

Octupole deformation in the nuclear chart based on the three dimensional mean field calculation

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Introduction

Main topic: Nuclear Deformation

How to investigate nuclear deformation? There are many aspects for the **Deformation**

Spontaneous symmetry breaking

The spherical symmetry is broken by the correlation between individual particle motions and quantum fluctuations (“single-particle” vs. “collectivity”).

S.P. states in deformed mean field

The degeneracy of single-particle states is solved in deformed mean field.

← **Neutron, Proton** states in the vicinity of Fermi surface have a key role to define the nuclear shape

Role of Pairing

Amplitude of pairing correlation which is sensitive to the **level density** plays important roles for the shape.

Many kinds of deformation

Quadrupole(prolate, oblate, triaxial), Octupole($l=3, m$), Hexadecapole(β_4)

How do we regard the nuclear deformation?

- Rotational bands ■ Large quadrupole moment ■ Changes of S.P. states
- Isomer in nuclear fission □ Changes of reaction cross section(?)

Method

S.E. et al., PRC**82**(2010) 034306, N.Tajima, et al., NPA**603** (1996) 23.

Procedure to calculate self-consistent HF+BCS state

Interaction (*ph*) : Skyrme (SkM*),
 (*pp, hh*) : Constant (monopole)

$$\Delta_k(t) = \sum_{l>0} G_{kl} \kappa_l(t) \quad G_{kl} = g f(\epsilon_k^0) f(\epsilon_l^0)$$

$f(\epsilon)$: cutoff function

1, HF calculation : Imaginary-time method

ϵ_k^0 : s.p. energy at g.s.

2, Unoccupied states are calculated up to the cutoff energy.

3, Occupation probabilities v_k are evaluated by BCS gap equation.

Iterative calculation
up to the convergence

3', Calculate density including $|v_k|^2$

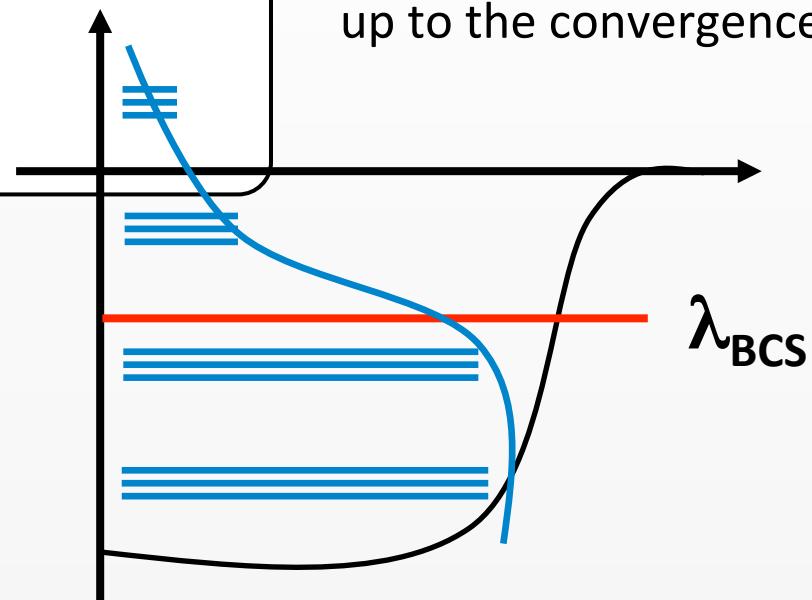
4, HF+BCS calculation : Imaginary-time method

$$f(\epsilon) = \left(1 + \exp \left[\frac{\epsilon - \epsilon_c}{0.5\text{MeV}} \right] \right)^{-1/2} \theta(e_c - \epsilon)$$

$$\epsilon_c = \tilde{\lambda} + 5.0\text{MeV} \quad e_c = \epsilon_c + 2.3\text{MeV}$$



Ave. of LUMO & HOMO



Calculation space for the self-consistent HF+BCS states

Fully 3D-Spherical meshed box:

Our subject is Nuclear chart. We calculate the even-even nuclei with $Z = 6 - 92$.

For light nuclei ($6 < Z < 20$),

we use the box has radius **12 [fm]** and meshed by **0.8 [fm]**.

For middle heavy nuclei ($20 < Z < 82$),

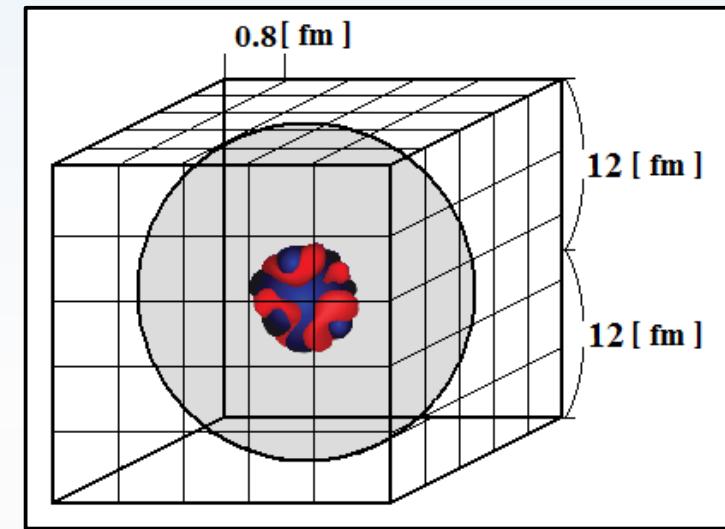
we use the box has radius **15 [fm]** and meshed by **1.0 [fm]**.

For heavy nuclei ($82 < Z < 92$),

we use the box has radius **20 [fm]** and meshed by **1.0 [fm]**.

$$\phi_l(\vec{r}, \sigma; t) \rightarrow \phi_l(x, y, z, \sigma; t)$$

$$x + y + z \sim 15,000 - 32,000$$



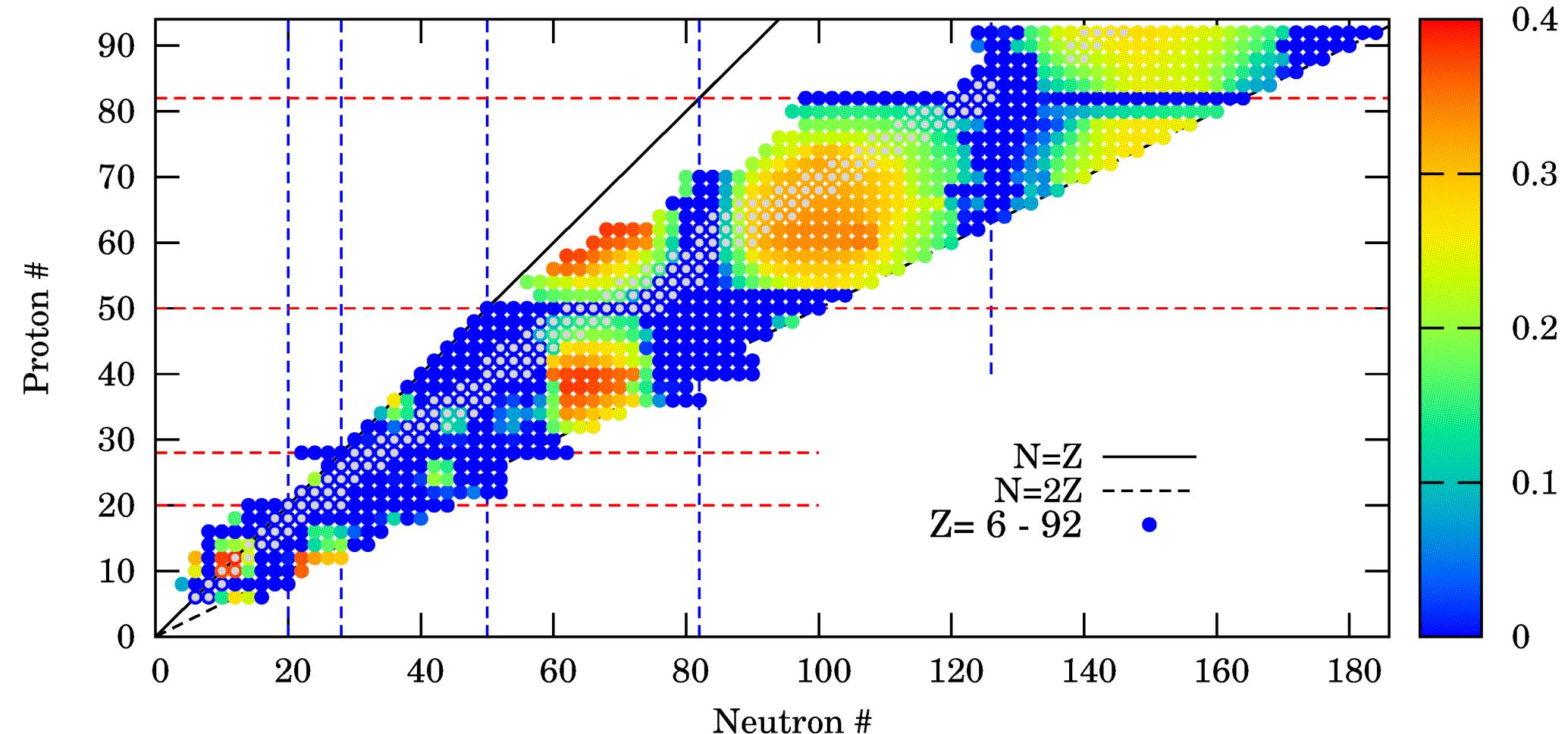
Each single-particle state has many lattice points to describe.

Results

3D HF+BCS Cal. w/ SkM*

From N=Z to N=2Z, Z=6-92 even-even (Total # 1005)

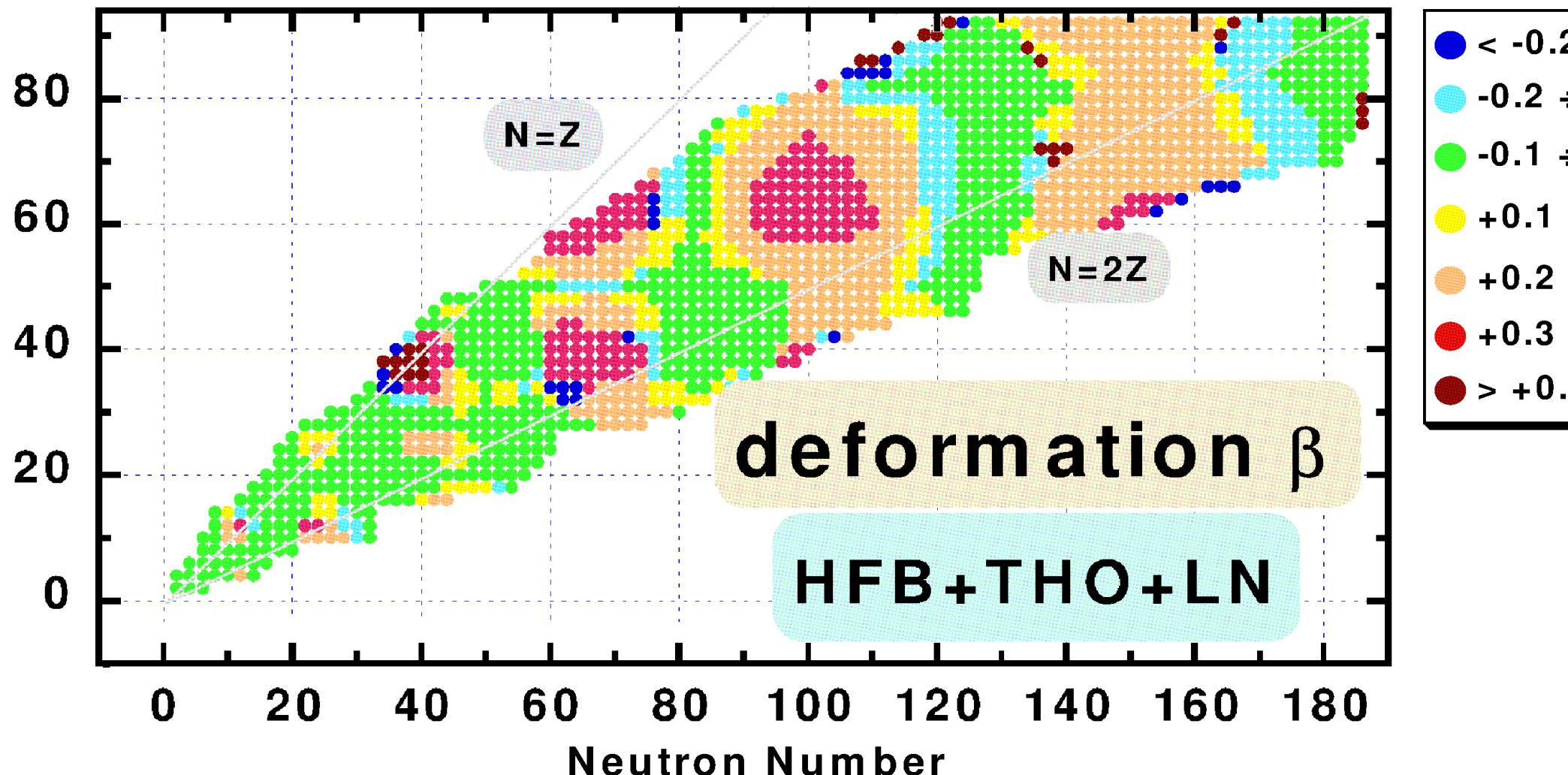
Quadrupole Deformation (# 1005)



Comparison

with M. V. Stoitsov, et. al. PRC**68** (2003) 054312

Axial deformed HFB+THO+LN Cal. w/ SkM* + δ -Vol. pairing

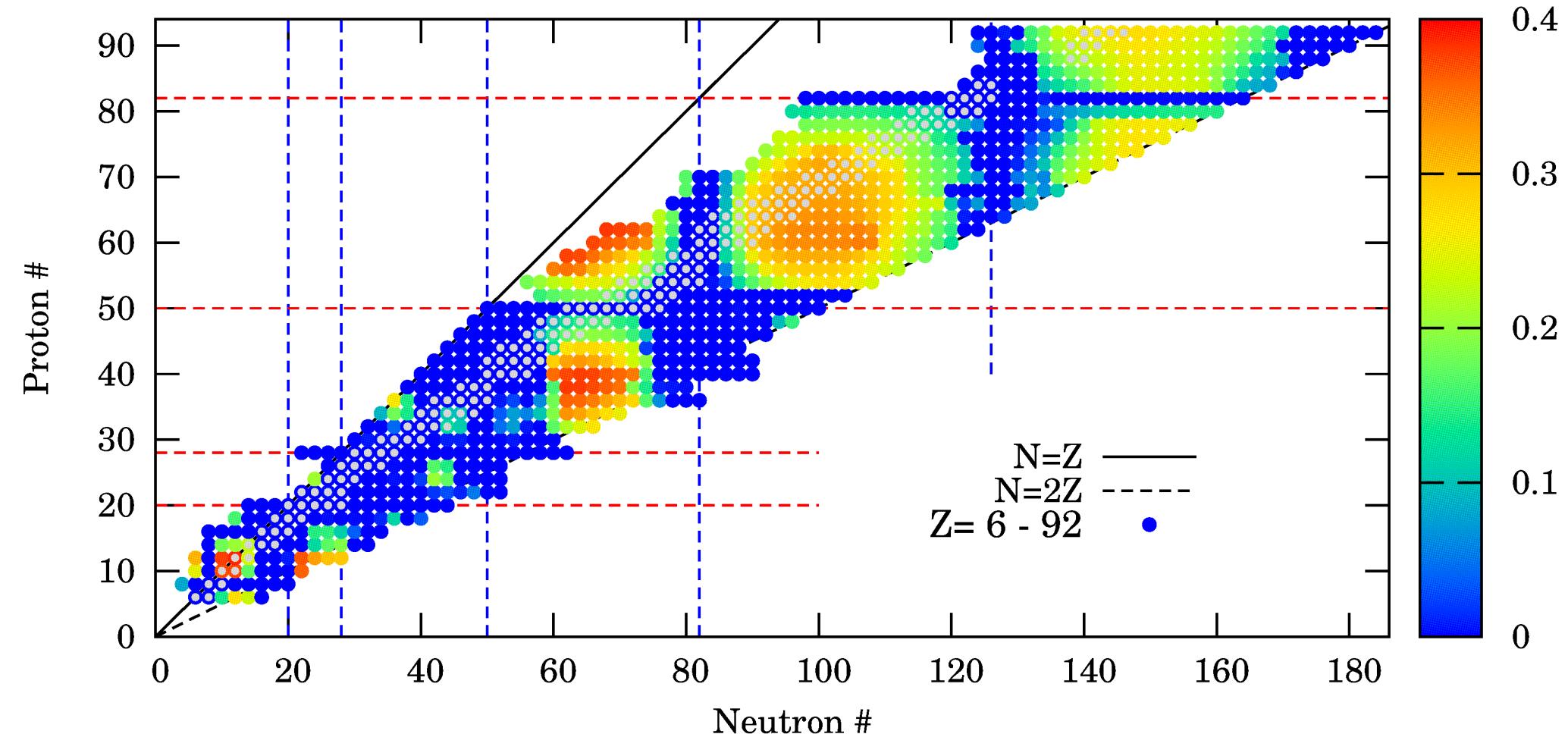


Results

3D HF+BCS Cal. w/ SkM*

From N=Z to N=2Z, Z=6-92 even-even (Total # 1005)

Quadrupole Deformation (# 1005)

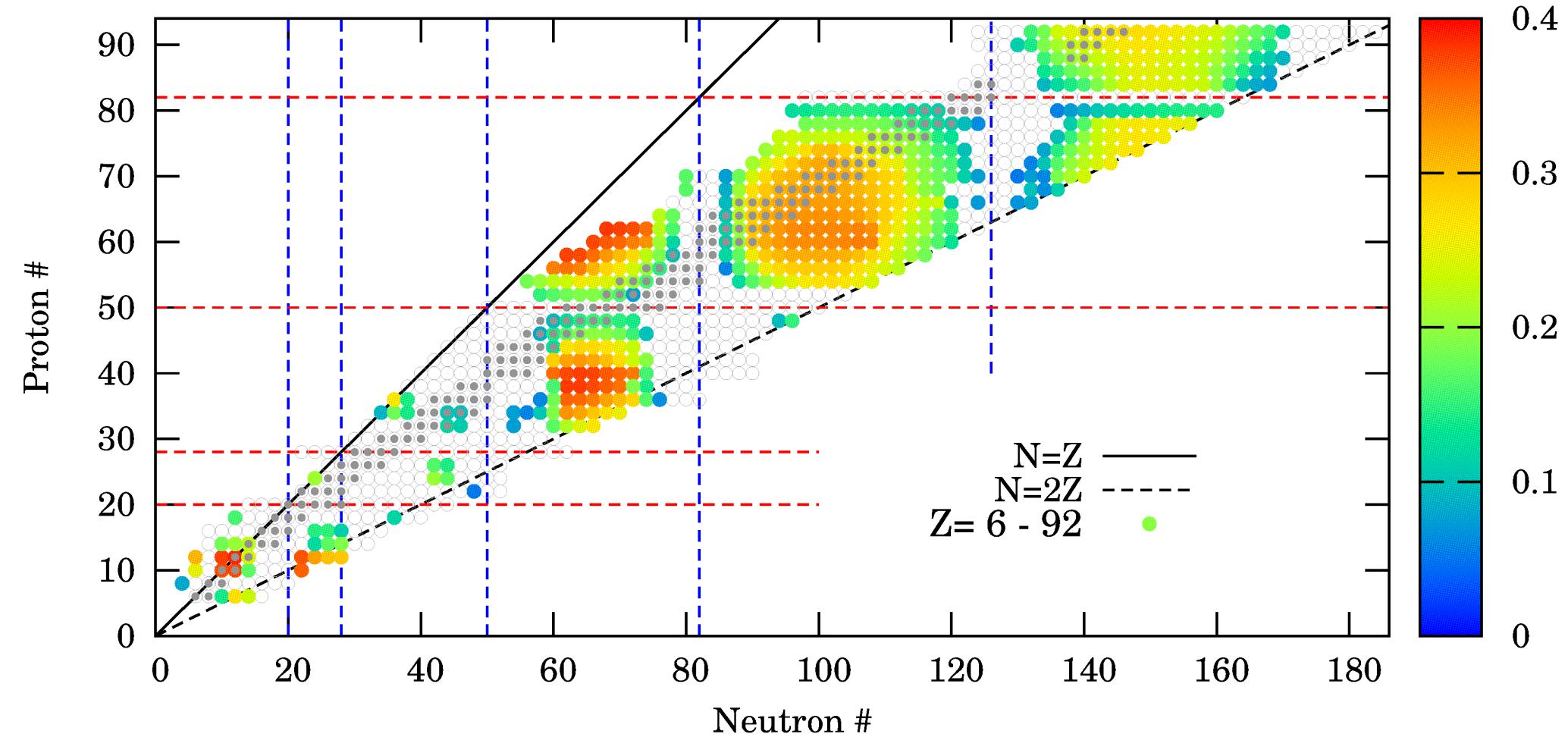


Results (Deformed)

3D HF+BCS Cal. w/ SkM*

Deformed nuclei: $|\beta_2| > 0.05$

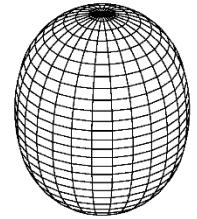
Quadrupole Deformation (# 546 / 1005)



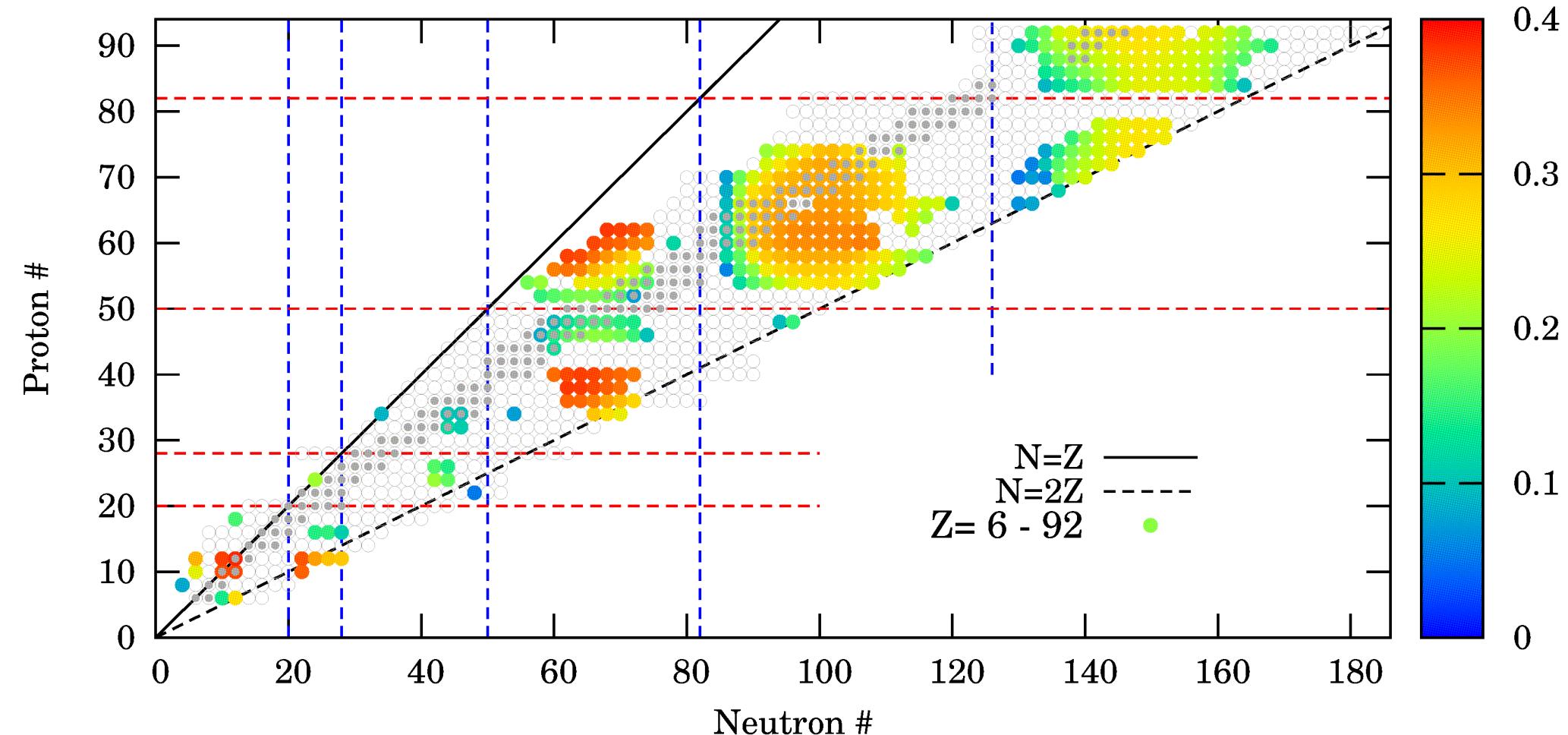
Results (Deformed: prolate)

3D HF+BCS Cal. w/ SkM*

Deformed nuclei: $|\beta_2| > 0.05, \gamma < 1.5^\circ$



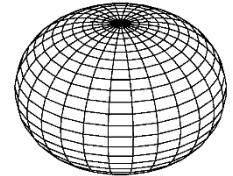
Quadrupole Deformation : Prolate (# 375 / 1005)



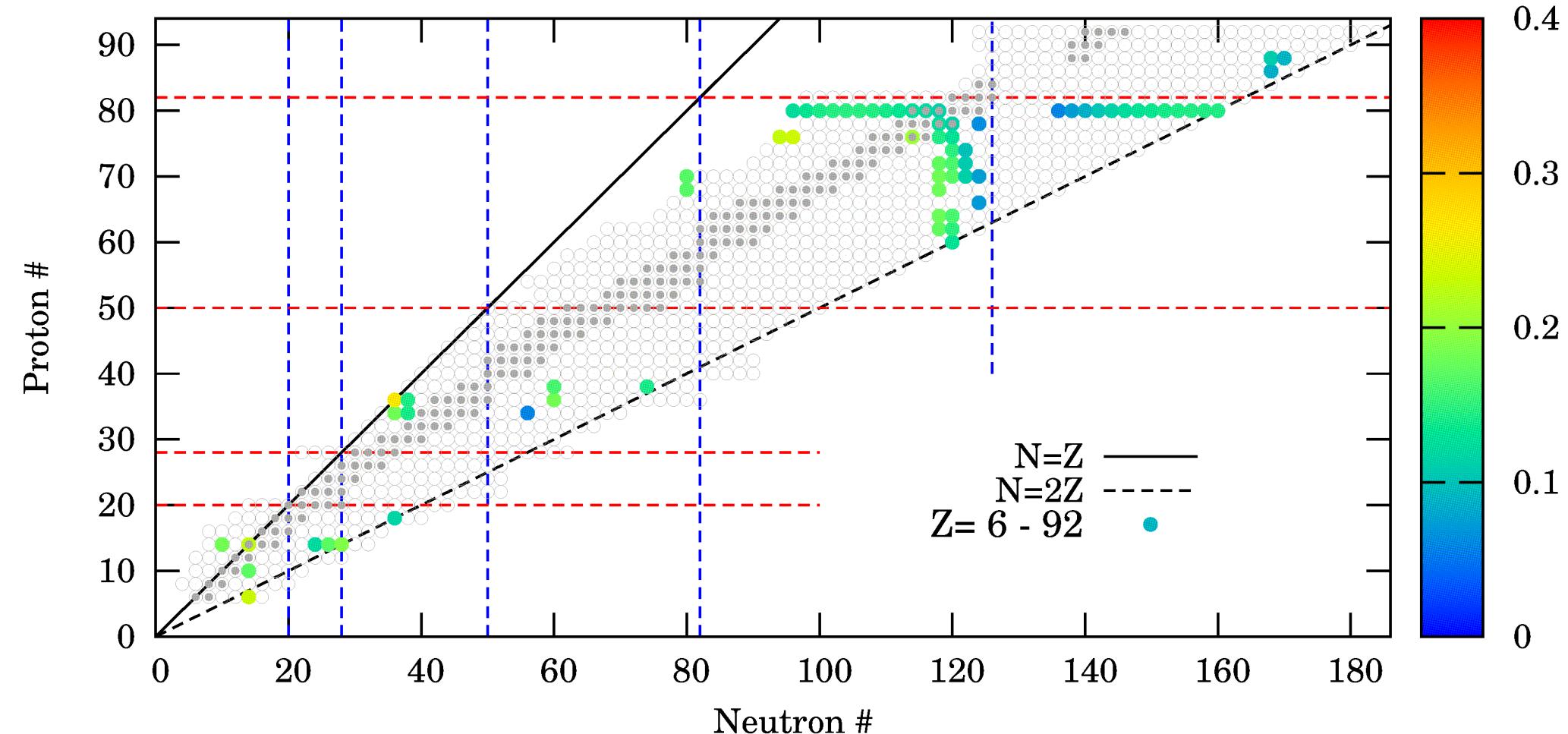
Results (Deformed: oblate)

3D HF+BCS Cal. w/ SkM*

Deformed nuclei: $|\beta_2| > 0.05$, $58.5^\circ < \gamma < 60^\circ$



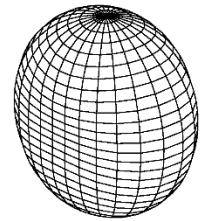
Quadrupole Deformation : Oblate (# 70 / 1005)



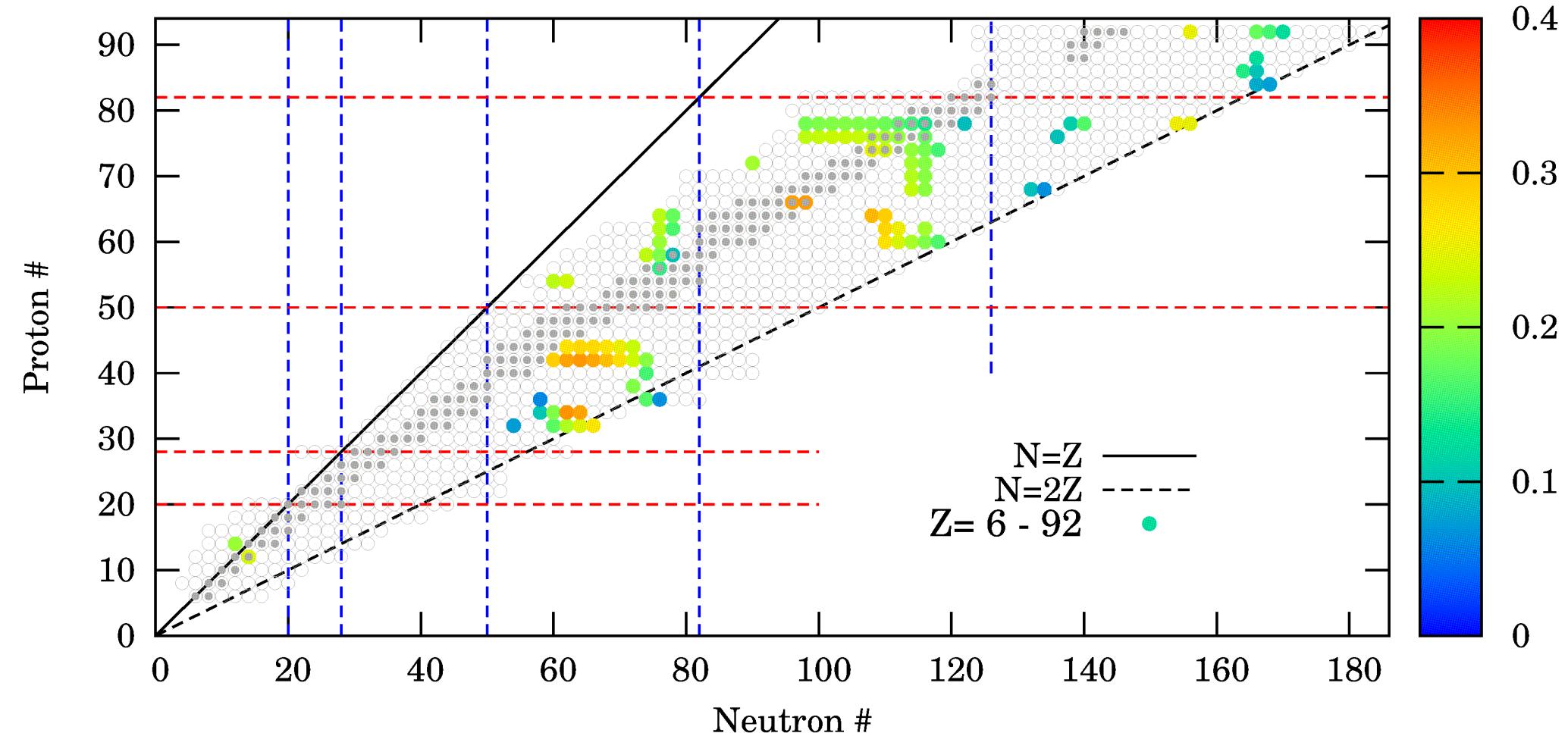
Results (Deformed: triaxial)

3D HF+BCS Cal. w/ SkM*

Deformed nuclei: $|\beta_2| > 0.05$, $1.5^\circ < \gamma < 58.5^\circ$



Quadrupole Deformation : Triaxial (# 101 / 1005)



Results (octupole)

3D HF+BCS Cal. w/ SkM* can describe octupole moments in the ground state.

The octupole deformation parameter is defined using the axis of rotational symmetry.
($\langle Q_{22} \rangle = \langle xy - yx \rangle = 0 \rightarrow z$ is regarded as the axial symmetric axis.)

Octupole deformation parameter β_3, β_{3m}

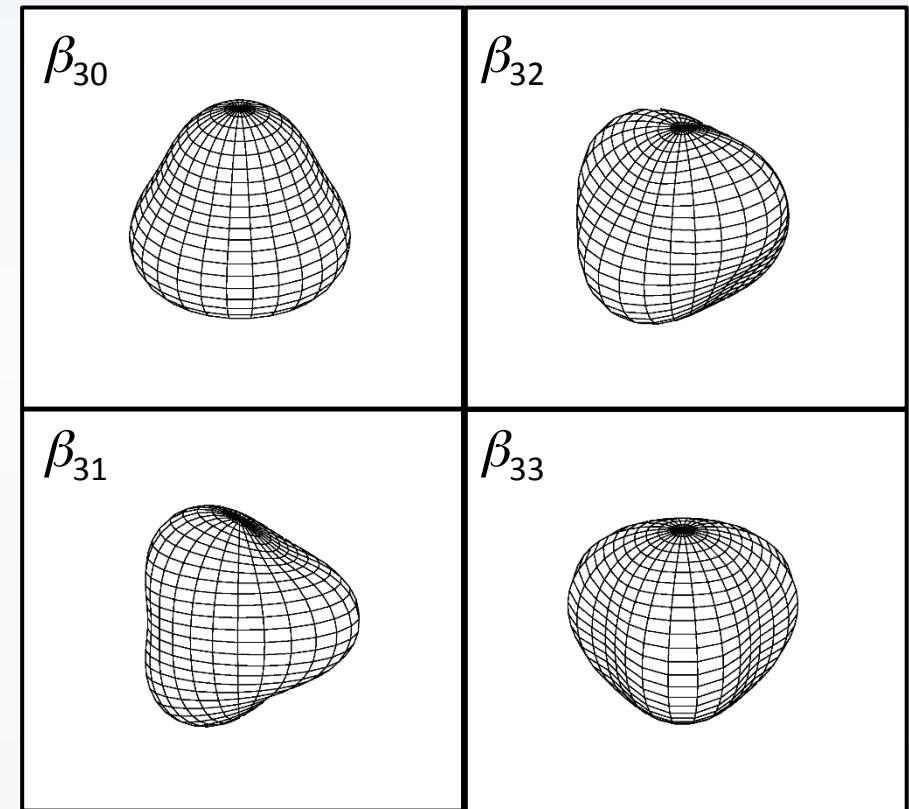
$$\beta_3 = \left(\sum_{m=-3}^3 \alpha_{3m}^2 \right)^{1/2}$$

$$\beta_{3m} = (\alpha_{3m}^2 + \alpha_{3-m}^2)^{1/2}$$

The mass-multipole moment

$$\alpha_{lm} \equiv \frac{4\pi}{3A\bar{R}^l} \int r^l X_{lm}(\Omega) \rho(r) d\mathbf{r}$$

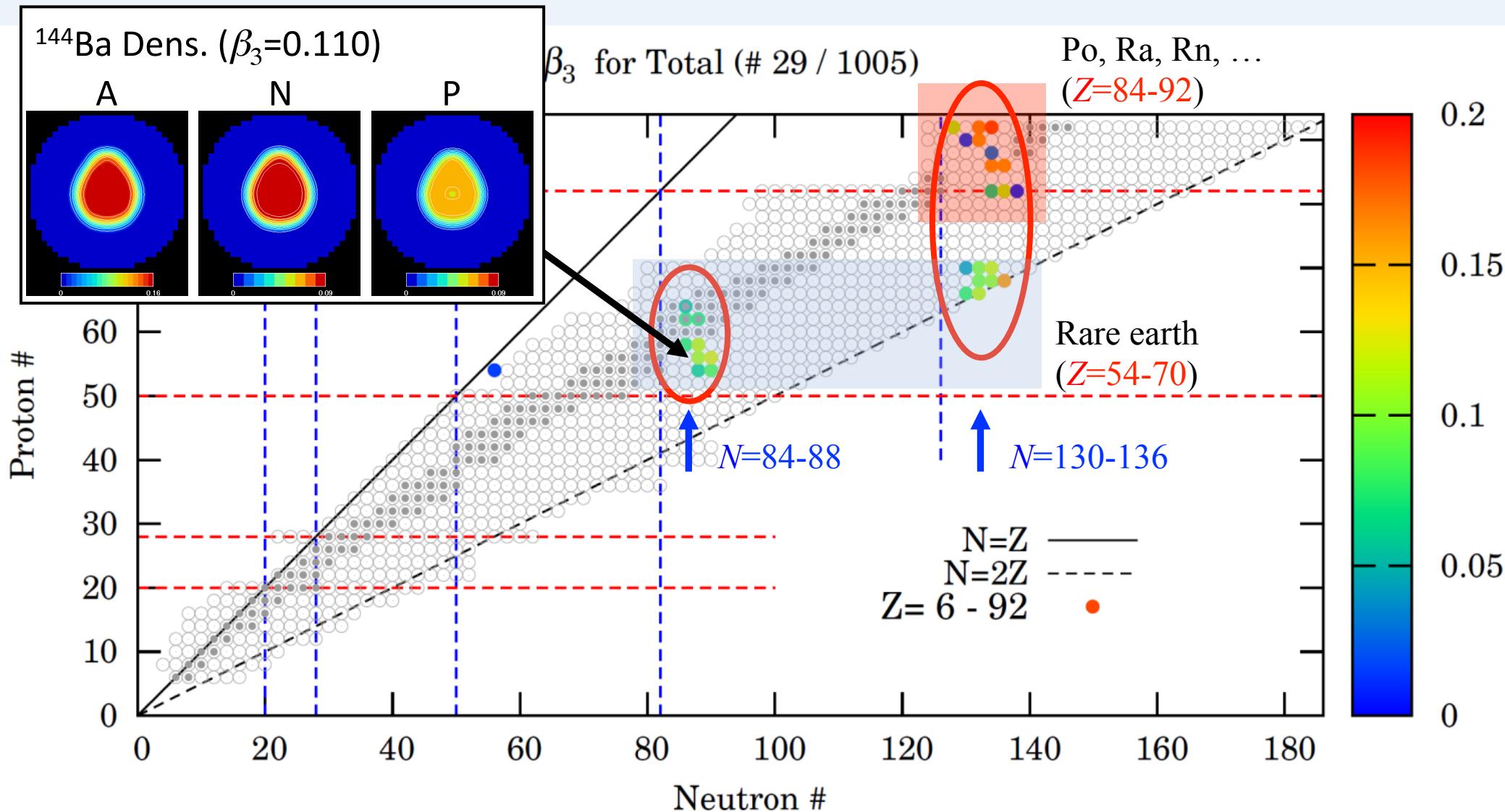
$$\bar{R} = \sqrt{5 \langle \sum_{i=1}^A r_i^2 \rangle / 3A}$$
$$X_{l0} = Y_{l0}$$
$$X_{lm} = \frac{1}{\sqrt{2}} (Y_{l-|m|} + Y_{l-|m|}^*)$$
$$X_{l-|m|} = \frac{-i}{\sqrt{2}} (Y_{l|m|} - Y_{l|m|}^*)$$



Results (Deformed: octupole)

3D HF+BCS Cal. w/ SkM*

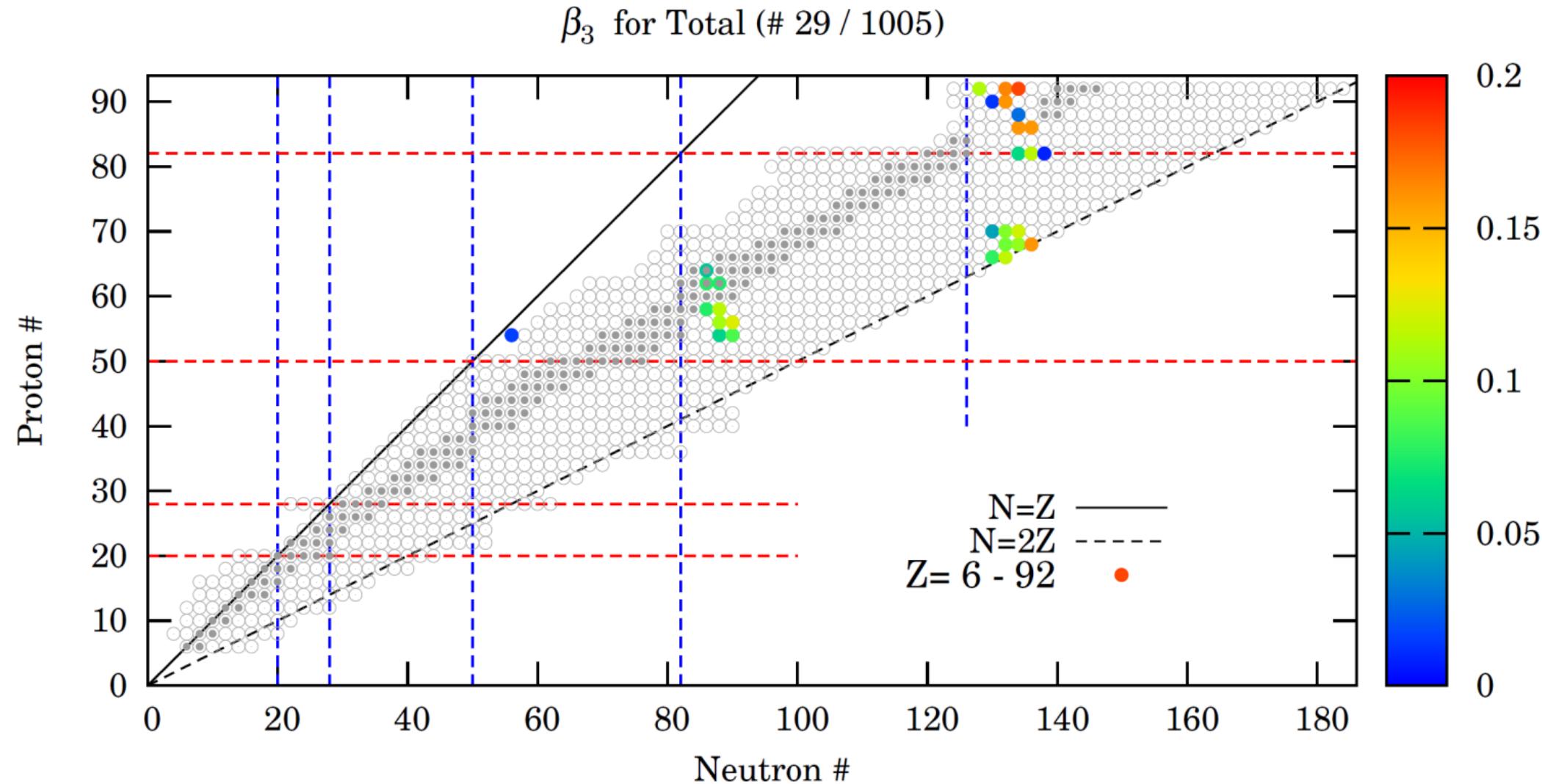
Octupole deformed nuclei: $|\beta_3| > 0.01$



Results (Deformed: octupole)

3D HF+BCS Cal. w/ SkM*

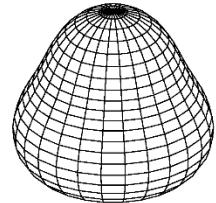
Octupole deformed nuclei: $|\beta_3| > 0.01$



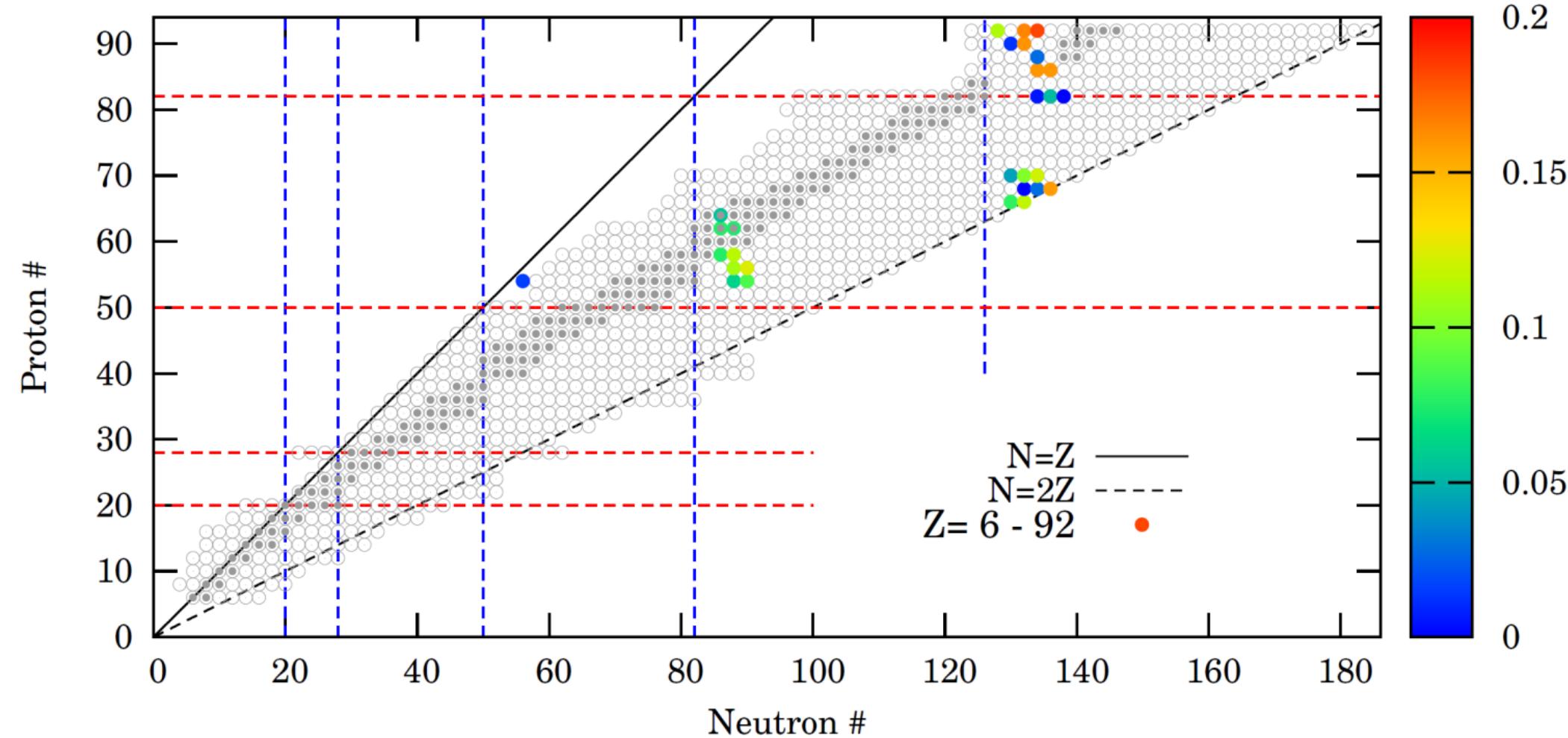
Results (Deformed: octupole β_{30})

3D HF+BCS Cal. w/ SkM*

Octupole deformed nuclei: $|\beta_3| > 0.01$



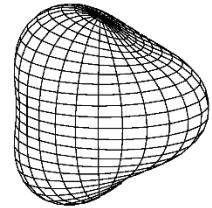
β_{30} for Total (# 1005)



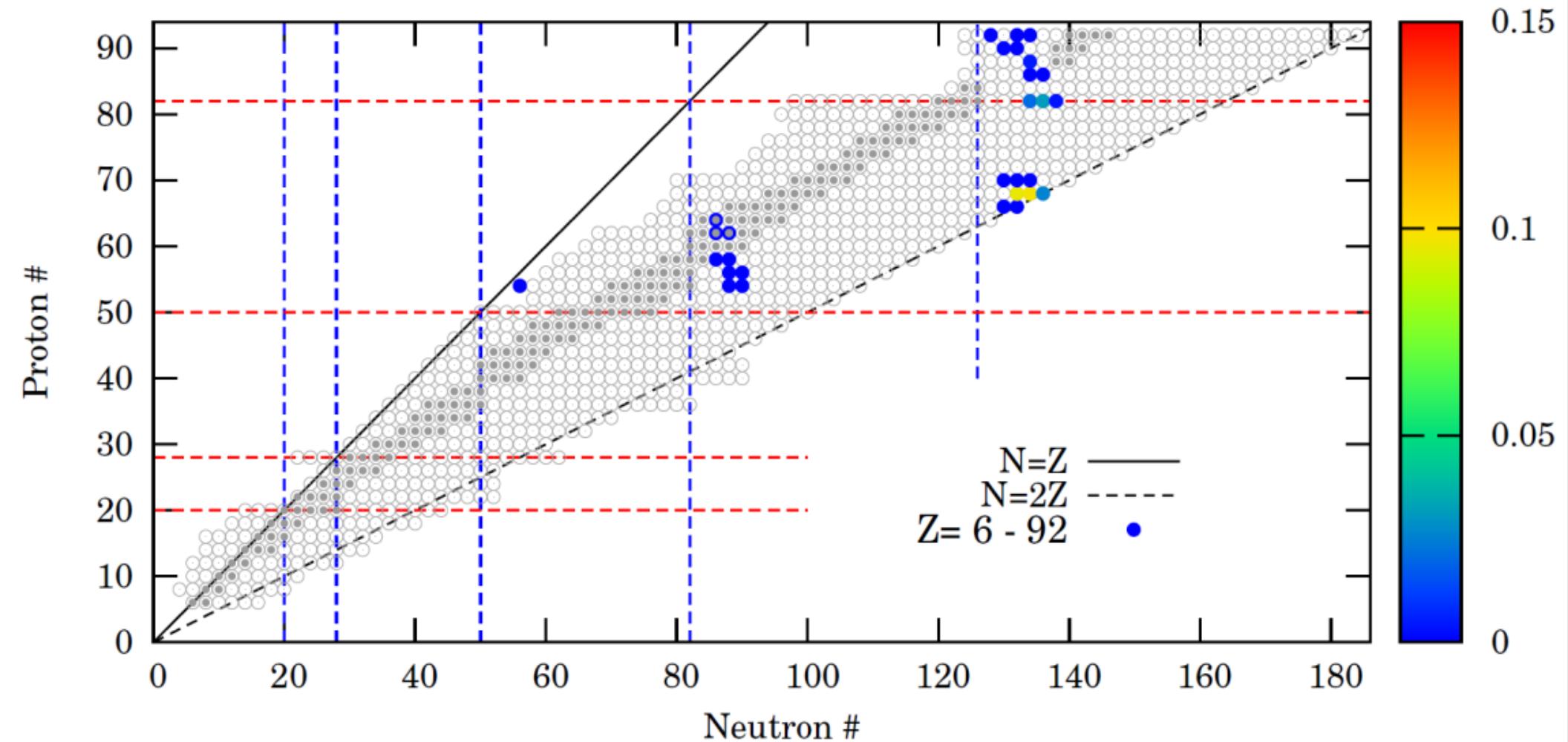
Results (Deformed: octupole β_{31})

3D HF+BCS Cal. w/ SkM*

Octupole deformed nuclei: $|\beta_3| > 0.01$



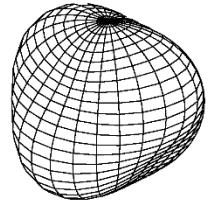
β_{31} for Total (# 1005)



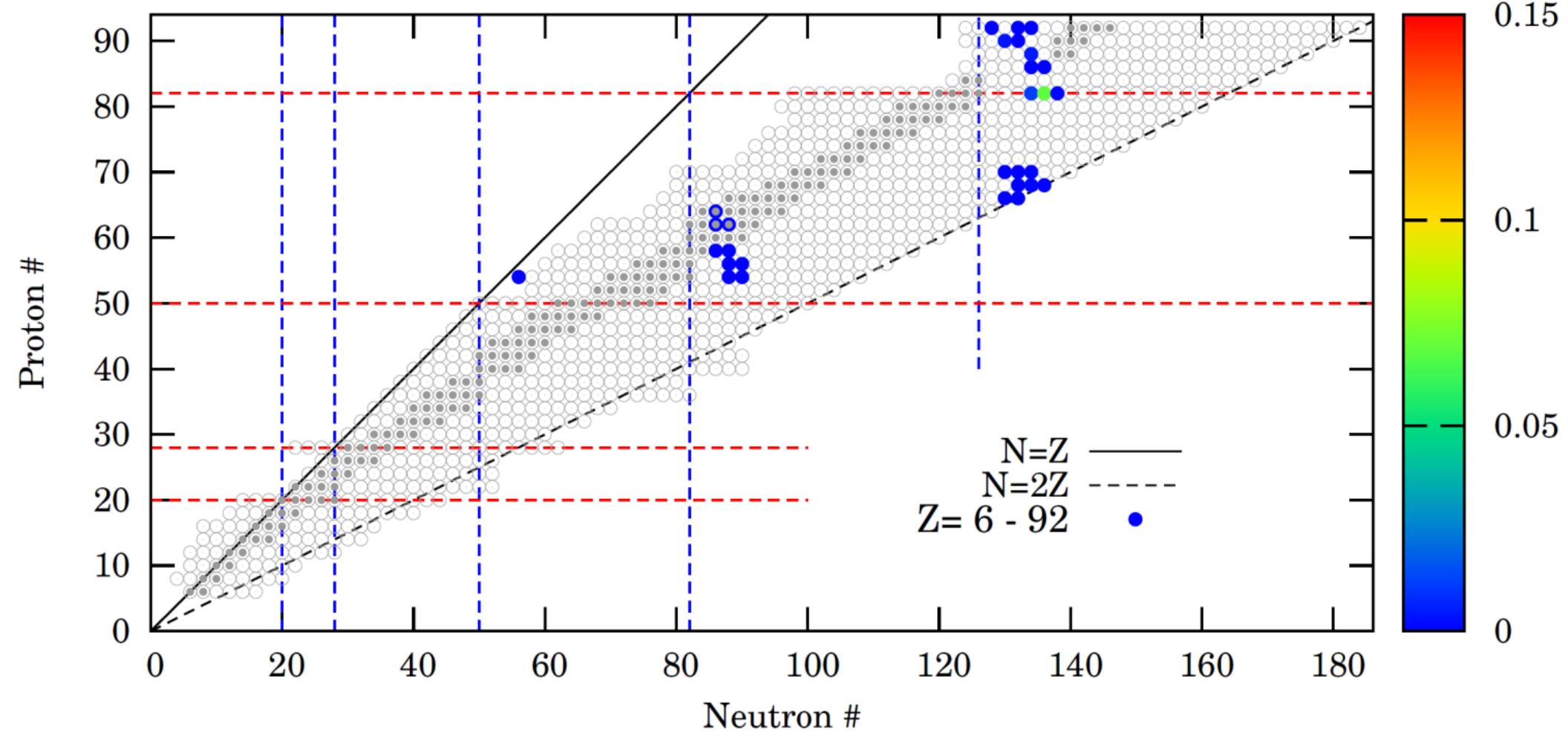
Results (Deformed: octupole β_{32})

3D HF+BCS Cal. w/ SkM*

Octupole deformed nuclei: $|\beta_3| > 0.01$



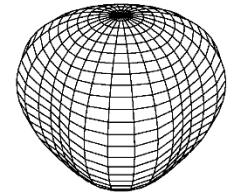
β_{32} for Total (# 1005)



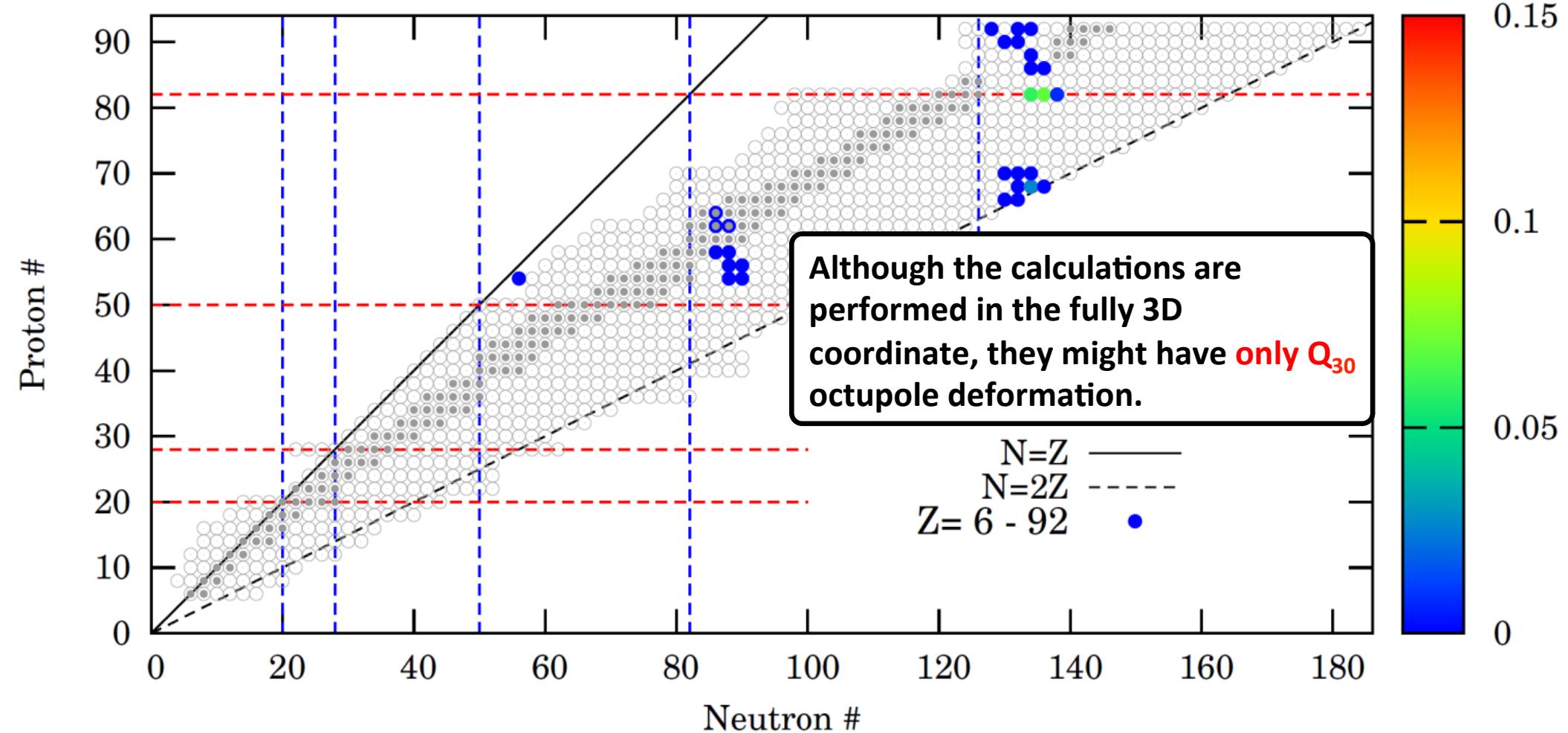
Results (Deformed: octupole β_{33})

3D HF+BCS Cal. w/ SkM*

Octupole deformed nuclei: $|\beta_3| > 0.01$

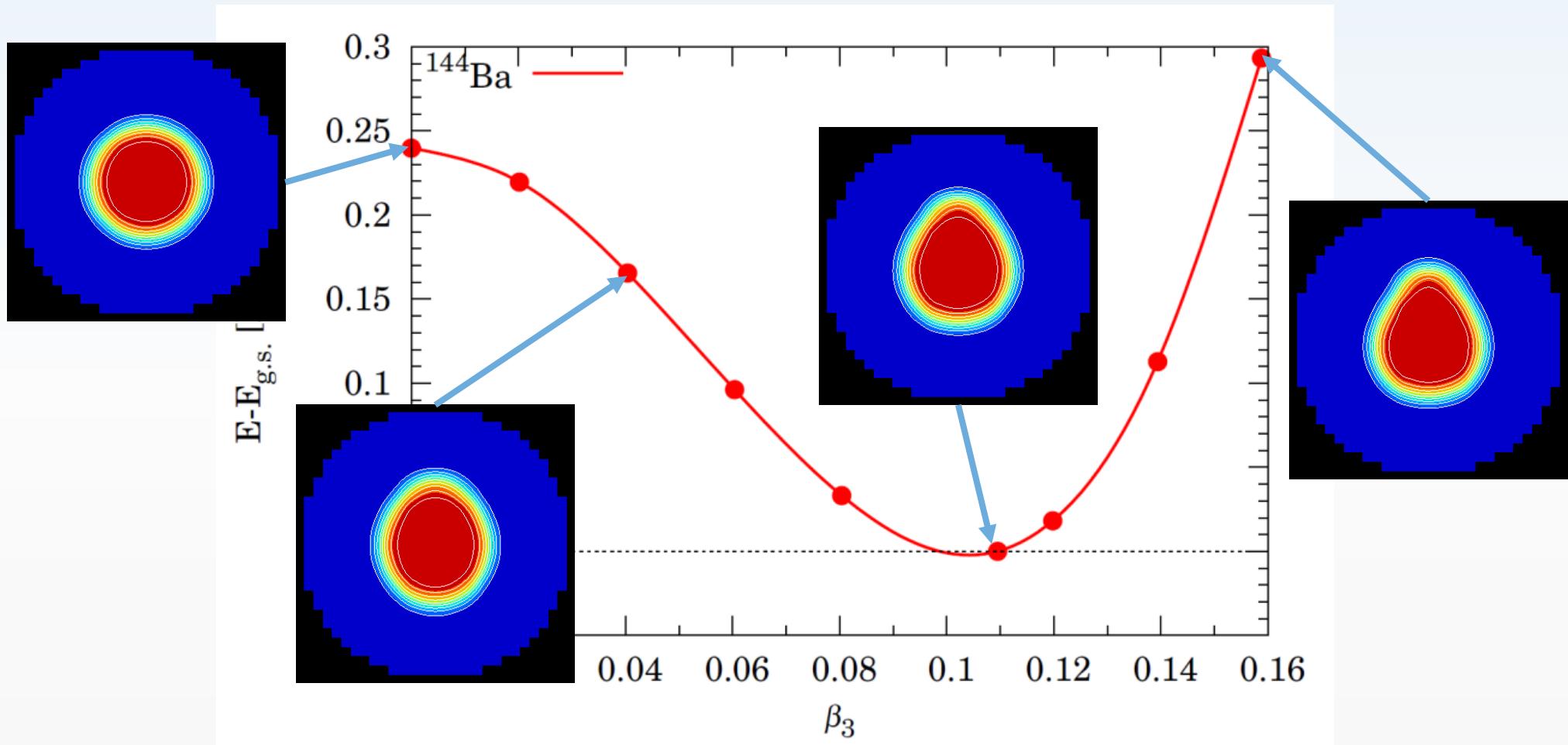


β_{33} for Total (# 1005)



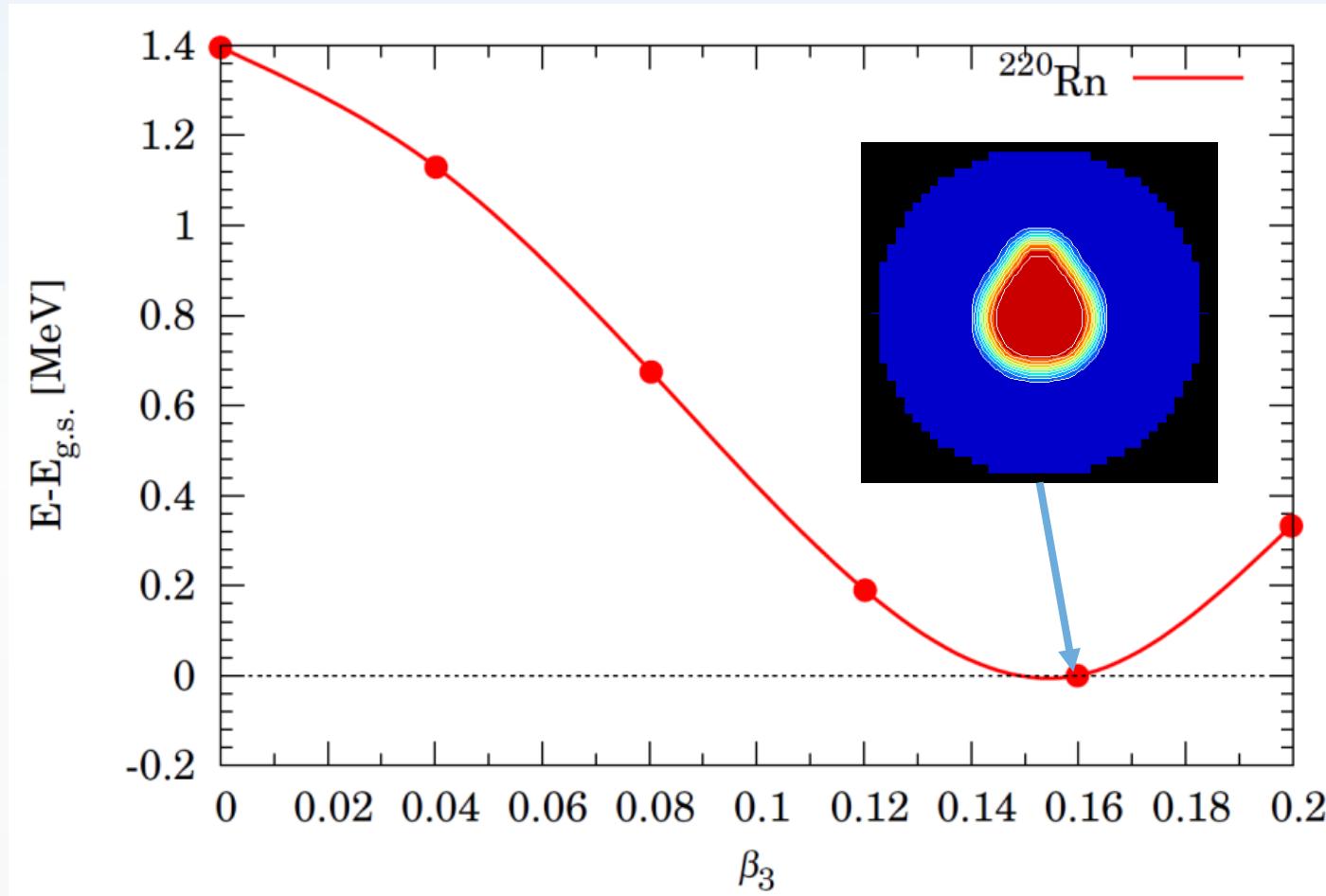
Results (Octupole correlations: ^{144}Ba) Skyrme HF + BCS + Constraints

w/ Q_{30} constraints

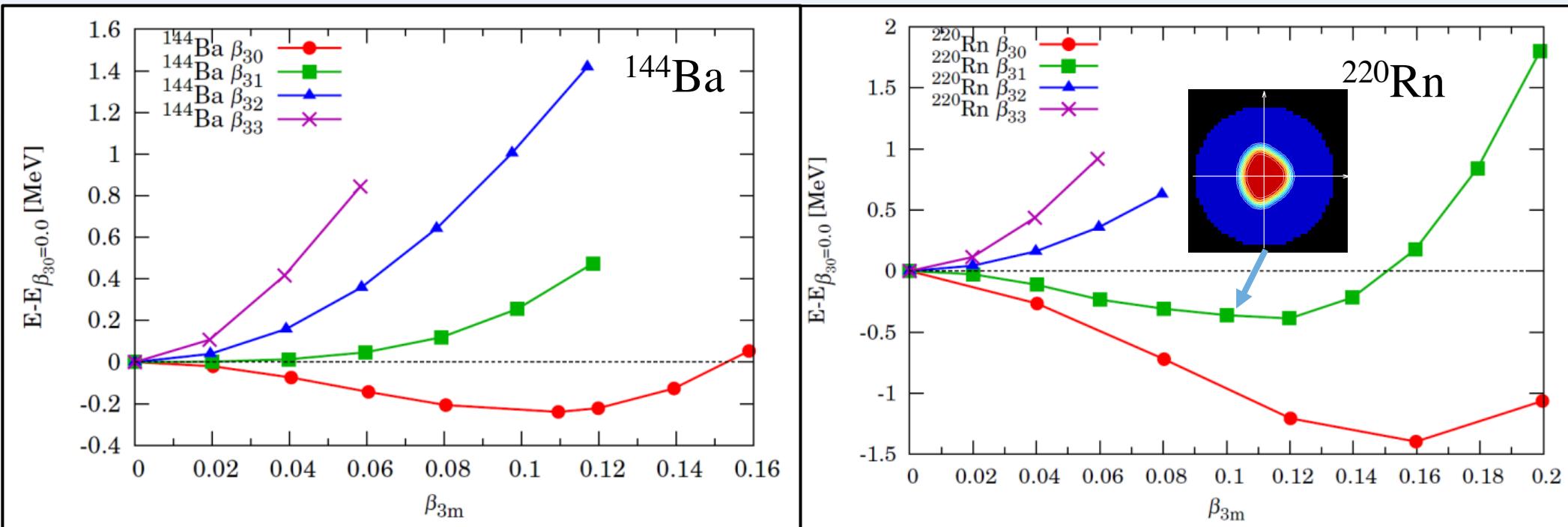


Results (Octupole correlations: ^{220}Rn) Skyrme HF + BCS + Constraints

w/ Q_{30} constraints



Results (β_{30} vs. β_{31} vs. β_{32} vs. β_{33} : ^{144}Ba , ^{220}Rn) Skyrme HF + BCS + Constraints w/ Q_{3m} constraints

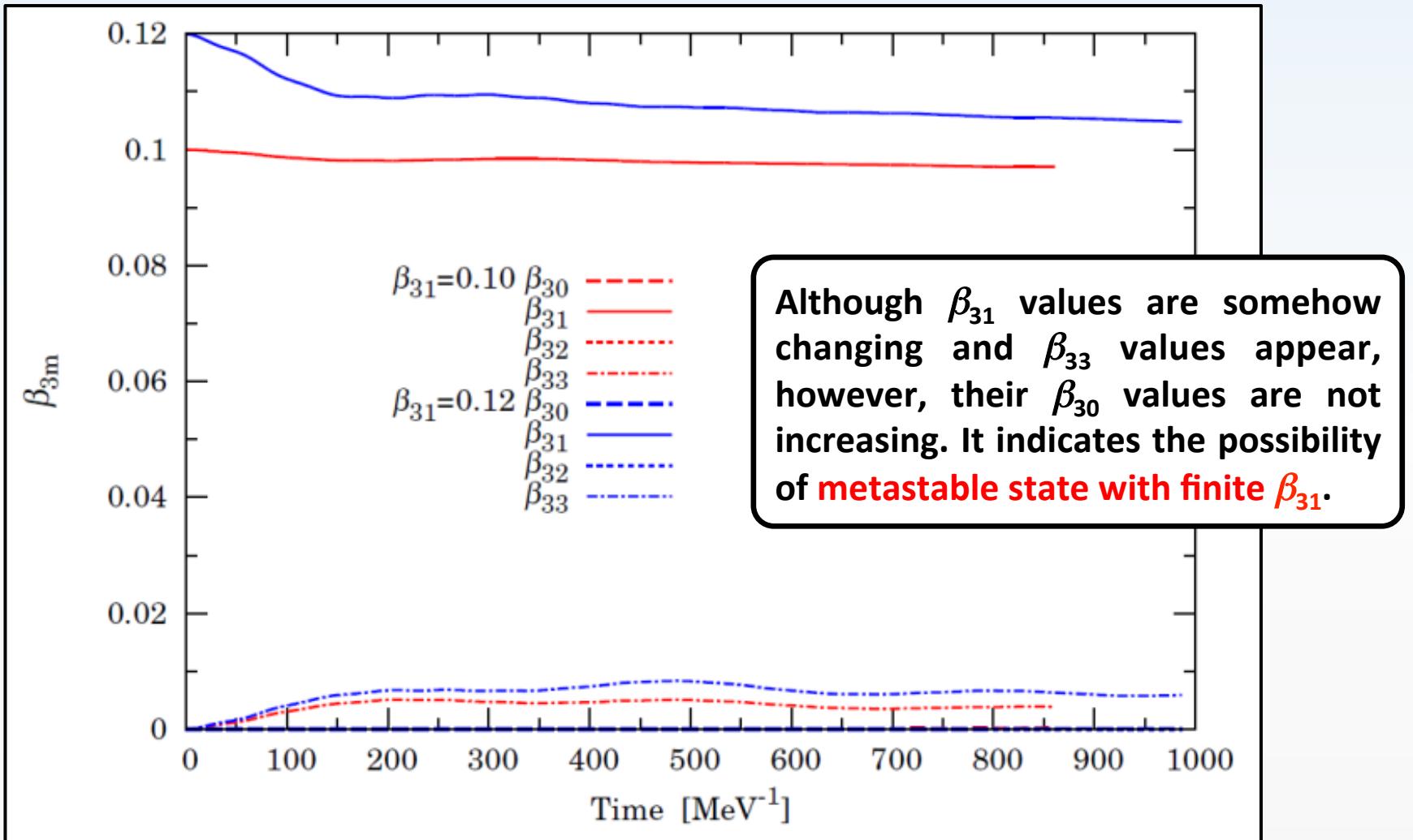


In both, β_{30} is lowest in the energy surface under the octupole constraints.

In this work, ^{144}Ba does not have local minimum on the β_{31} , β_{32} and β_{33} surfaces. Although we need more investigation, ^{220}Rn has some possibility of a local minimum on the β_{31} energy surface.

Results (check the pocket of β_{31} : ^{220}Rn)

Using real-time evolution (w/ Cb-TDHFB), check the stability of β_{31} potential pocket.



Summary

- ✓ We investigate the ground states for even-even nuclei in the nuclear chart by 3D HF+BCS. We found about 54% deformed nuclide in the chart.
- ✓ In the quadrupole deformed nuclei, the Prolate nuclei is 70%, Oblate is 12%, and Triaxial is 18%. We found the Prolate dominance.
- ✓ The nuclei with **octupole deformation** in their ground states are found (about 30 nuclei). They appear in the mass region with **Z=54-70, 86-92** and **N=86-90, 130-138**, which is consistent to the region of **$\Delta l=3$ correlation**. They might appear only with the correlations in both **proton** and **neutron**.
- ✓ The octupole deformed nuclei might have **only pear shape** (Q_{30} type), although this work are performed in the 3D coordinate.
- ✓ We investigate octupole correlations in ^{144}Ba and ^{220}Rn using **multi-constraints** for Q_{30} , Q_{31} , Q_{32} , and Q_{33} . **^{220}Rn metastable state might be indicated in Q_{31}** .

Remains to do ...

- Stable calculation for octupole deformed nuclei
- Parity projection
- Octupole correlation strength
 - Relation between deformed nuclei region and pairing strength
 - Space between single-particle orbits

Thank you !