

# Lattice QCD calculation of the $\rho$ meson decay width

at 第三者評価 2007/10/31

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We calculate  $\rho$  meson decay width  
from scattering phase shift for  $|l|=1 \pi\pi$  system.

Results have already been presented in arXiv:0708.3705.

# 1. Introduction

## Previous works of $\rho$ meson decay

- 1) S. Gottlieb, P.B. Mackenzie, H.B. Thacker, D. Weingarten      PLB134(1984)346.

Quench ,  $m_\pi/m_\rho = 0.84 - 0.91$

- 2) R.D. Loft, T.A. DeGrand      PRD9(1989)2692.

Quench ,  $m_\pi/m_\rho = 0.9$

- 3) C. McNeile, C. Michael + UKQCD      PLB556(2003)177.

$N_f = 2$  ,  $m_\pi/m_\rho = 0.578^{+13}_{-19}$

## problems

- 1) Quench

$$\rho \rightarrow \tilde{\pi} \tilde{\pi}$$

How can we extract physical decay width  
from “value in Quenched Theory” ?

- 2)  $m_\pi/m_\rho > 1/2$

$\rho$  meson behaves as “stable particle” !!

# This work

$$N_f = 2 \quad , \quad m_\pi/m_\rho = 0.42$$

$$L = 2.53 \text{ fm} \quad (La = 12) \quad , \quad Ta = 24$$

$$1/a = 0.91 \text{ GeV}$$

generated by CP-PACS col. PRD70(2004)074503.

Calc. of SC. phase shift for  $l=1 \pi\pi$  system  
from energy eigenvalue by Rummikainen - Gottlieb formula.

$$W \implies \tan \delta \longrightarrow \Gamma_\rho$$

All calc. is carried out with VPP5000  
at Information Processing Center of U. Tsukuba.

## 2. Method

### Moving frame

- CM. frame

$$\rho(0) \rightarrow \pi(p)\pi(-p) \quad p \neq 0$$

$$p_{\min.} = 2\pi/L$$

$$\sqrt{s} = 2 \cdot \sqrt{m_\pi^2 + p_{\min.}^2} \gg m_\rho \quad \text{in typical case}$$

- Moving frame with  $\mathbf{P} = (0, 0, 1) \times 2\pi/L$

$$\rho(P) \rightarrow \pi(P)\pi(0) \implies \rho(0) \rightarrow \pi(k)\pi(-k)$$

Lorentz trans.

$$k_{\min.} = 2\pi/L \times \frac{1}{(2\gamma)}$$

$$\sqrt{s} = 2 \cdot \sqrt{m_\pi^2 + k_{\min.}^2} \sim m_\rho$$

In our case ( $m_\pi/m_\rho = 0.42$ ,  $L = 2.53$  fm,  $\gamma \sim 1.2$ )

CM. frame       $\sqrt{s} / m_\rho = 1.47$

Moving frame     $\sqrt{s} / m_\rho = 0.97$

$W$  : energy in moving frame

$\tan \delta$  : scattering phase shift for  $I = 1$   $\pi\pi$  system

$$\tan \delta = Z(k \cdot L / (2\pi))$$

$$\frac{1}{Z(q)} = \frac{1}{2\pi^2 q \gamma} \sum_{\mathbf{r} \in \Gamma} \frac{1 + (3r_3^2 - r^2)/q^2}{r^2 - q^2}$$

$$\Gamma = \{ \mathbf{r} \mid \mathbf{r} = \gamma^{-1} [\mathbf{n} + 1/2 \cdot \mathbf{P} \cdot L / (2\pi)] , \mathbf{n} \in \mathbf{Z}^3 \}$$

$$\sqrt{s} = \sqrt{W^2 - P^2} = 2\sqrt{m_\pi^2 + k^2} , \quad \gamma = W/\sqrt{s}$$

## Diag. method

We set  $\mathbf{P} = (0, 0, 1) \times 2\pi/L$

$$\begin{aligned}\mathcal{O}_1(t) &\equiv \rho_3(\mathbf{P}, t) = \sum_{\mathbf{x}} \frac{1}{\sqrt{2}} \left( \bar{u}(\mathbf{x}, t) \gamma_3 u(\mathbf{x}, t) - \bar{d}(\mathbf{x}, t) \gamma_3 d(\mathbf{x}, t) \right) \cdot e^{i\mathbf{P}\cdot\mathbf{x}} \\ \mathcal{O}_2(t) &\equiv (\pi\pi)(\mathbf{P}, t) = \frac{1}{\sqrt{2}} \left( \pi^-(\mathbf{P}, t) \pi^+(\mathbf{0}, t) - \pi^+(\mathbf{P}, t) \pi^-(\mathbf{0}, t) \right)\end{aligned}$$

$$\begin{aligned}G_{ij}(t) &= \langle \mathcal{O}_i^\dagger(t) \mathcal{O}_j(0) \rangle \quad (\text{ : } 2 \times 2 \text{ matrix }) \\ &= \sum_{\alpha} V_{i\alpha} \lambda_{\alpha}(t) V_{\alpha j}^\dagger \quad \text{for large } t \\ &\quad \left( V_{i\alpha} = \langle 0 | \mathcal{O}_i^\dagger | \alpha \rangle , \quad \lambda_{\alpha}(t) = \exp(-W_{\alpha} \cdot t) \right)\end{aligned}$$

Assuming higher states ( $\alpha \geq 3$ ) are negligible,

$$\text{Ev}[ G(t) G^{-1}(t_0) ] = \lambda(t) \sim e^{-Wt} \quad \text{for large } t$$

## Calc. of $G(t)$

$$\gamma_{\pi\pi \rightarrow \pi\pi}(t) = \text{Diagram 1} - \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} - \text{Diagram 5} - \text{Diagram 6}$$

Diagrams:

- Diagram 1: Two vertical ovals with arrows pointing right. The left oval has labels  $-p$  at the top and  $p$  at the bottom. The right oval has labels  $0$  at the top and  $0$  at the bottom.
- Diagram 2: Four overlapping ovals forming a cross-like shape with arrows pointing in various directions.
- Diagram 3: A triangle with arrows pointing clockwise.
- Diagram 4: A triangle with arrows pointing counter-clockwise.
- Diagram 5: A square with arrows pointing clockwise.
- Diagram 6: A square with arrows pointing counter-clockwise.

$$\gamma_{\pi\pi \rightarrow \rho}(t) = \text{Diagram 7} - \text{Diagram 8}$$

Diagrams:

- Diagram 7: A triangle with vertices labeled  $-p$ ,  $p$ , and  $0$ . Arrows point from  $-p$  to  $p$  and from  $p$  to  $0$ .
- Diagram 8: A triangle with vertices labeled  $0$ ,  $0$ , and  $0$ . Arrows point from  $0$  to  $0$  and from  $0$  to  $0$ .

$$G_{\rho \rightarrow \pi\pi}(t) = \text{Diagram 9} - \text{Diagram 10}$$

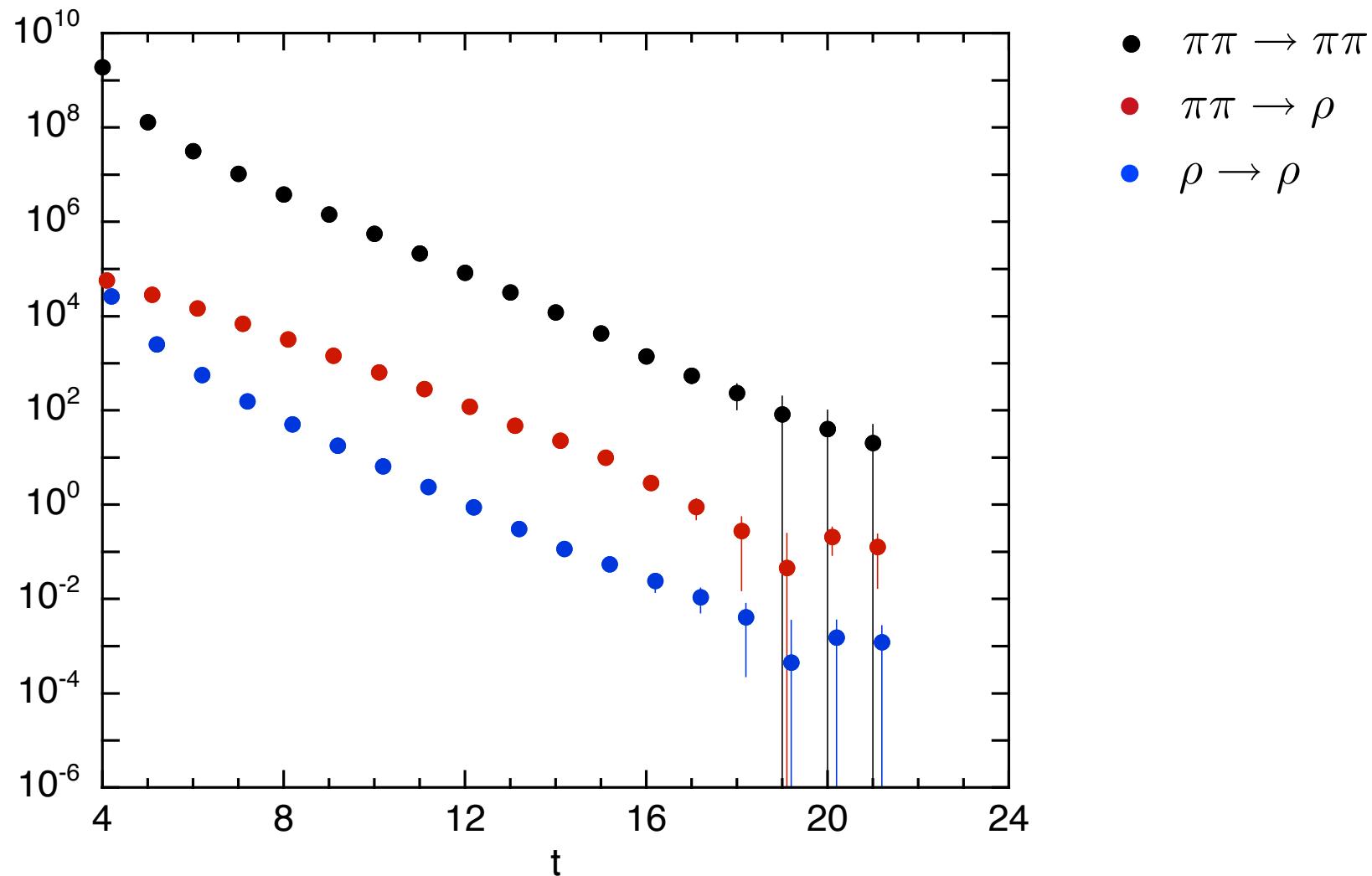
Diagrams:

- Diagram 9: A triangle with vertices labeled  $-p$ ,  $0$ , and  $p$ . Arrows point from  $-p$  to  $0$  and from  $0$  to  $p$ .
- Diagram 10: A triangle with vertices labeled  $0$ ,  $0$ , and  $0$ . Arrows point from  $0$  to  $0$  and from  $0$  to  $0$ .

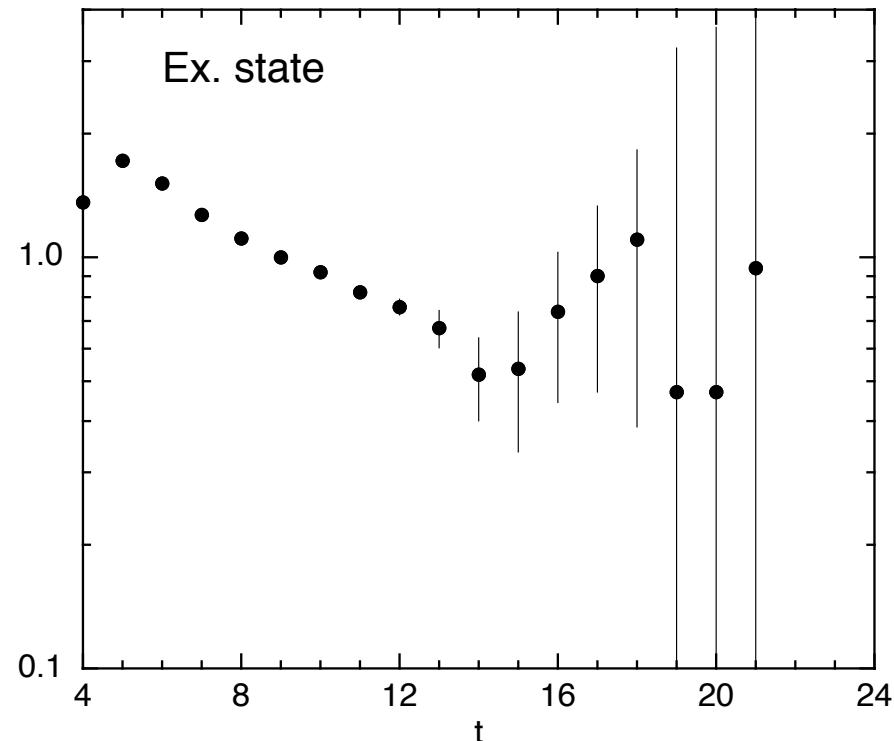
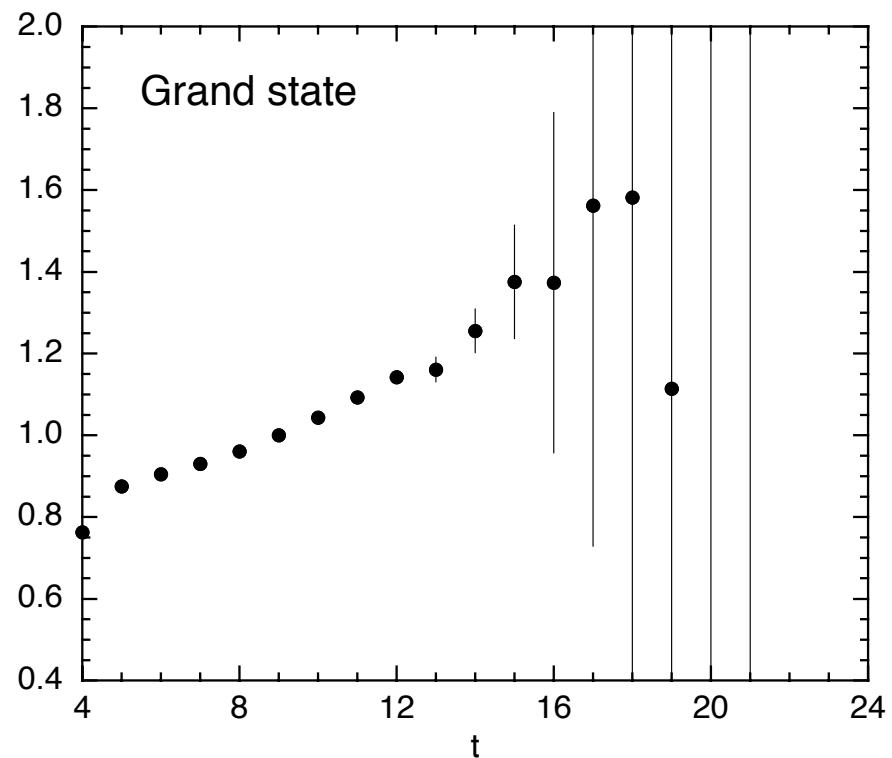
These are calculated by combination  
of the stochastic source and the source method.

# 3. Results

## Results. of $G(t)$



$$\text{Eigenvalue } \lambda(t) = \text{Ev} [ G(t) G^{-1}(t_0) ] \sim \exp(-Wt)$$



$$\lambda_1(t) / e^{-E_0 t} \sim e^{-\Delta W_1 \cdot t}$$

$$\lambda_2(t) / e^{-E_0 t} \sim e^{-\Delta W_2 \cdot t}$$

(  $E_0$  : energy of free two pions )

$$\Delta W_1 = -4.41(60) \times 10^{-2}$$

$$W_1 = E_0 + \Delta W_1 = 0.9364 \pm 0.0063$$

$$\Delta W_2 = 1.05(22) \times 10^{-1}$$

$$W_2 = E_0 + \Delta W_2 = 1.085 \pm 0.022$$

# Calc. of SC. phase shift

1) energy :  $W_1, W_2$

using two dispersion relations

- continuum ( Cont ) :

$$\sqrt{s} = \sqrt{W^2 - P^2}$$

- lattice ( Lat ) :

$$\cosh \sqrt{s} = \cosh W - 2 \cdot \sin^2 P/2$$

}

difference =  $O(a)$  error

2)  $\sqrt{s} \rightarrow$  RG. formula  $\rightarrow \tan \delta$

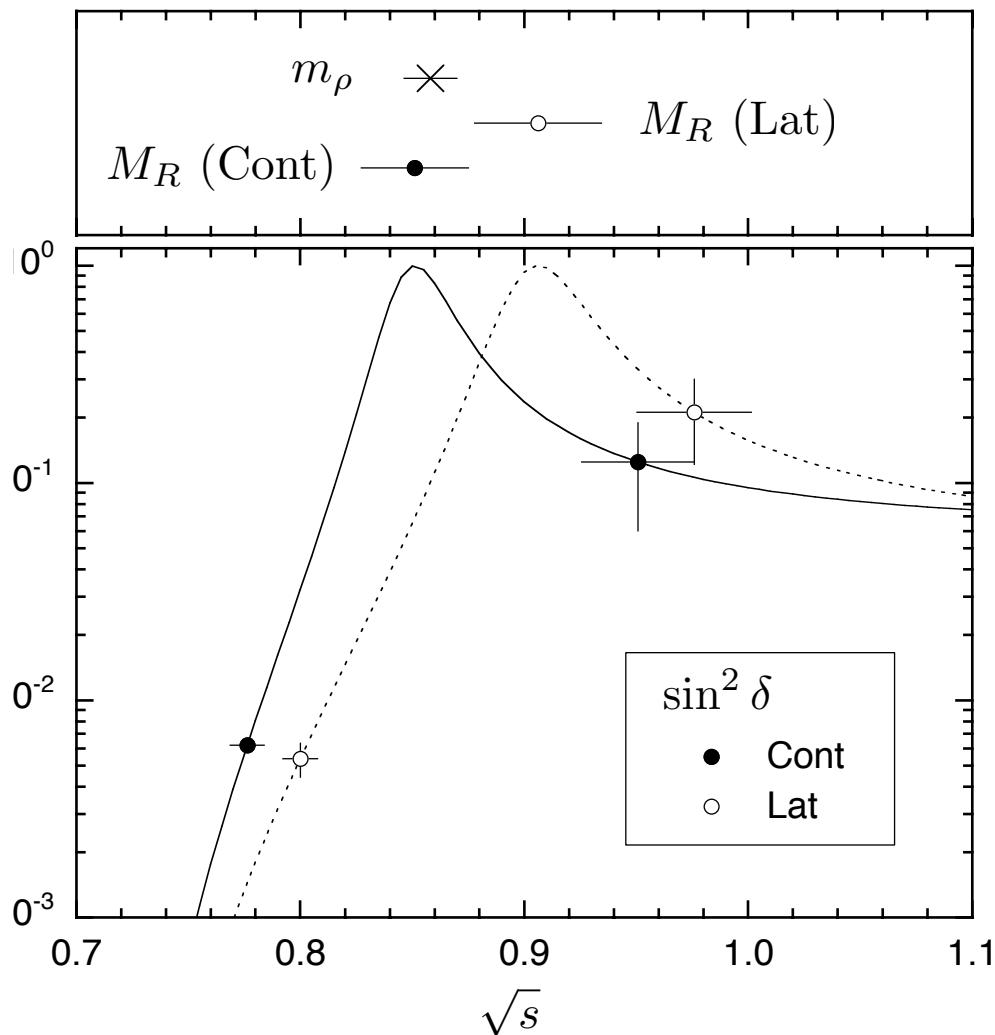
**Grand.** ( $\sqrt{s} < m_\rho$ )

	Cont	Lat
$\sqrt{s}$	0.7764(75)	0.8000(77)
$\tan \delta$	0.07906(93)	0.0736(66)

**Ex.** ( $\sqrt{s} > m_\rho$ )

	Cont	Lat
$\sqrt{s}$	0.951(25)	0.976(26)
$\tan \delta$	-0.38(11)	-0.52(14)

$$m_\rho = 0.858(12) \text{ ( from time correlator )}$$



$$\tan \delta = \frac{g_{\rho\pi\pi}^2}{6\pi} \frac{k^3}{\sqrt{s} (M_R^2 - s)}$$

$$\begin{aligned}\Gamma_\rho &= \frac{g_{\rho\pi\pi}^2}{6\pi} \frac{(k^{\text{Ph}})^3}{(m_\rho^{\text{Ph}})^2} \\ &= \frac{g_{\rho\pi\pi}^2}{6\pi} \times 4.128 \text{ MeV}\end{aligned}$$

assumption :  $g_{\rho\pi\pi}$  : const.

$$\text{Cont} : \Gamma_\rho = 162 \pm 35 \text{ MeV}$$

$$M_R/m_\rho = 0.992 \pm 0.033$$

$$\text{Lat} : \Gamma_\rho = 140 \pm 27 \text{ MeV}$$

$$M_R/m_\rho = 1.056 \pm 0.038$$

cf. Expt. :  $\Gamma_\rho = 150 \text{ MeV}$

## 4. Summary

We calculate  $\rho$  meson decay width  
from scattering phase shift for  $|l|=1 \pi\pi$  system.

We find

(1) Calculation is possible.

(2)  $\Gamma_\rho = 162 \pm 35$  MeV

$\Gamma_\rho = 140 \pm 27$  MeV      cf. Expt. :  $\Gamma_\rho = 150$  MeV

But

(1) Long extrapolation from  $m_\pi/m_\rho = 0.42$  to phys. point.

Calc. near phys. point !!

(2) Large O(a) error in each steps of analysis.

Calc. near cont. limit !!

(3) Huge stat. error ( about 25% for final results of decay width ) .

Consider more efficient method !!