Calculation of nuclear transition matrix elements of neutrinoless double-beta decay

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What is the neutrino mass?

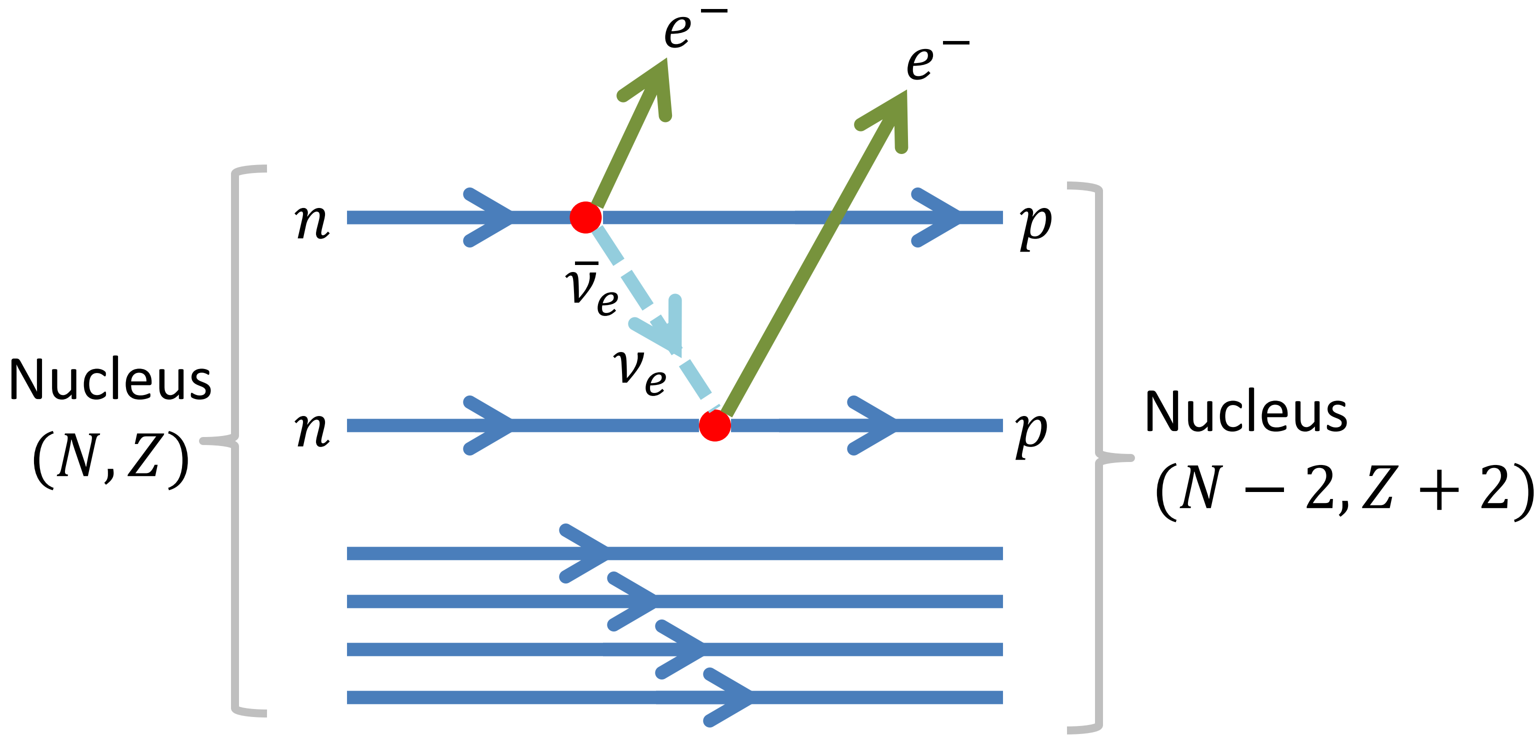
The neutrino is massless in the standard theory.

Other physical issues of neutrino

- Dirac or Majorana particle?
- Breaking of the lepton number conservation?
- How is the right-handed neutrino experimentally observed? Does that neutrino have an interaction?

One of the intensively studied few methods to determine the neutrino mass:

Application of neutrinoless double-beta ($0\nu\beta\beta$) decay of nuclei



Neutrino assumed to be Majorana particle.

The principle to determine the effective neutrino mass using $\boldsymbol{0\nu\beta\beta}$ decay

$$1/T_{0\nu}(0^+ \to 0^+) = \left|M^{(0\nu)}\right|^2 G_{01} \left(\frac{\langle m_\nu \rangle}{m_e}\right)^2$$

Exp. $\longrightarrow$ $T_{0\nu}$: half-life of the transition

Cal. $\longrightarrow$

- $M^{(0\nu)}$: transition matrix element of nucleus (nuclear matrix element)
- G_{01}: transition matrix element of electron (phase space factor)

$\langle m_\nu \rangle$: effective neutrino mass

m_e : electron mass

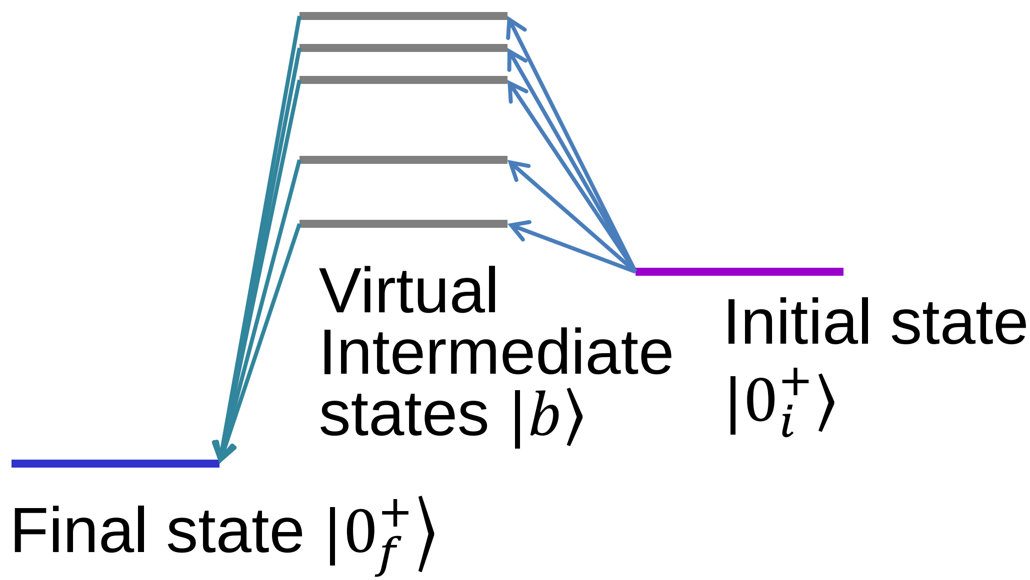
Nuclear matrix element

$$M^{(0\nu)} = \sum_{b} \sum_{pp'} \sum_{nn'} \langle pp' | V(r_{12}, E_b) | nn' \rangle \langle 0_f^+ | c_{p'}^\dagger c_{n'} | b \rangle \langle b | c_p^\dagger c_n | 0_i^+ \rangle$$

$$V(r_{12}, E_b) = h_+(r_{12}, E_b) \left\{ -\boldsymbol{\sigma}(1) \cdot \boldsymbol{\sigma}(2) + \frac{g_V^2}{g_A^2} \right\} \tau^+(1) \tau^+(2)$$

Fig. 3.2

M. Doi et al. Prog. Theor. Phys. Suppl. No. 83 (1985) 1



Path of $0\nu\beta\beta$ decay in the calculation via virtual intermediate states

Status

(Relativistic) quasiparticle random-phase approximation
Shell model
Interacting boson model-2
Projected Hartree-Fock-Bogoliubov
Generator-coordinate method + Particle number and angular momentum projection

Fig. 5.

A. Feassler, arXiv:2103.3648 (2012)

Status

Discrepancy problem of results by methods remains, though many improvements have been made.

Fig. 5.

A. Feassler, arXiv:2103.3648 (2012)

Nuclear matrix element

$$M^{(0\nu)} = \sum_{b} \sum_{pp'nn'} \langle pp' | V(r_{12}, E_b) | nn' \rangle \langle 0_f^+ | c_{p'}^\dagger c_{n'} | b \rangle \langle b | c_p^\dagger c_n | 0_i^+ \rangle$$

$$\cong \sum_{pp'nn'} \langle pp' | V(r_{12}, \bar{E}) | nn' \rangle \left\langle 0_f^+ \middle| c_{p'}^\dagger c_{n'} \sum_{b} | b \rangle \langle b | c_p^\dagger c_n \middle| 0_i^+ \right\rangle$$

: closure approximation,

g

Fig. 3.

M. Horoi et al., PRC 81, 024321 (2010)

$\bar{E}$ dependence of $M^{0\nu}$

Closure approximation is good.

QRPA

Approximation using only two-quasiparticle excitations $a_i^\dagger a_j^\dagger$ and $a_i a_j$ for the elementary mode of excitation

Considering only neutron-neutron and proton-proton quasiparticle pairs : like-particle QRPA

Considering only proton-neutron quasiparticle pairs : proton-neutron QRPA

Application of QRPA to nuclear matrix element

$$M^{(0\nu)} \cong \sum_{pp'nn'} \langle pp'|V(\bar{E})|nn'\rangle \left\langle 0_f^+ \middle| c_{p'}^\dagger c_{n'} c_p^\dagger c_n \middle| 0_i^+ \right\rangle$$

$$\boxed{\begin{aligned} 1 = |0_i^+\rangle\langle 0_i^+| &+ \sum_{b_i:\mathrm{pnQRPA}} |b_i\rangle\langle b_i| + \sum_{b_{i1}b_{i2}} |b_{i1}b_{i2}\rangle\langle b_{i1}b_{i2}| \\ &+ \sum_{b_{i1}b_{i2}b_{i3}} |b_{i1}b_{i2}b_{i3}\rangle\langle b_{i1}b_{i2}b_{i3}| + \cdots, \end{aligned}}$$

Note $\quad c_p^\dagger c_n \sim O_{bi}^\dagger + O_{bi}, \qquad |b_i\rangle = O_{bi}^\dagger |0_i^+\rangle$

$\langle b_{i1} b_{i2} \cdots \left| c_p^\dagger c_n \right| 0_i^+\rangle = 0$ in QRPA

Application of QRPA to nuclear matrix element

$$M^{(0\nu)} \cong \sum_{b_i b_f} \sum_{pp'nn'} \langle pp'|V(\bar{E})|nn'\rangle \langle 0_f^+|c_{p'}^\dagger c_{n'}\big|b_f\rangle\langle b_f\big|b_i\rangle \langle b_i|c_p^\dagger c_n|0_i^+\rangle$$

: usual equation in the QRPA approach

As long as the closure approximation is used, the application of the QRPA can be justified theoretically.

The QRPA is also used without the closure approximation.

The overlap of the QRPA states

The QRPA ground state is defined to be the vacuum to the QRPA quasiboson :

$$O_b|0_i^+\rangle \cong 0$$

O_b : annihilation operator of QRPA state b

$$|b_i\rangle = O_{bi}^{\dagger}|0_i^+\rangle = O_{bi}^{\dagger}\frac{1}{\mathcal{N}}e^{v}|0_{i\mathrm{HFB}}^+\rangle,$$

$\mathcal{N}$: normalization factor

$v{\sim}a^{\dagger}a^{\dagger}a^{\dagger}a^{\dagger}$: product of quasiparticle creation operators

$\propto$ backward amplitude Y_{ij}

$$O_b^{\dagger} = \sum_{ij}\left(X_{ij}a_i^{\dagger}a_j^{\dagger} - Y_{-i-j}a_{-j}a_{-i}\right), \qquad a_i|0_{i\mathrm{HFB}}^+\rangle = 0.$$

The overlap $\langle b_f|b_i\rangle$ is calculated using that definition of the QRPA ground state.

More about application of QRPA

$$M^{(0\nu)} \cong \sum_{pp'nn'} \langle pp'|V(\bar{E})|nn'\rangle \left\langle 0_f^+ \middle| c_{p'}^\dagger c_{n'} c_p^\dagger c_n \middle| 0_i^+ \right\rangle$$

$$\sum_{b_f:\mathrm{pnQRPA}} |b_f\rangle\langle b_f|$$

$$\sum_{b_i:\mathrm{pnQRPA}} |b_i\rangle\langle b_i|$$

More about application of QRPA

$$M^{(0\nu)} \cong \sum_{pp'nn'} \langle pp' | V(\bar{E}) | nn' \rangle \langle 0_f^+ | \underbrace{c_{p'}^\dagger c_{n'} c_p^\dagger c_n}_{-c_{p'}^\dagger c_p^\dagger c_{n'} c_n} | 0_i^+ \rangle$$

$$\boxed{\sum_{b_f:\text{likeQRPA}} |b_f\rangle\langle b_f|} \quad \boxed{\sum_{b_i:\text{likeQRPA}} |b_i\rangle\langle b_i|}$$

Application of the like-particle QRPA is possible, if the closure approximation is used.

The QRPA ground state

Fig. 1
J.T. Phys. Rev.C
87, 024316 (2013)

$$|0_i^+\rangle = \prod_{K\pi} \frac{1}{\mathcal{N}_{\mathrm{pn}i}^{K\pi}} \exp\left[v_{\mathrm{pn}i}^{(K\pi)}\right] \prod_{K'\pi'} \frac{1}{\mathcal{N}_{\mathrm{like}i}^{K'\pi'}} \exp\left[v_{\mathrm{like}i}^{(K'\pi')}\right] |0_{i\mathrm{HFB}}^+\rangle,$$

: consisting of the product of the operators of the pnQRPA and like-particle QRPA.

Check

$$
\begin{aligned}
&\langle b_i | c_p^\dagger c_n | 0_i^+ \rangle \\
&\quad \cong \overbrace{\prod_{K'\pi'} \frac{1}{\mathcal{N}_{\mathrm{like}i}^{K'\pi'\,2}} \langle 0_{i\mathrm{HFB}}^+ | \exp[v_{\mathrm{like}i}^{(K'\pi')\dagger}] \exp[v_{\mathrm{like}i}^{(K'\pi')}] | 0_{i\mathrm{HFB}}^+ \rangle}^{=1} \\
&\quad \times \prod_{K\pi} \frac{1}{\mathcal{N}_{\mathrm{pn}i}^{K\pi\,2}} \langle 0_{i\mathrm{HFB}}^+ | \exp[v_{\mathrm{pn}i}^{(K\pi)\dagger}]\; c_p^\dagger c_n \exp[v_{\mathrm{pn}i}^{(K\pi)}]\; | 0_{i\mathrm{HFB}}^+ \rangle .
\end{aligned}
$$

$$
\begin{aligned}
\langle b_f | b_i \rangle &\cong \prod_{K'\pi'} \frac{1}{\mathcal{N}_{\mathrm{pn}f}^{K'\pi'} \mathcal{N}_{\mathrm{pn}i}^{K'\pi'}} \frac{1}{\mathcal{N}_{\mathrm{like}f}^{K'\pi'} \mathcal{N}_{\mathrm{like}i}^{K'\pi'}} \\
&\quad \times \langle 0_{f\mathrm{HFB}}^+ | \exp[v_{\mathrm{pn}f}^{(K'\pi')\dagger}] \exp[v_{\mathrm{like}f}^{(K'\pi')\dagger}] O_{\mathrm{pn}f} O_{\mathrm{pn}i}^\dagger \\
&\quad \times \exp[v_{\mathrm{like}i}^{(K'\pi')}] \exp[v_{\mathrm{pn}i}^{(K'\pi')}] | 0_{i\mathrm{HFB}}^+ \rangle .
\end{aligned}
$$

The like-particle QRPA correlations cannot be removed from the overlap of the pnQRPA states of different nuclei.

Calculations that I have done

Test calculations of the overlap of like-particle QRPA states have been performed using ^{26}Mg-^{26}Si with the expansion with respect to $v_{\mathrm{l}i}^{(K\pi)}$ and $v_{\mathrm{l}p}^{(K\pi)}$.

J.T. PRC **86**, 021301(R) (2012); **87**, 024316 (2013)

One of conclusions : that overlap calculation is feasible.
The product g.s. wave function not yet used.

Fig. 6
J.T. Phys. Rev.C
87, 024316 (2013)

The expansion of the unnormalized overlap with respect to *v*

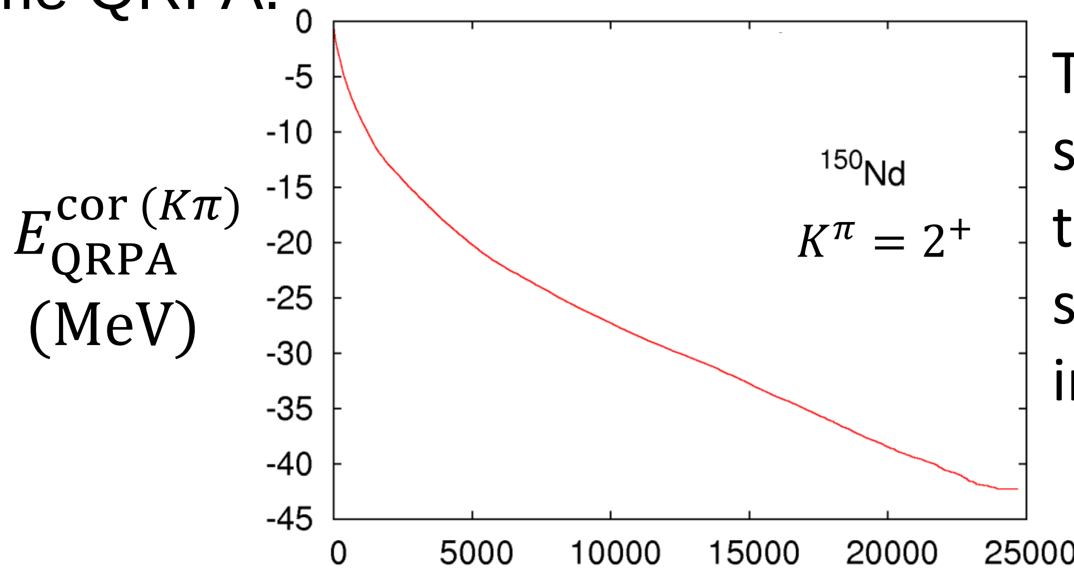
Interaction:
Skyrme SkM* and volume delta pairing

Calculation of $M^{(0\nu)}$ of ^{150}Nd-^{150}Sm via ^{148}Nd, SkM*+volume pairing

$\left|M^{(0\nu)}\right| \sim 0.02$

2.5 – 3.5 (QRPA , Tübingen),
1.8 – 3.5 including various approaches (the previous figure)

0.02 is too small, because the QRPA correlations are too large; it is known that the correlation energy diverges in the Skyrme QRPA.

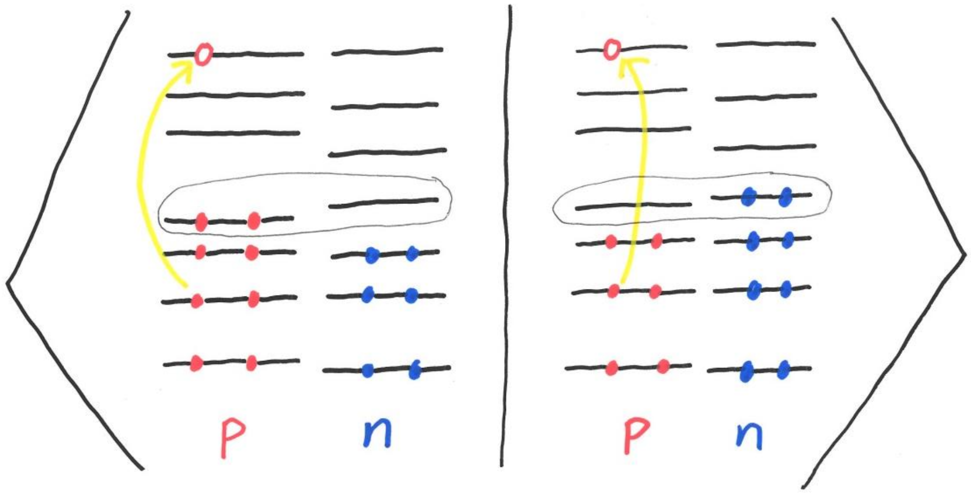


The lower-energy states are used than the other states not included.

QRPA states included in the correlation energy

Un-normalized overlap

$$\langle 0^+_{f\mathrm{HFB}}|\exp[v^{(K'\pi')\dagger}_{\mathrm{pn}f}]\exp[v^{(K'\pi')\dagger}_{\mathrm{like}f}]O_{\mathrm{pn}f}O^\dagger_{\mathrm{pn}i}$$
$$\times \exp[v^{(K'\pi')}_{\mathrm{like}i}]\exp[v^{(K'\pi')}_{\mathrm{pn}i}]|0^+_{i\mathrm{HFB}}\rangle.$$



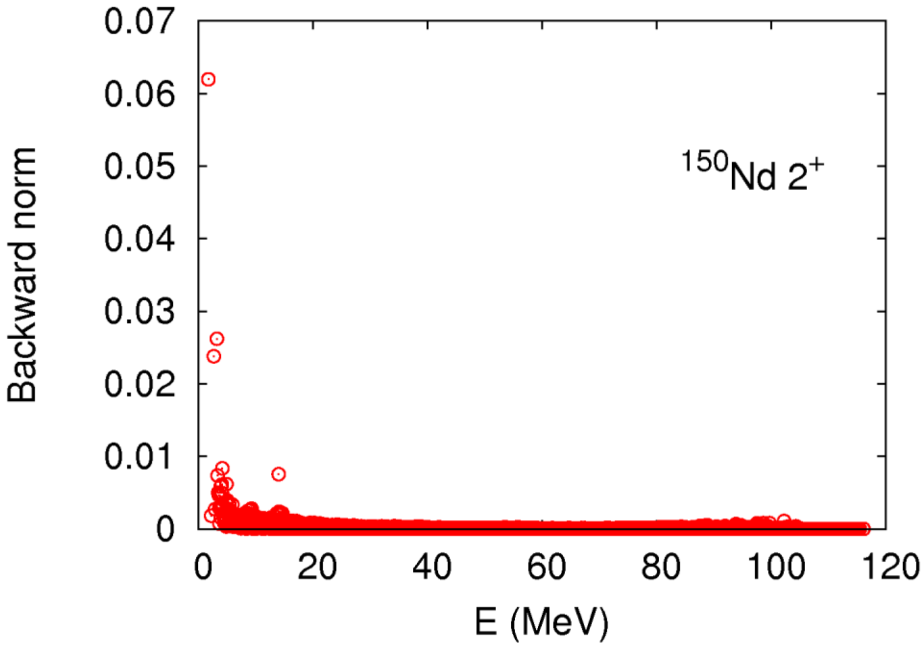
The QRPA correlations have an effect to decrease the nuclear matrix elements through the normalization factor in the overlap.

My prescription

To pick up the QRPA solutions having the largest backward amplitudes so as to get

$$E_{\mathrm{QRPA}}^{\mathrm{cor}} = E_{\mathrm{exp}} - E_{HFB}$$

and use only these states for calculating $v_{\mathrm{l}i}^{(K\pi)}$, $v_{\mathrm{l}f}^{(K\pi)}$, $v_{\mathrm{pn}i}^{(K\pi)}$, and $v_{\mathrm{pn}f}^{(K\pi)}$, i.e. for calculating the QRPA ground states.



10 like-particle QRPA states for

$E_{\mathrm{QRPA}}^{\mathrm{cor}} = -1.72\mathrm{MeV}$ (^{150}Nd) and

18 like-particle QRPA states for

$E_{\mathrm{QRPA}}^{\mathrm{cor}} = -3.44\mathrm{MeV}$ (^{150}Sm)

were picked up.

Revised calculations are in progress.

Many calculations are necessary.

Summary

- Brief history of studies of neutrino and several physical issues were shown.
- How the $0\nu\beta\beta$decay is used for determining the effective neutrino mass was explained.
- Status was shown of the calculations of the nuclear matrix elements

  — the discrepancy problem of the results by methods
- The closure approximation and the application of the QRPA were explained.
- The overlaps based on the definition of the quasiboson vacuum are used; the like-particle QRPA correlations affect the overlap of the pnQRPA states.
- The QRPA correlations have the effect to reduce the nuclear matrix element through the normalization factor in the overlap.