Vlasov-Poisson Simulation and Its Application to Neutrinos in Large-Scale Structure in the Universe

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Achievements (FY2008-FY2013)

- Non-equilibrium ionization and two-temperature structure in merging galaxy clusters

  Akahori, T., Yoshikawa, K. 2008, PASJ, 60, 19

  Akahori, T., Yoshikawa, K. 2010, PASJ, 62, 335

  Akahori, T., Yoshikawa, K. 2012, PASJ, 64, 12

- Vlasov-Poisson simulations for collisionless self-gravitating systems in 6D phase space

  Yoshikawa K., Yoshida, N., Umemura, M. 2013, ApJ, 762, 116

- Acceleration of N-body simulations with the SIMD instruction: Phantom GRAPE

  Tanikawa, W., Yoshikawa, K., Okamoto, T., Nitadori, K. 2012, New A., 17, 82

  Tanikawa, W., Yoshikawa, K., Nitadori, K., Okamoto, T. 2013, New A., 19, 74

- Novel algorithms for radiation transfer simulations: ARGOT & ART schemes

  Okamoto, T. Yoshikawa, K. Umemura, M. 2011, MNRAS, 419, 2855

Vlasov-Poisson Simulation in the 6D Phase Space

- Numerical Methodology
- Test Suites
- Advantage and Disadvantage / Vlasov vs N-body

Vlasov-Poisson Simulations

Vlasov-Poisson equations

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \frac{\partial f}{\partial \vec{x}} - \nabla \phi \cdot \frac{\partial f}{\partial \vec{v}} = 0$$

$$\nabla^2 \phi = 4\pi G \rho = 4\pi G \int f d^3 \vec{v}$$

- Alternative to N-body methods that simulates collisionless self-gravitating systems by integrating Vlasov-Poisson equations.

  Fujiwara (1981, 1983), Nishida et al. (1981, 1984), Hozumi (1997), Hozumi et al. (2000)

- Simulations in the 6D phase space require very large amount of memory and huge computational costs.

- First Vlasov-Poisson simulations in the 6D phase space

  Yoshikawa, Yoshida, Umemura 2013, ApJ, 762, 116

Numerical Methods

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \frac{\partial f}{\partial \vec{x}} - \nabla \phi \cdot \frac{\partial f}{\partial \vec{v}} = 0$$

$$\nabla^2 \phi = 4\pi G \rho = 4\pi G \int f d^3 \vec{v}$$

Schemes

- Each of 3D physical and velocity space is discretized with uniform regular mesh.
- Vlasov equation is solved with the directional splitting scheme
- Poisson equation is solved with convolution method using FFT

Parallelization

- 6D phase space is decomposed along the 3D physical space.

Advection Equation

- Vlasov equation is decomposed into 6 one-dimensional advection equations.

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} = 0$$

- Physical requirements

  - positivity
  - mass conservation
  - maximum principle

→ Positive Flux Conservative (PFC) method

Filbet, Sonnendrucker, Bertrand, J. Comp. Phys (2001) 172, 166-187

- satisfies all the physical requirements.
- effectively 3rd order scheme in space.

Time Integration

- 2nd order leapfrog scheme

$$f(\vec{x},\vec{v},t^{n+1}) = T_{v_x}(\Delta t/2)T_{v_y}(\Delta t/2)T_{v_z}(\Delta t/2)$$
$$T_x(\Delta t)T_y(\Delta t)T_z(\Delta t)$$
$$T_{v_x}(\Delta t/2)T_{v_y}(\Delta t/2)T_{v_z}(\Delta t/2) \quad f(\vec{x},\vec{v},t^{n})$$

  - Poisson equation is solved after updating the advection equation in physical space

- Timestep constrains

$$\Delta t = C\min(\Delta t_{\mathrm{v}}, \Delta t_{\mathrm{x}})$$

$$\Delta t_{\mathrm{x}} = \min\left(\frac{\Delta x}{V_x^{\max}}, \frac{\Delta y}{V_y^{\max}}, \frac{\Delta z}{V_z^{\max}}\right) \qquad \Delta t_{\mathrm{v}} = \min_i\left(\frac{\Delta v_x}{|a_{x,i}|}, \frac{\Delta v_y}{|a_{y,i}|}, \frac{\Delta v_z}{|a_{z,i}|}\right)$$

King Sphere

- **initial condition**

$$f_{\mathrm{K}}(\mathcal{E}) = \frac{\rho_0}{(2\pi\sigma^2)^{3/2}} \left[\exp(\mathcal{E}/\sigma^2) - 1\right] \qquad \mathcal{E} > 0$$

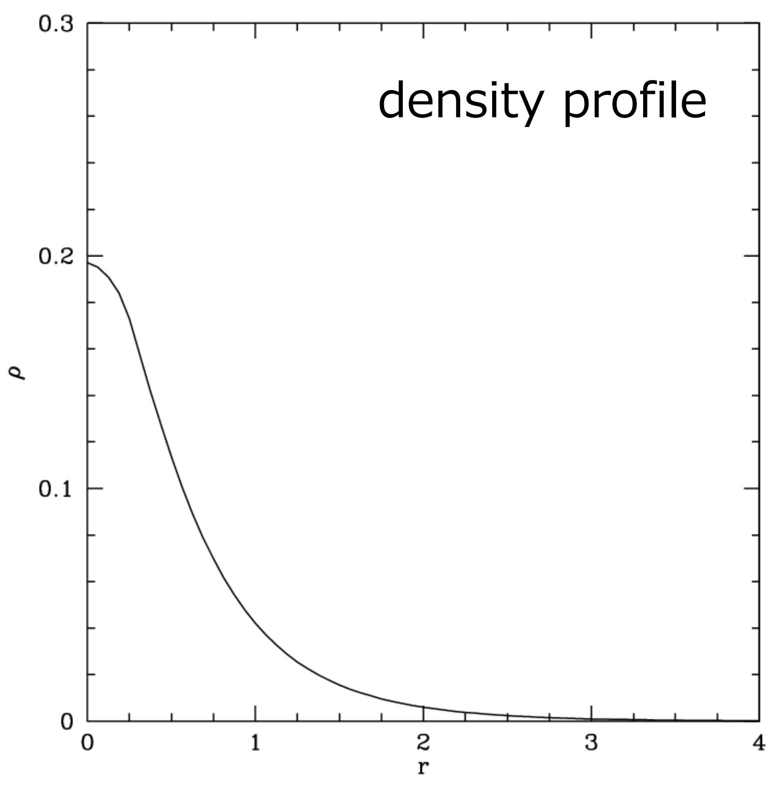
$$= 0 \qquad \mathcal{E} < 0$$

$$\mathcal{E} = \Psi - \frac{v^2}{2} \qquad \Psi(0)/\sigma^2 = 3$$

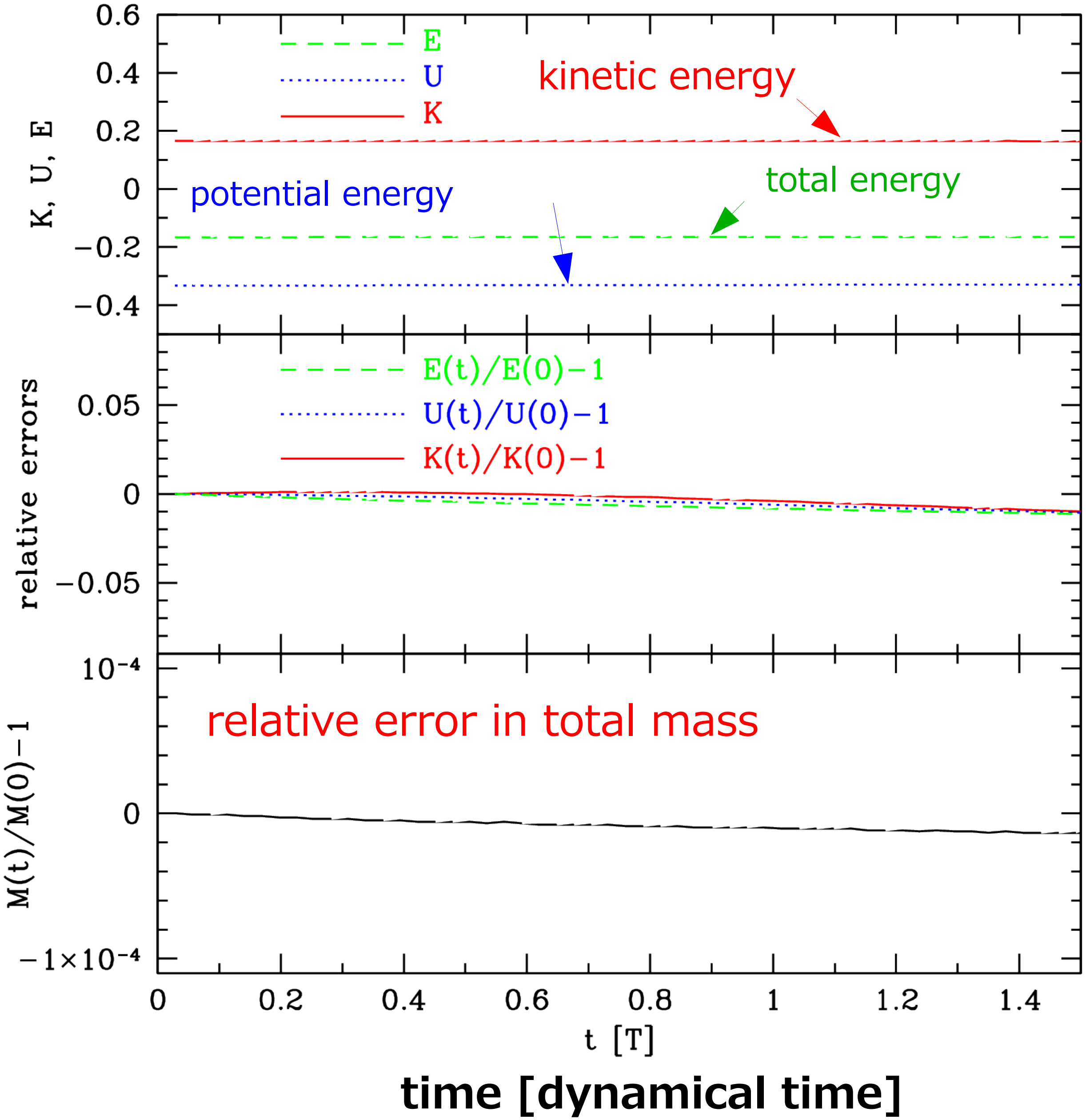
- **number of mesh grids :**

physical space 64^3
velocity space 64^3

- **stable solution of Vlasov-Poisson equation**

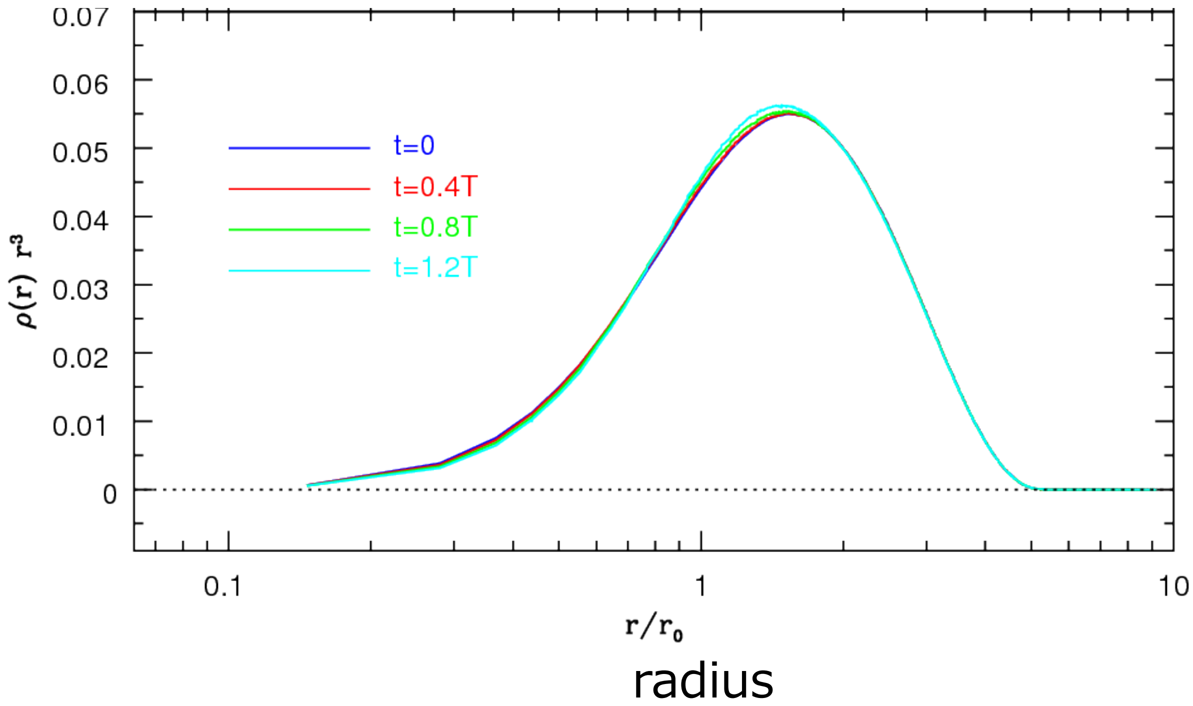


King Sphere



- time variation of kinetic and potential energies are sufficiently small.
- relative error of total energy is within 1%.
- total mass is also well conserved with sufficiently good accuracy.

King Sphere



- time “evolution” of the density profile of King sphere
- profiles are almost unchanged
- slight mass transfer from the center to the outskirts probably due to the numerical diffusion.

3-D Self-Gravitating System

- **initial condition**

$$f(\vec{x},\vec{v}) = \frac{(1+\delta)}{(2\pi\sigma^2)^{3/2}} \exp\left(-\frac{|\vec{v}|^2}{2\sigma^2}\right)$$

$$\rho = 1 + \delta$$

white-noise power spectrum for the density perturbation δ

- **number of mesh grids**

physical space : 64^3
velocity space : 64^3

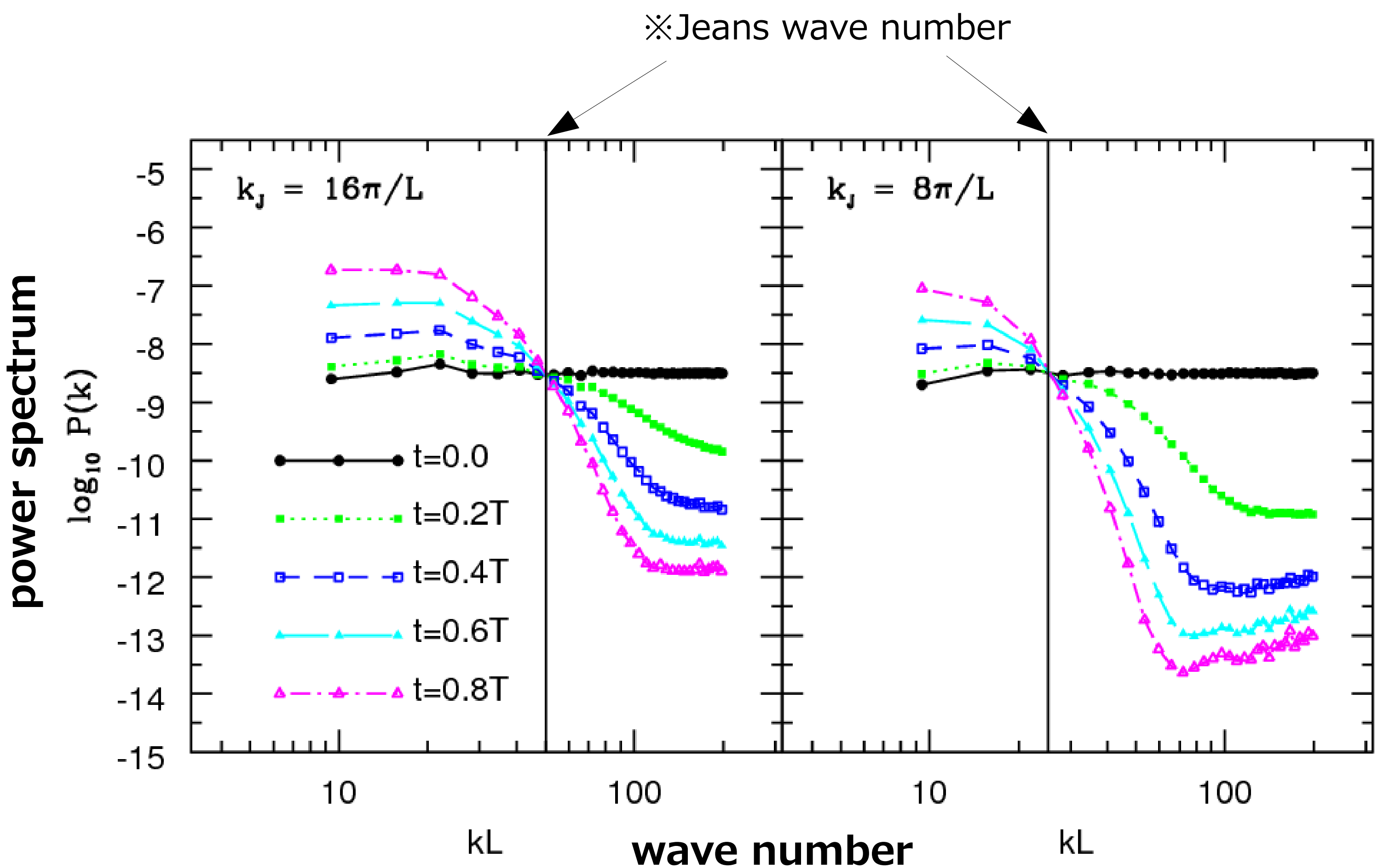
- **boundary condition** periodic boundary condition

Jeans wave number : $k_J^2 = \dfrac{4\pi G\rho}{\sigma^2}$

$k < k_J$ → gravitational instability

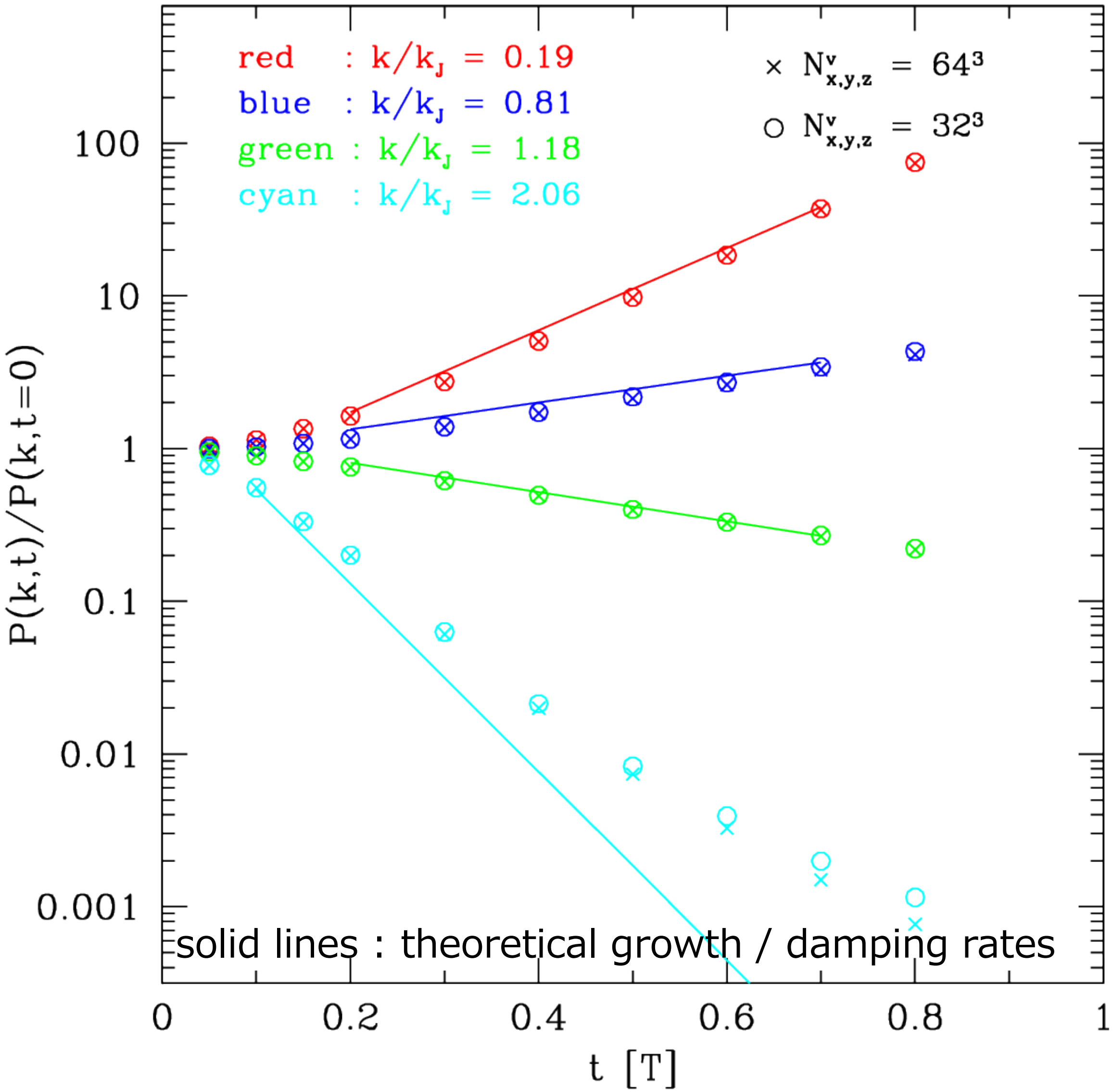
$k > k_J$ → collisionless damping

3-D Self-Gravitating System



- clear switching of gravitational instability and collisionless damping at the Jeans wave number.

3-D Self-Gravitating System



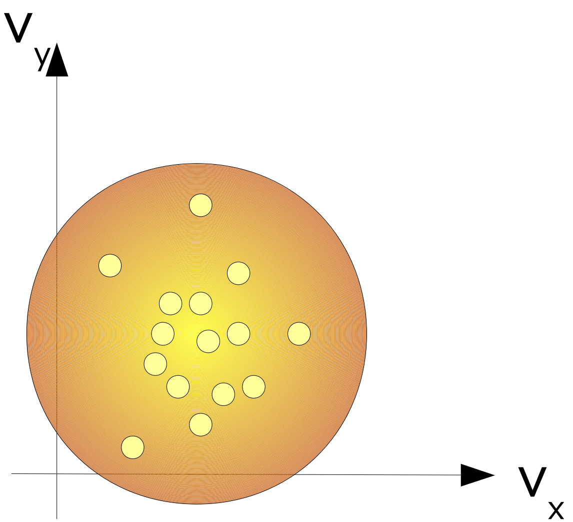
- growing / damping rates

  consistent with linear theory

- For a large k/k_J , damping rate gets smaller due to the non-linear landau damping

N-body vs Vlasov

- N-body simulation
  - particles sample the mass distribution in the 6D phase space volume in a Monte-Carlo manner, and are advanced along the characteristic lines of the Vlasov equation.
  - numerical results include intrinsic shot noise.
  - spatial resolution is adaptive and better in higher density regions
  - poor at solving collisionless damping (aka free streaming)



- Vlasov-Poisson simulation
  - treats matter as continuum fluid in the phase space

    ⇒ free from shot noise contamination
  - good at simulating collisionless damping
  - inevitably poor spatial resolution compared with N-body simulations

Application to Dynamics of Neutrinos in the Large-Scale Structure Formation

Neutrinos in Large-Scale Structure

- Observation of neutrino oscillation turns out that neutrinos are massive.

  → dynamical effect on the large-scale structure formation

- Ground-based experiments can only probe the mass difference between different flavors but not the absolute mass of neutrinos.

$$0.05\,\mathrm{eV} \leq \sum_{i=1}^{3} m_i \leq 1.4\,\mathrm{eV}$$

neutrino oscillation (lower bound) — WMAP 9yr (upper bound)

- The absolute mass and the mass hierarchy are important for theories beyond the standard model of elementary particles .
- The imprints of massive neutrinos on LSS in the universe is very important.

Collisionless Damping of Neutrinos

- very large velocity dispersion of neutrinos

$$\sigma_{\rm v} = 150(1+z)\left(\frac{1{\rm eV}}{m_\nu}\right)\ {\rm km/s}$$

- collisionless damping (free streaming)

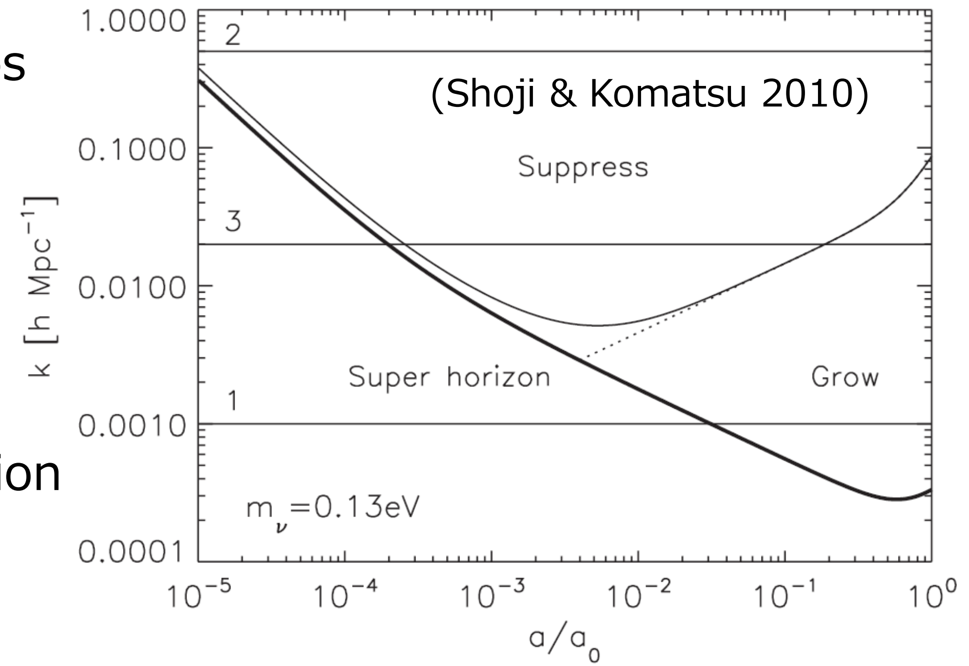
suppress the growth of density perturbation

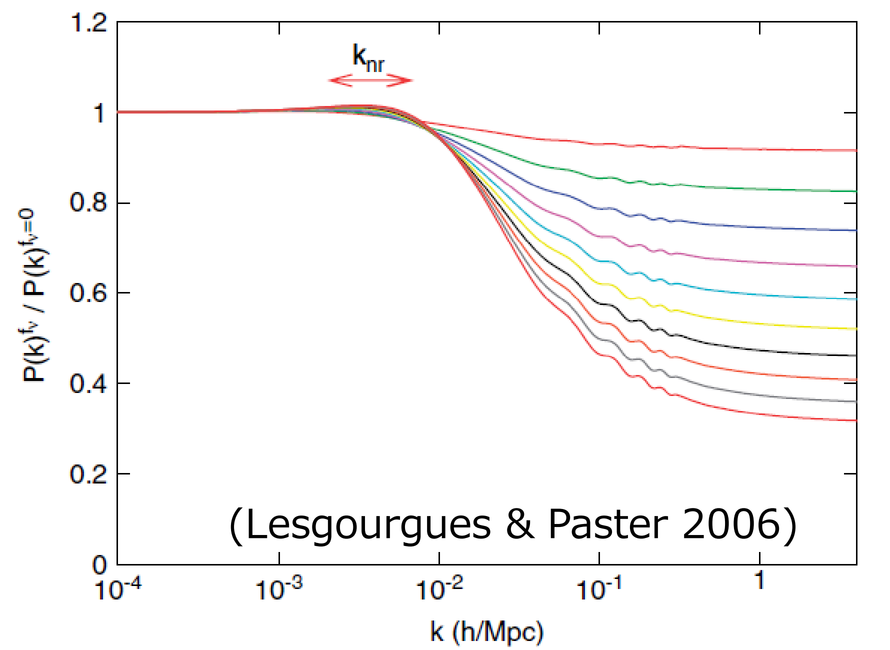
$$\Rightarrow \quad \delta \propto D_+(t)^{1-\frac{3}{5}f_\nu}$$

f_ν : neutrino mass fraction

- scale of collisionless damping

$$k_{\rm nr} = 0.018\Omega_{\rm m}^{1/2}\left(\frac{m_\nu}{1\,{\rm eV}}\right) h\,{\rm Mpc}^{-1}$$





LSS observations can probe the absolute mass of neutrinos.

Simulations of LSS formation with CDM+neutrinos

- Previous simulations based-on N-body methods

  neutrino dynamics are not solved in a consistent manner.

A hybrid of N-body and Vlasov-Poisson simulations

- CDM (Cold Dark Matter)

  very small thermal velocity dispersion ⇨ N-body simulations

- neutrino (Hot Dark Matter)

  collisionless damping due to large velocity dispersion

  ⇨ Vlasov-Poisson simulation

Vlasov Equation in the Cosmological Comoving Frame

- Vlasov equation in canonical coordinates

$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{a^2} \cdot \frac{\partial f}{\partial \vec{x}} - \nabla\phi \cdot \frac{\partial f}{\partial \vec{p}} = 0 \qquad \vec{p} = a^2 \dot{\vec{x}}$$

$$\nabla^2 \phi = 4\pi G a^2 \bar{\rho}\delta = 4\pi G a^2 \bar{\rho} \left[\int f d^3\vec{p} - 1 \right]$$

  - matter distribution in momentum space quickly overflows the predefined range of momentum.

- modified Vlasov equation in terms of peculiar velocity $\vec{v} = a\dot{\vec{x}}$

$$\frac{\partial f}{\partial t} + \frac{\vec{v}}{a} \cdot \frac{\partial f}{\partial \vec{x}} - \underline{\left[H\vec{v} + \frac{\nabla\phi}{a} \right]} \cdot \frac{\partial f}{\partial \vec{v}} = 0$$

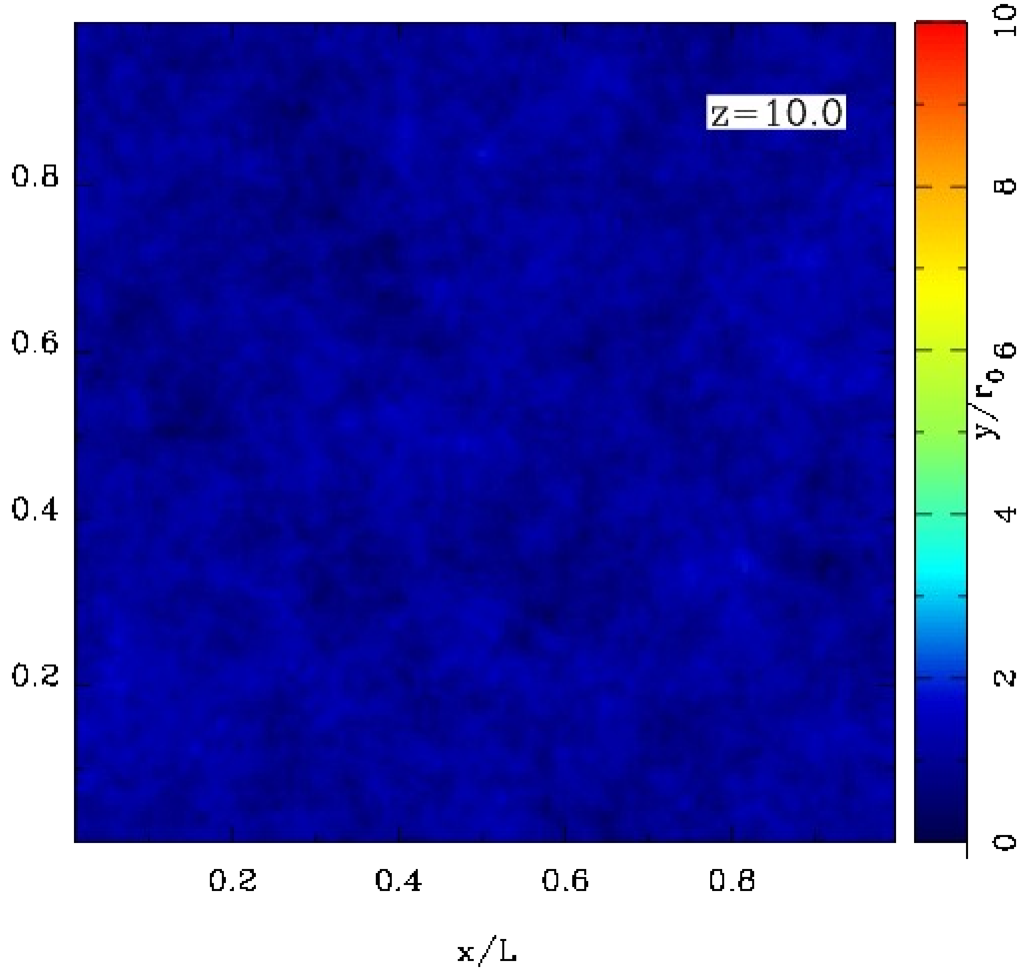
  - the extent of peculiar velocity distribution does not change a lot.

Cosmological Vlasov Simulation

ΛCDM universe with WMAP-9 yr cosmological parameters L=120 Mpc/h

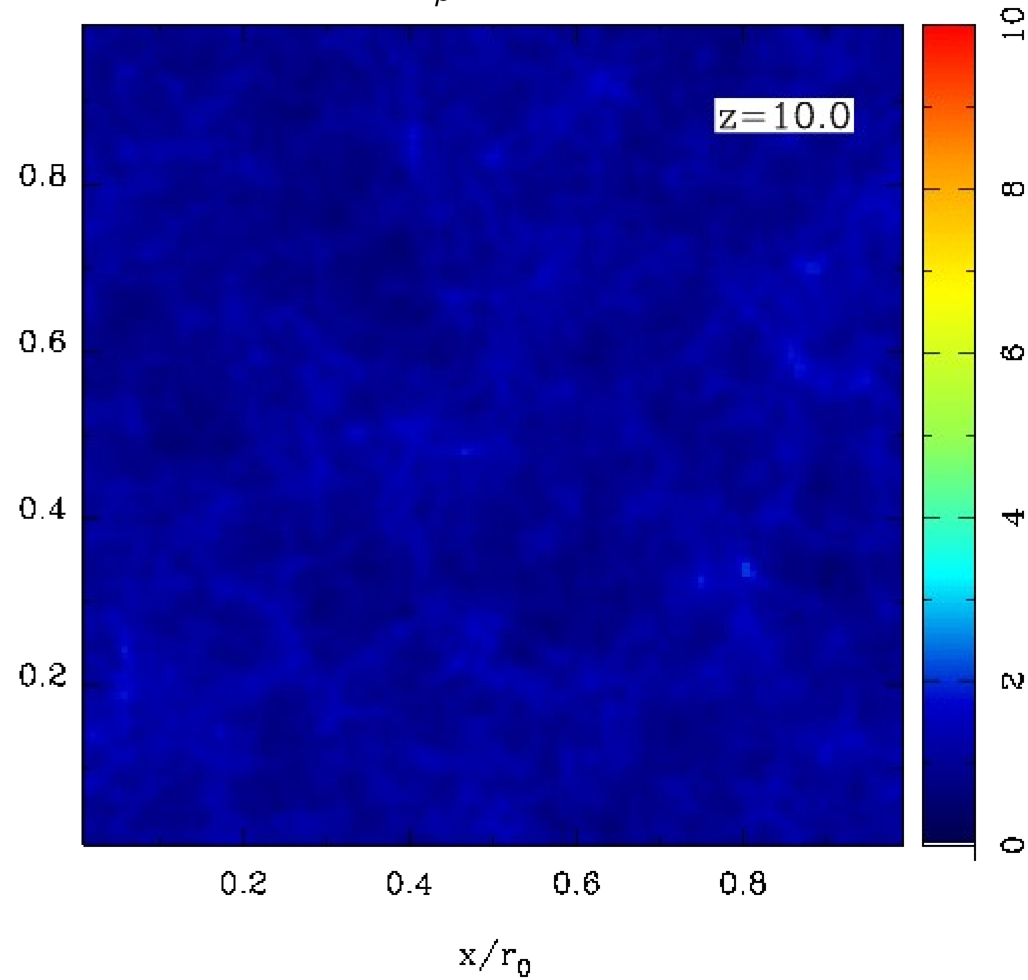
Vlasov-Poisson シミュレーション

$N_x = 128^3$ $N_v = 64^3$

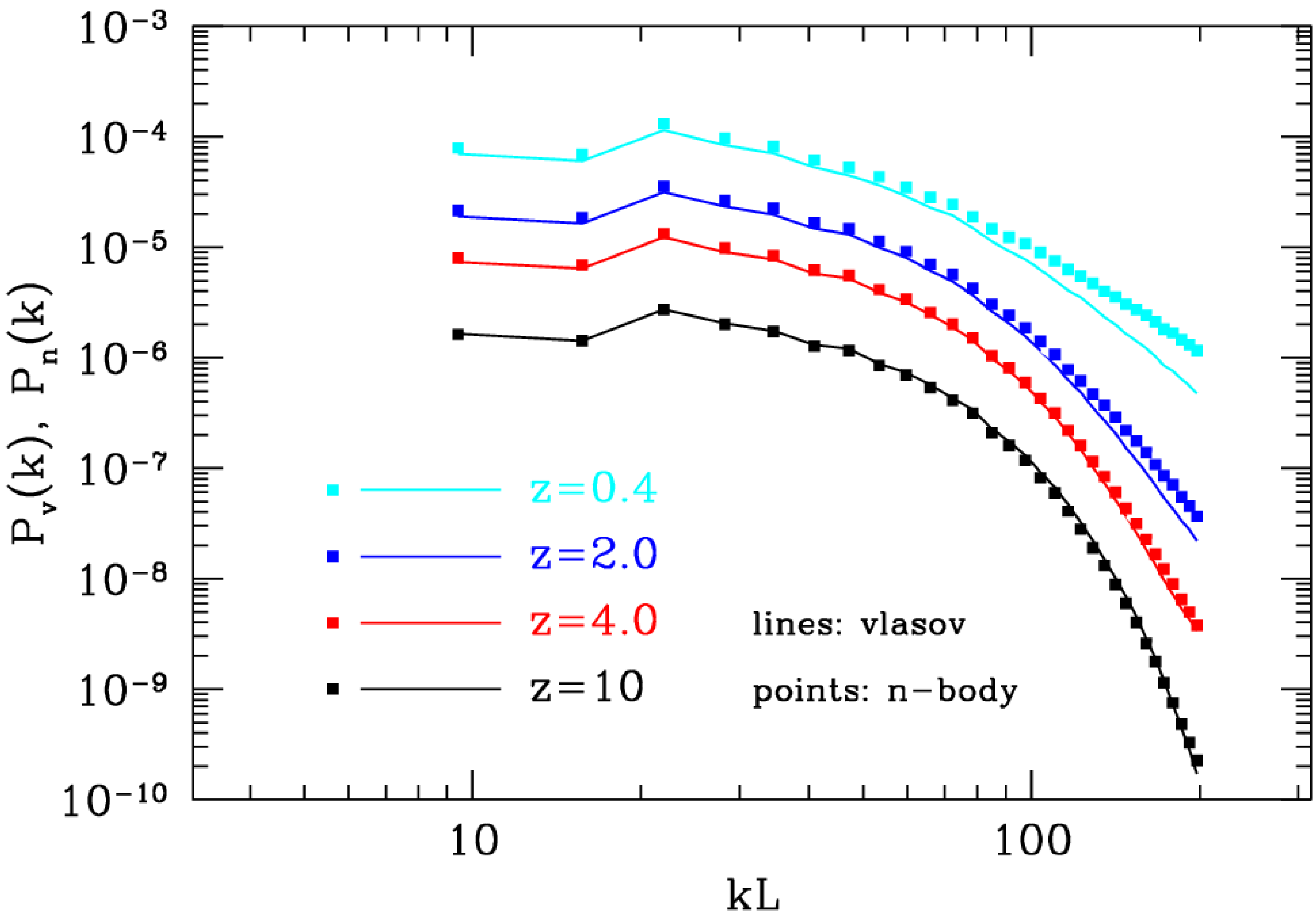


N体 シミュレーション

$N_p = 128^3$



Cosmological Vlasov Simulations



- consistent P(k) between N-body and Vlasov simulations
- Difference in P(k) on smaller scales due to numerical diffusion

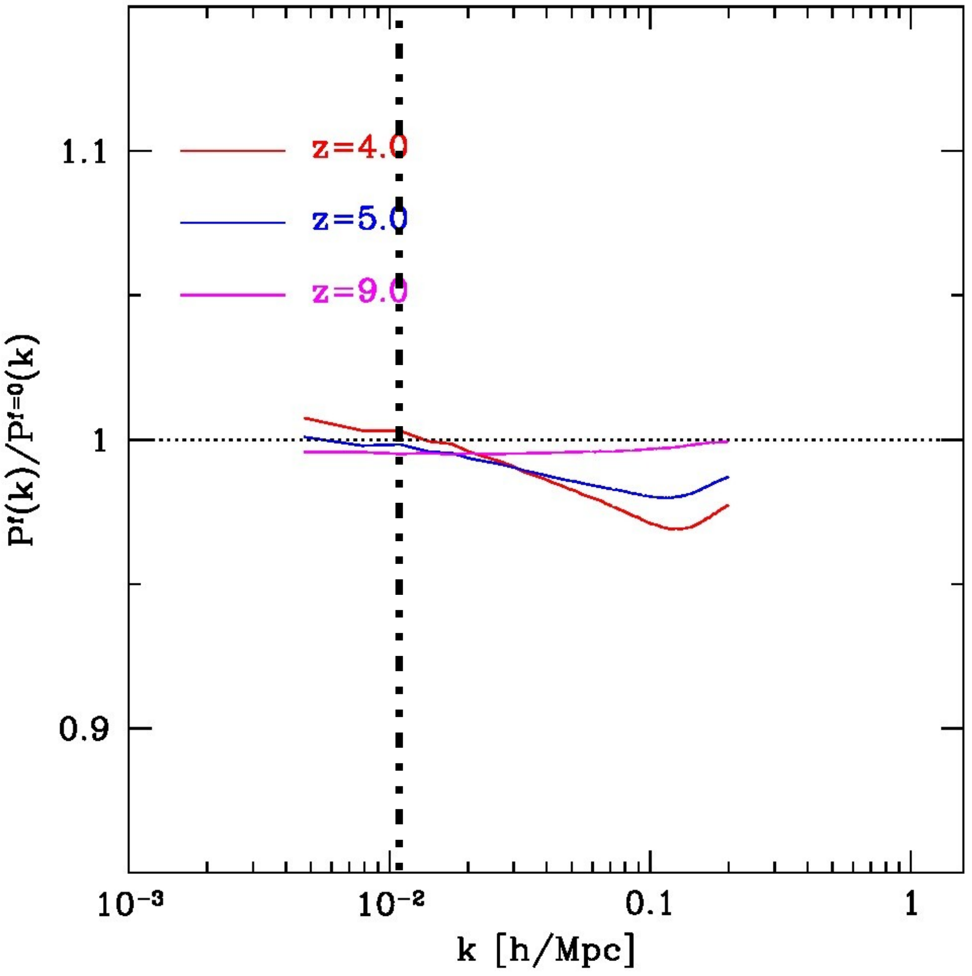
Preliminary results of CDM + neutrinos simulation

ΛCDM with massive neutrino

- L=2000 Mpc/h
- *PLANCK* 2013 cosmological parameters

$$\sum_i m_i = 1\,\mathrm{eV}$$

- $N_x=128^3$ and $N_v=64^3$ for Vlasov simulations
- $N_p=128^3$ for N-body simulations



Summary

- We, for the first time, performed the Vlasov-Poisson simulations of self-gravitating systems in 6D phase space volume.

- Several numerical tests have been checked in comparison with analytical models and N-body results.

- We apply the Vlasov-Poisson simulations to neutrino dynamics in the LSS formation to correctly simulate the collisionless damping of neutrinos.

- A new formulation and implementation of Vlasov-Poisson simulations in the cosmological comoving coordinate.

- We performed preliminary hybrid runs of N-body and Vlasov simulations.