Nonperturbative Renormalization in Schrödinger Functional Scheme

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for
PACS-CS collaboration

Position of renormalization works

Complement of the main project

- Nf=2+1 QCD at physical quark mass (2008-2010)
  - Strong coupling α_s for Nf=3 QCD
  - Quark mass renormalization factor for Nf=3 QCD

- A challenge to larger volume L～9fm (2011-)
  - ★ use of APE stout smeared link for Dirac operator
  - Clover term coefficient c_{SW}

Plan of the talk

✔ 1. Introduction

2. Strong coupling α_s for N_f=3 QCD

3. Quark mass renormalization factor

4. Clover term coefficient

5. Conclusion

Strong coupling αs for Nf=3 QCD

- Goal of Schrödinger functional scheme

Renormalize on the lattice and convert it to the $\overline{\rm MS}$ scheme

or renormalization group invariant (RGI) quantity

An Example: $\Lambda_{\rm QCD}$

can be evaluated precisely by perturbation theory

if renormalized coupling $\overline{g}(\mu)$ and renormalization scale μ are given precisely for small $\overline{g}(\mu)$

$$\Lambda_{\rm QCD} = \mu (b_0 \overline{g})^{-\frac{b_1}{2b_0^2}} \exp\left(-\frac{1}{2b_0\overline{g}}\right) \exp\left(-\int_0^{\overline{g}} dg \left(\frac{1}{\beta(g)} + \frac{1}{b_0 g^3} - \frac{b_1}{b_0^2 g}\right)\right)$$

Strong coupling αs for Nf=3 QCD

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An Example: Λ_{QCD}

can be evaluated precisely by perturbation theory

if renormalized coupling $\overline{g}(\mu)$ ↔ renormalization scale μ are given precisely for small $\overline{g}(\mu)$

on lattice: μ tends to be low energy, $\overline{g}(\mu)$ tends to be strong

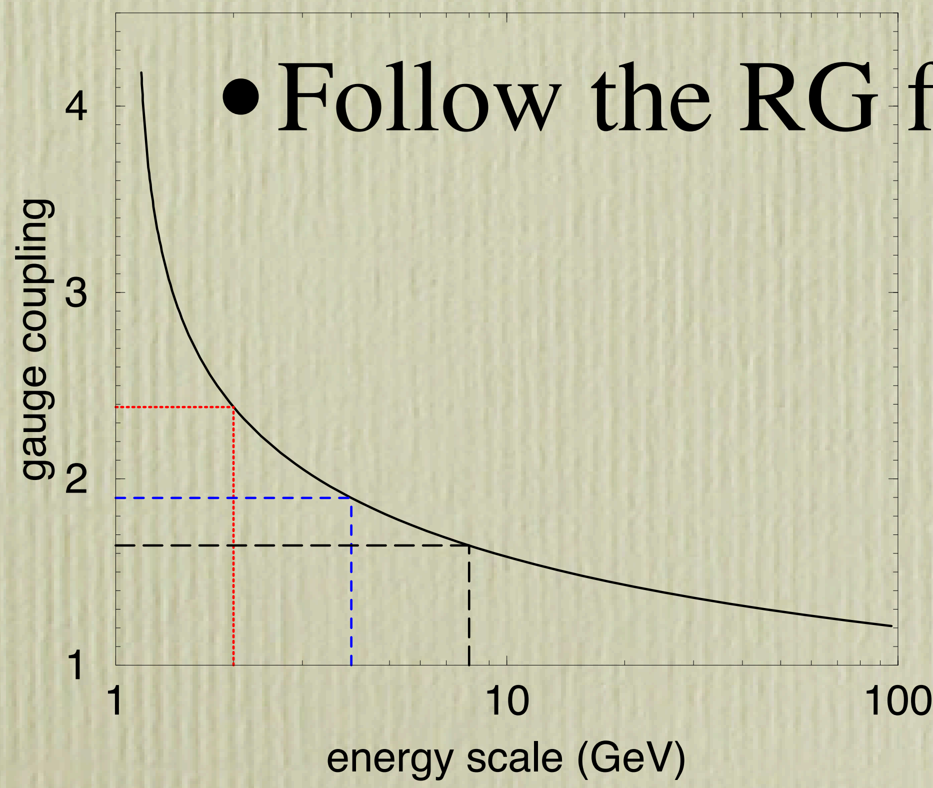
↑ use of SF scheme

Strong coupling αs for Nf=3 QCD

Key quantity: Step scaling function (SSF) (Luescher et al)

Discrete renormalization group flow: $\mu \to 2\mu$

$$\overline{g}(\mu) \longleftarrow \overline{g}(2\mu)$$



- Follow the RG flow in discretized way
  - Can reach at any high energy scale
  - Not a speciality of SF scheme
  - Well suited for SF scheme

Unique renormalization scale

Box size $\mu = 1/L$

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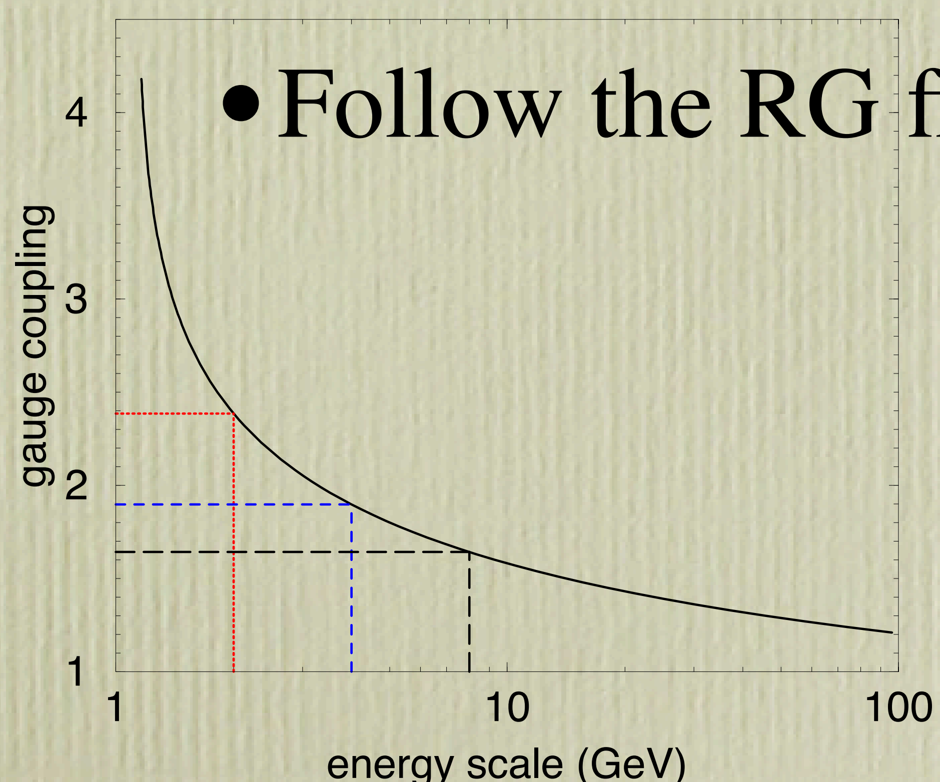

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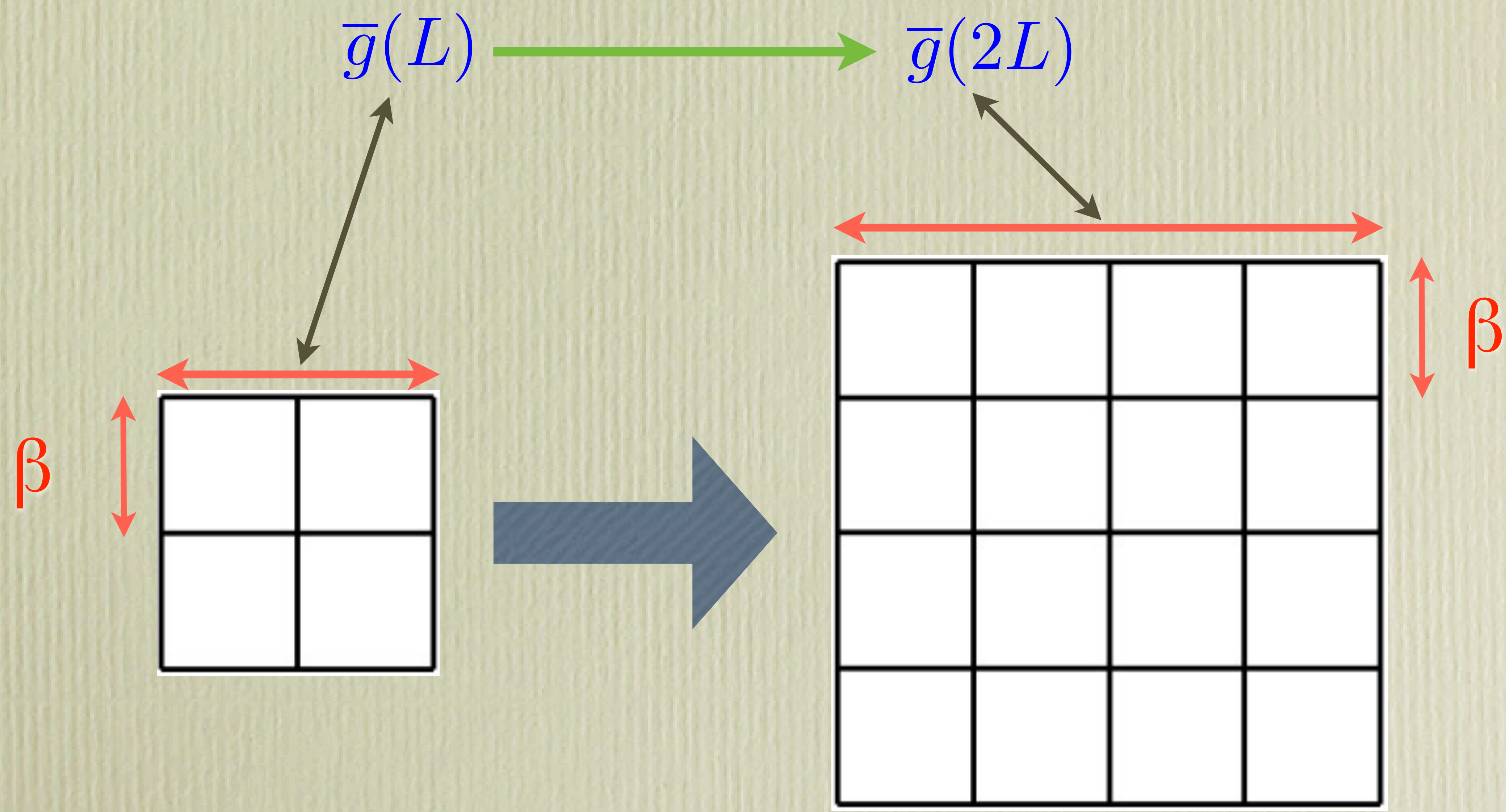

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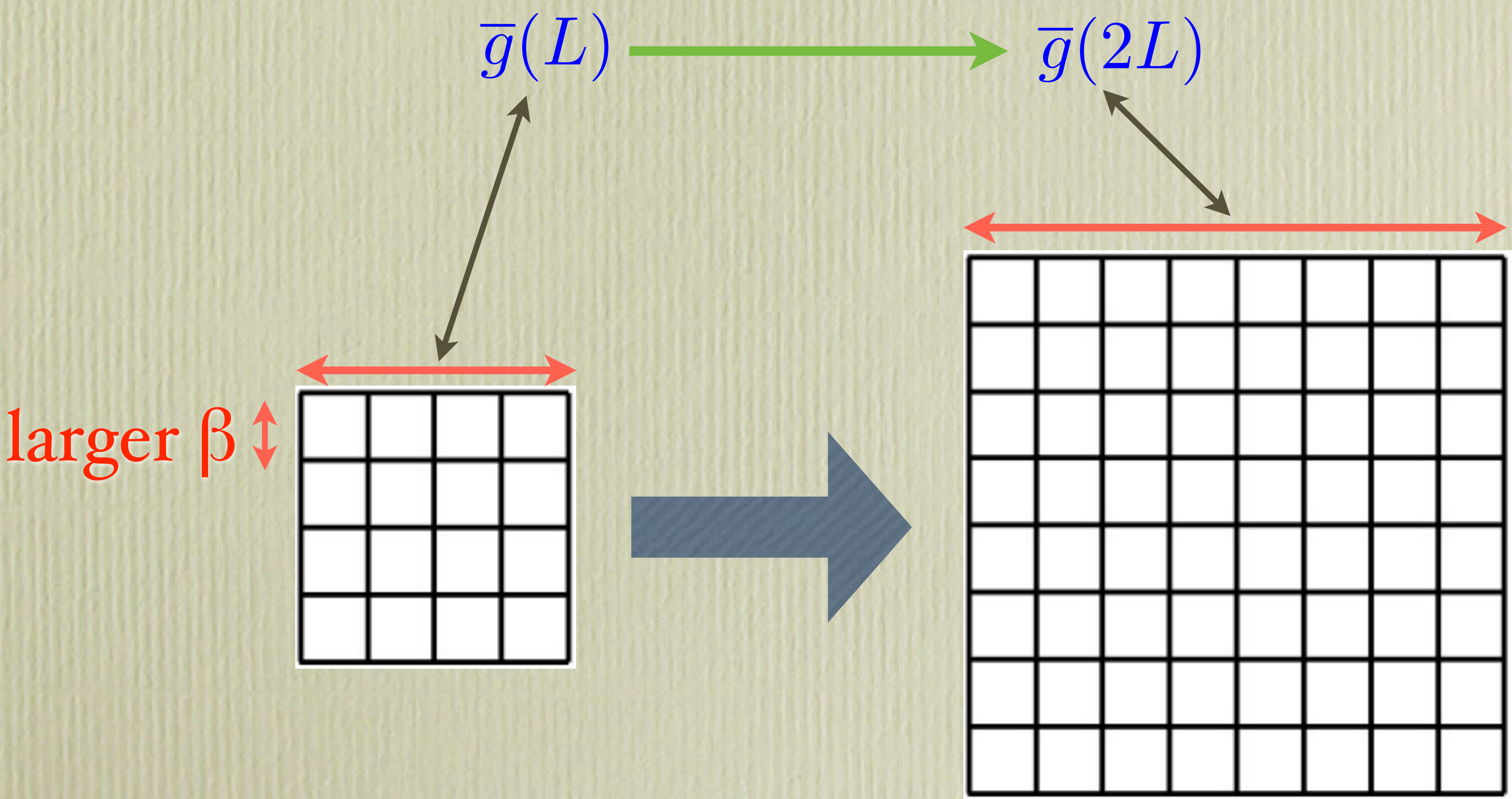


SSF in the continuum limit

Strong coupling αs for Nf=3 QCD

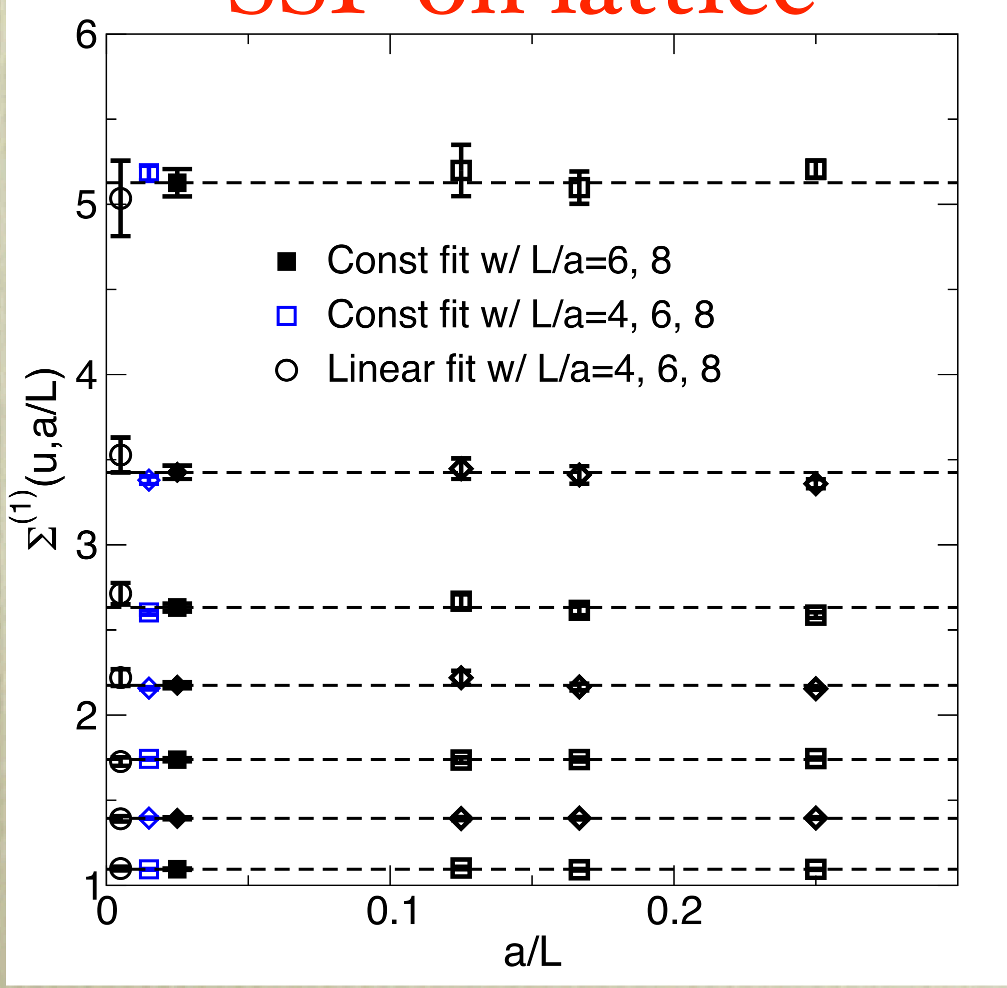
Step scaling function (SSF)

★ Iwasaki gauge action

★ Nonperturbatively improved Wilson fermion

- three massless quarks

SSF on lattice



★Consistency between three continuum extrapolations

- Constant extrapolation with two/three data
- Linear extrapolation

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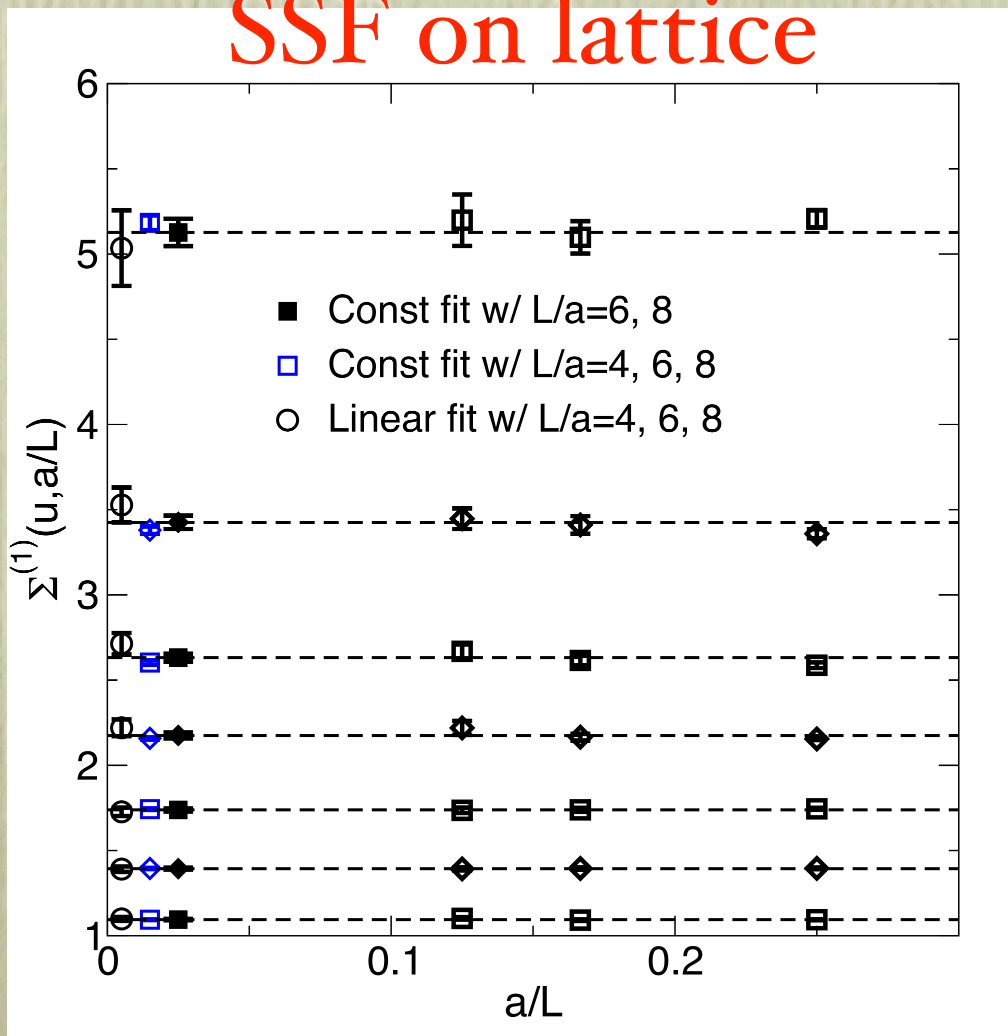
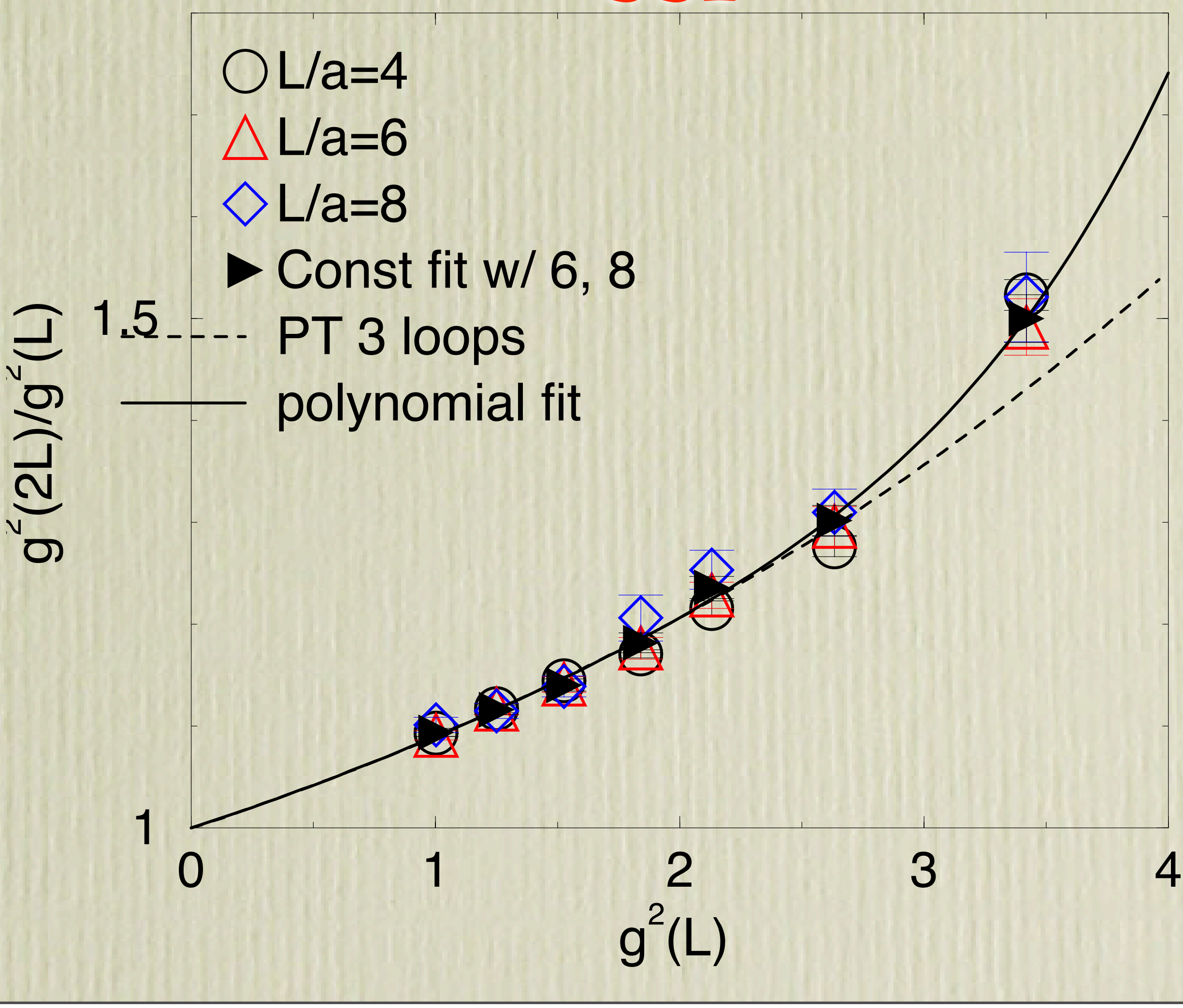


SSF



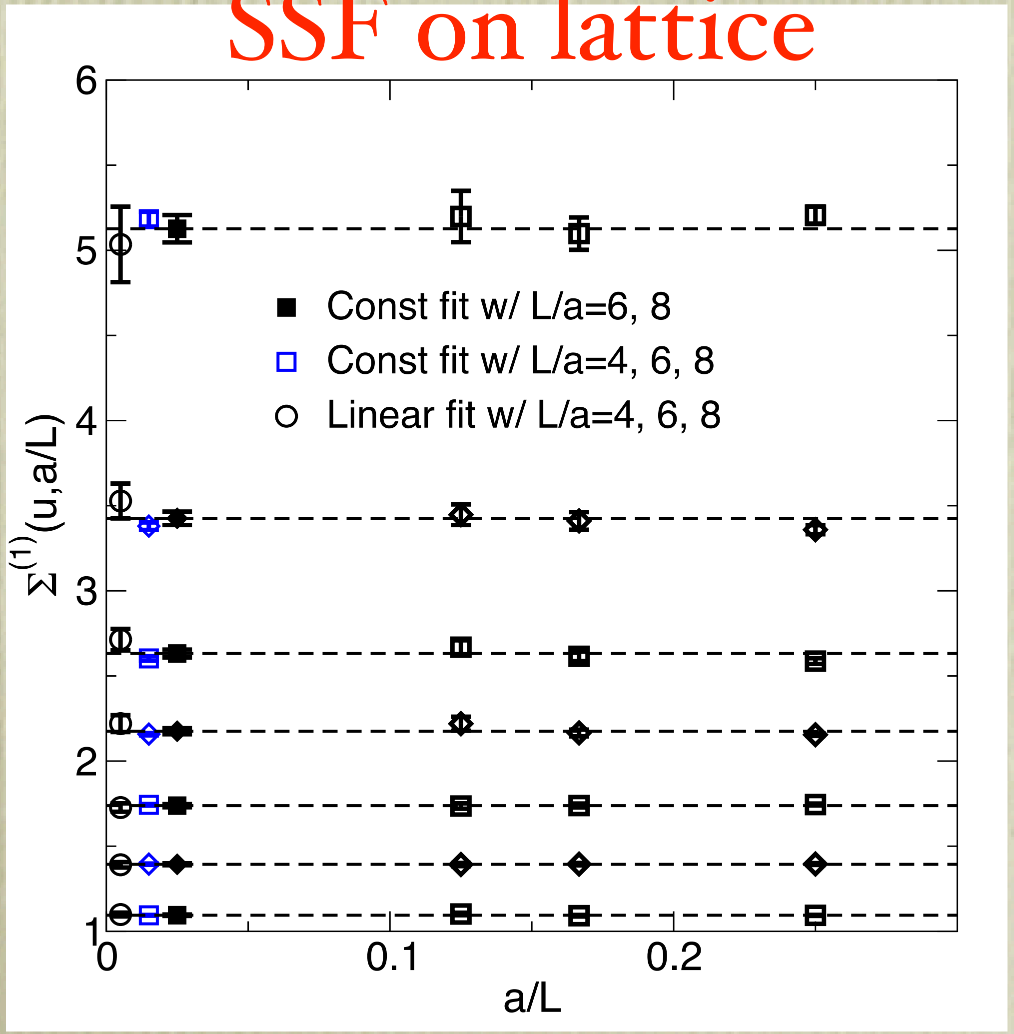
Strong coupling αs for Nf=3 QCD

Step scaling function (SSF)

★ Perturbative improvement of the SSF

- One loop PT is not enough

SSF on lattice



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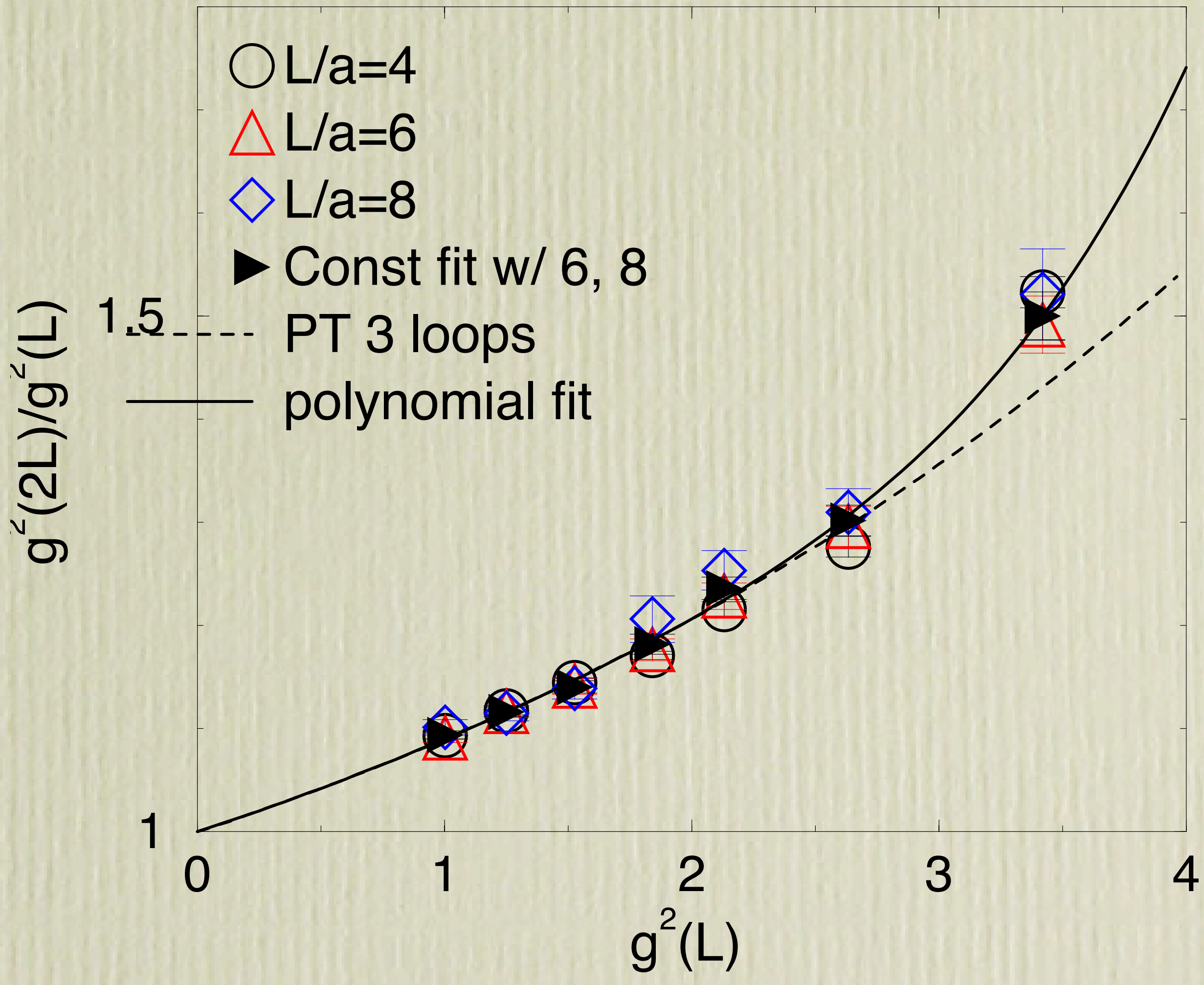

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Strong coupling αs for Nf=3 QCD

$\alpha_S(M_Z)$ and $\Lambda_{\overline{\rm MS}}$

Box size in physical unit $L_{\rm max} = aN_{\rm max}$ ← typically 4-6

$\bar{g}(L_{\rm max})$

β=1.90 a=0.08995(40) fm

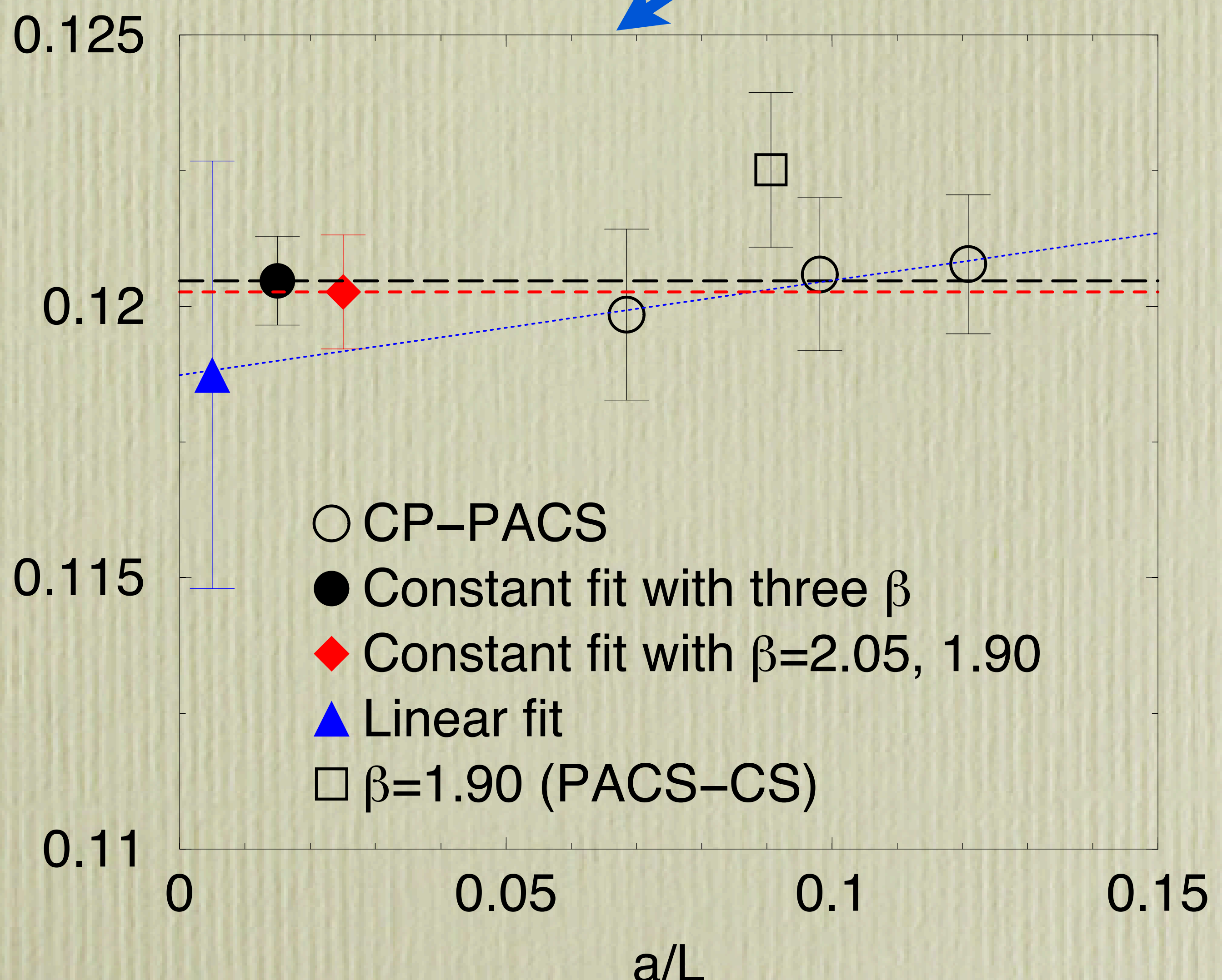
↓ perform step scaling n times

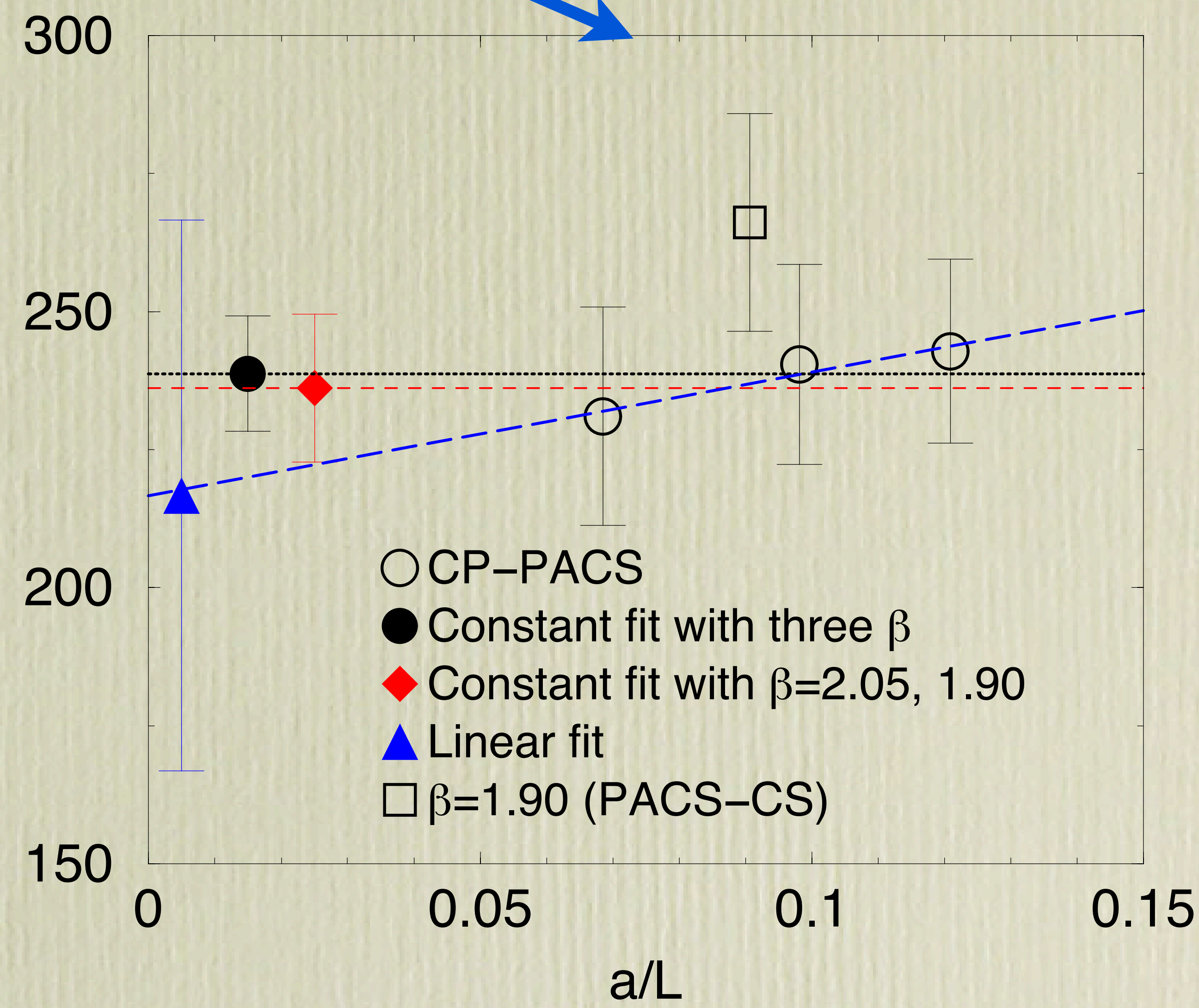
$\bar{g}(L_{\rm max}/2^n)$ and renormalization scale $L_{\rm max}/2^n$ are precisely determined

For n～10 perturbation theory is applicable

Strong coupling αs for Nf=3 QCD

$\alpha_S(M_Z)$ and $\Lambda_{\overline{\rm MS}}$





$$\alpha_s(M_Z) = 0.12047(81)(48)(-173)$$

$$\Lambda_{\overline{\rm MS}} = 239(10)(6)(-22)\,{\rm MeV}$$

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3. Quark mass renormalization factor

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Quark mass renormalization factor

Goal: RGI mass

$$M = \overline{m}(\mu)\left(2b_0\overline{g}^2(\mu)\right)^{-\frac{d_0}{2b_0}} \exp\left(-\int_0^{\overline{g}(\mu)} dg \left(\frac{\tau(g)}{\beta(g)} - \frac{d_0}{b_0 g}\right)\right)$$

renormalized mass — renormalized coupling

if evaluated at large μ

RGI mass M can be given perturbatively

SSF works well $\overline{m}(L) \longleftrightarrow \overline{m}(2L)$

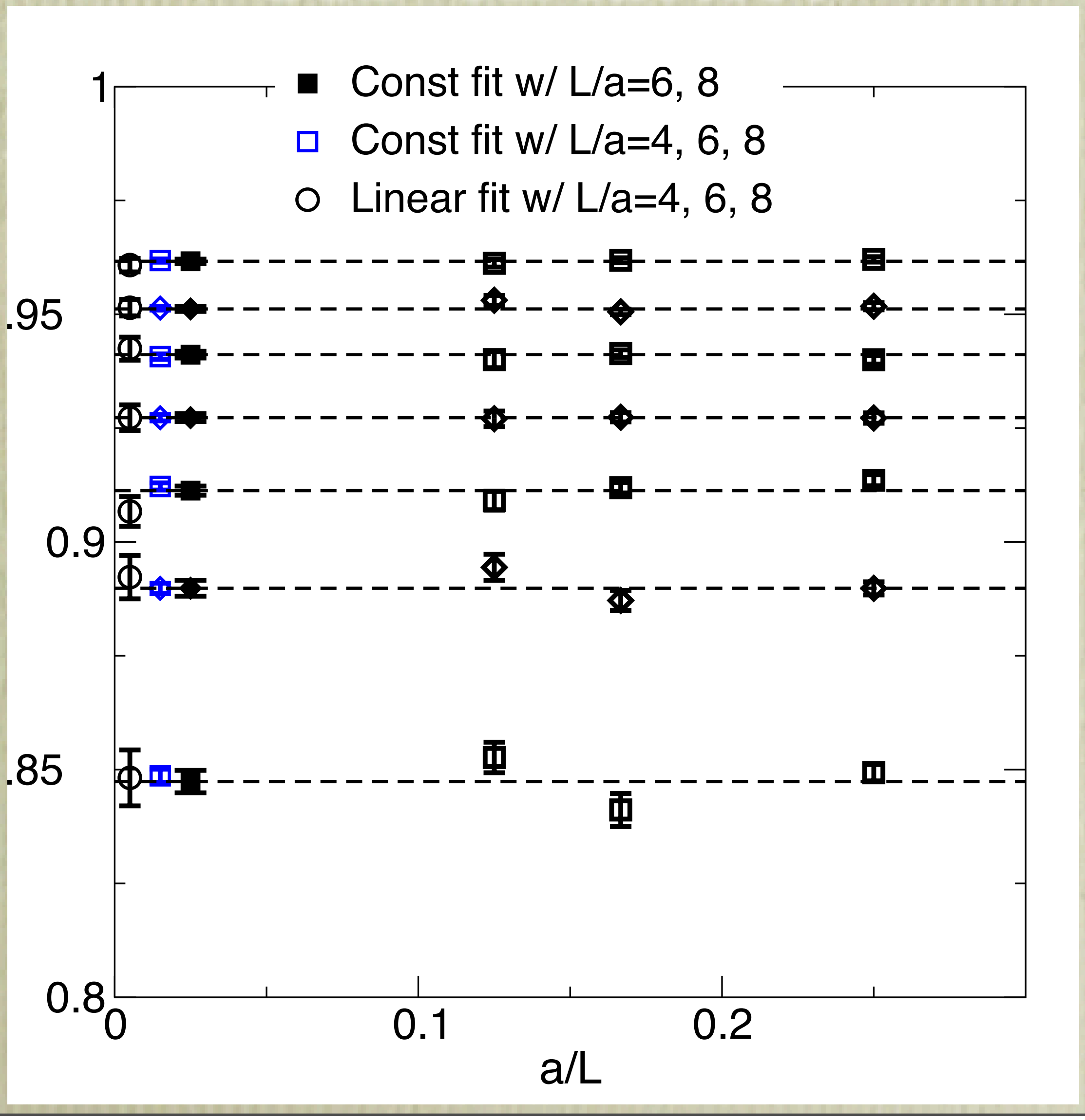
Applying iteratively $\overline{m}(L_{\mathrm{max}}) \Rightarrow \overline{m}(L_{\mathrm{max}}/2^n)$

Quark mass renormalization factor

Quark mass step scaling function (Luescher et al)

$$\Sigma_P\left(u, a/L\right) = \left.\frac{Z_P(g_0, 2L)}{Z_P(g_0, L)}\right|_{\overline{g}^2(L)=u, m=0}$$

SSF on lattice



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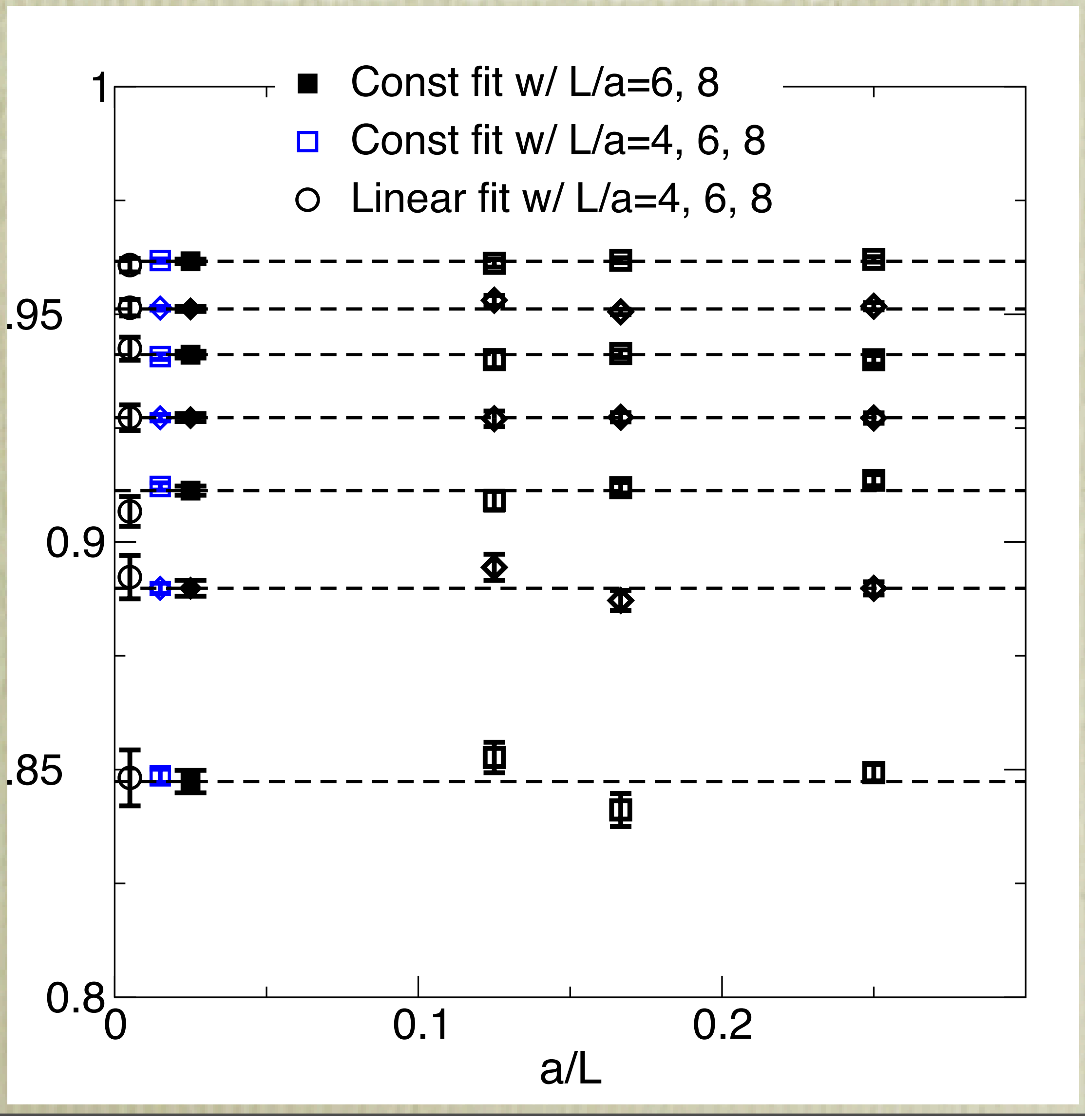

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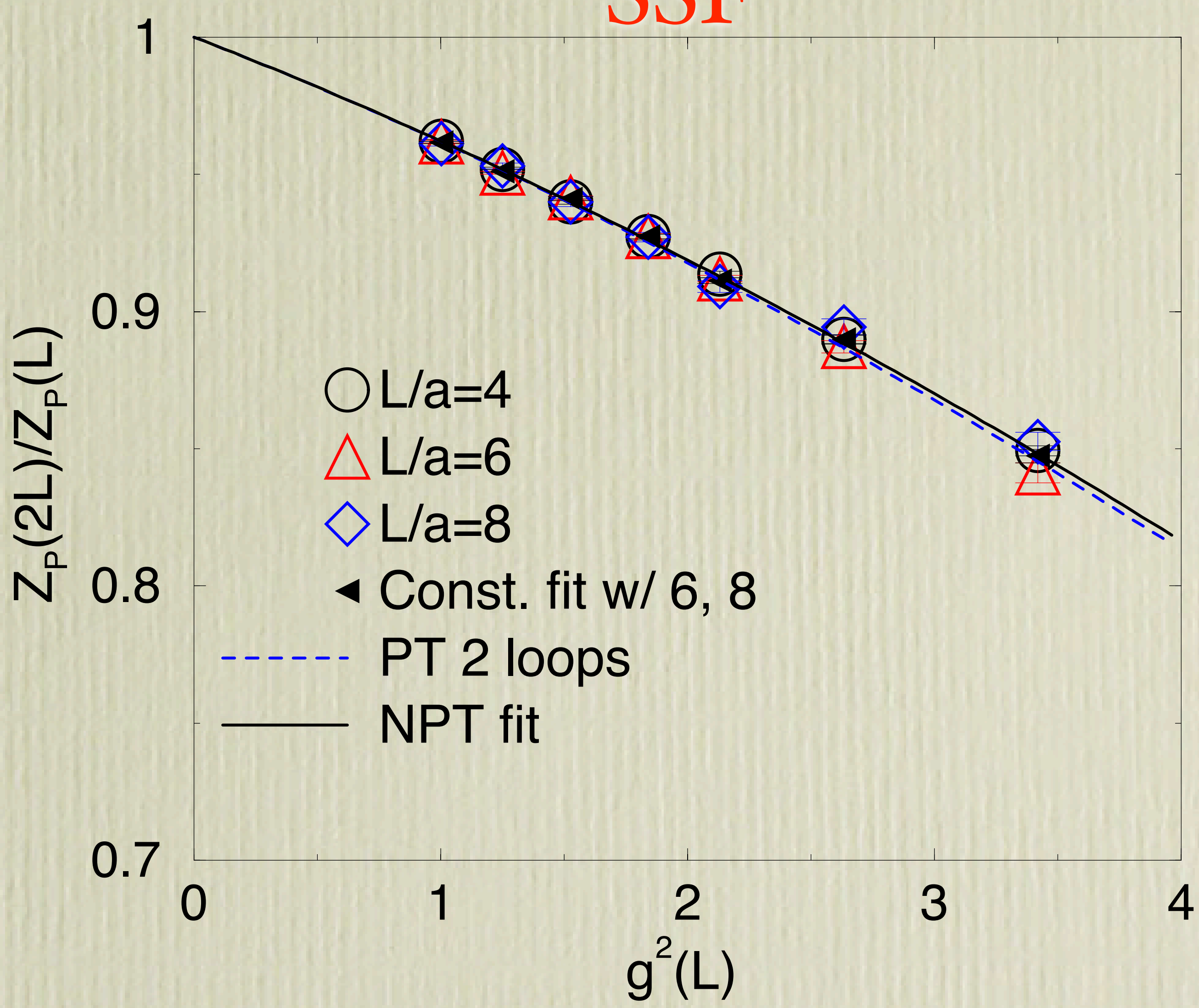

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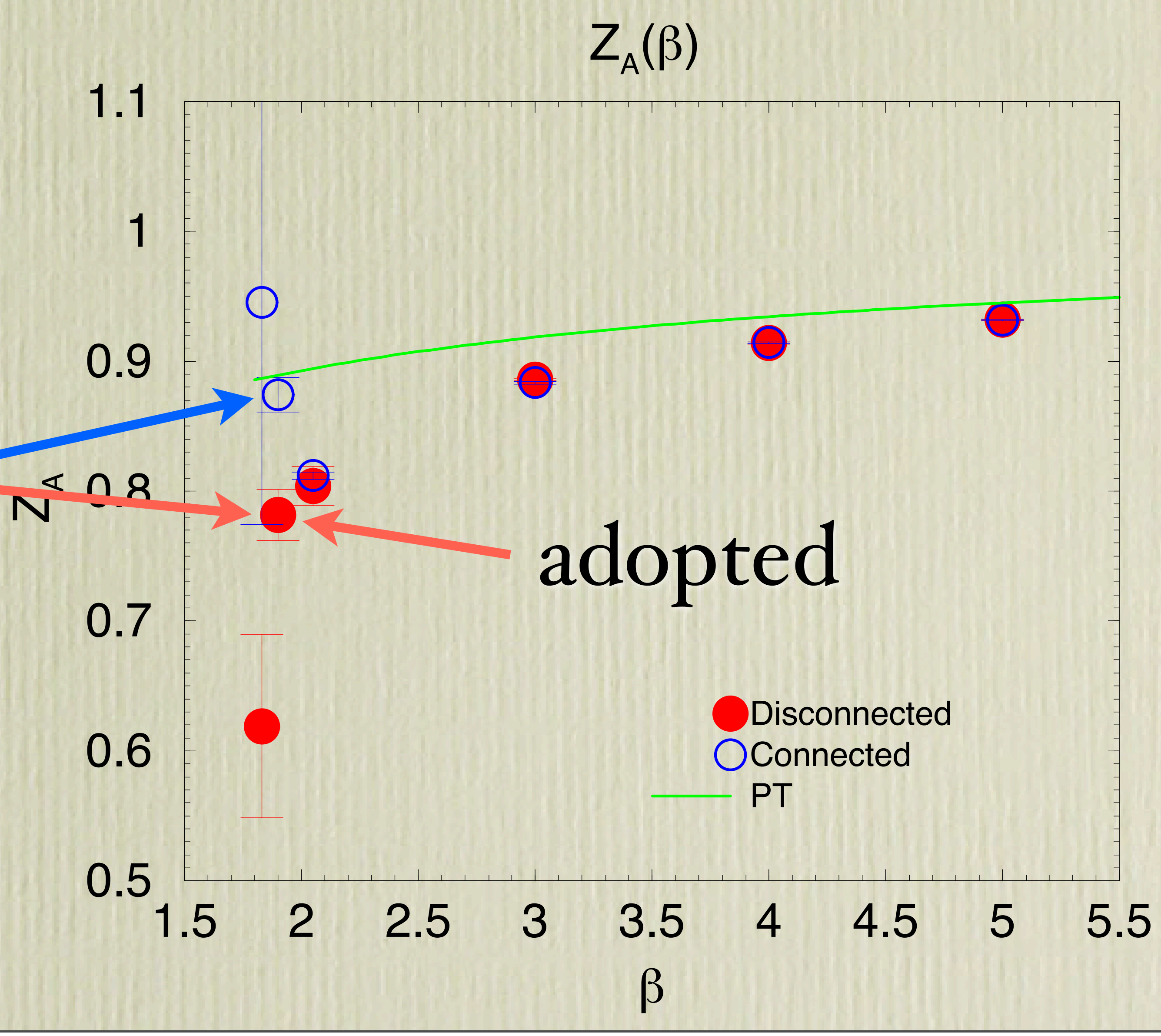
Quark mass renormalization factor

Renormalization factor of axial vector current

$$M = m_{\rm PCAC}^{\rm (bare)}(\beta) \left. \frac{Z_A(\beta)}{Z_P(\beta, a/L_{\rm max})} \right|_{a\to 0} \left. \frac{\overline{m}(L_{\rm max}/2^n)}{\overline{m}(L_{\rm max})} \right|_{\rm NP} \left. \frac{M}{\overline{m}(L_{\rm max}/2^n)} \right|_{\rm PT}$$

different definition of ZA with different boundary operator

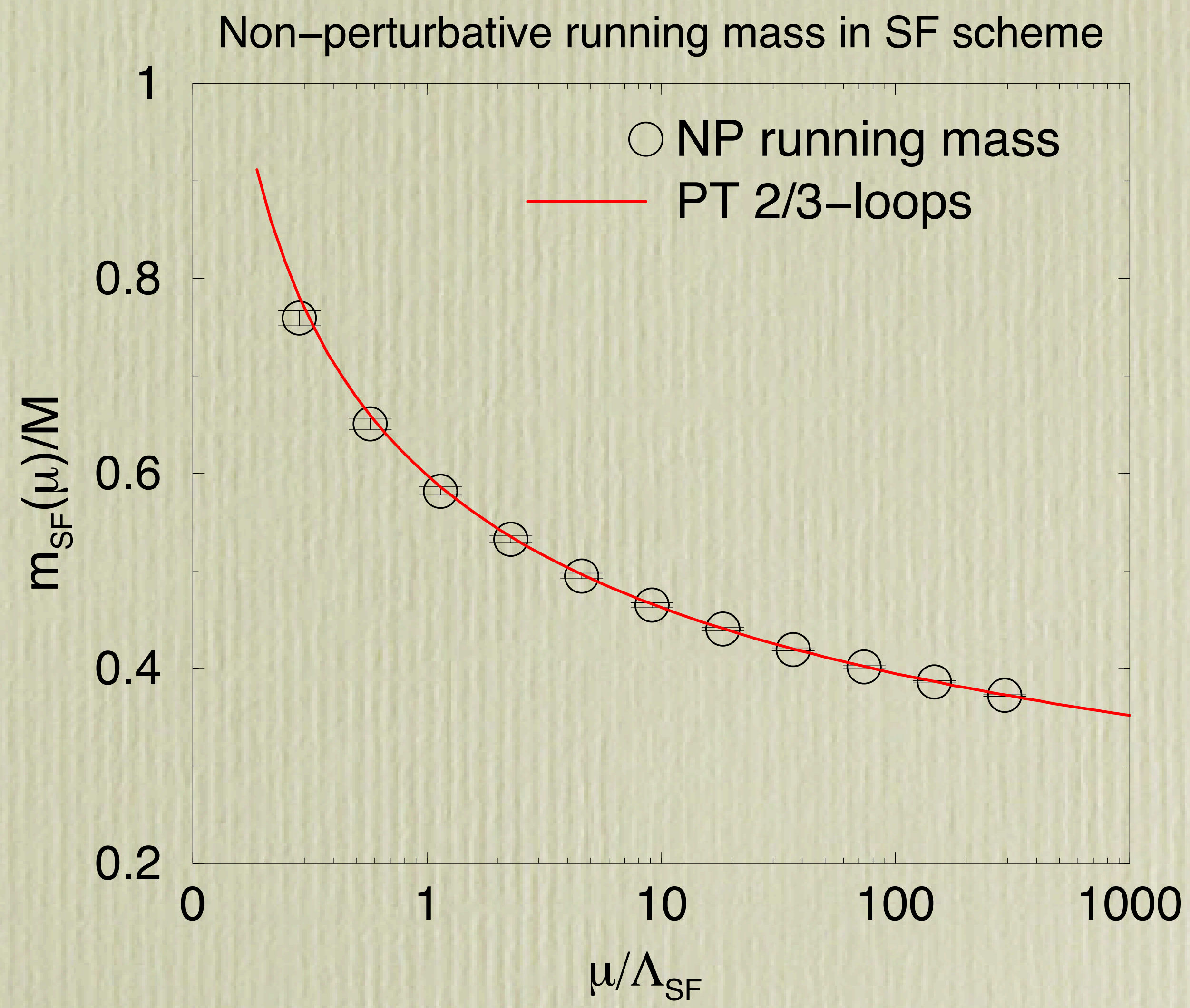
O(a) error



adopted

Quark mass renormalization factor

Non-perturbative running mass for Nf=3 QCD



$$m_{ud}^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = 2.78(27)\mathrm{MeV}$$

$$m_{s}^{\overline{\mathrm{MS}}} = 86.7(2.3)\mathrm{MeV}$$

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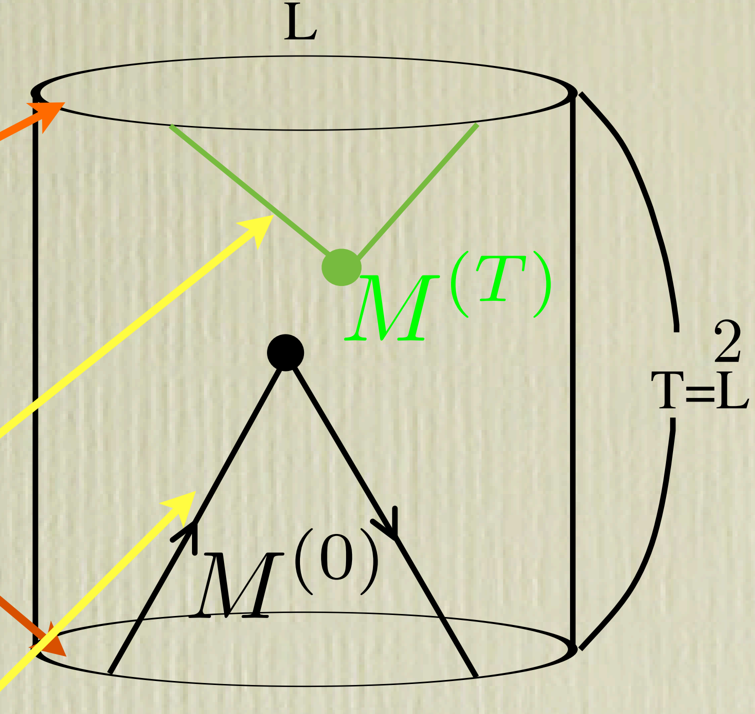
Evaluation of Csw by SF(Alpha)

Fix cSW in order that O(a) effect vanish

Two different Dirichlet BC's



Two improved current correlators

$$A_\mu^{(T)} + c_A \partial_\mu P^{(T)}$$

$$A_\mu^{(0)} + c_A \partial_\mu P^{(0)}$$

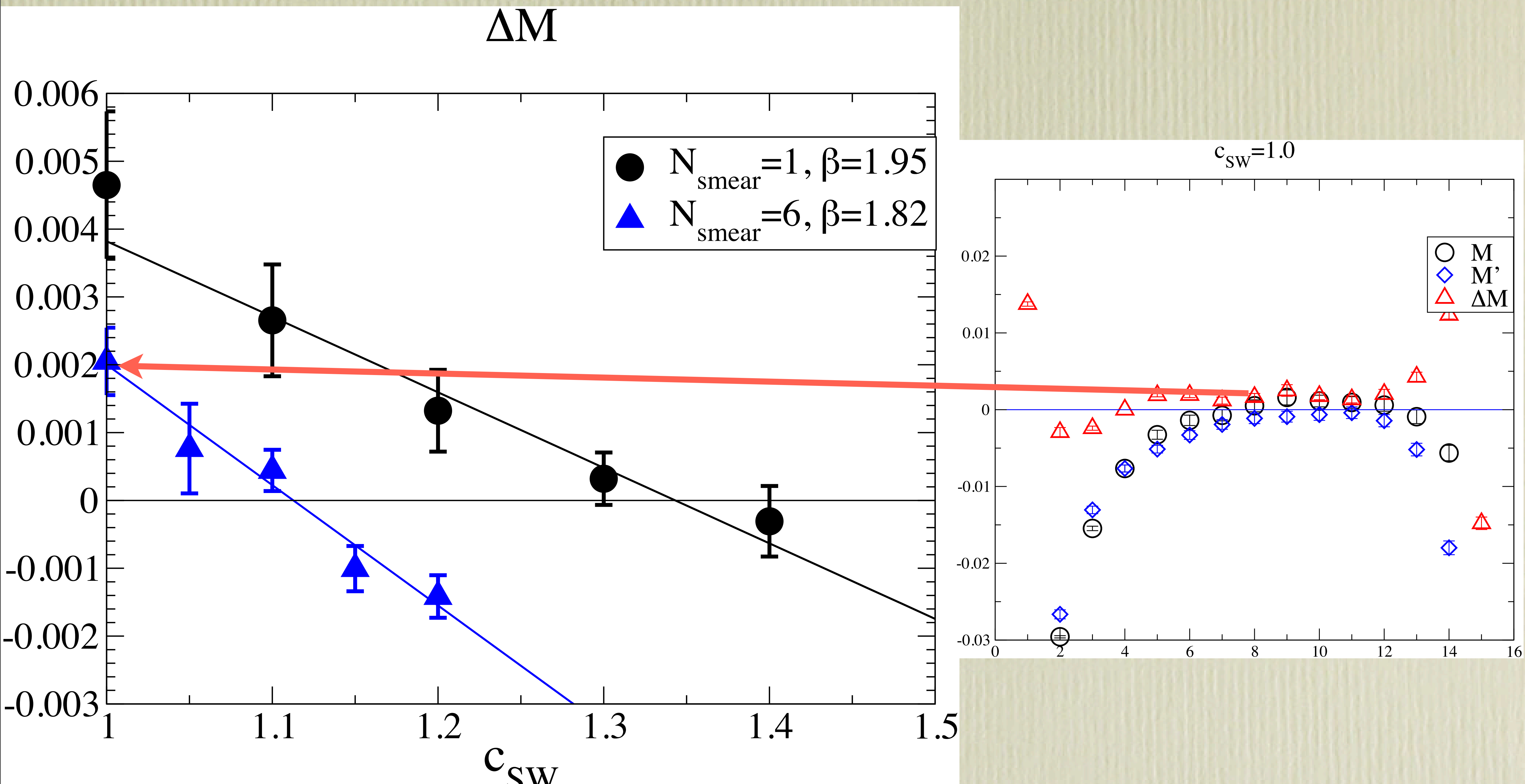
Two PCAC masses $M = 0 \to \kappa_c$

O(a) indicator $\Delta M = 0 \to c_{\rm SW}$

$\Delta M = M^{(0)} - M^{(T)}$ boundary + bulk O(a) effect

$O(10^{-4})$ at tree level

Csw dependence of O(a) indicator



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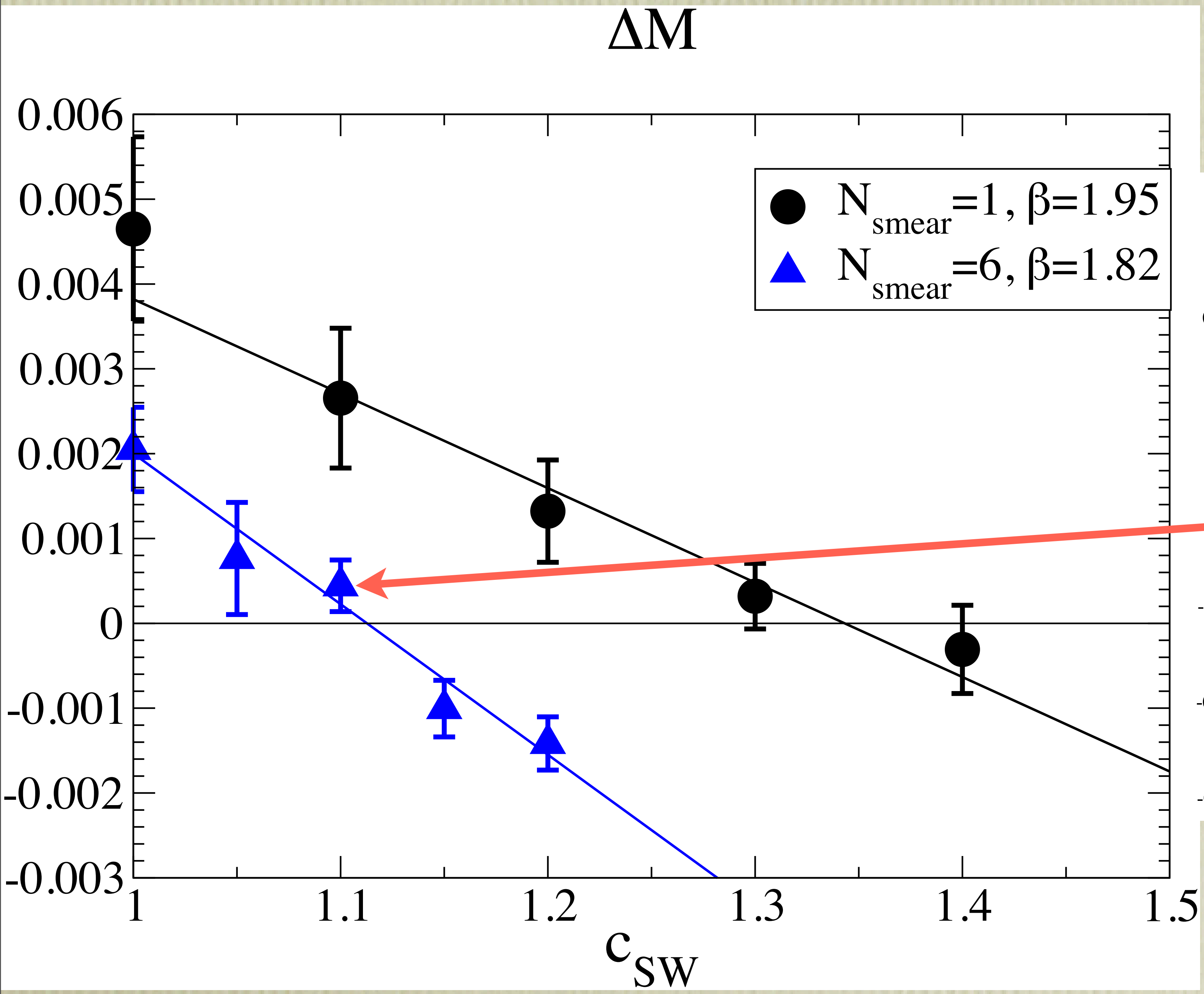

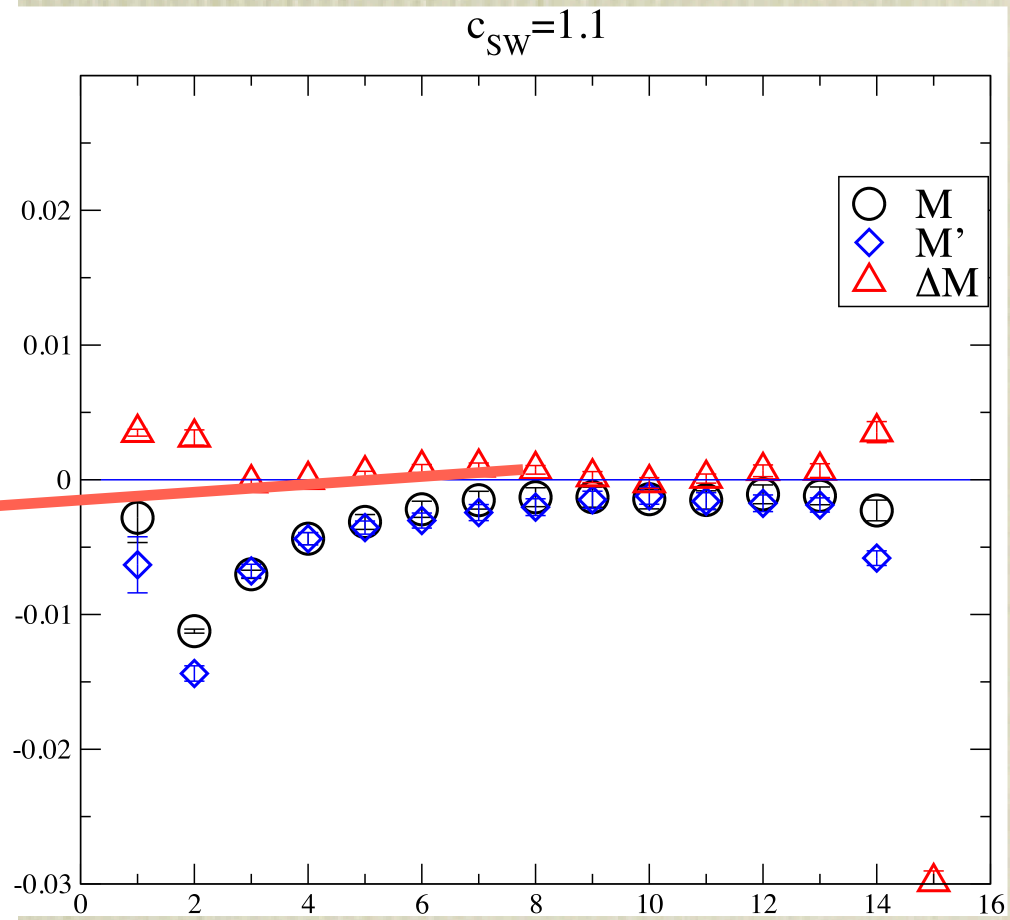

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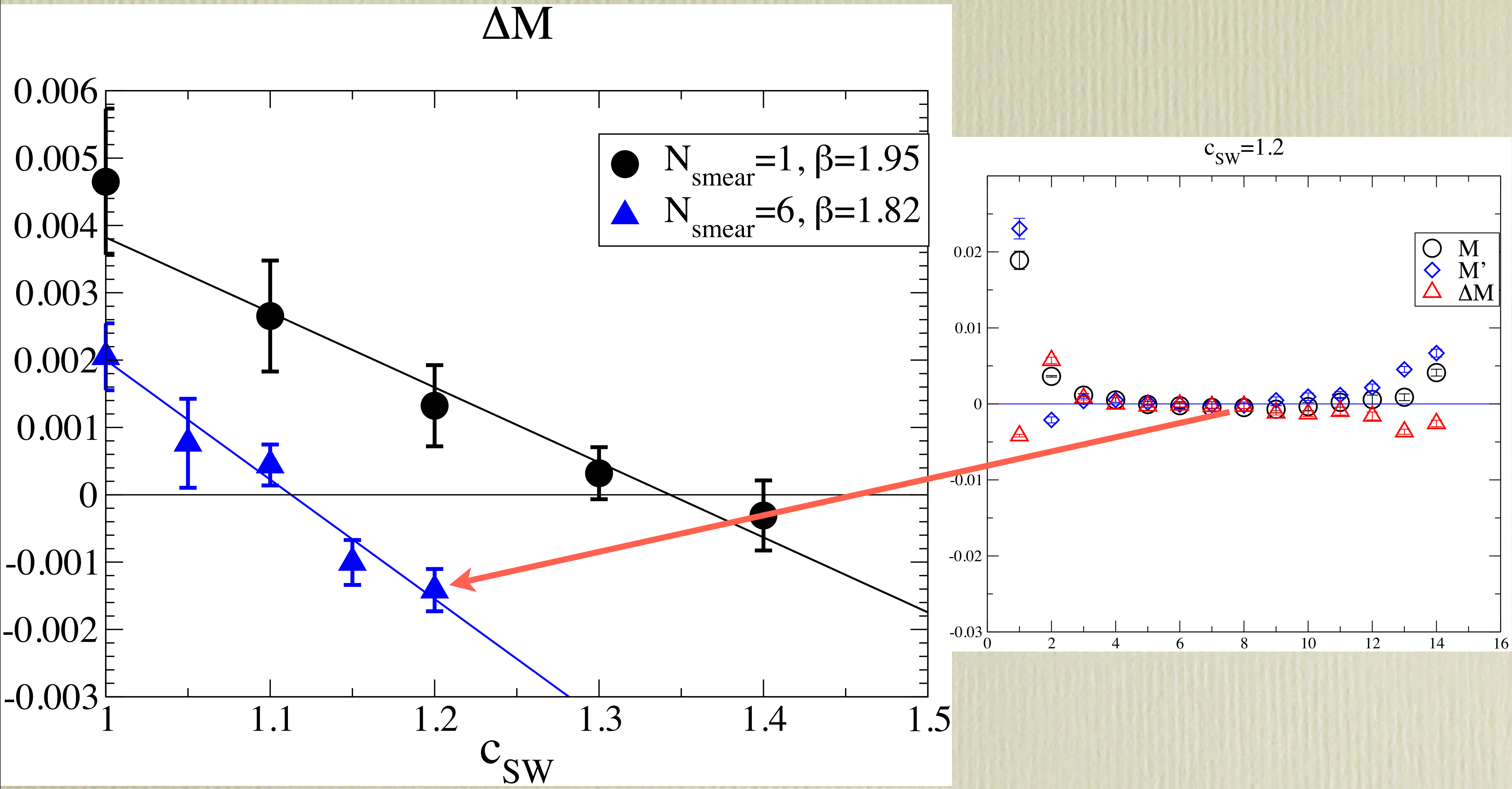
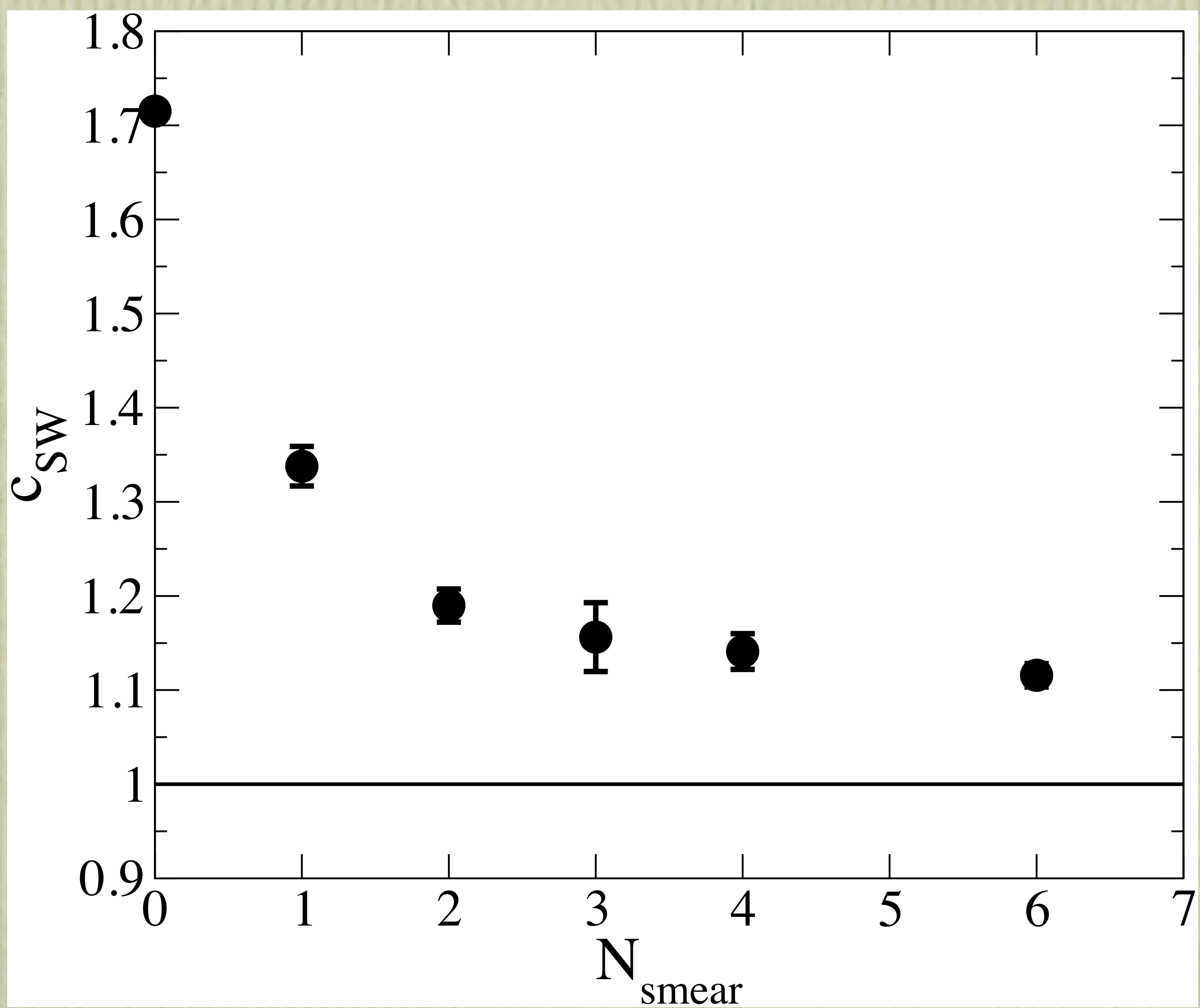
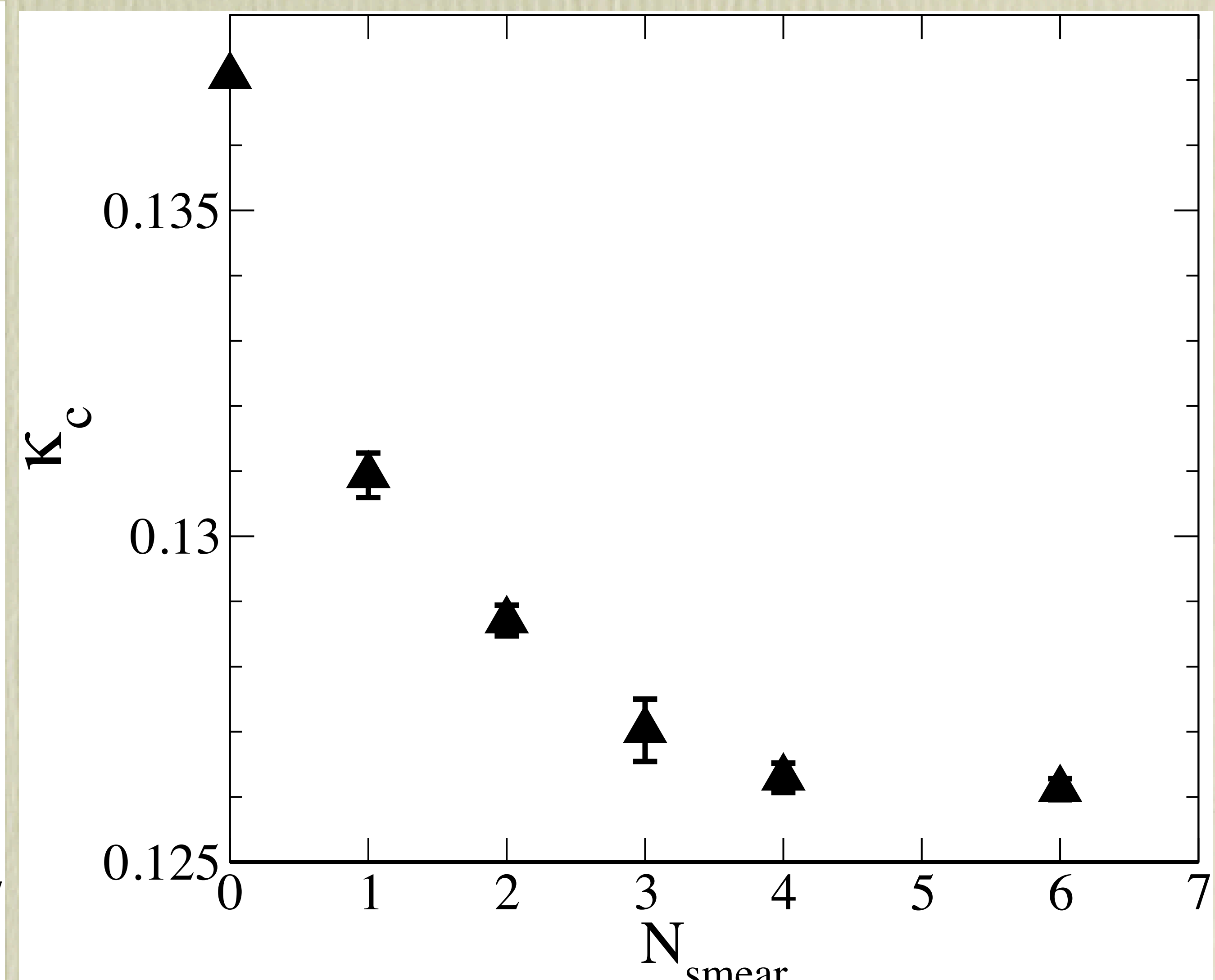


Clover term coefficient

CSW

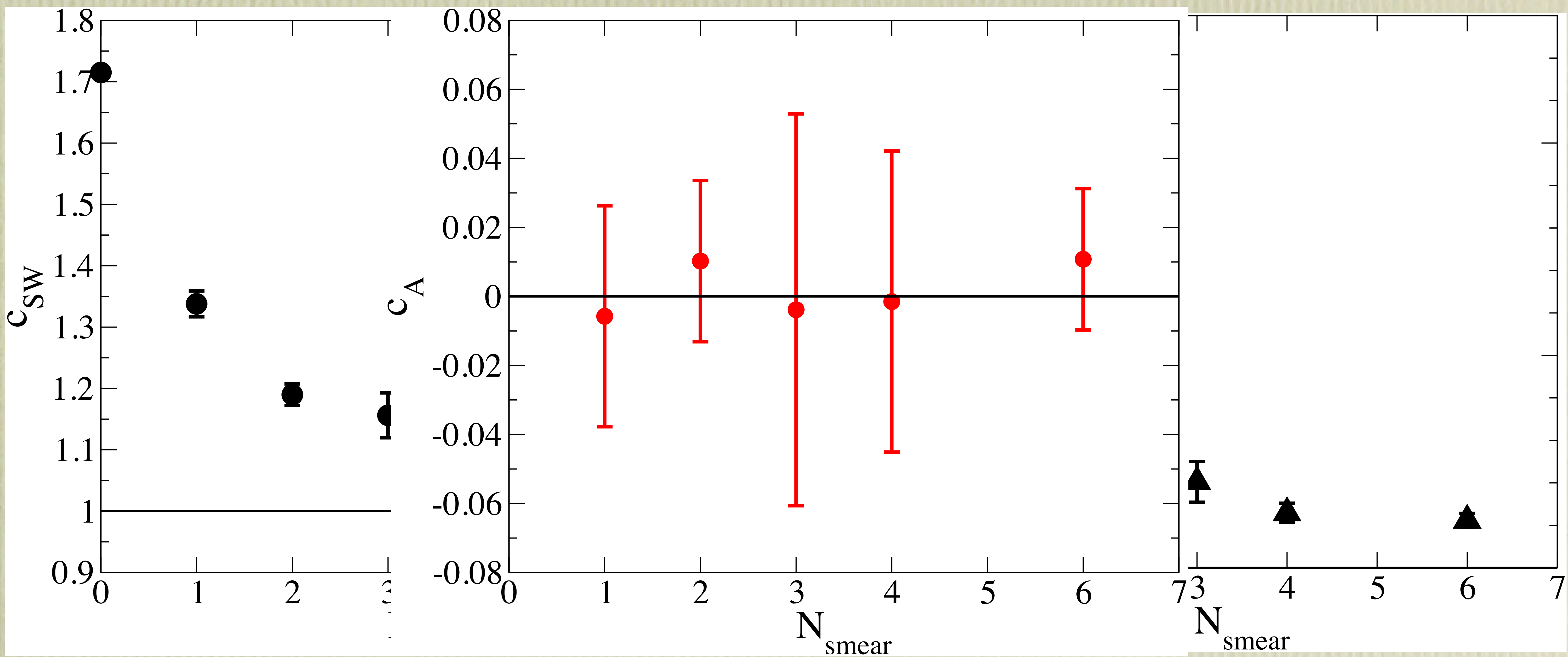


κ_c



Clover term coefficient

c_{SW} c_A κ_c



Conclusion

★Strong coupling α_s for Nf=3 QCD

- SF scheme works well.
- Need $a \to 0$ limit, but is not available.
- Smearing may reduce O(a) error.

★Quark mass renormalization factor

- Z_A has a large O(a) error.
- Smearing may reduce O(a) error.

★Clover term coefficient

- $C_{sw} \sim 1.1$

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- Csw∼1.1