# Current status for two baryon systems in lattice QCD I. Difficulties in the direct method

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on microscopic theories of nuclear structure and dynamics
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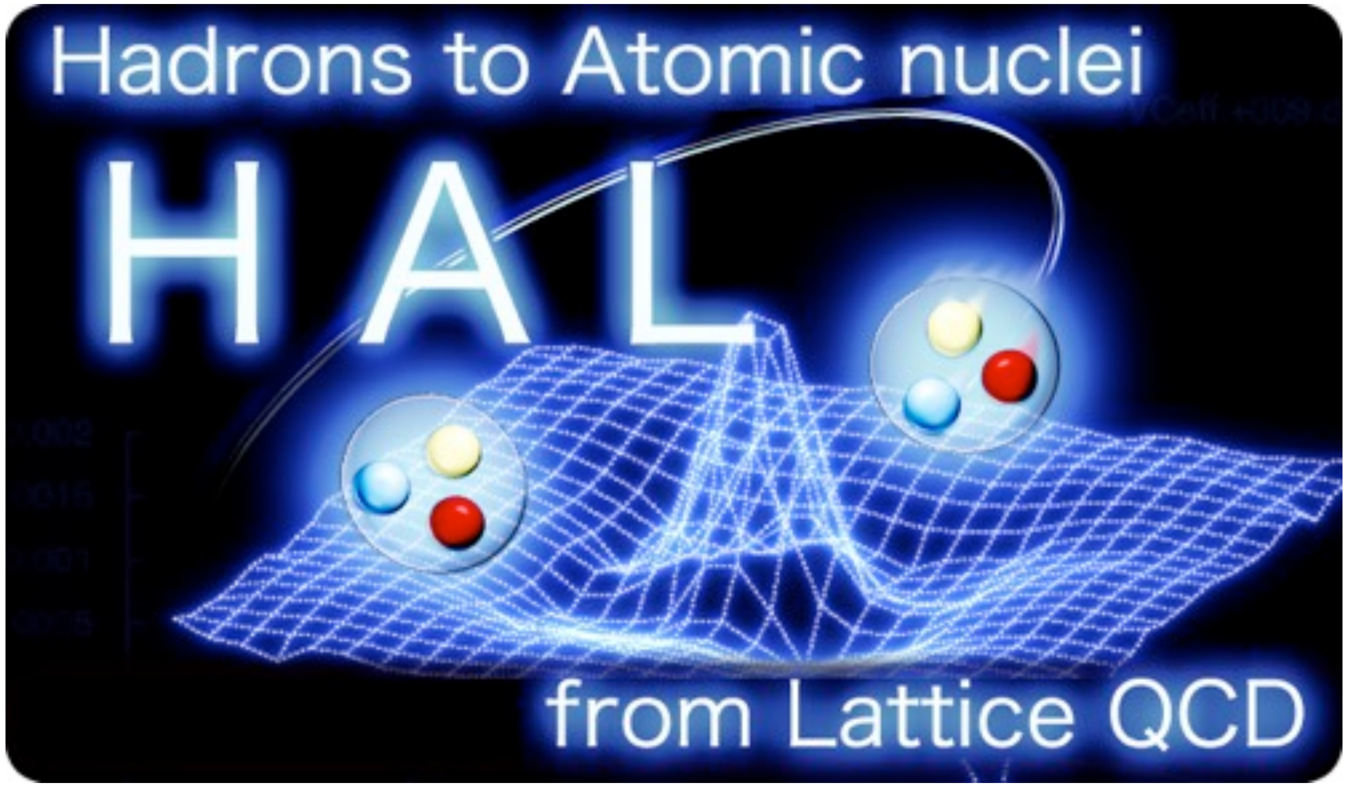
RIKEN Nishina Center/CCS, University of Tsukuba

# For
# HAL QCD Collaboration

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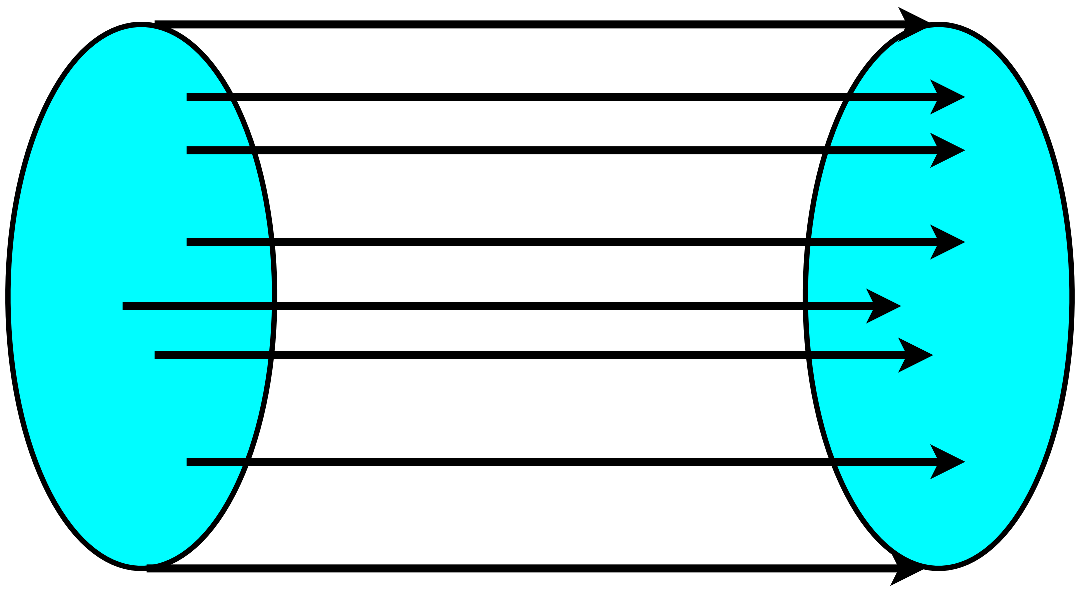
Riken: Takumi Doi, Tetsuo Hatsuda, Takumi Iritani
RCNP, Osaka: Yoichi Ikeda, Noriyoshi Ishii, Keiko Murano
Tsukuba: Hidekatsu Nemura
Nihon: Takashi Inoue
Tours, France: Sinya Gongyo
Birjand, Iran: Faisal Etminan

* PhD students

# Introduction

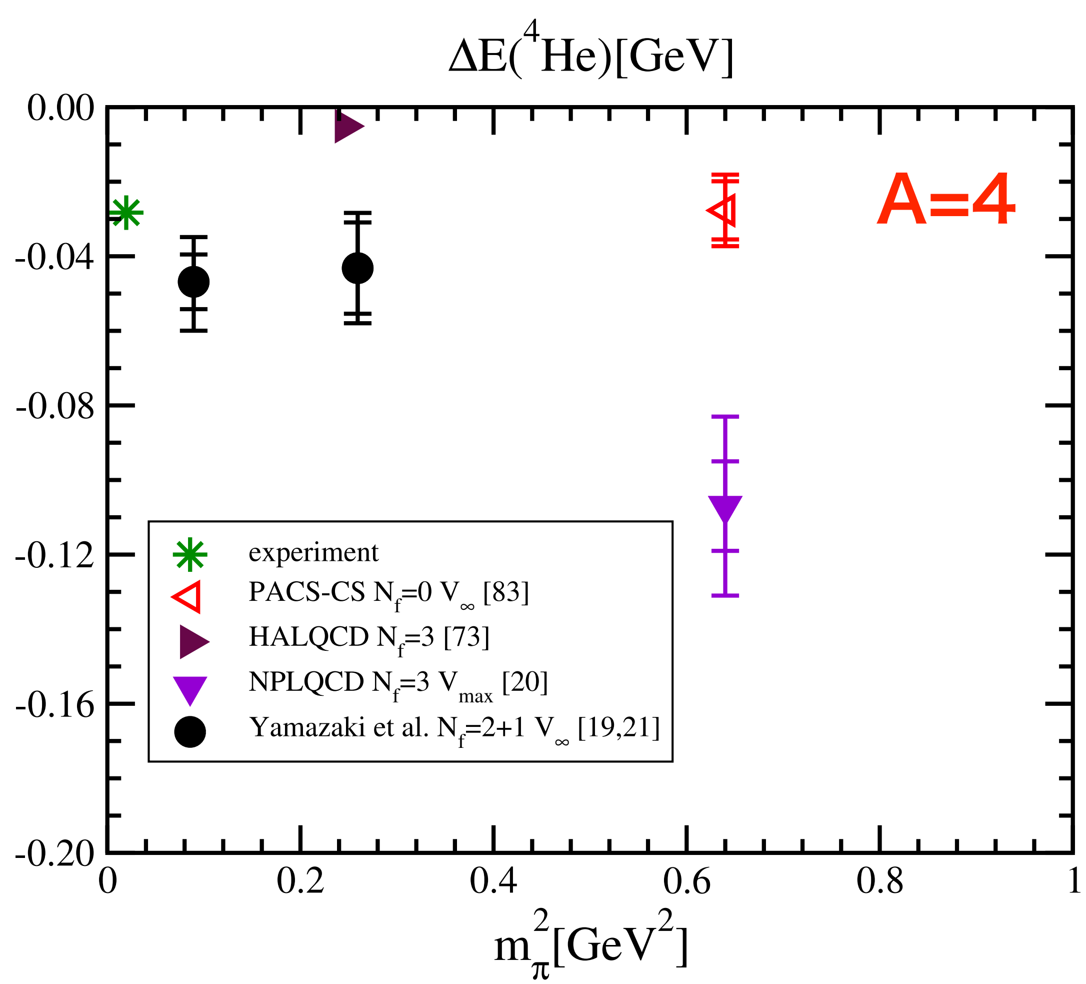
# Strategies for nuclear physics from (lattice) QCD

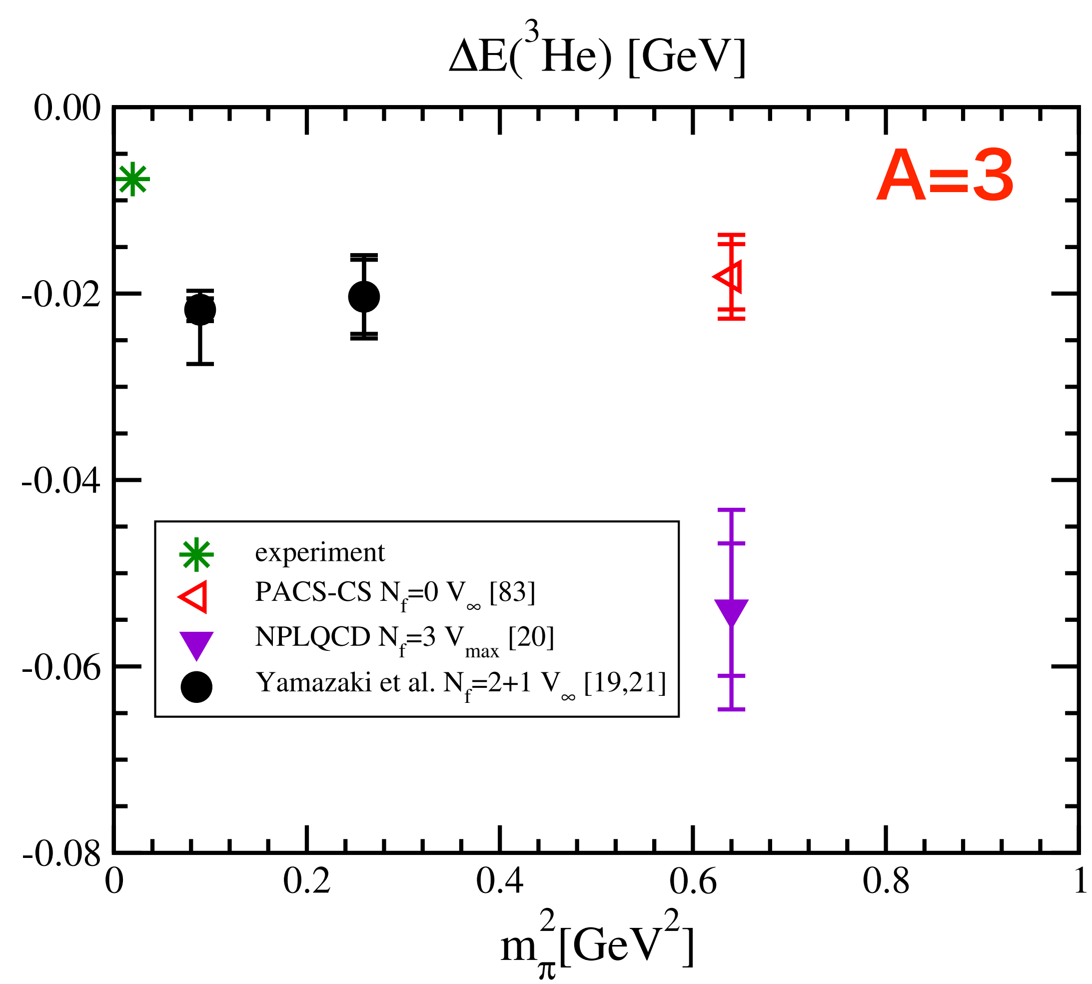
**Direct method** calculate nuclei directly from lattice QCD



$$\simeq e^{-m_A t} + \cdots$$

3A quark lines A: atomic number



Potential method (HALQCD method) Talk by T. Doi on 13th

$$G_{NN}(r,t) = \langle N(r,t)N(0,t)\bar{N}(0)\bar{N}(0)\rangle$$

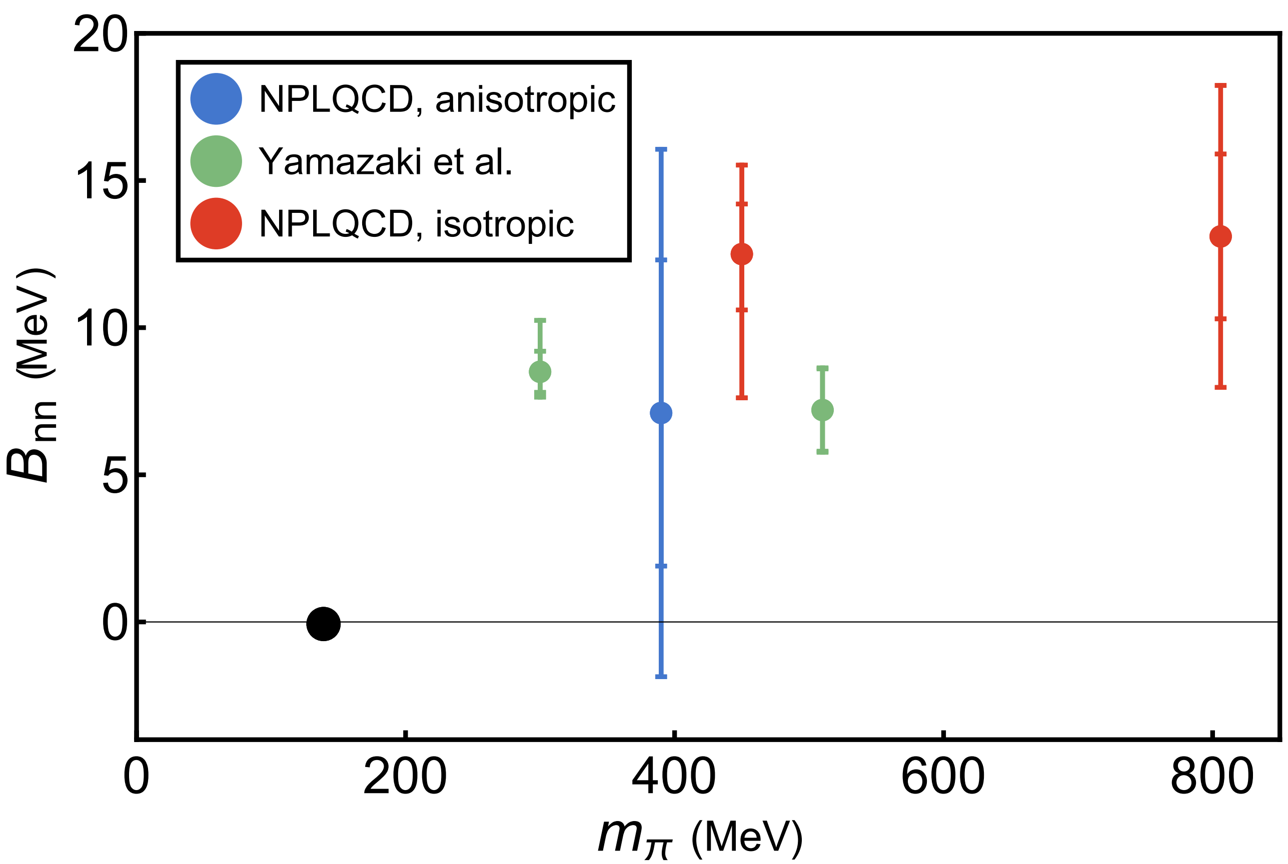
"potential" $+$ Schrödinger equation

phase shift

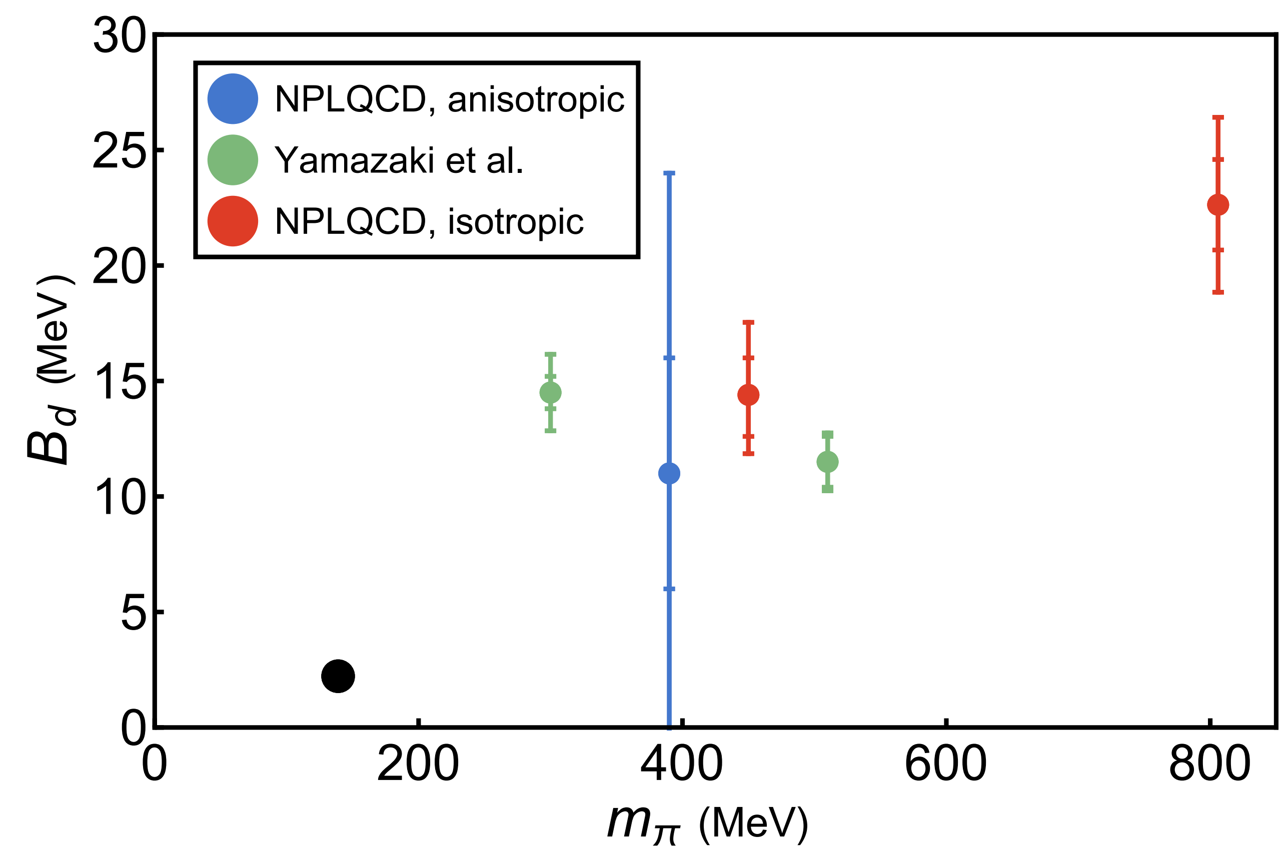
binding energy

This talk: reliability checks for the direct method using 2-baryon systems

B.E of di-neutron




B.E of deuteron




Next talk by Iritani: reliability checks for the potential method

Introduction

I. Direct method

II. Mirage problem (Operator dependence)

III. Sanity check

IV. Conclusion

# I. Direct method

# Extraction of energy shift

**Energy shift** $\Delta E \equiv E_{NN} - 2m_N$

$O(10\text{ MeV})$ $O(2\text{ GeV})$ $O(2\text{ GeV})$

large cancellation

0.5 % accuracy required

**Ratio** $R(t) = \dfrac{G_{NN}(t)}{G_N(t)^2} \sim e^{-\Delta E t}$

expect cancellation of both statistical and systematic errors

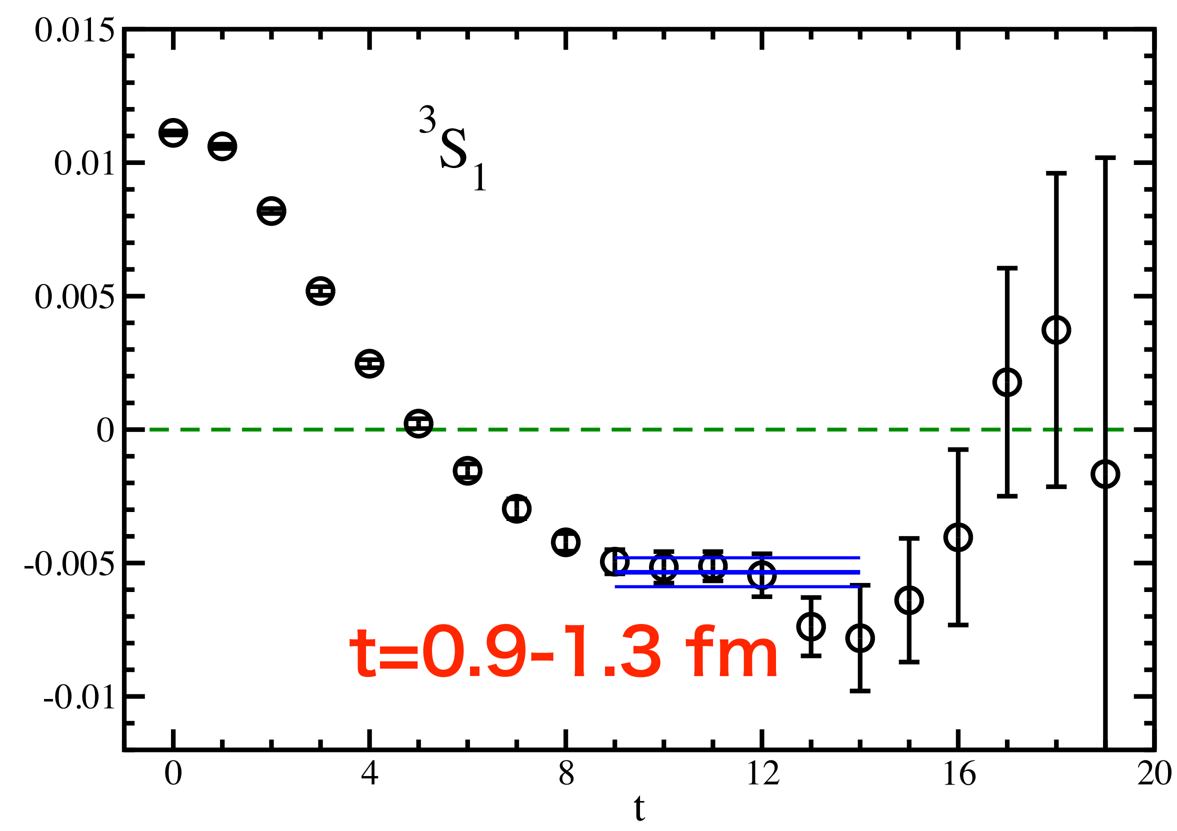
**Effective energy shift**

$$\Delta E(t) = \frac{1}{a}\log\frac{R(t)}{R(t+a)} \longrightarrow \Delta E, \qquad t \to \infty$$

**Plateau method**

We identify $\Delta E(t)$ as $\Delta E$, if it becomes constant.

YIKU 2012: PRD86(2012)074514



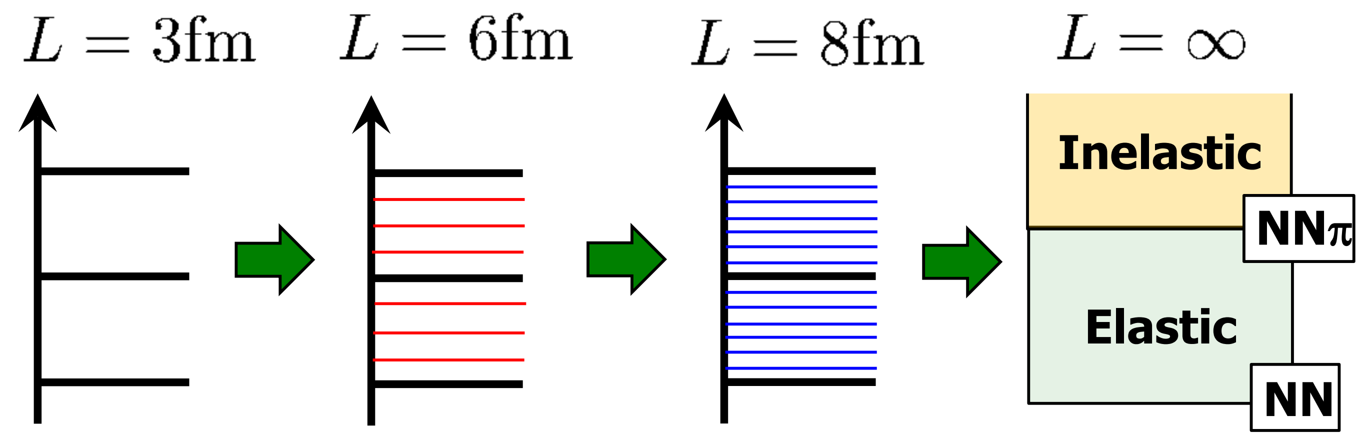
# Is the plateau method reliable ?

**Excitation energy** $E_1 - E_0$

binding energy: very small

finite volume effect for scattering state

$$\simeq \frac{1}{m_N}\frac{(2\pi)^2}{L^2}$$




$E_1 - E_0 \simeq 50$ MeV at $L = 4$ fm

$t \gg 1/(E_1 - E_0) \simeq 4$ fm is needed to suppress excited states.

Observing the plateau guarantees the ground state saturation even when $t \gg 1/(E_1 - E_0)$ is NOT satisfied.

claimed by Y(I)KU('11,'12,'15), NPL('12,'13,'15), CalLat('15)

# Examination of the statement

**Mock-up data** @ $m_\pi = 0.5$ GeV, $L = 4$ fm (setup of YIKU2012)

$$R(t) = e^{-\Delta E t} \left(1 + b\, e^{-\delta E_{\rm el} t} + c\, e^{-\delta E_{\rm inel} t}\right)$$

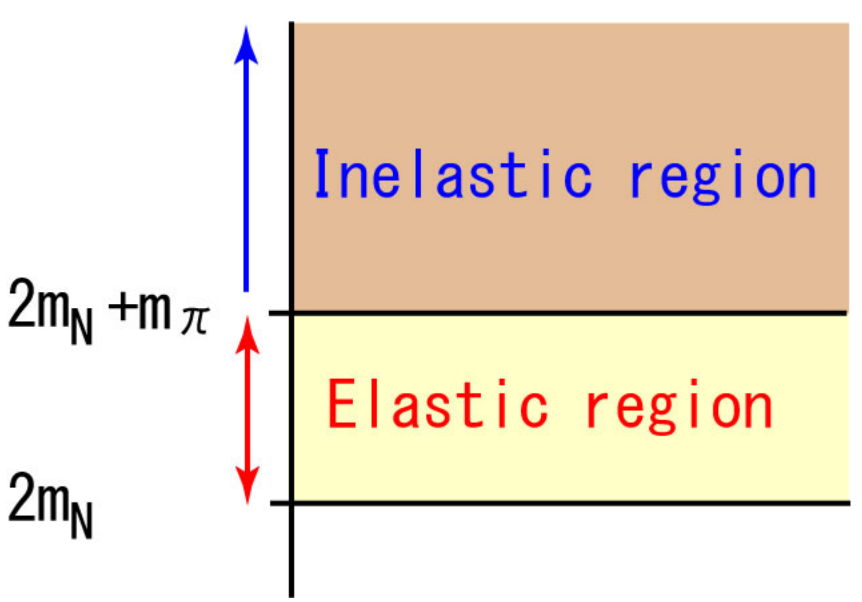
$\delta E_{\rm el} \propto \frac{1}{L^2}$ the lowest excitation energy of elastic scattering state

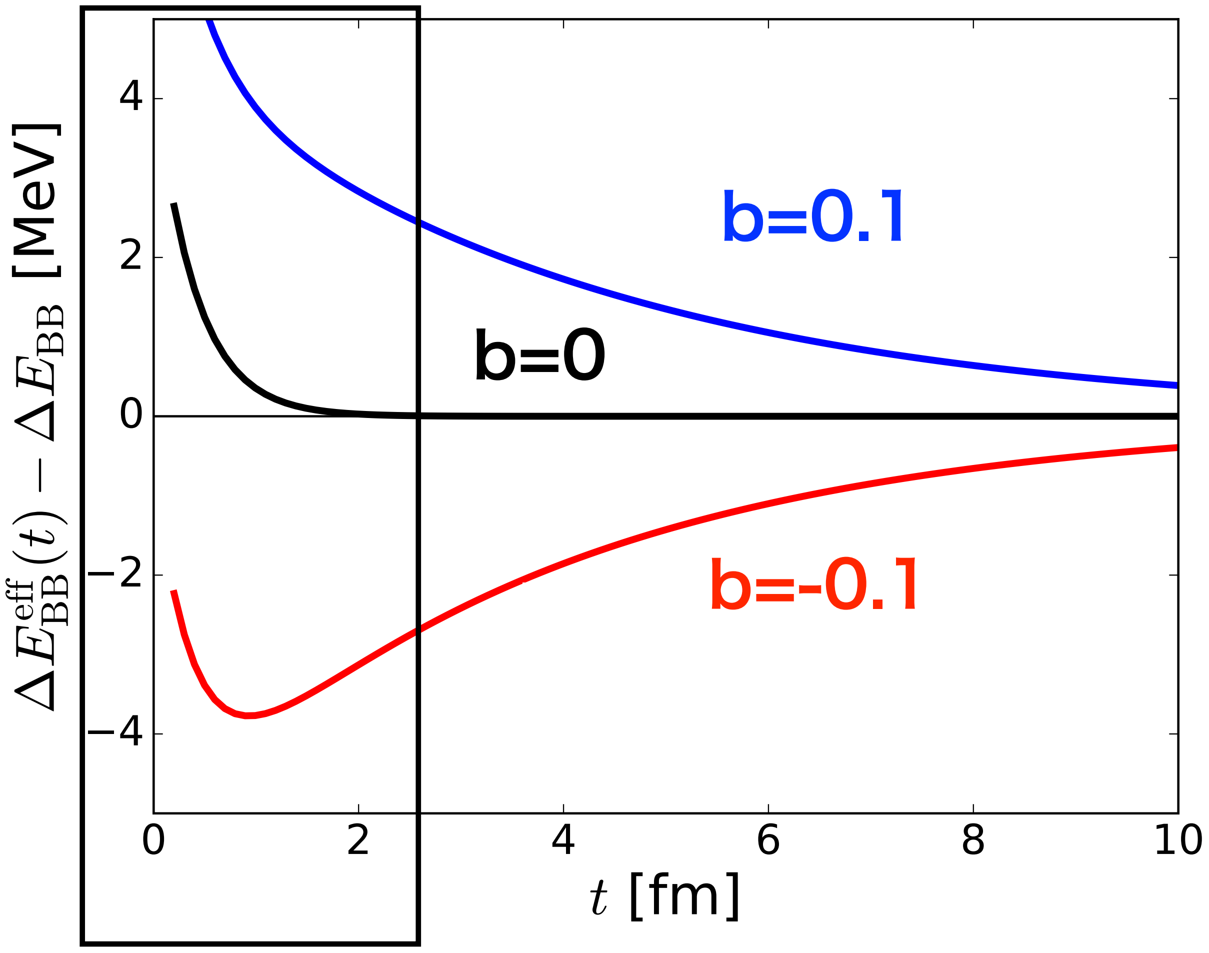
$\delta E_{\rm el} = 50$ MeV at $L \simeq 4$ fm

$b = \pm 0.1$ 10 % contamination $b = 0$ for a comparison

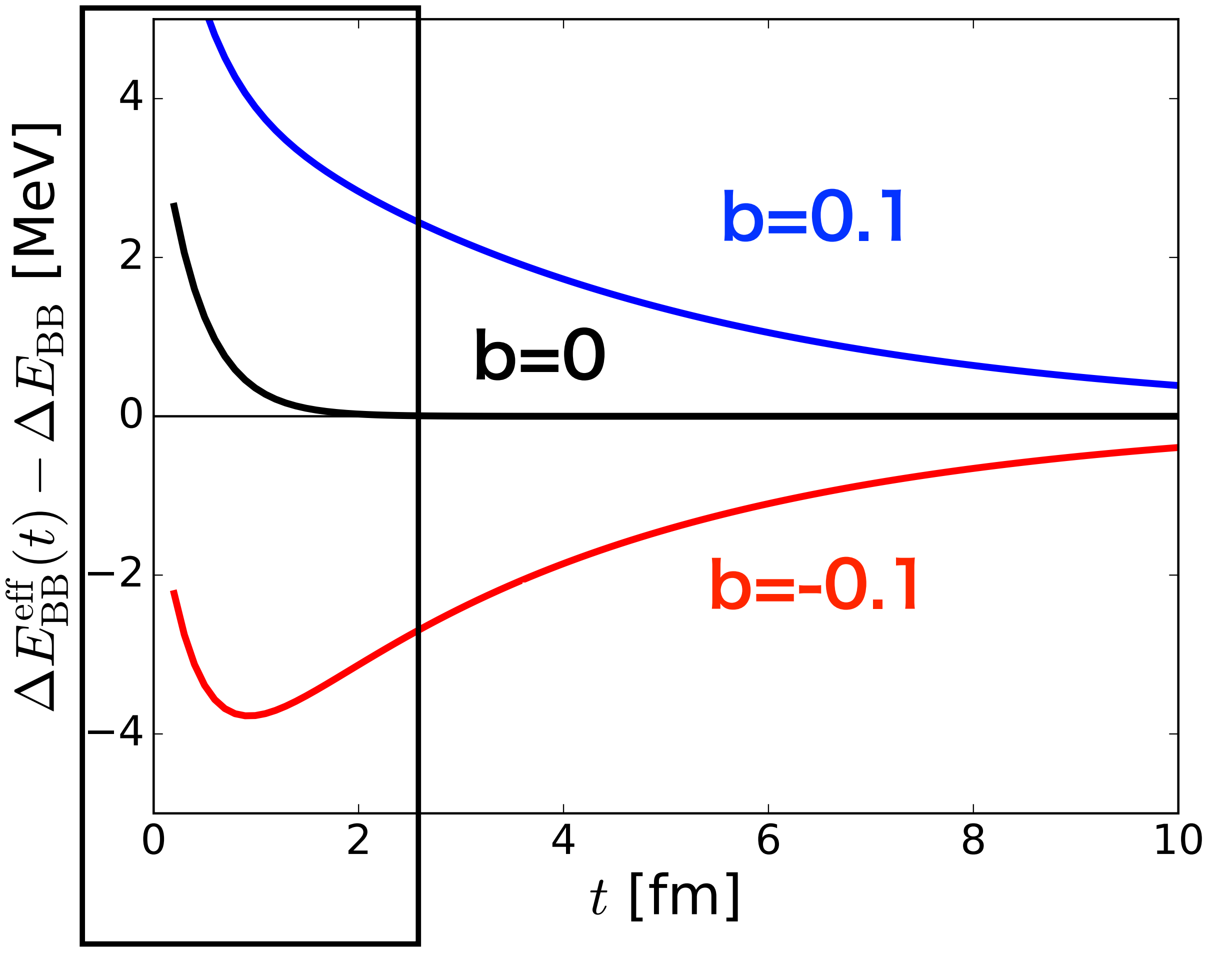
$\delta E_{\rm inel} = 500$ MeV the inelastic energy from heavy pions

$c = 0.01$ 1% contamination

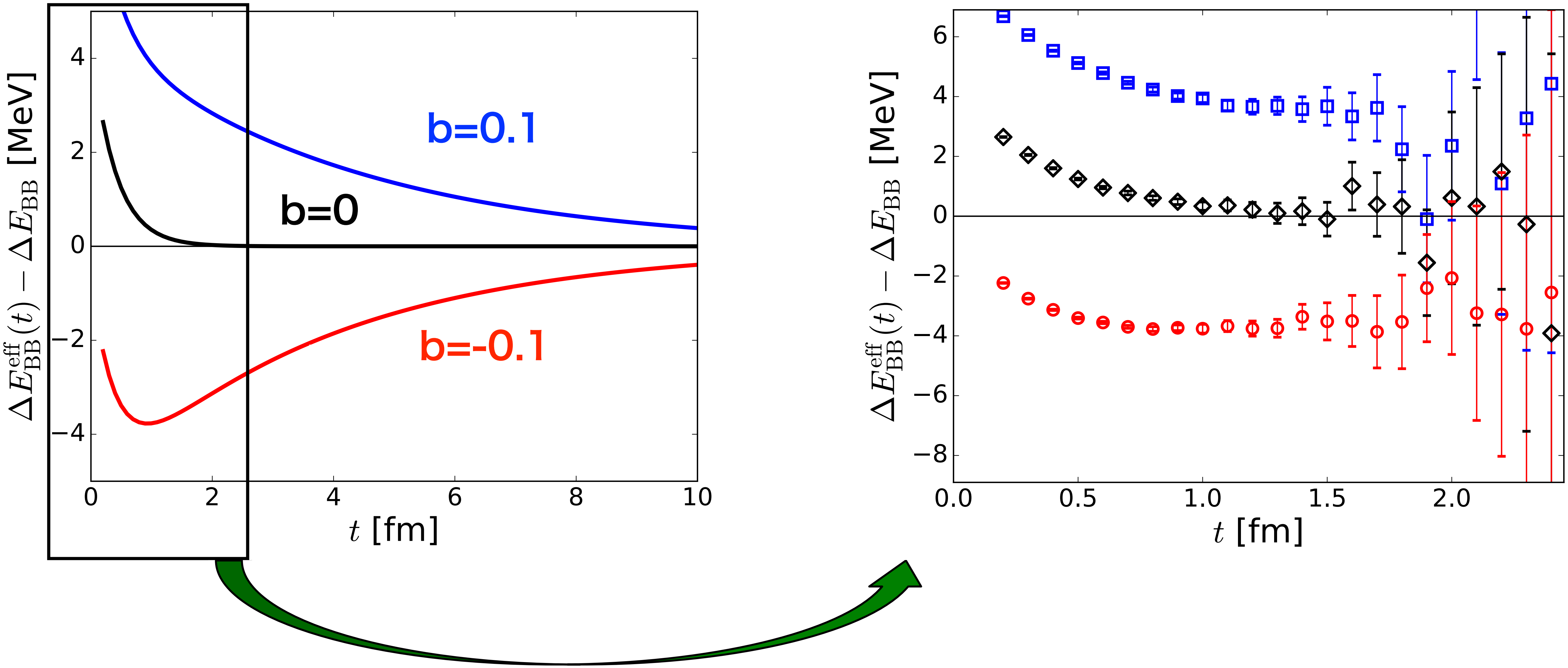




b=0.1
b=0
b=-0.1
$\Delta E^{\rm eff}_{\rm BB}(t) - \Delta E_{\rm BB}$ [MeV]
$t$ [fm]
4
2
0
−2
−4
0
2
4
6
8
10




Zoom + increasing errors and fluctuations

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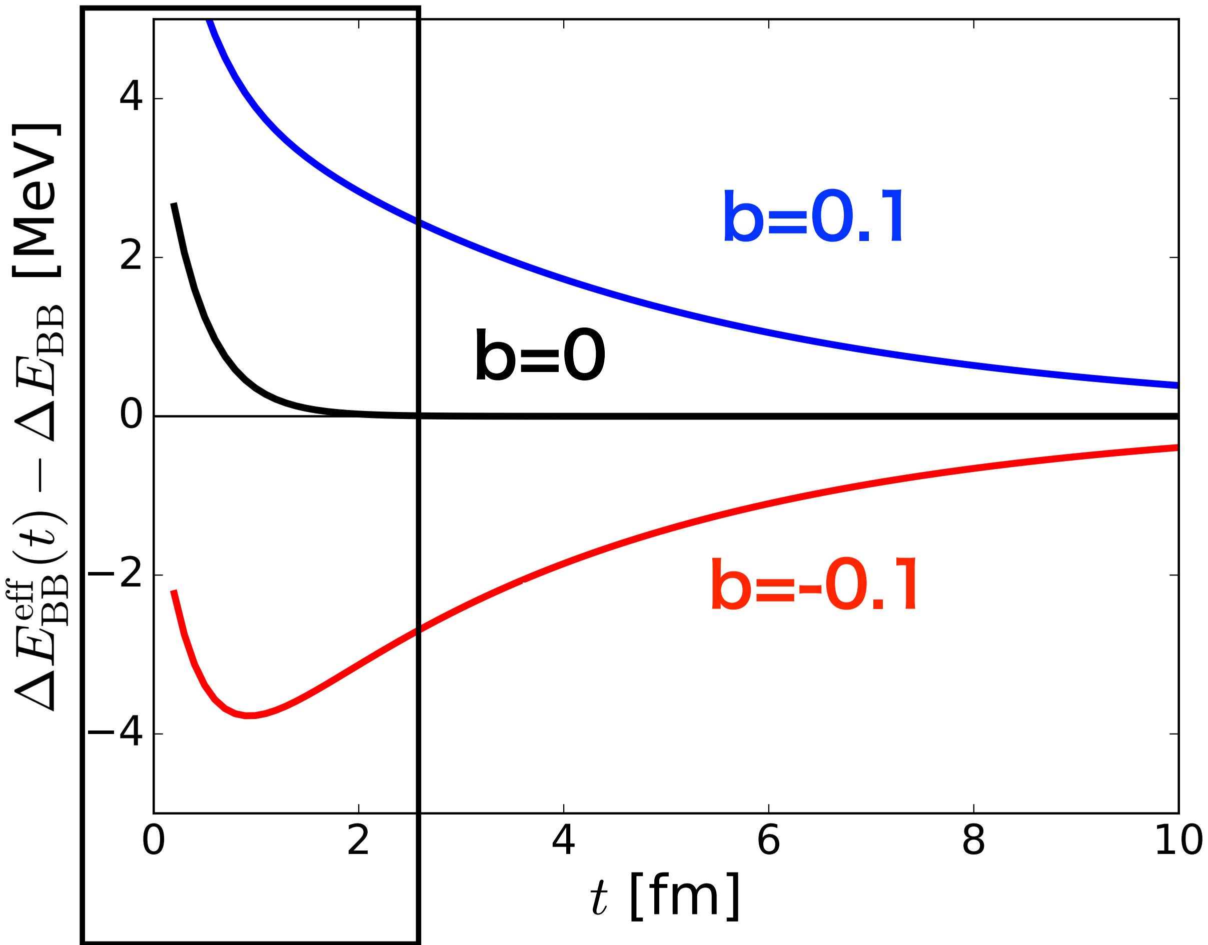


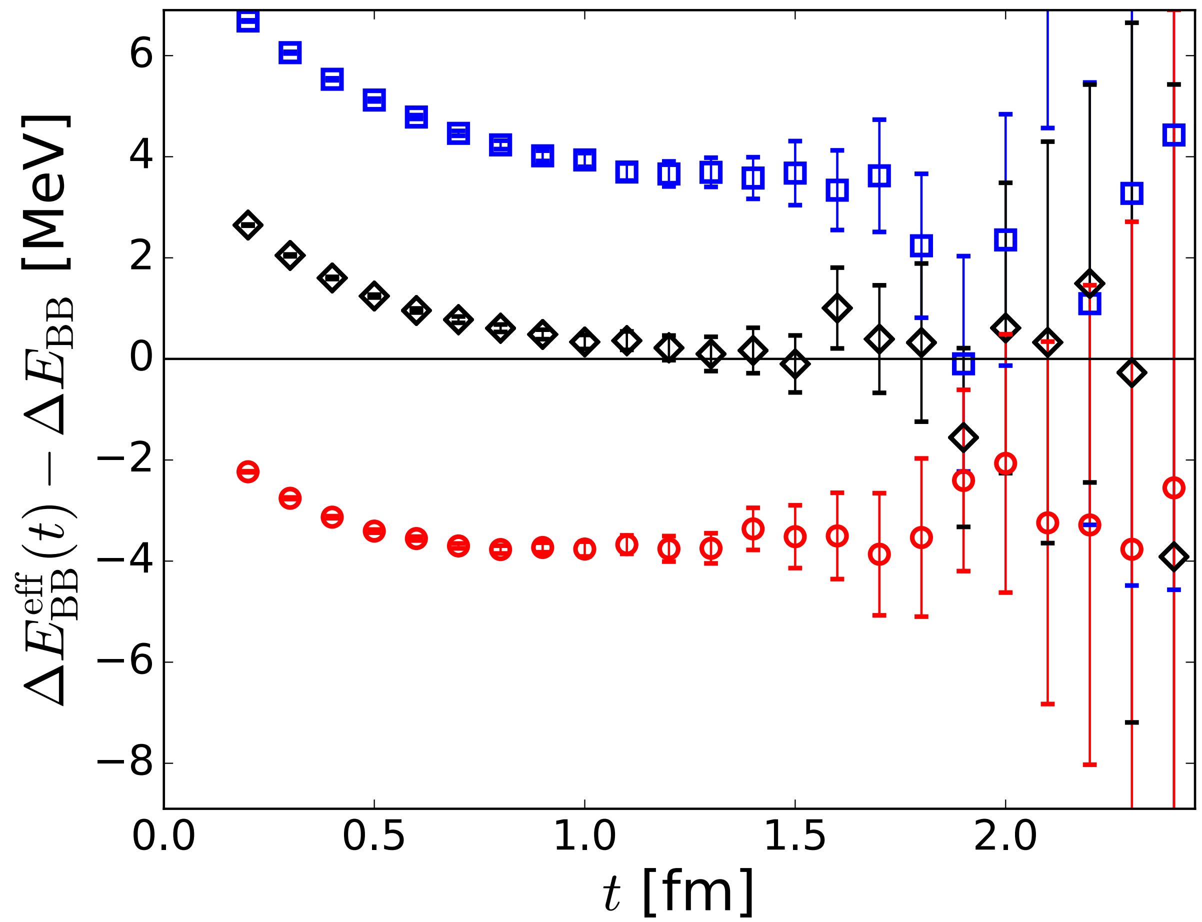


“Plateaux” at t ~ 1 fm
but they are fake (Mirage)

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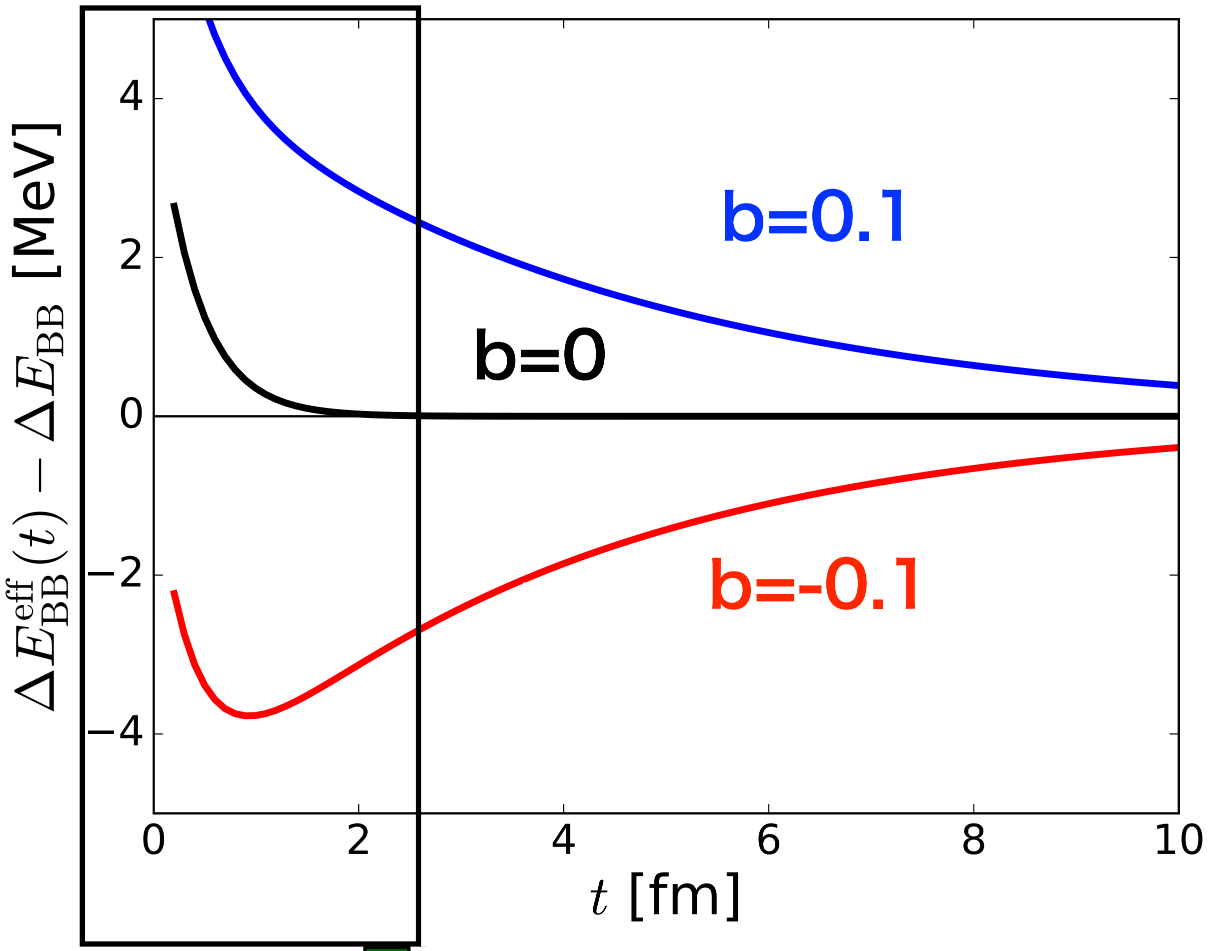


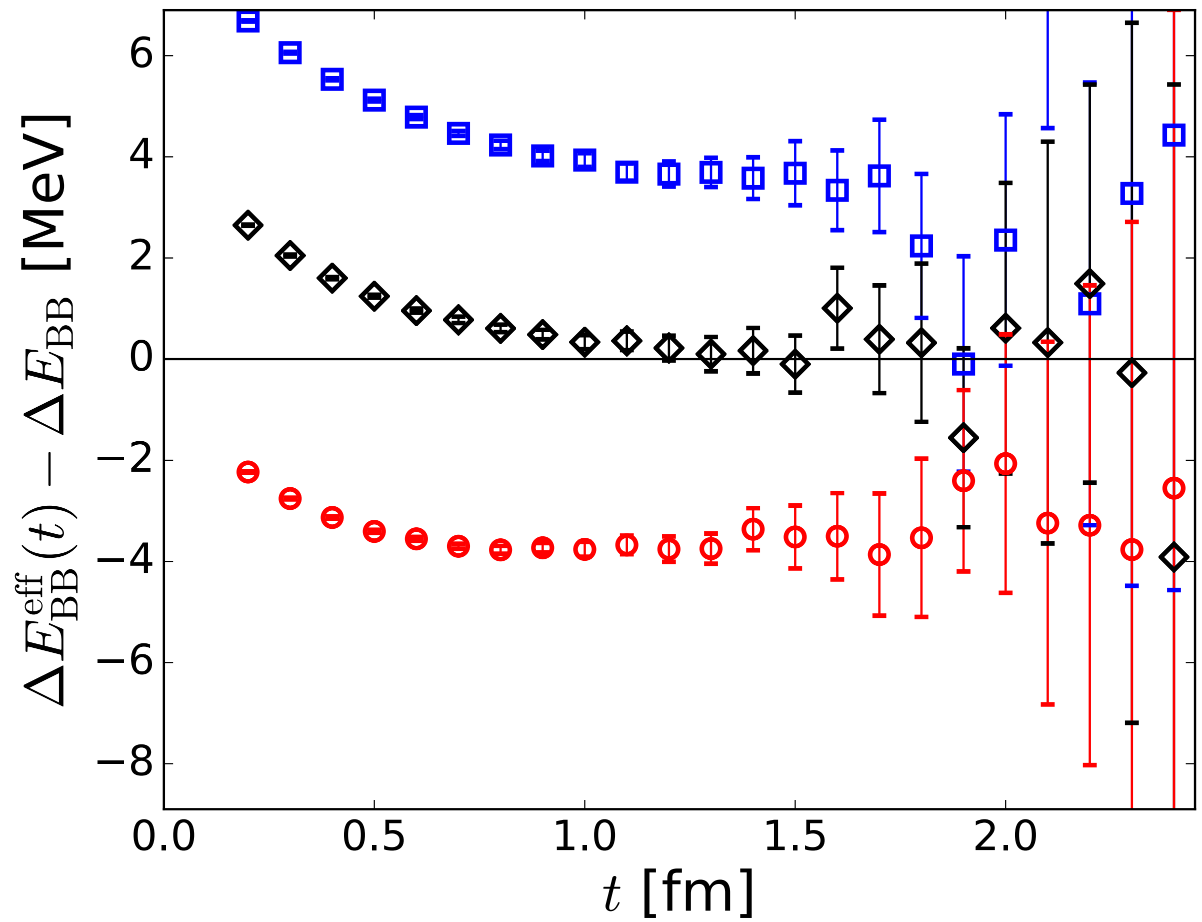


**Zoom + increasing errors and fluctuations**

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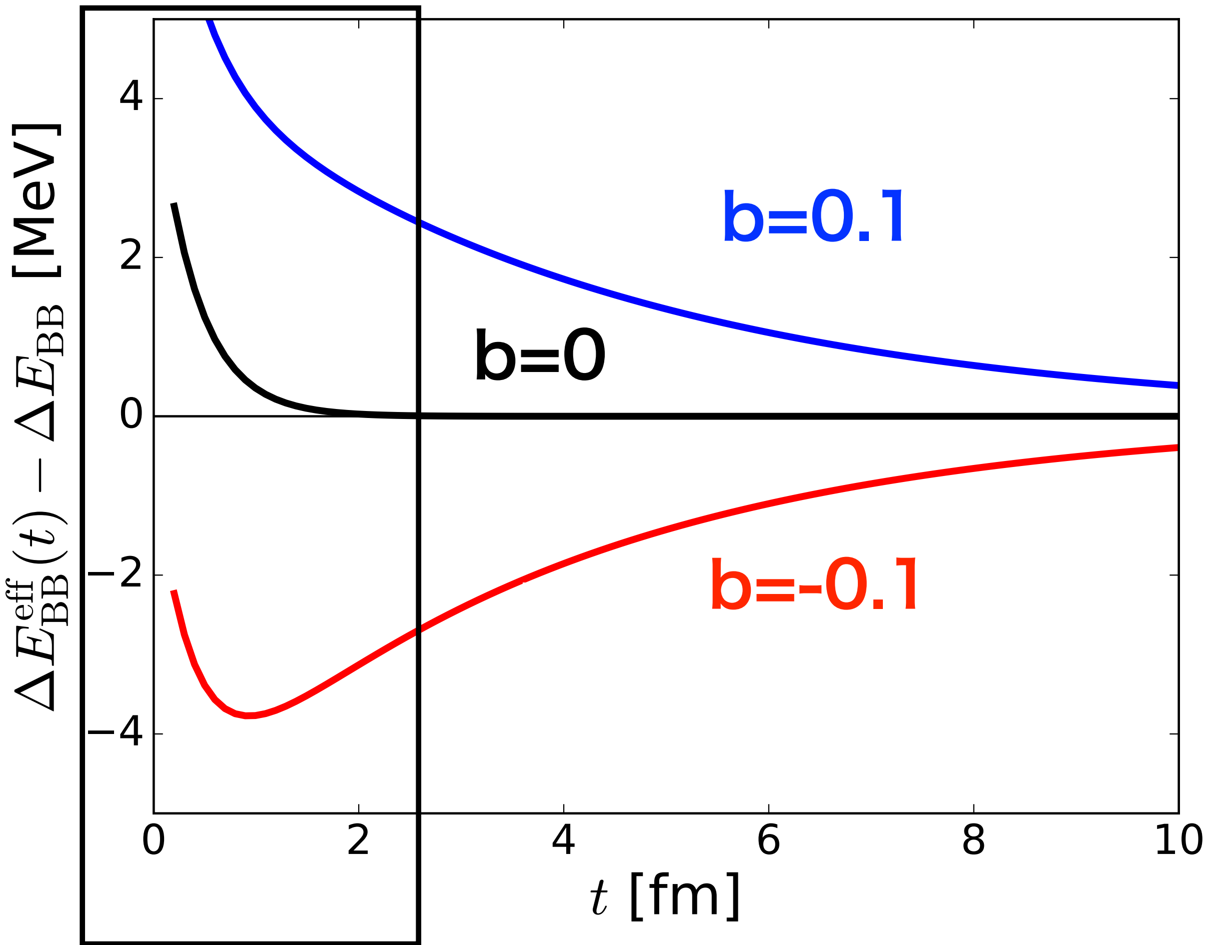
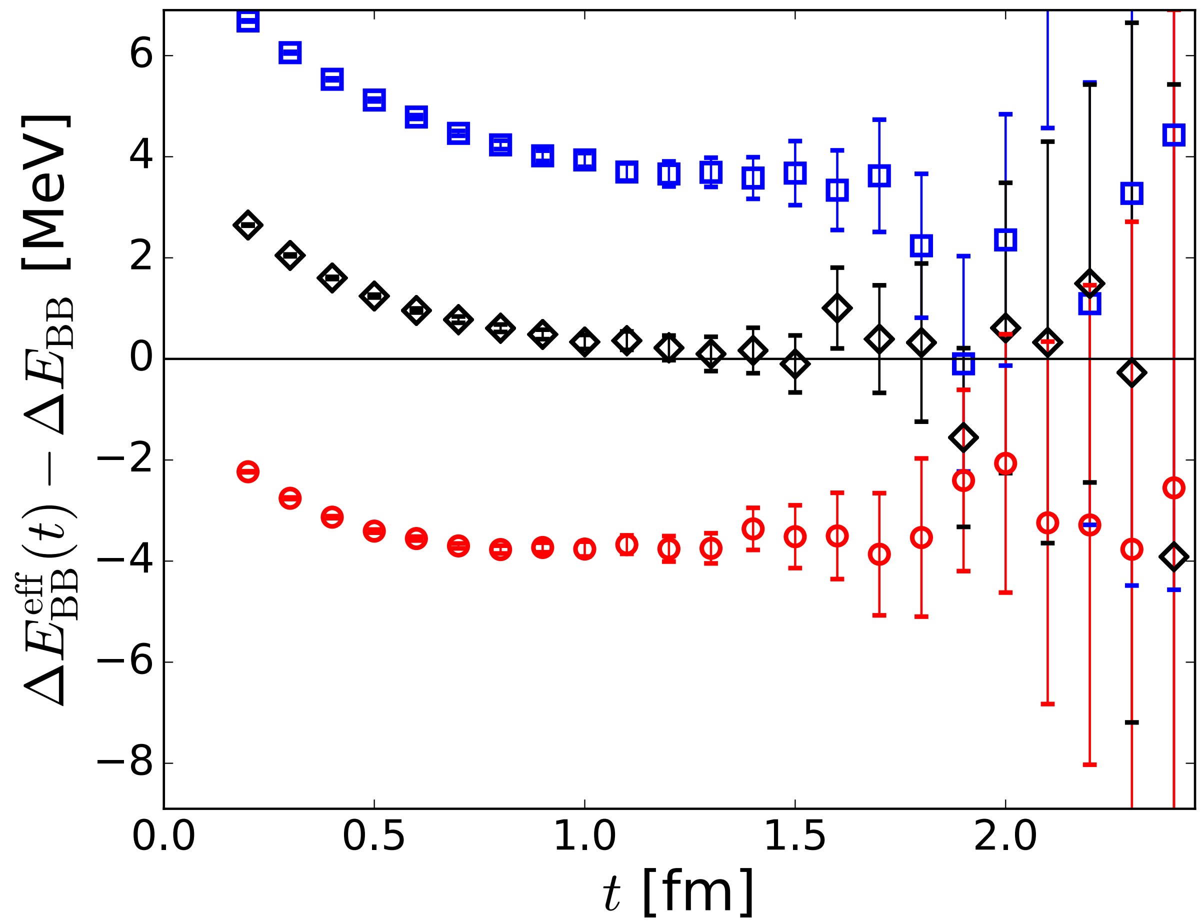
Observing the plateau guarantees the ground state saturation even when $t \gg 1/(E_1 - E_0)$ is NOT satisfied. claimed by Y(I)KU(‘11,’12,’15), NPL(’12,’13,’15), CalLat(’15)

**No ! We can not distinguish the real plateau from its mirages.**

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**The "looking for a plateau at small t" method does not work.**

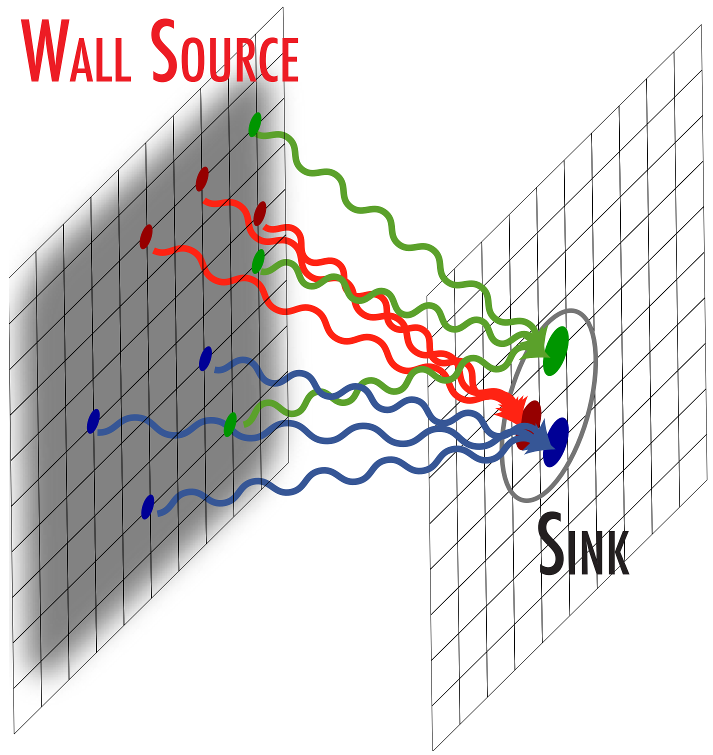
# II. Mirage problem
# (Operator dependence)

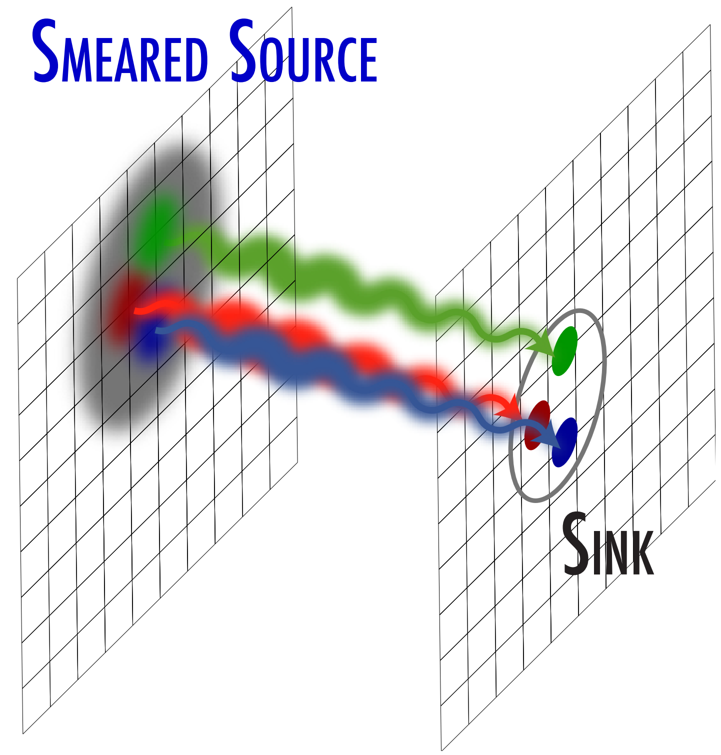
## - Manifestation of the problem I -

T. Iritani et al. (HAL QCD), JHEP1610(2016)101 (arXiv:1607.06371)

# Source operator dependence of plateaux

quark wall source vs quark smeared source




$$\sum_{\mathbf{y}} q(\mathbf{y}, t_0)$$

$$\sum_{\mathbf{y}} e^{-B|\mathbf{x}_0 - \mathbf{y}|} q(\mathbf{y}, t_0)$$

b are different between the two.

**Lattice setup** 2+1 flavor QCD

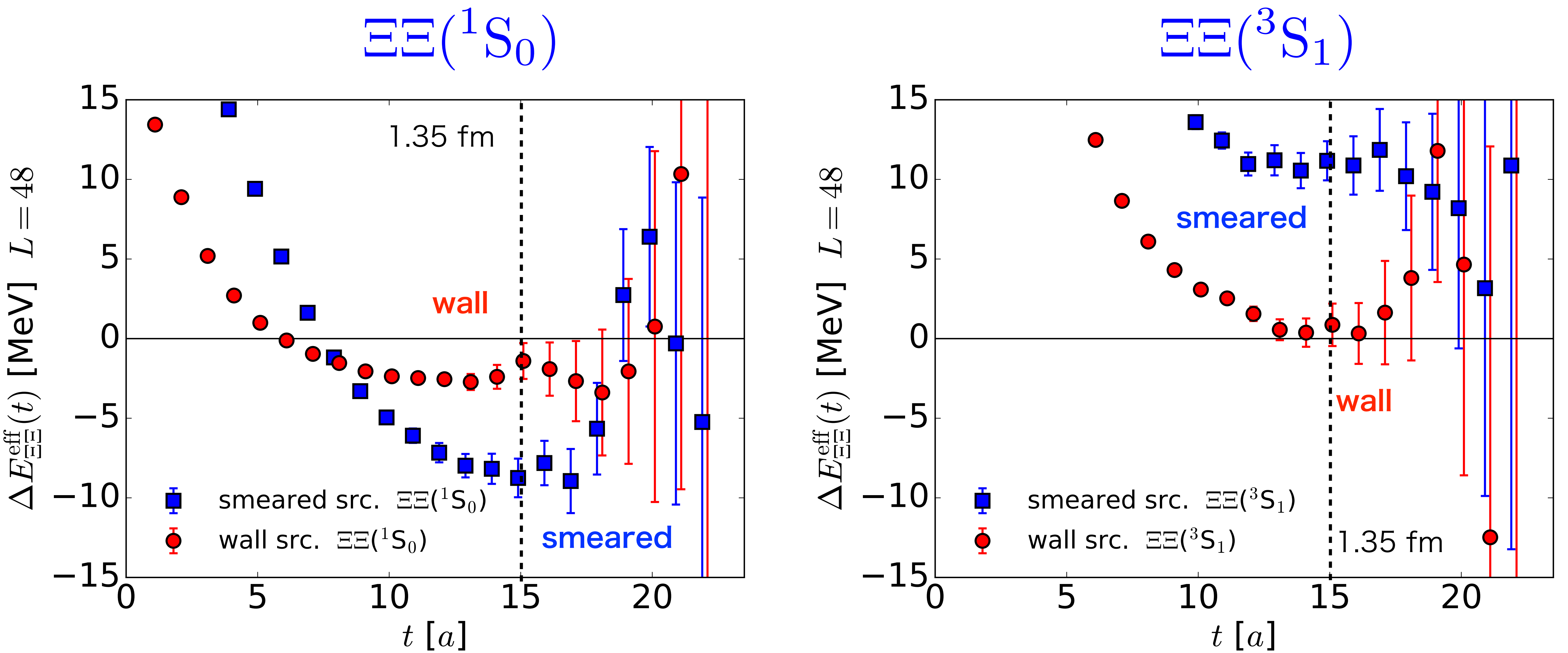
same gauge configurations of YIKU 2012

$$a = 0.09 \text{ fm } (a^{-1} = 2.2 \text{ GeV})$$

$$m_\pi = 0.51 \text{ GeV}, m_N = 1.32 \text{ GeV}, m_K = 0.62 \text{ GeV}, m_\Xi = 1.46 \text{ GeV}$$

# Energy shift of $\Xi\Xi$

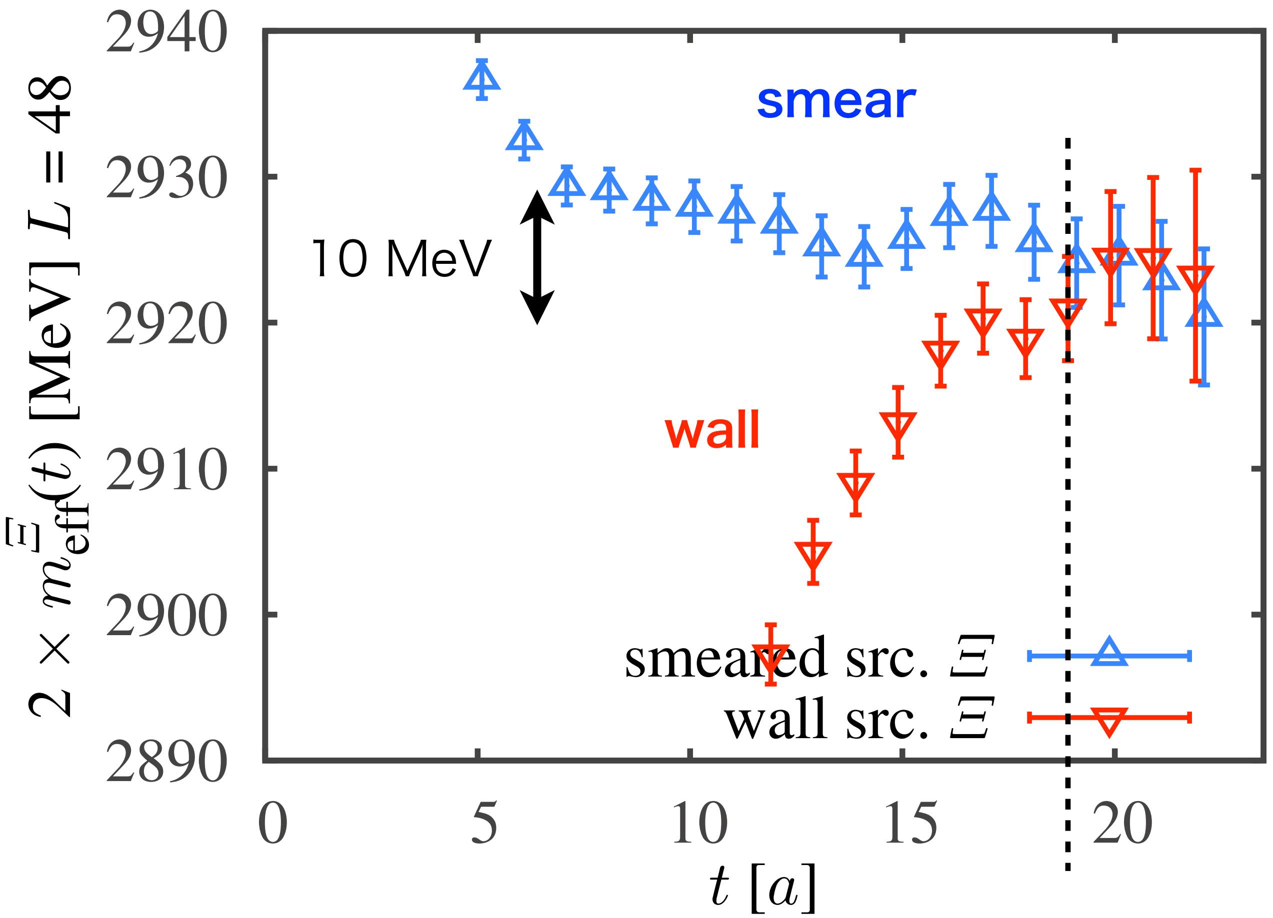
smaller statistical errors



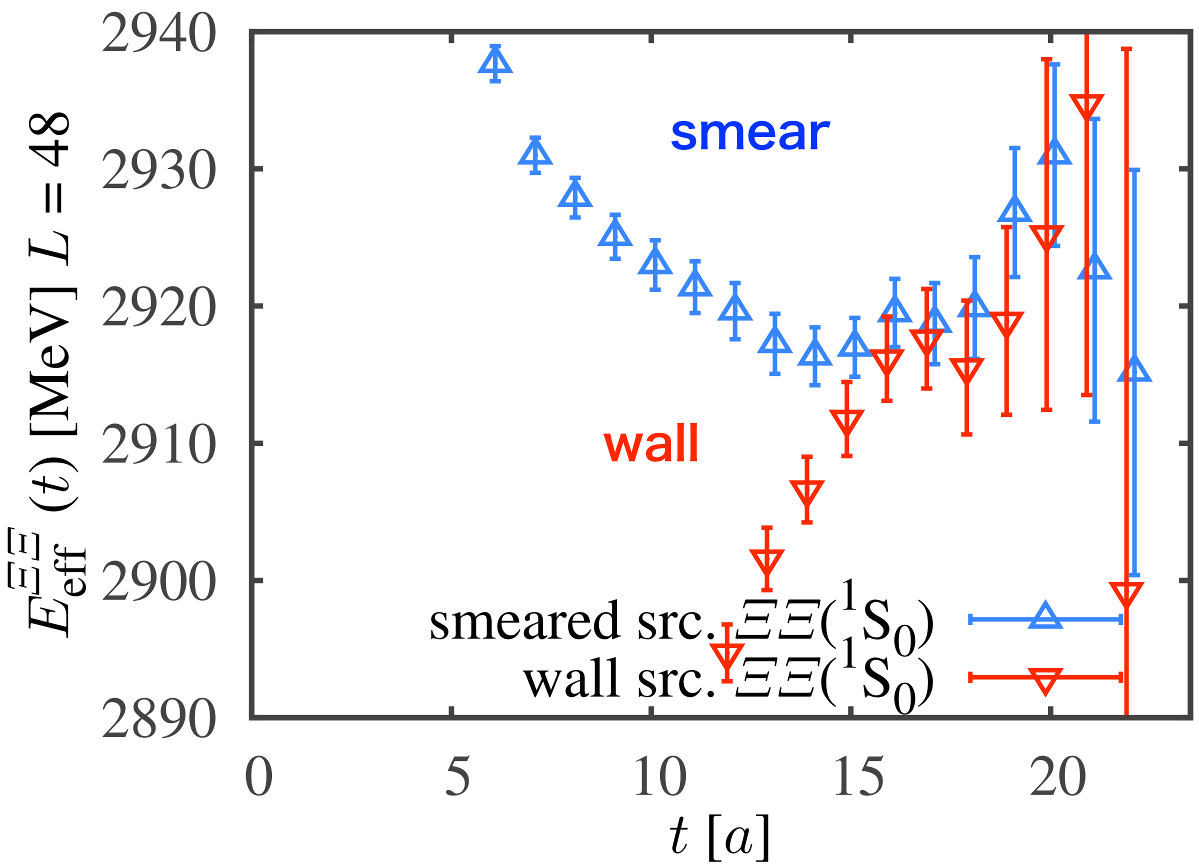

- Not surprisingly, two sources disagree.
- The mirage problem becomes reality.
- Plateau-like structures around t=1-1.5 fm are by no means trustable.
- Both might agree at t > 18a, but errors are too large.

# Numerator and denominator

$2m_\Xi$




$E_{\Xi\Xi}(^3\mathrm{S}_1)$




Variations of individuals are larger than the difference in the ratio.

The method must rely on cancellation of systematics in the ratio.

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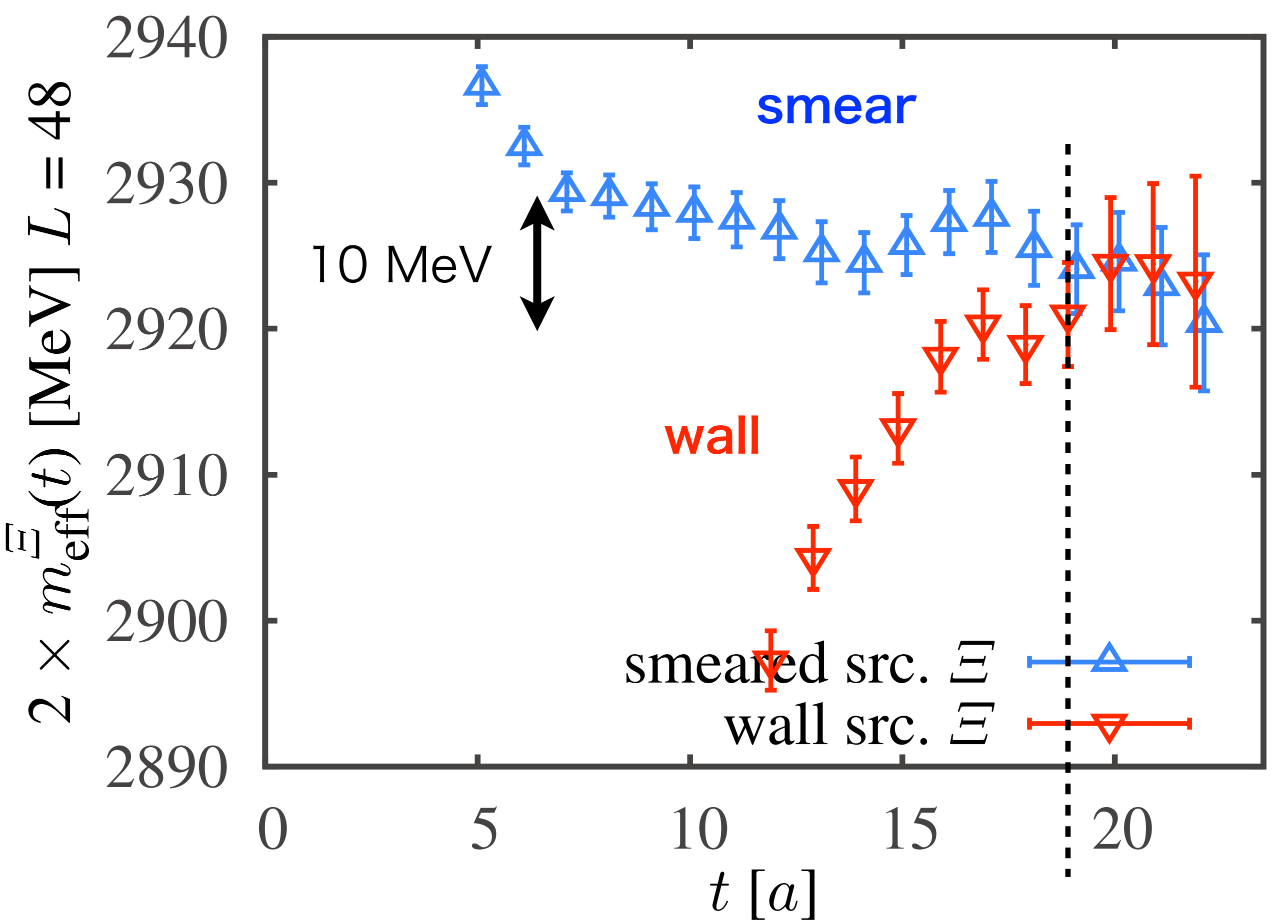


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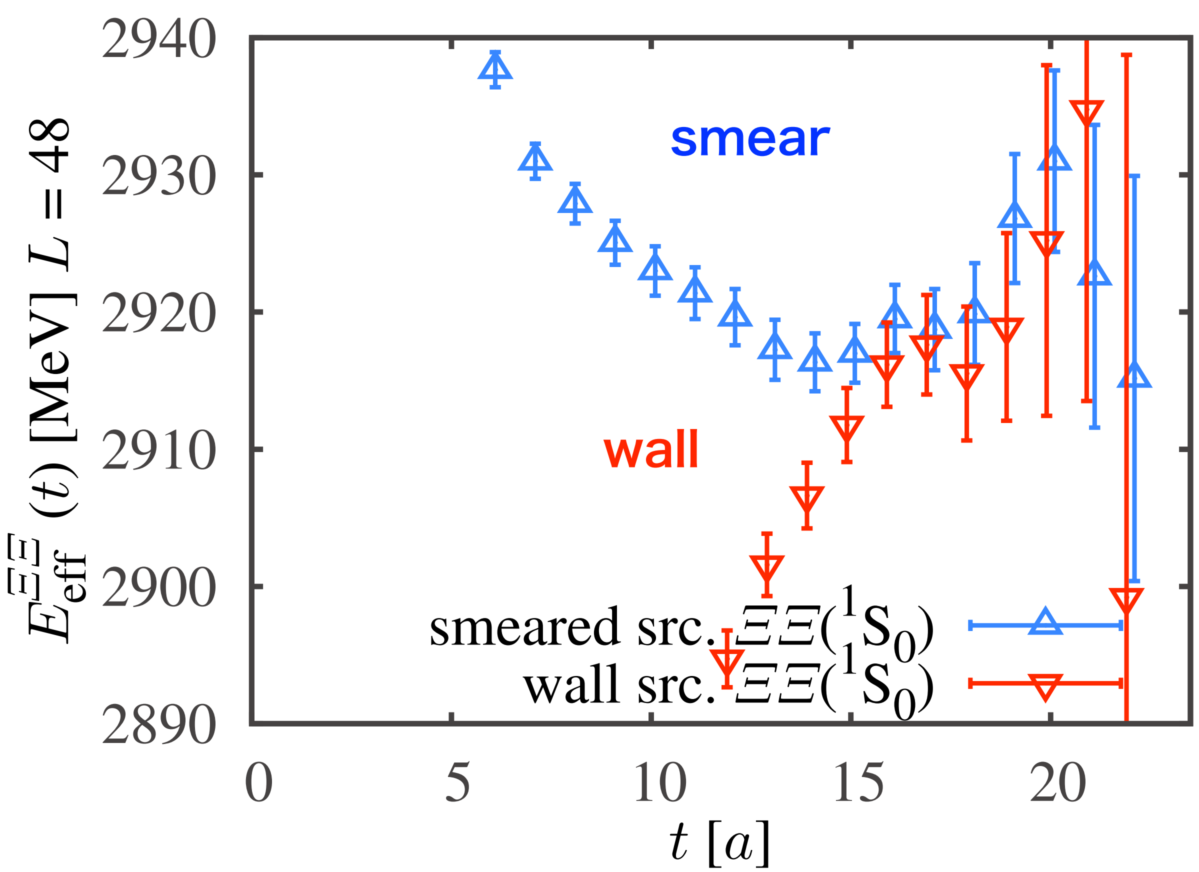


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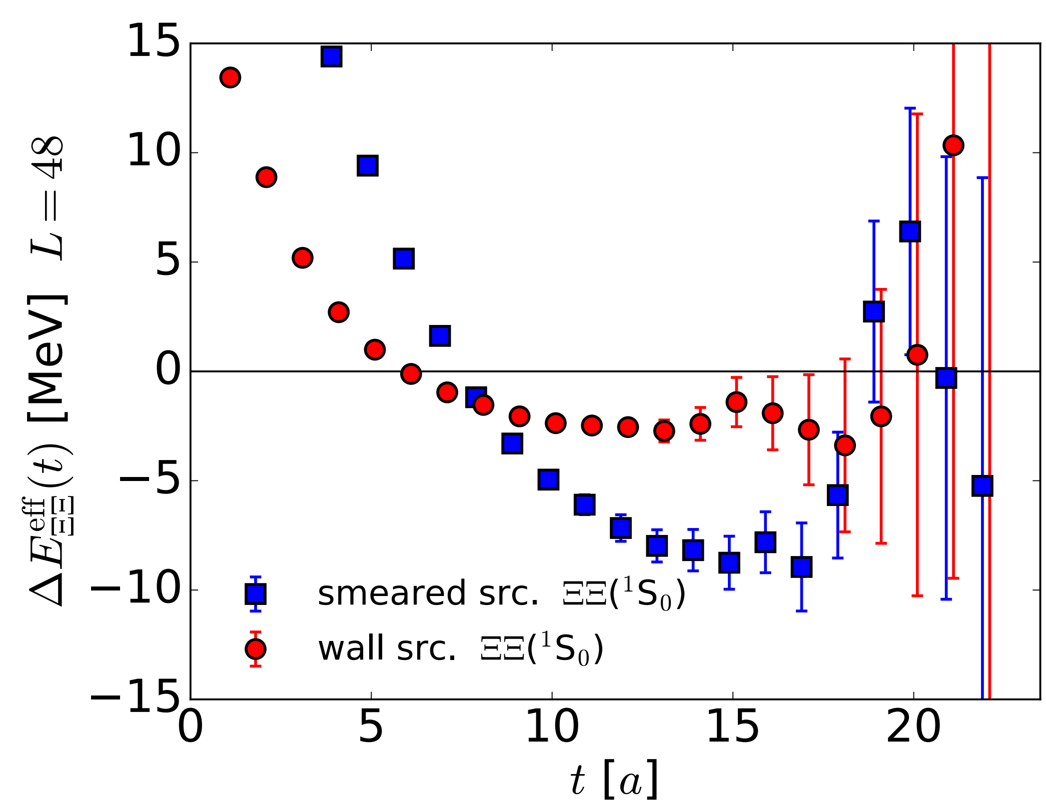
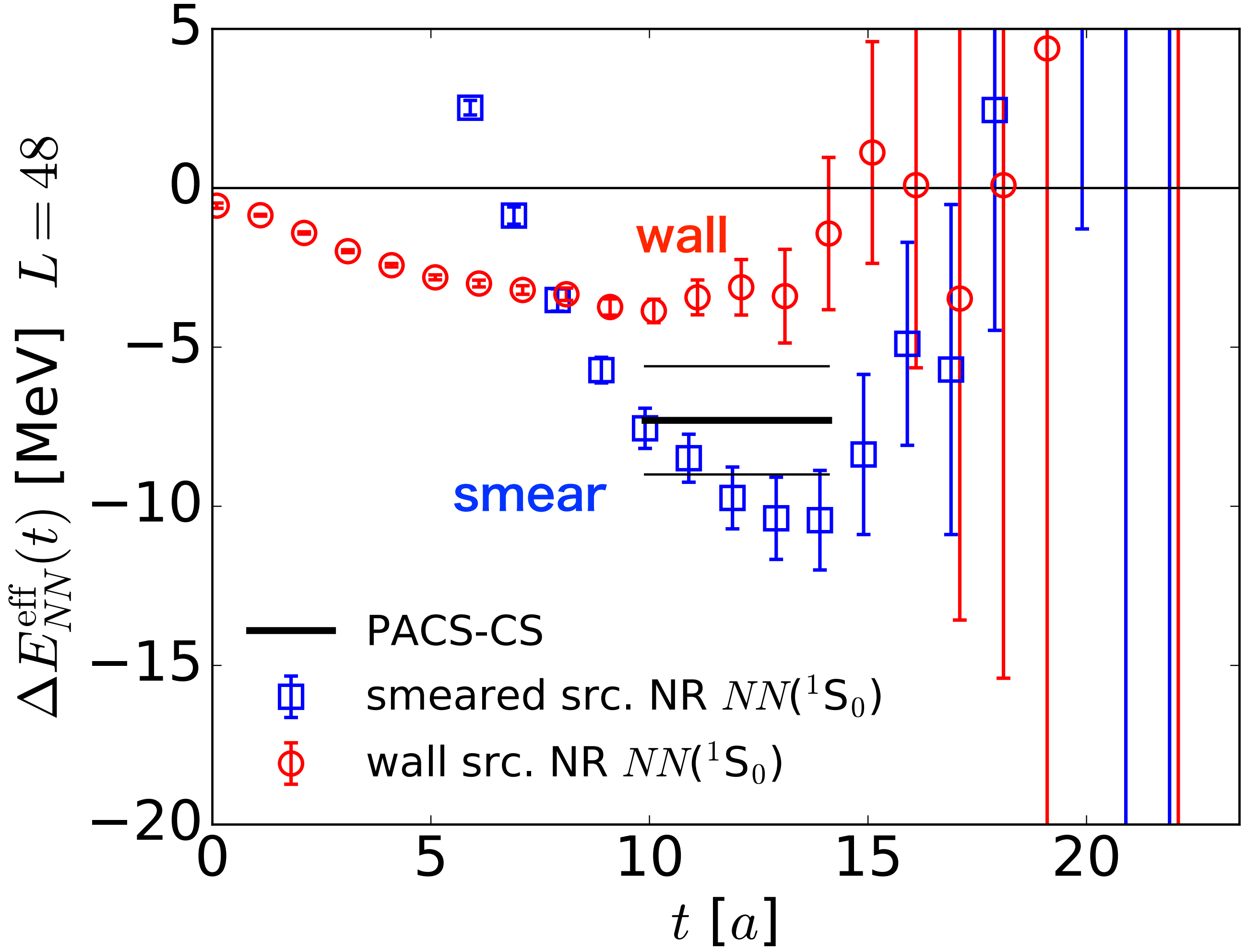
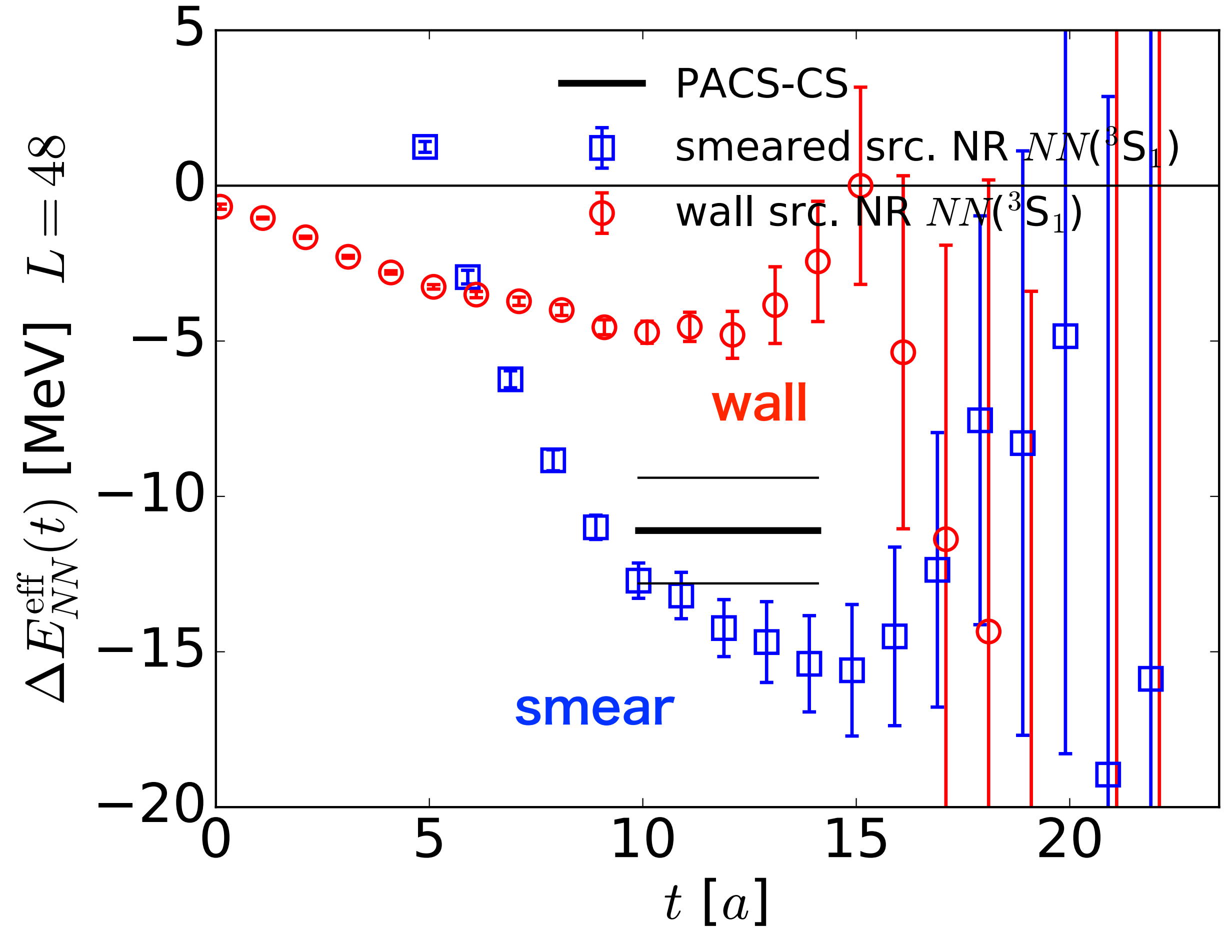


# Same problem also appears for NN

$NN(^1\mathrm{S}_0)$



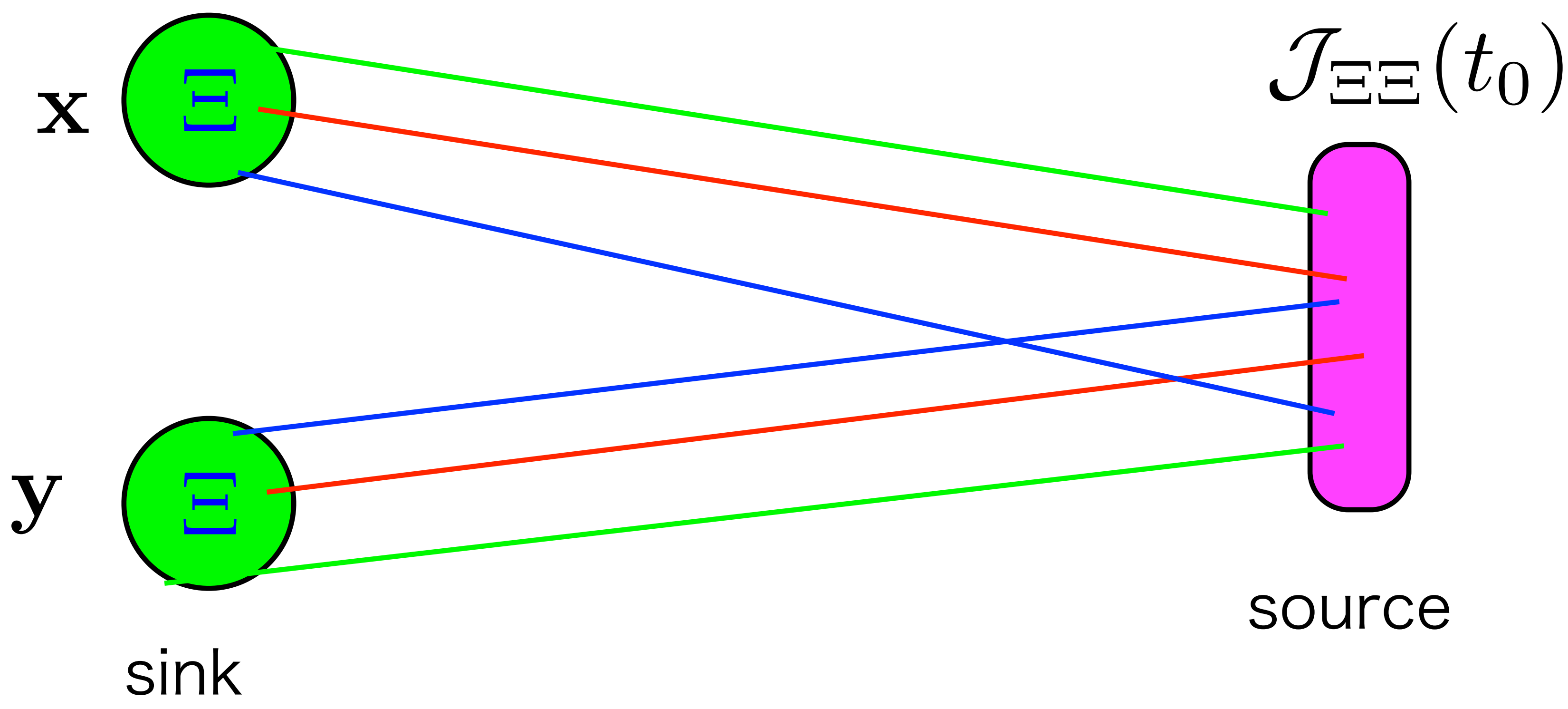

$NN(^3\mathrm{S}_1)$




With larger errors, disagreement also exists.

In addition, we may have

# Sink 2-baryon operator dependence of plateaux



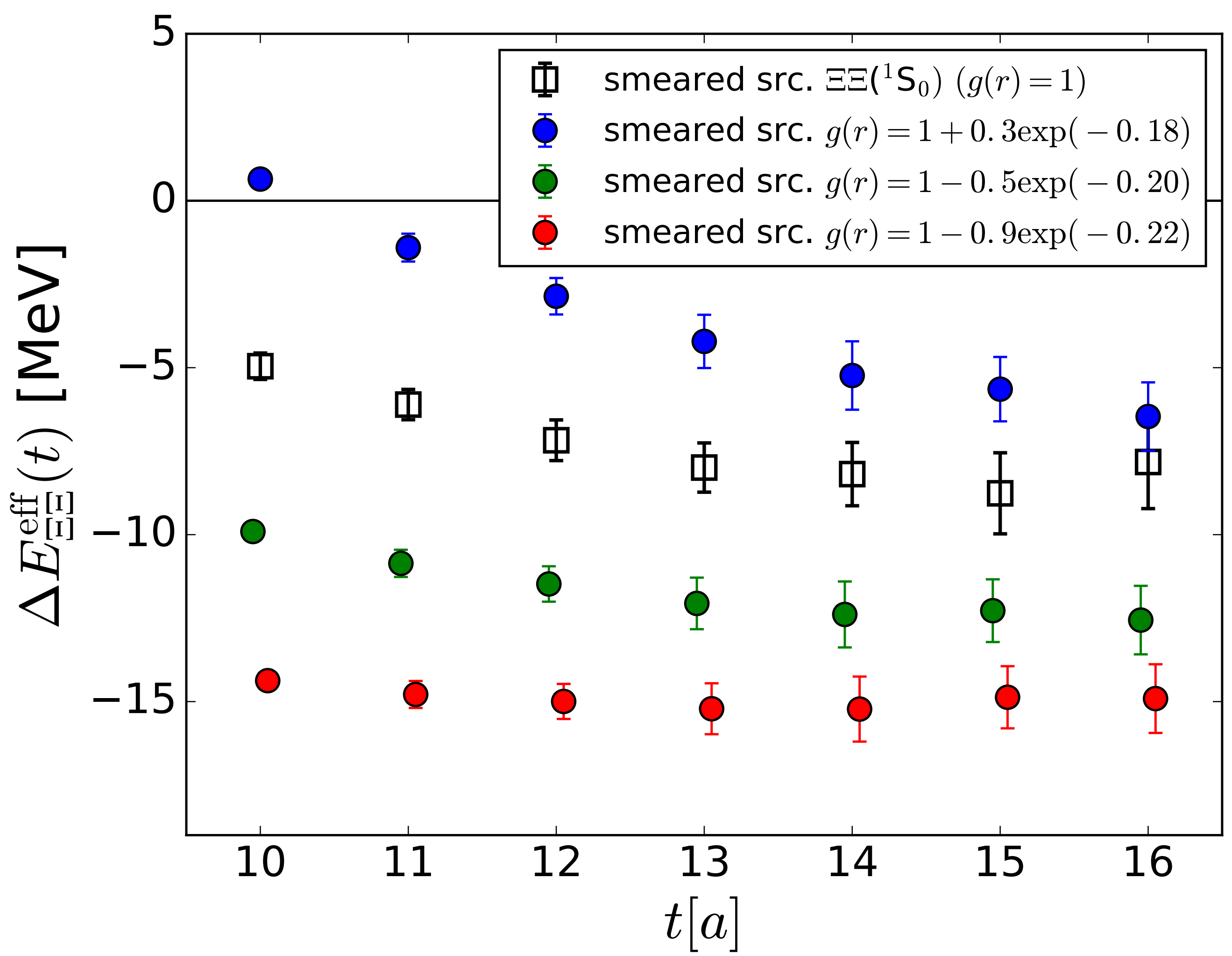

$$G_{\Xi\Xi}(t) = \sum_{\mathbf{x},\mathbf{y}} g(|\mathbf{x}-\mathbf{y}|)\langle \Xi(\mathbf{x},t)\Xi(\mathbf{y},t)\mathcal{J}_{\Xi\Xi}(t_0)\rangle$$

$$g(r) = 1 \; : \; \text{standrad sink operator}$$

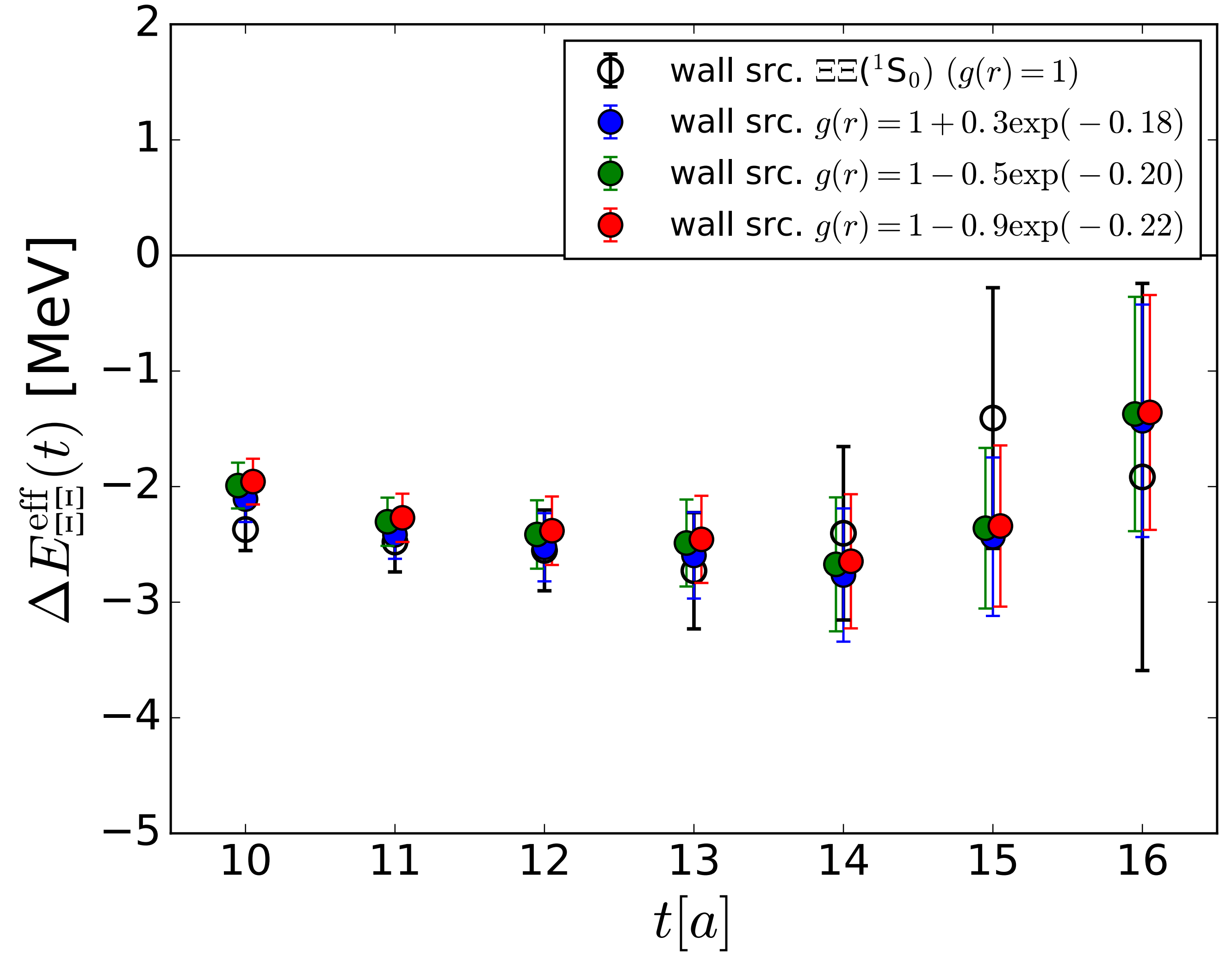
$$g(r) = 1 + A\exp(-Br) \; : \; \text{generalized sink operator}$$

The true plateau must NOT dependent on g(r).

## Smeared source




## Wall source




- smeared source is very sensitive to g(r).
  - Sometimes deeper and more stable.
  - one can produce an arbitrary value (within a certain range) by g(r).
- Wall source is insensitive to g(r).

- Dangers of fake plateaux exit in principle for the direct method.
- Problem becomes manifest in the strong source/sink operator dependences of plateau values in YIKU 2012.
- Are there any symptoms in other results ?
    - Study of source dependences requires additional simulations.
    - need simpler and easier check

# III. Sanity check

## - Manifestation of the problem II -

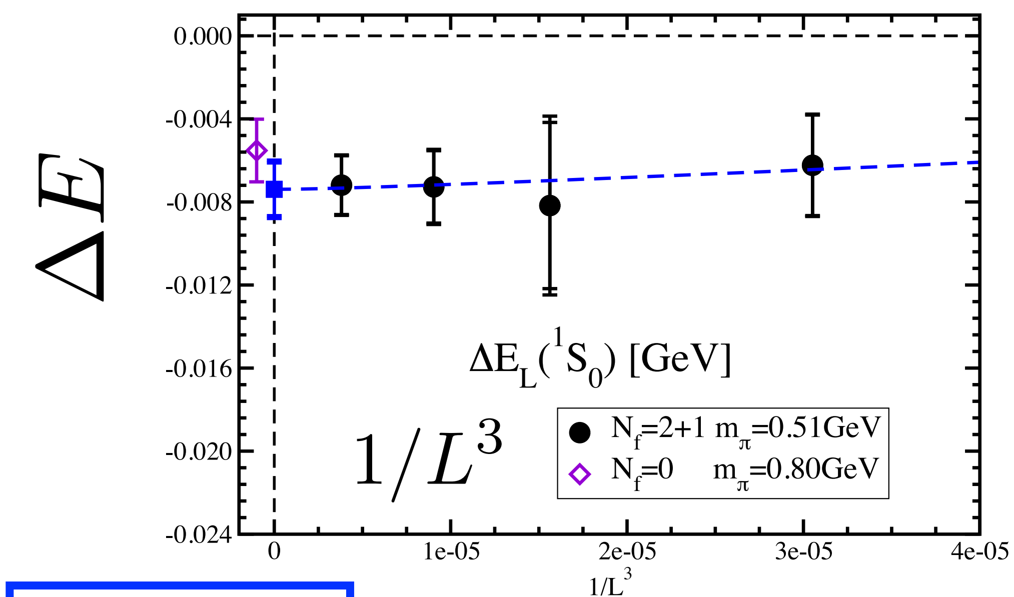
S. Aoki, T. Doi, T. Iritani, PoS(Lattice2016) 109 (aiXiv:1610:09763)

# Finite volume formula

Lüscher, NPB354(1991)531

## Direct method

YIKU2012

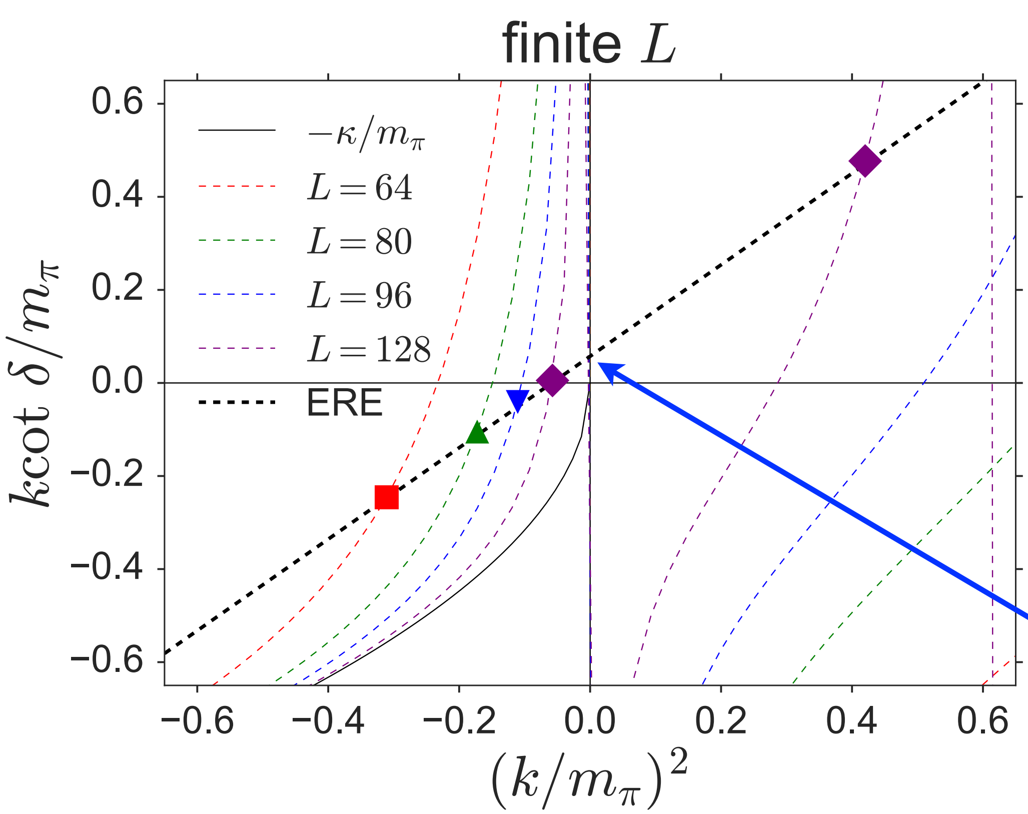



$$\Delta E = 2\sqrt{k^2 + m_N^2} - 2m_N, \quad q = \frac{kL}{2\pi}$$

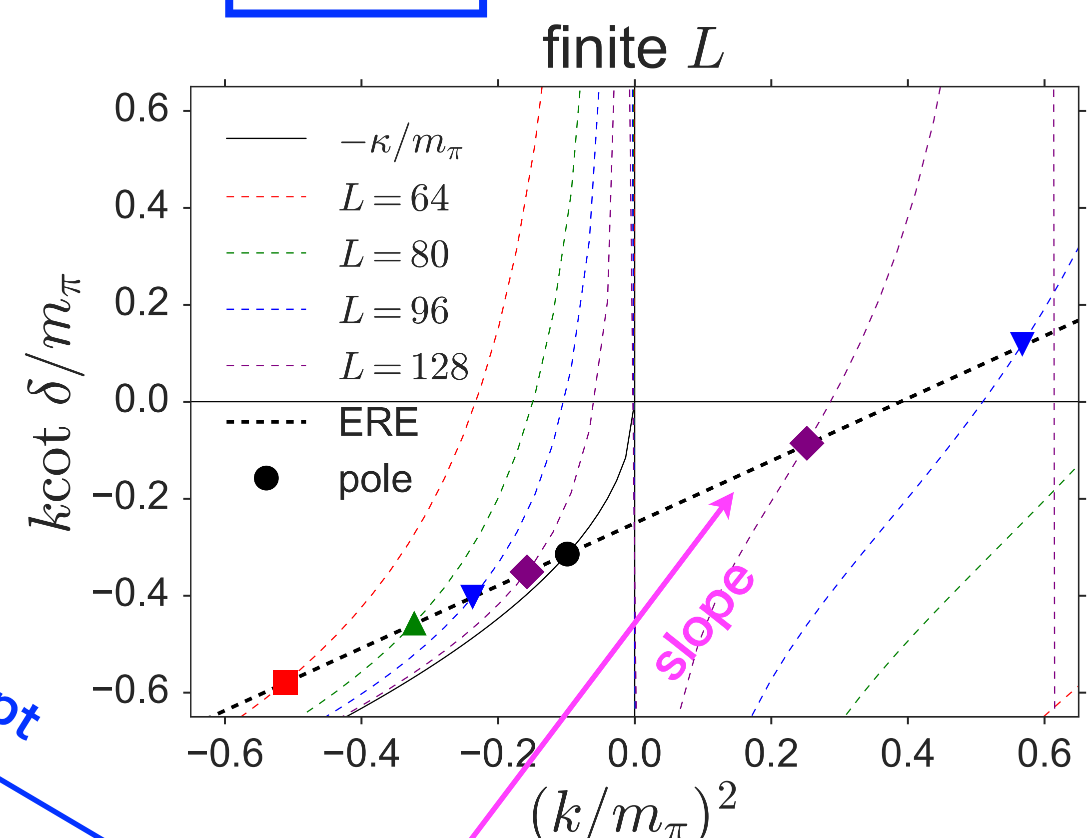
$$\boxed{k\cot\delta(k) = \frac{1}{\pi L}\sum_{\vec{n}\in\mathbb{Z}^3}\frac{1}{\vec{n}^2 - q^2}}$$

$\delta(k)$:scattering phase shift

## unbound




## bound



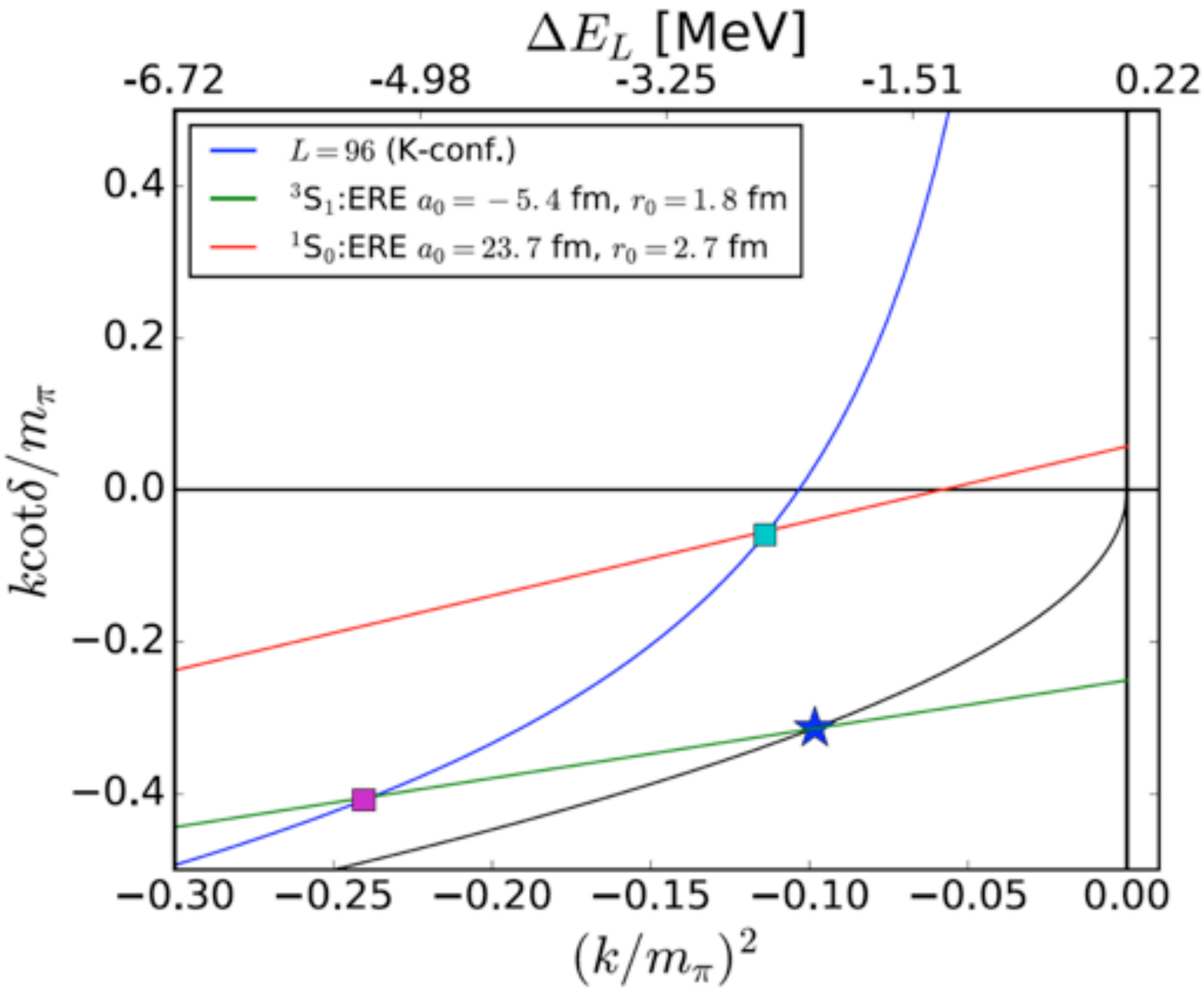

intercept

slope

## Effective Range Expansion (ERE)

$$k\cot\delta(k) = \frac{1}{a} + \frac{1}{2}rk^2 + \cdots$$

# ERE at physical pion mass



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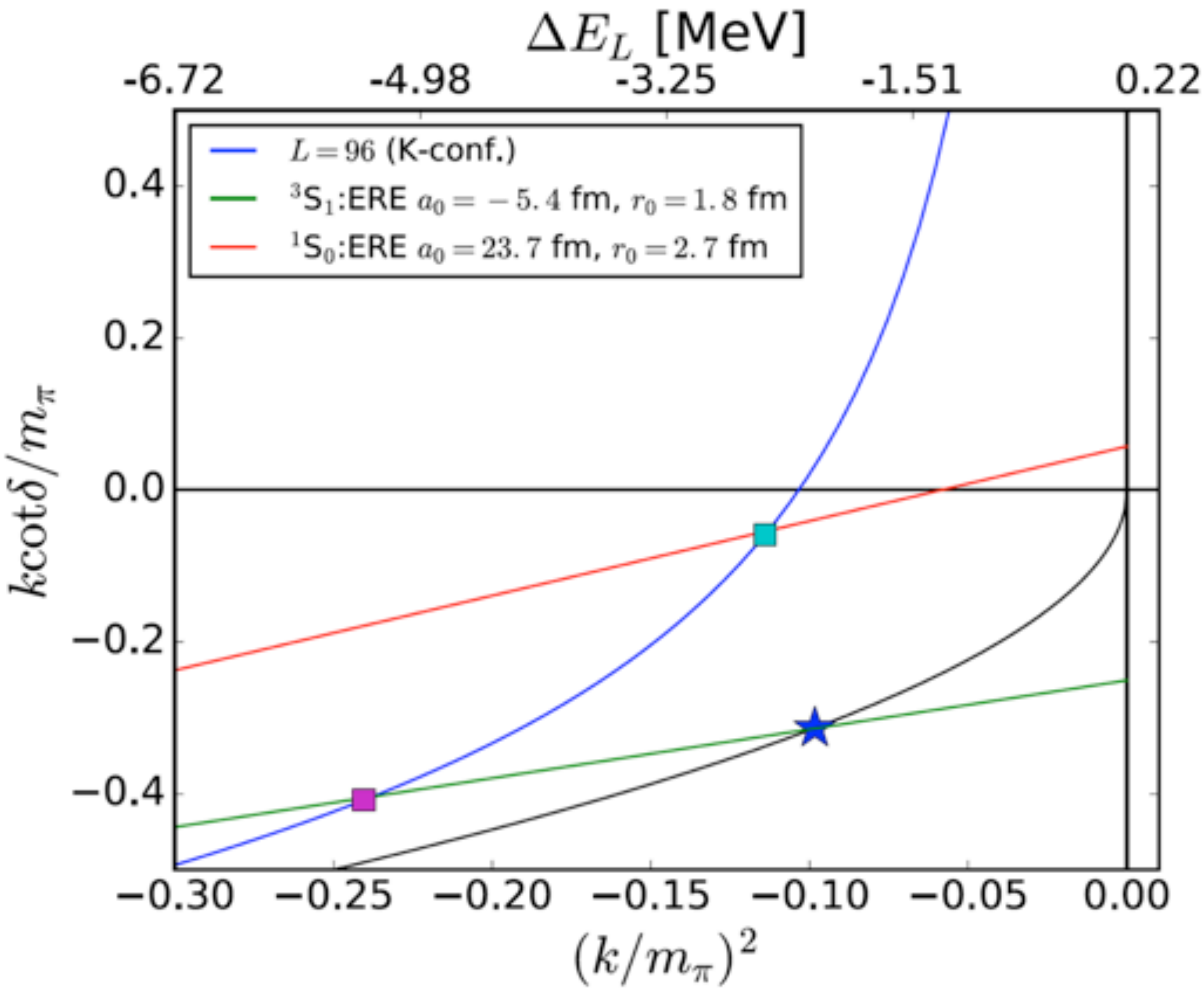


Instead, a behavior shown below indicates the problem in lattice QCD data.

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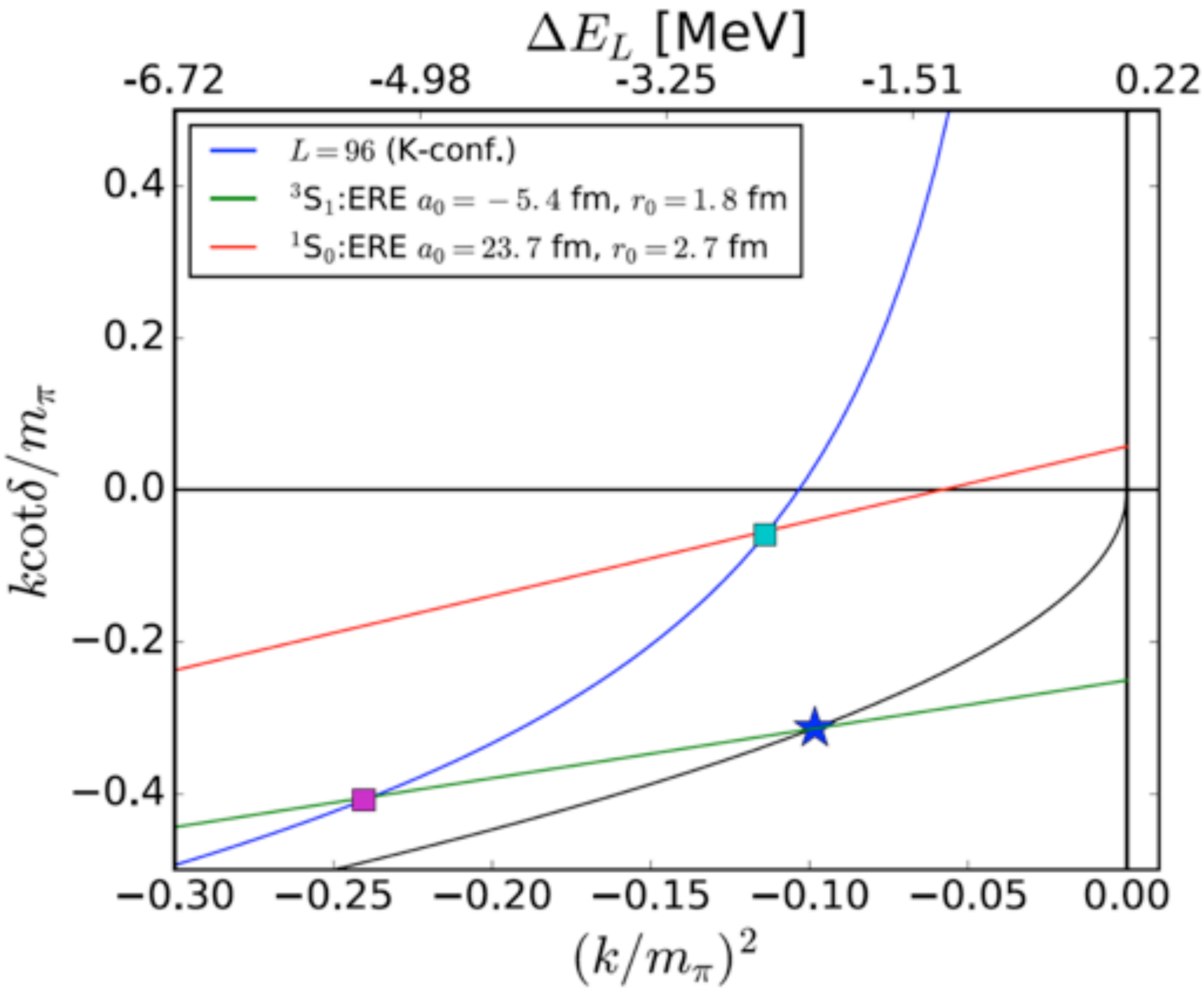


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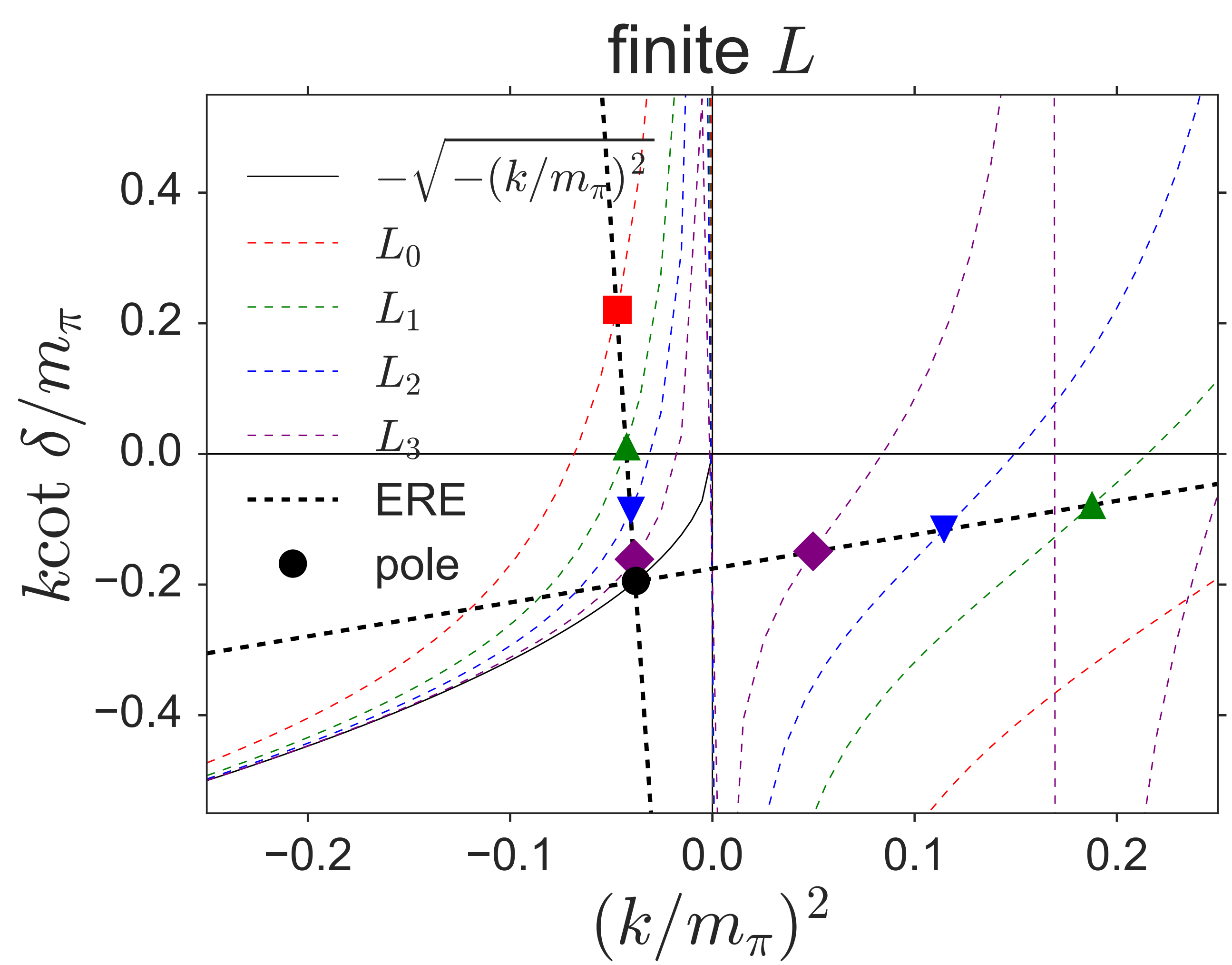
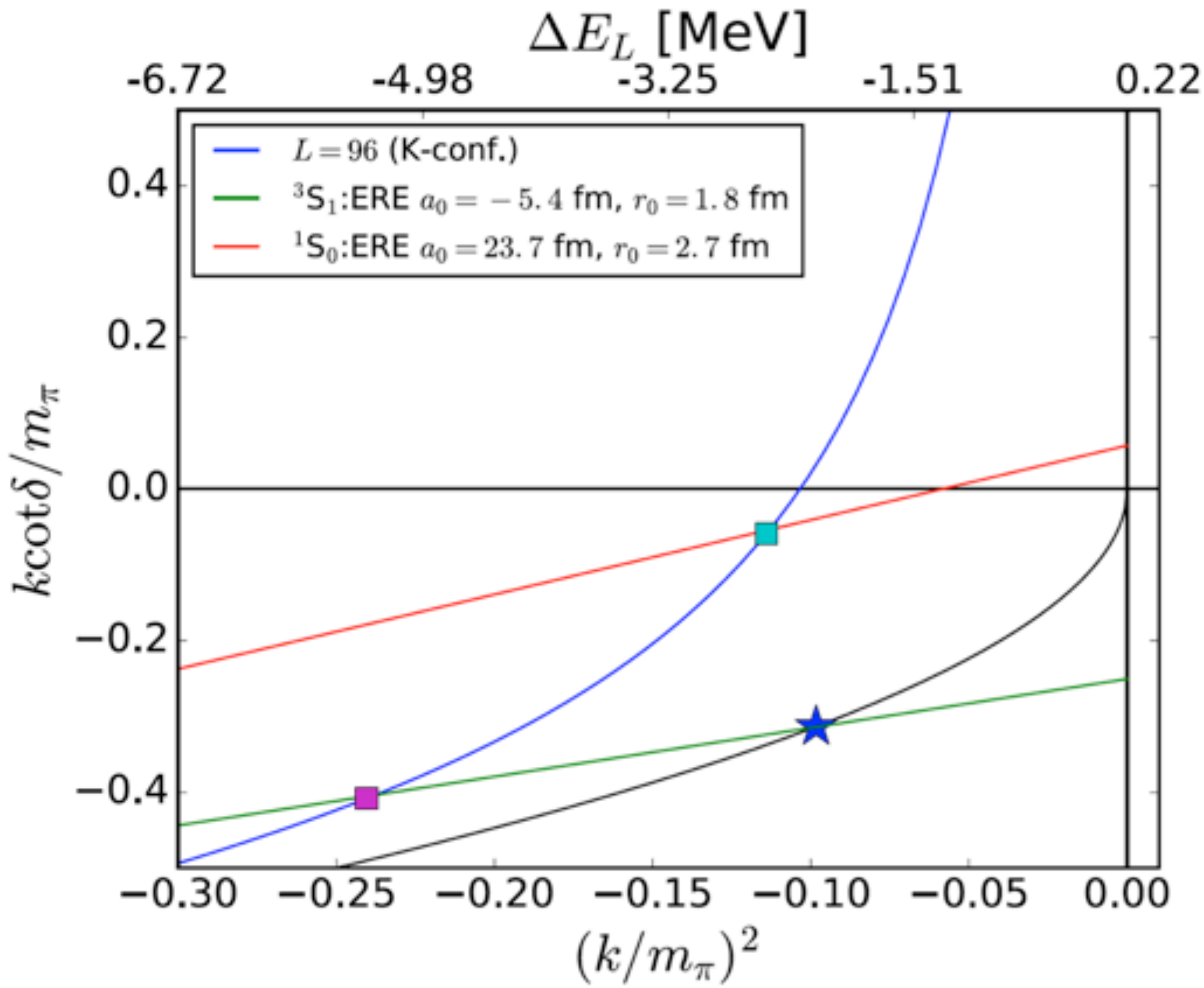


## ERE at physical pion mass

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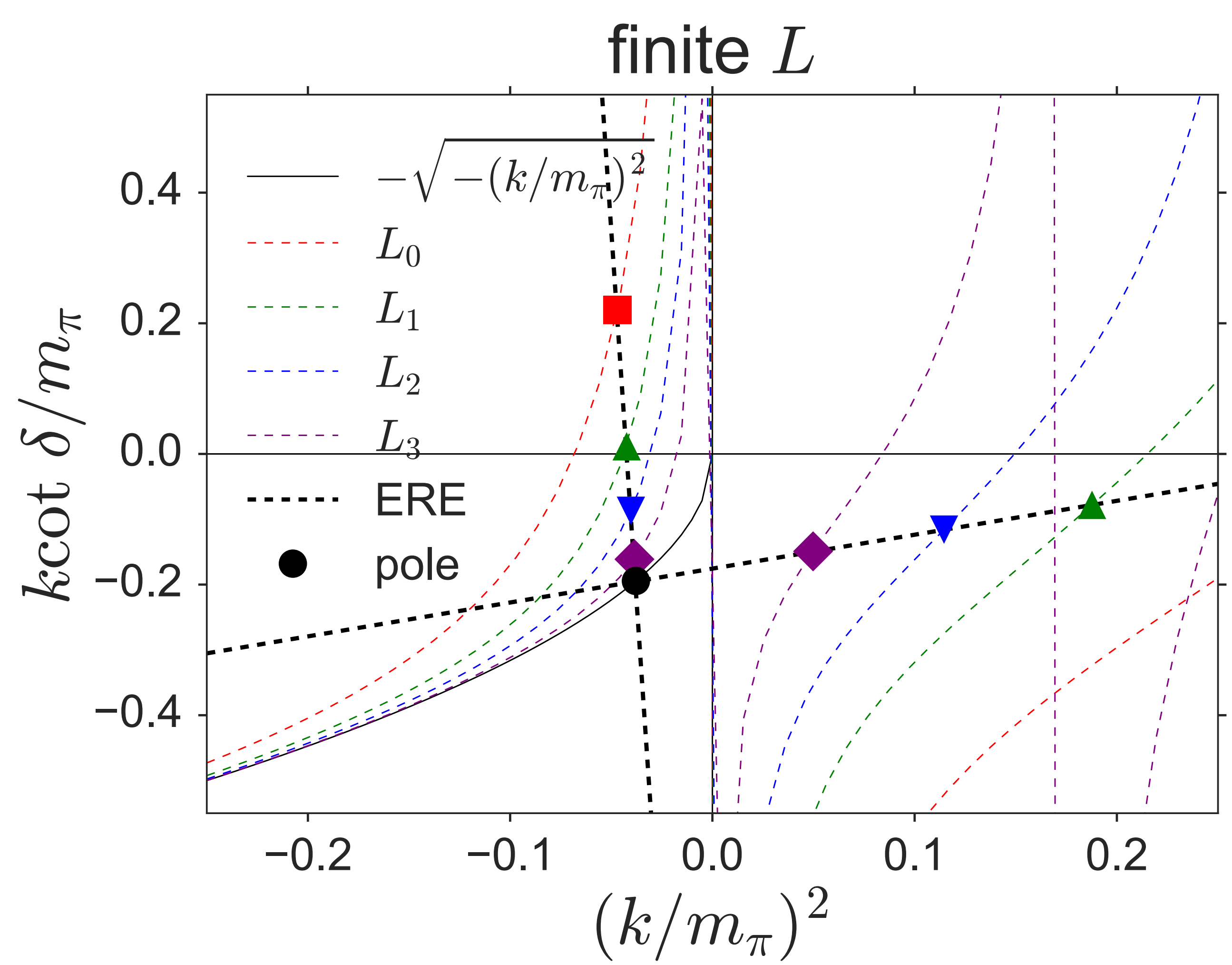
$$1/a \simeq -\infty, \quad r \simeq -\infty$$



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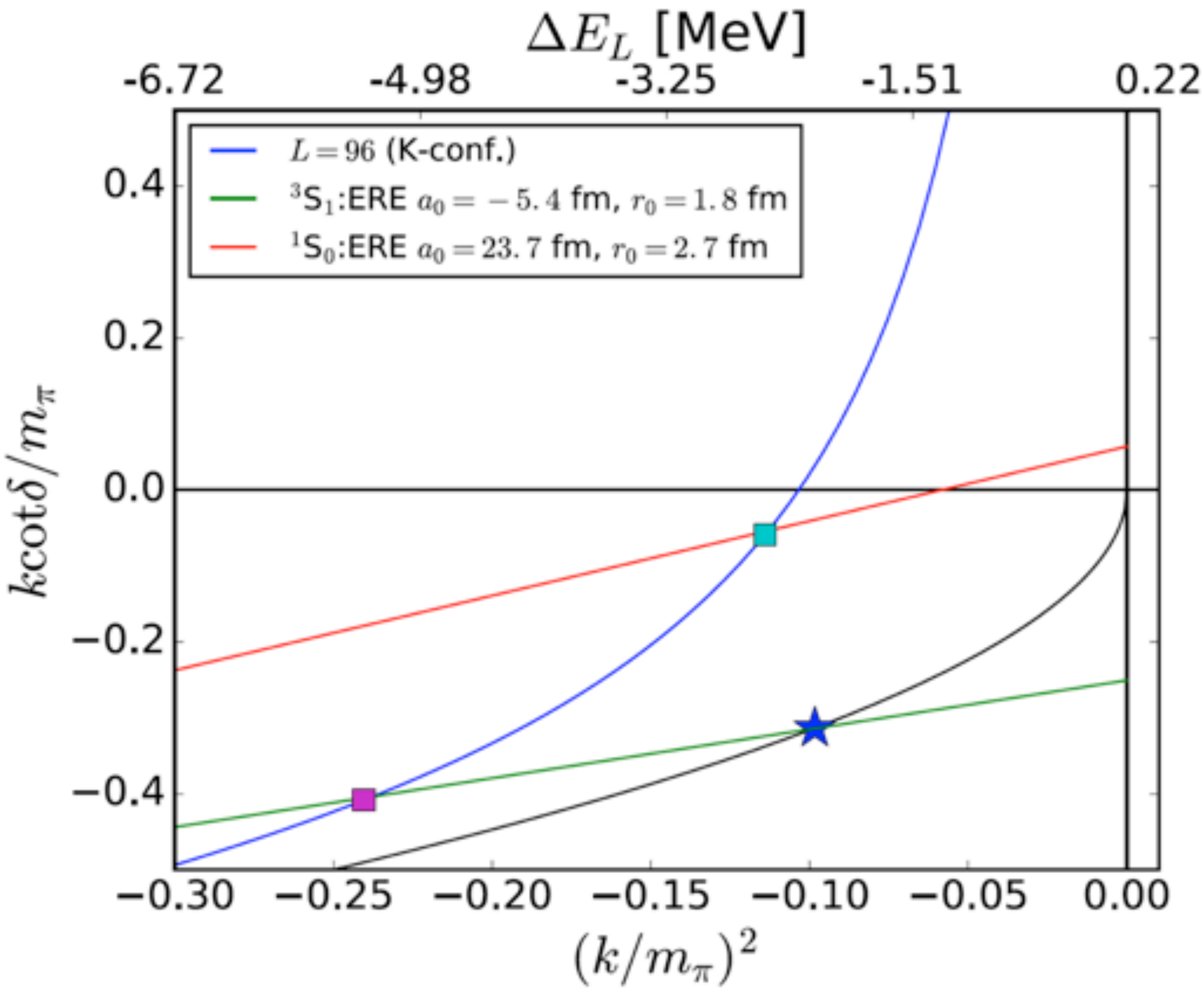


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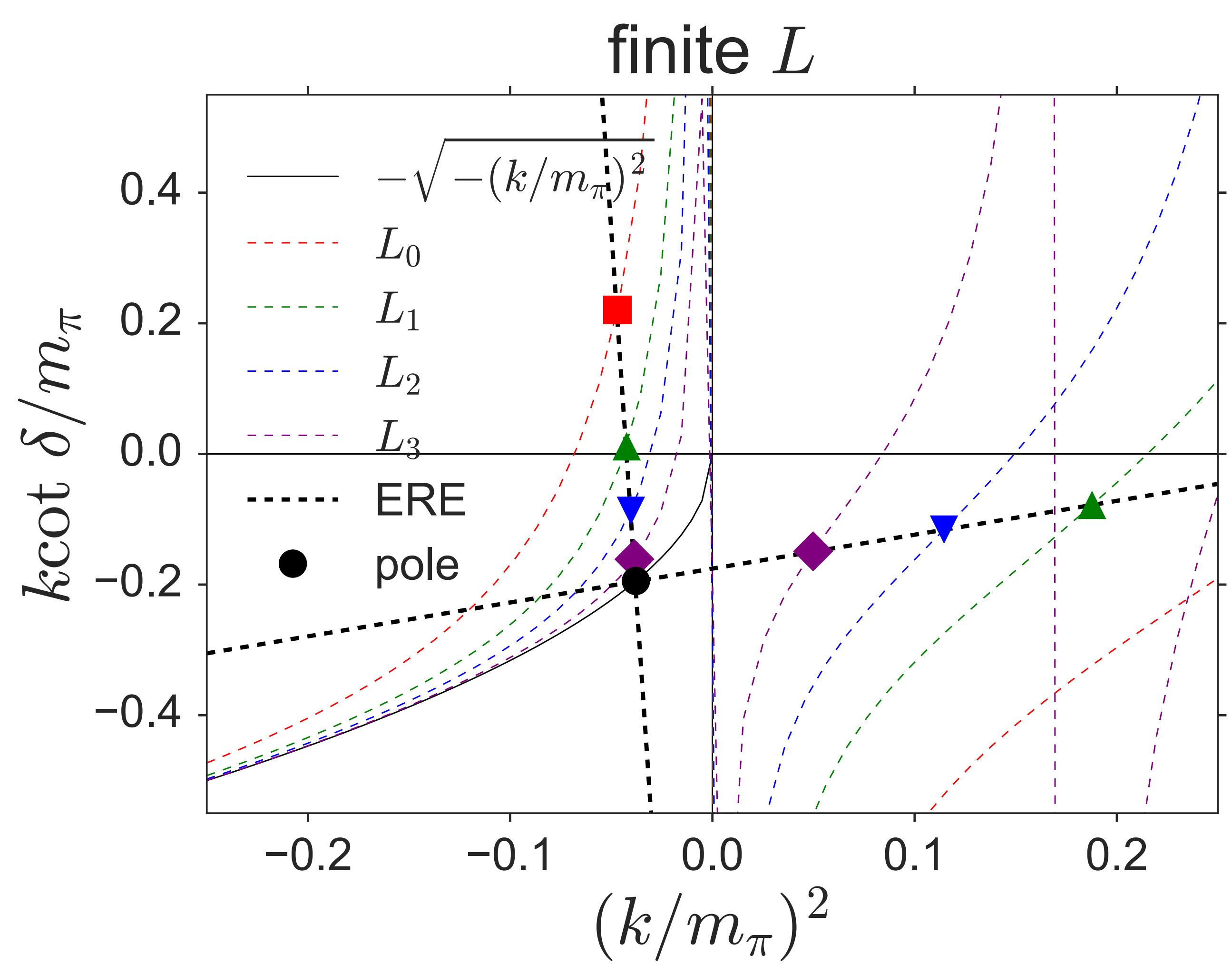
**"Sanity Check"**

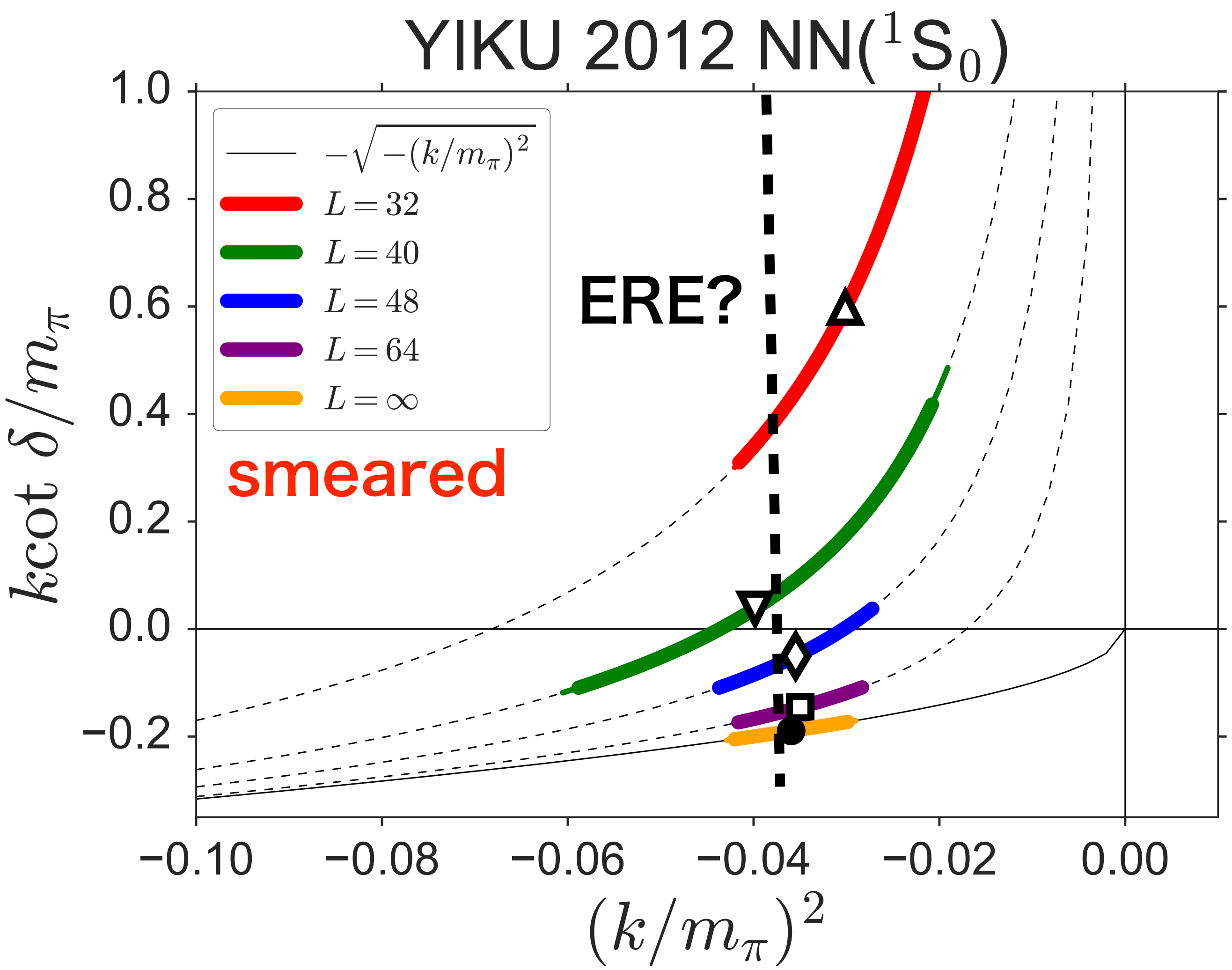


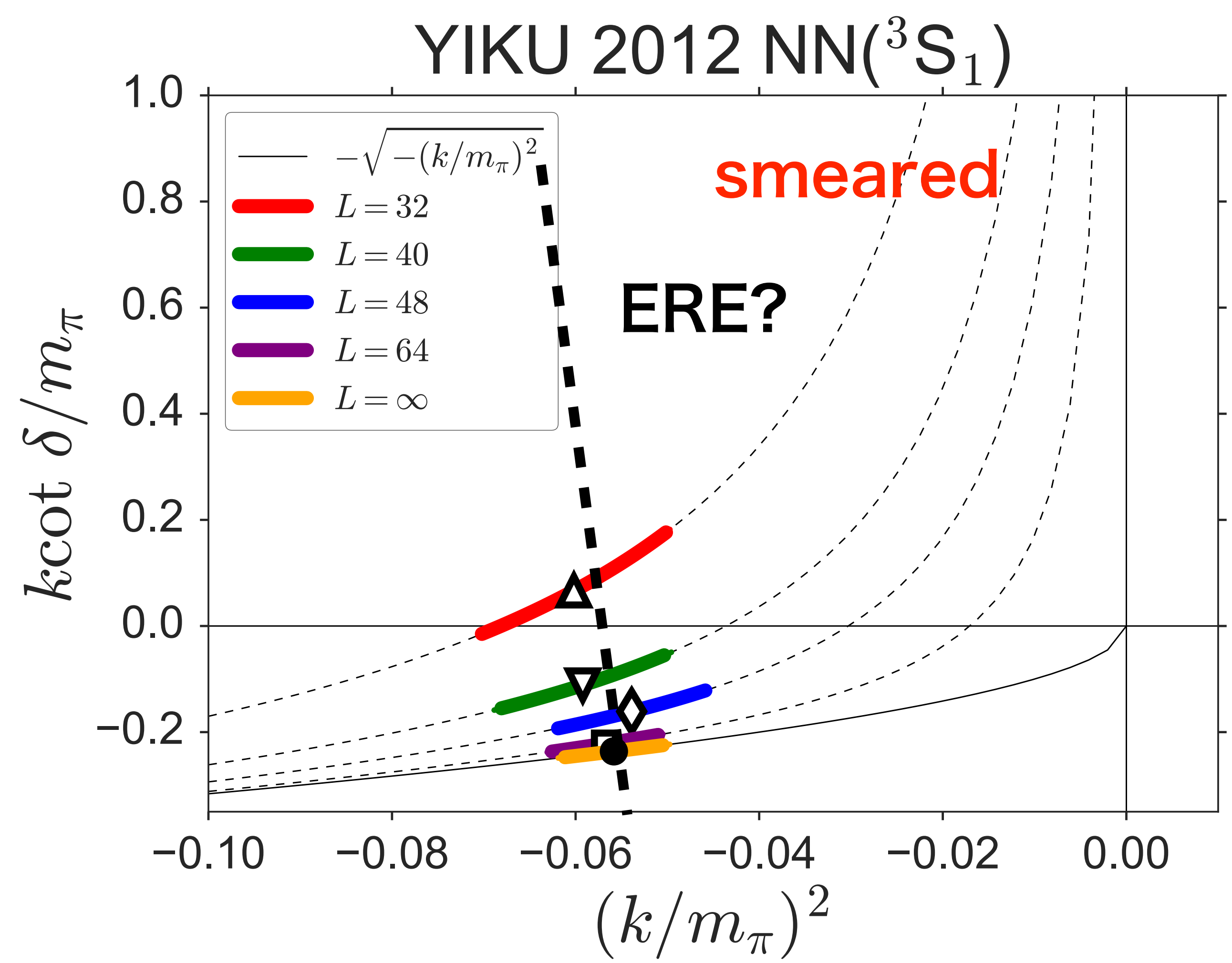
# YIKU2012 Yamazaki et al. PRD86(2012)074514

$m_\pi = 0.51$ GeV, $L = 2.9 - 5.8$ fm

$\Delta E_{NN}(^1\mathrm{S}_0) = -7.4(1.3)(0.6)$ MeV

$\Delta E_{NN}(^3\mathrm{S}_1) = -11.5(1.1)(0.6)$ MeV




singular behaviors

$\Delta E$ is almost independent on $L$, while it is shallow bound state.

“Not Sanity”

# IV. Conclusion

The direct method gives no reliable result for two(or more)-baryon systems so far, since systematic errors due to contaminations from excited (elastic) states are not under control.

**Do not be misled.**

## Check Table for NN

| | single baryon | double baryon | | | | |
|---|---|---|---|---|---|---|
| | plateau check | mirage plateau | src-dep check | sink-dep check | Effective Range expansion check | **Overall Verdict** |
| **YKU 2011** | ○ | × | △ | **Not checked** | × | **False** |
| **YIKU 2012** | ○ | × | × | × | × | **False** |
| **YIKU 2015** | ○ | × | **Not checked** | **Not checked** | × | **False** |
| **NPL 2012** | ○ | × | **Not checked** | **Not checked** | × | **False** |
| **NPL 2013** | ○ | × | **Not checked** | **Not checked** | △ | **False** |
| **NPL 2015** | △ | × | **Not checked** | **Not checked** | × | **False** |

3 or more baryons are more difficult, and therefore less reliable than this.

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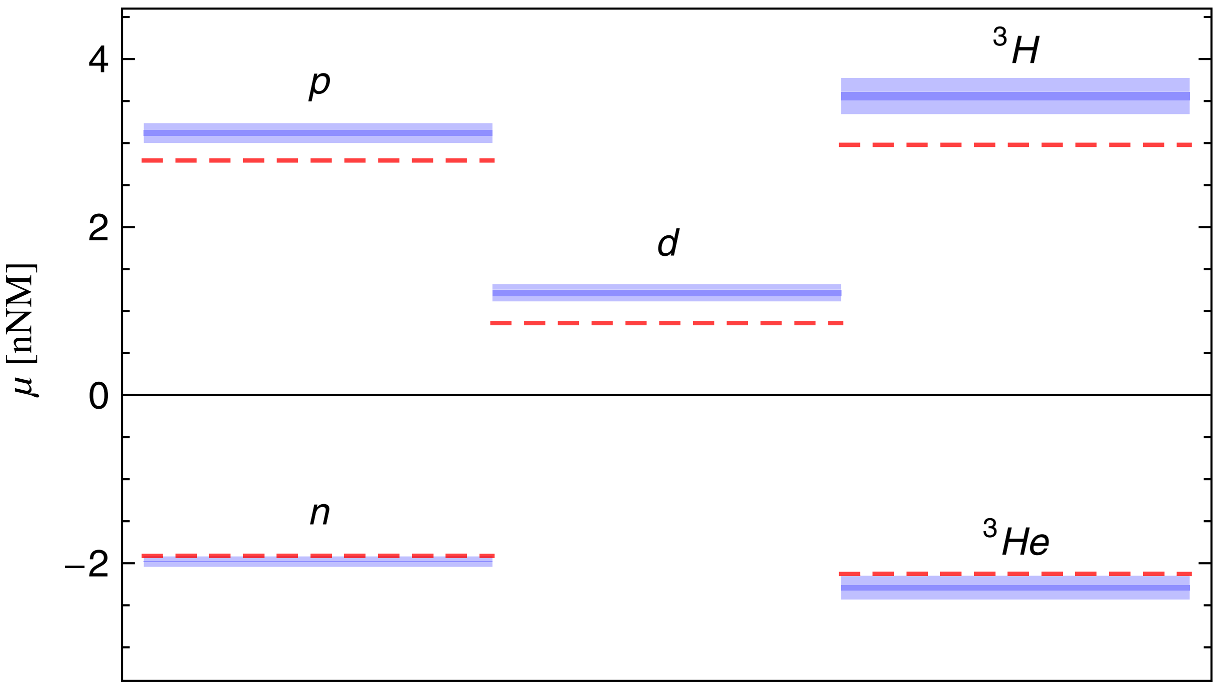
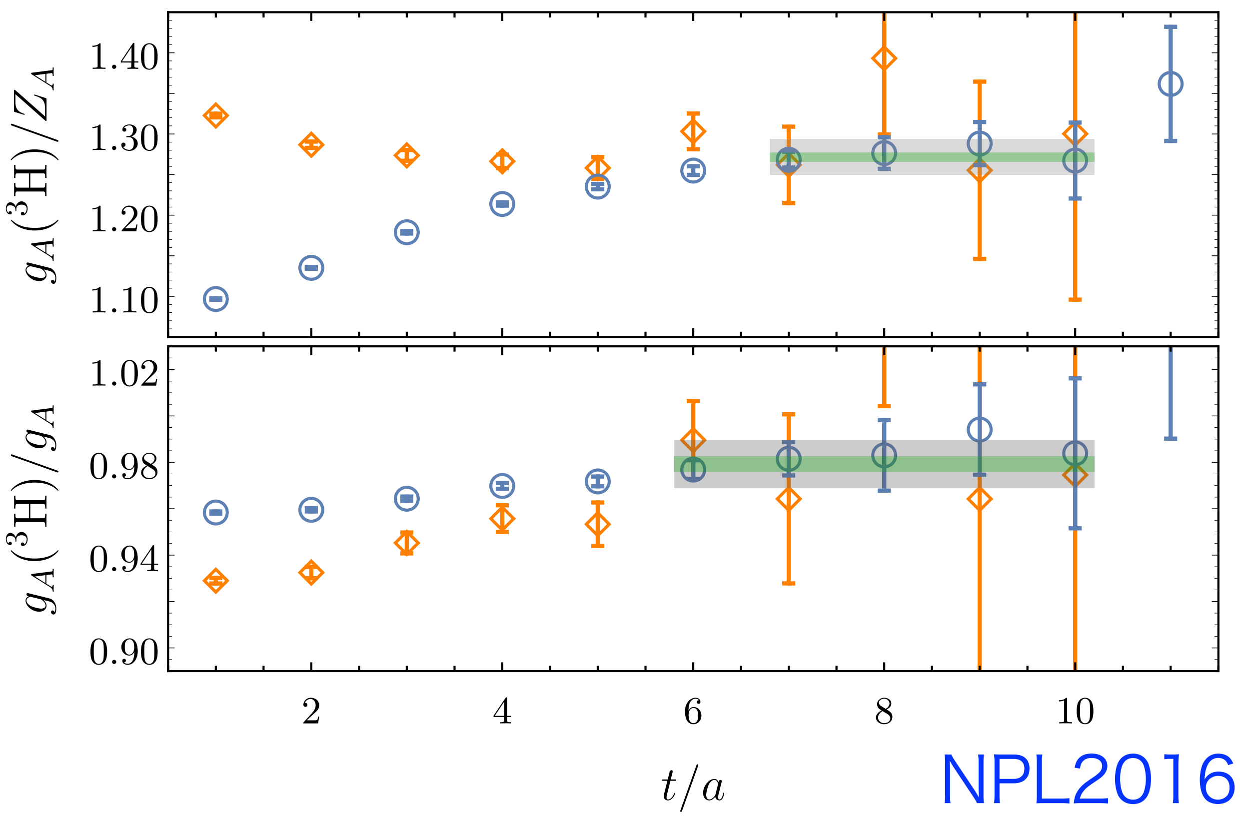
Magnetic moment of nuclei




NPL2014

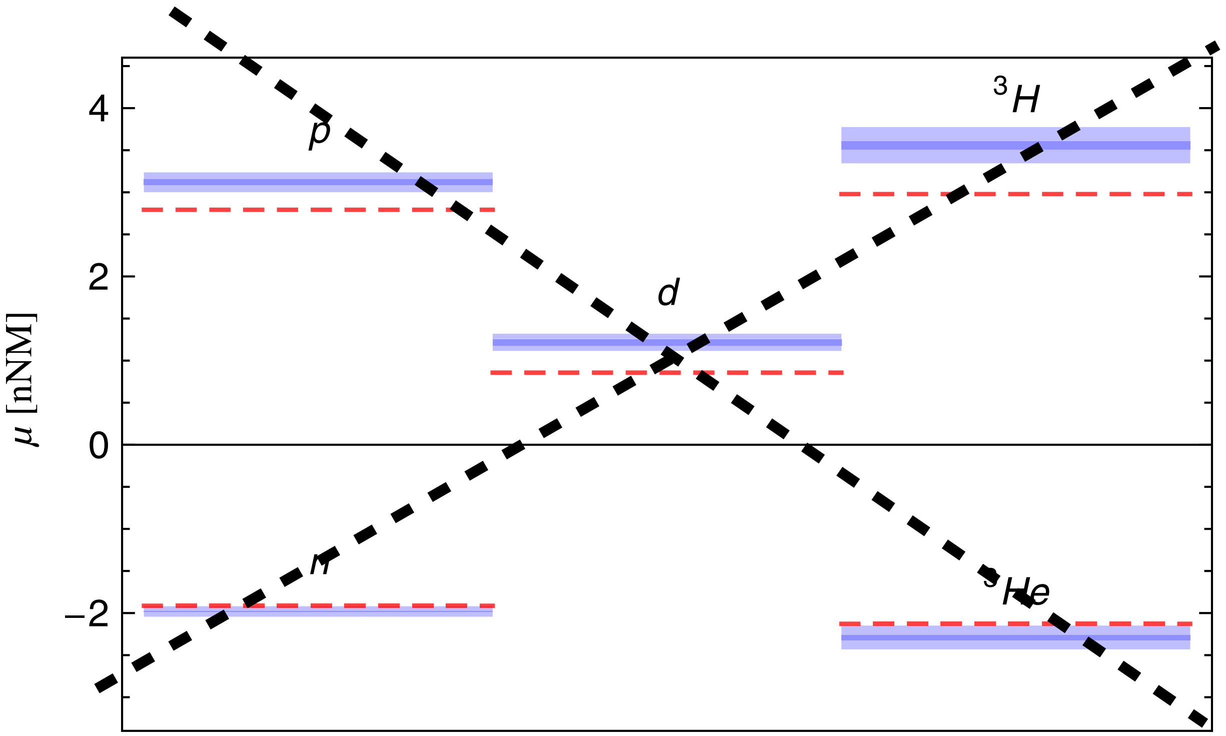
Axial matrix element of nuclei




NPL2016

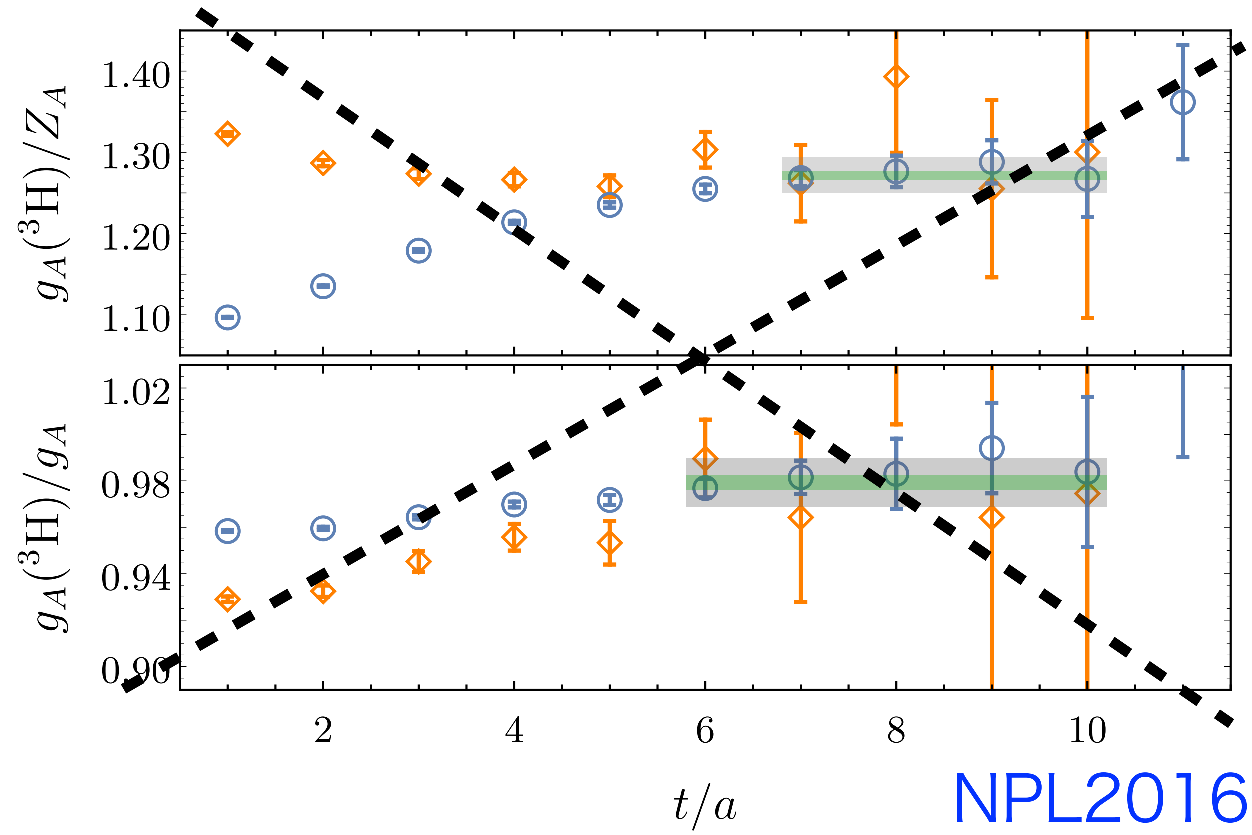
**These results also suffer the same difficulty, and are probably wrong.**

## Magnetic moment of nuclei




NPL2014

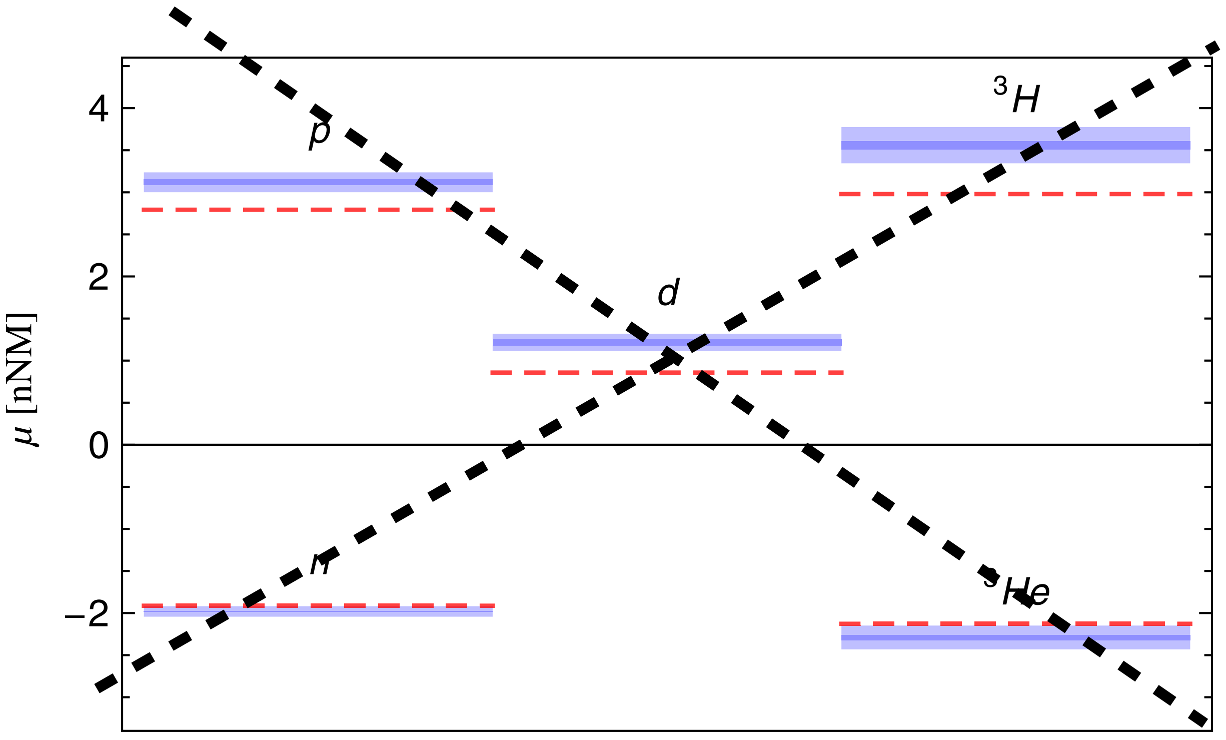
## Axial matrix element of nuclei




NPL2016

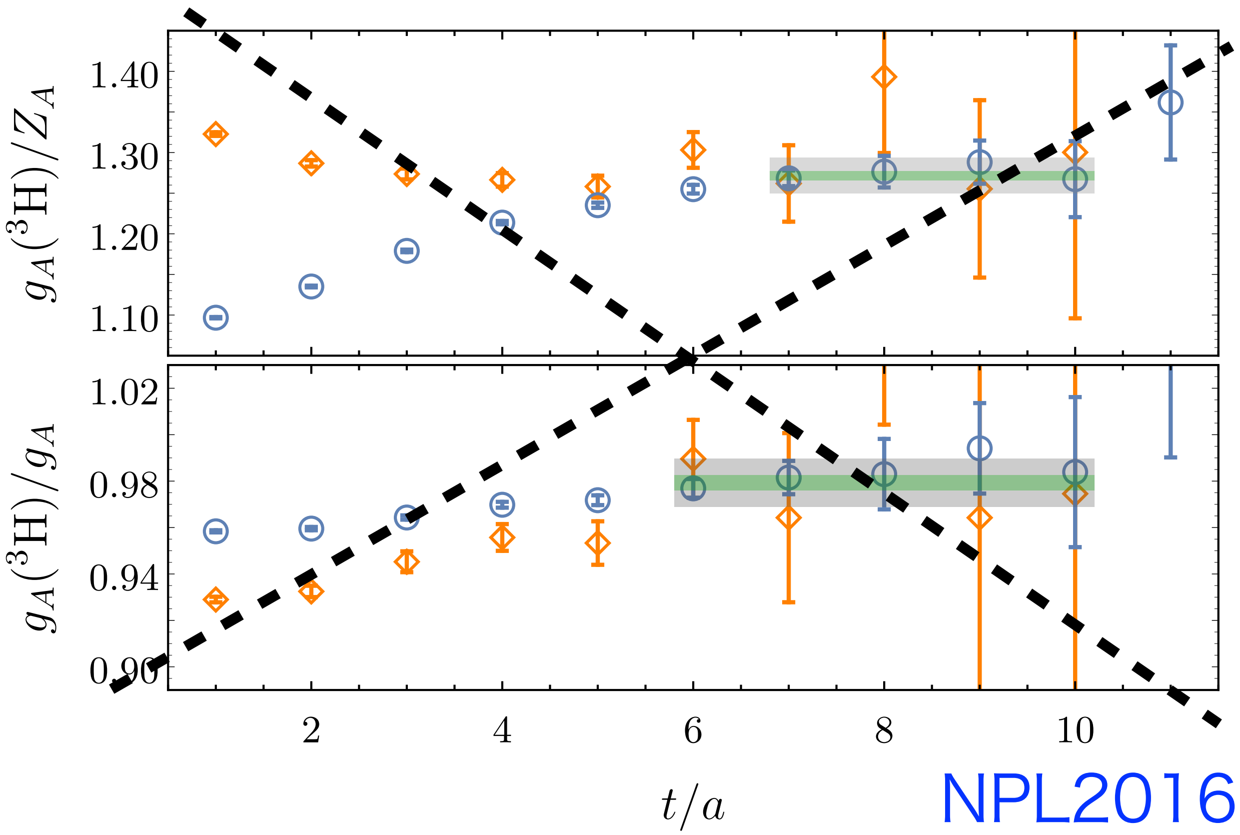
**These results also suffer the same difficulty, and are probably wrong.**

Magnetic moment of nuclei




NPL2014

Axial matrix element of nuclei

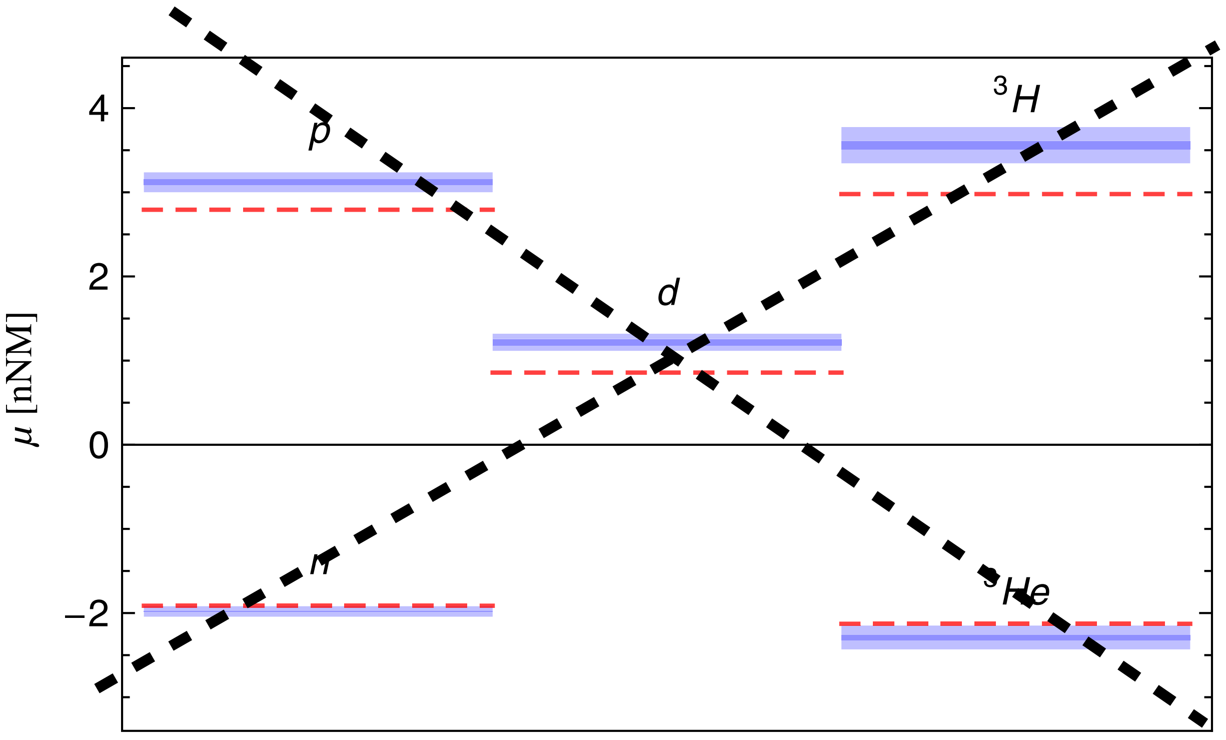



NPL2016

**These results also suffer the same difficulty, and are probably wrong.**

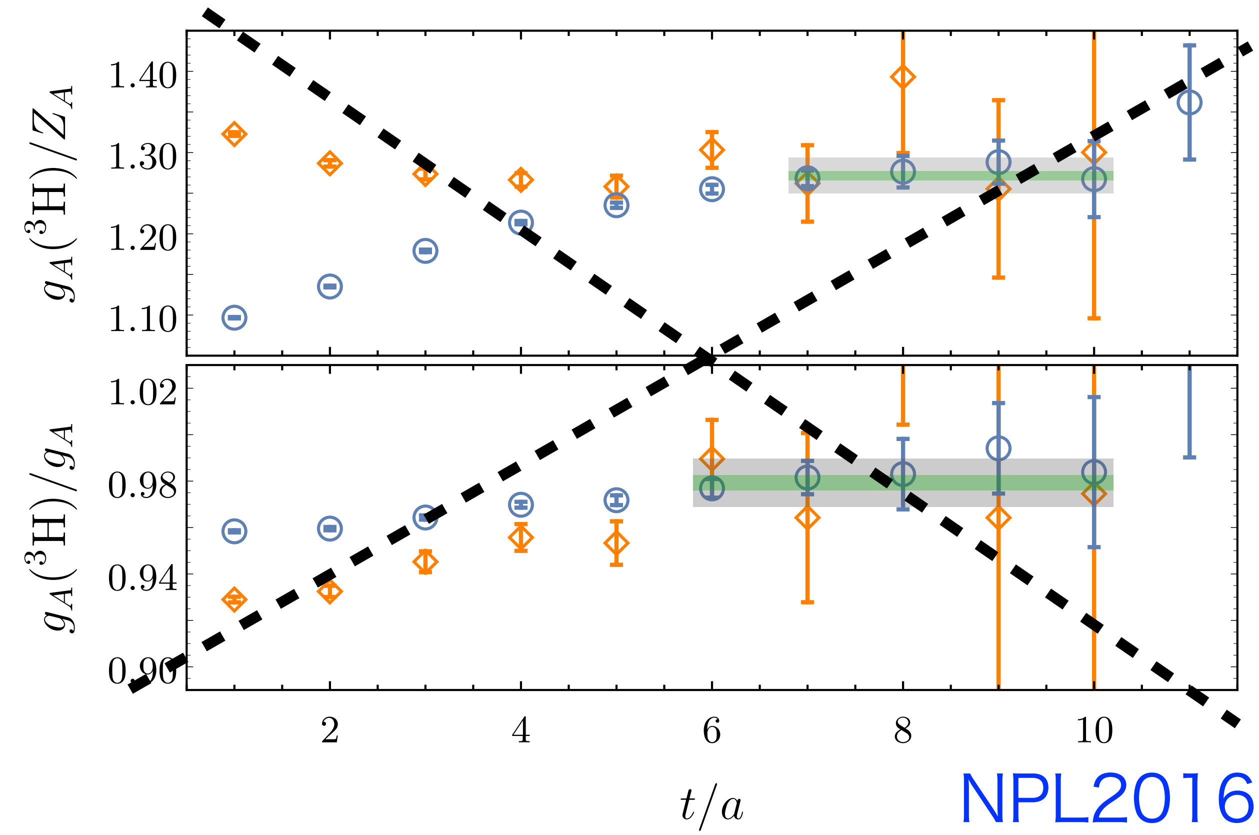
An Inconvenient Truth

Magnetic moment of nuclei




NPL2014

Axial matrix element of nuclei




NPL2016

**These results also suffer the same difficulty, and are probably wrong.**

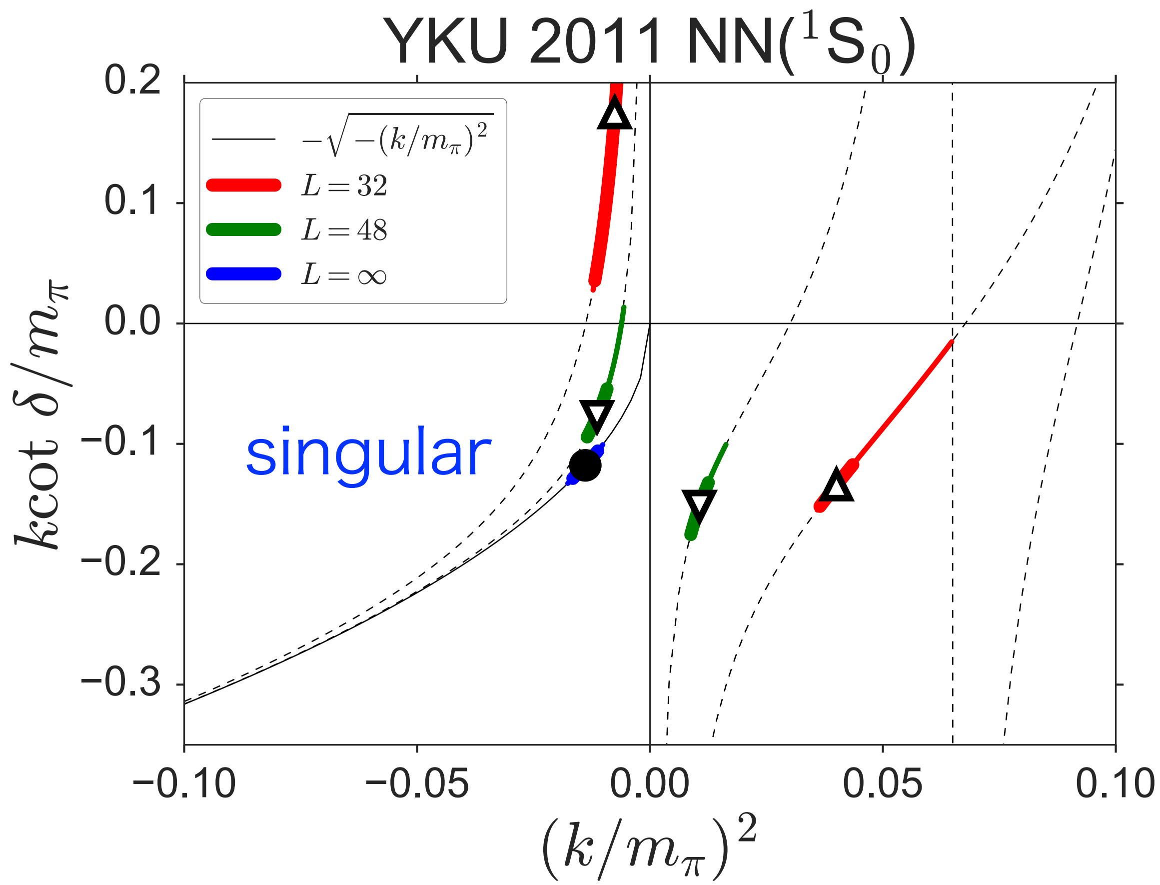
An Inconvenient Truth

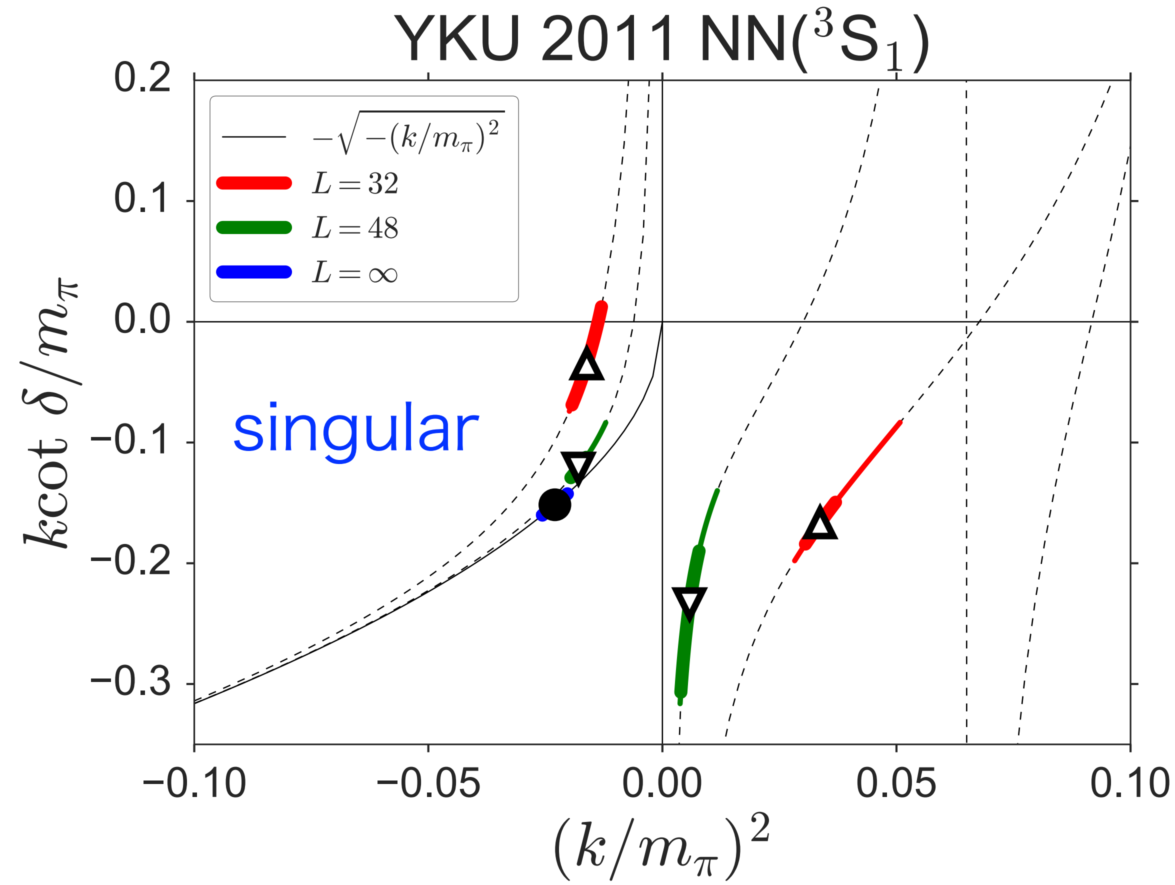
**The variational method using several operators are mandatory to overcome the difficulty.**

# Back-up slides

## Yamazaki et al. 2011 : PRD84(2011)054506

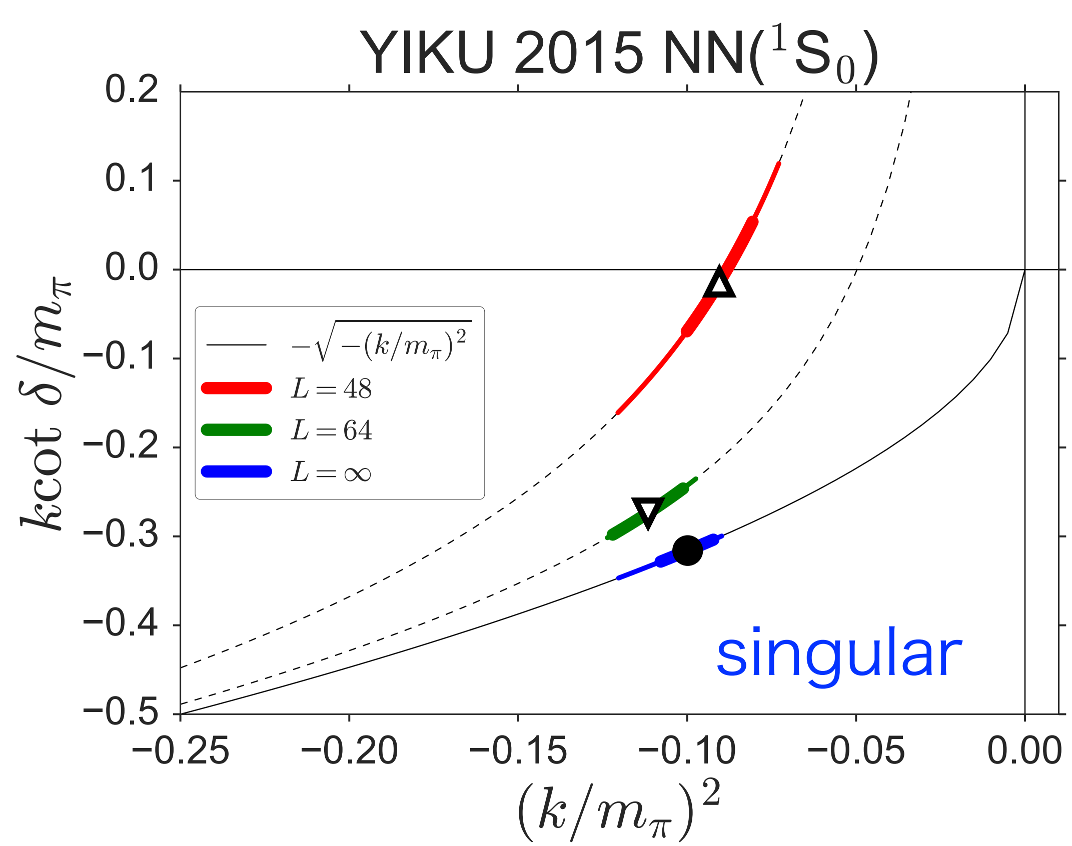
Quenched, $a \simeq 0.128$ fm, $m_\pi \simeq 800$ MeV

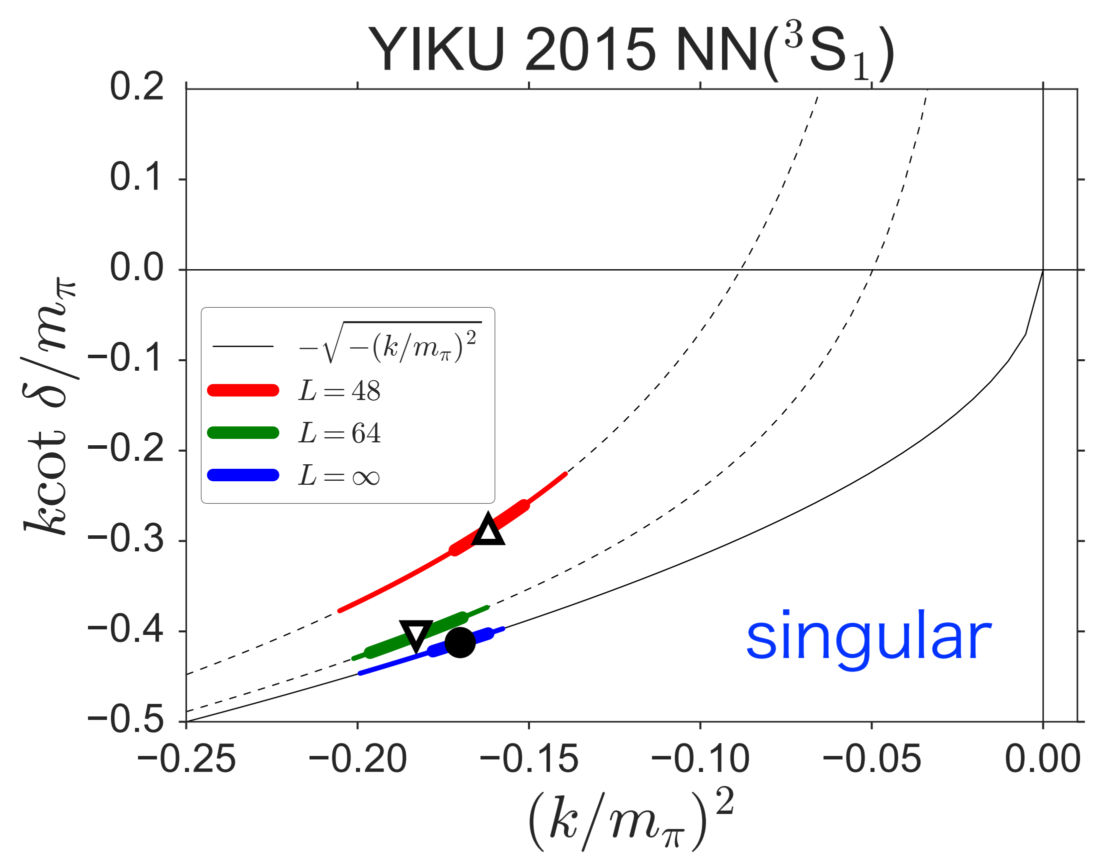


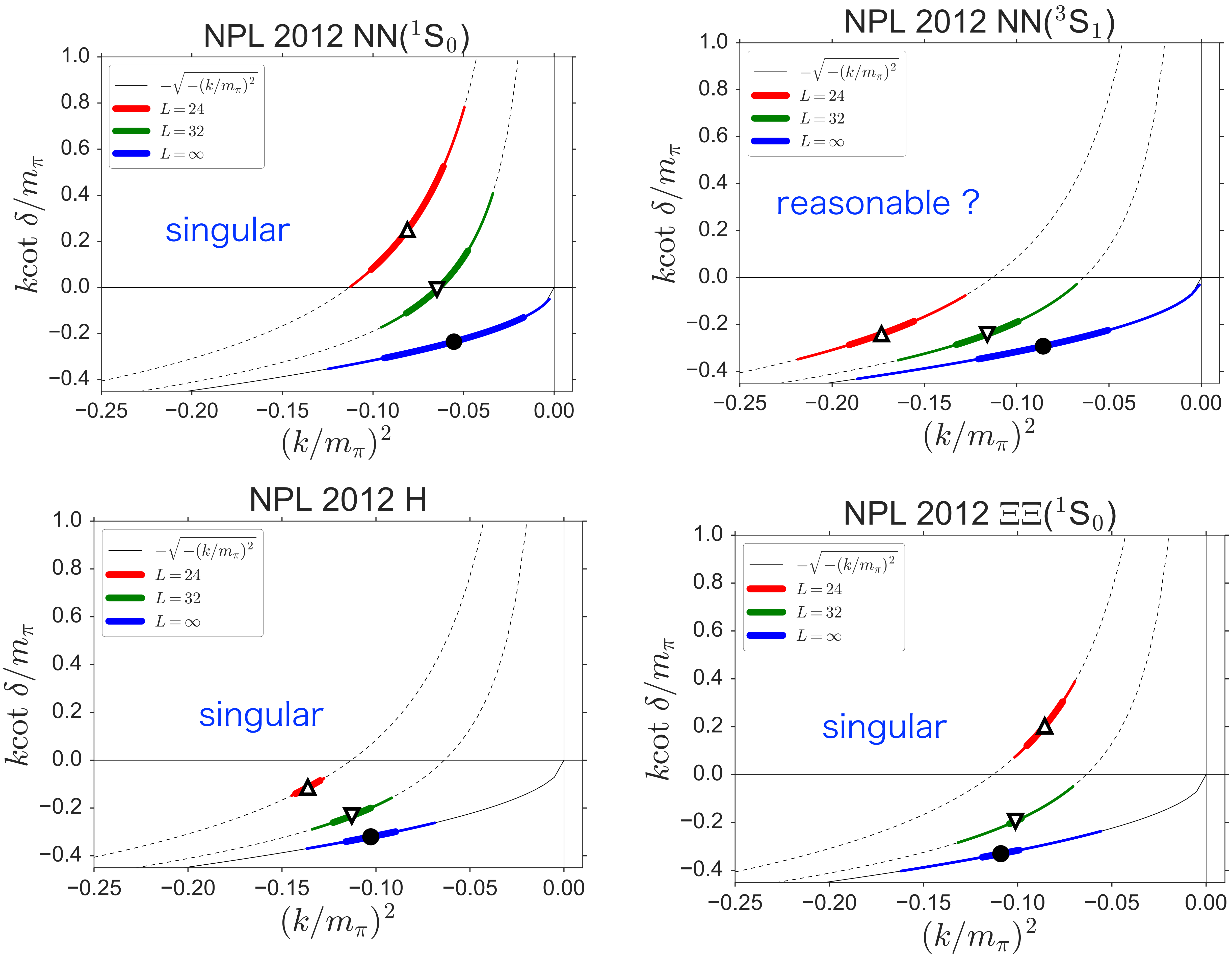

## Yamazaki et al. 2015 : PRD92(2015)014501

$N_f = 2+1$, $a \simeq 0.09$ fm, $m_\pi \simeq 300$ MeV

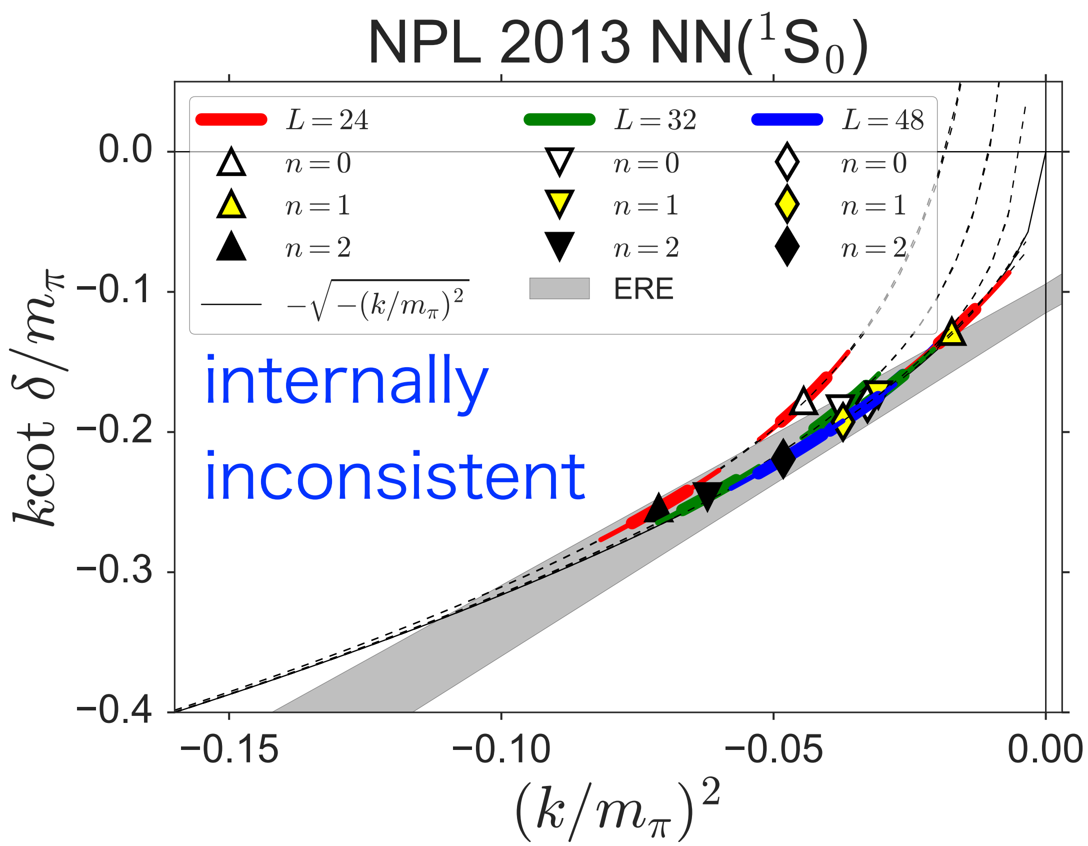


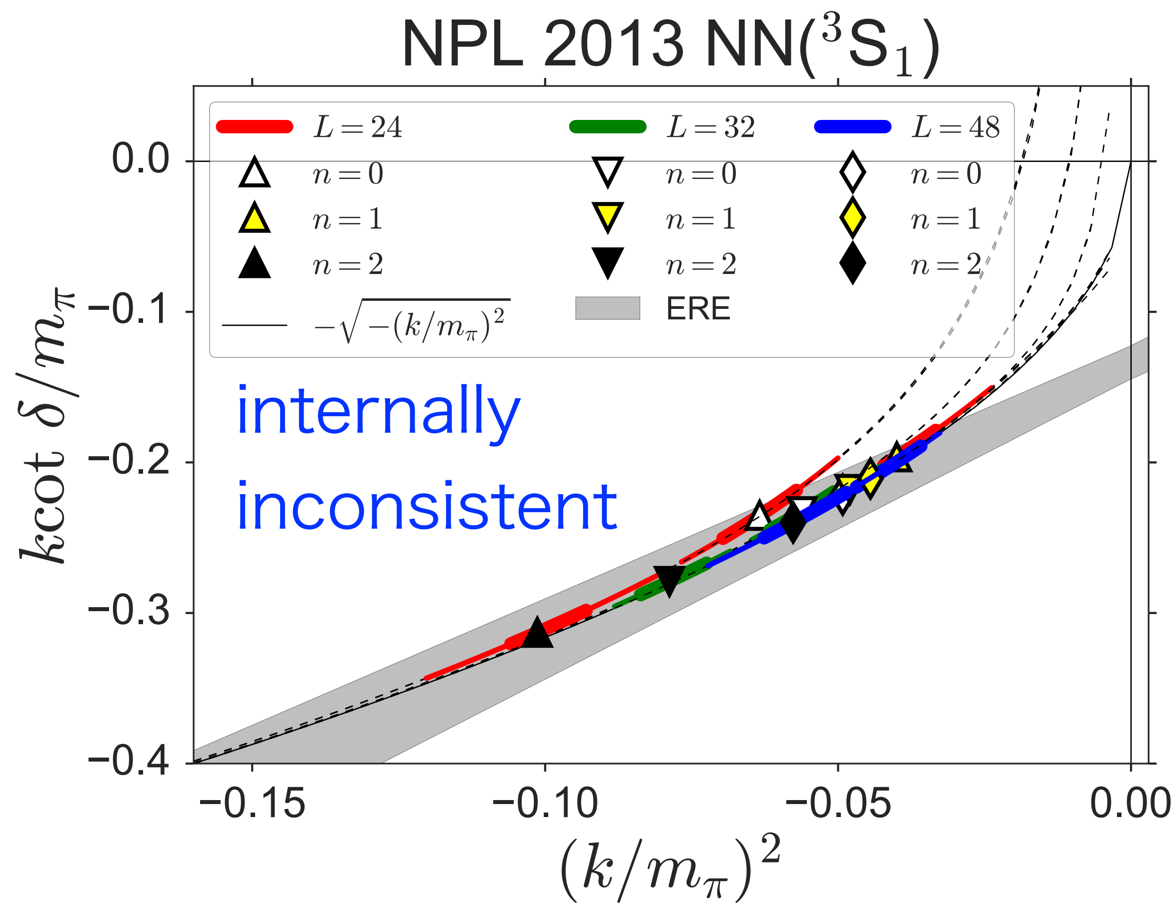



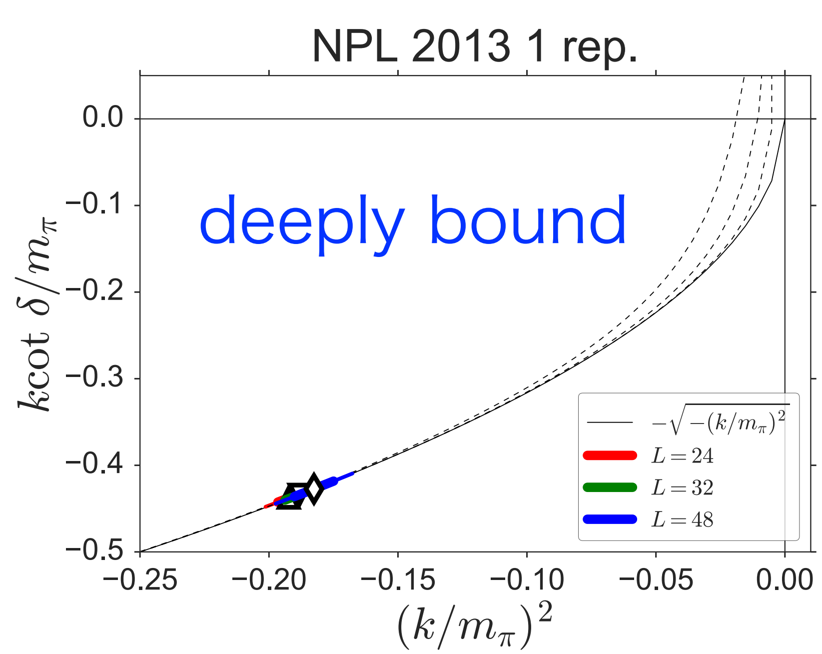
NPL 2012 : PRD85(2012)054511 $N_f = 2+1,\ a_s \simeq 0.123$ fm, $a_s/a_t \simeq 3.5,\ m_\pi \simeq 390$ MeV

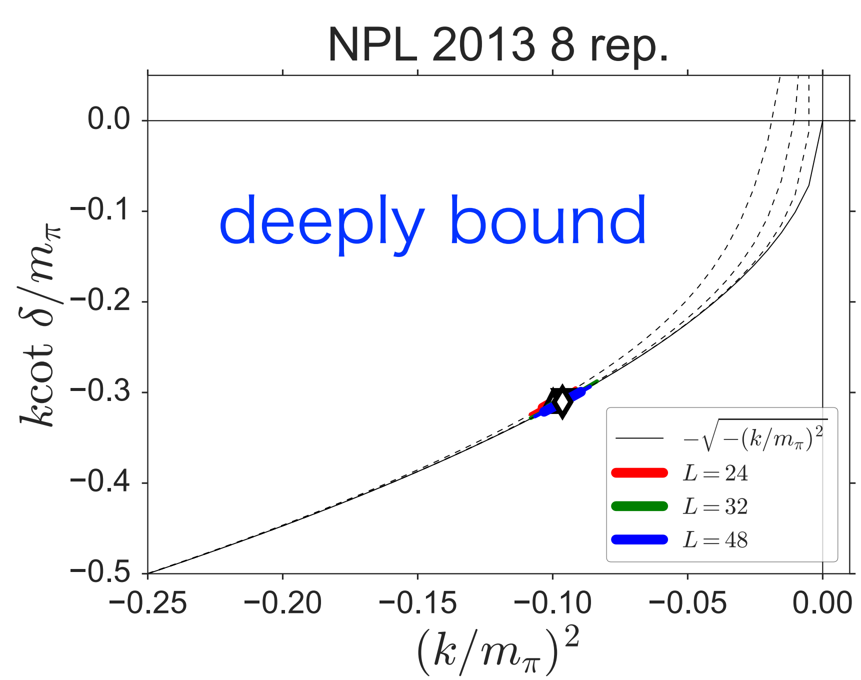


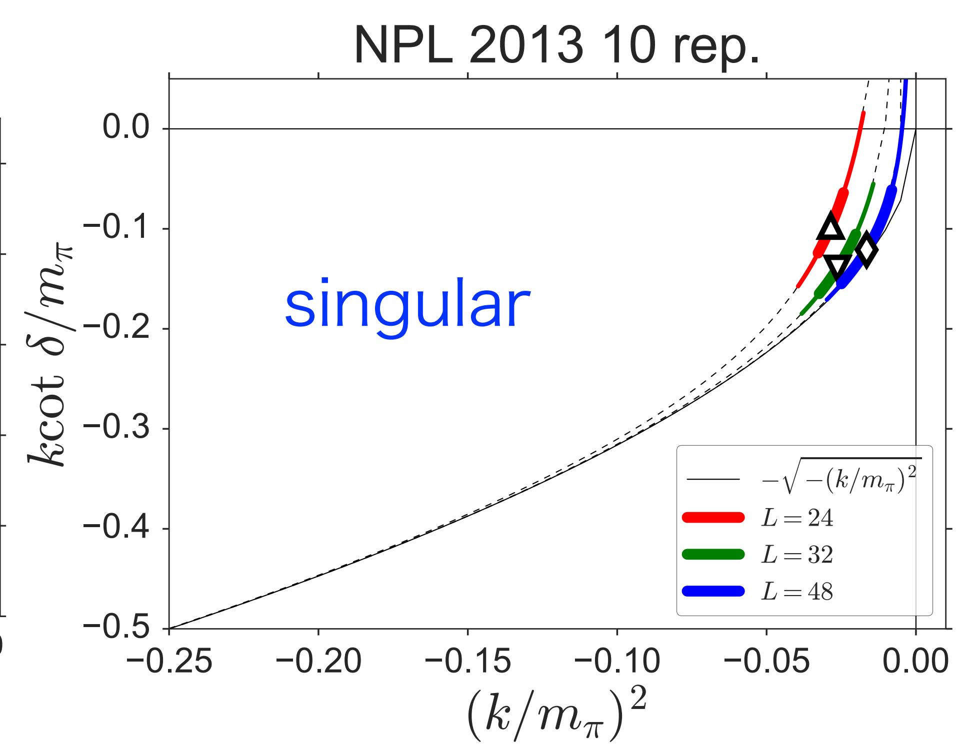
**NPL 2012 : PRC88(2013)024003** $N_f = 3$ (SU(3) limit), $a \simeq 0.145$ fm, $m_{\rm PS} \simeq 800$ MeV



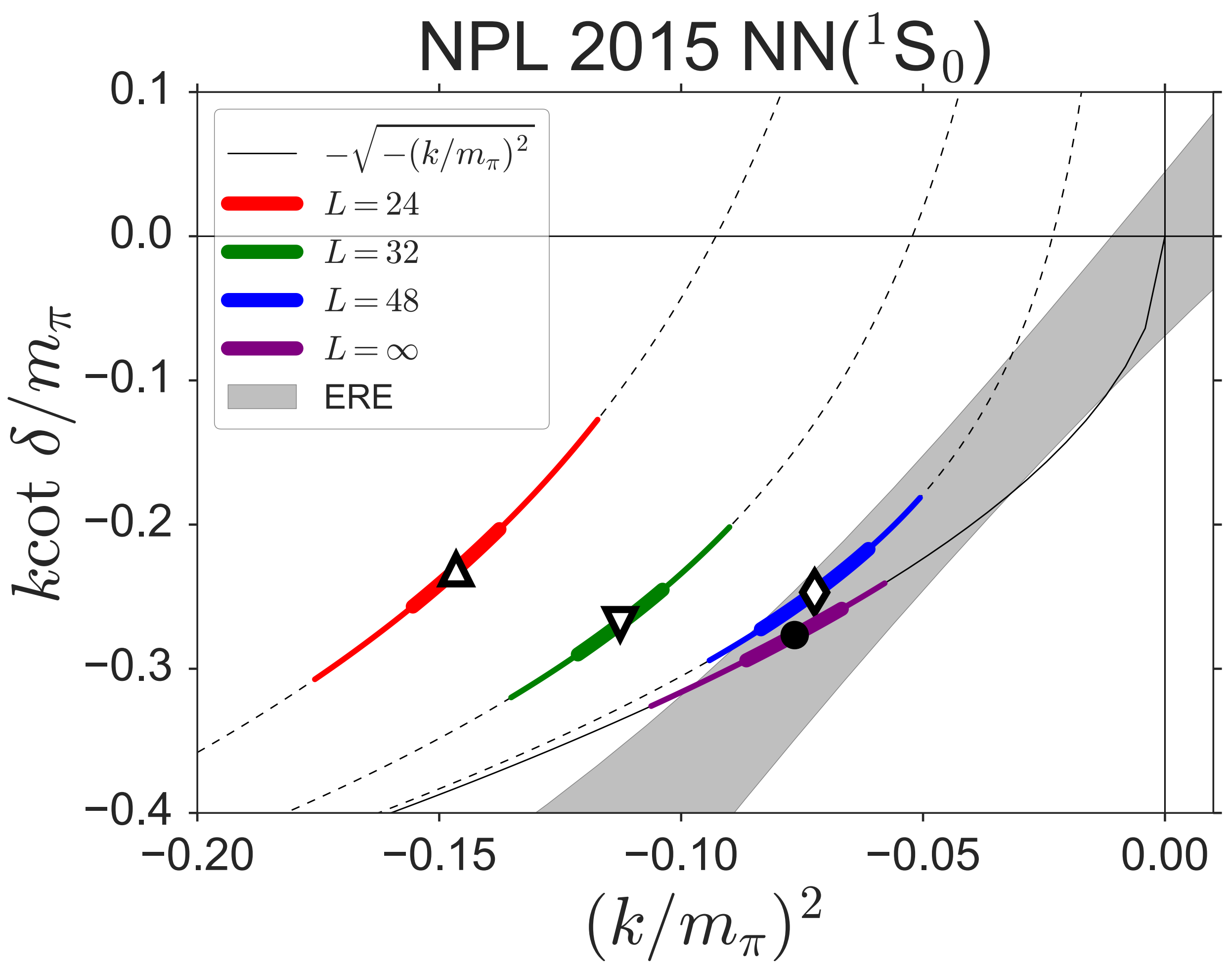



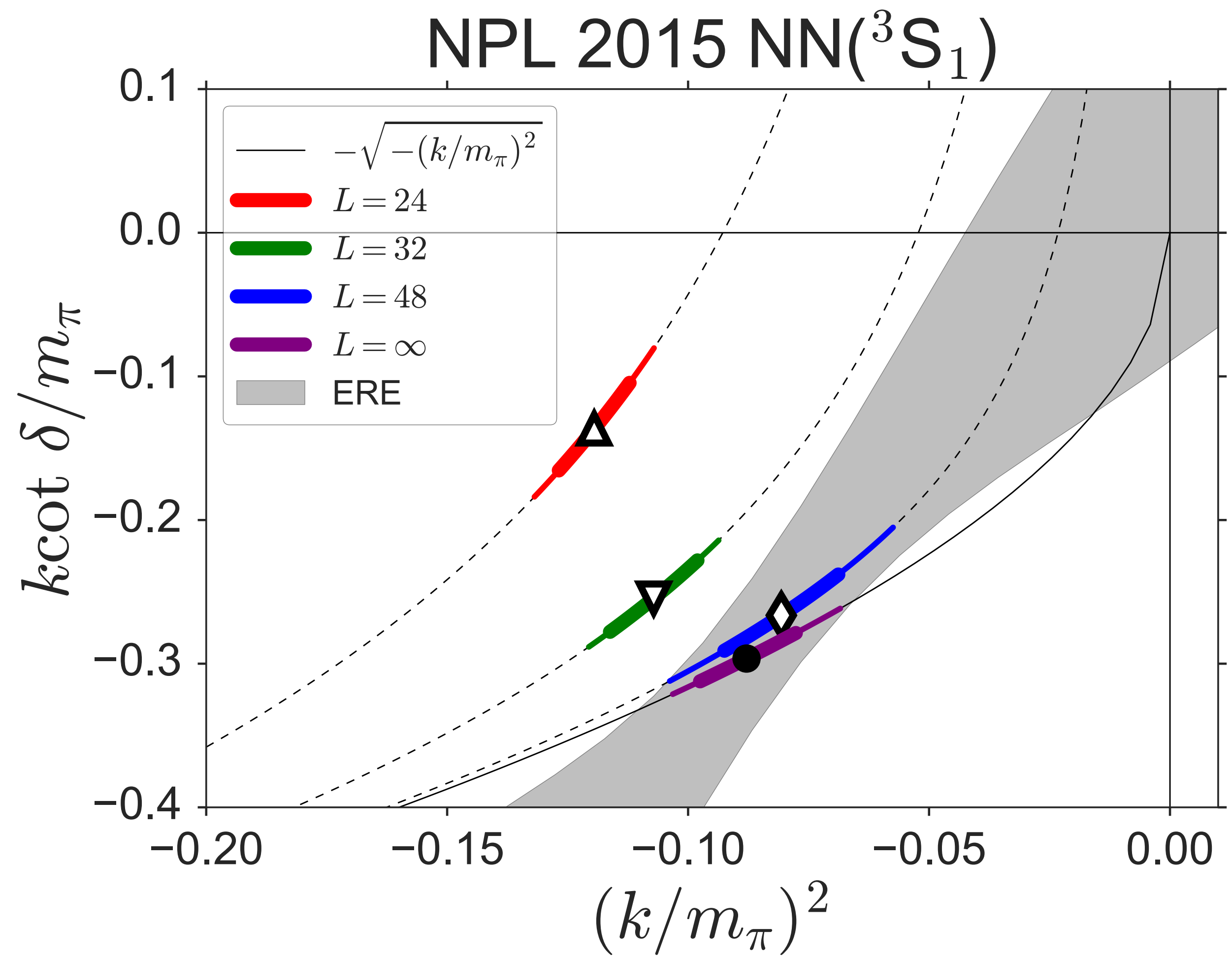



**NPL 2015 : PRD92(2015)114512**

$N_f = 2+1,\ a \simeq 0.1167\ \text{fm},\ m_\pi \simeq 450\ \text{MeV}$




internally inconsistent




singular and internally inconsistent