Nuclear Physics from Lattice QCD

Takumi Doi
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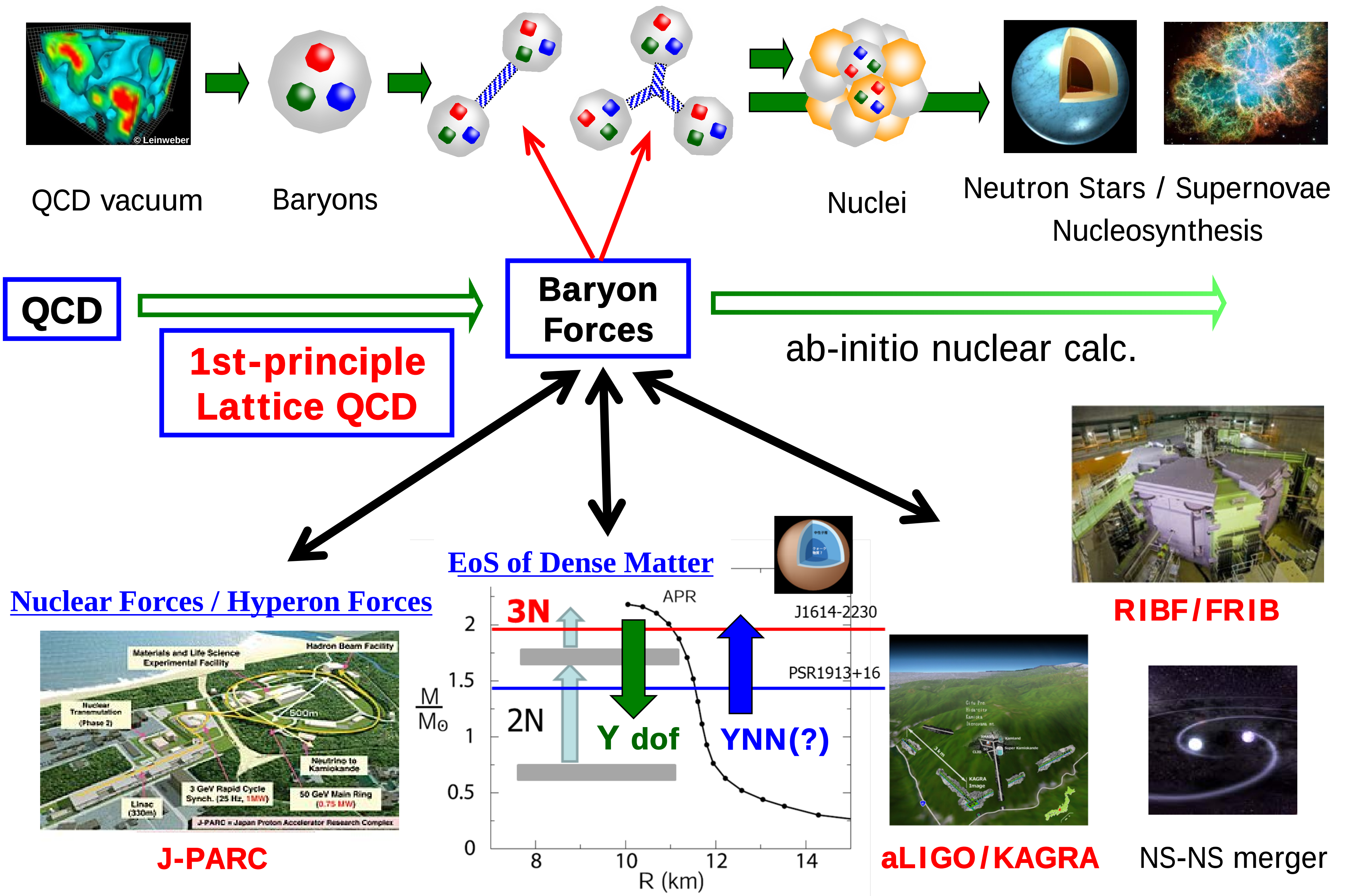
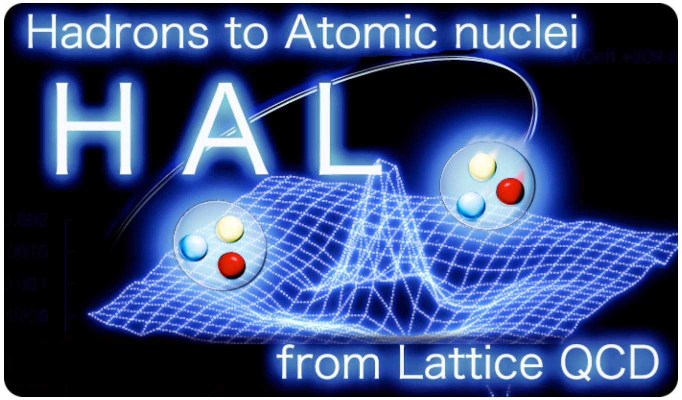
- Introduction
- Theoretical framework
- Results at physical quark masses
- Summary / Prospects

The Odyssey from Quarks to Universe



- **Outline**
  - Introduction
  - Theoretical framework (HAL QCD method)
  - Results at physical quark masses
  - Summary / Prospects

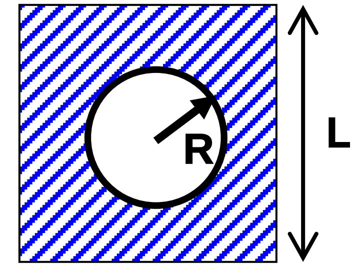


S. Aoki, D. Kawai,
T. Miyamato, K. Sasaki (YITP)
T. Doi, T. Hatsuda, T. Iritani (RIKEN)
F. Etminan (Univ. of Birjand)
S. Gongyo (Univ. of Tours)
Y. Ikeda, N. Ishii, K. Murano (RCNP)
T. Inoue (Nihon Univ.)
H. Nemura (Univ. of Tsukuba)

[HAL QCD method]

- Nambu-Bethe-Salpeter (NBS) wave function

$$\psi(\vec{r}) = \langle 0|\ N(\vec{r})N(\vec{0})\ |N(\vec{k})N(-\vec{k}); in\rangle$$



$$(\nabla^2 + k^2)\psi(\vec{r}) = 0, \quad r > R$$

- – phase shift at asymptotic region

$$\psi(r) \simeq A \frac{\sin(kr - l\pi/2 + \delta(k))}{kr}$$

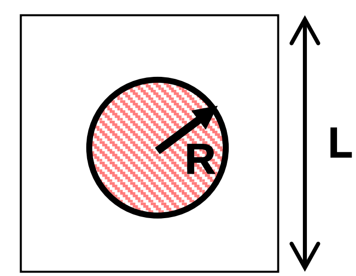
M.Luscher, NPB354(1991)531
C.-J.Lin et al., NPB619(2001)467
N.Ishizuka, PoS LAT2009 (2009) 119
CP-PACS Coll., PRD71(2005)094504

Extended to multi-particle systems

S. Aoki et al., PRD88(2013)014036

- Consider the wave function at “interacting region”

$$(\nabla^2 + k^2)\psi(\boldsymbol{r}) = m \int d\boldsymbol{r}' U(\boldsymbol{r}, \boldsymbol{r}')\psi(\boldsymbol{r}'), \quad r < R$$



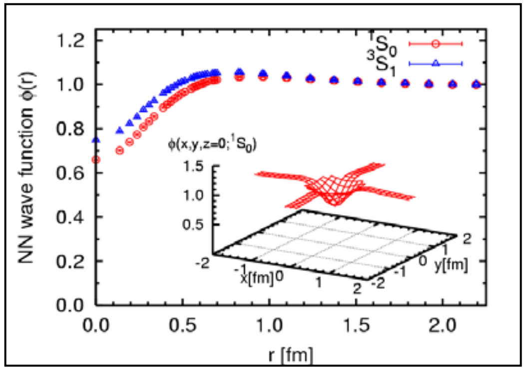
- – U(r,r’): faithful to the phase shift by construction
  - U(r,r’): E-independent, while non-local in general
    - – Non-locality ➔ derivative expansion

Aoki-Hatsuda-Ishii PTP123(2010)89

HAL QCD method

Lattice QCD

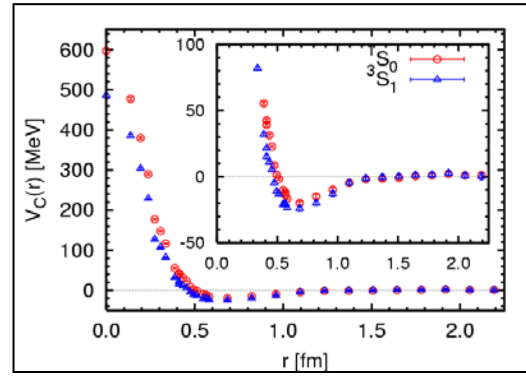
NBS wave func.



$$\psi_{NBS}(\vec{r}) = \langle 0|N(\vec{r})N(\vec{0})|N(\vec{k})N(-\vec{k}), in\rangle$$
$$\simeq A_k \sin(kr - l\pi/2 + \delta_l(k))/(kr)$$

(at asymptotic region)

Lat Baryon Force



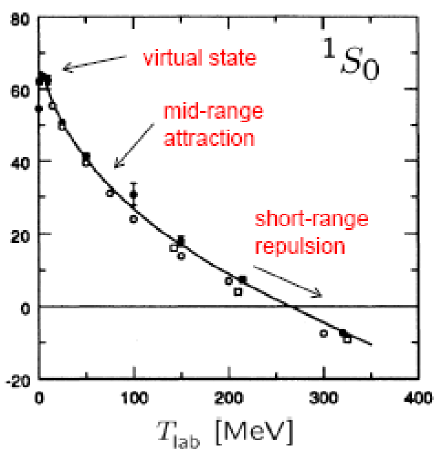
$$(k^2/m_N - H_0)\,\psi(\vec{r}) = \int d\vec{r}'\, U(\vec{r},\vec{r}')\psi(\vec{r}')$$

E-indep (& non-local) Potential:
Faithful to phase shifts

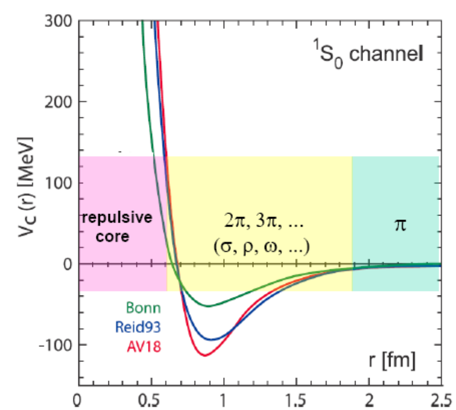
Analog to …

Scattering Exp.

Phase shifts



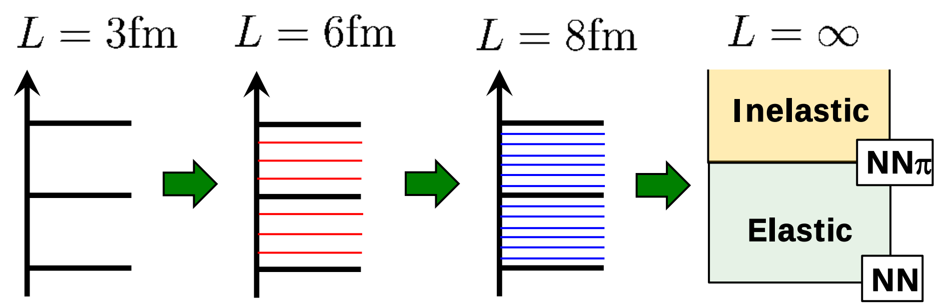
Phen. Potential



The Challenge in multi-baryons on the lattice

Almost No Excitation Energy

➔ LQCD method based on G.S. saturation unreliable



$S/N \propto$ 10^{-4} 10^{-13} 10^{-25}

QCD

Baryon Forces

HAL QCD method

"Time-dependent method" N.Ishii et al. PLB712(2012)437

G.S. saturation NOT required w/ E-indep pot

Direct method

G.S. saturation required

Yamazaki et al.
Savage et al. (NPL Coll.)

Experiments

The fate of the direct method (check on NN)

T. Iritani et al. (HAL Coll.) JHEP1610(2016)101 **[Talks by S. Aoki & T. Iritani (Fri.)]**

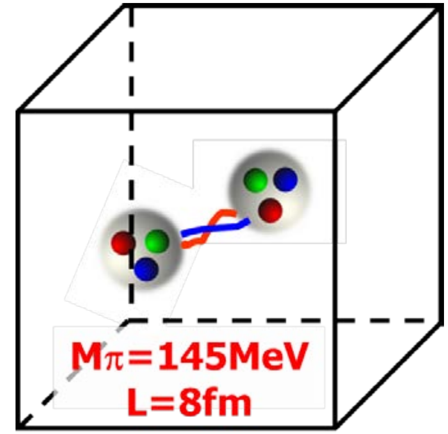
	single baryon	double baryon				
	plateau check	mirage plateau	src-dep check	sink-dep check	Effective Range expansion check	**Overall Verdict**
YKU 2011	○	×	△	**Not checked**	×	**False**
YIKU 2012	○	×	×	×	×	**False**
YIKU 2015	○	×	**Not checked**	**Not checked**	×	**False**
NPL 2012	○	×	**Not checked**	**Not checked**	×	**False**
NPL 2013	○	×	**Not checked**	**Not checked**	△	**False**
NPL 2015	△	×	**Not checked**	**Not checked**	×	**False**

- **Outline**
  - Introduction
  - Theoretical framework
  - Results at (almost) physical quark masses w/ HAL method
    - Nuclear forces and Hyperon forces
    - Impact on dense matter
  - Summary / Prospects

Lattice QCD Setup

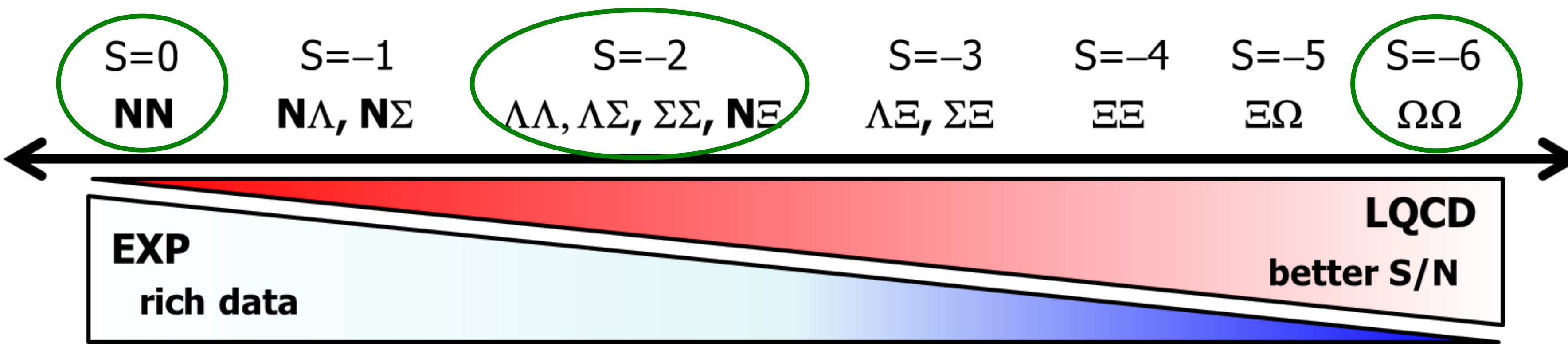
- **Nf = 2 + 1 gauge configs**
  - clover fermion + Iwasaki gauge
  - V=(8.1fm)^4, a=0.085fm (1/a = 2.3 GeV)
  - m(pi) ~= 145 MeV, m(K) ~= 525 MeV

K.I. Ishikawa et al., PoS LAT2015, 075



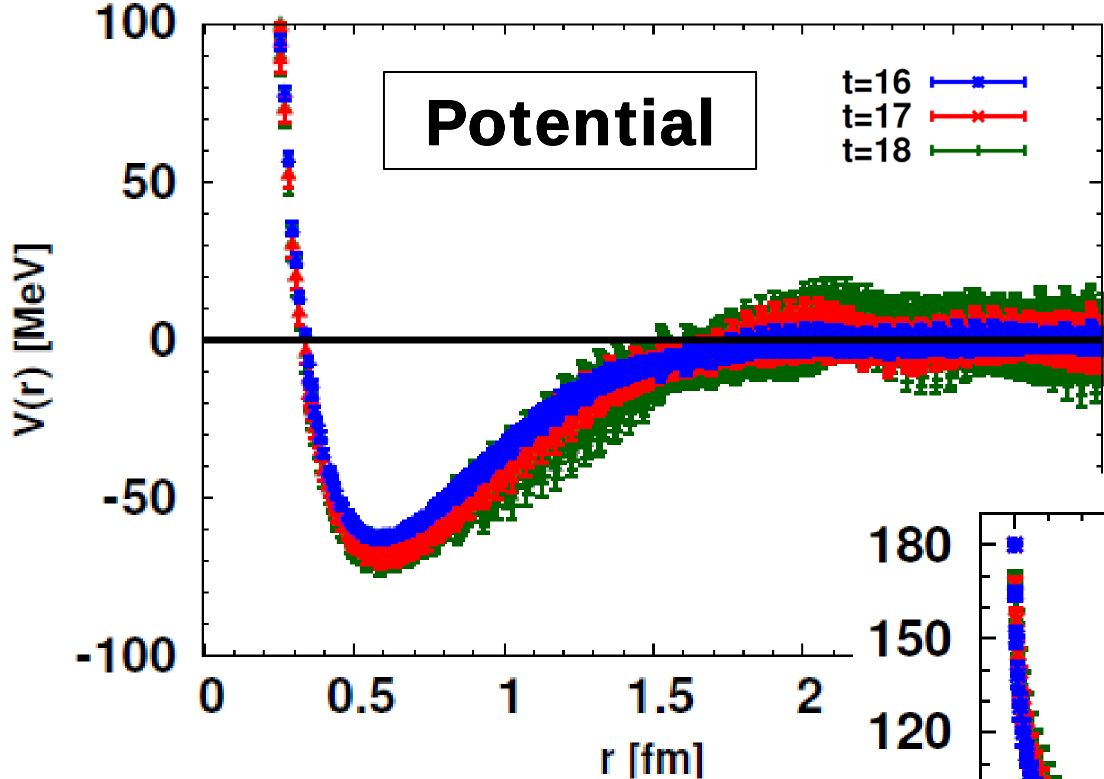
- **Measurement**
  - NN/YN/YY for central/tensor forces in P=(+) (S, D-waves)
  - Unified Contraction Algorithm (UCA) ➔ drastic speedup in calc

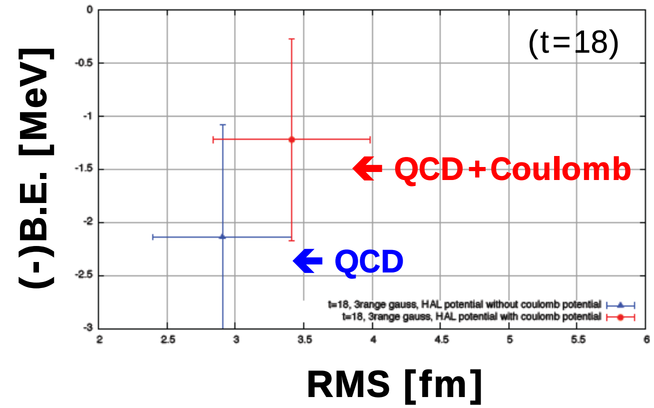
Hyperon forces provide precious predictions

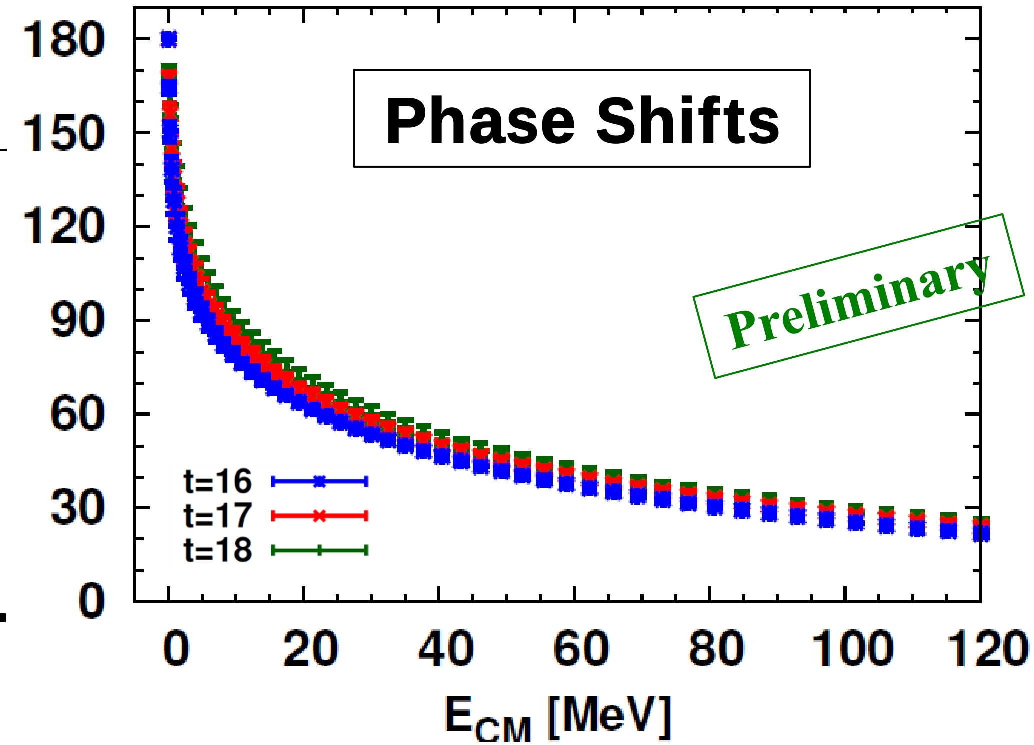


ΩΩ system ($^1S_0$)

(The "most strange" dibaryon system)







Strong Attraction

➔ Vicinity of bound / unbound

[~ Unitary limit]

⟺ ΩΩ correlation in HIC exp.

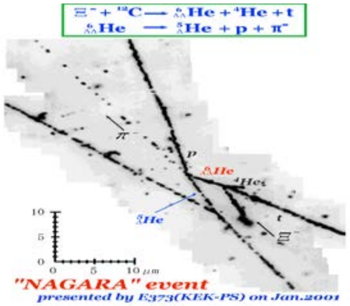
[S. Gongyo / K. Sasaki]

S = -2 channel (Coupled Channel)

H-dibaryon ($^1S_0$, $\Lambda\Lambda$–$N\Xi$–$\Sigma\Sigma$)

NAGARA-event (2001)

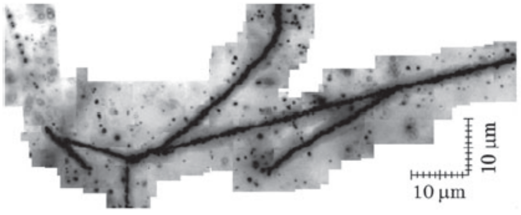
$$\Xi^- + {}^{12}\mathrm{C} \rightarrow {}_{\Lambda\Lambda}{}^{6}\mathrm{He} + {}^{4}\mathrm{He} + t$$



Ξ–hypernuclei

KISO-event (2014)

$$\Xi^- + {}^{14}\mathrm{N} \rightarrow {}_{\Lambda}{}^{10}\mathrm{Be} + {}_{\Lambda}{}^{5}\mathrm{He}$$

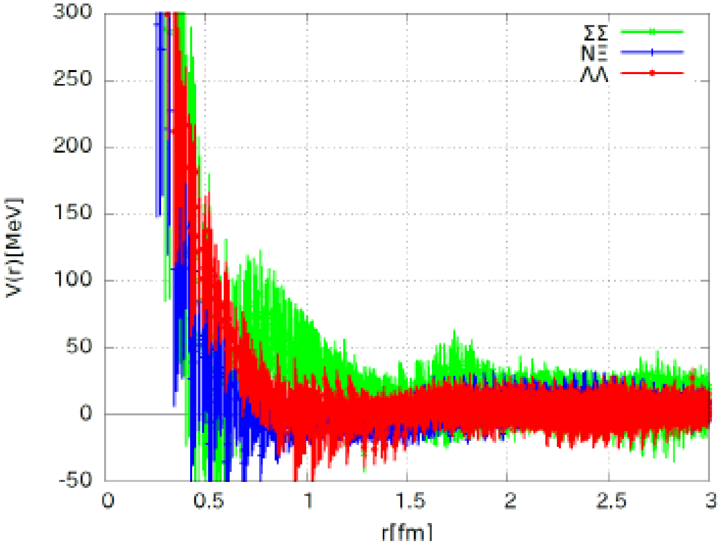


B.E. = 4.38(25) MeV
(or 1.11(25) MeV)

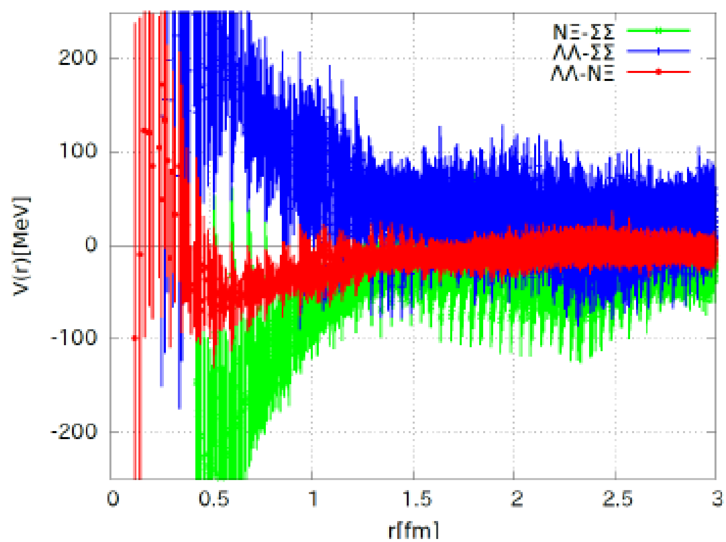
H-dibaryon @ Nf=2+1, m_π=146 MeV

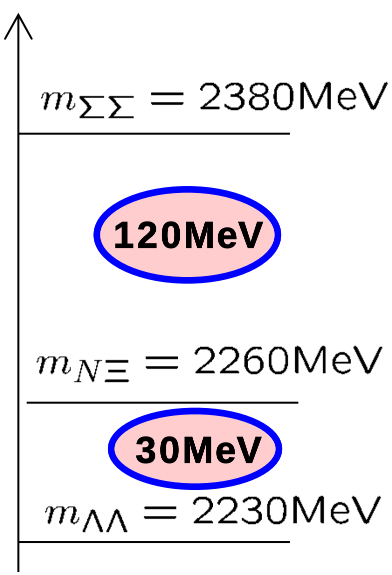
[K. Sasaki]

diagonal



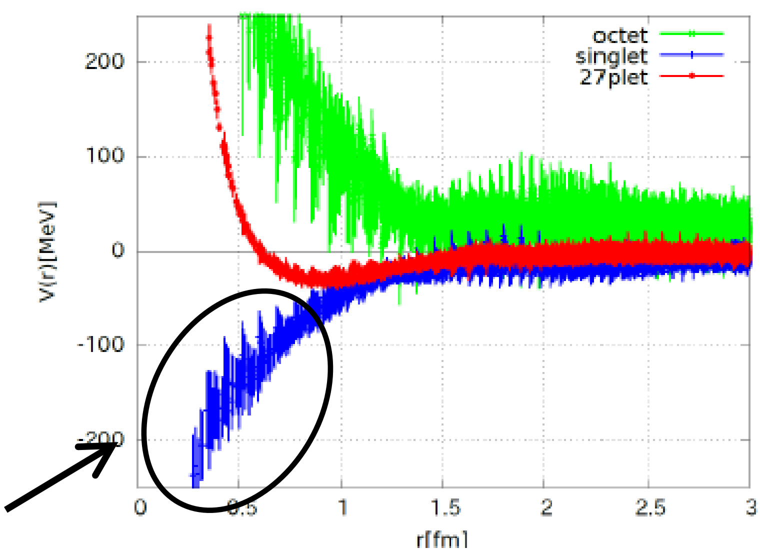
off-diagonal





diagonal in SU(3)-irrep base

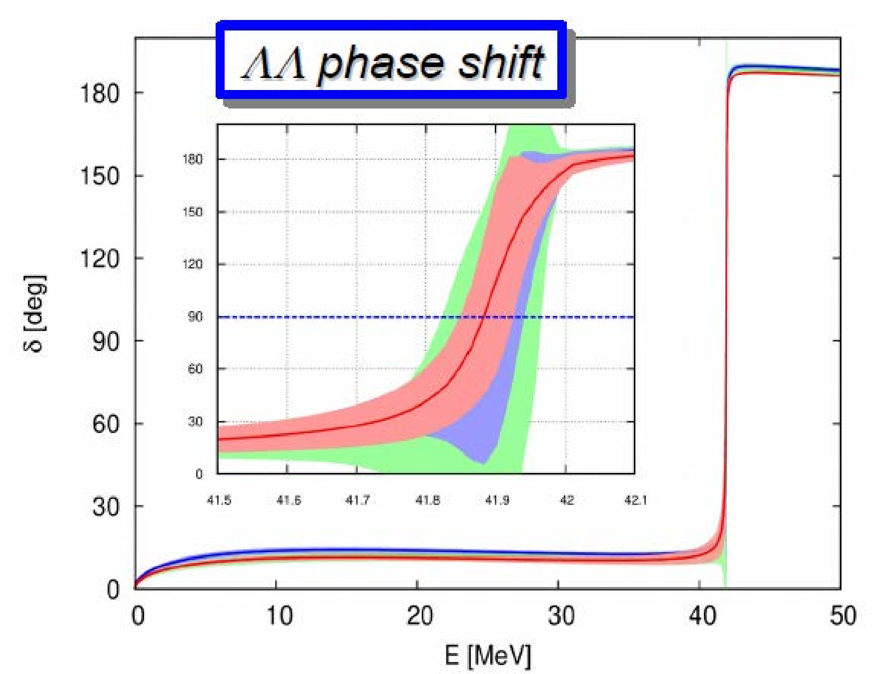
Strong Attraction in flavor-singlet channel

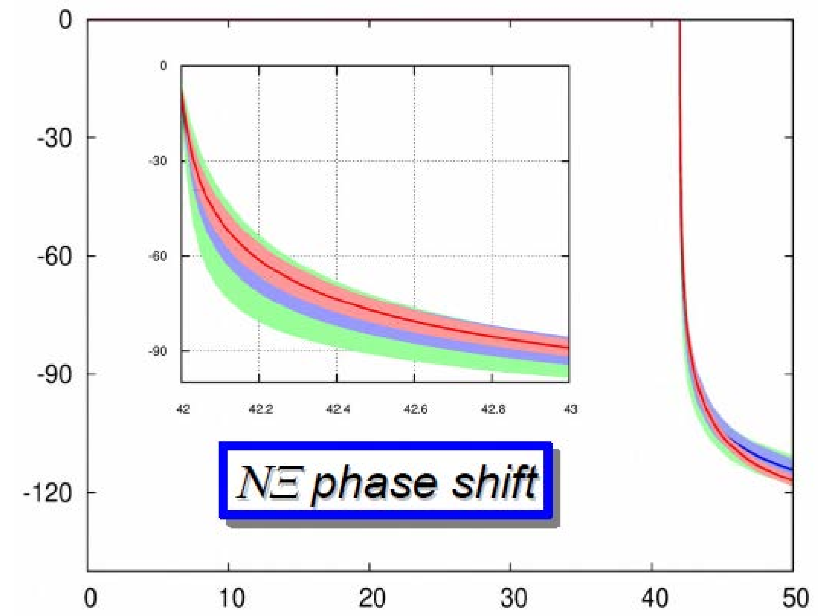


(400conf x 4rot x 28src, t=11)

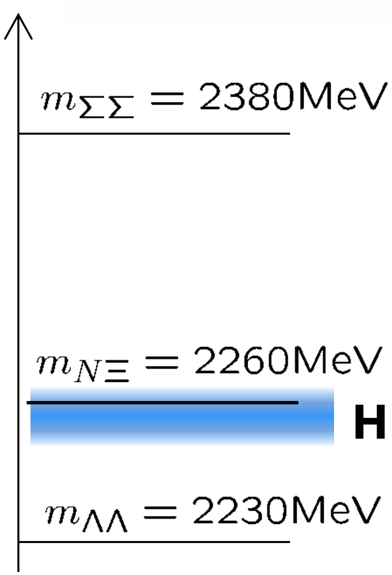
ΛΛ, NΞ (effective) 2x2 coupled channel analysis

Preliminary





(t=9,10,11)



H-resonance (?)

A Resonant Dihyperon (?)

pole analysis on going

➔ **J-PARC experiment (E42)**

N.B. t-dep should be checked;
single m_B has ~3% sys @ t=10

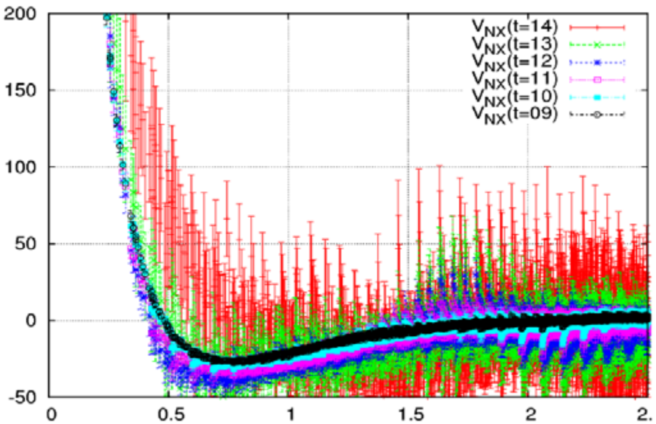
[K. Sasaki]

NΞ-Potentials

[K. Sasaki]

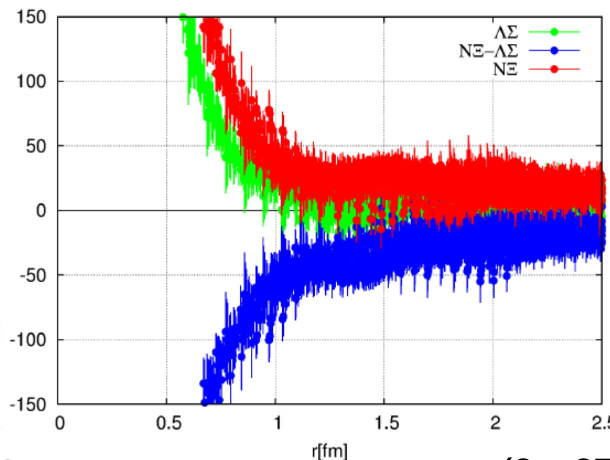
⇔ Ξ–hypernuclei

NΞ (I=0, $^3S_1$)	NΞ–ΛΣ (I=1, $^1S_0$)	NΞ–ΛΣ–ΣΣ (I=1, $^3S_1$)



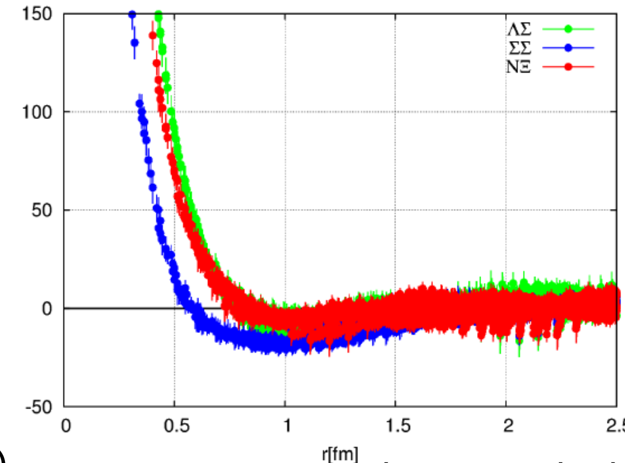
(8a)

Attractive



(8s, 27)

Repulsive



(8a, 10, 10bar)

Attractive

(ΛΛ–NΞ–ΣΣ (I=0, $^1S_0$))

Is interaction net attractive ? Stay tuned !

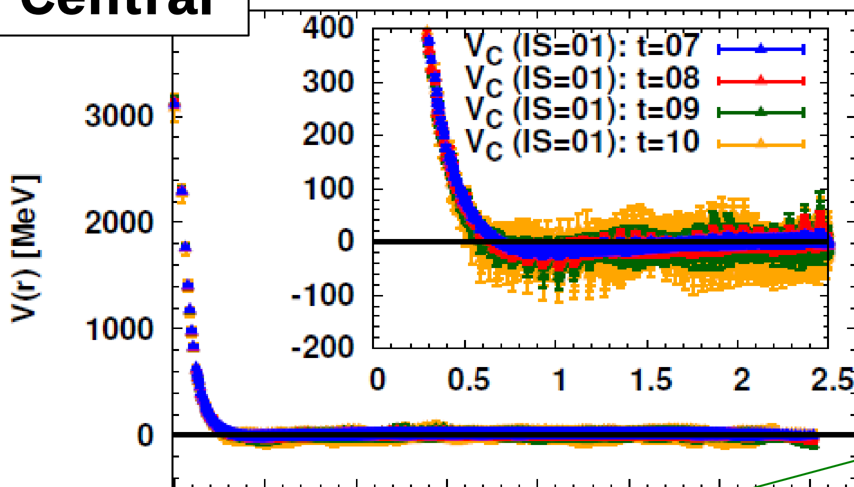
(200conf x 4rot x 20src, t=10)

NN system (S = 0)

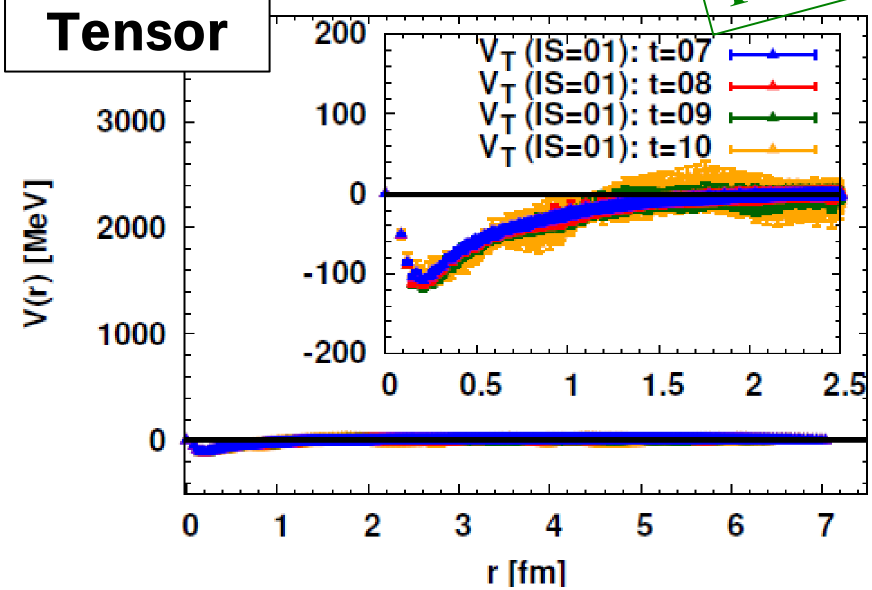
NN system ($^3S_1$-$^3D_1$)

Potentials

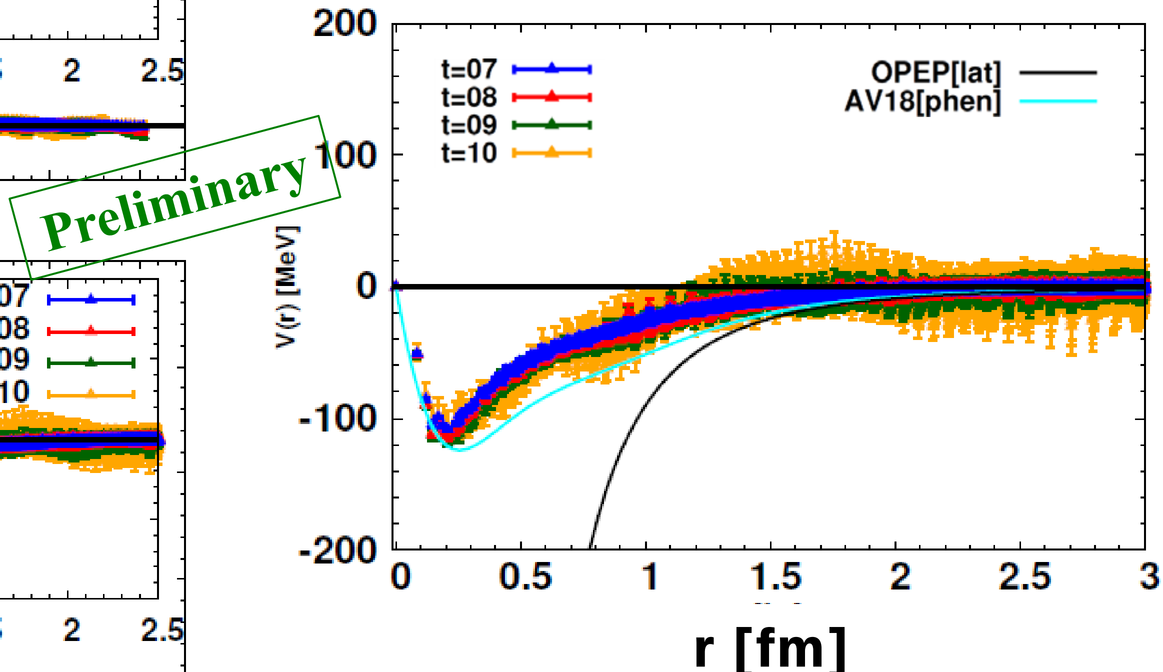
Central



Tensor



- **Vc: repulsive core + long-range attraction**
- **Vt: strong tensor force !**

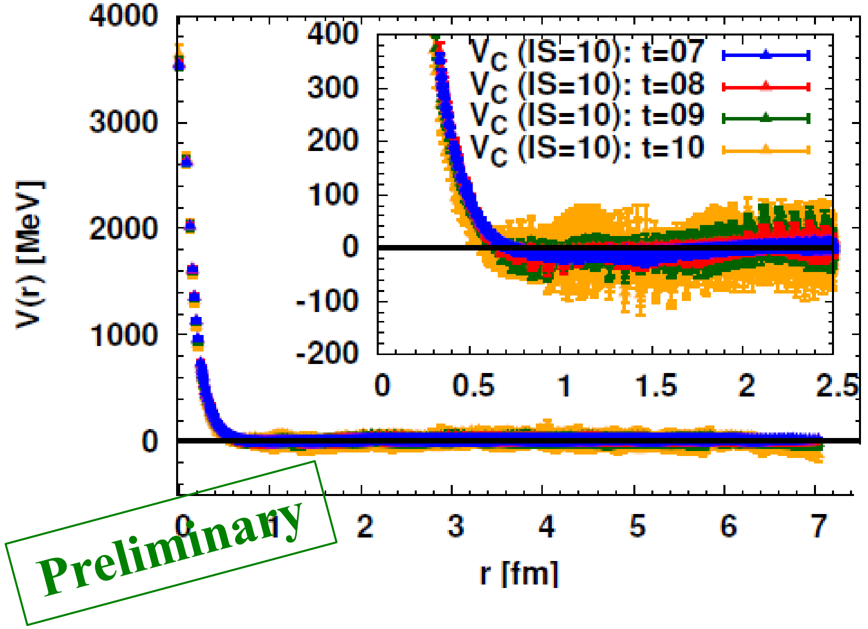


m(eff) for single N: ~2-4% sys err for t = 8-10

(400conf x 4rot x 48src)

NN system ($^1S_0$)

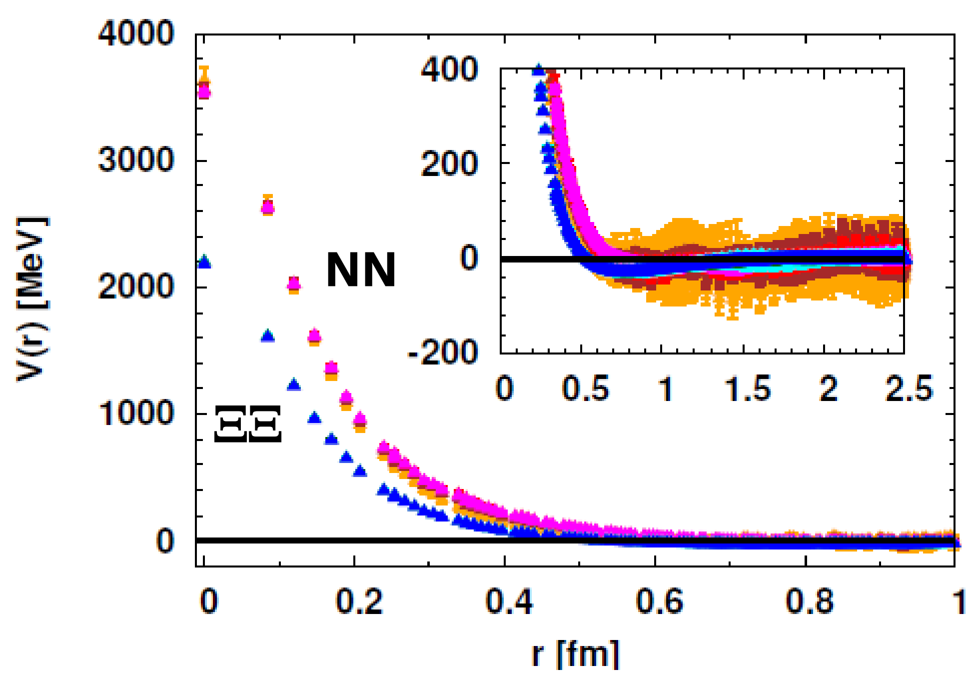
Potentials



- **Vc: repulsive core + long-range attraction**

The effect of SU(3)f breaking

NN($^1S_0$) and ΞΞ($^1S_0$) : 27-plet



Repulsive core enhanced
for lighter quark mass ? ←→ OGE ?

N.B. Sys error in NN may be underestimated

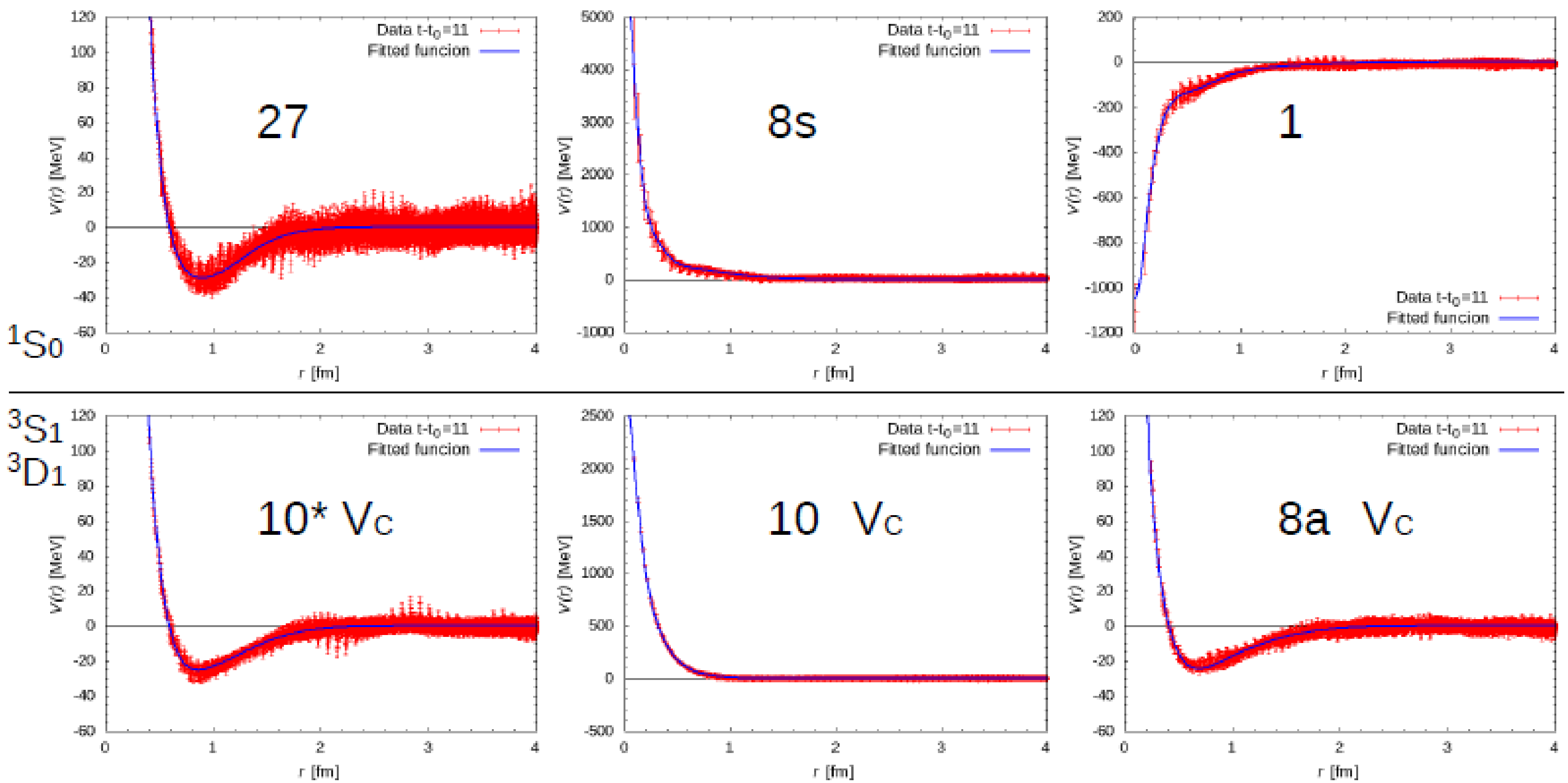
(400conf x 4rot x 48src)

Impact on dense matter

S=-2 interactions suitable to grasp whole NN/YN/YY interactions

Central Force in Irrep-base (diagonal)

$$8 \times 8 = \underbrace{27 + 8s + 1}_{^1S_0} + \underbrace{10^* + 10 + 8a}_{^3S_1,\ ^3D_1}$$



(off-diagonal component is small)

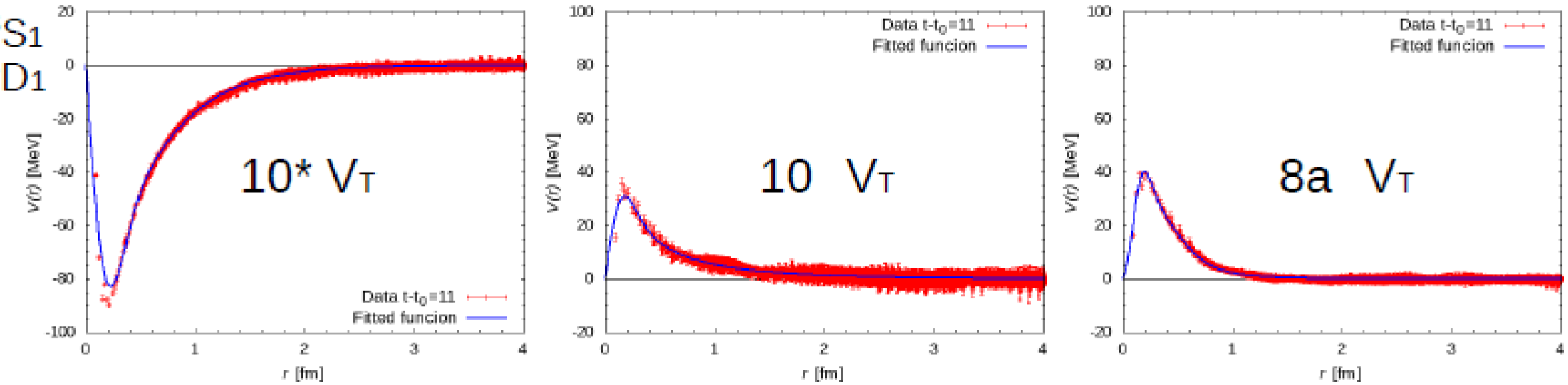
[K. Sasaki]

S=-2 interactions suitable to grasp whole NN/YN/YY interactions

Tensor Force in Irrep-base (diagonal)

$$8 \times 8 = \underbrace{27 + 8s + 1}_{^1S_0} + \underbrace{10^* + 10 + 8a}_{^3S_1,\ ^3D_1}$$

$^3S_1$
$^3D_1$



➔ **We calculate single-particle energy of hyperon in nuclear matter w/ LQCD baryon forces**

(off-diagonal component neglected)

We fit by

$$V(r) = a_1 e^{-a_2 r^2} + a_3 e^{-a_4 r^2} + a_5 \left[\left(1-e^{-a_6 r^2}\right) \frac{e^{-a_7 r}}{r} \right]^2 \quad \text{(central)}$$

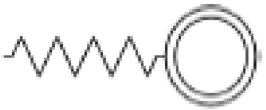
$$V(r) = a_1 \left(1-e^{-a_2 r^2}\right)^2 \left(1+\frac{3}{a_3 r}+\frac{3}{(a_3 r)^2}\right)\frac{e^{-a_3 r}}{r} + a_4 \left(1-e^{-a_5 r^2}\right)^2 \left(1+\frac{3}{a_6 r}+\frac{3}{(a_6 r)^2}\right)\frac{e^{-a_6 r}}{r} \quad \text{(tensor)}$$

Brueckner-Hartree-Fock LOBT

- Hyperon single-particle potential

M. Baldo, G.F. Burgio, H.-J. Schulze, Phys. Rev. C58, 3688 (1998)

$$U_Y(k) = \sum_{N=n,p} \sum_{SLJ} \sum_{k' \le k_F} \langle k\, k' |\, G^{SLJ}_{(YN)(YN)}\left(e_Y(k) + e_N(k')\right) |\, k\, k' \rangle$$



$${}^{2S+1}L_J \;=\; {}^1S_0\,,\ {}^3S_1\,,\ {}^3D_1\,, \;\Big|\; {}^1P_1\,,\ {}^3P_J\ \cdots$$

in our study

limitation

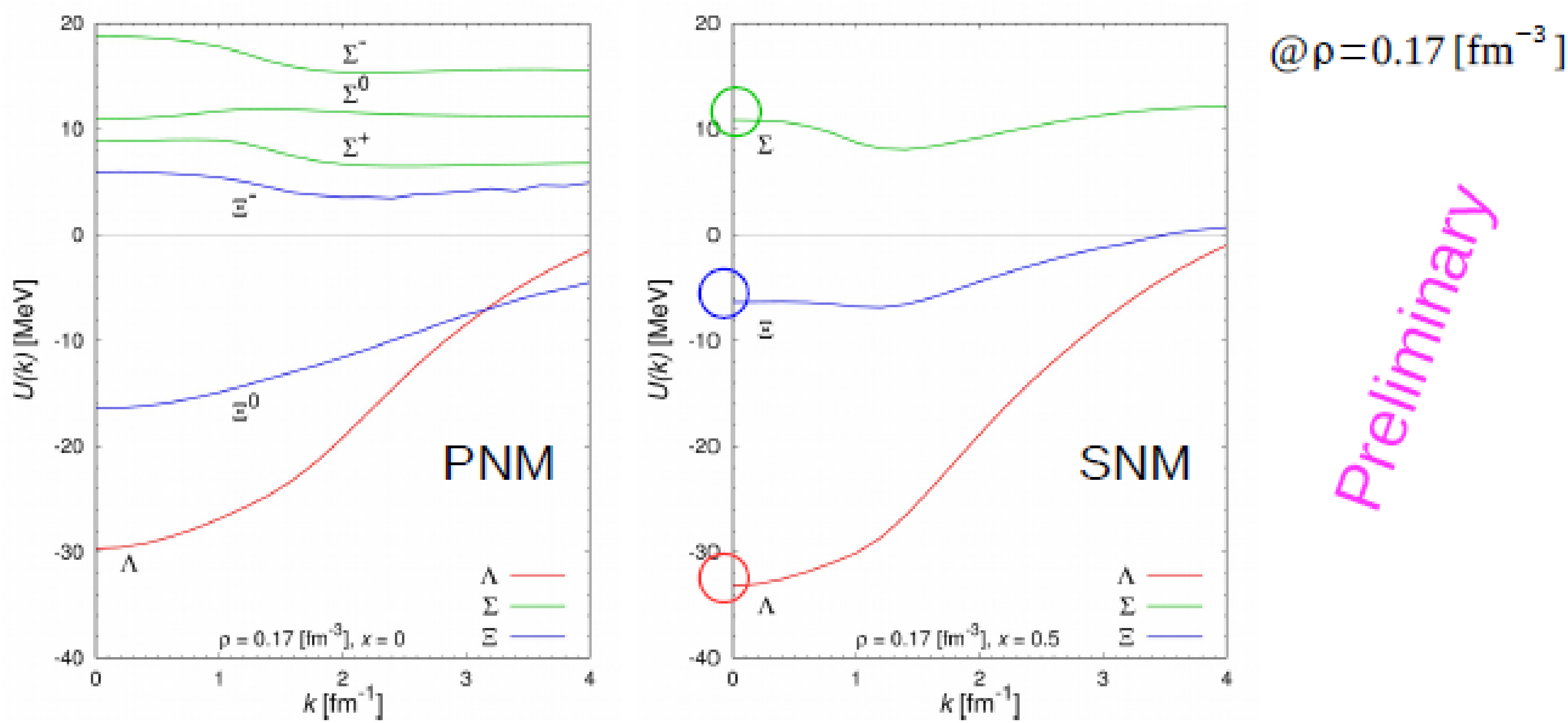
- YN G-matrix using $V^{\mathrm{LQCD}}_{S=-1}$, $M^{\mathrm{Phys}}_{N,Y}$, $U^{\mathrm{AV18,BHF}}_{n,p}$ and, U^{LQCD}_{Y}

Q=0 $\begin{pmatrix} G^{SLJ}_{(\Lambda n)(\Lambda n)} & G_{(\Lambda n)(\Sigma^0 n)} & G_{(\Lambda n)(\Sigma^- p)} \\ G_{(\Sigma^0 n)(\Lambda n)} & G_{(\Sigma^0 n)(\Sigma^0 n)} & G_{(\Sigma^0 n)(\Sigma^- p)} \\ G_{(\Sigma^- p)(\Lambda n)} & G_{(\Sigma^- p)(\Sigma^0 n)} & G_{(\Sigma^- p)(\Sigma^- p)} \end{pmatrix}$ Q=+1 $\begin{pmatrix} G^{SLJ}_{(\Lambda p)(\Lambda p)} & G_{(\Lambda p)(\Sigma^0 p)} & G_{(\Lambda p)(\Sigma^+ n)} \\ G_{(\Sigma^0 p)(\Lambda p)} & G_{(\Sigma^0 p)(\Sigma^0 p)} & G_{(\Sigma^0 p)(\Sigma^+ n)} \\ G_{(\Sigma^+ n)(\Lambda p)} & G_{(\Sigma^+ n)(\Sigma^0 p)} & G_{(\Sigma^+ n)(\Sigma^+ n)} \end{pmatrix}$

Q=−1 $G^{SLJ}_{(\Sigma^- n)(\Sigma^- n)}$ Q = +2 $G^{SLJ}_{(\Sigma^+ p)(\Sigma^+ p)}$

[T. Inoue]

Hyperon single-particle potentials



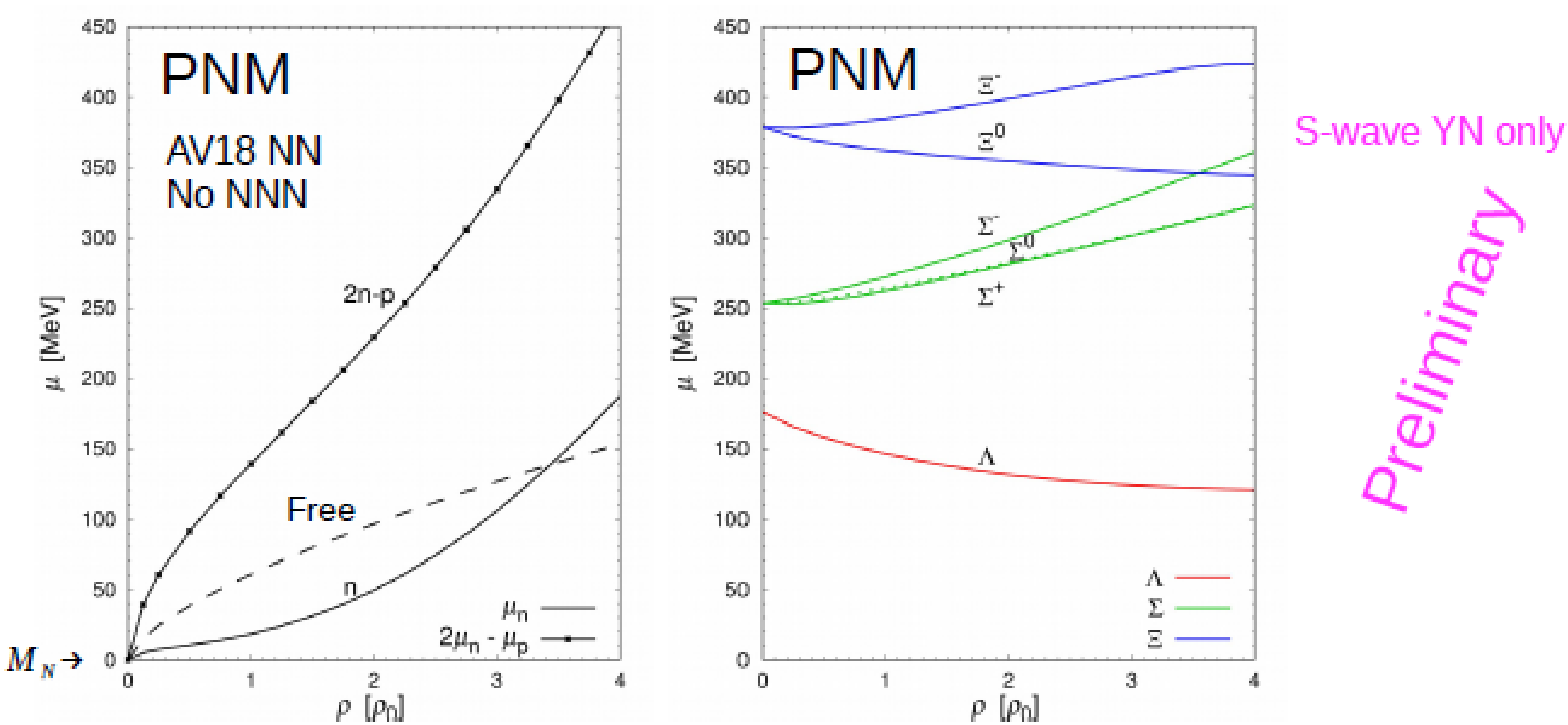
- obtained by using YN,YY forces form QCD.
- Results are compatible with experimental suggestion.

$$U_\Lambda^{\mathrm{Exp}}(0) \simeq -30, \quad U_\Xi(0)^{\mathrm{Exp}} \simeq -10, \quad U_\Sigma^{\mathrm{Exp}}(0) \geq +20 \quad \text{[MeV]}$$

attraction | attraction small | repulsion

[T. Inoue]

Chemical potentials



- Density dependence of chemical pot. of n and Y in PNM.

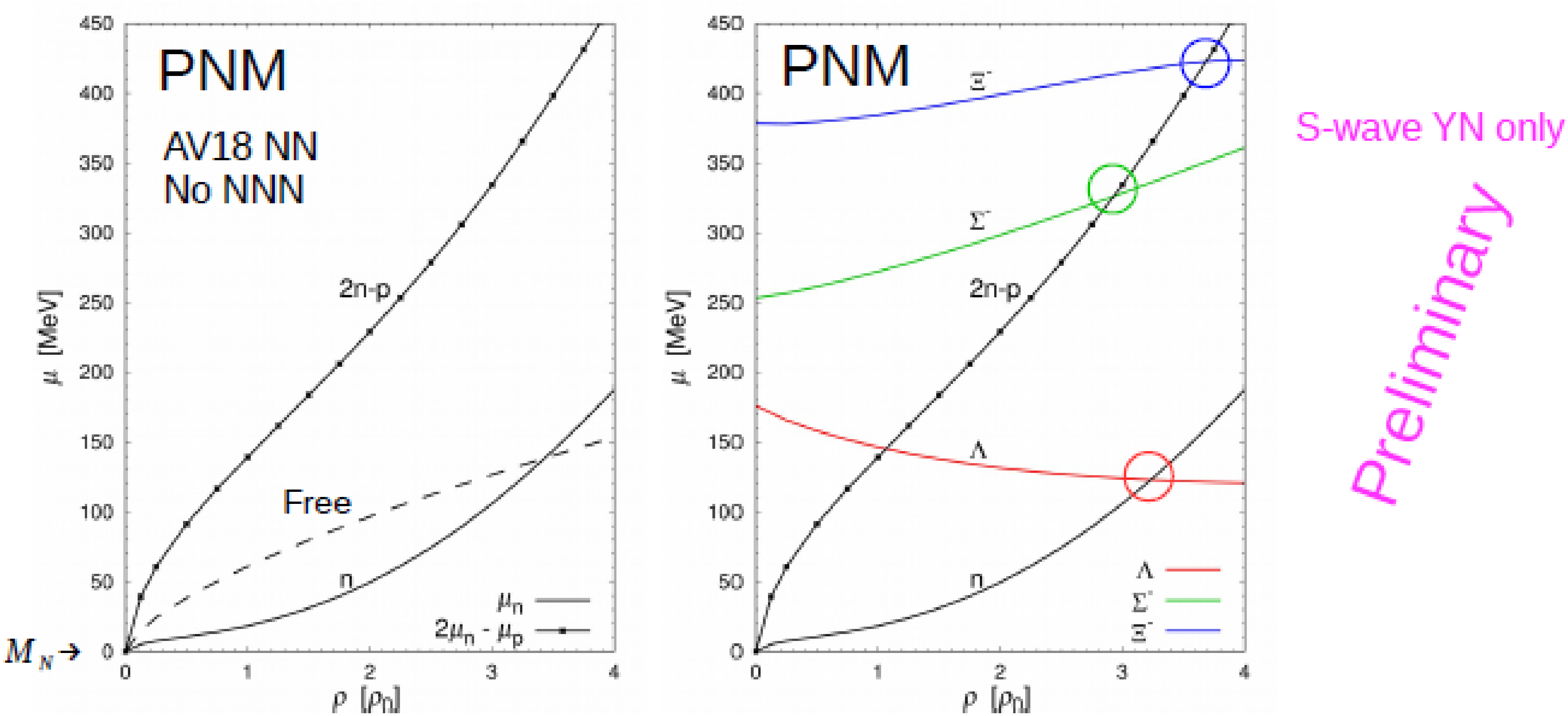
$$\mu_n(\rho) = \frac{k_F^2}{2M} + U_n(\rho; k_F), \quad \mu_Y(\rho) = M_Y - M_N + U_Y(\rho; 0)$$

- Hyperon appear as $n \to Y^0$ if $\mu_n > \mu_{Y^0}$

  $nn \to pY^-$ if $2\mu_n > \mu_p + \mu_{Y^-}$

[T. Inoue]

Hyperon onset (just for a demonstration)

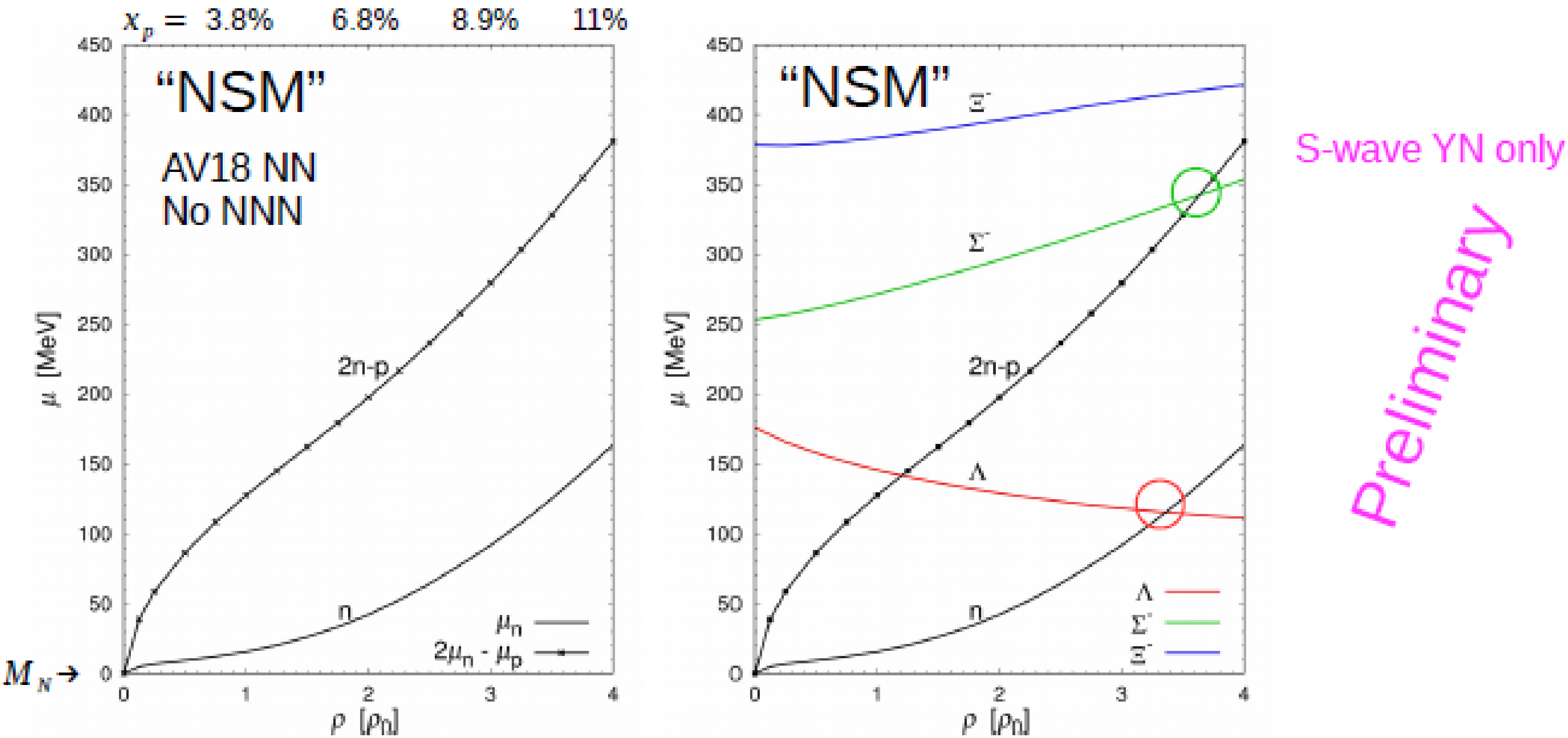


- First, Σ^- appear at 2.9 ρ_0. Next, Λ appear at 3.3 ρ_0.
  - NS matter is not PNM especially at high density.
  - We should compare with more sophisticated μ_n and μ_p.
  - P-wave YN force may be important at high density.

3

[T. Inoue]

Hyperon onset (just for a demonstration)



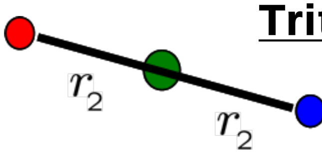
- "NSM" is matter w/ n, p, e, μ under β-eq and Q=0.

[Missing]
P-wave / LS forces
3-baryon forces

[T. Inoue]

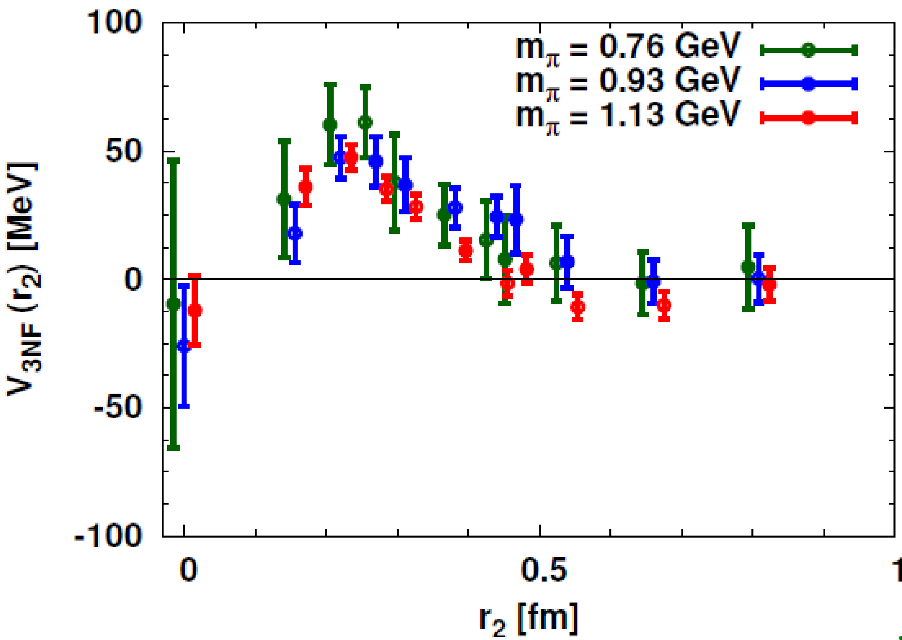
3N-forces (3NF)

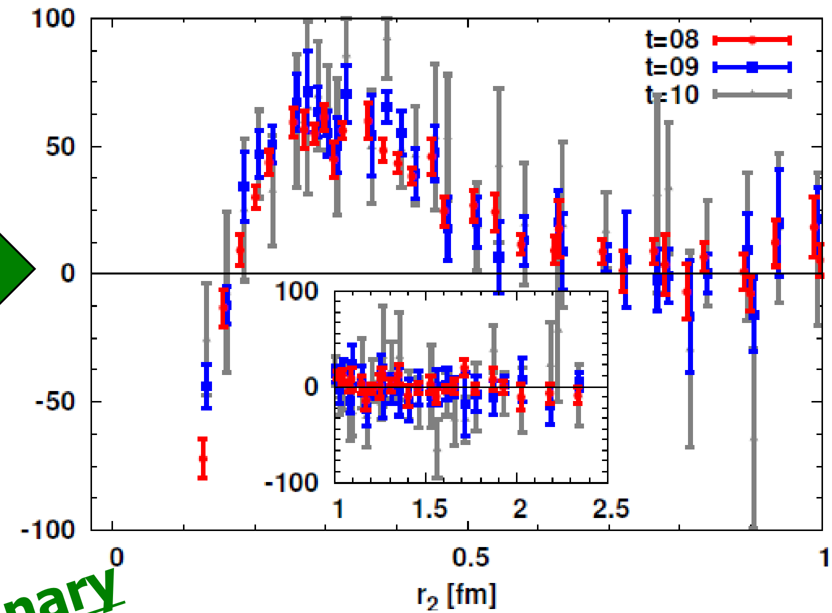
Triton channel



Nf=2, mπ=0.76-1.1 GeV

Nf=2+1, mπ=0.51 GeV





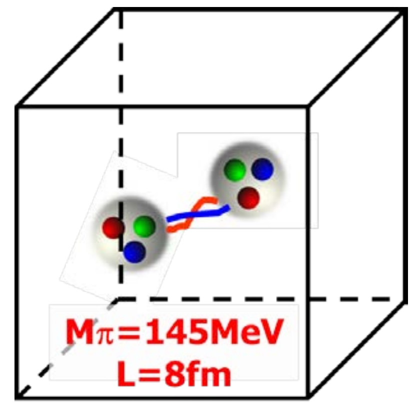
Preliminary

Magnitude of 3NF is similar for all masses
Range of 3NF tend to get longer (?) for m(pi)=0.5GeV

Kernel: ~50% efficiency achieved !

Summary

- Baryon forces: Bridge between particle/nuclear/astro-physics
- HAL QCD method crucial for a reliable calculation
  - Direct method suffers from excited state contaminations
- The 1st LQCD for Baryon Interactions at ~ phys. point
  - m(pi) ~= 145 MeV, L ~= 8fm, 1/a ~= 2.3GeV
  - Central/Tensor forces for NN/YN/YY in P=(+) channel



Nuclear Physics from LQCD
New Era is dawning !

- Prospects
  - Exascale computing Era ~ 2020
  - LS-forces, P=(-) channel, 3-baryon forces, etc., & EoS