

# Beyond-mean-field theory for multi-octupole excitations in $^{208}\text{Pb}$ and subbarrier fusion of $^{16}\text{O} + ^{208}\text{Pb}$



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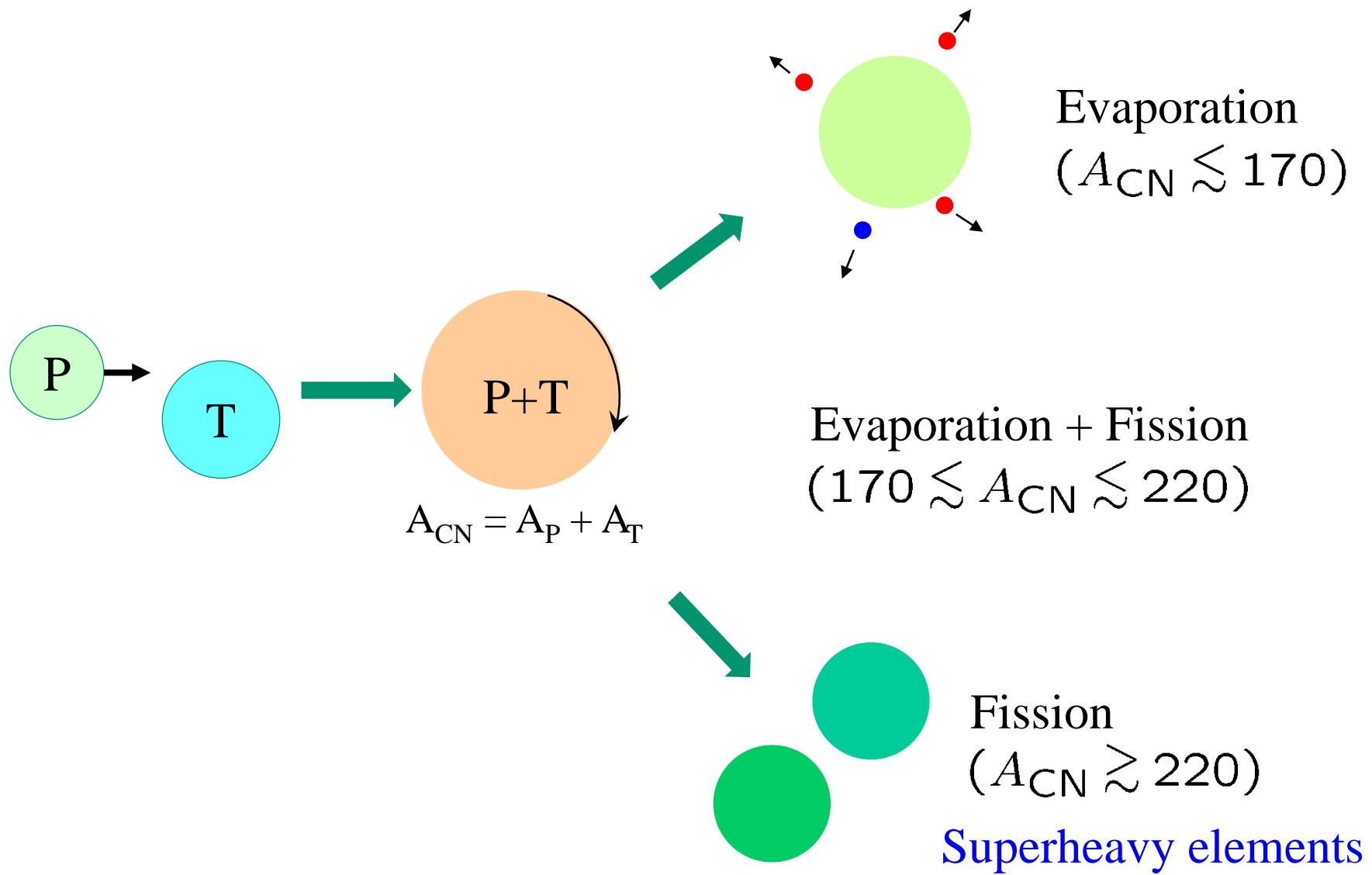
/ Tohoku → N.Carolina U.)



1. *Introduction: Heavy-ion subbarrier fusion reactions*
2. *Coupled-channels approach  
with “beyond-mean-field” method*
3. *Octupole excitations in fusion of  $^{16}\text{O} + ^{208}\text{Pb}$*
4. *Summary*

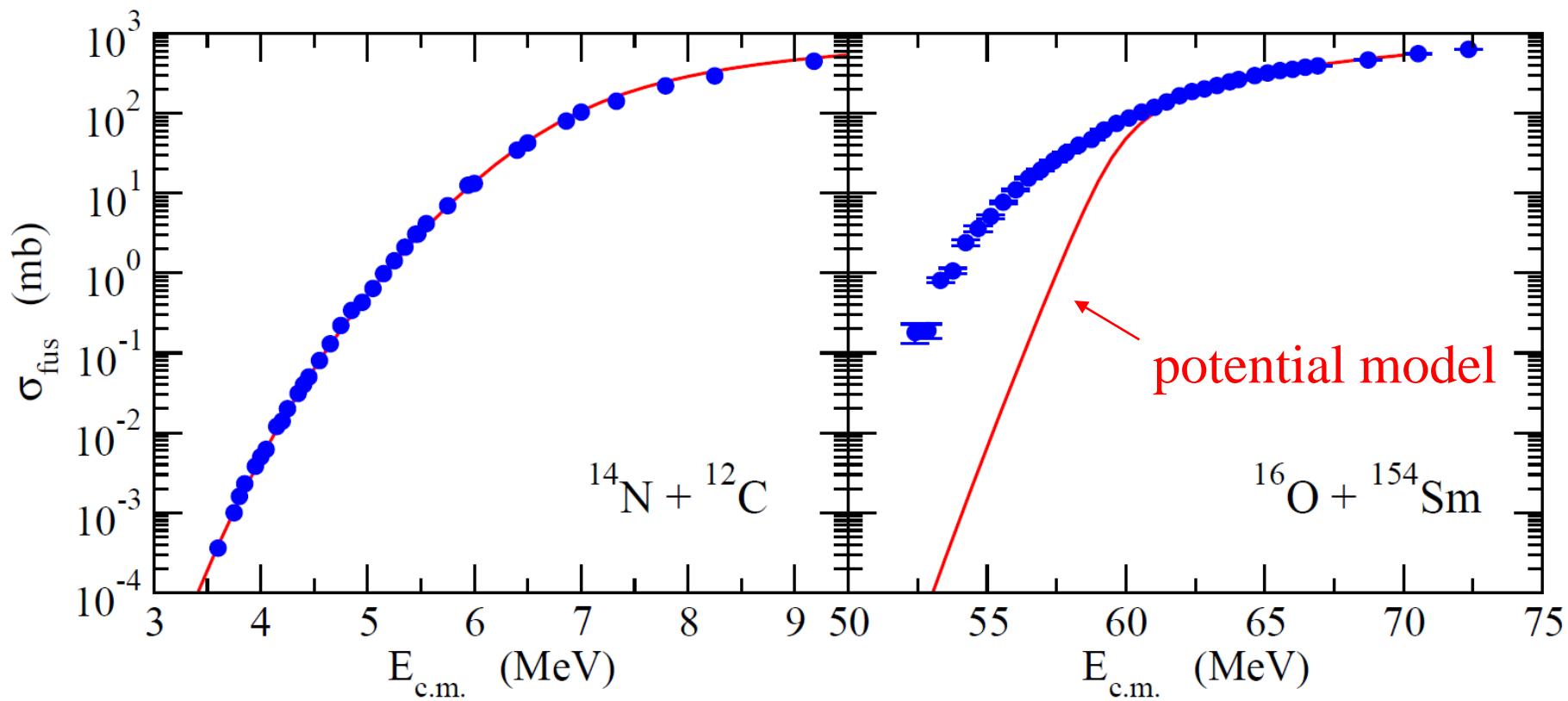
# Introduction: Heavy-ion fusion reactions

Fusion: compound nucleus formation



# Discovery of large sub-barrier enhancement of $\sigma_{\text{fus}}$

potential model:  $V(r) + \text{absorption}$

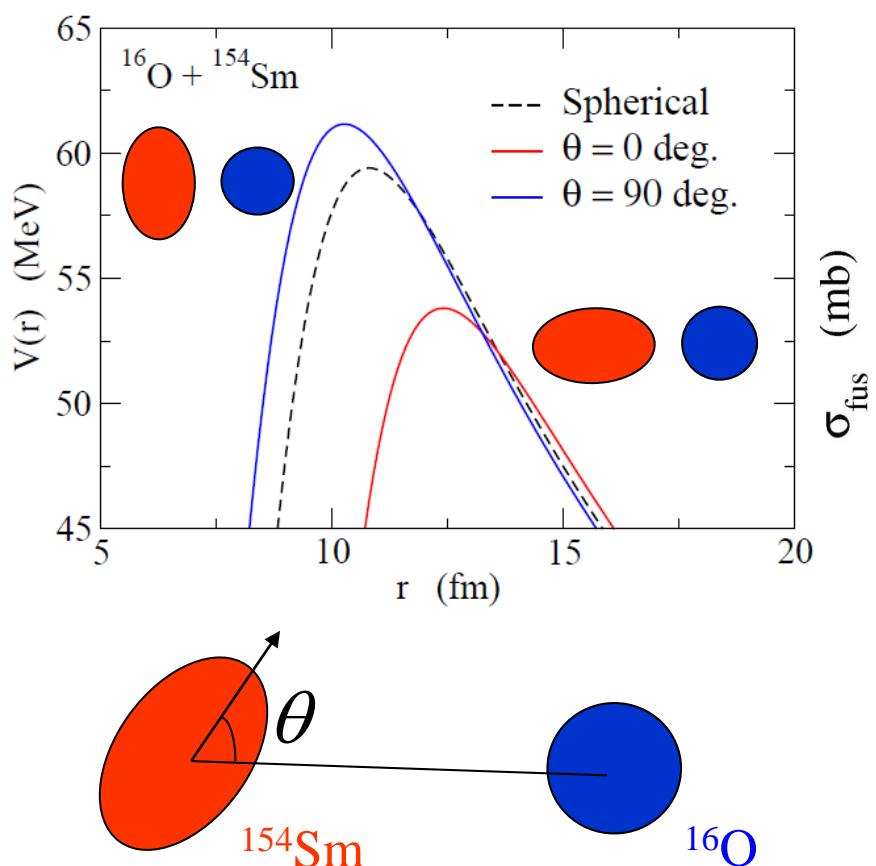


cf. seminal work:

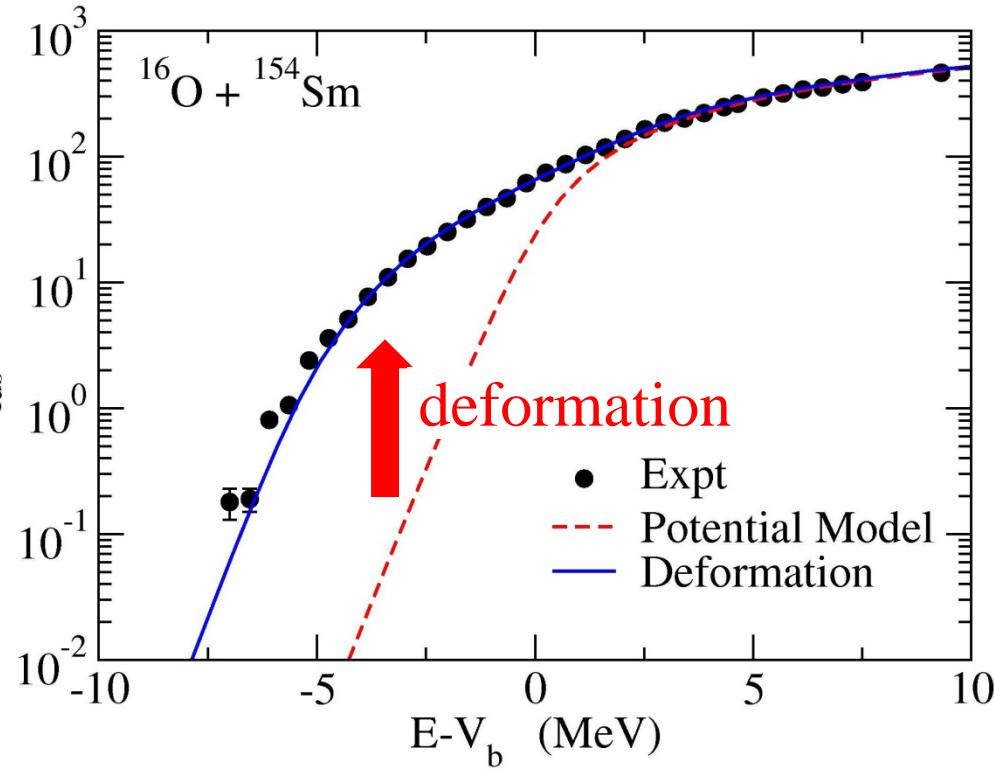
R.G. Stokstad et al., PRL41('78) 465

## Effect of nuclear deformation

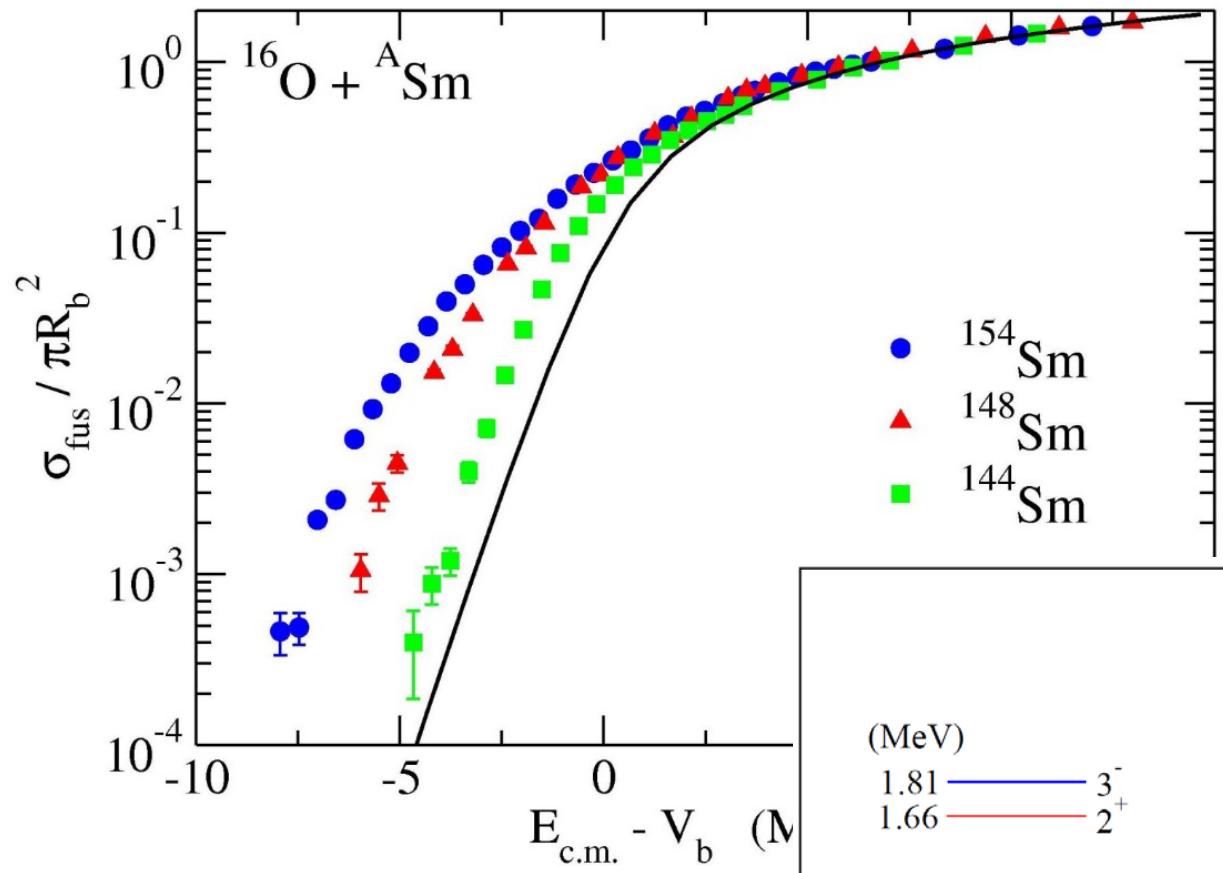
$^{154}\text{Sm}$  : a deformed nucleus with  $\beta_2 \sim 0.3$



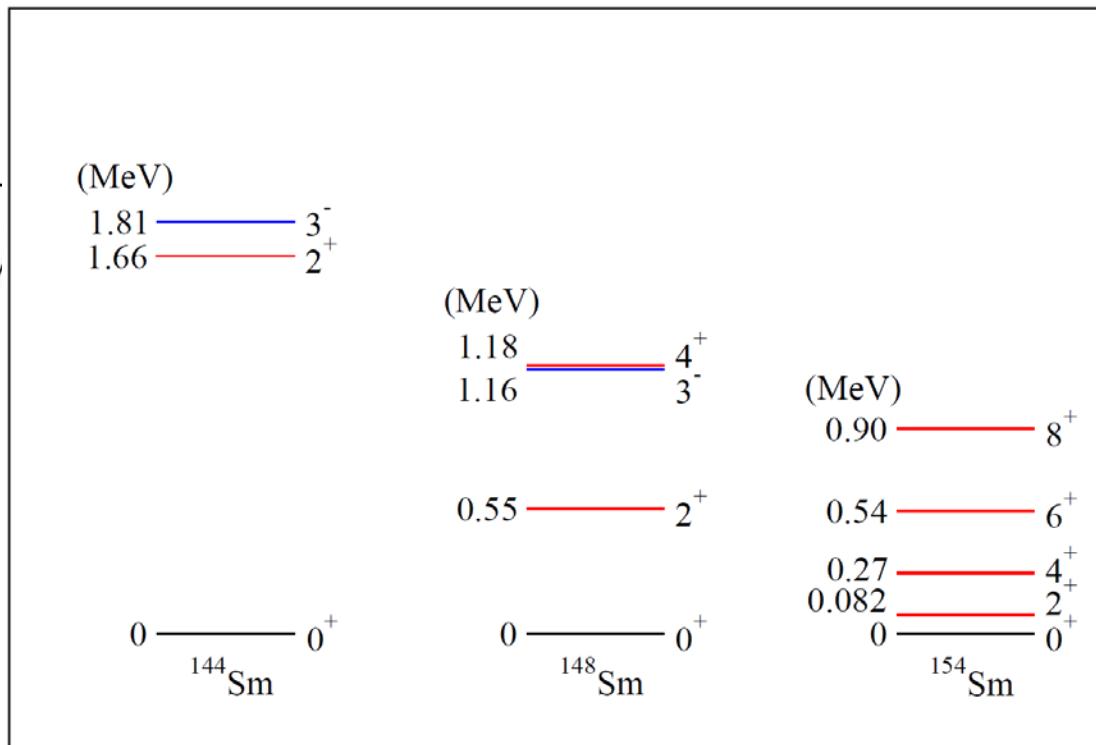
$$\sigma_{\text{fus}}(E) = \int_0^1 d(\cos \theta) \sigma_{\text{fus}}(E; \theta)$$



Fusion: strong interplay between nuclear structure and nuclear reaction

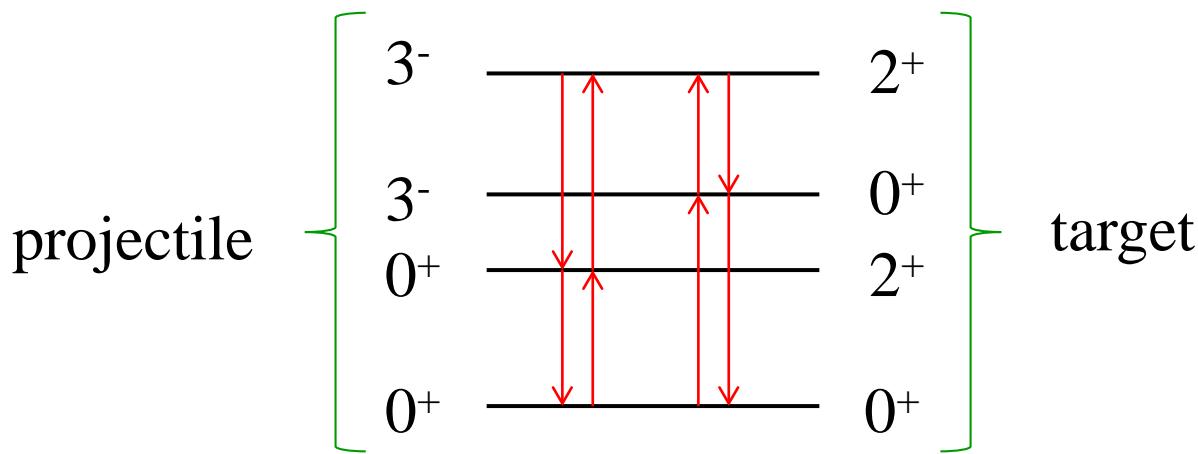
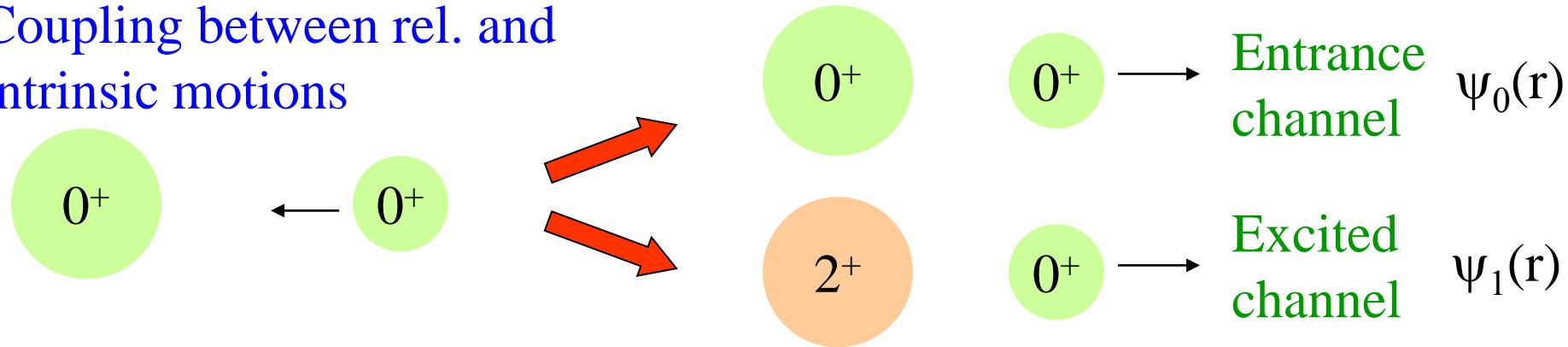


Strong target dependence  
at  $E < V_b$



# Coupled-Channels method

Coupling between rel. and intrinsic motions



$$\Psi(r, \xi) = \sum_k \psi_k(r) \phi_k(\xi)$$



coupled Schroedinger equations for  $\psi_k(r)$

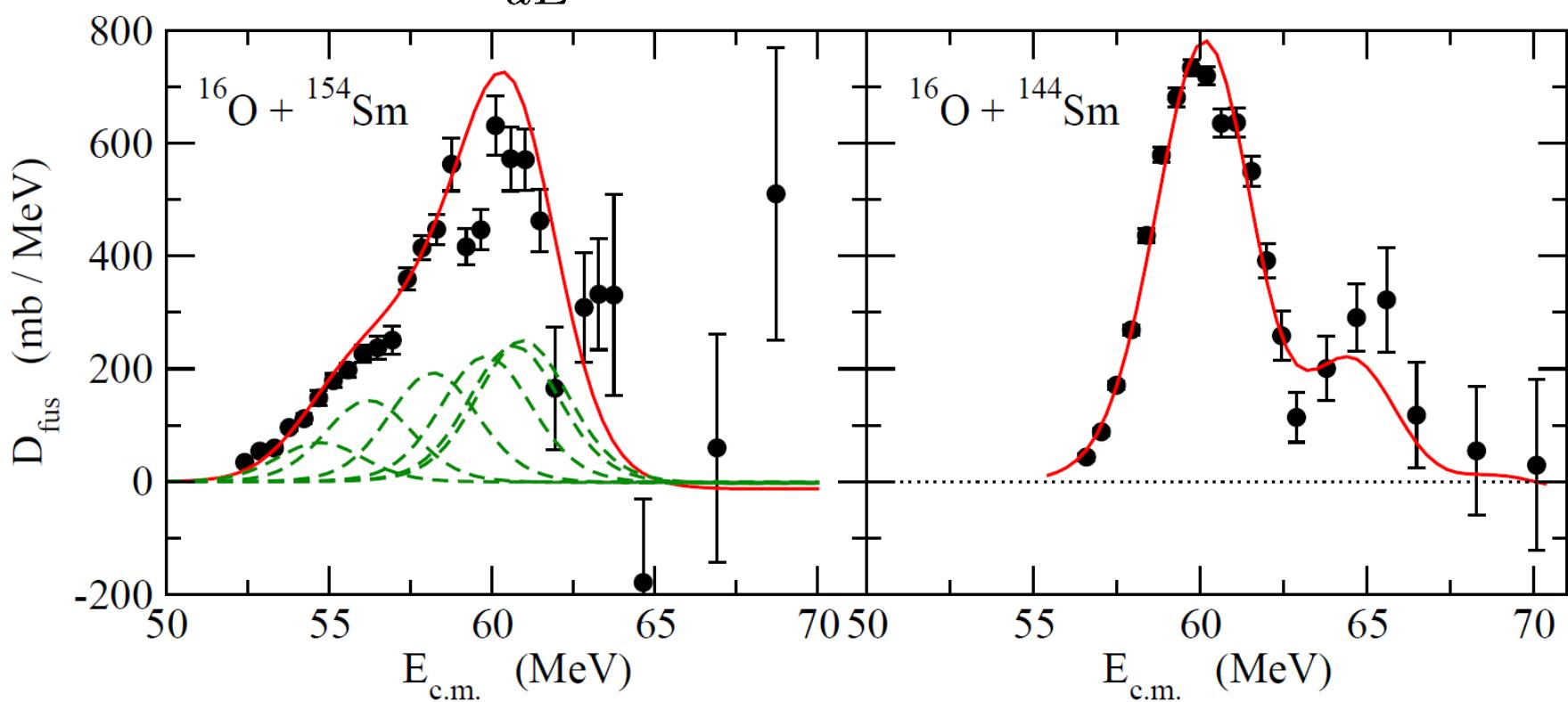
## C.C. approach: a standard tool for sub-barrier fusion reactions

cf. CCFULL (K.H., N. Rowley, A.T. Kruppa, CPC123 ('99) 143)

✓ Fusion barrier distribution (Rowley, Satchler, Stelson, PLB254('91))

$$D_{\text{fus}}(E) = \frac{d^2(E\sigma_{\text{fus}})}{dE^2}$$

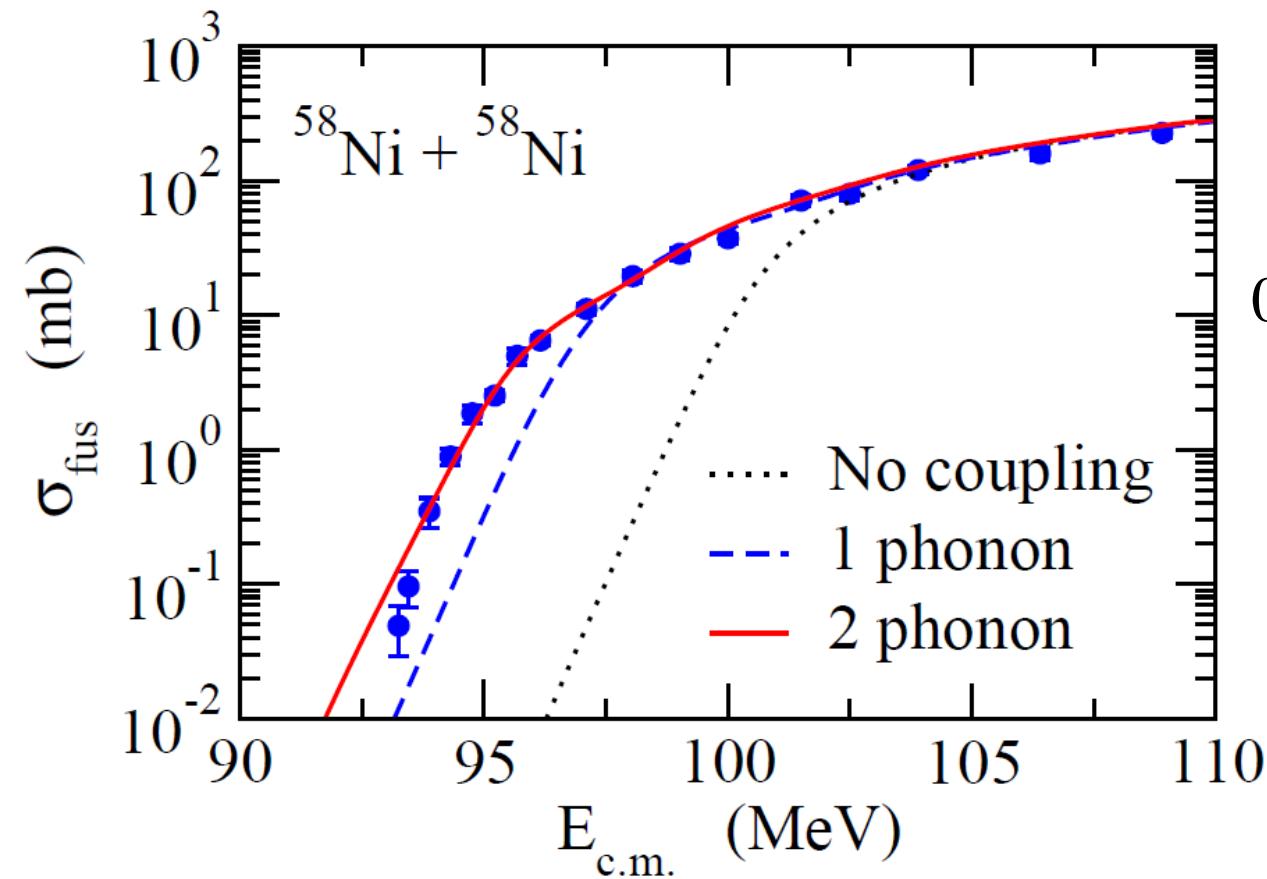
— c.c. calculations



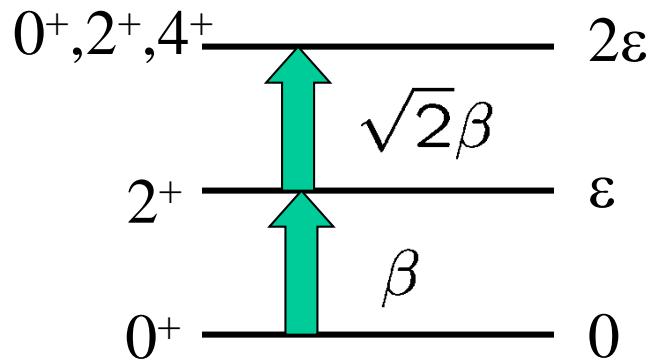
# Semi-microscopic modeling of sub-barrier fusion

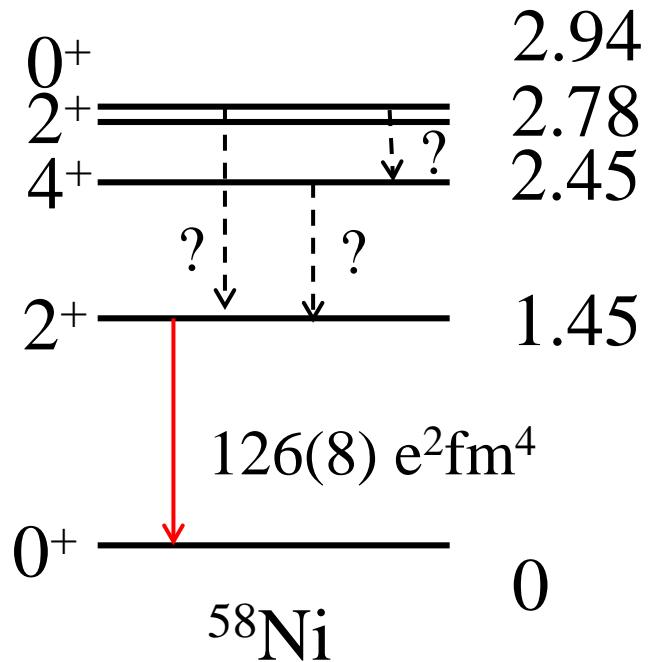
K.H. and J.M. Yao, PRC91('15) 064606

## multi-phonon excitations



simple harmonic oscillator





$$Q(2_1^+) = -10 \pm 6 \text{ efm}^2$$

Simple harmonic oscillator  
: justifiable?

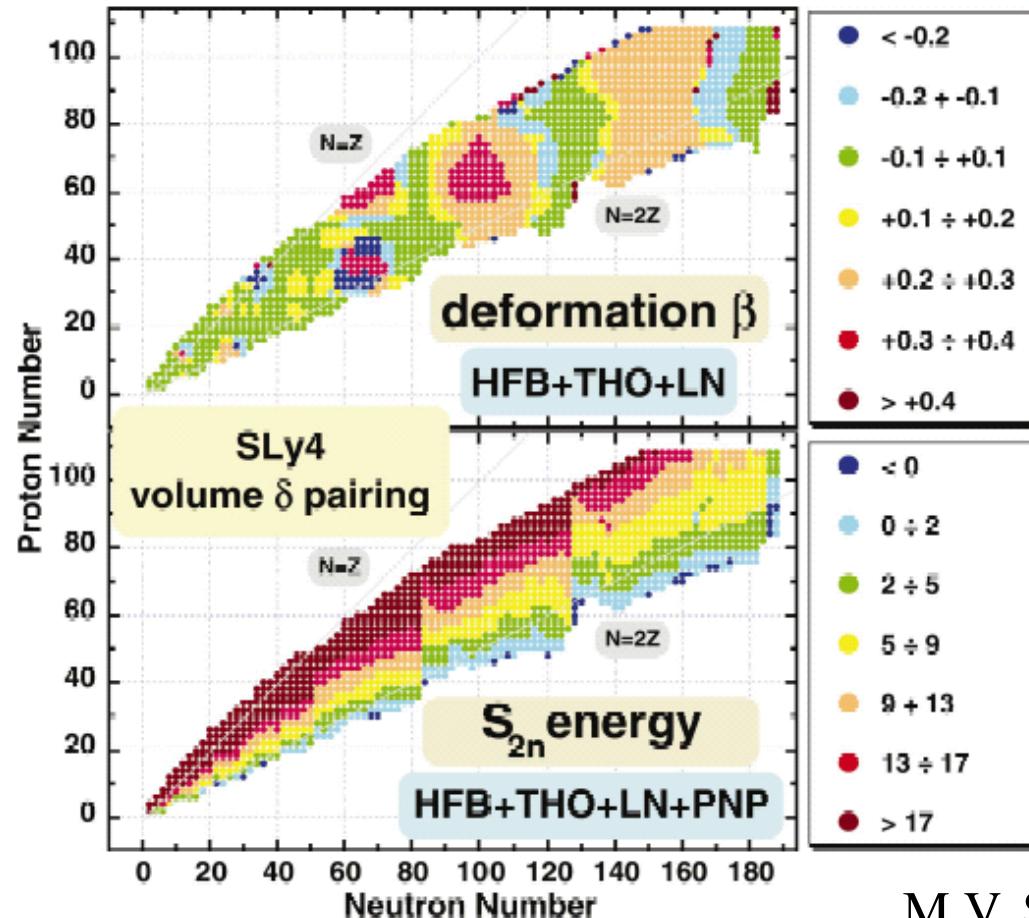
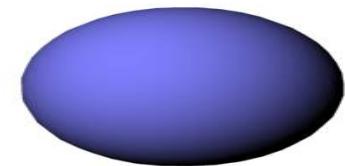


beyond mean-field approach  
to account for anharmonicity

cf. GCM: large amplitude  
collective motions

## Self-consistent mean-field (Hartree-Fock) method:

- independent particles in a mean-field potential
- global theory for **the whole nuclear chart**
- intuitive picture for nuclear deformation
- optimized shape can be automatically determined



# Mean-field approximation and beyond

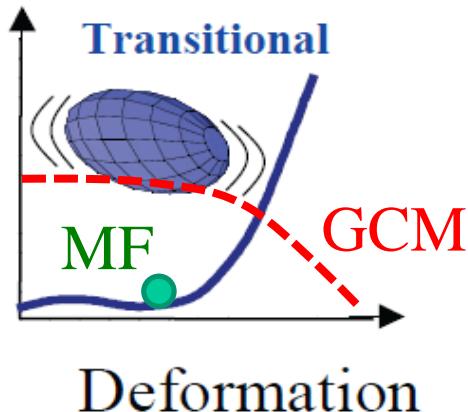
## Drawbacks of the mean-field approximation : nuclear spectrum

- ✓ body-fixed frame formalism → intuitive picture of nuclear def.
  - ✓ spectrum: lab-frame ← transformation from intrinsic to lab. frames
- nuclear spectrum: requires to go beyond the mean-field approximation

$$|\Psi_{IM}(\beta)\rangle = \hat{P}_{MK}^I \hat{P}^N \hat{P}^Z |\Psi_{\text{MF}}(\beta)\rangle$$

angular momentum + particle number projections

- ✓ quantum fluctuation



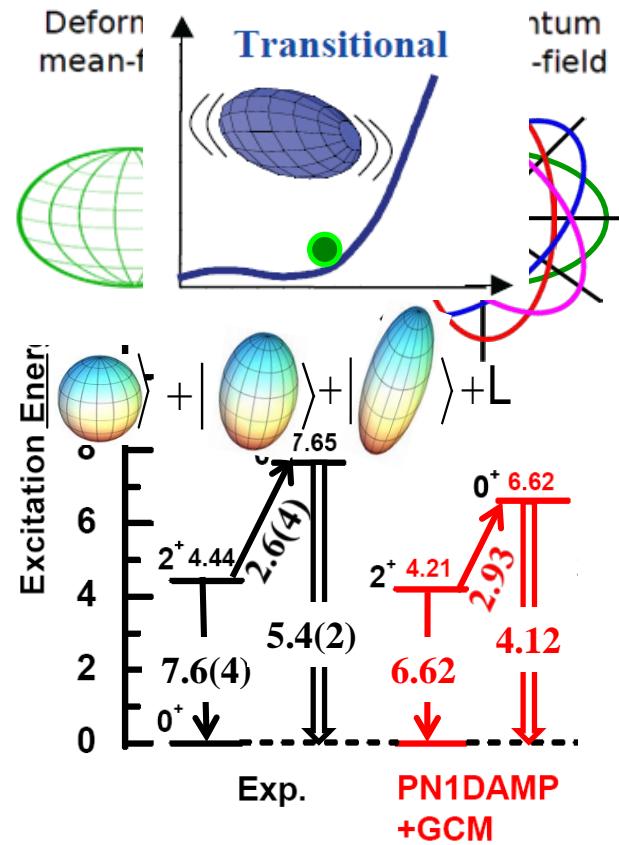
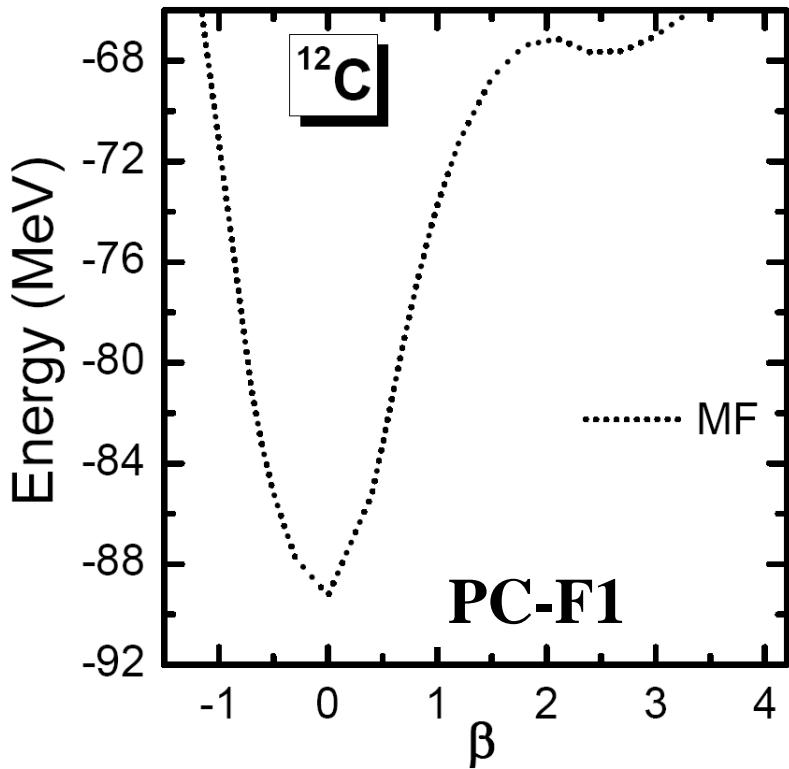
$$|\Phi_{IM}\rangle = \int d\beta f(\beta) |\Psi_{IM}(\beta)\rangle$$

generator coordinate method (GCM)

single → multi Slater determinants

□ Beyond MF: Illustration with  $^{12}\text{C}$  : (GCM+PNP+AMP)

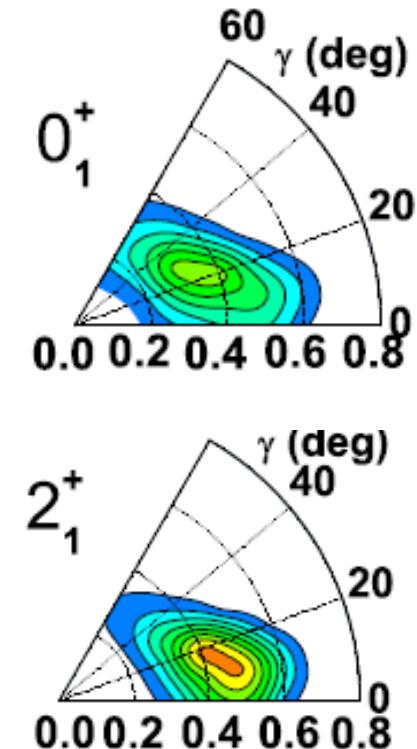
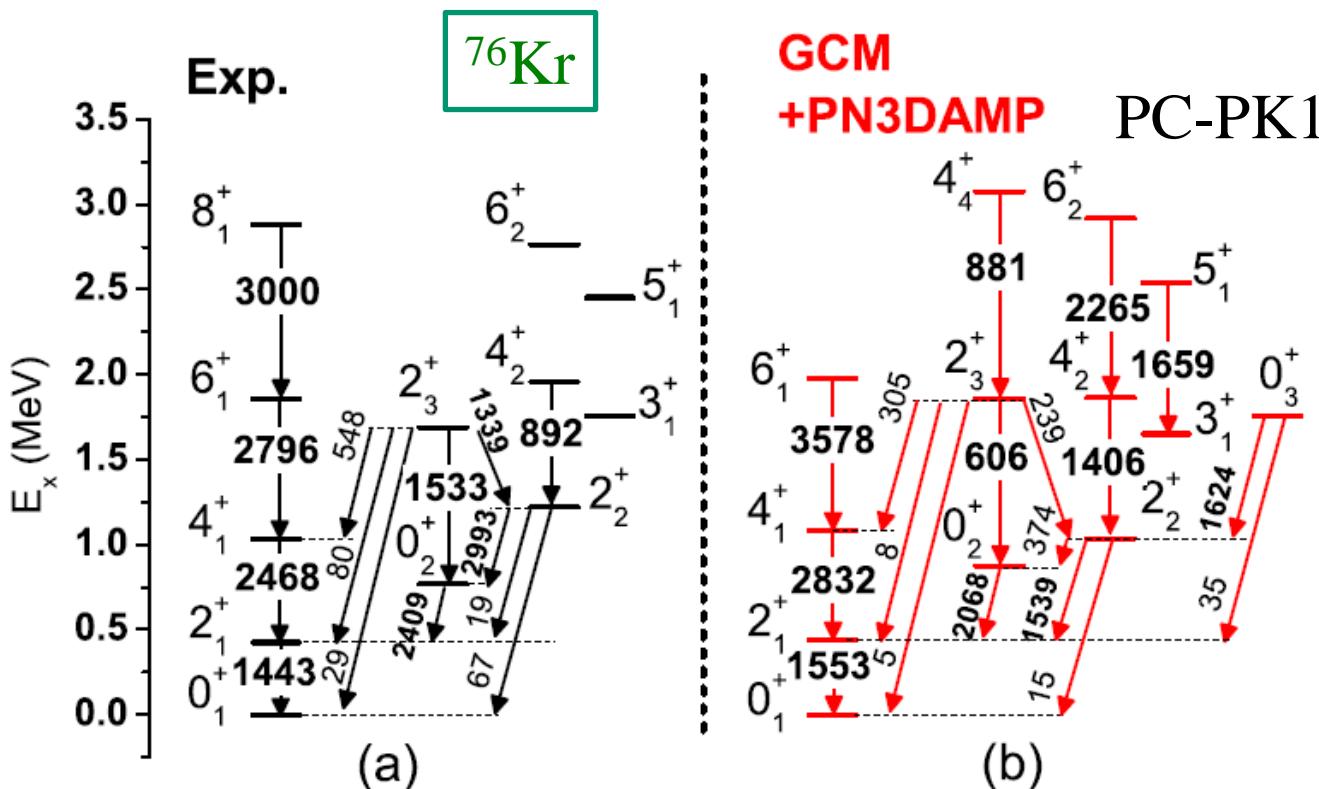
$$|\Phi_{IM_I}\rangle = \sum_{\beta} F^I(\beta) \hat{P}_{M_I K}^I \hat{P}^N \hat{P}^Z |\varphi(\beta)\rangle$$



➤ Low-lying spectrum is reproduced rather well.

## beyond mean-field approximation

- ✓ angular momentum + particle number projections
- ✓ quantum fluctuation (GCM)

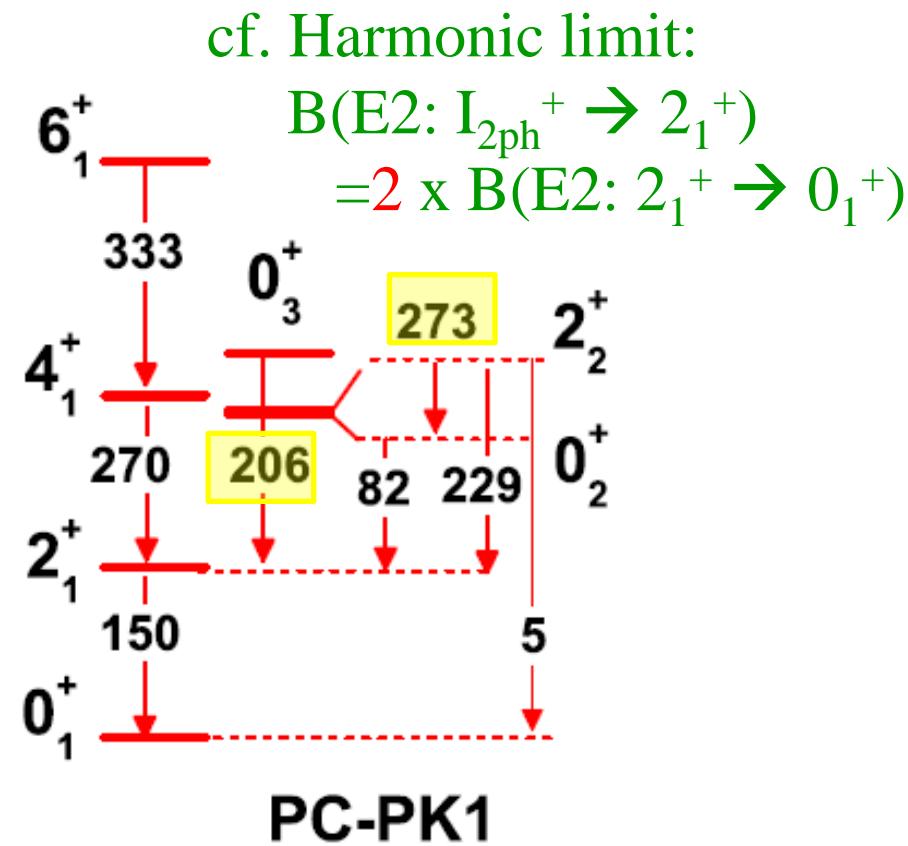
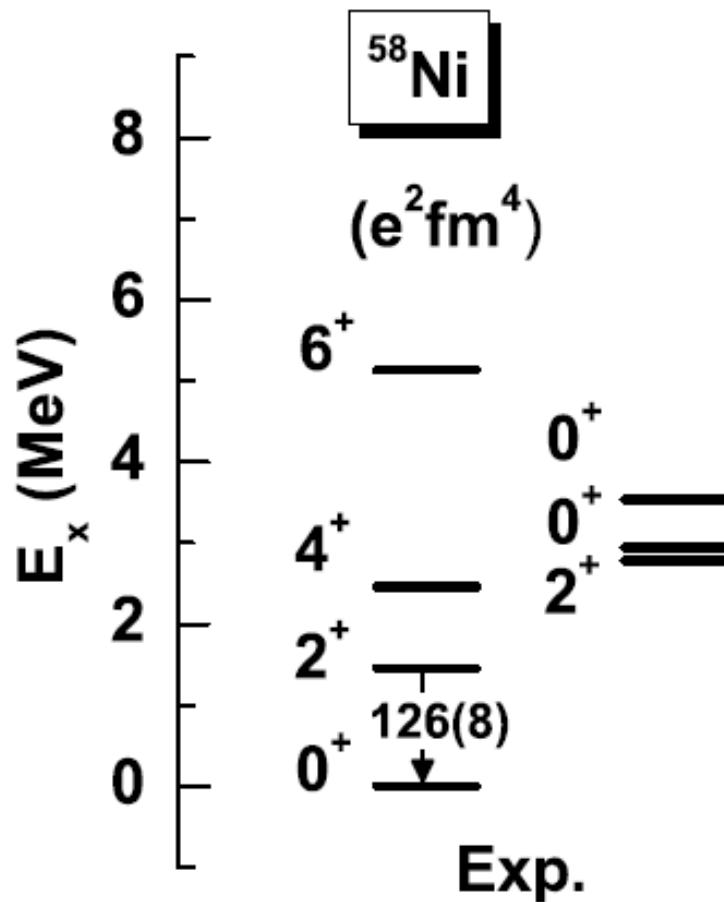


J.M. Yao, K.H., Z.P. Li, J. Meng, and P. Ring, PRC89 ('14) 054306

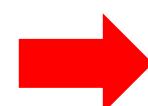
# Recent beyond-MF (MR-DFT) calculations for $^{58}\text{Ni}$

K.H. and J.M. Yao, PRC91 ('15) 064606

J.M. Yao, M. Bender, and P.-H. Heenen, PRC91 ('15) 024301



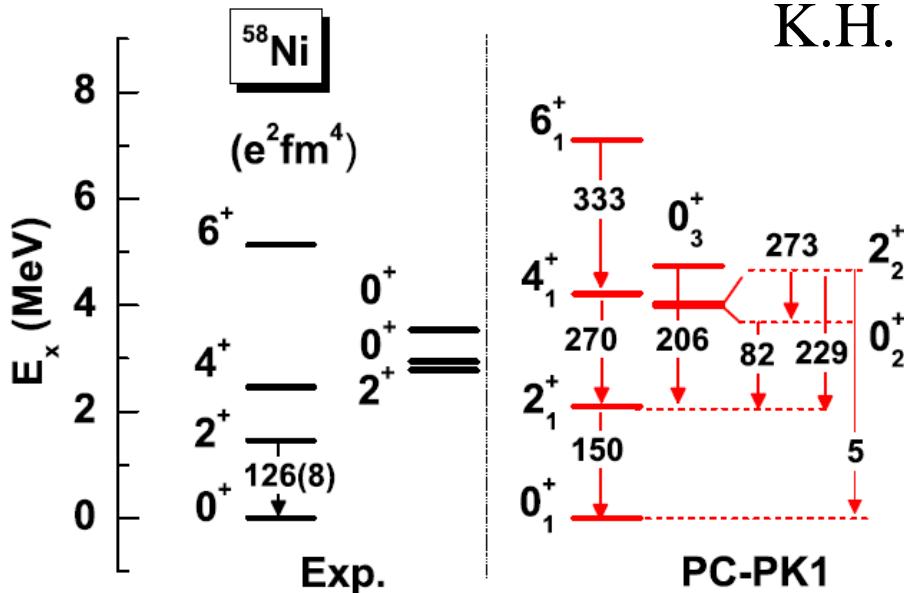
- ✓ A large fragmentation of  $(2^+ \times 2^+)_{J=0}$
- ✓ A strong transition from  $2_2^+$  to  $0_2^+$



effects on sub-barrier fusion?

# Semi-microscopic coupled-channels model for sub-barrier fusion

K.H. and J.M. Yao, PRC91 ('15) 064606

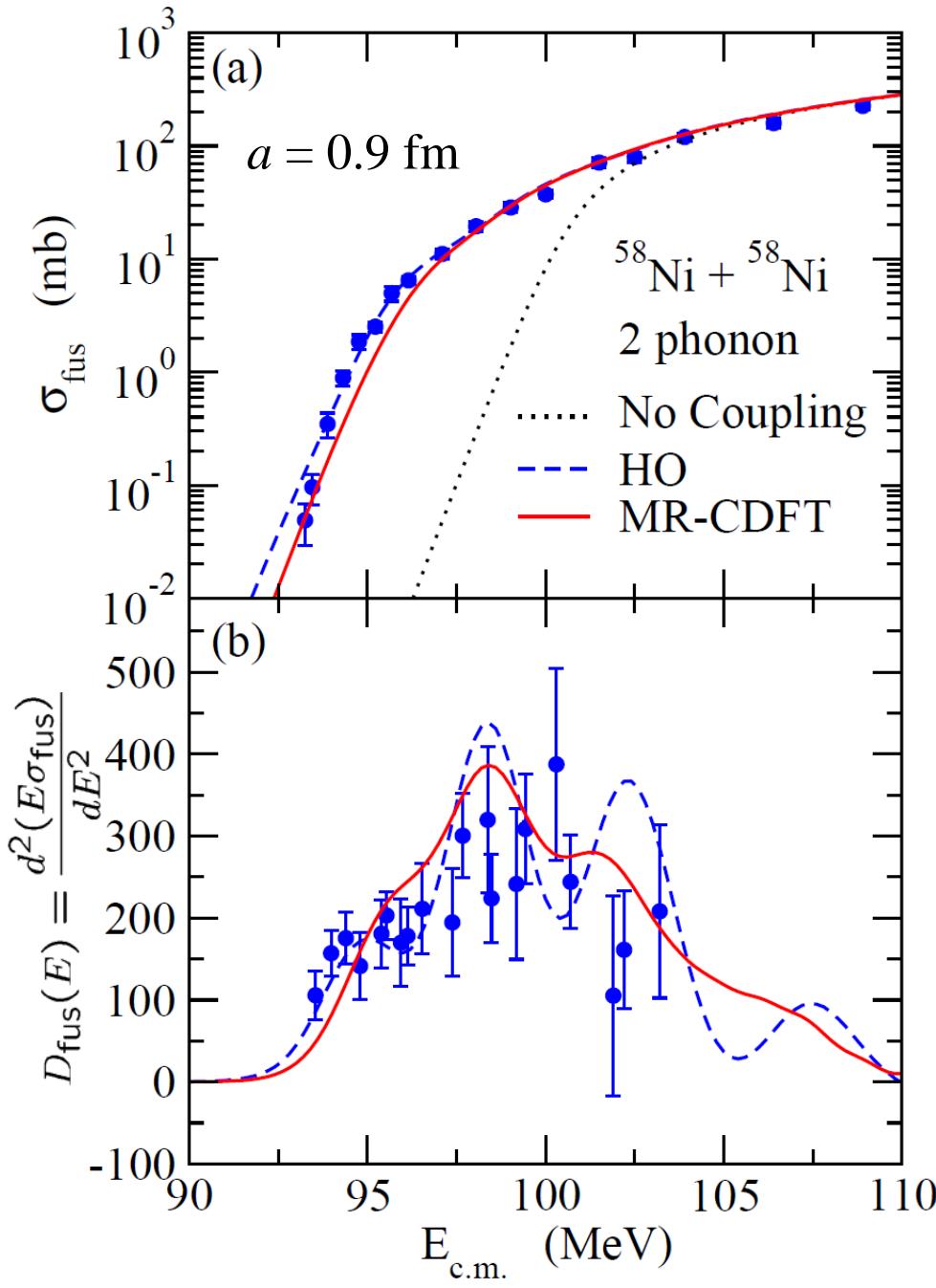


microscopic  
multi-pole operator



$$V_{\text{coup}}(r, \xi) \sim F_\lambda(r) Q_\lambda \cdot Y_\lambda(\hat{r})$$

- ✓  $M(\text{E}2)$  from MR-DFT calculation ← among higher members of phonon states
- ✓ scale to the empirical  $B(\text{E}2; 2_1^+ \rightarrow 0_1^+)$
- ✓ still use a phenomenological potential
- ✓ use the experimental values for  $E_x$
- ✓  $\beta_N$  and  $\beta_C$  from  $M_n/M_p$  for each transition
- ✓ axial symmetry (no  $3^+$  state)



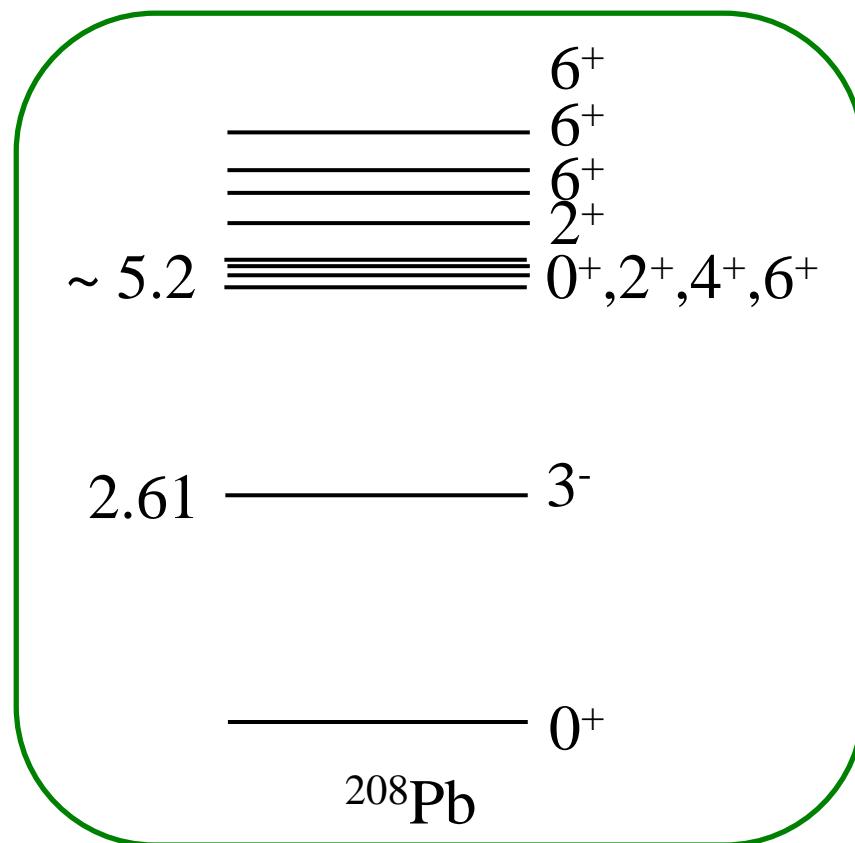
$^{58}\text{Ni} + ^{58}\text{Ni}$   
 anharmonicity of  $2^+$  phonon  
 $\rightarrow$  only a minor improvement



Next, more non-trivial case  
 with  $2^+ - 3^-$  coupling:  
 anharmonicity of oct. vib.  
 in  $^{208}\text{Pb}$

## Application to $^{16}\text{O} + ^{208}\text{Pb}$ fusion reaction

double-octupole phonon states in  $^{208}\text{Pb}$



M. Yeh, M. Kadi, P.E. Garrett et al., PRC57 ('98) R2085

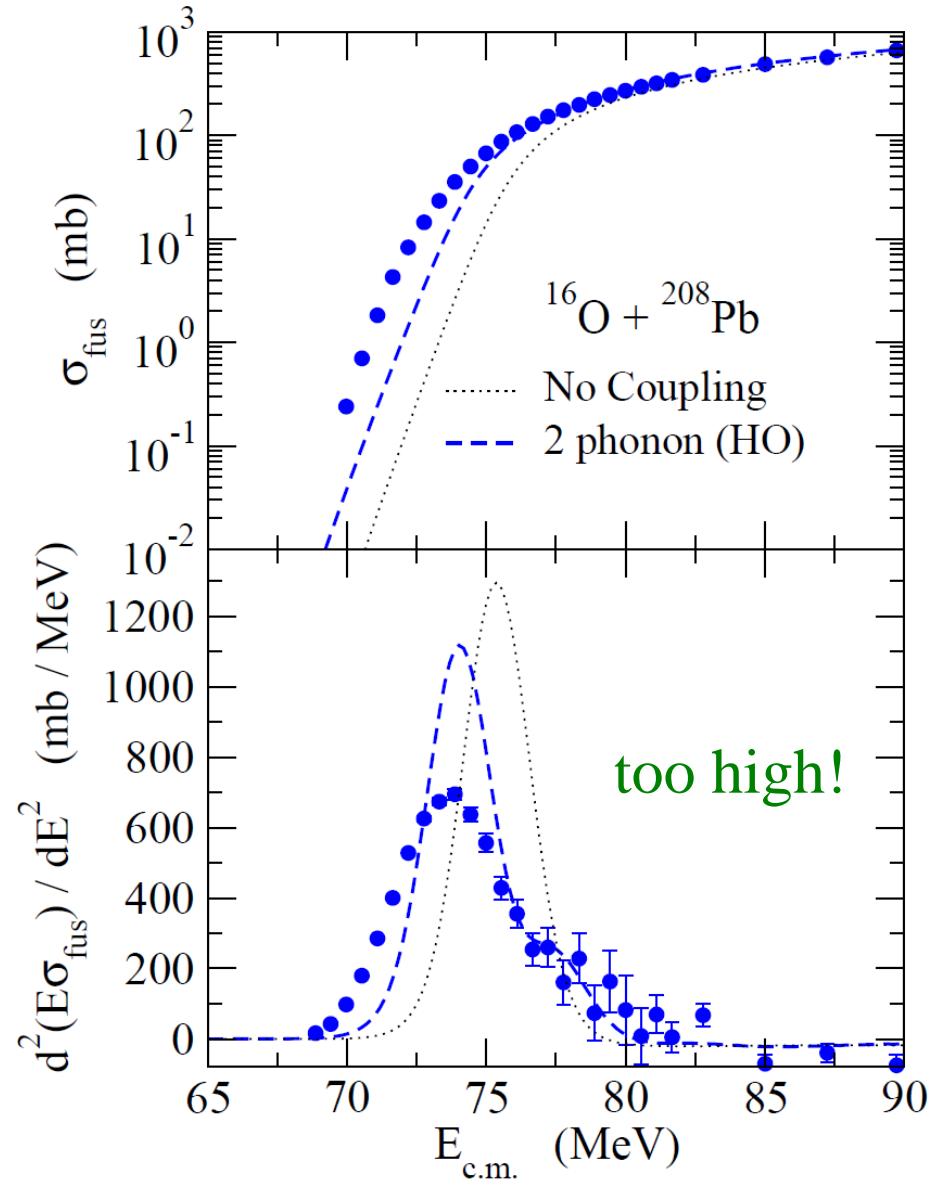
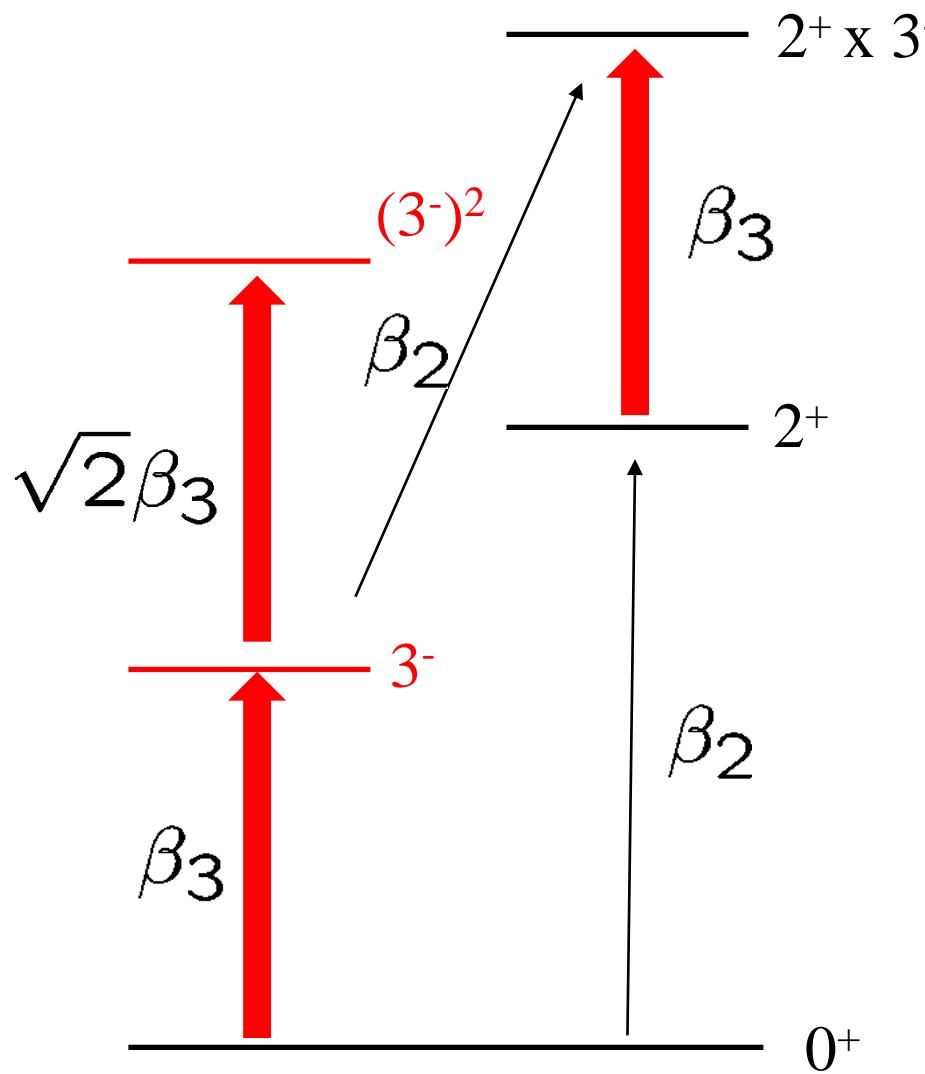
K. Vetter, A.O. Macchiavelli et al., PRC58 ('98) R2631

V. Yu. Pnomarev and P. von Neumann-Cosel, PRL82 ('99) 501

B.A. Brown, PRL85 ('00) 5300

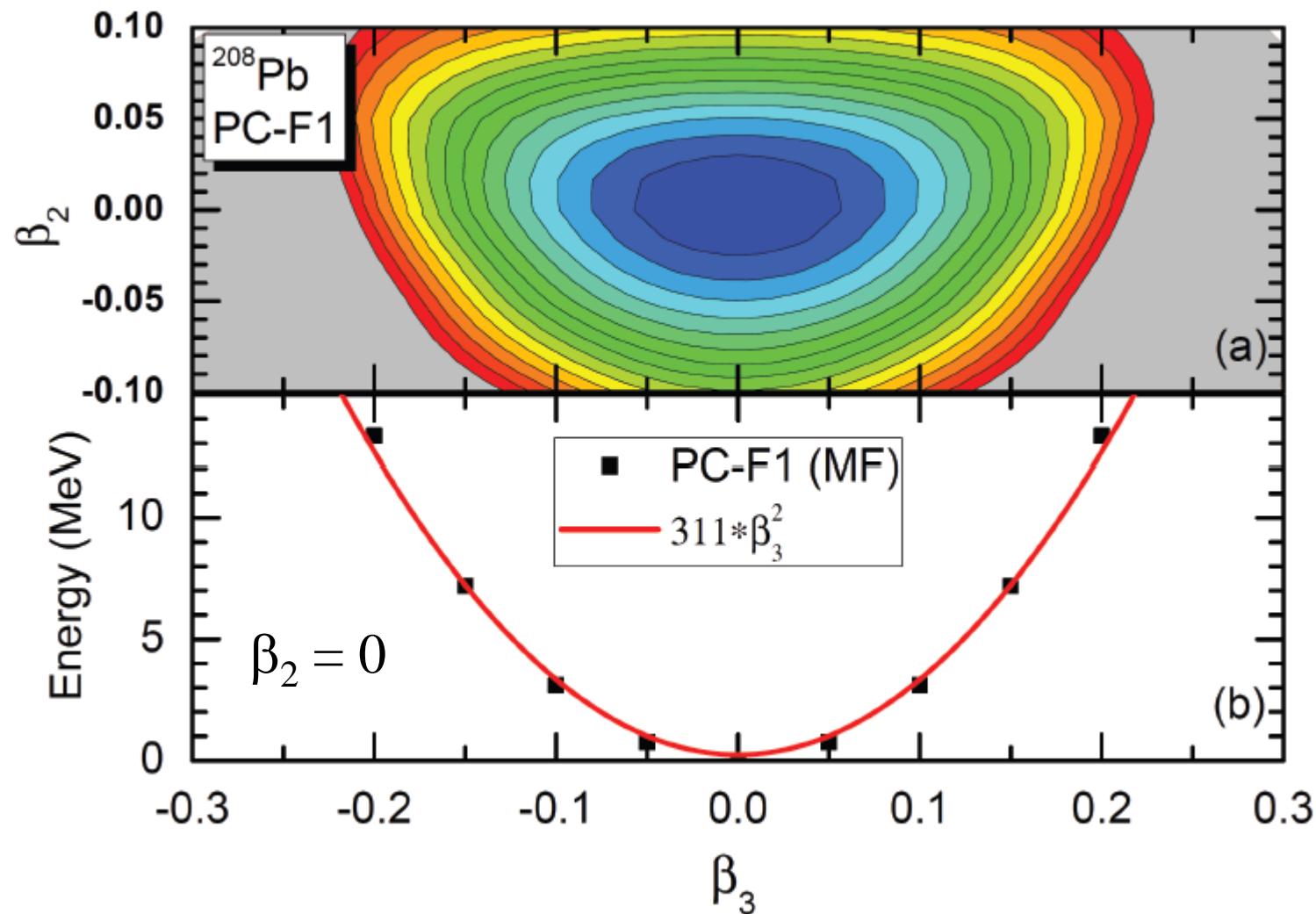
large fragmentations, especially  $6^+$  state

## Application to $^{16}\text{O} + ^{208}\text{Pb}$ fusion reaction

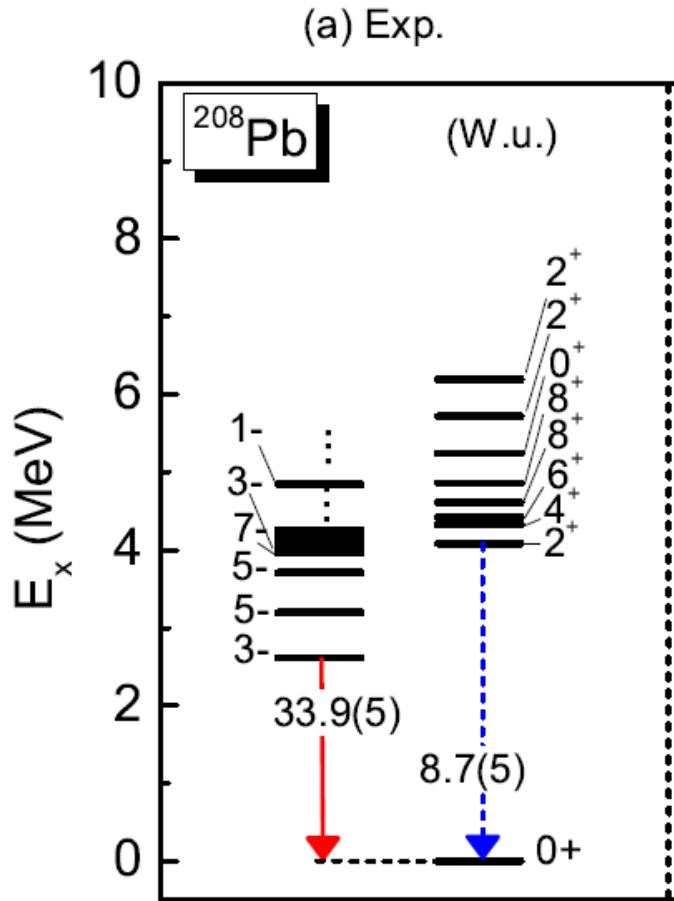


cf. C.R. Morton et al., PRC60('99) 044608

# potential energy surface of $^{208}\text{Pb}$ (RMF with PC-F1)

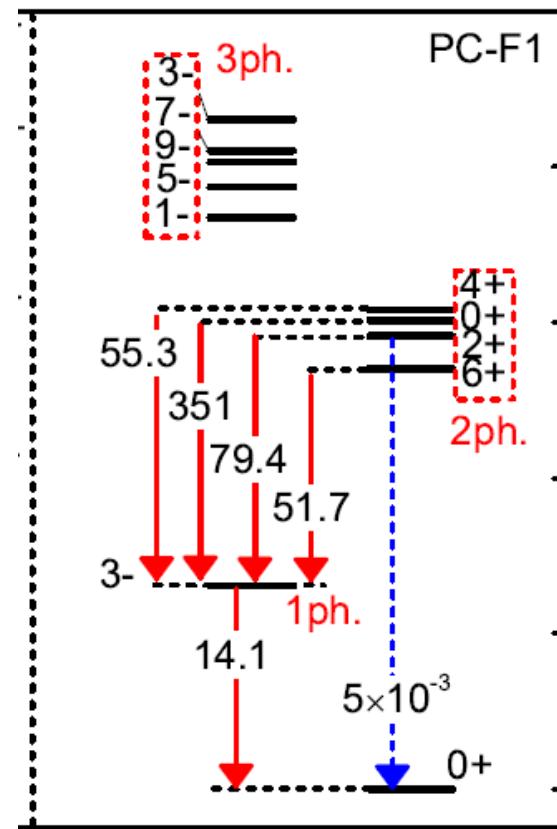


Expt. data



$\beta_2=0$ , fluctuation in  $\beta_3$

(c) GCM ( $\beta_3$ )



- $E_{2\text{ph}} \sim E_{1\text{ph}}$
- large anharmonicity in  $B(E3)$ ;  
cf. H.O.:  $B(E3: I_{2\text{ph}} \rightarrow 3_1^-) = 2 B(E3: 3_1^- \rightarrow \text{g.s.})$
- underestimate  $B(E3)$  (and  $B(E2)$ )

expt. data

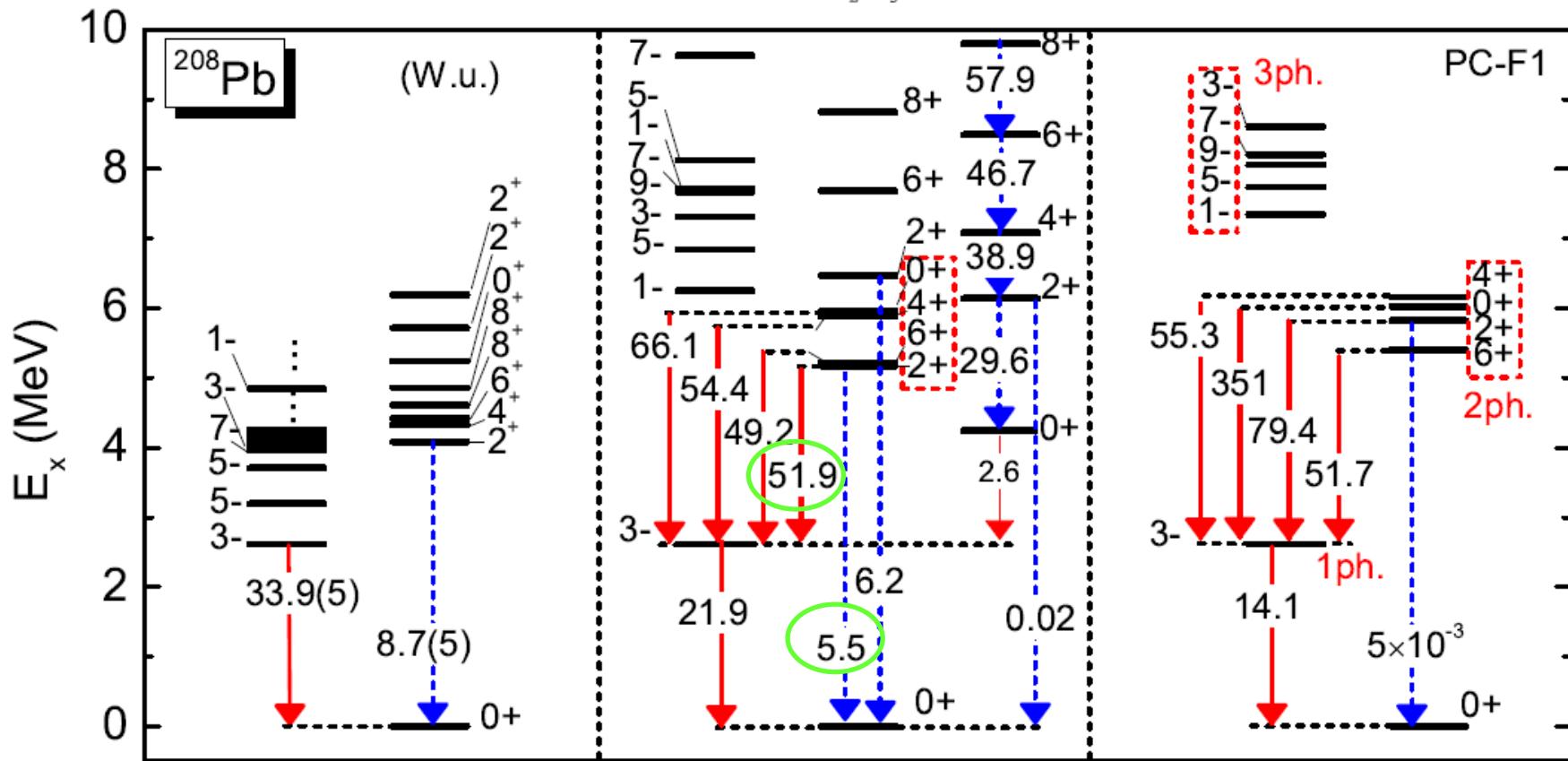
fluctuation both  
in  $\beta_3$  and  $\beta_2$

fluctuation in  $\beta_3$   
frozen at  $\beta_2=0$

(a) Exp.

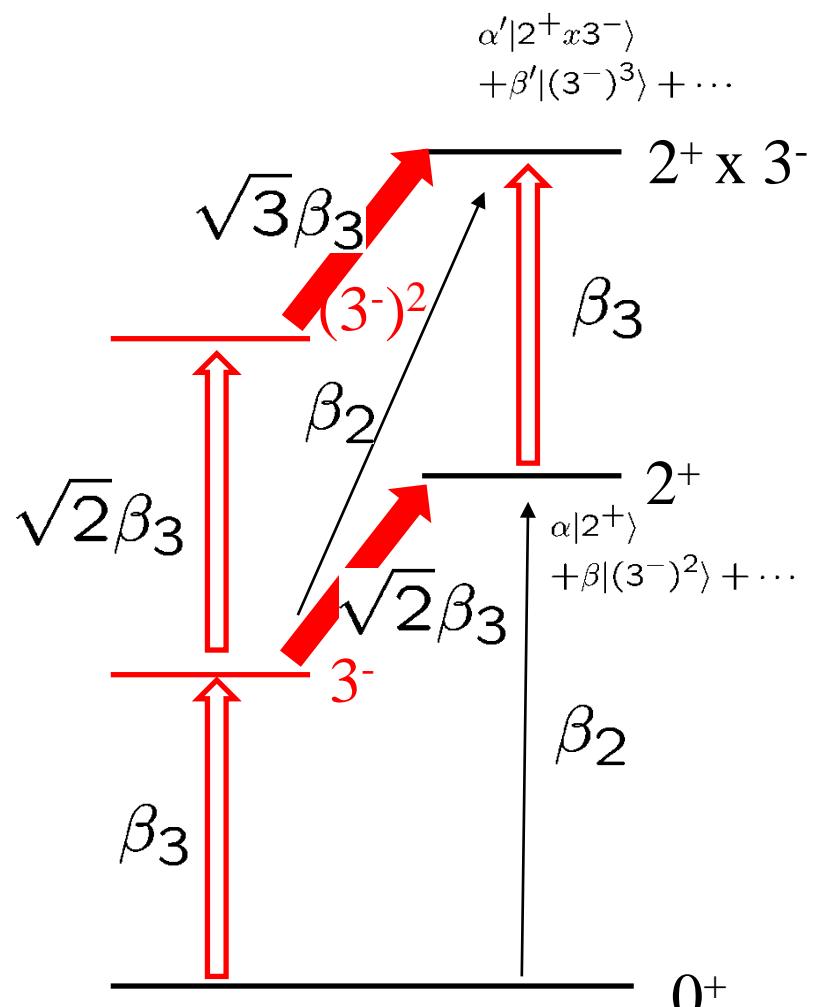
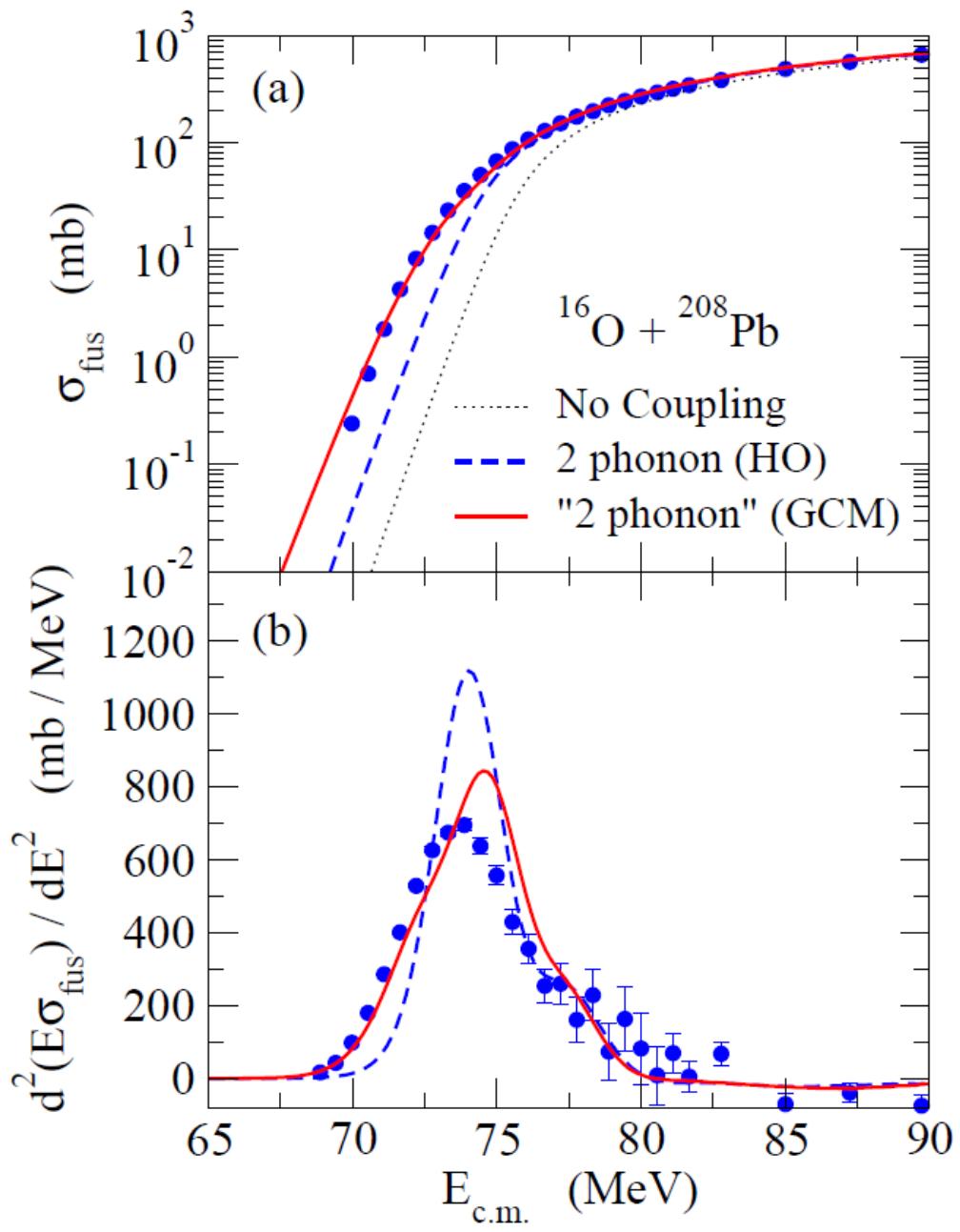
(b) GCM ( $\beta_2$ - $\beta_3$ )

(c) GCM ( $\beta_3$ )



$2_1^+$  state: strong coupling both to g.s. and  $3_1^-$

$$\longrightarrow |2_1^+\rangle = \alpha |2^+\rangle_{\text{HO}} + \beta |[3^- \otimes 3^-]^{(I=2)}\rangle_{\text{HO}} + \dots$$



J.M. Yao and K.H.,  
PRC94 ('16) 11303(R)

# Summary

## Heavy-ion subbarrier fusion reactions

- ✓ strong interplay between reaction and structure  
cf. fusion barrier distributions

### ➤ C.C. calculations combined with beyond-MF method

- ✓ anharmonicity
- ✓ octupole vibrations:  $^{16}\text{O} + ^{208}\text{Pb}$

more flexibility:

- application to transitional nuclei
- a good guidance to a Q-moment of excited states in spherical nuclei

C.C. with shell model?

