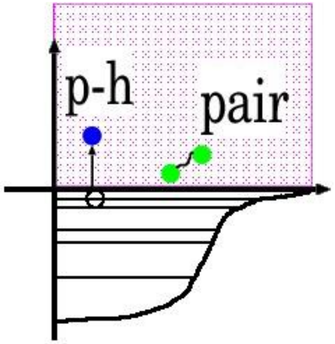
∗ Large-amplitude collective motions
∗ Collision process of two nuclei
← TDHF has been a powerful tool.

TDHFB equations were used to study the dynamical role of the *pairing correlations* in the process of fission and fusion.



Small-amplitude limit of the TDHF and TDHFB
→ random phase approximation (RPA),
quasi-particle RPA (QRPA)

Study of the effects of the pairing correlations and the continuum states in the neutron-rich unstable nuclei



← Skyrme-TDHFB + 3D spatial grids

- C.S.B. Avez and P. Chomaz, Phys. Rev. C78, 044318 (2008).
- S. Ebata, T. Nakatsukasa, T. Inakura, K. Yoshida, Y. Hashimoto, and K. Yabana, Phys. Rev. C82, 034306 (2010).
- Stetcu, A. Bulgac, P. Magierski, and K.J. Roche, Phys. Rev. C84, 051309 (2011).

another candidate:

i) Gogny-TDHFB + 3D harmonic oscillator (HO) eigenfunctions

ii) Gogny-TDHFB + spatial grids

- Y. Hashimoto, Eur. Phys. J. A48, 55 (2012)
- Y. Hashimoto, Phys. Rev. C88, 034307 (2013)

also see T. Matsuse, RIKEN Review 19, 18 (1998).

Recent results of the Gogny-TDHFB calculations

*** 3D HO eigenfunctions**

*** 2D HO + Lagrange mesh (z-axis)**

2. TDHFB equations

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} U(t) \\ V(t) \end{pmatrix} = \mathcal{H}\begin{pmatrix} U(t) \\ V(t) \end{pmatrix}, \qquad \mathcal{H} = \begin{pmatrix} h & \Delta \\ -\Delta^* & -h^* \end{pmatrix}$$

$$\beta_k^\dagger = \sum_\alpha \left(U_{\alpha k} C_\alpha^\dagger + V_{\alpha k} C_\alpha\right)$$

$$\beta_k = \sum_\alpha \left(U_{\alpha k}^* C_\alpha + V_{\alpha k}^* C_\alpha^\dagger\right)$$

$$i\hbar\frac{\partial}{\partial t}U_{\alpha k} = \sum_\beta \left(h_{\alpha\beta}U_{\beta k} + \Delta_{\alpha\beta}V_{\beta k}\right),$$

$$i\hbar\frac{\partial}{\partial t}V_{\alpha k} = -\sum_\beta \left(\Delta_{\alpha\beta}^* U_{\beta k} + h_{\alpha\beta}^* V_{\beta k}\right),$$

$$h_{\alpha\beta} = T_{\alpha\beta} + \Gamma_{\alpha\beta},$$

$$\Gamma_{\alpha\beta} = \sum_{\gamma\delta} \mathcal{V}_{\alpha\gamma\beta\delta}\rho_{\delta\gamma}, \quad \Delta_{\alpha\beta} = \frac{1}{2}\sum \mathcal{V}_{\alpha\beta\gamma\delta}\kappa_{\gamma\delta},$$

$$\begin{pmatrix} U \\ V \end{pmatrix}^{(n+1)} = \exp\left(-i\frac{\Delta t}{\hbar}\mathcal{H}^{(n+1/2)}\right)\begin{pmatrix} U \\ V \end{pmatrix}^{(n)}$$

$$i\frac{\partial}{\partial t}U_{\alpha k} = \sum_l \left\{U_{\alpha l}H_{11}(lk) - V_{\alpha l}^* H_{20}^*(lk)\right\},$$

$$i\frac{\partial}{\partial t}V_{\alpha k}^* = \sum_l \left\{U_{\alpha l}H_{20}(lk) - V_{\alpha l}^* H_{11}^*(lk)\right\},$$

$$H_{11}(kl) = \sum_{\alpha\beta} U_{\alpha k}^*\left(h_{\alpha\beta}U_{\beta l} + \Delta_{\alpha\beta}V_{\beta l}\right) - \sum_{\alpha\beta} V_{\alpha k}^*\left(h_{\alpha\beta}^* V_{\beta l} + \Delta_{\alpha\beta}^* U_{\beta l}\right),$$

$$H_{20}(kl) = \sum_{\alpha\beta} U_{\alpha k}^*\left(h_{\alpha\beta}V_{\beta l}^* + \Delta_{\alpha\beta}U_{\beta l}^*\right) - \sum_{\alpha\beta} V_{\alpha k}^*\left(h_{\alpha\beta}^* U_{\beta l}^* + \Delta_{\alpha\beta}^* V_{\beta l}^*\right)$$

$$\begin{pmatrix} U & V^* \end{pmatrix}^{(n+1)} = \begin{pmatrix} U & V^* \end{pmatrix}^{(n)} \exp\left(-i\frac{\Delta t}{\hbar}\bar{H}_{HFB}\right)$$

Gogny-D1S

$$V_{12} = \sum_{i=1}^{2} \exp\left[-\frac{|\vec{r}_1 - \vec{r}_2|^2}{\mu_i^2}\right] \cdot (W_i + B_i \hat{P}_\sigma - H_i \hat{P}_\tau - M_i \hat{P}_\sigma \hat{P}_\tau) +$$

Gauss part

$$+t_3(1 + x_0 \hat{P}_\sigma)\,\delta(\vec{r}_1 - \vec{r}_2)\left[\rho\left(\frac{\vec{r}_1 + \vec{r}_2}{2}\right)\right]^\gamma +$$

density dependent part

$$+iW_{\mathrm{LS}}(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \overleftarrow{\nabla}_{12} \times \delta(\vec{r}_1 - \vec{r}_2)\vec{\nabla}_{12} + V_{\mathrm{Coul.}}\,,$$

L-S part, Coulomb

$\mu_1 = 0.7$ fm	$\mu_2 = 1.2$ fm
$W_1 =$ -1720.3 MeV	$W_2 = 103.639$ MeV
$B_1 = 1300$ MeV	$B_2 =$ -163.483 MeV
$H_1 =$ -1813.53 MeV	$H_2 = 162.812$ MeV
$M_1 = 1397.60$ MeV	$M_2 =$ -223.934 MeV
$t_3 = 1390.60$ MeV fm$^{3(1+\gamma)}$	$x_0 = 1$
$\gamma = 1/3$	$W_{LS} = 130$ MeV fm^5

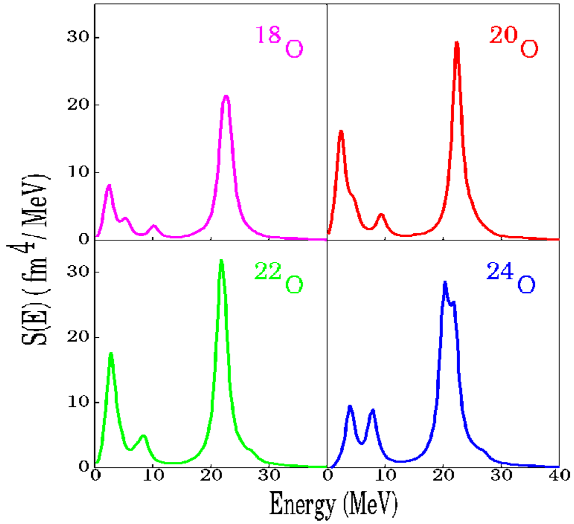
- basis function：three-dimensional harmonic oscillator wave functions
- space： $N_{\mathrm{shell}} = n_x + n_y + n_z \leq 5$

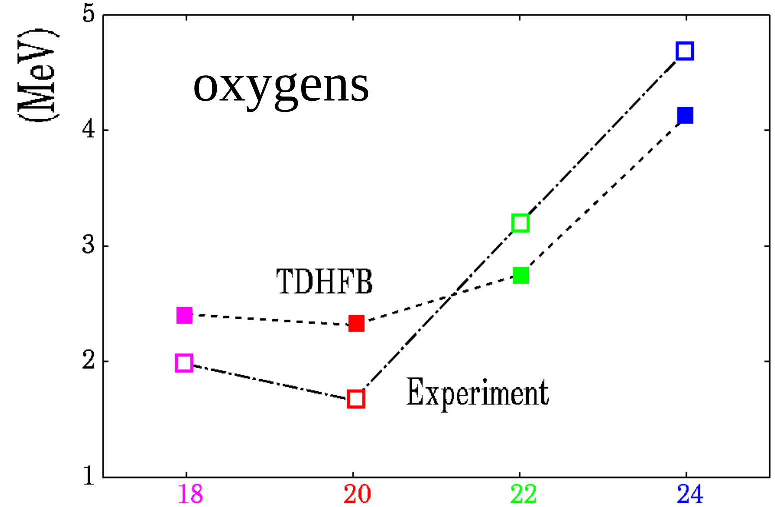
3. Results

3 - I. TDHFB calculations with the three-dimensional (3D) harmonic oscillator basis

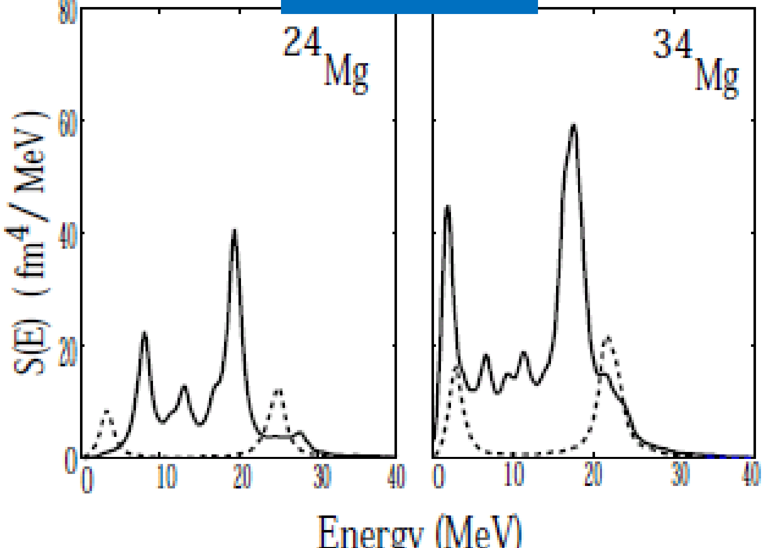
Strength functions & energies from linear response

quadrupole

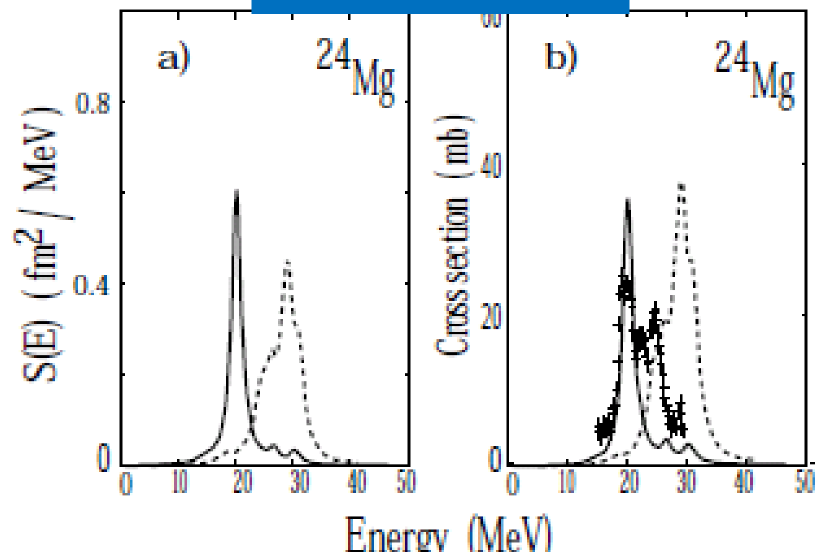




quadrupole



isovector dipole

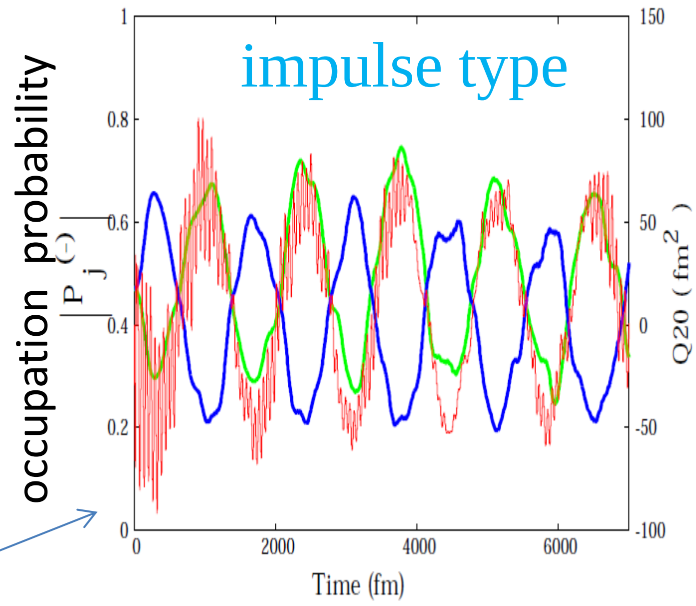


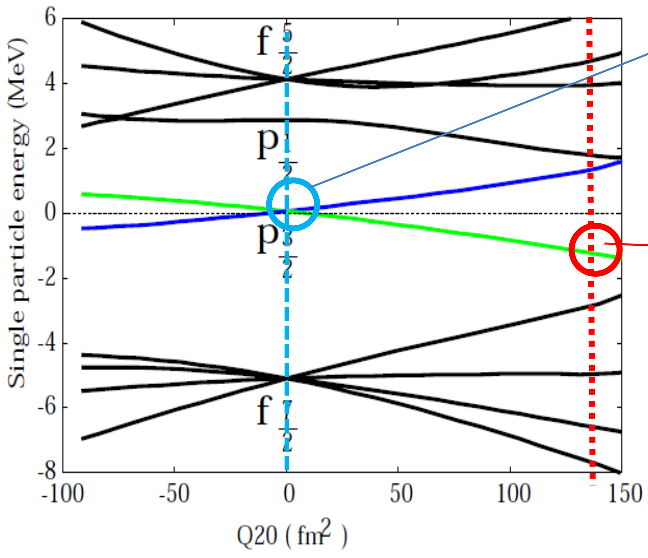
Variation of occupation probability in quadrupole vibration

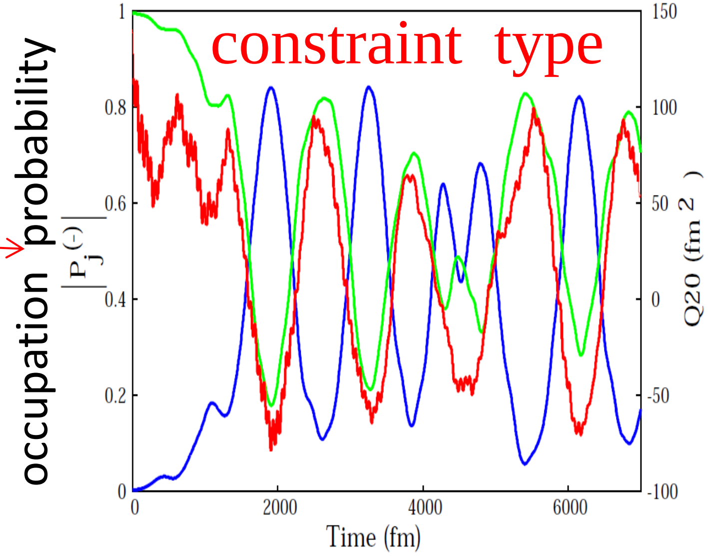
^{52}Ti

occupation probability of orbital (k):

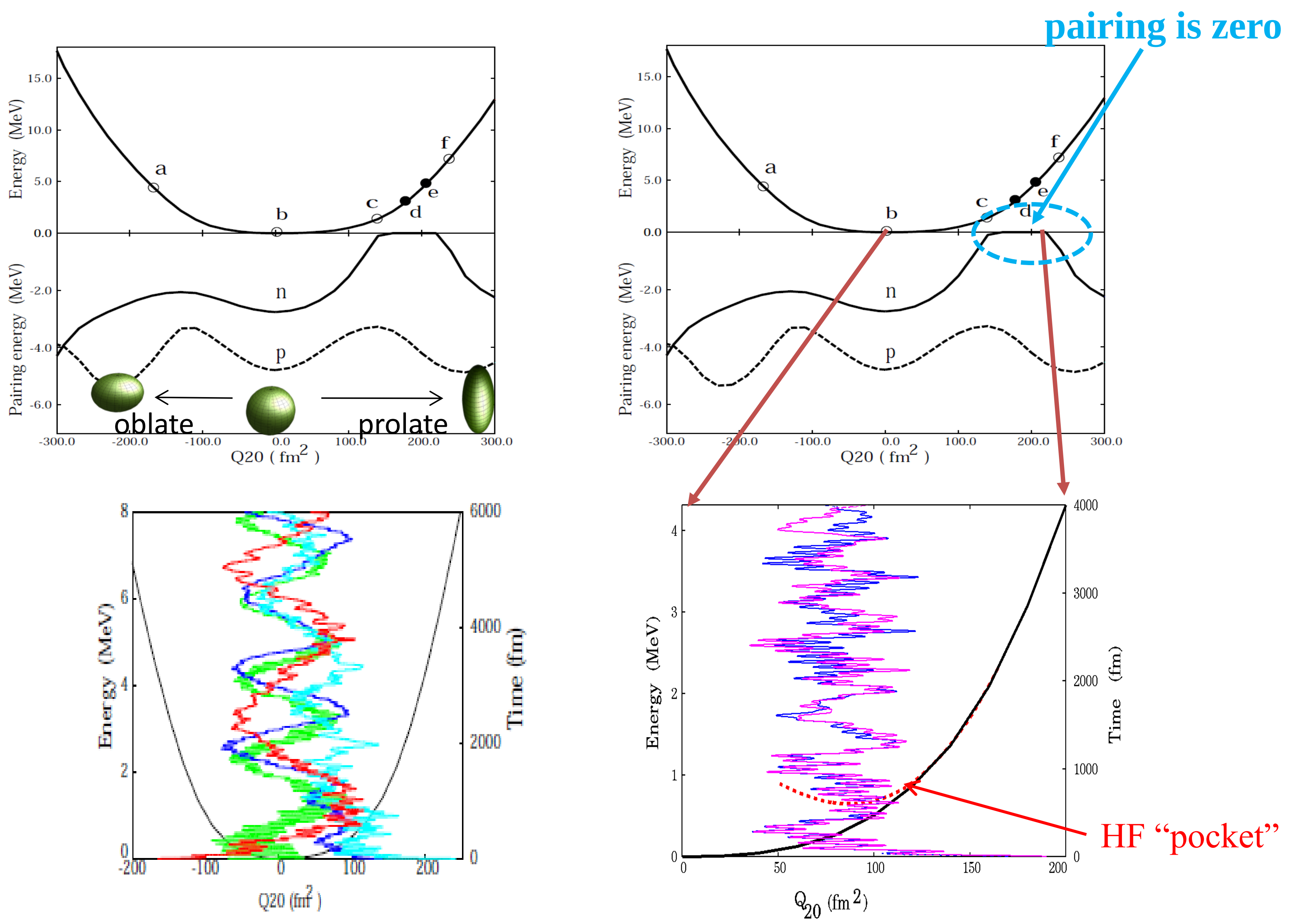
$$p(k) = \sum_\alpha V^*_{\alpha k} V_{\alpha k}$$





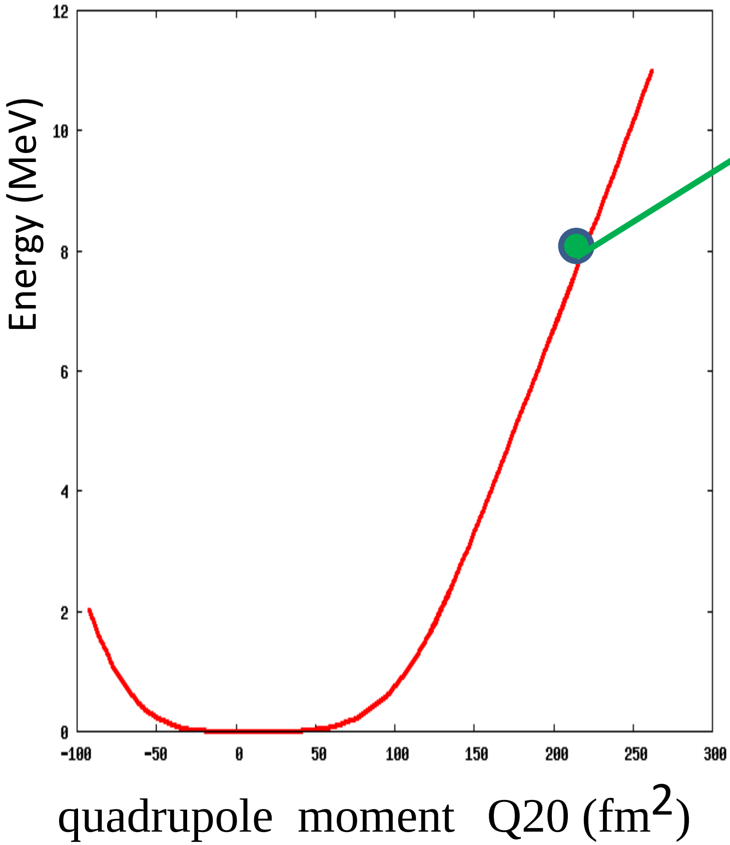


Nonlinear quadrupole vibration and pairing in ^{52}Ti

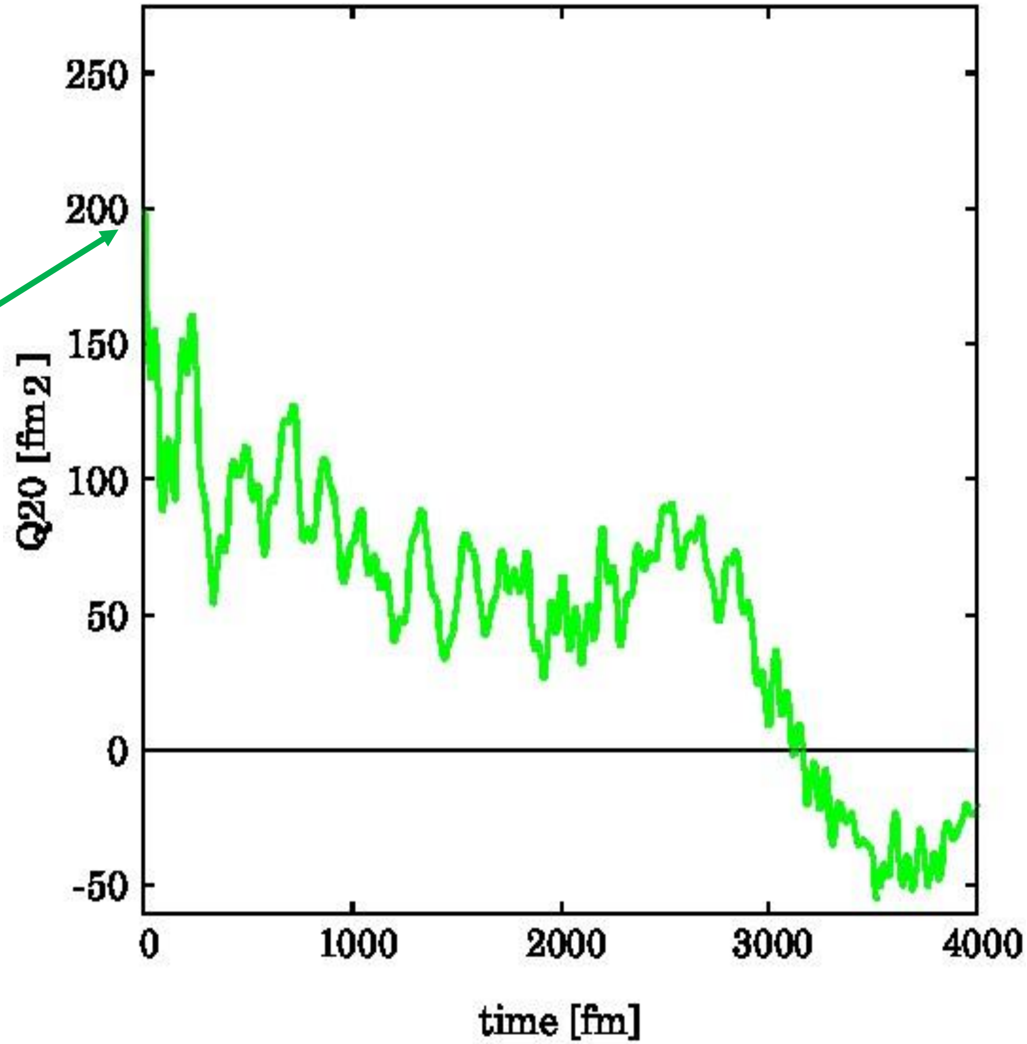


Relaxation of nonlinear quadrupole vibration of ^{44}Ti

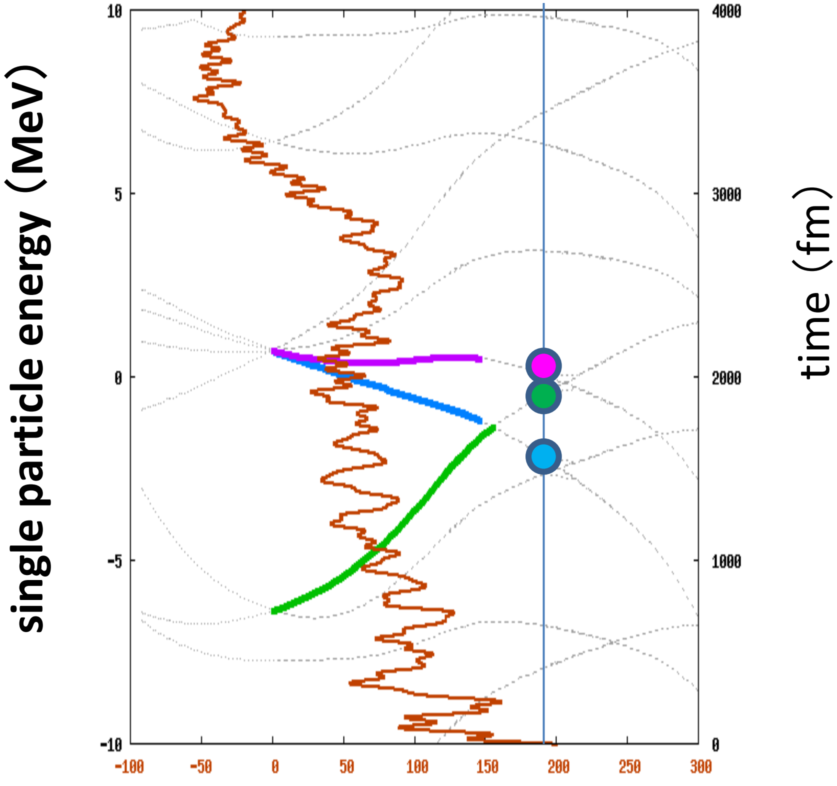
^{44}Ti Energy vs Q20



Quadrupole moment vs time



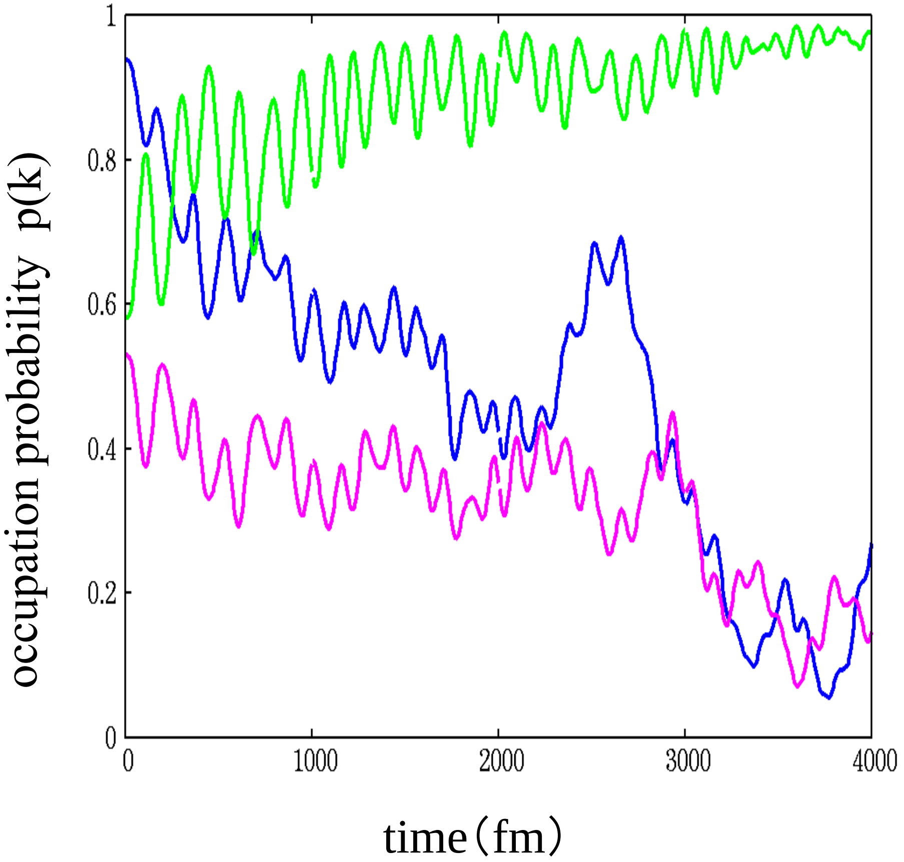
Relaxation of quadrupole oscillation （^{44}Ti ）



quadrupole moment

occupation probability p(k)

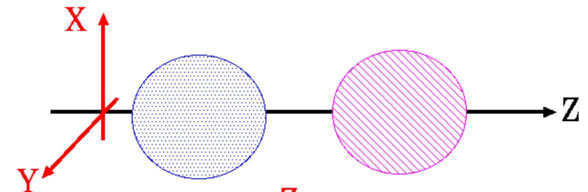
(protons)



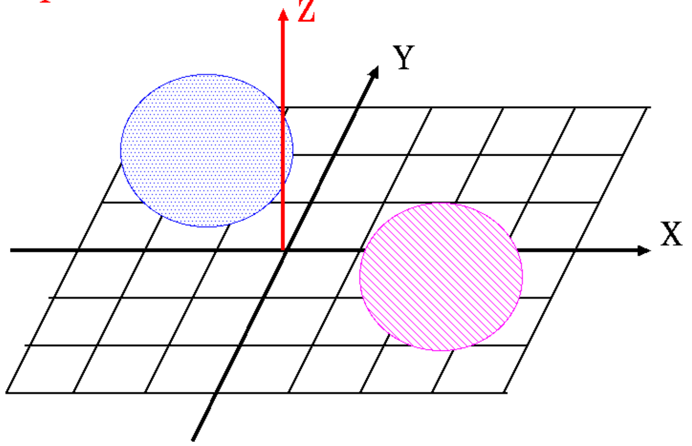
3 - II. TDHFB calculations with a Lagrange mesh

-- one-dimensional (1D) case --

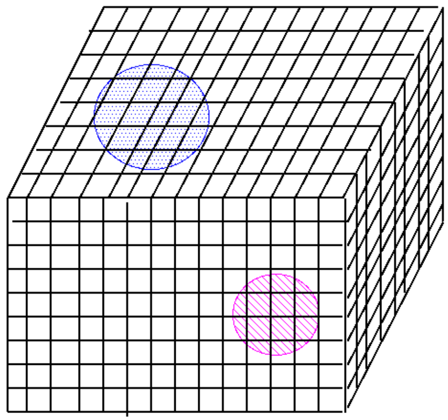
harmonic oscillator	Lagrange mesh
x, y	z



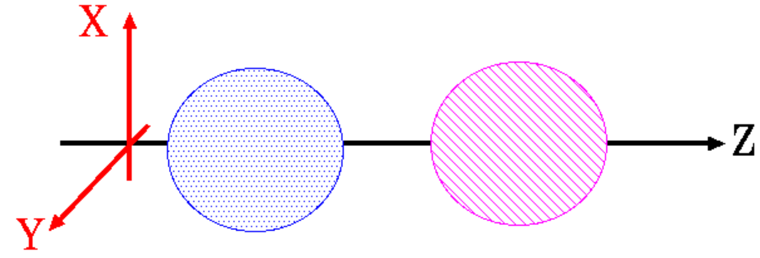
harmonic oscillator	Lagrange mesh
z	x, y



harmonic oscillator	Lagrange mesh
———	x, y, z



harmonic oscillator	Lagrange mesh
x, y	z



$$\left\{\phi_{n_x}(x), \phi_{n_y}(y), \phi_{n_z}(z)\right\} \Longrightarrow \left\{\phi_{n_x}(x), \phi_{n_y}(y), \underline{f_{n_z}(z)}\right\}$$

harmonic oscillator

Lagrange mesh

$$f_l(z) = \frac{1}{N}\frac{\sin\left(\pi\left(z - z_l\right)/h\right)}{\sin\left(\pi\left(z - z_l\right)/L\right)}$$

$$L = Nh$$

$$f_k(z_{k'}) = \delta_{kk'}$$

$$\int_{-L/2}^{L/2} f_l(z) f_{l'}(z) dz = h\,\delta_{ll'}$$

$$\int_{-L/2}^{L/2} f_l(z) W(z) f_{l'}(z) dz = h\,W(z_l)\,\delta_{ll'}$$

D. Baye and P. Heenen,
J. Phys. A 19, 2041 (1986).

* HFB : gradient method , diagonalization, number constraints

* time evolution:

$$h_{\alpha\beta} = T_{\alpha\beta} + \Gamma_{\alpha\beta},$$

$$\Gamma_{\alpha\beta} = \sum_{\gamma\delta} \mathcal{V}_{\alpha\gamma\beta\delta}\rho_{\delta\gamma}, \quad \Delta_{\alpha\beta} = \frac{1}{2}\sum \mathcal{V}_{\alpha\beta\gamma\delta}\kappa_{\gamma\delta},$$

$$i\hbar \frac{\partial}{\partial t}\begin{pmatrix} U(t) \\ V(t) \end{pmatrix} = \mathcal{H}\begin{pmatrix} U(t) \\ V(t) \end{pmatrix} \qquad \mathcal{H} = \begin{pmatrix} h & \Delta \\ -\Delta^* & -h^* \end{pmatrix}.$$

$$i\hbar\frac{\partial}{\partial t}\left(U(t)\ V^*(t)\right) = \left(U(t)\ V^*(t)\right)\bar{H}_{\mathrm{HFB}} \qquad \bar{H}_{\mathrm{HFB}} = \begin{pmatrix} H_{11} & H_{20} \\ -H_{20}^* & -H_{11}^* \end{pmatrix}$$

$$H_{11}(kl) = \sum_{\alpha\beta} U_{\alpha k}^* \left(h_{\alpha\beta}U_{\beta l} + \Delta_{\alpha\beta}V_{\beta l}\right) - \sum_{\alpha\beta} V_{\alpha k}^* \left(h_{\alpha\beta}^* V_{\beta l} + \Delta_{\alpha\beta}^* U_{\beta l}\right)$$

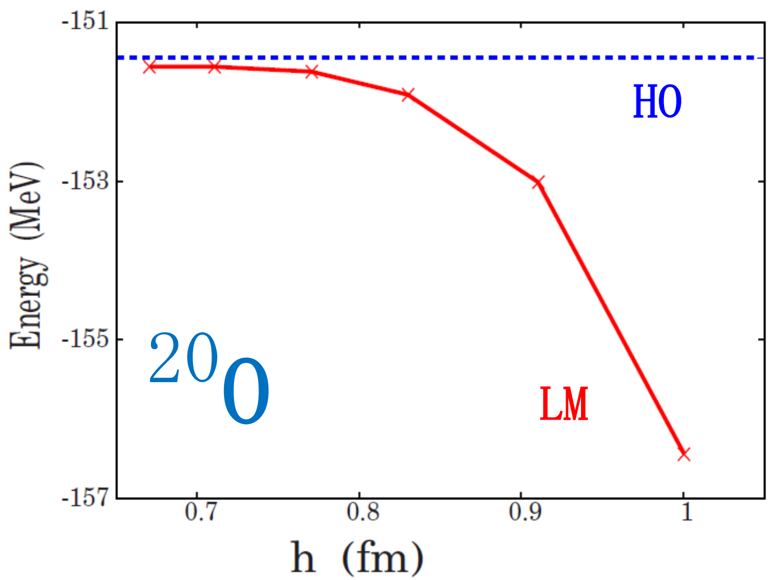
$$H_{20}(kl) = \sum_{\alpha\beta} U_{\alpha k}^* \left(h_{\alpha\beta}V_{\beta l}^* + \Delta_{\alpha\beta}U_{\beta l}^*\right) - \sum_{\alpha\beta} V_{\alpha k}^* \left(h_{\alpha\beta}^* U_{\beta l}^* + \Delta_{\alpha\beta}^* V_{\beta l}^*\right)$$

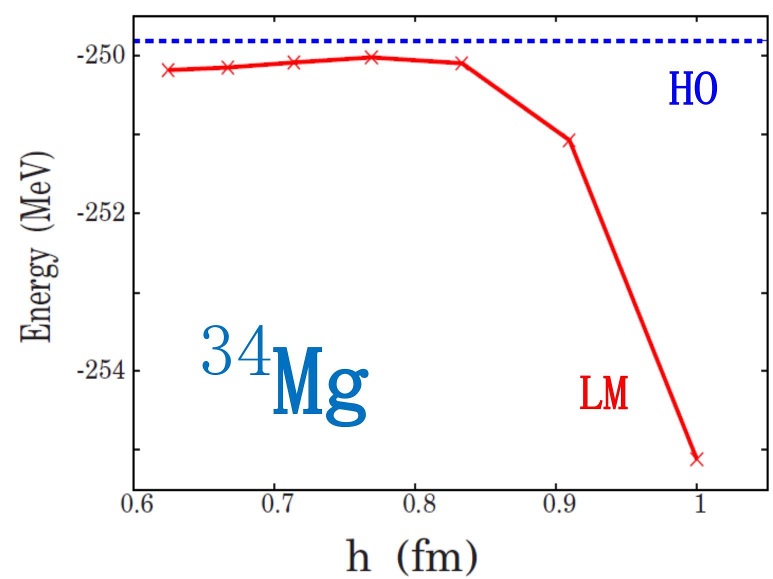
$$(U\ \ V^*)^{(n+1)} = (U\ \ V^*)^{(n)} \exp\left(-i\frac{\Delta t}{\hbar}\bar{H}_{\mathrm{HFB}}\right)$$

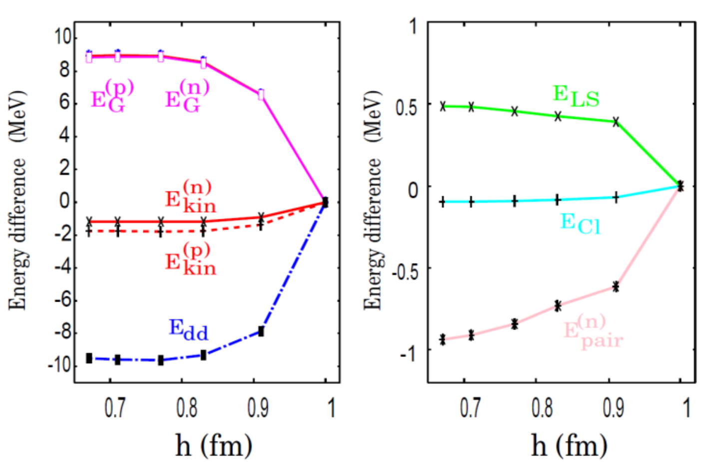
Examples of
HFB and TDHFB calculations
with a Lagrange mesh

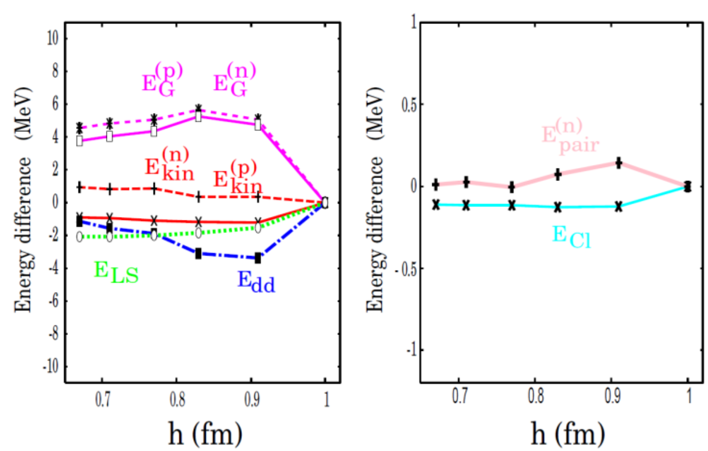
HFB: ^{20}O and ^{34}Mg

Nx_max = Ny_max = 4,
Nmsh = 23, L = 20 fm
quasiparticle rbitals = 70

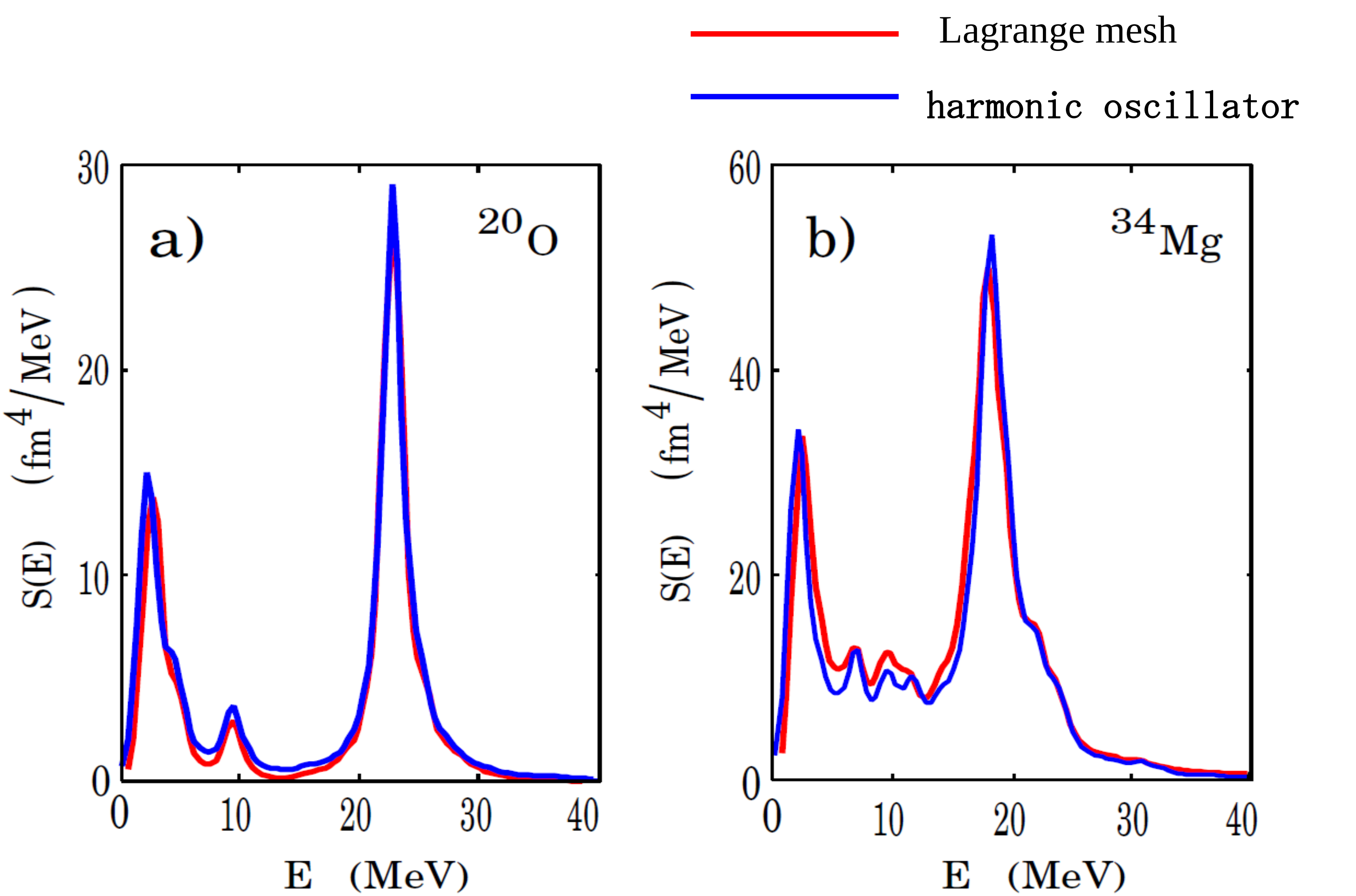






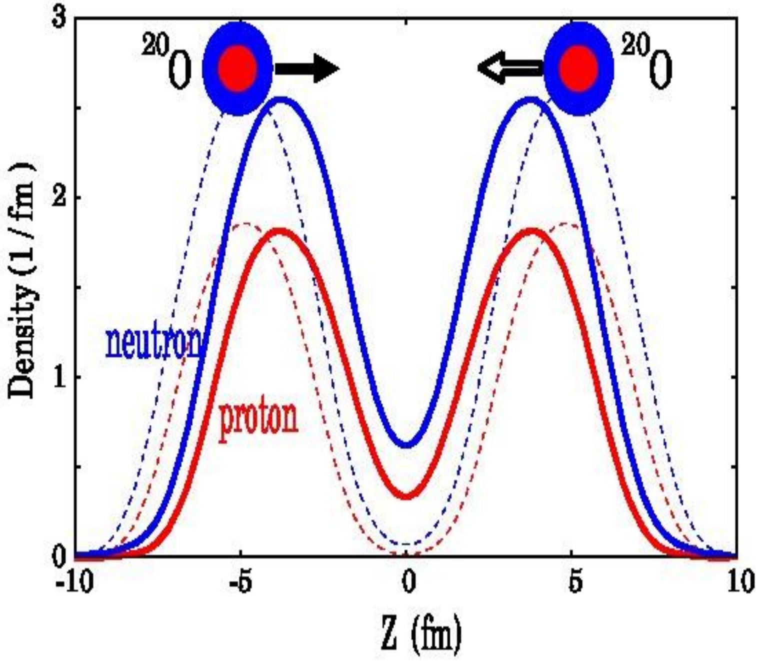


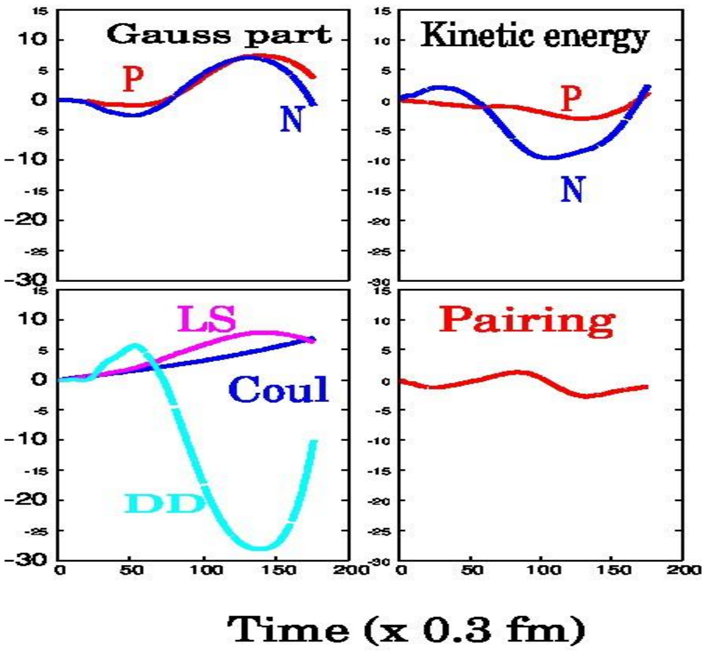
Strength functions of quadrupole [K=0] vibrations



3 - III. TDHFB calculations with a Lagrange mesh

-- Application to head-on collision $^{20}O + ^{20}O$ --





Now, the calculations are running on a computer.

4. Summary

1． (HF, TDHF,) HFB, TDHFB calculations
with Gogny interaction
* 3D harmonic oscillator basis
* 2D harmonic oscillator basis ＋ Lagrange mesh

○ strength functions
○ adiabatic change of occupation probabilities
across energy-crossing point

2. Applications to the nucleus-nucleus head-on
collisions are in progress.

3. Extension to the full 3D spatial mesh is in progress.