Spectral Energetics Analysis of the General Circulation of the Atmosphere in the Vertical Wavenumber Domain

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INTRODUCTION

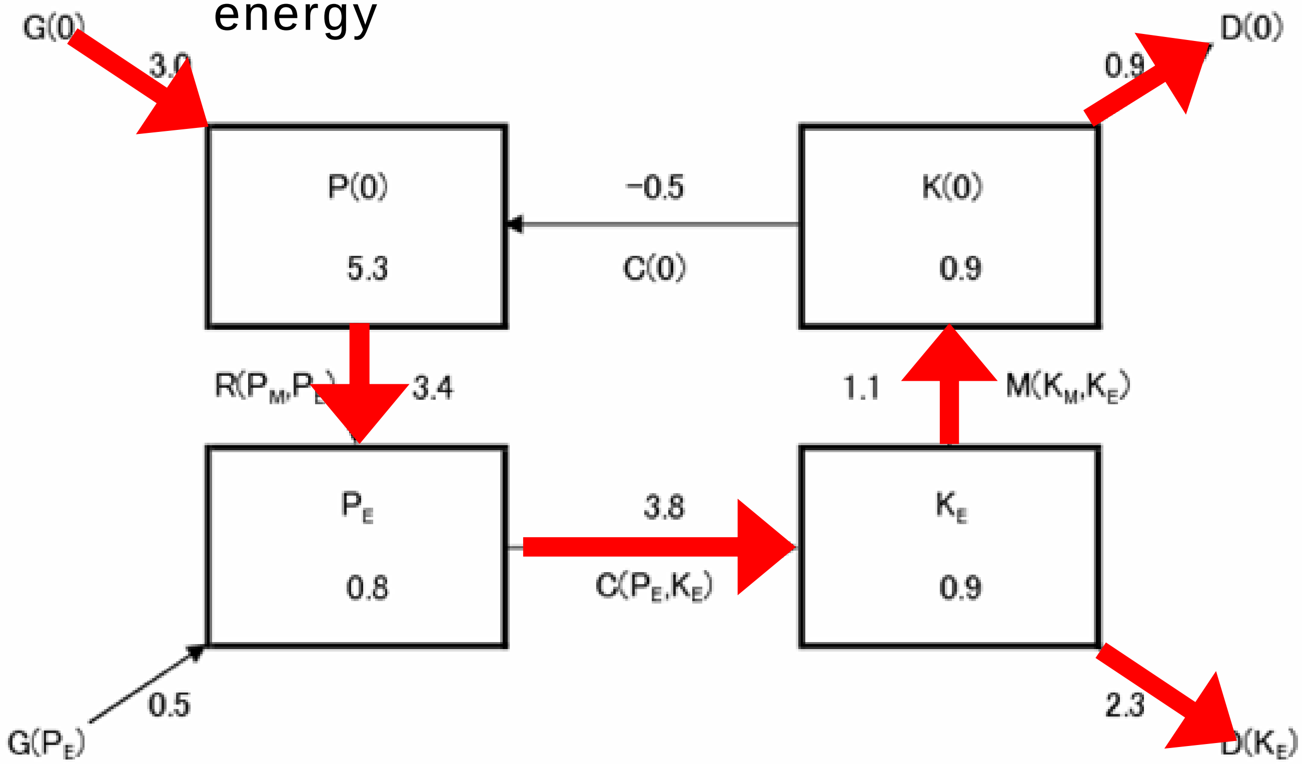
Energetics in zonal wavenumber domain
Lorenz Energy Cycle

Available potential energy

Kinetic Energy

zonal

eddy

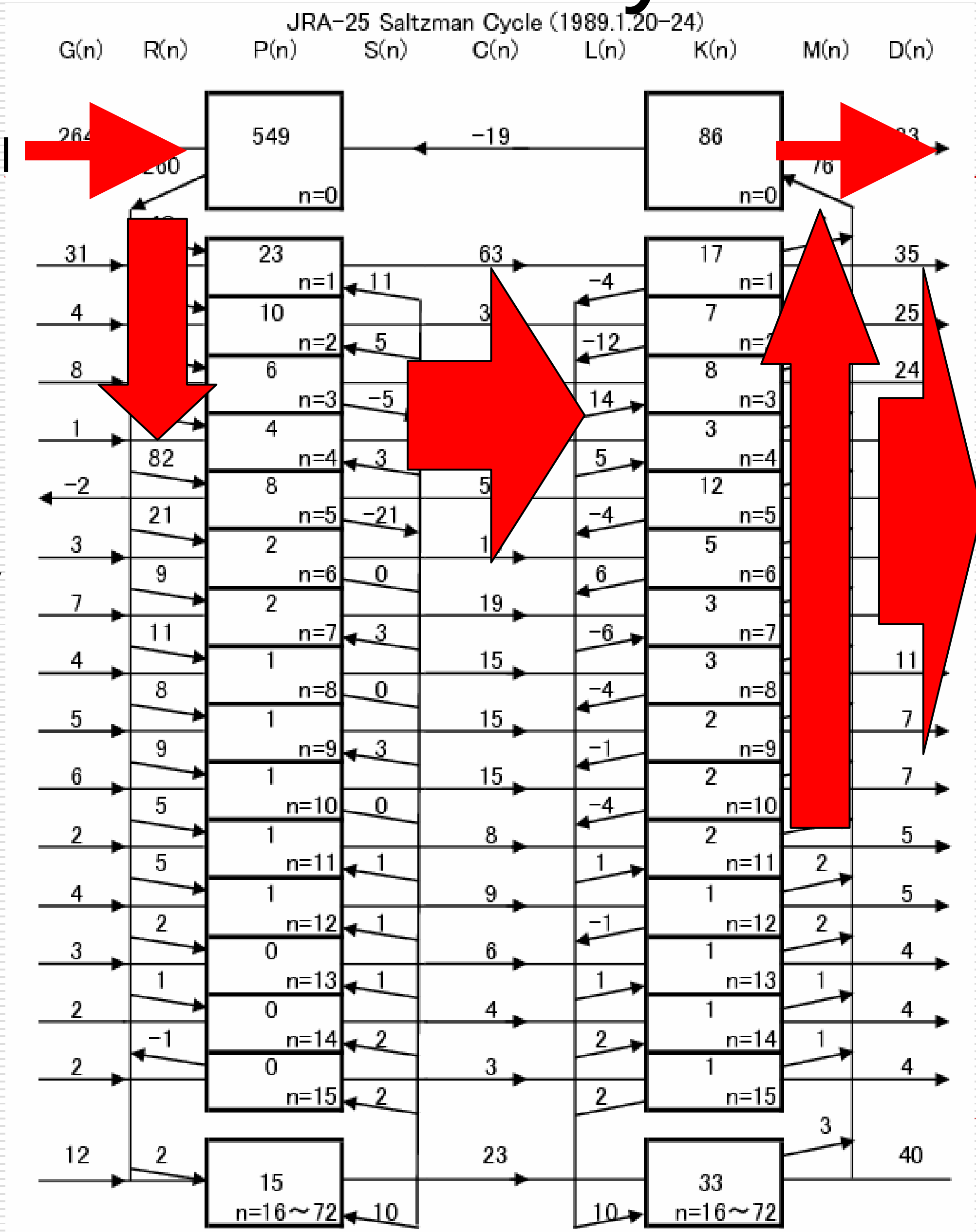


(Lorenz, 1954)

Saltzman Cycle

zonal

eddy



G: Generation of APE

R: zonal-wave interaction of APE

P: Available Potential Energy (APE)

S: wave-wave interaction of APE

C: Conversion from APE to KE

L: wave-wave interaction of KE

K: Kinetic Energy (KE)

M: zonal-wave interaction of KE

D: Dissipation

(Saltzman, 1957 & 1970)

- Lorenz and Saltzman investigated the energy flow of the atmospheric general circulation in the zonal wavenumber domain.

  (Fourier transform)

Purpose of this study

We can investigate the energy flow between baroclinic component of the atmosphere, if vertical transform is applied to the primitive equation.

(Vertical structure function)

METHOD

Vertical structure function

Vertical structure equation

$$\frac{\partial}{\partial \sigma}\left(\sigma^2 \frac{\partial G_m}{\partial \sigma}\right) + \frac{R\gamma}{g h_m} G_m = 0$$

Constant

Euler Equation

G_m : vertical structure function

R : gas constant

h_m : equivalent height

constant

γ : static stability parameter

constant

Eigenvalue problem of the Sturm-Liouville type

$$\frac{\partial G_m}{\partial \sigma} = 0, \qquad \sigma = \varepsilon$$

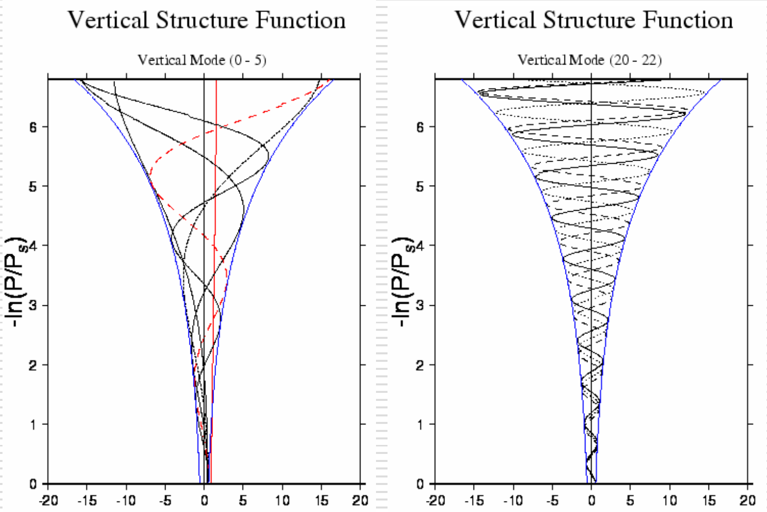
$$\frac{\partial G_m}{\partial \sigma} + \alpha G_m = 0, \qquad \sigma = 1$$

Obtain the vertical structure function analytically …

Analytical Solution of the Vertical Structure Equation

✓Barotropic Component

$$G_0(\sigma) = C_1 \sigma^{r_1} + C_2 \sigma^{r_2}$$



✓Baroclinic Component

$$G_m(\sigma) = \sigma^{-\frac{1}{2}} \left\{ C_1 \cos\left(\mu_m \ln \sigma\right) + C_2 \sin\left(\mu_m \ln \sigma\right) \right\}$$

Primitive equations

- Equation of motions

$$\frac{\partial u}{\partial t} - 2\Omega \sin\theta v + \frac{1}{a\cos\theta}\frac{\partial \phi}{\partial \lambda} = -\mathbf{V}\cdot\nabla u - \omega\frac{\partial u}{\partial p} + \frac{\tan\theta}{a}uv + F_u \tag{1}$$

$$\frac{\partial v}{\partial t} + 2\Omega \sin\theta u + \frac{1}{a}\frac{\partial \phi}{\partial \theta} = -\mathbf{V}\cdot\nabla v - \omega\frac{\partial v}{\partial p} - \frac{\tan\theta}{a}uu + F_v \tag{2}$$

- Thermodynamic equation

$$\frac{\partial c_p T}{\partial t} + \mathbf{V}\cdot\nabla c_p T + \omega\frac{\partial c_p T}{\partial p} = \omega\alpha + Q \tag{3}$$

- Law of mass conservation

$$\frac{1}{a\cos\theta}\frac{\partial u}{\partial \lambda} + \frac{1}{a\cos\theta}\frac{\partial v\cos\theta}{\partial \theta} + \frac{\partial \omega}{\partial p} = 0 \tag{4}$$

- State equation

$$p\alpha = RT \tag{5}$$

- Hydrostatic equation

$$\frac{\partial \phi}{\partial p} = -\alpha \tag{6}$$

Kinetic Energy Equation

$$
\begin{aligned}
\frac{\partial K_m}{\partial t} = -\sum_{l,n} \frac{p_s}{g} \Bigg[& \frac{r_{nlm}}{a\cos\theta}\left(U_m U_l \frac{\partial U_n}{\partial \lambda} + V_m U_l \frac{\partial V_n}{\partial \lambda} \right) \\
& + \frac{r_{nlm}}{a}\left(U_m V_l \frac{\partial U_n}{\partial \theta} + V_m V_l \frac{\partial V_n}{\partial \theta} \right) \\
& - \frac{\tan\theta}{a} r_{nlm}\left(U_m U_l V_n - V_m U_l U_n \right) + r_{nl'm}\left(U_m \Omega_l U_n + V_m \Omega_l V_n \right) \Bigg] \\
& - \frac{p_s}{a\cos\theta} U_m \frac{\partial A_m}{\partial \lambda} - \frac{p_s}{a} V_m \frac{\partial A_m}{\partial \theta} \\
& - D_m
\end{aligned}
$$

Kinetic energy interaction

Generation of kinetic energy

(conversion from APE)

Dissipation

Available potential energy (APE) equation

$$\frac{\partial P_m}{\partial t} = \sum_{l,n} \frac{g p_s}{R\gamma} R_1 \left[\frac{A_m}{a\cos\theta} U_l \frac{\partial A_n}{\partial \lambda} + \frac{A_m}{a} V_l \frac{\partial A_n}{\partial \theta} \right] + \sum_{l,n} \frac{g p_s}{R\gamma} R_2 \Omega_l A_n A_m$$

Available potential energy interaction

$$+ \frac{p_s}{a\cos\theta} U_m \frac{\partial A_m}{\partial \lambda} + \frac{p_s}{a} V_m \frac{\partial A_m}{\partial \theta}$$

Conversion to Kinetic energy

$$+ \frac{p_s}{C_p \gamma} A_m \left(H_m + \sum_n r_{pn'm} H_n \right)$$

Generation of APE and radiative cooling

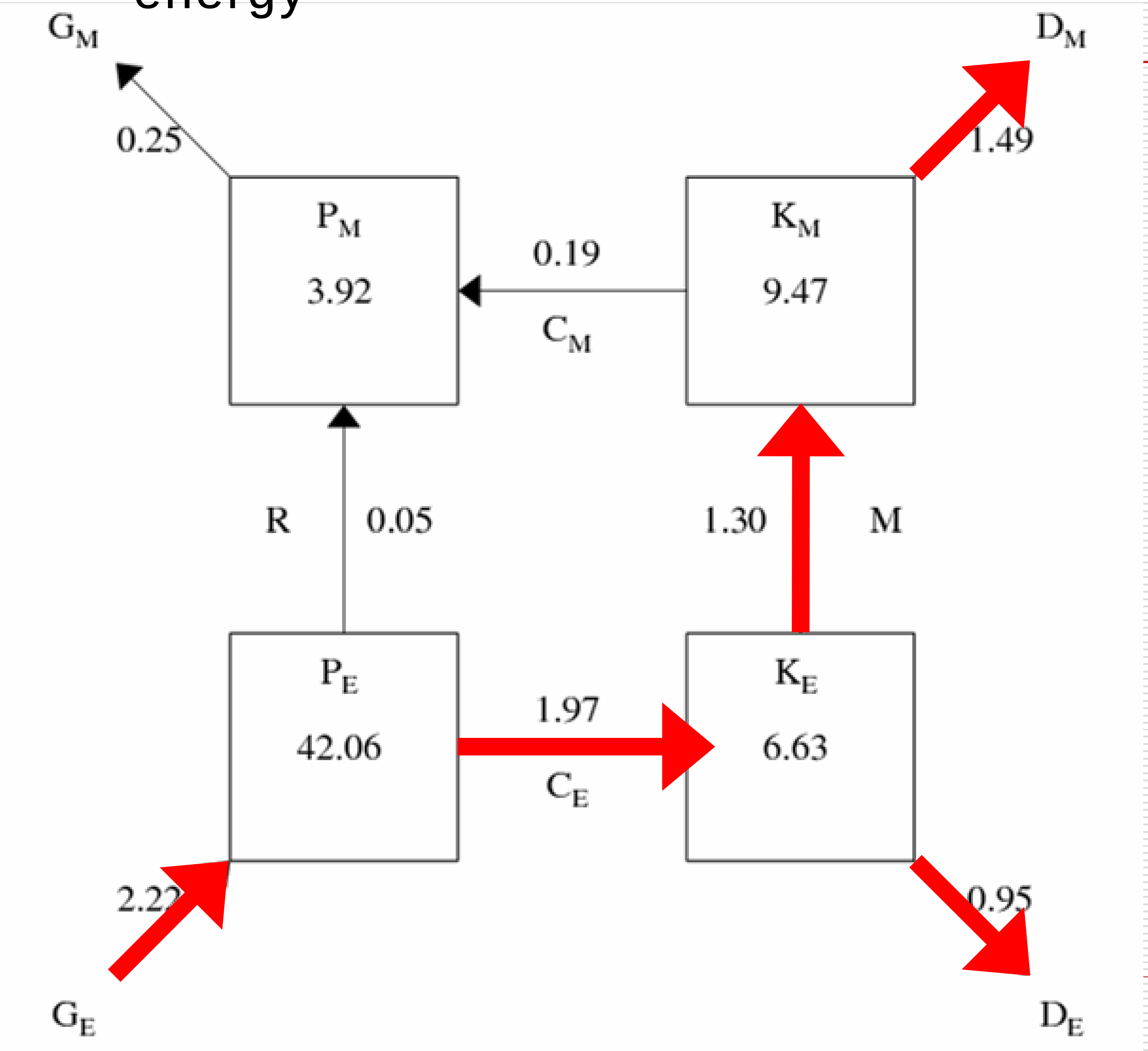
RESULTS

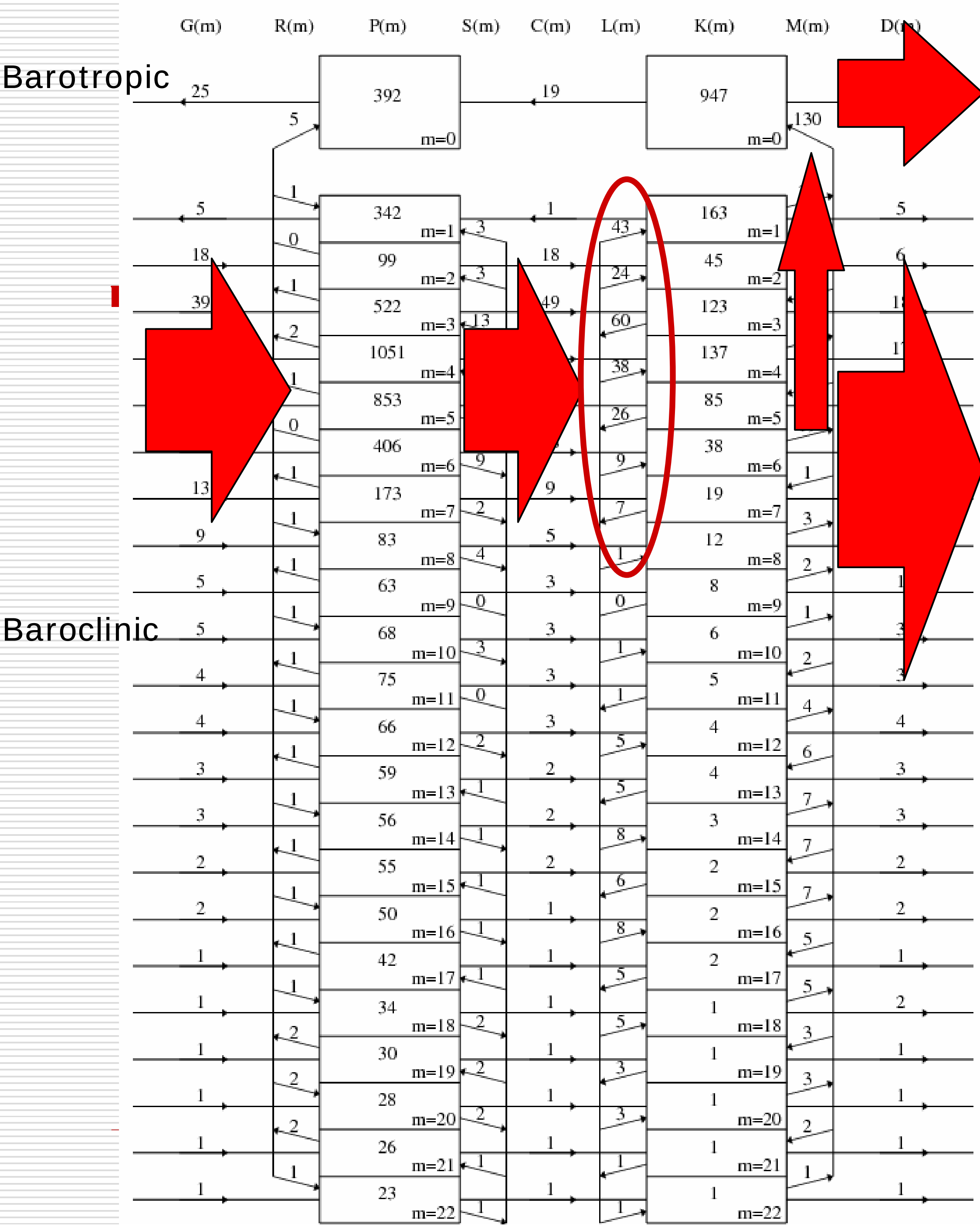
Available potential energy

Kinetic energy

Barotropic component

Baroclinic component





G: Generation of APE

R: barotropic-baroclinic interactions of APE

P: Available Potential Energy (APE)

S: baroclinic-baroclinic interactions of APE

C: Baroclinic conversion

L: baroclinic-baroclinic interactions of KE

K: Kinetic Energy (KE)

M: barotropic-baroclinic interactions of KE

D: Dissipation

CONCLUSIONS

- The atmospheric energy flow in the vertical wavenumber domain was calculated, with expanding the primitive equations using analytical vertical structure functions.

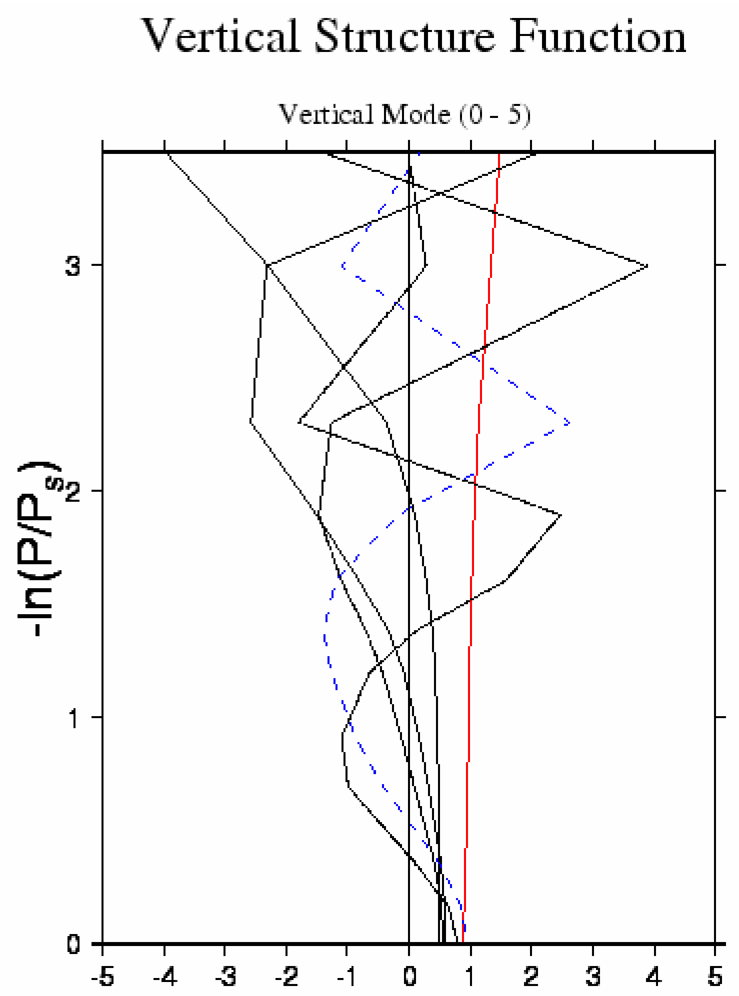
- ✓ The energy interactions between available potential energy in the vertical wavenumber domain are comparatively small.
- ✓ The kinetic energy converted from available potential energy is interacting among baroclinic modes.

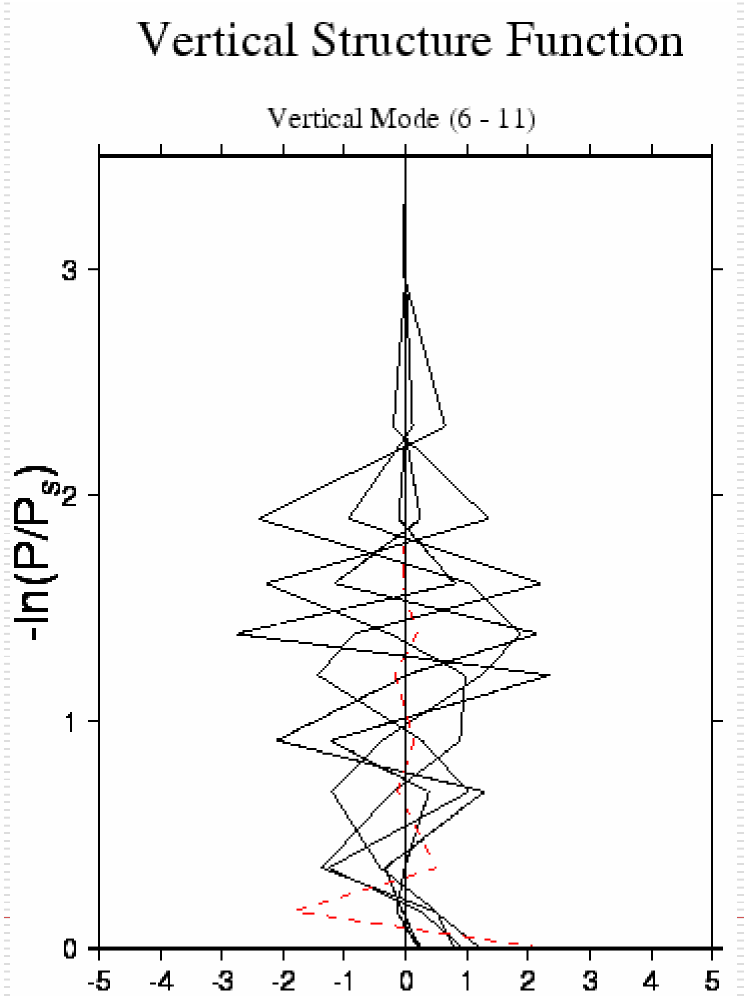
Future study

- ✓ More analysis about baroclinic-baroclinic interactions of kinetic energy.

Thank you for listening!

Numerical Solution





Vertical expansion

東西風 ： $u(p) = \sum_{m=0}^{M} U_m G_m(p)$

南北風 ： $v(p) = \sum_{m=0}^{M} V_m G_m(p)$

気温 ： $T(p) = \sum_{m=0}^{M} B_m G_m(p)$

高度 ： $z(p) = \sum_{m=0}^{M} A_m G_m(p)$

鉛直風 ： $\omega(p) = \sum_{m=0}^{M} \Omega_m G_m(p)$

➢Vertical expansion

$$f(p) = \sum_{m=0}^{\infty} F_m G_m(p)$$

F_m : Coefficient of G_m

p_s : Surface pressure (1000hPa)

$$F_m = \frac{1}{p_s} \int_0^{p_s} f(p) G_m(p) dp$$

➢Orthogonality of vertical structure function

$$\frac{1}{p_s} \int_0^{p_s} G_m G_n dp = \delta_{mn}$$

Kinetic energy

$$K = \frac{1}{g}\int_0^{p_s} \frac{u^2 + v^2}{2} dp$$

$$= \frac{1}{g}\int_0^{p_s} \frac{1}{2}\left[\sum_m U_m G_m \sum_n U_n G_n + \sum_m V_m G_m \sum_n V_n G_n\right] dp$$

$$= \frac{1}{g}\sum_m \sum_n \frac{U_m U_n + V_m V_n}{2}\int_0^{p_s} G_m G_n dp$$

$$= \frac{p_s}{g}\sum_m \frac{U_m^{\ 2} + V_m^{\ 2}}{2}$$

$$\frac{\partial K_m}{\partial t} = \frac{p_s}{g}\left(U_m \frac{\partial U_m}{\partial t} + V_m \frac{\partial V_m}{\partial t}\right)$$

Available potential energy

$$P = \frac{1}{g}\int_0^{p_s} \frac{g^2 p^2}{2R\gamma}\left(\frac{\partial z'}{\partial p}\right)^2 dp$$

$$= \frac{1}{g}\int_0^{p_s} -\frac{1}{2} z' \frac{\partial}{\partial p}\left(\frac{g^2 p^2}{R\gamma}\frac{\partial z'}{\partial p}\right) dp$$

$$= \int_0^{p_s} \frac{1}{2h_m} z'^2 dp$$

$$= \sum_m \frac{p_s}{2h_m} A_m{}^2$$

$$\frac{\partial P_m}{\partial t} = \frac{p_s}{h_m}\left(A_m \frac{\partial A_m}{\partial t}\right)$$