CCS-RIKEN workshop, Dec.12, 2016

An application of the CSM+SVM with the complex-range Gaussian basis function to the four-body resonances

1. Method (CSM, SVM, CG)
2. Application of CSM+SVM+CG to the excited resonances of ^{4}He

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S. Aoyama, PTEP, in press.

Selected for a Viewpoint in Physics
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Candidate Resonant Tetraneutron State Populated by the ^{4}He(^{8}He,^{8}Be) Reaction

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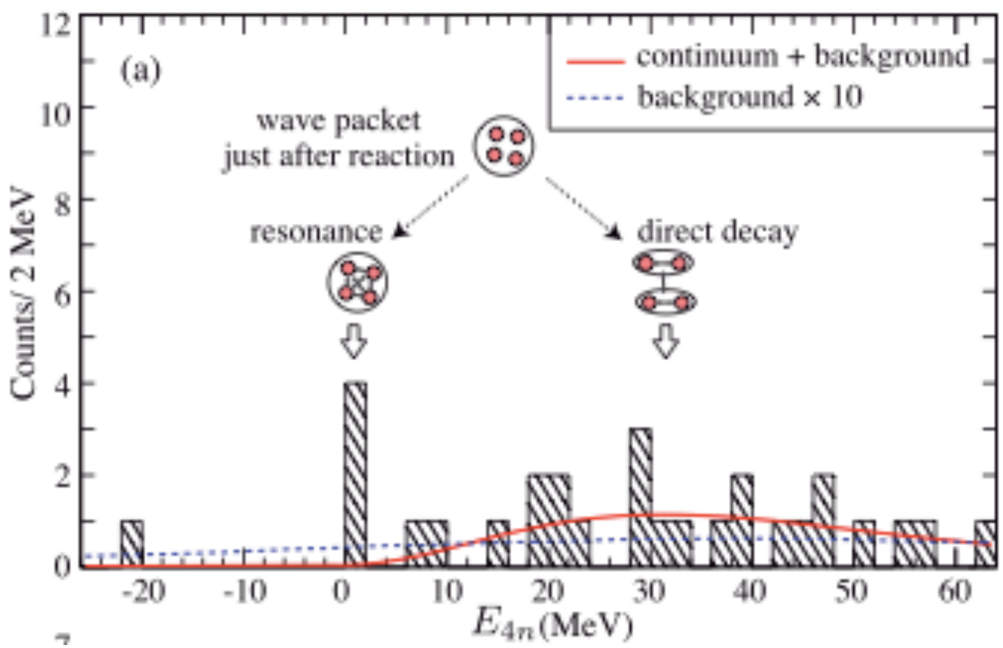


E=0.83±0.65(stat)±1.25(syst) MeV

Complex Scaling Method

Review
S. Aoyama, T. Myo, K. Kato, K.Ikeda, Prog. Theor. Phys. 116(2006).

The complex scaling for the coordinate is described as

$$U(\theta)\boldsymbol{r} = \boldsymbol{r}e^{i\theta}, \qquad U(\theta)\boldsymbol{p} = \boldsymbol{p}e^{-i\theta}.$$

The Schrödinger equation is rewritten as

$$H(\theta)U(\theta)\Psi = E(\theta)U(\theta)\Psi,$$
$$H(\theta)\Psi(\theta) = E(\theta)\Psi(\theta),$$

where

$$H(\theta) = U(\theta)HU^{-1}(\theta).$$

The complex eigenvalue is obtained by the diagonalization of the Hamiltonian matrix.

$$\sum_j C_j \langle\psi_i|H(\theta)|\psi_j\rangle = E(\theta)\sum_j C_j\langle\psi_j|\psi_j\rangle,$$

$$|\Psi(\theta)) = \sum_j C_j(E,\theta)|\psi_j\rangle.$$

J. Aguilar and J. M. Combes, Commun. Math. Phys. 22 (1971).
E. Balslev and J. M. Combes, Commun. Math. Phys. 22(1971).

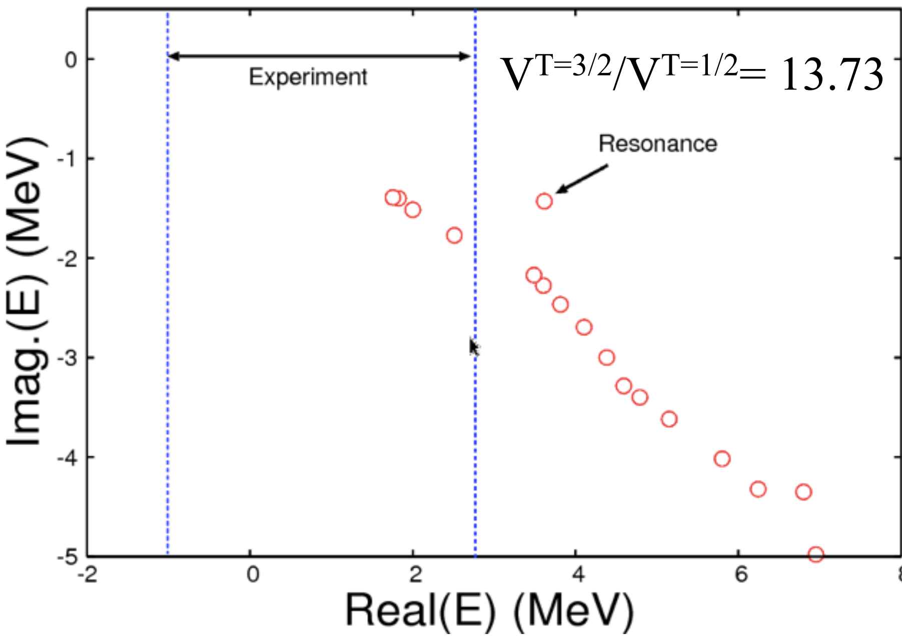
According to the ABC-theorem,

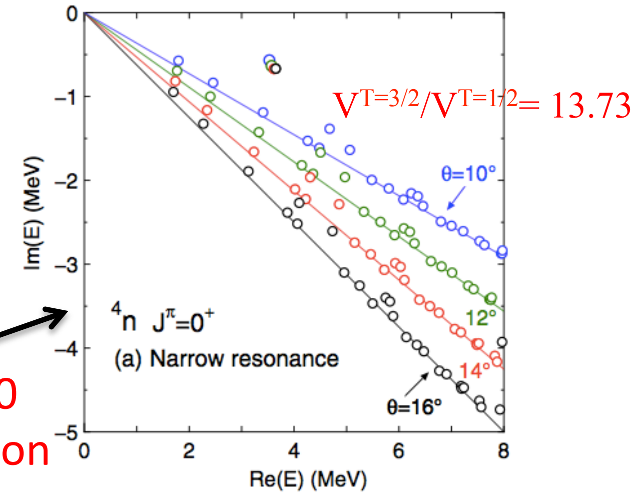
i)The energies of bound states and resonances ($\theta > tan^{-1}(\Gamma/2E_r)$) are not changed by scaling.

ii)The continuum spectra are obtained along 2θ lines, which start at the threshold energies of the sub-systems.

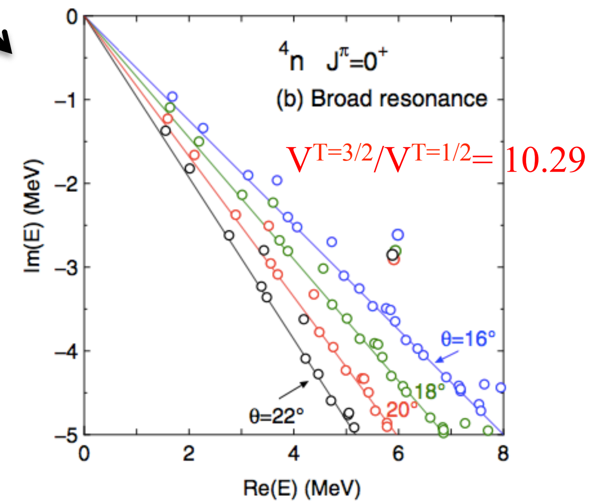
Complex Scaling Method for tetra neutron ^{4}n

E.Hiyama, R.Lazauskas, J.Carbonell, M.Kamimura, PRC93(2016)044004





~10,000 dimension



AV8'+3NF

Typical number of dimension of Hamiltonian matrix for 4-nucleon systems is estimated as one million!
=> It's beyond the computational ability of standard super computers.

Stochastic Variational Method

K. Varga, Y. Suzuki, Y. Ohbayashi, Phys. Rev. C50, 189 (1994)

In order to treat the large model space, we use SVM (Stochastic Variational Method).

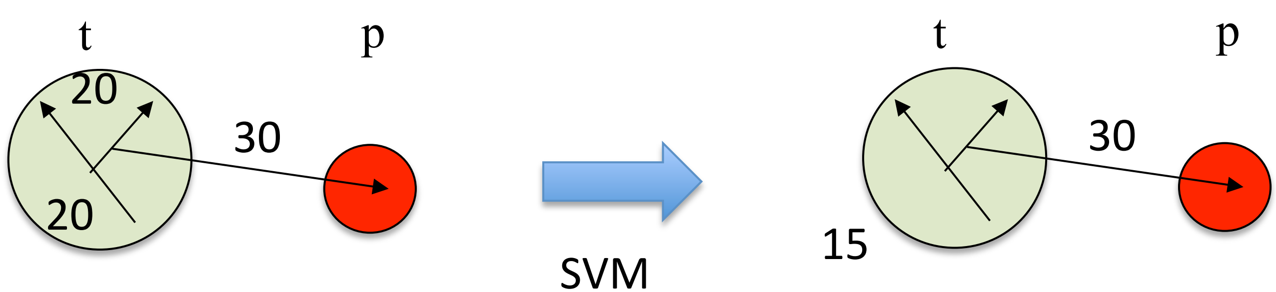
MN

	Present					Ref. [24]		Exp.
cluster	E MeV	T MeV	V_N MeV	V_C MeV	R_{rms} fm	E MeV	R_{rms} fm	E MeV
$t(\frac{1}{2}^+)$	−8.38	27.24	-35.62	0	1.70	−8.38	1.71	−8.48
$h(\frac{1}{2}^+)$	−7.70	26.73	-35.11	0.67	1.72	−7.71	1.74	−7.72
$d(1^+)$	−2.20	10.49	-12.69	0	1.94	−2.20	1.95	−2.22

15 base → t, h
5 base → d

[24]Y. Suzuki *et al.* FBS42(2008)33

Example for a t+p channel



N_b=20X20X30=12000bases

N_b=15X20=450bases

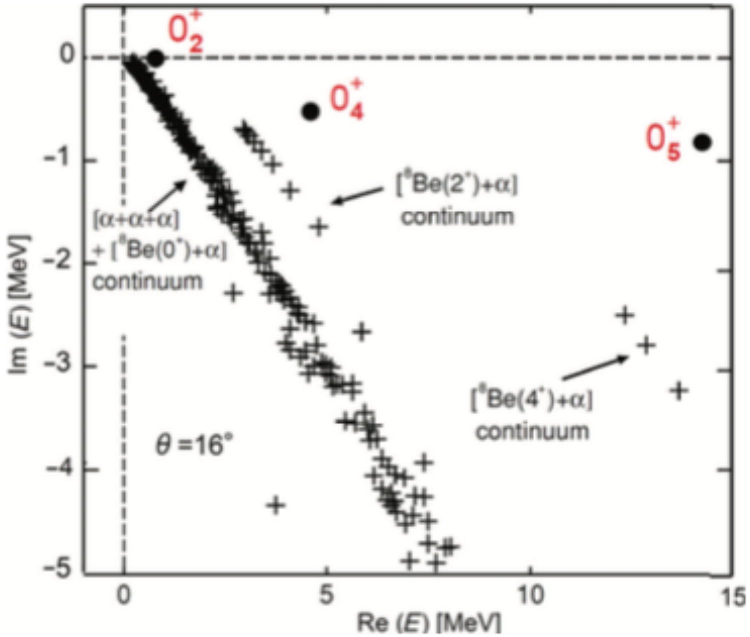
Computational time is proportional to N_bXN_b (or N_bXN_bXN_b for larger dimension)

Complex-range Gaussian basis function

Oscillating part is introduced to the Gaussian-type basis function.

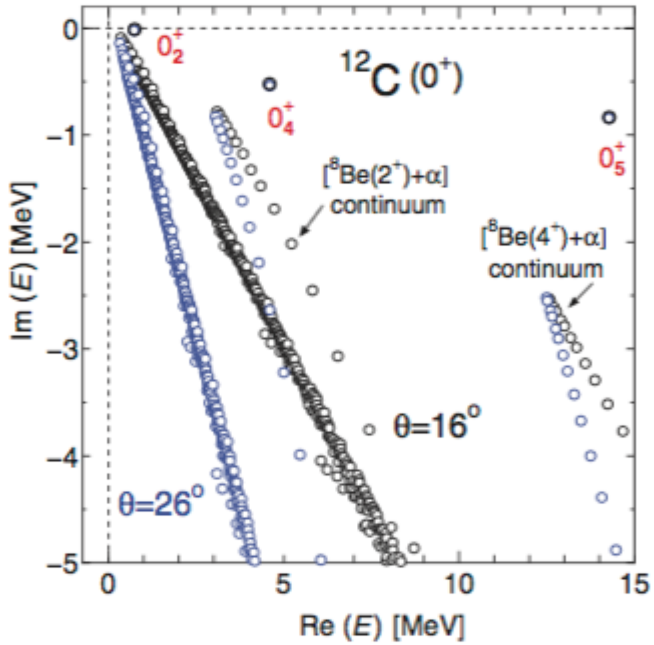
$$u_l(a_i, r) = r^l \exp(-(1 + i\omega)a_i r^2),$$
$$u_l^*(a_i, r) = r^l \exp(-(1 - i\omega)a_i r^2).$$

Real-range Gaussian



Kurokawa and Kato, PRC71(2005)

Complex-range Gaussian



Ohtsubo, Fukushima, Kamimura, Hiyama, PTEP073D02(2013)

The binding energy of triton by using CSM+SVM

The bound state solution should not depend on the scaling angle θ.

Standard CSM with geometric progression

Precise solution

	RG	CG(OG)	CG(OG)	GP
N_b	15	30	30	800
ω	0.0	0.6	$1.57(=\frac{\pi}{2})$	0.0
$E(\theta=0.0)$(MeV)	-8.38	-8.30	-4.77	-8.385
$E(\theta=0.2)$(MeV)	$-8.51-i0.04$	$-8.34-i0.03$	$-5.23-i1.10$	$-8.385-i0.872\times10^{-4}$
$E(\theta=0.4)$(MeV)	$-8.89-i0.48$	$-8.47+i0.02$	$-6.35-i1.85$	$-8.388-i0.006$

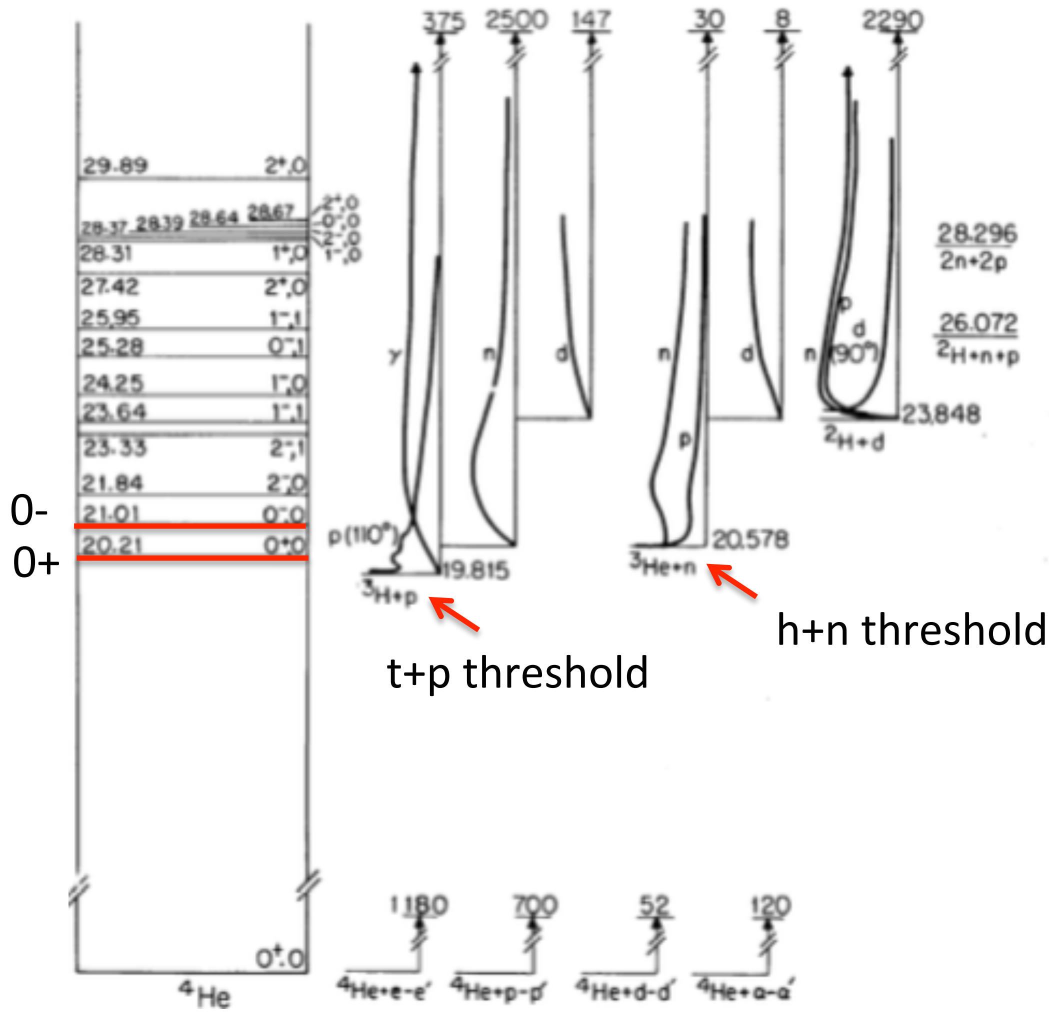
θ-dependence is small for complex-range Gaussian

θ-dependence is large for real-range Gaussian

800 dimension -> 30 dimension

Standard CSM　　　　CSM+SVM+CG

Energy Levels of ^{4}He



D.R. Tilley, H.R. Weller, G.M. Hale, Nucl Phys A 541(1992).

Hamiltonian and basis function

The Hamiltonian is

$$\widehat{H} = \sum_{i=1}^{4} \widehat{T}_i - \widehat{T}_{\rm cm} + \sum_{i<j}^{4} \widehat{V}_{ij} + \sum_{i<j}^{4} \widehat{V}_{ij}^{C}.$$

where $\widehat{V}_{ij}$ is the two-nucleon potential (Minnesota), $\widehat{V}_{ij}^{C}$ is the Coulomb potential.

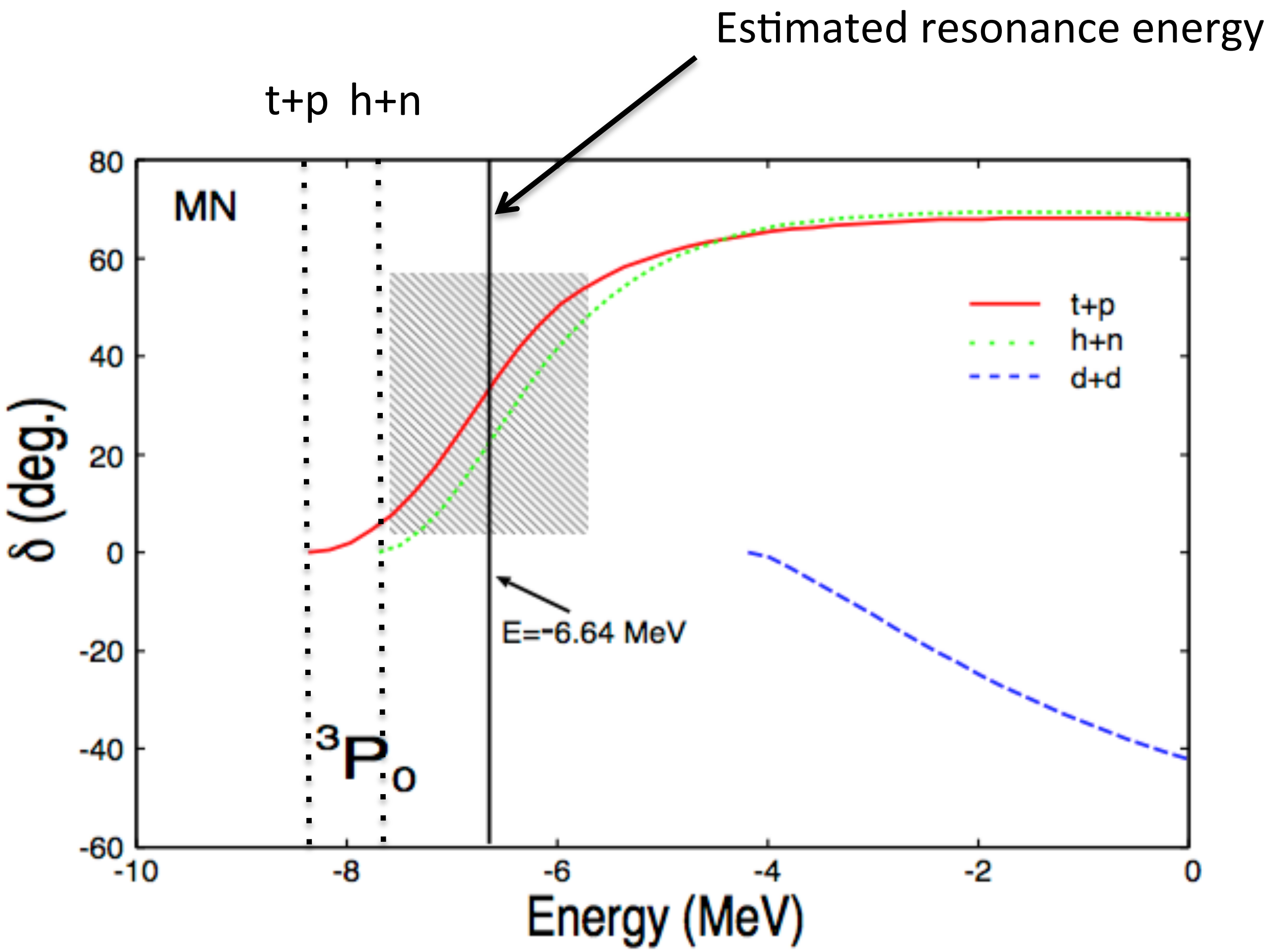
The basis function of ^{4}He is written as

$$\Psi_{JM} = \sum_{\alpha} c_\alpha \Phi_{JM}^{\alpha}(K) + \sum_{\beta} d_\beta \Phi_{JM}^{\beta}(H),$$

$$\begin{aligned}\Phi_{JM}(K) = \mathcal{A}\Big[\Big[\Big[\psi_{L_1}^{(1)}(\boldsymbol{x}_1)\psi_{L_2}^{(2)}(\boldsymbol{x}_2)\Big]_{L_{12}} \psi_{L_3}^{(3)}(\boldsymbol{x}_3)\Big]_L \Big[\Big[\big[\chi^{(1)}\chi^{(2)}\big]_{S_{12}} \chi^{(3)}\Big]_{S_{123}} \chi^{(4)}\Big]_S\Big]_{JM} \\ \times \Big[\big[\tau^{(1)}\tau^{(2)}\big]_{T_{12}} \tau^{(3)}\Big]_{T_{123}M_{123}} \tau_{\frac{1}{2}M_4}^{(4)},\end{aligned}$$

$$\begin{aligned}\Phi_{JM}(H) = \mathcal{A}\Big[\Big[\Big[\psi_{L_1}^{(1)}(\boldsymbol{y}_1)\psi_{L_2}^{(2)}(\boldsymbol{y}_2)\Big]_{L_{12}} \psi_{L_3}^{(3)}(\boldsymbol{y}_3)\Big]_L \Big[\big[\chi^{(1)}\chi^{(2)}\big]_{S_{12}} \big[\chi^{(3)}\chi^{(4)}\big]_{S_{34}}\Big]_S\Big]_{JM} \\ \times \big[\tau^{(1)}\tau^{(2)}\big]_{T_{12}M_{12}} \big[\tau^{(3)}\tau^{(4)}\big]_{T_{34}M_{34}}.\end{aligned}$$

$^3P_0$ phase shift with MRM



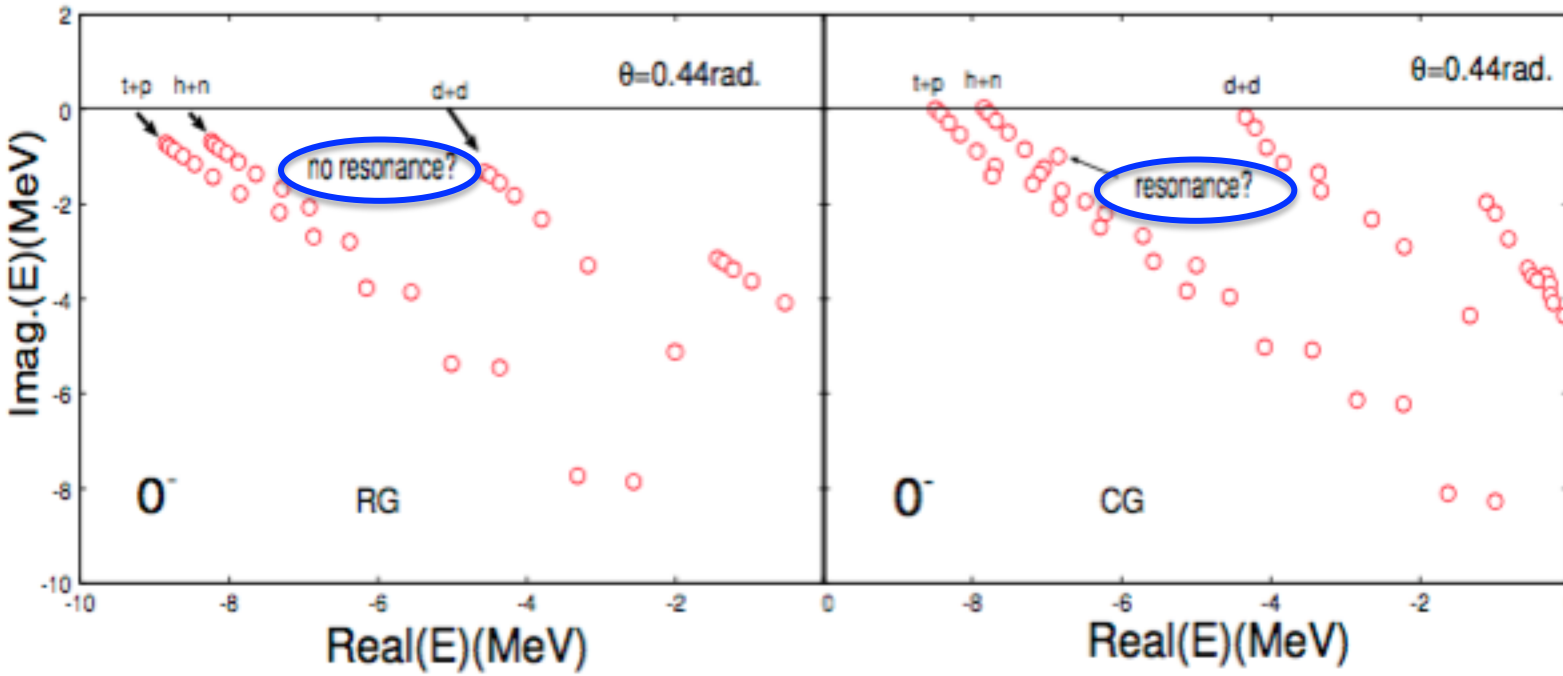
S. Aoyama, K. Arai, Y. Suzuki, P.Descouvemont, and D. Baye, Few-body Syst. 52(2012).

Complex Eigenvalues of 0^- in ^{4}He

CSM+SVM

Real Range (RG) Gaussian

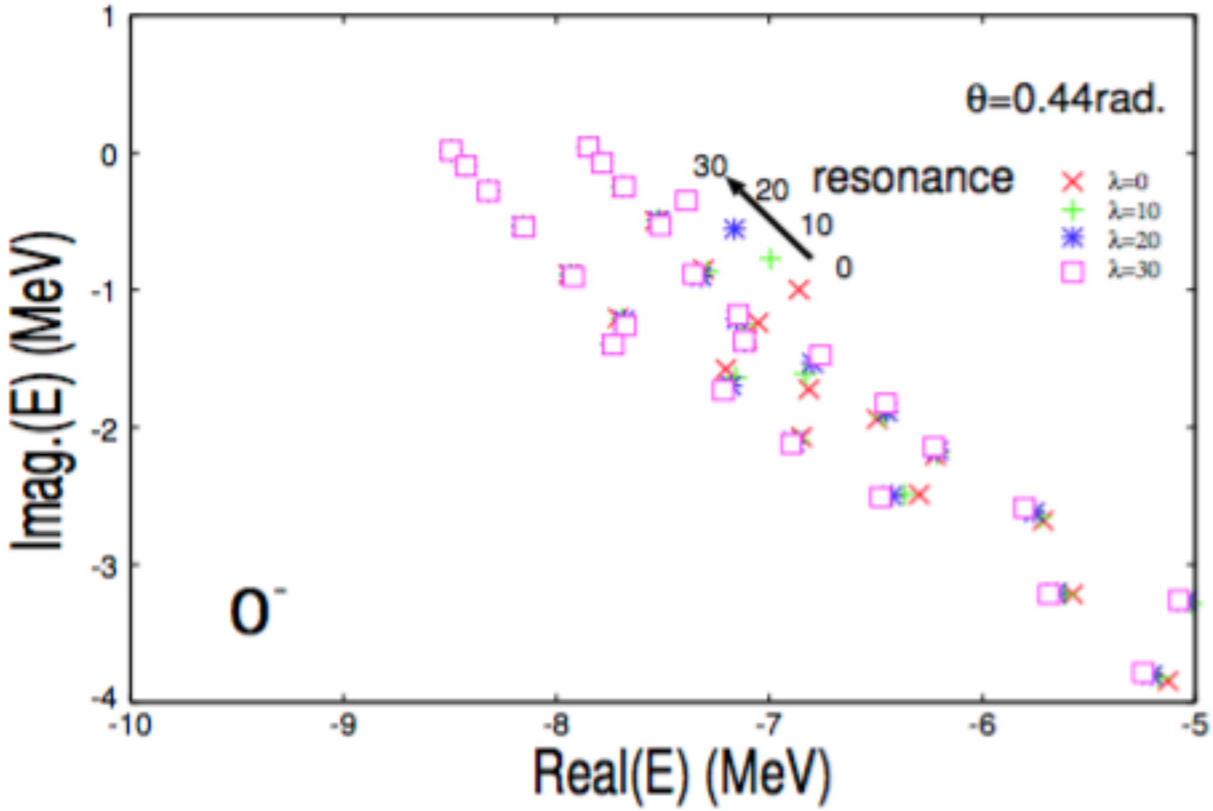
Complex Range (GG) Gaussian



N_b=15 for t and ^{3}He
N_b=5 for deutron

b_3^{max}=50fm and N=30 for 3N+N
b_3^{max}=40fm for N=20 for 2N+2N

λ-trajectory of Complex Eigenvalues of 0⁻ in ^{4}He



$$\widehat{H}(\lambda) = \widehat{H} - \lambda V^{4N},$$

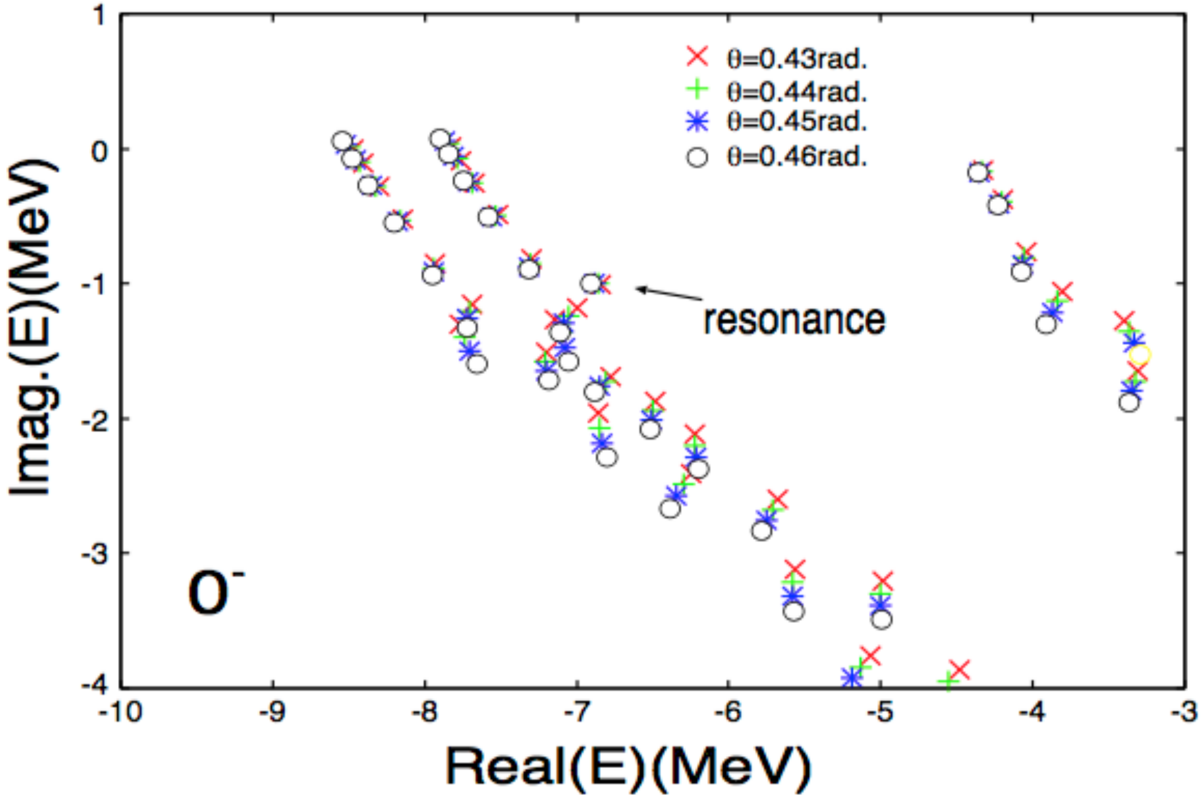
$$V^{4N} = \exp\left(-\frac{(r_1-r_2)^2+(r_2-r_3)^2+(r_3-r_4)^2+(r_4-r_1)^2}{b^2}\right), \quad b = 4\text{fm},$$

ACCC+CSM

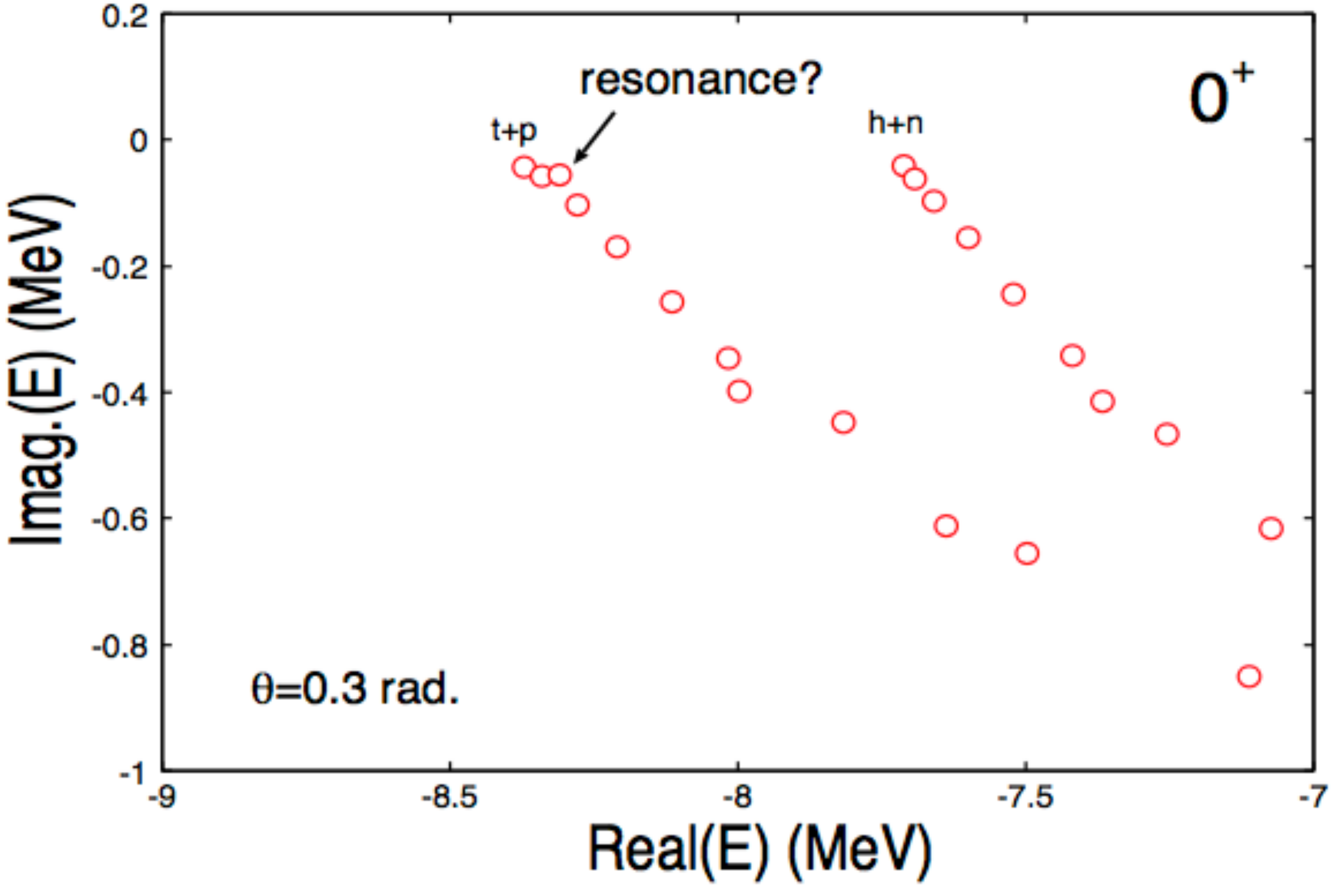
S. Aoyama, Phys. Rev. C 68 (2003).

S. Aoyama, Phys. Rev. Lett. 89 052501 (2002).

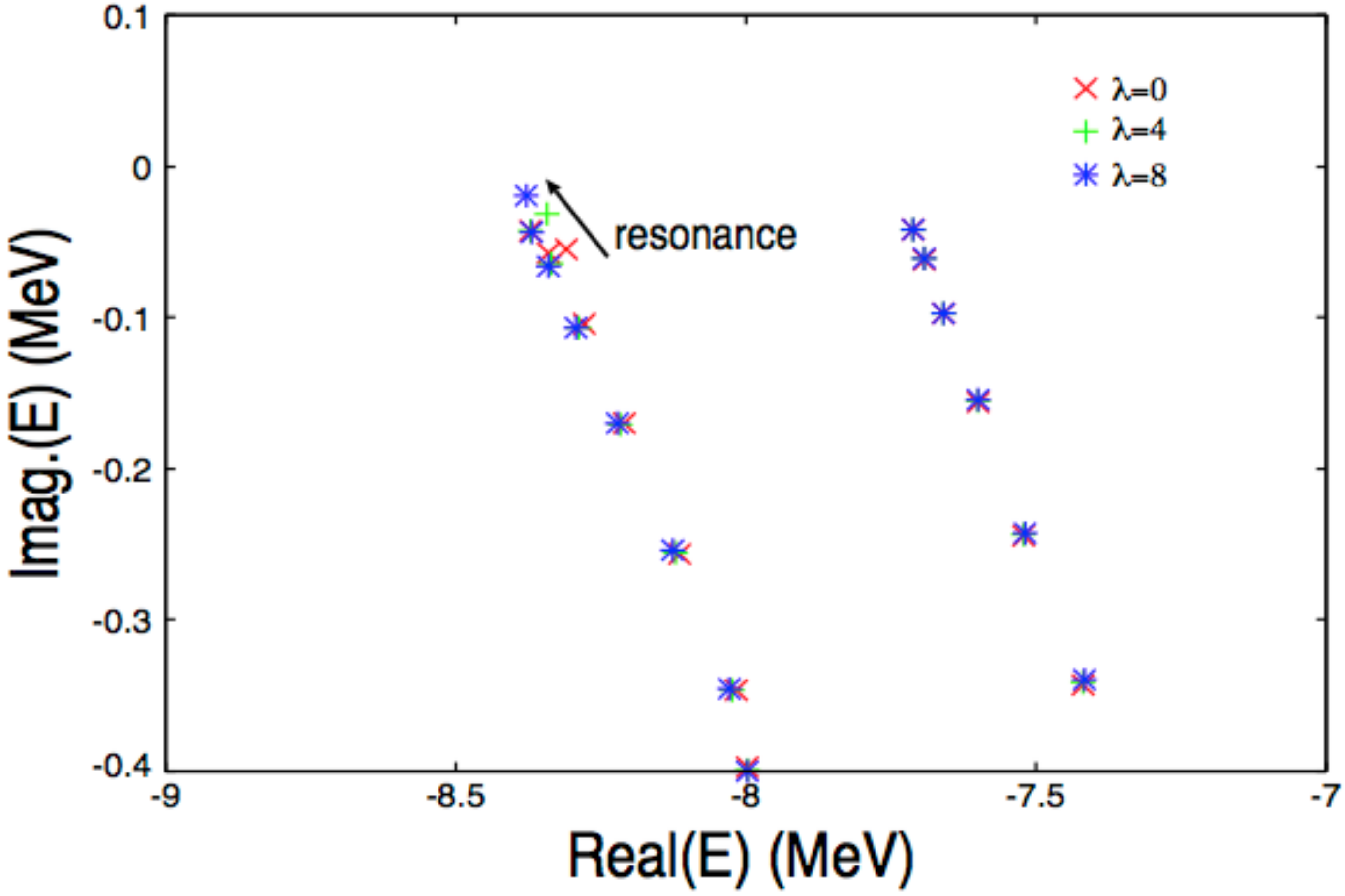
θ-trajectory of Complex Eigenvalues of 0^- in ^{4}He



Complex Eigenvalues of 0^+ in ^{4}He



λ-trajectory of Complex Eigenvalues of 0^+ in ^{4}He



Summary

CSM+SVM is proposed and applied to the excited resonance of ^{4}He (0±).

CSM+SVM with the RG basis function can not reproduce the excited resonance of ^{4}He.

CSM+SVM with the CG basis function can reproduce well the excited resonance of ^{4}He.

Typical computational time with CSM+SVM is 100 times faster than the conventional CSM(GEM) at least.

Future problem

Systematic investigation of four-nucleon resonances with realistic interactions.